CHAPTER 9

CENTRE OF MASS, LINEAR MOMENTUM, COLLISION

9.1 CENTRE OF MASS

Suppose a spin bowler throws a cricket ball vertically upward. Being a spinner, his fingers turn while throwing the ball and the ball goes up spinning rapidly. Focus your attention to a particular point on the surface of the ball. How does it move in the space ? Because the ball is spinning as well as rising up, in general, the path of a particle at the surface is complicated, not confined to a straight line or to a plane. The centre of the ball, however, still goes on the vertical straight line and the spinner's fingers could not make its path complicated. If he does not throw the ball vertically up, rather passes it to his fellow fielder, the centre of the ball goes in a parabola.

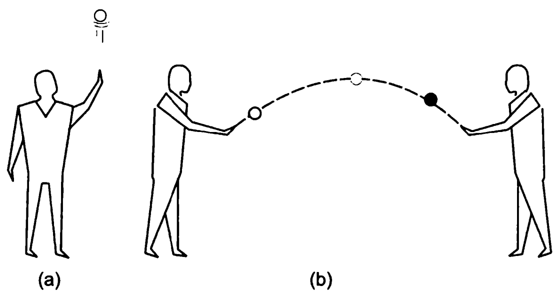


Figure 9.1

All the points of the ball do not go in parabolic paths. If the ball is spinning, the paths of most of the particles of the ball are complicated. But the centre of the ball always goes in a parabola irrespective of how the ball is thrown. (In fact the presence of air makes the path of the centre slightly different from a parabola and bowlers utilise this deviation. This effect will be discussed in a later chapter. At present we neglect it.)

The centre of the ball is a very special point which is called the *centre of mass* of the ball. Its motion is just like the motion of a single particle thrown.

Definition of Centre of Mass

Let us consider a collection of N particles (Figure 9.2). Let the mass of the ith particle be m_i and its coordinates with reference to the chosen axes be x_i , y_i , z_i . Write the product $m_i x_i$ for each of the particles and add them to get $\sum m_i x_i$. Similarly get $\sum_i m_i y_i$ and $\sum_i m_i z_i$. Then find

$$X = \frac{1}{M} \sum_i m_i x_i , \; Y = \frac{1}{M} \sum_i m_i y_i \text{ and } Z = \frac{1}{M} \sum_i m_i z_i$$

where $M = \sum_i m_i$ is the total mass of the system.

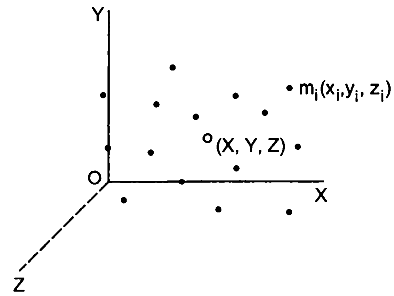


Figure 9.2

Locate the point with coordinates (X, Y, Z). This point is called the *centre of mass* of the given collection of the particles. If the position vector of the i th particle is $\vec{r_i}$, the centre of mass is defined to have the position vector

$$\vec{R}_{CM} = \frac{1}{M} \sum_i m_i \vec{r_i} . \qquad \dots (9.1)$$

Taking x, y, z components of this equation, we get the coordinates of centre of mass as defined above

$$X = \frac{1}{M} \sum_i m_i x_i , \; Y = \frac{1}{M} \sum_i m_i y_i , \; Z = \frac{1}{M} \sum_i m_i z_i \quad \dots (9.2)$$

Example 9.1

Four particles A, B, C and D having masses m, 2m, 3m and 4m respectively are placed in order at the corners of a square of side a. Locate the centre of mass.

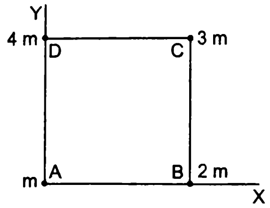


Figure 9.3

Solution : Take the axes as shown in figure (9.3). The coordinates of the four particles are as follows :

Particle	mass	x-coordinate	y-coordinate
A	m	0	0
B	$2m$	a	0
C	$3m$	a	a
D	$4m$	0	a

Hence, the coordinates of the centre of mass of the four-particle system are

$$X = \frac{m \cdot 0 + 2\,m\,a + 3\,m\,a + 4\,m \cdot 0}{m + 2\,m + 3\,m + 4\,m} = \frac{a}{2}$$

$$Y = \frac{m \cdot 0 + 2\,m \cdot 0 + 3\,m\,a + 4\,m\,a}{m + 2\,m + 3\,m + 4\,m} = \frac{7a}{10}$$

The centre of mass is at $\left(\frac{a}{2}, \frac{7a}{10}\right)$.

Centre of Mass of Two Particles

As the simplest example, consider a system of two particles of masses m_1 and m_2 separated by a distance d (figure 9.4). Where is the centre of mass of this system ?

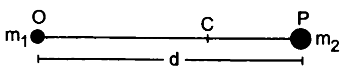


Figure 9.4

Take the origin at m_1 and the X-axis along the line joining m_1 and m_2. The coordinates of m_1 are (0, 0, 0) and of m_2 are (d, 0, 0). So,

$$\sum_i m_i\,x_i = m_1\,0 + m_2\,d = m_2\,d,\ \sum_i m_i\,y_i = 0,\ \sum_i m_i\,z_i = 0.$$

The total mass is $M = m_1 + m_2$. By definition, the centre of mass will be at $\left(\frac{m_2\,d}{m_1 + m_2}, 0, 0\right)$. We find that the centre of mass of a system of two particles is situated on the line joining the particles. If O, C, P be the positions of m_1, the centre of mass and m_2 respectively, we have

$$OC = \frac{m_2\,d}{m_1 + m_2} \quad \text{and} \quad CP = \frac{m_1\,d}{m_1 + m_2}$$

so that $m_1(OC) = m_2(CP)$ ... (9.3)

The centre of mass divides internally the line joining the two particles in inverse ratio of their masses.

Centre of Mass of Several Groups of Particles

Consider a collection of $N_1 + N_2$ particles. We call the group of N_1 particles as the first part and the other group of N_2 particles as the second part. Suppose the first part has its centre of mass at C_1 and the total mass M_1 (figure 9.5). Similarly the second part has its centre of mass at C_2 and the total mass M_2 . Where is the centre of mass of the system of $N_1 + N_2$ particles ?

Figure 9.5

The x-coordinate of the centre of mass is

$$X = \frac{\sum\limits_{i=1}^{N_1+N_2} m_i\,x_i}{M_1 + M_2} = \frac{\sum\limits_{i=1}^{N_1} m_i\,x_i + \sum\limits_{i=N_1+1}^{N_1+N_2} m_i\,x_i}{M_1 + M_2}. \quad \text{... (9.4)}$$

If X_1 , X_2 are the x-coordinates of C_1 and C_2 , then by the definition of centre of mass, $\sum m_i\,x_i$ for the first part is $M_1 X_1$ and $\sum m_i\,x_i$ for the second part is $M_2 X_2$. Hence equation (9.4) becomes,

$$X = \frac{M_1\,X_1 + M_2\,X_2}{M_1 + M_2}$$

Similarly, $$Y = \frac{M_1\,Y_1 + M_2\,Y_2}{M_1 + M_2}$$

and $$Z = \frac{M_1\,Z_1 + M_2\,Z_2}{M_1 + M_2}.$$

But this is also the centre of mass of two point particles of masses M_1 and M_2 placed at C_1 and C_2 respectively. Thus, we obtain a very useful result. *If we know the centres of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centres of mass.*

Example 9.2

Two identical uniform rods AB and CD, each of length L are jointed to form a T-shaped frame as shown in figure (9.6). Locate the centre of mass of the frame. The centre of mass of a uniform rod is at the middle point of the rod.

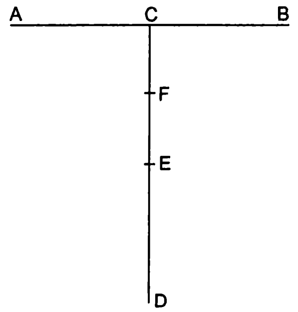


Figure 9.6

Solution : Let the mass of each rod be m. Take the centre C of the rod AB as the origin and CD as the Y-axis. The rod AB has mass m and its centre of mass is at C. For the calculation of the centre of mass of the combined system, AB may be replaced by a point particle of mass m placed at the point C. Similarly the rod CD may be replaced by a point particle of mass m placed at the centre E of the rod CD. Thus, the frame is equivalent to a system of two particles of equal masses m each, placed at C and E. The centre of mass of this pair of particles will be at the middle point F of CE.

The centre of mass of the frame is, therefore, on the rod CD at a distance $L/4$ from C.

9.2 CENTRE OF MASS OF CONTINUOUS BODIES

If we consider the body to have continuous distribution of matter, the summation in the formula of centre of mass should be replaced by integration. So, we do not talk of the ith particle, rather we talk of a small element of the body having a mass dm. If x, y, z are the coordinates of this small mass dm, we write the coordinates of the centre of mass as

$$X = \frac{1}{M}\int x\,dm\,,\quad Y = \frac{1}{M}\int y\,dm\,,\quad Z = \frac{1}{M}\int z\,dm. \quad \ldots \text{(9.5)}$$

The integration is to be performed under proper limits so that as the integration variable goes through the limits, the elements cover the entire body. We illustrate the method with three examples.

(a) Centre of Mass of a Uniform Straight Rod

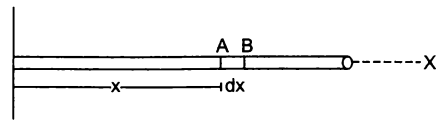


Figure 9.7

Let M and L be the mass and the length of the rod respectively. Take the left end of the rod as the origin and the X-axis along the rod (figure 9.7). Consider an element of the rod between the positions x and $x + dx$. If $x = 0$, the element is at the left end of the rod. If $x = L$, the element is at its right end. So as x varies from 0 through L, the elements cover the entire rod. As the rod is uniform, the mass per unit length is M/L and hence the mass of the element is $dm = (M/L)\,dx$. The coordinates of the element are $(x, 0, 0)$. (The coordinates of different points of the element differ, but the difference is less than dx and that much is harmless as integration will automatically correct it. So x-coordinate of the left end of the element may be called the "x-coordinate of the element.")

The x-coordinate of the centre of mass of the rod is

$$X = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\left(\frac{M}{L}dx\right)$$

$$= \frac{1}{L}\left[\frac{x^2}{2}\right]_0^L = \frac{L}{2}\,.$$

The y-coordinate is

$$Y = \frac{1}{M}\int y\,dm = 0$$

and similarly $Z = 0$. The centre of mass is at $\left(\frac{L}{2}, 0, 0\right)$, i.e., at the middle point of the rod.

(b) Centre of Mass of a Uniform Semicircular Wire

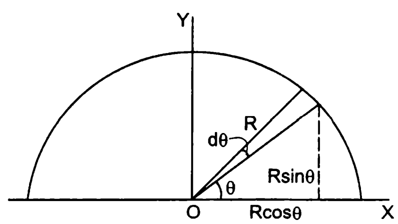


Figure 9.8

Let M be the mass and R the radius of a uniform semicircular wire. Take its centre as the origin, the line joining the ends as the X-axis, and the Y-axis in the plane of the wire (figure 9.8). The centre of mass must be in the plane of the wire i.e., in the X-Y plane.

How do we choose a small element of the wire? First, the element should be so defined that we can vary the element to cover the whole wire. Secondly, if we are interested in $\int x\,dm$, the x-coordinates of different parts of the element should only infinitesimally differ in range. We select the element as follows. Take a radius making an angle θ with the X-axis and rotate it further by an angle $d\theta$. Note the points of intersection of the radius with the wire during this rotation. This gives an element of length $R\,d\theta$. When we take $\theta = 0$, the element is situated near the right edge of the wire. As θ is gradually increased to π, the element takes all positions on the wire i.e., the whole wire is covered. The "coordinates of the element" are $(R\cos\theta,\ R\sin\theta)$. Note that the coordinates of different parts of the element differ only by an infinitesimal amount.

As the wire is uniform, the mass per unit length of the wire is $\frac{M}{\pi R}\cdot$ The mass of the element is, therefore,

$$dm = \left(\frac{M}{\pi R}\right)(R\,d\theta) = \frac{M}{\pi}\,d\theta.$$

The coordinates of the centre of mass are

$$X = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^{\pi}(R\cos\theta)\left(\frac{M}{\pi}\right)d\theta = 0$$

and

$$Y = \frac{1}{M}\int y\,dm = \frac{1}{M}\int_0^{\pi}(R\sin\theta)\left(\frac{M}{\pi}\right)d\theta = \frac{2R}{\pi}$$

The centre of mass is at $\left(0, \frac{2R}{\pi}\right)$.

(c) Centre of Mass of a Uniform Semicircular Plate

This problem can be worked out using the result obtained for the semicircular wire and that any part of the system (semicircular plate) may be replaced by a point particle of the same mass placed at the centre of mass of that part.

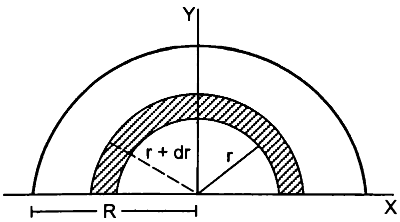


Figure 9.9

Figure (9.9) shows the semicircular plate. We take the origin at the centre of the semicircular plate, the X-axis along the straight edge and the Y-axis in the plane of the plate. Let M be the mass and R be its radius. Let us draw a semicircle of radius r on the plate with the centre at the origin. We increase r to $r + dr$ and draw another semicircle with the same centre. Consider the part of the plate between the two semicircles of radii r and $r + dr$. This part, shown shaded in figure (9.9), may be considered as a semicircular wire.

If we take $r = 0$, the part will be formed near the centre and if $r = R$, it will be formed near the edge of the plate. Thus, if r is varied from 0 to R, the elemental parts will cover the entire semicircular plate.

We can replace the semicircular shaded part by a point particle of the same mass at its centre of mass for the calculation of the centre of mass of the plate.

The area of the shaded part $= \pi\,r dr$. The area of the plate is $\pi R^2/2$. As the plate is uniform, the mass per unit area is $\frac{M}{\pi R^2/2}\cdot$ Hence the mass of the semicircular element

$$\frac{M}{\pi R^2/2}(\pi\,r\,dr) = \frac{2\,M\,rdr}{R^2}.$$

The y-coordinate of the centre of mass of this wire is $2r/\pi$. The y-coordinate of the centre of mass of the plate is, therefore,

$$Y = \frac{1}{M}\int_0^R\left(\frac{2\,r}{\pi}\right)\left(\frac{2\,M\,r}{R^2}\,dr\right) = \frac{1}{M}\cdot\frac{4\,M}{\pi R^2}\,\frac{R^3}{3} = \frac{4\,R}{3\,\pi}.$$

The x-coordinate of the centre of mass is zero by symmetry.

9.3 MOTION OF THE CENTRE OF MASS

Consider two particles A and B of masses m_1 and m_2 respectively. Take the line joining A and B as the X-axis. Let the coordinates of the particles at time t be x_1 and x_2. Suppose no external force acts on the two-particle-system. The particles A and B, however, exert forces on each other and the particles accelerate along the line joining them. Suppose the particles are initially at rest and the force between them is attractive. The particles will then move along the line AB as shown in figure (9.10).

Figure 9.10

The centre of mass at time t is situated at

$$X = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}.$$

As time passes, x_1, x_2 change and hence X changes and the centre of mass moves along the X-axis. Velocity of the centre of mass at time t is

$$V_{CM} = \frac{dx}{dt} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}. \quad \ldots (9.6)$$

The acceleration of the centre of mass is

$$a_{CM} = \frac{dV_{CM}}{dt} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2}. \quad \ldots (9.7)$$

Suppose the magnitude of the forces between the particles is F. As the only force acting on A is F towards B, its acceleration is $a_1 = F/m_1$. The force on B is $(-F)$ and hence $a_2 = -F/m_2$

Putting in (9.7),

$$a_{CM} = \frac{m_1(F/m_1) + m_2(-F/m_2)}{m_1 + m_2} = 0.$$

That means, the velocity of the centre of mass does not change with time. But as we assumed, initially the particles are at rest. Thus, $v_1 = v_2 = 0$ and from (9.5) $V_{CM} = 0$. Hence the centre of mass remains fixed and does not change with time.

Thus, *if no external force acts on a two-particle-system and its centre of mass is at rest (say in the inertial frame A) initially, it remains fixed (in the inertial frame A) even when the particles individually move and accelerate*. Let us now generalise this result.

Consider a system of N particles, the ith particle having a mass m_i and the position vector $\vec{r_i}$ with respect to an inertial frame. Each particle is acted upon by forces due to all other $(N-1)$ particles and forces due to the sources outside the system. The acceleration of the ith particle is

$$\vec{a_i} = \frac{1}{m_i}\left(\sum_{j \neq i} \vec{F_{ij}} + \vec{F_i}^{ext} \right) \text{ (Newton's second law)}$$

$$\text{or, } m_i \vec{a_i} = \left(\sum_{j \neq i} \vec{F_{ij}} + \vec{F_i}^{ext} \right).$$

Here $\vec{F_{ij}}$ is the force on the ith particle due to the jth particle and $\vec{F_i}^{ext}$ is the vector sum of the forces acting on the ith particle by the external sources. Summing over all the particles

$$\sum_i m_i \vec{a_i} = \sum_{i \neq j} \sum_j \vec{F_{ij}} + \sum_i \vec{F_i}^{ext} = \vec{F}^{ext}. \quad \ldots (9.8)$$

The internal forces $\vec{F_{ij}}$ add up to zero as they cancel in pairs, $(\vec{F_{ij}} + \vec{F_{ji}} = 0)$ by Newton's third law. $\vec{F}^{ext}$ is the sum of all the forces acting on all the particles by the external sources.

Now $\sum_i m_i \vec{r_i} = M \vec{R}_{CM}$ giving $\sum_i m_i \vec{a_i} = M \vec{a}_{CM}$

Putting in (9.8),

$$M \vec{a}_{CM} = \vec{F}^{ext}. \quad \ldots (9.9)$$

If the external forces acting on the system add to zero, $\vec{a}_{CM} = 0$ and hence the velocity of the centre of mass is constant. If initially the centre of mass was at rest with respect to an inertial frame, it will continue to be at rest with respect to that frame. The individual particles may go on complicated paths changing their positions, but the centre of mass will be obtained at the same position.

If the centre of mass was moving with respect to the inertial frame at a speed v along a particular direction, it will continue its motion along the same straight line with the same speed. Thus, *the motion of the centre of mass of the system is not affected by the internal forces. If the external forces add up to zero, the centre of mass has no acceleration.*

Example 9.3

Two charged particles of masses m and 2m are placed a distance d apart on a smooth horizontal table. Because of their mutual attraction, they move towards each other and collide. Where will the collision occur with respect to the initial positions ?

Solution : As the table is smooth, there is no friction. The weight of the particles and the normal force balance each other as there is no motion in the vertical direction. Thus, taking the two particles as constituting the system, the sum of the external forces acting on the system is zero. The forces of attraction between the particles are the internal forces as we have included both the particles in the system. Therefore, the centre of mass of the system will have no acceleration.

Initially, the two particles are placed on the table and their velocities are zero. The velocity of the centre of mass is, therefore, zero. As time passes, the particles move, but the centre of mass will continue to be at the same place. At the time of collision, the two particles are at one place and the centre of mass will also be at that place. As the centre of mass does not move, the collision will take place at the centre of mass.

The centre of mass will be at a distance $2d/3$ from the initial position of the particle of mass m towards the other particle and the collision will take place there.

When the external forces do not add up to zero, the centre of mass is accelerated and the acceleration is given by equation (9.9)

$$\vec{a}_{CM} = \frac{\vec{F}^{ext}}{M}.$$

If we have a single particle of mass M on which a force $\vec{F}^{ext}$ acts, its acceleration would be the same as

$\frac{\vec{F}^{ext}}{M}$ · Thus *the motion of the centre of mass of a system is identical to the motion of a single particle of mass equal to the mass of the given system, acted upon by the same external forces that act on the system.*

To explain this statement, once again consider the spinning ball of figure (9.1b). The ball is spinning and at the same time moving under gravity. To find the motion of the centre of mass of the ball, which is actually the centre of the ball, we imagine a particle of mass equal to that of the ball. We throw this particle with the velocity v, which the centre of mass had at the time of projection. What is the motion of this single particle of mass M subjected to the force Mg downward, thrown initially with velocity v? It is a parabolic motion, given by,

$$x = v_x t, \quad y = v_y t - \frac{1}{2} g t^2. \qquad \dots (9.10)$$

The centre of the ball exactly traces this curve with coordinates given by this equation only.

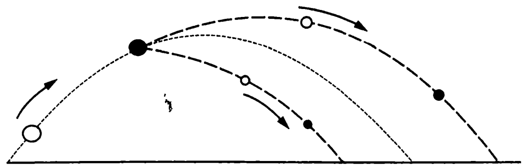


Figure 9.11

Next, suppose the ball breaks up into two parts (figure 9.11) because of some internal stress, while moving along the parabola. The two parts go on two different parabolae because the velocities of the parts change at the instant of breaking. Locate the two parts at some instant t and calculate the position of the centre of mass of the combination at that instant. It will be found at the same point on the original parabola where the centre would have been at the instant t according to equation (9.10).

9.4 LINEAR MOMENTUM AND ITS CONSERVATION PRINCIPLE

The (linear) momentum of a particle is defined as $\vec{p} = m\vec{v}$. The momentum of an N-particle system is the (vector) sum of the momenta of the N particles i.e.,

$$\vec{P} = \sum_i \vec{p}_i = \sum_i m_i \vec{v}_i .$$

But $\sum_i m_i \vec{v}_i = \frac{d}{dt} \sum_i m_i \vec{r}_i = \frac{d}{dt} M \vec{R}_{CM} = M \vec{V}_{CM}$.

Thus, $$\vec{P} = M \vec{V}_{CM} . \qquad \dots (9.11)$$

As we have seen, if the external forces acting on the system add up to zero, the centre of mass moves with constant velocity, which means $\vec{P}$ = constant. Thus the linear momentum of a system remains constant (in magnitude and direction) if the external forces acting on the system add up to zero. This is known as *the principle of conservation of linear momentum.*

Consider a trivial example of a single particle on which no force acts (imagine a practical situation where this can be achieved). Looking from an inertial frame, the particle is moving with uniform velocity and so its momentum remains constant as time passes.

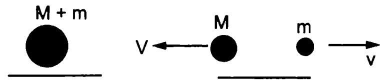


Figure 9.12

As a different example, consider a radioactive nucleus at rest which emits an alpha particle along the X-axis. Let m and M be the masses of the alpha particle and the residual nucleus respectively. Take the entire nucleus as the system. The alpha particle is ejected from the nucleus because of the forces between the neutrons and protons of the nucleus (this is the nuclear force and not gravitational or electromagnetic). There is no external force acting on the system and hence its linear momentum should not change. The linear momentum before the emission was zero as the nucleus was at rest. After the emission, the system is broken up into two parts, the alpha particle and the residual nucleus. If the alpha particle is emitted with a speed v, the residual nucleus must recoil in the opposite direction with a speed V, so that

$$M\vec{V} + m\vec{v} = 0 \quad \text{or,} \quad \vec{V} = -\frac{m}{M}\vec{v}.$$

9.5 ROCKET PROPULSION

In a rocket, the fuel burns and produces gases at high temperatures. These gases are ejected out of the rocket from a nozzle at the backside of the rocket. The ejecting gas exerts a forward force on the rocket which helps it in accelerating.

Suppose, a rocket together with its fuel has a mass M_0 at $t = 0$. Let the gas be ejected at a constant rate $r = -\frac{dM}{dt}$ · Also suppose, the gas is ejected at a constant velocity u with respect to the rocket.

At time t, the mass of the rocket together with the remaining fuel is

$$M = M_0 - rt.$$

If the velocity of the rocket at time t is v, the linear momemtum of this mass M is

$$P = Mv = (M_0 - rt)v. \qquad \dots \text{(i)}$$

Consider a small time interval Δt. A mass $\Delta M = r\Delta t$ of the gas is ejected in this time and the velocity of the rocket becomes $v + \Delta v$. The velocity of the gas with respect to ground is

$$\vec{v}_{\text{gas, ground}} = \vec{v}_{\text{gas, rocket}} + \vec{v}_{\text{rocket, ground}}$$
$$= -u + v$$

in the forward direction.

The linear momentum of the mass M at $t + \Delta t$ is,

$$(M - \Delta M)(v + \Delta v) + \Delta M(v - u). \quad \ldots \text{(ii)}$$

Assuming no external force on the rocket-fuel system, from (i) and (ii),

$$(M - \Delta M)(v + \Delta v) + \Delta M(v - u) = Mv$$

or, $$(M - \Delta M)(\Delta v) = (\Delta M)u$$

or, $$\Delta v = \frac{(\Delta M)u}{M - \Delta M}$$

or, $$\frac{\Delta v}{\Delta t} = \frac{\Delta m}{\Delta t}\frac{u}{M - \Delta m} = \frac{r\,u}{M - r\Delta t}$$

Taking the limit as $\Delta t \to 0$,

$$\frac{dv}{dt} = \frac{ru}{M} = \frac{ru}{M_0 - rt}.$$

This gives the acceleration of the rocket. We see that the acceleration keeps on increasing as time passes. If the rocket starts at $t = 0$ and we neglect any external force such as gravity,

$$\int_0^v dv = ru\int_0^t \frac{dt}{M_0 - rt}$$

or, $$v = ru\left(-\frac{1}{r}\right)\ln\frac{M_0 - rt}{M_0}$$

or, $$v = u\ln\frac{M_0}{M_0 - rt}.$$

9.6 COLLISION

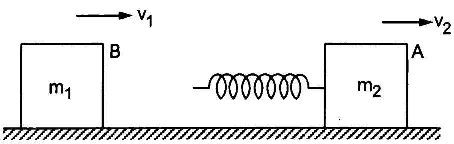


Figure 9.13

Consider the situation shown in figure (9.13). Two blocks of masses m_1 and m_2 are moving on the same straight line on a frictionless horizontal table. The block m_2, which is ahead of m_1, is going with a speed v_2 smaller than the speed v_1 of m_1. A spring is attached to the rear end of m_2. Since $v_1 > v_2$, the block m_1 will touch the rear of the spring at some instant, say t_1. Then onwards, the velocity of the left end of the spring will be equal to the velocity of m_1 (as they are in contact). The velocity of the right end of the spring will be same as that of m_2 (as they are in contact). Since m_1 moves faster than m_2, the length of the spring will decrease. The spring will be compressed. As it is compressed, it pushes back both the blocks with forces kx where x is the compression and k, the spring constant. This force is in the direction of the velocity of m_2, hence m_2 will accelerate. However, this is opposite to the velocity of m_1 and so m_1 will decelerate. The velocity of the front block A (which was slower initially) will gradually increase, and the velocity of the rear block B (which was faster initially) will gradually decrease. The spring will continue to become more and more compressed as long as the rear block B is faster than the front block A. There will be an instant $t_1 + \Delta t_1$, when the two blocks will have equal velocities. At this instant, both the ends of the spring will move with the same velocity and no further compression will take place. This corresponds to the maximum compression of the spring. Thus, "the spring-compression is maximum when the two blocks attain equal velocities".

Now, the spring being already compressed, it continues to push back the two blocks. Thus, the front block A will still be accelerated and the rear block B will still be decelerated. At $t_1 + \Delta t_1$ the velocities were equal and hence, after $t_1 + \Delta t_1$ the front block will move faster than the rear block. And so do the ends of the spring as they are in contact with the blocks. The spring will thus increase its length. This process will continue till the spring acquires its natural length, say at a time $t_1 + \Delta t_1 + \Delta t_2$. Once the spring regains its natural length, it stops exerting any force on the blocks. As the two blocks are moving with different velocities by this time, the rear one slower, the rear block will leave contact with the spring and the blocks will move with constant velocities. Their separation will go on increasing.

During the whole process, the momentum of the two-blocks system remains constant. The momentum before the instant t_1 was $m_1v_1 + m_2v_2 = P$. At time $t_1 + \Delta t_1$, the two blocks have equal velocities say V and we have $m_1V + m_2V = P$. After the contact is broken, the blocks finally attain constant velocities v_1' and $v_2'(v_2' > v_1')$ and the momentum will be $m_1v_1' + m_2v_2' = P$. In fact, take the velocities of the blocks at any instant, before the collision, during the collision or after the collision; the momentum will be equal to P. This is because there is no resultant external force acting on the system. Note that the spring being massless, exerts equal and opposite forces on the blocks.

Next, consider the energy of the system. As there is no friction anywhere, the sum of the kinetic energy and the elastic potential energy remains constant. The gravitational potential energy does not come into the picture, as the motion is horizontal. The elastic potential energy is $\frac{1}{2}kx^2$ when the spring is compressed by x. If u_1 and u_2 are the speeds at this time, we have,

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 + \frac{1}{2}kx^2 = E$$

where E is the total energy of the system.

At and before $t = t_1$, the spring is at its natural length so that,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = E. \qquad \text{... (i)}$$

At time $t = t_1 + \Delta t_1$, $u_1 = u_2 = V$ and the compression of the spring is maximum. Thus,

$$\frac{1}{2}(m_1 + m_2)V^2 + \frac{1}{2}kx_{max}^2 = E.$$

At and after $t = t_1 + \Delta t_1 + \Delta t_2$, the spring acquires its natural length, so that,

$$\frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 = E. \qquad \text{... (ii)}$$

From (i) and (ii),

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2.$$

The kinetic energy before the collision is the same as the kinetic energy after the collision. However, we can not say that the kinetic energy remains constant because it changes as a function of time, during the interval t_1 to $t_1 + \Delta t_1 + \Delta t_2$.

Example 9.4

Each of the blocks shown in figure (9.14) has mass 1 kg. *The rear block moves with a speed of* 2 m/s *towards the front block kept at rest. The spring attached to the front block is light and has a spring constant* 50 N/m. *Find the maximum compression of the spring.*

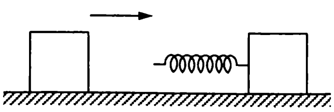


Figure 9.14

Solution : Maximum compression will take place when the blocks move with equal velocity. As no net external force acts on the system of the two blocks, the total linear momentum will remain constant. If V is the common speed at maximum compression, we have,

$$(1\text{ kg})(2\text{ m/s}) = (1\text{ kg})V + (1\text{ kg})V$$

or, $V = 1$ m/s.

Initial kinetic energy $= \frac{1}{2}(1\text{ kg})(2\text{ m/s})^2 = 2$ J

Final kinetic energy

$$= \frac{1}{2}(1\text{ kg})(1\text{ m/s})^2 + \frac{1}{2}(1\text{ kg})(1\text{ m/s})^2 = 1\text{ J}.$$

The kinetic energy lost is stored as the elastic energy in the spring.

Hence, $\frac{1}{2}(50\text{ N/m})x^2 = 2\text{ J} - 1\text{ J} = 1\text{ J}$

or, $x = 0{\cdot}2$ m.

Almost similar is the situation when two balls collide with each other and no spring is put between them (figure 9.15). At the instant they come into contact, the rear ball has a larger velocity v_1 and the front ball has a smaller velocity v_2. But the surfaces in contact must move equal distance in any time interval as long as they remain in contact. The balls have to be deformed at the contact.

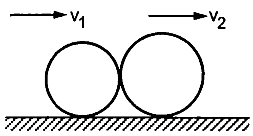


Figure 9.15

The deformed balls push each other and the velocities of the two balls change. The total kinetic energy of the two balls decreases as some energy is converted into the elastic potential energy of the deformed balls. The deformation is maximum (and the kinetic energy minimum) when the two balls attain equal velocities. Total momentum of the balls remains constant. The behaviour of the balls after this depends on the nature of the materials of the balls. If the balls are perfectly elastic, forces may develop inside them so that the balls try to regain their original shapes. In this case, the balls continue to push each other, the velocity of the front ball increases while that of the rear ball decreases and thus the balls separate. After separation, the balls regain their original shapes so that the elastic potential energy is completely converted back into kinetic energy. Thus, although the kinetic energy is not constant, the initial kinetic energy is equal to the final kinetic energy. Such a collision is called an *elastic collision.*

On the contrary, if the materials of the balls are *perfectly inelastic,* the balls have no tendency to regain their original shapes after maximum deformation. As a result, they do not push each other and continue to move with the common velocity with their deformed shapes. The kinetic energy decreases at the time of deformation and thereafter remains constant at this decreased value. Such a collision is called an *inelastic collision.*

If the material is *partially elastic,* the balls try to regain their original shapes, they push each other, even after maximum deformation. The velocities further change, the balls separate but the shapes are not completely recovered. Some energy remains inside the deformed ball. The final kinetic energy is, therefore, less than the initial kinetic energy. But the loss of kinetic energy is not as large as that in the case of a perfectly inelastic collision.

Thus, for an elastic collision,

$$\left.\begin{aligned} m_1\vec{v}_1 + m_2\vec{v}_2 &= m_1\vec{v}_1{}' + m_2\vec{v}_2{}' \\ \text{and } \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 &= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 \end{aligned}\right| \quad \dots (9.12)$$

i.e., $K_f = K_i$.

For an inelastic collision, $\vec{v}_1{}' = \vec{v}_2{}' = \vec{V}$,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{V} + m_2\vec{V} \quad \dots (9.13)$$

and $K_f < K_i$.

For a partially elastic collision,

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

$$K_f < K_i, \quad \Delta K = K_i - K_f$$

is the loss of kinetic energy. It is less than that in the case of a perfectly inelastic collision. For one dimensional collision (head-on collision) the vector sign may be removed.

9.7 ELASTIC COLLISION IN ONE DIMENSION

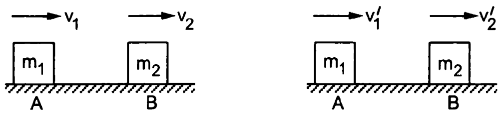


Figure 9.16

Consider two elastic bodies A and B moving along the same line (figure 9.16). The body A has a mass m_1 and moves with a velocity v_1 towards right and the body B has a mass m_2 and moves with a velocity v_2 in the same direction. We assume $v_1 > v_2$ so that the two bodies may collide. Let v_1' and v_2' be the final velocities of the bodies after the collision. The total linear momentum of the two bodies remains constant, so that,

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2' \quad \dots \text{(i)}$$

$$\text{or,} \quad m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$$

$$\text{or,} \quad m_1(v_1 - v_1') = m_2(v_2' - v_2). \quad \dots \text{(ii)}$$

Also, since the collision is elastic, the kinetic energy before the collision is equal to the kinetic energy after the collision. Hence,

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$

$$\text{or,} \quad m_1v_1^2 - m_1v_1'^2 = m_2v_2'^2 - m_2v_2^2$$

$$\text{or,} \quad m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2). \quad \dots \text{(iii)}$$

Dividing (iii) by (ii),

$$v_1 + v_1' = v_2' + v_2$$

$$\text{or,} \quad v_1 - v_2 = v_2' - v_1'. \quad \dots \text{(iv)}$$

Now, $(v_1 - v_2)$ is the rate at which the separation between the bodies decreases before the collision. Similarly, $(v_2' - v_1')$ is the rate of increase of separation after the collision. So the equation (iv) may be written as

Velocity of separation (after collision)

= Velocity of approach (before collision). ... (9.14)

This result is very useful in solving problems involving elastic collision. The final velocities v_1' and v_2' may be obtained from equation (i) and (iv). Multiply equation (iv) by m_2 and subtract from equation (i).

$$2m_2v_2 + (m_1 - m_2)v_1 = (m_1 + m_2)v_1'$$

$$\text{or,} \quad v_1' = \frac{(m_1 - m_2)}{m_1 + m_2}v_1 + \frac{2m_2}{m_1 + m_2}v_2. \quad \dots (9.15)$$

Now multiply equation (iv) by m_1 and add to equation (i),

$$2m_1v_1 - (m_1 - m_2)v_2 = (m_2 + m_1)v_2'$$

$$\text{or,} \quad v'_2 = \frac{2m_1v_1}{m_1 + m_2} - \frac{(m_1 - m_2)v_2}{m_1 + m_2}. \quad \dots (9.16)$$

Equations, (9.15) and (9.16) give the final velocities in terms of the initial velocities and the masses.

Special cases :

(a) Elastic collision between a heavy body and a light body :

Let $m_1 >> m_2$. A heavy body hits a light body from behind.

We have,

$$\frac{m_1 - m_2}{m_1 + m_2} \approx 1, \quad \frac{2m_2}{m_1 + m_2} \approx 0$$

$$\text{and} \quad \frac{2m_1}{m_1 + m_2} \approx 2.$$

With these approximations the final velocities of the bodies are, from (9.15) and (9.16),

$$v_1' \approx v_1 \text{ and } v_2' \approx 2v_1 - v_2.$$

The heavier body continues to move with almost the same velocity. If the lighter body were kept at rest $v_2 = 0$, $v_2' = 2v_1$ which means the lighter body, after getting a push from the heavier body will fly away with a velocity double the velocity of the heavier body.

Next suppose $m_2 >> m_1$. A light body hits a heavy body from behind.

We have,

$$\frac{m_1 - m_2}{m_1 + m_2} \approx -1$$

$$\frac{2\,m_2}{m_1 + m_2} \approx 2$$

and $$\frac{2\,m_1}{m_1 + m_2} \approx 0$$

The final velocities of the bodies are, from (9.15) and (9.16),

$$v_1' \approx -v_1 + 2v_2 \quad \text{and} \quad v_2' \approx v_2 .$$

The heavier body continues to move with almost the same velocity, the velocity of the lighter body changes. If the heavier body were at rest, $v_2 = 0$ then $v_1' = -v_1$, the lighter body returns after collision with almost the same speed. This is the case when a ball collides elastically with a fixed wall and returns with the same speed.

(b) Elastic collision of two bodies of equal mass :

Putting $m_1 = m_2$ in equation (9.15) and (9.16)

$$v_1' = v_2 \quad \text{and} \quad v_2' = v_1 .$$

When two bodies of equal mass collide elastically, their velocities are mutually interchanged.

9.8 PERFECTLY INELASTIC COLLISION IN ONE DIMENSION

Final Velocity

When perfectly inelastic bodies moving along the same line collide, they stick to each other. Let m_1 and m_2 be the masses, v_1 and v_2 be their velocities before the collision and V be the common velocity of the bodies after the collision. By the conservation of linear momentum,

$$m_1 v_1 + m_2 v_2 = m_1 V + m_2 V$$

or, $$V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} . \qquad \dots \text{(i)}$$

Loss in Kinetic Energy

The kinetic energy before the collision is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

and that after the collision is $\frac{1}{2}(m_1 + m_2)V^2$. Using equation (i), the loss in kinetic energy due to the collision is

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 - \frac{1}{2}(m_1 + m_2)\,V^2$$

$$= \frac{1}{2}\left[m_1 v_1^2 + m_2 v_2^2 - \frac{(m_1 v_1 + m_2 v_2)^2}{m_1 + m_2} \right]$$

$$= \frac{1}{2}\left[\frac{m_1 m_2 (v_1^2 + v_2^2 - 2 v_1 v_2)}{m_1 + m_2} \right]$$

$$= \frac{m_1 m_2 (v_1 - v_2)^2}{2\,(m_1 + m_2)} .$$

We see that the loss in kinetic energy is positive.

Example 9.5

A cart A of mass 50 kg *moving at a speed of* 20 km/h *hits a lighter cart B of mass* 20 kg *moving towards it at a speed of* 10 km/h. *The two carts cling to each other. Find the speed of the combined mass after the collision.*

Solution : This is an example of inelastic collision. As the carts move towards each other, their momenta have opposite sign. If the common speed after the collision is V, momentum conservation gives

$$(50\text{ kg})\,(20\text{ km/h}) - (20\text{ kg})\,(10\text{ km/h}) = (70\text{ kg})\,V$$

or $$V = \frac{80}{7}\text{ km/h}.$$

9.9 COEFFICIENT OF RESTITUTION

We have seen that for a perfectly elastic collision

velocity of separation = velocity of approach

and for a perfectly inelastic collision

velocity of separation = 0.

In general, the bodies are neither perfectly elastic nor perfectly inelastic. In that case we can write

velocity of separation = e (velocity of approach),

where $0 < e < 1$. The constant e depends on the materials of the colliding bodies. This constant is known as *coefficient of restitution*. If $e = 1$, the collision is perfectly elastic and if $e = 0$, the collision is perfectly inelastic.

Example 9.6

A block of mass m moving at speed v collides with another block of mass 2 m *at rest. The lighter block comes to rest after the collision. Find the coefficient of restitution.*

Solution : Suppose the second block moves at speed v' towards right after the collision. From the principle of conservation of momentum,

$mv = 2mv'$ or $v' = v/2$.

Hence, the velocity of separation $= v/2$ and the velocity of approach $= v$. By definition,

$$e = \frac{\text{velocity of the separation}}{\text{velocity of approach}} = \frac{v/2}{v} = \frac{1}{2} .$$

9.10 ELASTIC COLLISION IN TWO DIMENSIONS

Consider two objects A and B of mass m_1 and m_2 kept on the X-axis (figure 9.17). Initially, the object

B is at rest and A moves towards B with a speed u_1. If the collision is not *head-on* (the force during the collision is not along the initial velocity), the objects move along different lines. Suppose the object A moves with a velocity $\vec{v}_1$ making an angle θ with the X-axis and the object B moves with a velocity $\vec{v}_2$ making an angle Φ with the same axis. Also, suppose $\vec{v}_1$ and $\vec{v}_2$ lie in X-Y plane. Using conservation of momentum in X and Y directions, we get

$$m_1 u_1 = m_1 v_1 \cos\theta + m_2 v_2 \cos\Phi \qquad \dots \text{(i)}$$

and
$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\Phi. \qquad \dots \text{(ii)}$$

If the collision is elastic, the final kinetic energy is equal to the initial kinetic energy. Thus,

$$\frac{1}{2} m_1 u_1^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2. \qquad \dots \text{(iii)}$$

We have four unknowns v_1, v_2, θ and Φ to describe the final motion whereas there are only three relations. Thus, the final motion cannot be uniquely determined with this information.

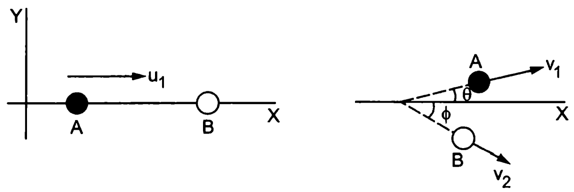


Figure 9.17

In fact, the final motion depends on the angle between the line of force during the collision and the direction of initial velocity. The momentum of each object must be individually conserved in the direction perpendicular to the force. The motion along the line of force may be treated as a one-dimensional collision.

9.11 IMPULSE AND IMPULSIVE FORCE

When two bodies collide, they exert forces on each other while in contact. The momentum of each body is changed due to the force on it exerted by the other. On an ordinary scale, the time duration of this contact is very small and yet the change in momentum is sizeable. This means that the magnitude of the force must be large on an ordinary scale. Such large forces acting for a very short duration are called *impulsive forces*. The force may not be uniform while the contact lasts.

The change in momentum produced by such an implusive force is

$$\vec{P}_f - \vec{P}_i = \int_{P_i}^{P_f} d\vec{P} = \int_{t_i}^{t_f} \frac{d\vec{P}}{dt}\, dt = \int_{t_i}^{t_f} \vec{F}\, dt. \qquad \dots (9.17)$$

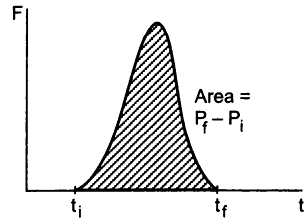


Figure 9.18

This quantity $\int_{t_i}^{t_f} \vec{F}\, dt$ is known as the impluse of the force $\vec{F}$ during the time interval t_i to t_f and is equal to the change in the momentum of the body on which it acts. Obviously, it is the area under the $F-t$ curve for one-dimensional motion (figure 9.18).

Worked Out Examples

1. *Three particles of masses* 0·50 kg, 1·0 kg *and* 1·5 kg *are placed at the three corners of a right-angled triangle of sides* 3·0 cm, 4·0 cm *and* 5·0 cm *as shown in figure (9-W1). Locate the centre of mass of the system.*

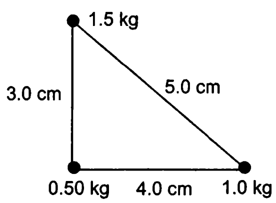


Figure 9-W1.

Solution : Let us take the 4·0 cm line as the X-axis and the 3·0 cm line as the Y-axis. The coordinates of the three particles are as follows :

m	x	y
0·50 kg	0	0
1·0 kg	4·0 cm	0
1·5 kg	0	3·0 cm

The x-coordinate of the centre of mass is

$$X = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(0{\cdot}50 \text{ kg}) \,.\, 0 + (1{\cdot}0 \text{ kg}) \,.\, (4{\cdot}0 \text{ cm}) + (1{\cdot}5 \text{ kg}) \,.\, 0}{0{\cdot}50 \text{ kg} + 1{\cdot}0 \text{ kg} + 1{\cdot}5 \text{ kg}}$$

$$= \frac{4\ \text{kg-cm}}{3\ \text{kg}} = 1{\cdot}3\ \text{cm}.$$

The y-coordinate of the centre of mass is

$$Y = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(0{\cdot}50\ \text{kg}) \cdot 0 + (1{\cdot}0\ \text{kg}) \cdot 0 + (1{\cdot}5\ \text{kg})(3{\cdot}0\ \text{cm})}{0{\cdot}50\ \text{kg} + 1{\cdot}0\ \text{kg} + 1{\cdot}5\ \text{kg}}$$

$$= \frac{4{\cdot}5\ \text{kg-cm}}{3\ \text{kg}} = 1{\cdot}5\ \text{cm}.$$

Thus, the centre of mass is 1·3 cm right and 1·5 cm above the 0·5 kg particle.

2. *Half of the rectangular plate shown in figure (9-W2) is made of a material of density ρ_1 and the other half of density ρ_2. The length of the plate is L. Locate the centre of mass of the plate.*

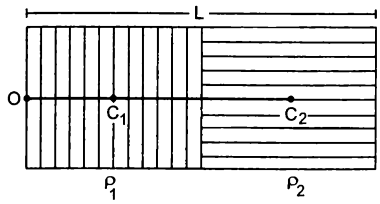


Figure 9-W2

Solution : The centre of mass of each half is located at the geometrical centre of that half. Thus, the left half may be replaced by a point particle of mass $K\rho_1$ placed at C_1 and the right half may be replaced by a point particle of mass $K\rho_2$ placed at C_2. This replacement is for the specific purpose of locating the combined centre of mass. Take the middle point of the left edge to be the origin. The x-coordinate of C_1 is $L/4$ and that of C_2 is $3L/4$. Hence, the x-coordinate of the centre of mass is

$$X = \frac{(K\rho_1)\frac{L}{4} + (K\rho_2)\frac{3L}{4}}{K\rho_1 + K\rho_2}$$

$$= \frac{(\rho_1 + 3\rho_2)}{4(\rho_1 + \rho_2)} L.$$

The combined centre of mass is this much to the right of the assumed origin.

3. *The density of a linear rod of length L varies as $\rho = A + Bx$ where x is the distance from the left end. Locate the centre of mass.*

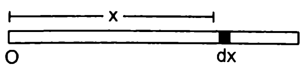


Figure 9-W3

Solution : Let the cross-sectional area be α. The mass of an element of length dx located at a distance x away from the left end is $(A + Bx)\,\alpha\,dx$. The x-coordinate of the centre of mass is given by

$$X_{CM} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^L x\,(A + Bx)\,\alpha\,dx}{\int_0^L (A + Bx)\,\alpha\,dx}$$

$$= \frac{A\frac{L^2}{2} + B\frac{L^3}{3}}{AL + B\frac{L^2}{2}} = \frac{3AL + 2BL^2}{3(2A + BL)}.$$

4. *A cubical block of ice of mass m and edge L is placed in a large tray of mass M. If the ice melts, how far does the centre of mass of the system "ice plus tray" come down ?*

Solution : Consider figure (9-W4). Suppose the centre of mass of the tray is a distance x_1 above the origin and that of the ice is a distance x_2 above the origin. The height of the centre of mass of the ice-tray system is

$$x = \frac{m\,x_2 + M\,x_1}{m + M}.$$

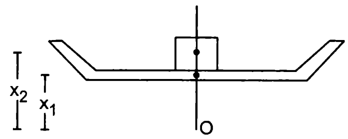


Figure 9-W4

When the ice melts, the water of mass m spreads on the surface of the tray. As the tray is large, the height of water is negligible. The centre of mass of the water is then on the surface of the tray and is at a distance $x_2 - L/2$ above the origin. The new centre of mass of the ice-tray system will be at the height

$$x' = \frac{m\left(x_2 - \frac{L}{2}\right) + M\,x_1}{m + M}.$$

The shift in the centre of mass $= x - x' = \dfrac{mL}{2(m + M)}$.

5. *Consider a two-particle system with the particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved so as to keep the centre of mass at the same position ?*

Solution : Consider figure (9-W5). Suppose the distance of m_1 from the centre of mass C is x_1 and that of m_2 from C is x_2. Suppose the mass m_2 is moved through a distance d' towards C so as to keep the centre of mass at C.

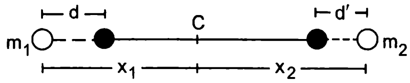


Figure 9-W5

Then,

$$m_1 x_1 = m_2 x_2 \quad \ldots \text{(i)}$$

and $$m_1 (x_1 - d) = m_2 (x_2 - d'). \quad \ldots \text{(ii)}$$

Subtracting (ii) from (i)

$$m_1 d = m_2 d'$$

or, $$d' = \frac{m_1}{m_2} d.$$

6. *A body of mass* 2·5 kg *is subjected to the forces shown in figure (9-W6). Find the acceleration of the centre of mass.*

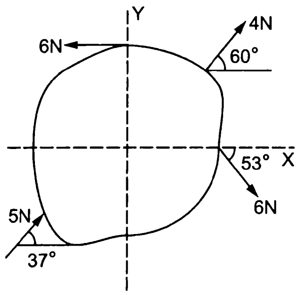


Figure 9-W6

Solution : Take the X and Y axes as shown in the figure. The x-component of the resultant force is

$$F_x = -6\text{ N} + (5\text{ N})\cos 37° + (6\text{ N})\cos 53° + (4\text{ N})\cos 60°$$

$$= -6\text{ N} + (5\text{ N})\cdot(4/5) + (6\text{ N})\cdot(3/5) + (4\text{ N})\cdot(1/2) = 3{\cdot}6\text{ N}.$$

Similarly, the y-component of the resultant force is

$$F_y = 5\text{ N}\sin 37° - (6\text{ N})\sin 53° + 4\text{ N}\sin 60°$$

$$= (5\text{ N})\cdot(3/5) - (6\text{ N})\cdot(4/5) + (4\text{ N})\cdot(\sqrt{3}/2) = 1{\cdot}7\text{ N}\cdot$$

The magnitude of the resultant force is

$$F = \sqrt{{F_x}^2 + {F_y}^2} = \sqrt{(3{\cdot}6\text{ N})^2 + (1{\cdot}7\text{ N})^2} \approx 4{\cdot}0\text{ N}.$$

The direction of the resultant force makes an angle θ with the X-axis where

$$\tan\theta = \frac{F_y}{F_x} = \frac{1{\cdot}7}{3{\cdot}6} = 0{\cdot}47.$$

The acceleration of the centre of mass is

$$a_{CM} = \frac{F}{M} = \frac{4{\cdot}0\text{ N}}{2{\cdot}5\text{ kg}} = 1{\cdot}6\text{ m/s}^2$$

in the direction of the resultant force.

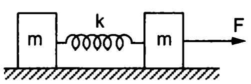


Figure 9-W7

7. *Two blocks of equal mass m are connected by an unstretched spring and the system is kept at rest on a frictionless horizontal surface. A constant force F is applied on one of the blocks pulling it away from the other as shown in figure (9-W7). (a) Find the position of the centre of mass at time t. (b) If the extension of the spring is x_0 at time t, find the displacement of the two blocks at this instant.*

Solution : (a) The acceleration of the centre of mass is given by

$$a_{CM} = \frac{F}{M} = \frac{F}{2m}\cdot$$

The position of the centre of mass at time t is

$$x = \frac{1}{2} a_{CM}\, t^2 = \frac{F t^2}{4 m}\cdot$$

(b) Suppose the displacement of the first block is x_1 and that of the second is x_2. As the centre of mass is at x, we should have

$$x = \frac{m x_1 + m x_2}{2 m}$$

or, $$\frac{F t^2}{4 m} = \frac{x_1 + x_2}{2}$$

or, $$x_1 + x_2 = \frac{F t^2}{2 m}\cdot \quad \ldots \text{(i)}$$

The extension of the spring is $x_2 - x_1$. Therefore,

$$x_2 - x_1 = x_0\,. \quad \ldots \text{(ii)}$$

from (i) and (ii), $$x_1 = \frac{1}{2}\left(\frac{F t^2}{2 m} - x_0\right)$$

and $$x_2 = \frac{1}{2}\left(\frac{F t^2}{2 m} + x_0\right).$$

8. *A projectile is fired at a speed of* 100 m/s *at an angle of* 37° *above the horizontal. At the highest point, the projectile breaks into two parts of mass ratio* 1 : 3, *the smaller coming to rest. Find the distance from the launching point to the point where the heavier piece lands.*

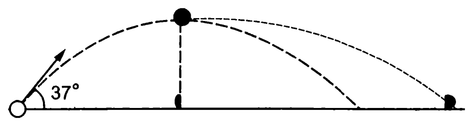


Figure 9-W8

Solution : See figure (9-W8). At the highest point, the projectile has horizontal velocity. The lighter part comes to rest. Hence the heavier part will move with increased horizontal velocity. In vertical direction, both parts have zero velocity and undergo same acceleration, hence they will cover equal vertical displacements in a given time. Thus, both will hit the ground together. As internal forces do not affect the motion of the centre of mass, the centre of mass hits the ground at the position where the original projectile would have landed. The range of the original projectile is

$$x_{CM} = \frac{2 u^2 \sin\theta\cos\theta}{g} = \frac{2\times 10^4 \times \frac{3}{5}\times\frac{4}{5}}{10}\text{ m}$$

$$= 960\text{ m}.$$

The centre of mass will hit the ground at this position. As the smaller block comes to rest after breaking, it falls down vertically and hits the ground at half of the range i.e., at $x = 480$ m. If the heavier block hits the ground at x_2, then

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

or, $$960 \text{ m} = \frac{\frac{M}{4} \times 480 \text{ m} + \frac{3M}{4} \times x_2}{M}$$

or, $$x_2 = 1120 \text{ m}.$$

9. *A block of mass M is placed on the top of a bigger block of mass* 10 *M as shown in figure (9-W9). All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.*

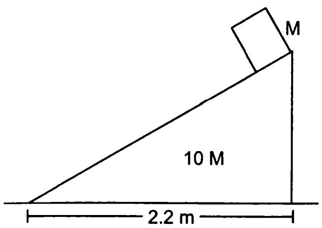


Figure 9-W9

Solution : If the bigger block moves towards right by a distance X, the smaller block will move towards left by a distance $(2{\cdot}2 \text{ m} - X)$. Taking the two blocks together as the system, there is no horizontal external force on it. The centre of mass, which was at rest initially, will remain at the same horizontal position.
Thus,

$$M\,(2{\cdot}2 \text{ m} - X) = 10\,MX$$

or, $$2{\cdot}2 \text{ m} = 11\,X$$

or, $$X = 0{\cdot}2 \text{ m}.$$

10. *The hero of a stunt film fires* 50 g *bullets from a machine gun, each at a speed of* 1·0 km/s. *If he fires* 20 *bullets in* 4 seconds, *what average force does he exert against the machine gun during this period ?*

Solution : The momentum of each bullet

$$= (0{\cdot}050 \text{ kg})\,(1000 \text{ m/s}) = 50 \text{ kg-m/s}.$$

The gun is imparted this much of momentum by each bullet fired. Thus, the rate of change of momentum of the gun

$$= \frac{(50 \text{ kg-m/s}) \times 20}{4 \text{ s}} = 250 \text{ N}$$

In order to hold the gun, the hero must exert a force of 250 N against the gun.

11. *A block moving horizontally on a smooth surface with a speed of* 20 m/s *bursts into two equal parts continuing in the same direction. If one of the parts moves at* 30 m/s, *with what speed does the second part move and what is the fractional change in the kinetic energy ?*

Solution : There is no external force on the block. Internal forces break the block in two parts. The linear momentum of the block before the break should, therefore, be equal to the linear momentum of the two parts after the break. As all the velocities are in same direction, we get,

$$M\,(20 \text{ m/s}) = \frac{M}{2}\,(30 \text{ m/s}) + \frac{M}{2}\,v$$

where v is the speed of the other part. From this equation $v = 10$ m/s. The change in kinetic energy is

$$\frac{1}{2}\frac{M}{2}\,(30 \text{ m/s})^2 + \frac{1}{2}\frac{M}{2}\,(10 \text{ m/s})^2 - \frac{1}{2}M\,(20 \text{ m/s})^2$$

$$= \frac{M}{2}\,(450 + 50 - 400)\,\frac{\text{m}^2}{\text{s}^2} = \left(50\,\frac{\text{m}^2}{\text{s}^2}\right)M.$$

Hence, the fractional change in the kinetic energy

$$= \frac{M\left(50\,\frac{\text{m}^2}{\text{s}^2}\right)}{\frac{1}{2}M\,(20 \text{ m/s})^2} = \frac{1}{4}.$$

12. *A car of mass M is moving with a uniform velocity v on a horizontal road when the hero of a Hindi film drops himself on it from above. Taking the mass of the hero to be m, what will be the velocity of the car after the event ?*

Solution : Consider the car plus the hero as the system. In the horizontal direction, there is no external force. Since the hero has fallen vertically, so his initial horizontal momentum = 0.

Initial horizontal momemtum of the system $= Mv$ towards right.

Finally the hero sticks to the roof of the car, so they move with equal horizontal velocity say V. Final horizontal momentum of the system

$$= (M + m)\,V$$

Hence, $$M\,v = (M + m)\,V$$

or, $$V = \frac{M\,v}{M + m}.$$

13. *A space shuttle, while travelling at a speed of* 4000 km/h *with respect to the earth, disconnects and ejects a module backward, weighing one fifth of the residual part. If the shuttle ejects the disconnected module at a speed of* 100 km/h *with respect to the state of the shuttle before the ejection, find the final velocity of the shuttle.*

Solution : Suppose the mass of the shuttle including the module is M. The mass of the module will be $M/6$. The total linear momentum before disconnection

$$= M\,(4000 \text{ km/h}).$$

The velocity of the ejected module with respect to the earth = its velocity with respect to the shuttle + the velocity of the shuttle with respect to the earth

$$= -100 \text{ km/h} + 4000 \text{ km/h} = 3900 \text{ km/h}.$$

If the final velocity of the shuttle is V then the total final linear momentum

$$= \frac{5M}{6} V + \frac{M}{6} \times 3900 \text{ km/h}.$$

By the principle of conservation of linear momentum,

$$M\,(4000 \text{ km/h}) = \frac{5M}{6} V + \frac{M}{6} \times 3900 \text{ km/h}$$

or, $V = 4020$ km/h .

14. *A boy of mass* 25 kg *stands on a board of mass* 10 kg *which in turn is kept on a frictionless horizontal ice surface. The boy makes a jump with a velocity component* 5 m/s *in a horizontal direction with respect to the ice. With what velocity does the board recoil? With what rate are the boy and the board separating from each other?*

Solution : Consider the "board + boy" as a system. The external forces on this system are (a) weight of the system and (b) normal contact force by the ice surface. Both these forces are vertical and there is no external force in horizontal direction. The horizontal component of linear momentum of the "board + boy" system is, therefore, constant.

If the board recoils at a speed v,

$$0 = (25 \text{ kg}) \times (5 \text{ m/s}) - (10 \text{ kg})v$$

or, $v = 12{\cdot}5$ m/s.

The boy and the board are separating with a rate

5 m/s + 12·5 m/s = 17·5 m/s.

15. *A man of mass m is standing on a platform of mass M kept on smooth ice. If the man starts moving on the platform with a speed v relative to the platform, with what velocity relative to the ice does the platform recoil?*

Solution : Consider the situation shown in figure (9-W10). Suppose the man moves at a speed w towards right and the platform recoils at a speed V towards left, both relative to the ice. Hence, the speed of the man relative to the platform is $V + w$. By the question,

$$V + w = v, \text{ or } w = v - V. \qquad \dots \text{(i)}$$

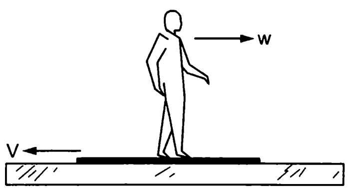


Figure 9-W10

Taking the platform and the man to be the system, there is no external horizontal force on the system. The linear momentum of the system remains constant. Initially, both the man and the platform were at rest. Thus,

$$0 = MV - mw$$

or, $MV = m\,(v - V)$ [Using (i)]

or, $V = \dfrac{m\,v}{M + m}$.

16. *A ball of mass m, moving with a velocity v along X-axis, strikes another ball of mass 2m kept at rest. The first ball comes to rest after collision and the other breaks into two equal pieces. One of the pieces starts moving along Y-axis with a speed v_1. What will be the velocity of the other piece?*

Solution : The total linear momentum of the balls before the collision is mv along the X-axis. After the collision, momentum of the first ball = 0, momentum of the first piece $= m\,v_1$ along the Y-axis and momentum of the second piece $= m\,v_2$ along its direction of motion where v_2 is the speed of the second piece. These three should add to mv along the X-axis, which is the initial momentum of the system.

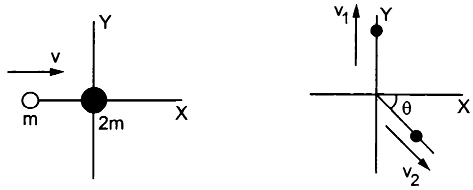


Figure 9-W11

Taking components along the X-axis,

$$m\,v_2 \cos\theta = m\,v \qquad \dots \text{(i)}$$

and taking components along the Y-axis,

$$m\,v_2 \sin\theta = m\,v_1. \qquad \dots \text{(ii)}$$

From (i) and (ii),

$$v_2 = \sqrt{v^2 + v_1^2} \text{ and } \tan\theta = v_1 / v.$$

17. *A bullet of mass* 50 g *is fired from below into the bob of mass* 450 g *of a long simple pendulum as shown in figure (9-W12). The bullet remains inside the bob and the bob rises through a height of* 1·8 m. *Find the speed of the bullet. Take* $g = 10$ m/s^2.

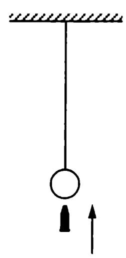


Figure 9-W12

Solution : Let the speed of the bullet be v. Let the common velocity of the bullet and the bob, after the bullet is embedded into the bob, is V. By the principle of conservation of linear momentum,

$$V = \frac{(0{\cdot}05 \text{ kg})v}{0{\cdot}45 \text{ kg} + 0{\cdot}05 \text{ kg}} = \frac{v}{10}.$$

The string becomes loose and the bob will go up with a deceleration of $g = 10 \text{ m/s}^2$. As it comes to rest at a height of 1·8 m, using the equation $v^2 = u^2 + 2ax$,

$$1{\cdot}8 \text{ m} = \frac{(v/10)^2}{2 \times 10 \text{ m/s}^2}$$

or, $$v = 60 \text{ m/s}.$$

18. *A light spring of spring constant k is kept compressed between two blocks of masses m and M on a smooth horizontal surface (figure 9-W13). When released, the blocks acquire velocities in opposite directions. The spring loses contact with the blocks when it acquires natural length. If the spring was initially compressed through a distance x, find the final speeds of the two blocks.*

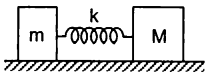


Figure 9-W13

Solution : Consider the two blocks plus the spring to be the system. No external force acts on this system in horizontal direction. Hence, the linear momentum will remain constant. As the spring is light, it has no linear momentum. Suppose the block of mass M moves with a speed V and the other block with a speed v after losing contact with the spring. As the blocks are released from rest, the initial momentum is zero. The final momentum is $MV - mv$ towards right. Thus,

$$MV - mv = 0 \quad \text{or,} \quad V = \frac{m}{M}v. \qquad \text{... (i)}$$

Initially, the energy of the system $= \frac{1}{2}kx^2$.

Finally, the energy of the system $= \frac{1}{2}mv^2 + \frac{1}{2}MV^2$.

As there is no friction,

$$\frac{1}{2}mv^2 + \frac{1}{2}MV^2 = \frac{1}{2}kx^2. \qquad \text{... (ii)}$$

Using (i) and (ii),

$$mv^2\left(1 + \frac{m}{M}\right) = kx^2$$

or, $$v = \sqrt{\frac{kM}{m(M+m)}}\,x$$

and $$V = \sqrt{\frac{km}{M(M+m)}}\,x.$$

19. *A block of mass m is connected to another block of mass M by a massless spring of spring constant k. The blocks are kept on a smooth horizontal plane. Initially, the blocks are at rest and the spring is unstretched when a constant force F starts acting on the block of mass M to pull it. Find the maximum extension of the spring.*

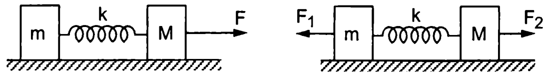


Figure 9-W14

Solution : Let us take the two blocks plus the spring as the system. The centre of mass of the system moves with an acceleration $a = \frac{F}{m+M}$. Let us work from a reference frame with its origin at the centre of mass. As this frame is accelerated with respect to the ground we have to apply a pseudo force ma towards left on the block of mass m and Ma towards left on the block of mass M. The net external force on m is

$$F_1 = ma = \frac{mF}{m+M} \text{ towards left}$$

and the net external force on M is

$$F_2 = F - Ma = F - \frac{MF}{m+M} = \frac{mF}{m+M} \text{ towards right.}$$

The situation from this frame is shown in figure (9-W14b). As the centre of mass is at rest in this frame, the blocks move in opposite directions and come to instantaneous rest at some instant. The extension of the spring will be maximum at this instant. Suppose the left block is displaced through a distance x_1 and the right block through a distance x_2 from the initial positions. The total work done by the external forces F_1 and F_2 in this period are

$$W = F_1x_1 + F_2x_2 = \frac{mF}{m+M}(x_1 + x_2).$$

This should be equal to the increase in the potential energy of the spring as there is no change in the kinetic energy. Thus,

$$\frac{mF}{m+M}(x_1 + x_2) = \frac{1}{2}k(x_1 + x_2)^2$$

or, $$x_1 + x_2 = \frac{2mF}{k(m+M)}.$$

This is the maximum extension of the spring.

20. *The two balls shown in figure (9-W15) are identical, the first moving at a speed v towards right and the second staying at rest. The wall at the extreme right is fixed. Assume all collisions to be elastic. Show that the speeds of the balls remain unchanged after all the collisions have taken place.*

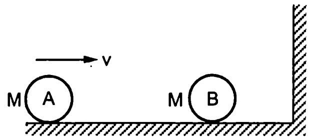


Figure 9-W15

Solution : **1st collision :** As the balls have equal mass and make elastic collision, the velocities are interchanged. Hence, after the first collision, the ball A comes to rest and the ball B moves towards right at a speed v.

2nd collision : The ball B moving with a speed v, collides with the wall and rebounds. As the wall is rigid and may be taken to be of infinite mass, momentum conservation gives no useful result. Velocity of separation should be equal to the velocity of approach. Hence, the ball rebounds at the same speed v towards left.

3rd collision : The ball B moving towards left at the speed v again collides with the ball A kept at rest. As the masses are equal and the collision is elastic, the velocities are interchanged. Thus, the ball B comes to rest and the ball A moves towards left at a speed v. No further collision takes place. Thus, the speeds of the balls remain the same as their initial values.

21. *A block of mass m moving at a velocity v collides head on with another block of mass 2m at rest. If the coefficient of restitution is 1/2, find the velocities of the blocks after the collision.*

Solution : Suppose after the collision the block of mass m moves at a velocity u_1 and the block of mass $2m$ moves at a velocity u_2. By conservation of momentum,

$$mv = mu_1 + 2mu_2 . \quad \dots \text{(i)}$$

The velocity of separation is $u_2 - u_1$ and the velocity of approach is v.

So, $$u_2 - u_1 = v/2. \quad \dots \text{(ii)}$$

From (i) and (ii), $u_1 = 0$ and $u_2 = v/2$.

22. *A block of mass* 1·2 kg *moving at a speed of* 20 cm/s *collides head-on with a similar block kept at rest. The coefficient of restitution is* 3/5. *Find the loss of kinetic energy during the collision.*

Solution : Suppose the first block moves at a speed v_1 and the second at v_2 after the collision. Since the collision is head-on, the two blocks move along the original direction of motion of the first block.

By conservation of linear momentum,

$$(1{\cdot}2\text{ kg})\,(20\text{ cm/s}) = (1{\cdot}2\text{ kg})\,v_1 + (1{\cdot}2\text{ kg})\,v_2$$

or, $$v_1 + v_2 = 20\text{ cm/s}. \quad \dots \text{(i)}$$

The velocity of separation is $v_2 - v_1$ and the velocity of approach is 20 cm/s. As the coefficient of restitution is 3/5, we have,

$$v_2 - v_1 = (3/5) \times 20\text{ cm/s} = 12\text{ cm/s}. \quad \dots \text{(ii)}$$

By (i) and (ii),

$$v_1 = 4\text{ cm/s} \quad \text{and} \quad v_2 = 16\text{ cm/s}.$$

The loss in kinetic energy is

$$\frac{1}{2}(1{\cdot}2\text{ kg})[(20\text{ cm/s})^2 - (4\text{ cm/s})^2 - (16\text{ cm/s})^2]$$

$$= (0{\cdot}6\text{ kg})\,[0{\cdot}04\text{ m}^2/\text{s}^2 - 0{\cdot}0016\text{ m}^2/\text{s}^2 - 0{\cdot}0256\text{ m}^2/\text{s}^2]$$

$$= (0{\cdot}6\text{ kg})\,(0{\cdot}0128\text{ m}^2/\text{s}^2) = 7{\cdot}7 \times 10^{-3}\text{ J}.$$

23. *A ball of mass m hits the floor with a speed v making an angle of incidence θ with the normal. The coefficient of restitution is e. Find the speed of the reflected ball and the angle of reflection of the ball.*

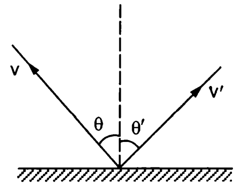


Figure 9-W16

Solution : See figure (9-W16). Suppose the angle of reflection is θ' and the speed after the collision is v'. The floor exerts a force on the ball along the normal during the collision. There is no force parallel to the surface. Thus, the parallel component of the velocity of the ball remains unchanged. This gives

$$v' \sin\theta' = v \sin\theta. \quad \dots \text{(i)}$$

For the components normal to the floor,

the velocity of separation $= v' \cos\theta'$

and the velocity of approach $= v \cos\theta$.

Hence, $$v' \cos\theta' = e\, v \cos\theta. \quad \dots \text{(ii)}$$

From (i) and (ii),

$$v' = v\sqrt{\sin^2\theta + e^2 \cos^2\theta}$$

and $$\tan\theta' = \frac{\tan\theta}{e}.$$

For elastic collision, $e = 1$ so that $\theta' = \theta$ and $v' = v$.

24. *A block of mass m and a pan of equal mass are connected by a string going over a smooth light pulley as shown in figure (9-W17). Initially the system is at rest when a particle of mass m falls on the pan and sticks to it. If the particle strikes the pan with a speed v find the speed with which the system moves just after the collision.*

Solution : Let the required speed be V.

As there is a sudden change in the speed of the block, the tension must change by a large amount during the collision.

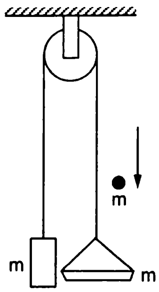


Figure 9-W17

Let N = magnitude of the contact force between the particle and the pan

T = tension in the string

Consider the impulse imparted to the particle. The force is N in upward direction and the impulse is $\int N\,dt$. This should be equal to the change in its momentum.

Thus, $$\int N\,dt = mv - mV. \quad \text{... (i)}$$

Similarly considering the impulse imparted to the pan,

$$\int (N - T)dt = mV \quad \text{... (ii)}$$

and that to the block,

$$\int T\,dt = mV. \quad \text{... (iii)}$$

Adding (ii) and (iii),

$$\int N\,dt = 2\,mV.$$

Comparing with (i),

$$mv - mV = 2mV$$

or, $$V = v/3.$$

□

QUESTIONS FOR SHORT ANSWER

1. Can the centre of mass of a body be at a point outside the body ?
2. If all the particles of a system lie in X-Y plane, is it necessary that the centre of mass be in X-Y plane ?
3. If all the particle of a system lie in a cube, is it neccesary that the centre of mass be in the cube ?
4. The centre of mass is defined as $\vec{R} = \frac{1}{M}\sum_i m_i \vec{r_i}$. Suppose we define "centre of charge" as $\vec{R}_C = \frac{1}{Q}\sum_i q_i \vec{r_i}$ where q_i represents the ith charge placed at $\vec{r_i}$ and Q is the total charge of the system.
(a) Can the centre of charge of a two-charge system be outside the line segment joining the charges ?
(b) If all the charges of a system are in X-Y plane, is it necessary that the centre of charge be in X-Y plane ?
(c) If all the charges of a system lie in a cube, is it necessary that the centre of charge be in the cube ?
5. The weight Mg of an extended body is generally shown in a diagram to act through the centre of mass. Does it mean that the earth does not attract other particles ?
6. A bob suspended from the ceiling of a car which is accelerating on a horizontal road. The bob stays at rest with respect to the car with the string making an angle θ with the vertical. The linear momentum of the bob as seen from the road is increasing with time. Is it a violation of conservation of linear momentum ? If not, where is the external force which changes the linear momentum ?
7. You are waiting for a train on a railway platform. Your three-year-old niece is standing on your iron trunk containing the luggage. Why does the trunk not recoil as she jumps off on the platform ?
8. In a head-on collision between two particles, is it necessary that the particles will acquire a common velocity at least for one instant ?
9. A collision experiment is done on a horizontal table kept in an elevator. Do you expect a change in the results if the elevator is accelerated up or down because of the noninertial character of the frame ?
10. Two bodies make an elastic head-on collision on a smooth horizontal table kept in a car. Do you expect a change in the result if the car is accelerated on a horizontal road because of the noninertial character of the frame ? Does the equation "Velocity of separation = Velocity of approach" remain valid in an accelerating car ? Does the equation "final momentum = initial momentum" remain valid in the accelerating car ?
11. If the total mechanical energy of a particle is zero, is its linear momentum necessarily zero ? Is it necessarily nonzero ?
12. If the linear momentum of a particle is known, can you find its kinetic energy ? If the kinetic energy of a particle is known can you find its linear momentum ?
13. What can be said about the centre of mass of a uniform hemisphere without making any calculation ? Will its distance from the centre be more than $r/2$ or less than $r/2$?
14. You are holding a cage containing a bird. Do you have to make less effort if the bird flies from its position in the cage and manages to stay in the middle without touching the walls of the cage ? Does it make a difference whether the cage is completely closed or it has rods to let air pass ?

15. A fat person is standing on a light plank floating on a calm lake. The person walks from one end to the other on the plank. His friend sitting on the shore watches him and finds that the person hardly moves any distance because the plank moves backward about the same distance as the person moves on the plank. Explain.

16. A high-jumper successfully clears the bar. Is it possible that his centre of mass crossed the bar from below it ? Try it with appropriate figures.

17. Which of the two persons shown in figure (9-Q1) is more likely to fall down ? Which external force is responsible for his falling down ?

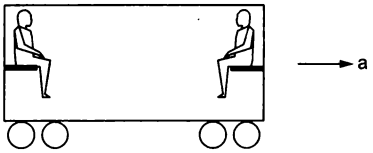


Figure 9-Q1

18. Suppose we define a quantity 'Linear Pomentum' as
linear pomentum = mass × speed.
The linear pomentum of a system of particles is the sum of linear pomenta of the individual particles. Can we state a principle of conservation of linear pomentum as "linear pomentum of a system remains constant if no external force acts on it" ?

19. Use the definition of linear pomentum from the previous question. Can we state the principle of conservation of linear pomentum for a single particle ?

20. To accelerate a car we ignite petrol in the engine of the car. Since only an external force can accelerate the centre of mass, is it proper to say that "the force generated by the engine accelerates the car" ?

21. A ball is moved on a horizontal table with some velocity. The ball stops after moving some distance. Which external force is responsible for the change in the momentum of the ball ?

22. Consider the situation of the previous problem. Take "the table plus the ball" as the system. Friction between the table and the ball is then an internal force. As the ball slows down, the momentum of the system decreases. Which external force is responsible for this change in the momentum ?

23. When a nucleus at rest emits a beta particle, it is found that the velocities of the recoiling nucleus and the beta particle are not along the same straight line. How can this be possible in view of the principle of conservation of momentum ?

24. A van is standing on a frictionless portion of a horizontal road. To start the engine, the vehicle must be set in motion in the forward direction. How can the persons sitting inside the van do it without coming out and pushing from behind ?

25. In one-dimensional elastic collision of equal masses, the velocities are interchanged. Can velocities in a one-dimensional collision be interchanged if the masses are not equal ?

OBJECTIVE I

1. Consider the following two equations :

$$\text{(A)}\quad \vec{R} = \frac{1}{M}\sum_i m_i \vec{r_i}$$

and

$$\text{(B)}\quad \vec{a}_{CM} = \frac{\vec{F}}{M}.$$

In a noninertial frame
(a) both are correct (b) both are wrong
(c) A is correct but B is wrong
(d) B is correct but A is wrong.

2. Consider the following two statements :
(A) Linear momentum of the system remains constant.
(B) Centre of mass of the system remains at rest.
(a) A implies B and B implies A.
(b) A does not imply B and B does not imply A.
(c) A implies B but B does not imply A.
(d) B implies A but A does not imply B.

3. Consider the following two statements :
(A) Linear momentum of a system of particles is zero.
(B) Kinetic energy of a system of particles is zero.
(a) A implies B and B implies A.
(b) A does not imply B and B does not imply A.
(c) A implies B but B does not imply A.
(d) B implies A but A does not imply B.

4. Consider the following two statements :
(A) The linear momentum of a particle is independent of the frame of reference.
(B) The kinetic energy of a particle is independent of the frame of reference.
(a) Both A and B are true. (b) A is true but B is false.
(c) A is false but B is true. (d) both A and B are false.

5. All the particles of a body are situated at a distance R from the origin. The distance of the centre of mass of the body from the origin is
(a) $= R$ (b) $\leq R$ (c) $> R$ (d) $\geq R$.

6. A circular plate of diameter d is kept in contact with a square plate of edge d as shown in figure (9-Q2). The density of the material and the thickness are same

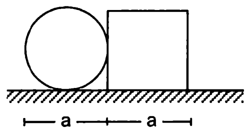


Figure 9-Q2

everywhere. The centre of mass of the composite system will be

(a) inside the circular plate (b) inside the square plate
(c) at the point of contact (d) outside the system.

7. Consider a system of two identical particles. One of the particles is at rest and the other has an acceleration $\vec{a}$. The centre of mass has an acceleration

(a) zero (b) $\frac{1}{2}\vec{a}$ (c) $\vec{a}$ (d) $2\vec{a}$.

8. Internal forces can change
(a) the linear momentum but not the kinetic energy
(b) the kinetic energy but not the linear momentum
(c) linear momentum as well as kinetic energy
(d) neither the linear momentum nor the kinetic energy.

9. A bullet hits a block kept at rest on a smooth horizontal surface and gets embedded into it. Which of the following does not change ?
(a) linear momentum of the block
(b) kinetic energy of the block
(c) gravitational potential energy of the block
(d) temperature of the block.

10. A uniform sphere is placed on a smooth horizontal surface and a horizontal force F is applied on it at a distance h above the surface. The acceleration of the centre
(a) is maximum when $h = 0$
(b) is maximum when $h = R$
(c) is maximum when $h = 2R$
(d) is independent of h.

11. A body falling vertically downwards under gravity breaks in two parts of unequal masses. The centre of mass of the two parts taken together shifts horizontally towards
(a) heavier piece (b) lighter piece
(c) does not shift horizontally
(d) depends on the vertical velocity at the time of breaking.

12. A ball kept in a closed box moves in the box making collisions with the walls. The box is kept on a smooth surface. The velocity of the centre of mass
(a) of the box remains constant
(b) of the box plus the ball system remains constant
(c) of the ball remains constant
(d) of the ball relative to the box remains constant.

13. A body at rest breaks into two pieces of equal masses. The parts will move
(a) in same direction (b) along different lines
(c) in opposite directions with equal speeds
(d) in opposite directions with unequal speeds.

14. A heavy ring of mass m is clamped on the periphery of a light circular disc. A small particle having equal mass is clamped at the centre of the disc. The system is rotated in such a way that the centre moves in a circle of radius r with a uniform speed v. We conclude that an external force
(a) $\frac{mv^2}{r}$ must be acting on the central particle
(b) $\frac{2mv^2}{r}$ must be acting on the central particle
(c) $\frac{2mv^2}{r}$ must be acting on the system
(d) $\frac{2mv^2}{r}$ must be acting on the ring.

15. The quantities remaining constant in a collision are
(a) momentum, kinetic energy and temperature
(b) momentum and kinetic energy but not temperature
(c) momentum and temperature but not kinetic energy
(d) momentum, but neither kinetic energy nor temperature.

16. A nucleus moving with a velocity $\vec{v}$ emits an α-particle. Let the velocities of the α-particle and the remaining nucleus be $\vec{v}_1$ and $\vec{v}_2$ and their masses be m_1 and m_2.
(a) $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ must be parallel to each other.
(b) None of the two of $\vec{v}$, $\vec{v}_1$ and $\vec{v}_2$ should be parallel to each other.
(c) $\vec{v}_1 + \vec{v}_2$ must be parallel to $\vec{v}$.
(d) $m_1\vec{v}_1 + m_2\vec{v}_2$ must be parallel to $\vec{v}$.

17. A shell is fired from a cannon with a velocity V at an angle θ with the horizontal direction. At the highest point in its path, it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. The speed of the other piece immediately after the explosion is

(a) $3V\cos\theta$ (b) $2V\cos\theta$ (c) $\frac{3}{2}V\cos\theta$ (d) $V\cos\theta$.

18. In an elastic collision
(a) the initial kinetic energy is equal to the final kinetic energy
(b) the final kinetic energy is less than the initial kinetic energy
(c) the kinetic energy remains constant
(d) the kinetic energy first increases then decreases.

19. In an inelastic collsion
(a) the initial kinetic energy is equal to the final kinetic energy
(b) the final kinetic energy is less than the initial kinetic energy
(c) the kinetic energy remains the constant
(d) the kinetic energy first increases then decreases.

OBJECTIVE II

1. The centre of mass of a system of particles is at the origin. It follows that
(a) the number of particles to the right of the origin is equal to the number of particles to the left
(b) the total mass of the particles to the right of the origin is same as the total mass to the left of the origin

(c) the number of particles on X-axis should be equal to the number of particles on Y-axis
(d) if there is a particle on the positive X-axis, there must be at least one particle on the negative X-axis.

2. A body has its centre of mass at the origin. The x-coordinates of the particles
(a) may be all positive (b) may be all negative
(c) may be all non-negative
(d) may be positive for some case and negative in other cases.

3. In which of the following cases the centre of mass of a rod is certainly not at its centre ?
(a) the density continuously increases from left to right
(b) the density continuously decreases from left to right
(c) the density decreases from left to right upto the centre and then increases
(d) the density increases from left to right upto the centre and then decreases.

4. If the external forces acting on a system have zero resultant, the centre of mass
(a) must not move (b) must not accelerate
(c) may move (d) may accelerate.

5. A nonzero external force acts on a system of particles. The velocity and the acceleration of the centre of mass are found to be v_0 and a_0 at an instant t. It is possible that
(a) $v_0 = 0$, $a_0 = 0$ (b) $v_0 = 0$, $a_0 \neq 0$,
(c) $v_0 \neq 0$, $a_0 = 0$ (d) $v_0 \neq 0$, $a_0 \neq 0$.

6. Two balls are thrown simultaneously in air. The acceleration of the centre of mass of the two balls while in air
(a) depends on the direction of the motion of the balls
(b) depends on the masses of the two balls
(c) depends on the speeds of the two balls
(d) is equal to g.

7. A block moving in air breaks in two parts and the parts separate
(a) the total momentum must be conserved
(b) the total kinetic energy must be conserved
(c) the total momentum must change
(d) the total kinetic energy must change.

8. In an elastic collision
(a) the kinetic energy remains constant
(b) the linear momentum remains constant
(c) the final kinetic energy is equal to the initial kinetic energy
(d) the final linear momentum is equal to the initial linear momentum.

9. A ball hits a floor and rebounds after an inelastic collision. In this case
(a) the momentum of the ball just after the collision is same as that just before the collision
(b) the mechanical energy of the ball remains the same during the collision
(c) the total momentum of the ball and the earth is conserved
(d) the total energy of the ball and the earth remains the same.

10. A body moving towards a finite body at rest collides with it. It is possible that
(a) both the bodies come to rest
(b) both the bodies move after collision
(c) the moving body comes to rest and the stationary body starts moving
(d) the stationary body remains stationary, the moving body changes its velocity.

11. In a head-on elastic collision of two bodies of equal masses
(a) the velocities are interchanged
(b) the speeds are interchanged
(c) the momenta are interchanged
(d) the faster body slows down and the slower body speeds up.

EXERCISES

1. Three particles of masses 1·0 kg, 2·0 kg and 3·0 kg are placed at the corners A, B and C respectively of an equilateral triangle ABC of edge 1 m. Locate the centre of mass of the system.

2. The structure of a water molecule is shown in figure (9-E1). Find the distance of the centre of mass of the molecule from the centre of the oxygen atom.

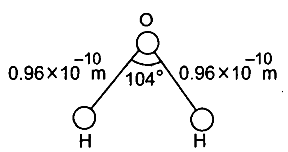


Figure 9-E1

3. Seven homogeneous bricks, each of length L, are arranged as shown in figure (9-E2). Each brick is displaced with respect to the one in contact by $L/10$. Find the x-coordinate of the centre of mass relative to the origin shown.

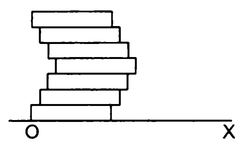


Figure 9-E2

4. A uniform disc of radius R is put over another uniform disc of radius $2R$ of the same thickness and density. The peripheries of the two discs touch each other. Locate the centre of mass of the system.

5. A disc of radius R is cut out from a larger disc of radius $2R$ in such a way that the edge of the hole touches the edge of the disc. Locate the centre of mass of the residual disc.

6. A square plate of edge d and a circular disc of diameter d are placed touching each other at the midpoint of an edge of the plate as shown in figure (9-Q2). Locate the centre of mass of the combination, assuming same mass per unit area for the two plates.

7. Calculate the velocity of the centre of mass of the system of particles shown in figure (9-E3).

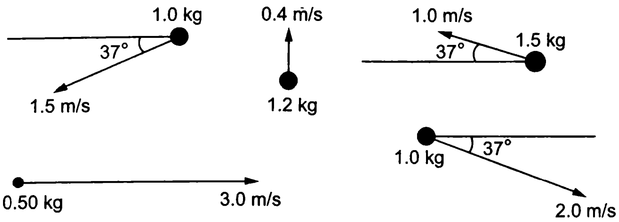
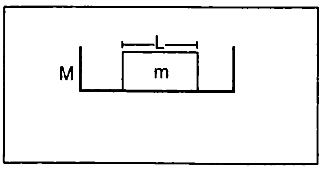


Figure 9-E3

8. Two blocks of masses 10 kg and 20 kg are placed on the X-axis. The first mass is moved on the axis by a distance of 2 cm. By what distance should the second mass be moved to keep the position of the centre of mass unchanged ?

9. Two blocks of masses 10 kg and 30 kg are placed along a vertical line. The first block is raised through a height of 7 cm. By what distance should the second mass be moved to raise the centre of mass by 1 cm ?

10. Consider a gravity-free hall in which a tray of mass M, carrying a cubical block of ice of mass m and edge L, is at rest in the middle (figure 9-E4). If the ice melts, by what distance does the centre of mass of "the tray plus the ice" system descend ?



Gravity-free hall

Figure 9-E4

11. Find the centre of mass of a uniform plate having semicircular inner and outer boundaries of radii R_1 and R_2 (figure 9-E5).

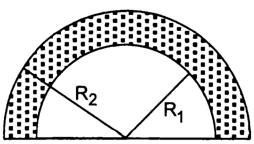


Figure 9-E5

12. Mr. Verma (50 kg) and Mr. Mathur (60 kg) are sitting at the two extremes of a 4 m long boat (40 kg) standing still in water. To discuss a mechanics problem, they come to the middle of the boat. Neglecting friction with water, how far does the boat move on the water during the process ?

13. A cart of mass M is at rest on a frictionless horizontal surface and a pendulum bob of mass m hangs from the roof of the cart (figure 9-E6). The string breaks, the bob falls on the floor, makes several collisions on the floor and finally lands up in a small slot made in the floor. The horizontal distance between the string and the slot is L. Find the displacement of the cart during this process.

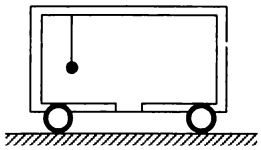


Figure 9-E6

14. The balloon, the light rope and the monkey shown in figure (9-E7) are at rest in the air. If the monkey reaches the top of the rope, by what distance does the balloon descend ? Mass of the balloon = M, mass of the monkey = m and the length of the rope ascended by the monkey = L.

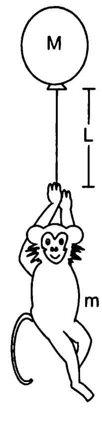


Figure 9-E7

15. Find the ratio of the linear momenta of two particles of masses 1·0 kg and 4·0 kg if their kinetic energies are equal.

16. A uranium-238 nucleus, initially at rest, emits an alpha particle with a speed of $1{\cdot}4 \times 10^{7}$ m/s. Calculate the recoil speed of the residual nucleus thorium-234. Assume that the mass of a nucleus is proportional to the mass number.

17. A man of mass 50 kg starts moving on the earth and acquires a speed of 1·8 m/s. With what speed does the earth recoil ? Mass of earth = 6×10^{24} kg.

18. A neutron initially at rest, decays into a proton, an electron and an antineutrino. The ejected electron has a momentum of $1{\cdot}4 \times 10^{-26}$ kg-m/s and the antineutrino

$6{\cdot}4 \times 10^{-27}$ kg-m/s. Find the recoil speed of the proton (a) if the electron and the antineutrino are ejected along the same direction and (b) if they are ejected along perpendicular directions. Mass of the proton $= 1{\cdot}67 \times 10^{-27}$ kg.

19. A man of mass M having a bag of mass m slips from the roof of a tall building of height H and starts falling vertically (figure 9-E8). When at a height h from the ground, he notices that the ground below him is pretty hard, but there is a pond at a horizontal distance x from the line of fall. In order to save himself he throws the bag horizontally (with respect to himself) in the direction opposite to the pond. Calculate the minimum horizontal velocity imparted to the bag so that the man lands in the water. If the man just succeeds to avoid the hard ground, where will the bag land?

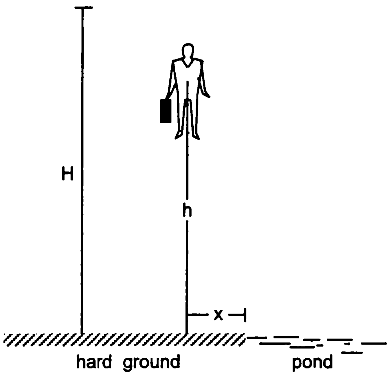


Figure 9-E8

20. A ball of mass 50 g moving at a speed of $2{\cdot}0$ m/s strikes a plane surface at an angle of incidence 45°. The ball is reflected by the plane at equal angle of reflection with the same speed. Calculate (a) the magnitude of the change in momentum of the ball (b) the change in the magnitude of the momentum of the ball.

21. Light in certain cases may be considered as a stream of particles called photons. Each photon has a linear momentum h/λ where h is the Planck's constant and λ is the wavelength of the light. A beam of light of wavelength λ is incident on a plane mirror at an angle of incidence θ. Calculate the change in the linear momentum of a photon as the beam is reflected by the mirror.

22. A block at rest explodes into three equal parts. Two parts start moving along X and Y axes respectively with equal speeds of 10 m/s. Find the initial velocity of the third part.

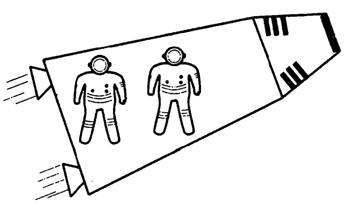


Figure 9-E9

23. Two fat astronauts each of mass 120 kg are travelling in a closed spaceship moving at a speed of 15 km/s in the outer space far removed from all other material objects. The total mass of the spaceship and its contents including the astronauts is 660 kg. If the astronauts do slimming exercise and thereby reduce their masses to 90 kg each, with what velocity will the spaceship move?

24. During a heavy rain, hailstones of average size $1{\cdot}0$ cm in diameter fall with an average speed of 20 m/s. Suppose 2000 hailstones strike every square meter of a $10 \text{ m} \times 10 \text{ m}$ roof perpendicularly in one second and assume that the hailstones do not rebound. Calculate the average force exerted by the falling hailstones on the roof. Density of a hailstone is 900 kg/m^3.

25. A ball of mass m is dropped onto a floor from a certain height. The collision is perfectly elastic and the ball rebounds to the same height and again falls. Find the average force exerted by the ball on the floor during a long time interval.

26. A railroad car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. If the car recoils with a speed v backward on the rails, with what velocity is the man approaching the engine?

27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is $50\,m$ where m is the mass of one shell. If the velocity of the shell with respect to the gun (in its state before firing) is 200 m/s, what is the recoil speed of the car after the second shot? Neglect friction.

28. Two persons each of mass m are standing at the two extremes of a railroad car of mass M resting on a smooth track (figure 9-E10). The person on left jumps to the left with a horizontal speed u with respect to the state of the car before the jump. Thereafter, the other person jumps to the right, again with the same horizontal speed u with respect to the state of the car before his jump. Find the velocity of the car after both the persons have jumped off.

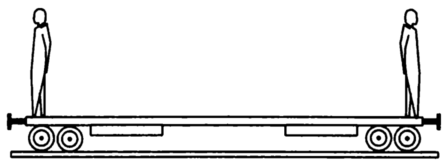


Figure 9-E10

29. Figure (9-E11) shows a small block of mass m which is started with a speed v on the horizontal part of the bigger block of mass M placed on a horizontal floor. The curved part of the surface shown is semicircular. All the surfaces are frictionless. Find the speed of the bigger block when the smaller block reaches the point A of the surface.

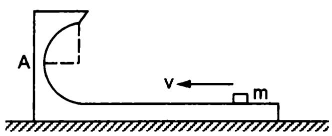


Figure 9-E11

30. In a typical Indian *Bugghi* (a luxury cart drawn by horses), a wooden plate is fixed on the rear on which one person can sit. A bugghi of mass 200 kg is moving at a speed of 10 km/h. As it overtakes a school boy walking at a speed of 4 km/h, the boy sits on the wooden plate. If the mass of the boy is 25 kg, what will be the new velocity of the bugghi ?

31. A ball of mass 0·50 kg moving at a speed of 5·0 m/s collides with another ball of mass 1·0 kg. After the collision the balls stick together and remain motionless. What was the velocity of the 1·0 kg block before the collision ?

32. A 60 kg man skating with a speed of 10 m/s collides with a 40 kg skater at rest and they cling to each other. Find the loss of kinetic energy during the collision.

33. Consider a head-on collision between two particles of masses m_1 and m_2. The initial speeds of the particles are u_1 and u_2 in the same direction. The collision starts at $t = 0$ and the particles interact for a time interval Δt. During the collision, the speed of the first particle varies as

$$v(t) = u_1 + \frac{t}{\Delta t}(v_1 - u_1).$$

Find the speed of the second particle as a function of time during the collision.

34. A bullet of mass m moving at a speed v hits a ball of mass M kept at rest. A small part having mass m' breaks from the ball and sticks to the bullet. The remaining ball is found to move at a speed v_1 in the direction of the bullet. Find the velocity of the bullet after the collision.

35. A ball of mass m moving at a speed v makes a head-on collision with an identical ball at rest. The kinetic energy of the balls after the collision is three fourths of the original. Find the coefficient of restitution.

36. A block of mass 2·0 kg moving at 2·0 m/s collides head on with another block of equal mass kept at rest. (a) Find the maximum possible loss in kinetic energy due to the collision. (b) If the actual loss in kinetic energy is half of this maximum, find the coefficient of restitution.

37. A particle of mass 100 g moving at an initial speed u collides with another particle of same mass kept initially at rest. If the total kinetic energy becomes 0·2 J after the collision, what could be the minimum and the maximum value of u.

38. Two friends A and B (each weighing 40 kg) are sitting on a frictionless platform some distance d apart. A rolls a ball of mass 4 kg on the platform towards B which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back and forth between A and B. The ball has a fixed speed of 5 m/s on the platform. (a) Find the speed of A after he rolls the ball for the first time. (b) Find the speed of A after he catches the ball for the first time. (c) Find the speeds of A and B after the ball has made 5 round trips and is held by A. (d) How many times can A roll the ball ? (e) Where is the centre of mass of the system "A + B + ball" at the end of the nth trip ?

39. A ball falls on the ground from a height of 2·0 m and rebounds up to a height of 1·5 m. Find the coefficient of restitution.

40. In a gamma decay process, the internal energy of a nucleus of mass M decreases, a gamma photon of energy E and linear momentum E/c is emitted and the nucleus recoils. Find the decrease in internal energy.

41. A block of mass 2·0 kg is moving on a frictionless horizontal surface with a velocity of 1·0 m/s (figure 9-E12) towards another block of equal mass kept at rest. The spring constant of the spring fixed at one end is 100 N/m. Find the maximum compression of the spring.

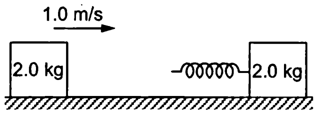


Figure 9-E12

42. A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10·0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the friction coefficient between the block and the surface (figure 9-E13).

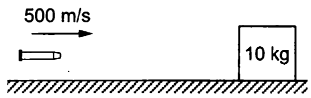


Figure 9-E13

43. A projectile is fired with a speed u at an angle θ above a horizontal field. The coefficient of restitution of collision between the projectile and the field is e. How far from the starting point, does the projectile makes its second collision with the field ?

44. A ball falls on an inclined plane of inclination θ from a height h above the point of impact and makes a perfectly elastic collision. Where will it hit the plane again ?

45. Solve the previous problem if the coefficient of restitution is e. Use θ = 45°, $e = \frac{3}{4}$ and h = 5 m.

46. A block of mass 200 g is suspended through a vertical spring. The spring is stretched by 1·0 cm when the block is in equilibrium. A particle of mass 120 g is dropped on the block from a height of 45 cm. The particle sticks to the block after the impact. Find the maximum extension of the spring. Take g = 10 m/s^2.

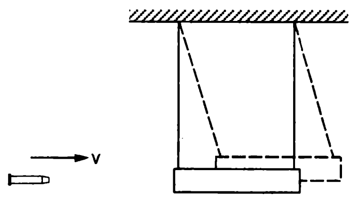


Figure 9-E14

47. A bullet of mass 25 g is fired horizontally into a ballistic pendulum of mass 5·0 kg and gets embedded in it (figure 9-E14). If the centre of the pendulum rises by a distance of 10 cm, find the speed of the bullet.

48. A bullet of mass 20 g moving horizontally at a speed of 300 m/s is fired into a wooden block of mass 500 g suspended by a long string. The bullet crosses the block and emerges on the other side. If the centre of mass of the block rises through a height of 20·0 cm, find the speed of the bullet as it emerges from the block.

49. Two masses m_1 and m_2 are connected by a spring of spring constant k and are placed on a frictionless horizontal surface. Initially the spring is stretched through a distance x_0 when the system is released from rest. Find the distance moved by the two masses before they again come to rest.

50. Two blocks of masses m_1 and m_2 are connected by a spring of spring constant k (figure 9-E15). The block of mass m_2 is given a sharp impulse so that it acquires a velocity v_0 towards right. Find (a) the velocity of the centre of mass, (b) the maximum elongation that the spring will suffer.

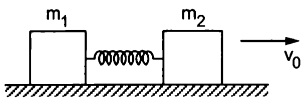


Figure 9-E15

51. Consider the situation of the previous problem. Suppose each of the blocks is pulled by a constant force F instead of any impulse. Find the maximum elongation that the spring will suffer and the distances moved by the two blocks in the process.

52. Consider the situation of the previous problem. Suppose the block of mass m_1 is pulled by a constant force F_1 and the other block is pulled by a constant force F_2. Find the maximum elongation that the spring will suffer.

53. Consider a gravity-free hall in which an experimenter of mass 50 kg is resting on a 5 kg pillow, 8 ft above the floor of the hall. He pushes the pillow down so that it starts falling at a speed of 8 ft/s. The pillow makes a perfectly elastic collision with the floor, rebounds and reaches the experimenter's head. Find the time elapsed in the process.

54. The track shown in figure (9-E16) is frictionless. The block B of mass $2m$ is lying at rest and the block A of mass m is pushed along the track with some speed. The collision between A and B is perfectly elastic. With what velocity should the block A be started to get the sleeping man awakened ?

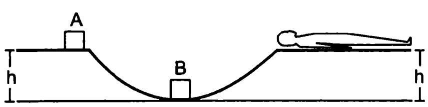


Figure 9-E16

55. A bullet of mass 10 g moving horizontally at a speed of $50\sqrt{7}$ m/s strikes a block of mass 490 g kept on a frictionless track as shown in figure (9-E17). The bullet remains inside the block and the system proceeds towards the semicircular track of radius 0·2 m. Where will the block strike the horizontal part after leaving the semicircular track ?

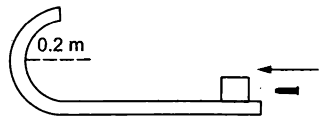


Figure 9-E17

56. Two balls having masses m and $2m$ are fastened to two light strings of same length l (figure 9-E18). The other ends of the strings are fixed at O. The strings are kept in the same horizontal line and the system is released from rest. The collision between the balls is elastic. (a) Find the velocities of the balls just after their collision. (b) How high will the balls rise after the collision ?

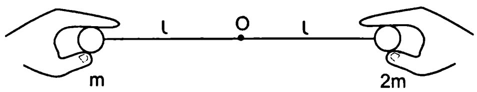


Figure 9-E18

57. A uniform chain of mass M and length L is held vertically in such a way that its lower end just touches the horizontal floor. The chain is released from rest in this position. Any portion that strikes the floor comes to rest. Assuming that the chain does not form a heap on the floor, calculate the force exerted by it on the floor when a length x has reached the floor.

58. The blocks shown in figure (9-E19) have equal masses. The surface of A is smooth but that of B has a friction coefficient of 0·10 with the floor. Block A is moving at a speed of 10 m/s towards B which is kept at rest. Find the distance travelled by B if (a) the collision is perfectly elastic and (b) the collision is perfectly inelastic. Take $g = 10\ \text{m/s}^2$.

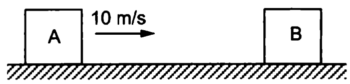


Figure 9-E19

59. The friction coefficient between the horizontal surface and each of the blocks shown in figure (9-E20) is 0·20. The collision between the blocks is perfectly elastic. Find the separation between the two blocks when they come to rest. Take $g = 10\ \text{m/s}^2$.

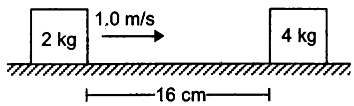


Figure 9-E20

60. A block of mass m is placed on a triangular block of mass M, which in turn is placed on a horizontal surface as shown in figure (9-E21). Assuming frictionless surfaces find the velocity of the triangular block when the smaller block reaches the bottom end.

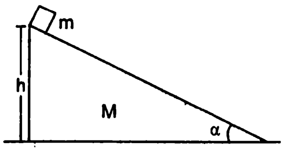


Figure 9-E21

61. Figure (9-E22) shows a small body of mass m placed over a larger mass M whose surface is horizontal near the smaller mass and gradually curves to become vertical. The smaller mass is pushed on the longer one at a speed v and the system is left to itself. Assume that all the surfaces are frictionless. (a) Find the speed of the larger block when the smaller block is sliding on the vertical part. (b) Find the speed of the smaller mass when it breaks off the larger mass at height h. (c) Find the maximum height (from the ground) that the smaller mass ascends. (d) Show that the smaller mass will again land on the bigger one. Find the distance traversed by the bigger block during the time when the smaller block was in its flight under gravity.

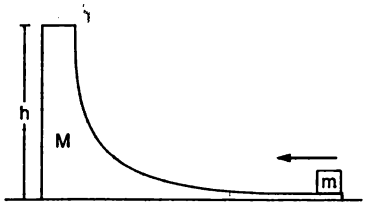


Figure 9-E22

62. A small block of superdense material has a mass of 3×10^{24} kg. It is situated at a height h (much smaller than the earth's radius) from where it falls on the earth's surface. Find its speed when its height from the earth's surface has reduced to $h/2$. The mass of the earth is 6×10^{24} kg.

63. A body of mass m makes an elastic collision with another identical body at rest. Show that if the collision is not head-on, the bodies go at right angle to each other after the collision.

64. A small particle travelling with a velocity v collides elastically with a spherical body of equal mass and of radius r initially kept at rest. The centre of this spherical body is located a distance $\rho(< r)$ away from the direction of motion of the particle (figure 9-E23). Find the final velocities of the two particles.

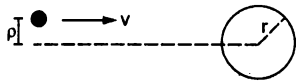


Figure 9-E23

[**Hint :** The force acts along the normal to the sphere through the contact. Treat the collision as one-dimensional for this direction. In the tangential direction no force acts and the velocities do not change].

□

ANSWERS

OBJECTIVE I

1. (c)	2. (d)	3. (d)	4. (d)	5. (b)	6. (b)
7. (b)	8. (b)	9. (c)	10. (d)	11. (c)	12. (b)
13. (c)	14. (c)	15. (d)	16. (d)	17. (a)	18. (a)
19. (b)					

OBJECTIVE II

1. none	2. (c), (d)	3. (a), (b)
4. (b), (c)	5. (b), (d)	6. (d)
7. (a), (d)	8. (b), (c), (d)	9. (c), (d)
10. (b), (c)	11. all	

EXERCISES

1. Taking AB as the x-axis and A as the origin, the centre of mass is at $(7/12 \text{ m}, \sqrt{3}/4 \text{ m})$

2. $6{\cdot}6 \times 10^{-12}$ m

3. 22 L/35

4. At R/5 from the centre of the bigger disc towards the centre of the smaller disc

5. At R/3 from the centre of the original disc away from the centre of the hole

6. $\dfrac{4d}{4+\pi}$ right to the centre of the disc

7. 0·20 m/s at 45° below the direction towards right

8. 1 cm

9. 1 cm downward

10. zero

11. $\dfrac{4(R_1^2 + R_1R_2 + R_2^2)}{3\pi(R_1 + R_2)}$ above the centre

12. 13 cm

13. $mL/(m + M)$

14. $mL/(m + M)$

15. 1 : 2

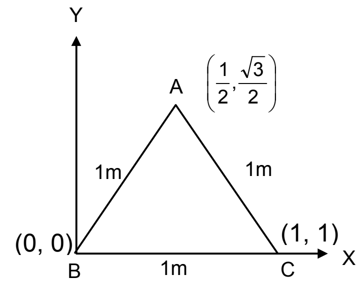
16. $2{\cdot}4 \times 10^5$ m/s

17. $1{\cdot}5 \times 10^{-23}$ m/s
18. (a) 12·2 m/s (b) 9·2 m/s
19. $\dfrac{Mx\sqrt{g}}{m\,[\sqrt{2H} - \sqrt{2(H-h)}]}$, Mx/m left to the line of fall
20. (a) 0·14 kg-m/s (b) zero
21. $2h\cos\theta/\lambda$
22. $10\sqrt{2}$ m/s 135° below the X-axis
23. 15 km/s
24. 1900 N
25. mg
26. $\left(1+\dfrac{M}{m}\right)v$
27. $200\left(\dfrac{1}{49}+\dfrac{1}{48}\right)$ m/s
28. $\dfrac{m^2u}{M(M+m)}$ towards left
29. $\dfrac{mv}{M+m}$
30. $\dfrac{28}{3}$ km/h
31. 2·5 m/s opposite to the direction of motion of the first ball
32. 1200 J
33. $u_2 - \dfrac{m_1}{m_2}\dfrac{t}{\Delta t}(v_1 - u_1)$
34. $\dfrac{mv-(M-m')v_1}{m+m'}$ in the initial direction
35. $1/\sqrt{2}$
36. $2\,\text{J}$, $1/\sqrt{2}$
37. 2 m/s, $2\sqrt{2}$ m/s
38. (a) 0·5 m/s (b) $\dfrac{10}{11}$ m/s (c) $\dfrac{50}{11}$ m/s, 5 m/s (d) 6 (e) $\dfrac{10}{21}d$ away from the initial position of A towards B
39. $\sqrt{3}/2$
40. $E + \dfrac{E^2}{2Mc^2}$
41. 10 cm
42. 0·16
43. $\dfrac{(1+e)\,u^2\sin 2\theta}{g}$
44. $8h\sin\theta$ along the incline
45. 18·5 m along the incline
46. 6·1 cm
47. 280 m/s
48. 250 m/s
49. $\dfrac{2m_2x_0}{m_1+m_2}$, $\dfrac{2m_1x_0}{m_1+m_2}$
50. (a) $\dfrac{m_2v_0}{m_1+m_2}$ (b) $v_0\left[\dfrac{m_1m_2}{(m_1+m_2)k}\right]^{1/2}$
51. $2F/k$, $\dfrac{2Fm_2}{k(m_1+m_2)}$, $\dfrac{2Fm_1}{k(m_1+m_2)}$
52. $\dfrac{2(m_1F_2+m_2F_1)}{k(m_1+m_2)}$
53. 2·22 s
54. Greater than $\sqrt{2{\cdot}5\,gh}$
55. At the junction of the straight and the curved parts
56. (a) Light ball $\dfrac{\sqrt{50\,gl}}{3}$ towards left, heavy ball $\dfrac{\sqrt{2\,gl}}{3}$ towards right (b) Light ball $2l$ and heavy ball $\dfrac{l}{9}$
57. $3\,Mgx/L$
58. (a) 50 m (b) 25 m
59. 5 cm
60. $\left[\dfrac{2m^2gh\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$
61. (a) $\dfrac{mv}{M+m}$ (b) $\left[\dfrac{(M^2+Mm+m^2)}{(M+m)^2}v^2 - 2gh\right]^{1/2}$
(c) $\dfrac{Mv^2}{2g(M+m)}$ (d) $\dfrac{2mv[Mv^2-2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$
62. $\sqrt{\dfrac{2gh}{3}}$
64. The small particle goes along the tangent with a speed of $v\rho/r$ and the spherical body goes perpendicular to the smaller particle with a speed of $\dfrac{v}{r}\sqrt{r^2-\rho^2}$

□

SOLUTIONS TO CONCEPTS
CHAPTER 9

1. $m_1 = 1kg$, $m_2 = 2kg$, $m_3 = 3kg$,
$x_1 = 0$, $x_2 = 1$, $x_3=1/2$
$y_1 = 0$, $y_2 = 0$, $y_3 = \sqrt{3}/2$
The position of centre of mass is

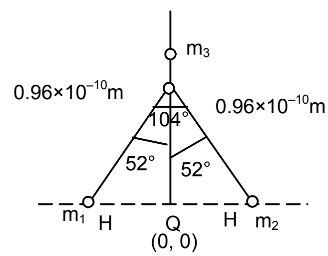


$$C.M = \left(\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}, \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}\right)$$

$$= \left(\frac{(1\times 0)+(2\times 1)+(3\times 1/2)}{1+2+3}, \frac{(1\times 0)+(2\times 0)+(3\times(\sqrt{3}/2))}{1+2+3}\right)$$

$= \left(\frac{7}{12}, \frac{3\sqrt{3}}{12}\right)$ from the point B.

2. Let θ be the origin of the system
In the above figure
$m_1 = 1gm$, $x_1 = -(0.96\times10^{-10})\sin 52°$ $y_1 = 0$
$m_2 = 1gm$, $x_2 = -(0.96\times10^{-10})\sin 52°$ $y_2 = 0$
$x_3 = 0$ $y_3 = (0.96 \times 10^{-10})\cos 52°$
The position of centre of mass

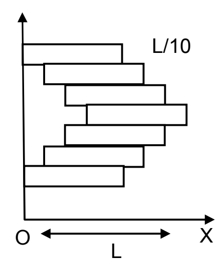


$$\left(\frac{m_1x_1+m_2x_2+m_3x_3}{m_1+m_2+m_3}, \frac{m_1y_1+m_2y_2+m_3y_3}{m_1+m_2+m_3}\right)$$

$$= \left(\frac{-(0.96\times10^{-10})\times\sin 52+(0.96\times10^{-10})\sin 52+16\times 0}{1+1+16}, \frac{0+0+16y_3}{18}\right)$$

$$= \left(0,\ (8/9)0.96\times10^{-10}\cos 52^{\circ}\right)$$

3. Let 'O' (0,0) be the origin of the system.
Each brick is mass 'M' & length 'L'.
Each brick is displaced w.r.t. one in contact by 'L/10'
∴The X coordinate of the centre of mass

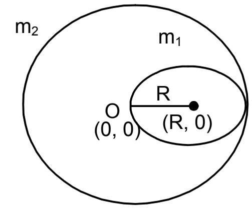


$$\overline{X}_{cm} = \frac{m\left(\frac{L}{2}\right)+m\left(\frac{L}{2}+\frac{L}{10}\right)+m\left(\frac{L}{2}+\frac{2L}{10}\right)+m\left(\frac{L}{2}+\frac{3L}{10}\right)+m\left(\frac{L}{2}+\frac{3L}{10}-\frac{L}{10}\right)+m\left(\frac{L}{2}+\frac{L}{10}\right)+m\left(\frac{L}{2}\right)}{7m}$$

$$= \frac{\frac{L}{2}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{3L}{10}+\frac{L}{2}+\frac{L}{5}+\frac{L}{2}+\frac{L}{10}+\frac{L}{2}}{7}$$

$$= \frac{\frac{7L}{2}+\frac{5L}{10}+\frac{2L}{5}}{7} = \frac{35L+5L+4L}{10\times 7} = \frac{44L}{70} = \frac{11}{35}L$$

4. Let the centre of the bigger disc be the origin.
2R = Radius of bigger disc
R = Radius of smaller disc
$m_1 = \pi R^2 \times T \times \rho$
$m_2 = \pi(2R)^2$ I $T \times \rho$
where T = Thickness of the two discs
ρ = Density of the two discs
∴ The position of the centre of mass



$$\left(\frac{m_1x_1+m_2x_2}{m_1+m_2}, \frac{m_1y_1+m_2y_2}{m_1+m_2}\right)$$

$x_1 = R \qquad y_1 = 0$

$x_2 = 0 \qquad y_2 = 0$

$$\left(\frac{\pi R^2 T\rho R + 0}{\pi R^2 T\rho + \pi(2R)^2 T\rho}, \frac{0}{m_1+m_2}\right) = \left(\frac{\pi R^2 T\rho R}{5\pi R^2 T\rho}, 0\right) = \left(\frac{R}{5}, 0\right)$$

At R/5 from the centre of bigger disc towards the centre of smaller disc.

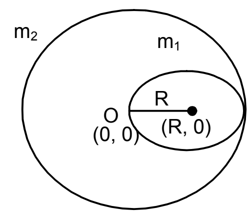
5. Let '0' be the origin of the system.

R = radius of the smaller disc

2R = radius of the bigger disc

The smaller disc is cut out from the bigger disc

As from the figure



$m_1 = \pi R^2 T\rho \qquad x_1 = R \qquad y_1 = 0$

$m_2 = \pi(2R)^2 T\rho \qquad x_2 = 0 \qquad y_2 = 0$

The position of C.M. = $\left(\frac{-\pi R^2 T\rho R + 0}{-\pi R^2 T\rho + \pi(2R)^2 T\rho R}, \frac{0+0}{m_1+m_2}\right)$

$$= \left(\frac{-\pi R^2 T\rho R}{3\pi R^2 T\rho}, 0\right) = \left(-\frac{R}{3}, 0\right)$$

C.M. is at R/3 from the centre of bigger disc away from centre of the hole.

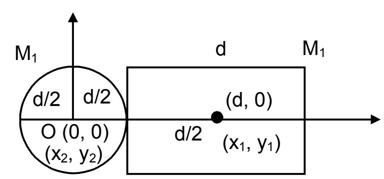
6. Let m be the mass per unit area.

$\therefore$ Mass of the square plate = $M_1 = d^2 m$

Mass of the circular disc = $M_2 = \frac{\pi d^2}{4} m$

Let the centre of the circular disc be the origin of the system.

$\therefore$ Position of centre of mass



$$= \left(\frac{d^2md + \pi(d^2/4)m \times 0}{d^2m + \pi(d^2/4)m}, \frac{0+0}{M_1+M_2}\right) = \left(\frac{d^3 m}{d^2 m\left(1+\frac{\pi}{4}\right)}, 0\right) = \left(\frac{4d}{\pi+4}, 0\right)$$

The new centre of mass is $\left(\frac{4d}{\pi+4}\right)$ right of the centre of circular disc.

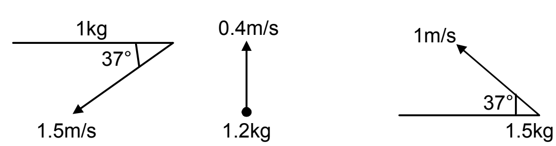
7. m_1 = 1kg. $\qquad \vec{v}_1 = -1.5 \cos 37\ \hat{i} - 1.55 \sin 37\ \hat{j} = -1.2\,\hat{i} - 0.9\,\hat{j}$

m_2 = 1.2kg. $\qquad \vec{v}_2 = 0.4\,\hat{j}$

m_3 = 1.5kg $\qquad \vec{v}_3 = -0.8\,\hat{i} + 0.6\,\hat{j}$

m_4 = 0.5kg $\qquad \vec{v}_4 = 3\,\hat{i}$

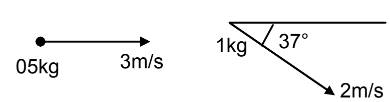
m_5 = 1kg $\qquad \vec{v}_5 = 1.6\,\hat{i} - 1.2\,\hat{j}$



So, $\vec{v}_c = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + m_4\vec{v}_4 + m_5\vec{v}_5}{m_1+m_2+m_3+m_4+m_5}$

$$= \frac{1(-1.2\hat{i} - 0.9\hat{j}) + 1.2(0.4\hat{j}) + 1.5(-0.8\hat{i} + 0.6\hat{j}) + 0.5(3\hat{i}) + 1(1.6\hat{i} - 1.2\hat{j})}{5.2}$$

$$= \frac{-1.2\hat{i} - 0.9\hat{j} + 4.8\hat{j} - 1.2\hat{i} + .90\hat{j} + 1.5\hat{i} + 1.6\hat{i} - 1.2\hat{j}}{5.2}$$



$$= \frac{0.7\hat{i}}{5.2} - \frac{0.72\hat{j}}{5.2}$$

8. Two masses m_1 & m_2 are placed on the X-axis
$m_1 = 10$ kg, $m_2 = 20$kg.
The first mass is displaced by a distance of 2 cm

$$\therefore \overline{X}_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{10 \times 2 + 20x_2}{30}$$

$$\Rightarrow 0 = \frac{20 + 20x_2}{30} \Rightarrow 20 + 20x_2 = 0$$

$\Rightarrow 20 = -20x_2 \Rightarrow x_2 = -1$.
$\therefore$ The 2nd mass should be displaced by a distance 1cm towards left so as to kept the position of centre of mass unchanged.

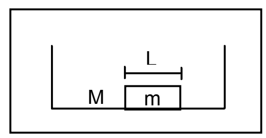
9. Two masses m_1 & m_2 are kept in a vertical line
$m_1 = 10$kg, $m_2 = 30$kg
The first block is raised through a height of 7 cm.
The centre of mass is raised by 1 cm.

$$\therefore 1 = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{10 \times 7 + 30y_2}{40}$$

$$\Rightarrow 1 = \frac{70 + 30y_2}{40} \Rightarrow 70 + 30y_2 = 40 \Rightarrow 30y_2 = -30 \Rightarrow y_2 = -1.$$

The 30 kg body should be displaced 1cm downward inorder to raise the centre of mass through 1 cm.

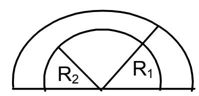
10. As the hall is gravity free, after the ice melts, it would tend to acquire a spherical shape. But, there is no external force acting on the system. So, the centre of mass of the system would not move.



11. The centre of mass of the blate will be on the symmetrical axis.

$$\Rightarrow \overline{y}_{cm} = \frac{\left(\frac{\pi R_2^2}{2}\right)\left(\frac{4R_2}{3\pi}\right) - \left(\frac{\pi R_1^2}{2}\right)\left(\frac{4R_1}{3\pi}\right)}{\frac{\pi R_2^2}{2} - \frac{\pi R_1^2}{2}}$$

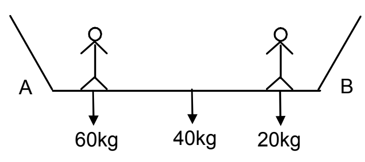
$$= \frac{(2/3)R_2^3 - (2/3)R_1^3}{\pi/2(R_2^2 - R_1^2)} = \frac{4}{3\pi}\frac{(R_2 - R_1)(R_2^2 + R_1^2 + R_1R_2)}{(R_2 - R_1)(R_2 + R_1)}$$



$$= \frac{4}{3\pi}\frac{(R_2^2 + R_1^2 + R_1R_2)}{R_1 + R_2}$$ above the centre.

12. $m_1 = 60$kg, $m_2 = 40$kg , $m_3 = 50$kg,
Let A be the origin of the system.
Initially Mr. Verma & Mr. Mathur are at extreme position of the boat.
$\therefore$ The centre of mass will be at a distance

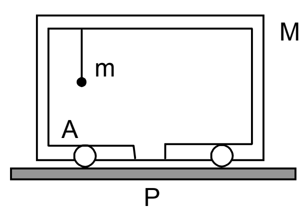
$$= \frac{60 \times 0 + 40 \times 2 + 50 \times 4}{150} = \frac{280}{150} = 1.87\text{m from 'A'}$$



When they come to the mid point of the boat the CM lies at 2m from 'A'.
$\therefore$ The shift in CM = 2 – 1.87 = 0.13m towards right.
But as there is no external force in longitudinal direction their CM would not shift.
So, the boat moves 0.13m or 13 cm towards right.

13. Let the bob fall at A,. The mass of bob = m.
The mass of cart = M.
Initially their centre of mass will be at

$$\frac{m \times L + M \times 0}{M + m} = \left(\frac{m}{M + m}\right)L$$



Distance from P
When, the bob falls in the slot the CM is at a distance 'O' from P.

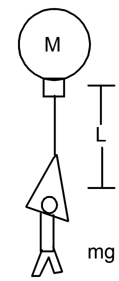
Shift in CM = $0 - \frac{mL}{M+m} = -\frac{mL}{M+m}$ towards left

$= \frac{mL}{M+m}$ towards right.

But there is no external force in horizontal direction.

So the cart displaces a distance $\frac{mL}{M+m}$ towards right.

14. Initially the monkey & balloon are at rest.
    So the CM is at 'P'
    When the monkey descends through a distance 'L'
    The CM will shift



$t_o = \frac{m \times L + M \times 0}{M+m} = \frac{mL}{M+m}$ from P

So, the balloon descends through a distance $\frac{mL}{M+m}$

15. Let the mass of the to particles be m_1 & m_2 respectively
    m_1 = 1kg, m_2 = 4kg
    ∴ According to question
    $\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2^2}{v_1^2} \Rightarrow \frac{v_2}{v_1} = \sqrt{\frac{m_1}{m_2}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}}$

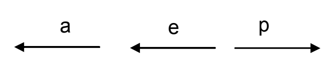
Now, $\frac{m_1 v_1}{m_2 v_2} = \frac{m_1}{m_2} \times \sqrt{\frac{m_2}{m_1}} = \frac{\sqrt{m_1}}{\sqrt{m_2}} = \frac{\sqrt{1}}{\sqrt{4}}$ = 1/2

$\Rightarrow \frac{m_1 v_1}{m_2 v_2}$ = 1 : 2

16. As uranium 238 nucleus emits a α-particle with a speed of 1.4×10^7m/sec. Let v_2 be the speed of the residual nucleus thorium 234.
    $\therefore m_1 v_1 = m_2 v_2$
    $\Rightarrow 4 \times 1.4 \times 10^7 = 234 \times v_2$
    $\Rightarrow v_2 = \frac{4 \times 1.4 \times 10^7}{234} = 2.4 \times 10^5$ m/sec.

17. $m_1 v_1 = m_2 v_2$
    $\Rightarrow 50 \times 1.8 = 6 \times 10^{24} \times v_2$
    $\Rightarrow v_2 = \frac{50 \times 1.8}{6 \times 10^{24}} = 1.5 \times 10^{-23}$ m/sec
    so, the earth will recoil at a speed of 1.5×10^{-23}m/sec.

18. Mass of proton = 1.67×10^{-27}
    Let 'V_p' be the velocity of proton



Given momentum of electron = 1.4×10^{-26}kg m/sec
Given momentum of antineutrino = 6.4×10^{-27} kg m/sec

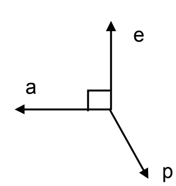
a) The electron & the antineutrino are ejected in the same direction. As the total momentum is conserved the proton should be ejected in the opposite direction.

$1.67 \times 10^{-27} \times V_p = 1.4 \times 10^{-26} + 6.4 \times 10^{-27} = 20.4 \times 10^{-27}$

$\Rightarrow V_p$ = (20.4 /1.67) = 12.2 m/sec in the opposite direction.

b) The electron & antineutrino are ejected $\perp^r$ to each other.

Total momentum of electron and antineutrino,

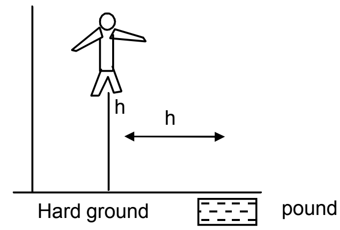


$= \sqrt{(14)^2 + (6.4)^2} \times 10^{-27}$ kg m/s = 15.4×10^{-27} kg m/s

Since, $1.67 \times 10^{-27} V_p = 15.4 \times 10^{-27}$ kg m/s

So V_p = 9.2 m/s

19. Mass of man = M, Initial velocity = 0
Mass of bad = m
Let the throws the bag towards left with a velocity v towards left. So, there is no external force in the horizontal direction.
The momentum will be conserved. Let he goes right with a velocity



$mv = MV \Rightarrow V = \frac{mv}{M} \Rightarrow v = \frac{MV}{m}$ ..(i)

Let the total time he will take to reach ground = $\sqrt{2H/g} = t_1$

Let the total time he will take to reach the height h = $t_2 = \sqrt{2(H-h)/g}$

Then the time of his flying = $t_1 - t_2 = \sqrt{2H/g} - \sqrt{2(H-h)/g} = \sqrt{2/g}\left(\sqrt{H} - \sqrt{H-h}\right)$

Within this time he reaches the ground in the pond covering a horizontal distance x

$\Rightarrow x = V \times t \Rightarrow V = x / t$

$\therefore v = \frac{M}{m}\frac{x}{t} = \frac{M}{m} \times \frac{\sqrt{g}}{\sqrt{2}\left(\sqrt{H} - \sqrt{H-h}\right)}$

As there is no external force in horizontal direction, the x-coordinate of CM will remain at that position.

$\Rightarrow 0 = \frac{M \times (x) + m \times x_1}{M + m} \Rightarrow x_1 = -\frac{M}{m}x$

$\therefore$ The bag will reach the bottom at a distance (M/m) x towards left of the line it falls.

20. Mass = 50g = 0.05kg

$v = 2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

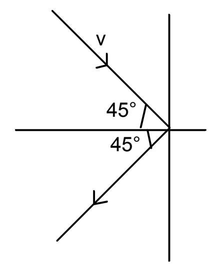
$v_1 = -2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}$

a) change in momentum = $m\vec{v} - m\vec{v}_1$

$= 0.05\,(2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j}) - 0.05\,(-2\cos 45°\,\hat{i} - 2\sin 45°\,\hat{j})$

$= 0.1\cos 45°\,\hat{i} - 0.1\sin 45°\,\hat{j} + 0.1\cos 45°\,\hat{i} + 0.1\sin 45°\,\hat{j}$

$= 0.2\cos 45°\,\hat{i}$



$\therefore$ magnitude = $\sqrt{\left(\frac{0.2}{\sqrt{2}}\right)^2} = \frac{0.2}{\sqrt{2}}$ = 0.14 kg m/s

c) The change in magnitude of the momentum of the ball

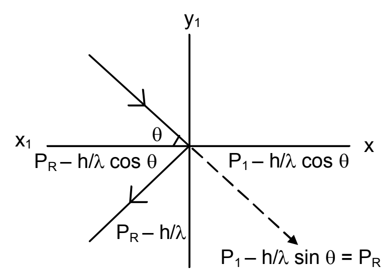
$-\left|\vec{P}_i\right| - \left|\vec{P}_f\right| = 2 \times 0.5 - 2 \times 0.5 = 0.$

21. $\vec{P}_{incidence} = (h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

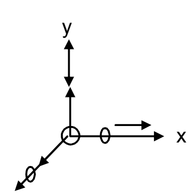
$P_{Reflected} = -(h/\lambda)\cos\theta\,\hat{i} - (h/\lambda)\sin\theta\,\hat{j}$

The change in momentum will be only in the x-axis direction. i.e.

$|\Delta P| = (h/\lambda)\cos\theta - ((h/\lambda)\cos\theta) = (2h/\lambda)\cos\theta$



22. As the block is exploded only due to its internal energy. So net external force during this process is 0. So the centre mass will not change.
Let the body while exploded was at the origin of the co-ordinate system.
If the two bodies of equal mass is moving at a speed of 10m/s in + x & +y axis direction respectively,



$\sqrt{10^2 + 10^2 + 2 10.10\cos 90^o} = 10\sqrt{2}$ m/s 45° w.r.t. + x axis

If the centre mass is at rest, then the third mass which have equal mass with other two, will move in the opposite direction (i.e. 135° w.r.t. + x- axis) of the resultant at the same velocity.

23. Since the spaceship is removed from any material object & totally isolated from surrounding, the missions by astronauts couldn't slip away from the spaceship. So the total mass of the spaceship remain unchanged and also its velocity.

24. d = 1cm, v = 20 m/s, u = 0, ρ = 900 kg/m^3 = 0.9gm/cm^3
volume = $(4/3)\pi r^3 = (4/3)\ \pi\ (0.5)^3 = 0.5238\text{cm}^3$
$\therefore$ mass = vρ = 0.5238 × 0.9 = 0.4714258gm
$\therefore$ mass of 2000 hailstone = 2000 × 0.4714 = 947.857
$\therefore$ Rate of change in momentum per unit area = 947.857 × 2000 = 19N/m^3
$\therefore$ Total force exerted = 19 × 100 = 1900 N.

25. A ball of mass m is dropped onto a floor from a certain height let 'h'.
$\therefore v_1 = \sqrt{2gh}$, $v_1 = 0$, $v_2 = -\sqrt{2gh}$ & $v_2 = 0$
$\therefore$ Rate of change of velocity :-
$$F = \frac{m \times 2\sqrt{2gh}}{t}$$
$\therefore v = \sqrt{2gh}$, s = h, v = 0
$\Rightarrow$ v = u + at
$$\Rightarrow \sqrt{2gh} = g\,t \Rightarrow t = \sqrt{\frac{2h}{g}}$$
$$\therefore \text{Total time } 2\sqrt{\frac{2h}{t}}$$
$$\therefore F = \frac{m \times 2\sqrt{2gh}}{2\sqrt{\frac{2h}{g}}} = mg$$

26. A railroad car of mass M is at rest on frictionless rails when a man of mass m starts moving on the car towards the engine. The car recoils with a speed v backward on the rails.
Let the mass is moving with a velocity x w.r.t. the engine.
$\therefore$The velocity of the mass w.r.t earth is (x – v) towards right
V_{cm} = 0 (Initially at rest)
$\therefore$ 0 = –Mv + m(x – v)
$$\Rightarrow Mv = m(x - v) \Rightarrow mx = Mv + mv \Rightarrow x = \left(\frac{M+m}{m}\right)v \Rightarrow x = \left(1 + \frac{M}{m}\right)v$$

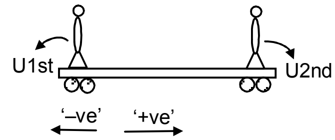
27. A gun is mounted on a railroad car. The mass of the car, the gun, the shells and the operator is 50m where m is the mass of one shell. The muzzle velocity of the shells is 200m/s.
Initial, V_{cm} = 0.
$$\therefore 0 = 49\ m \times V + m \times 200 \Rightarrow V = \frac{-200}{49}\ \text{m/s}$$
$\therefore \frac{200}{49}$ m/s towards left.
When another shell is fired, then the velocity of the car, with respect to the platform is,
$\Rightarrow$ V\` = $\frac{200}{49}$ m/s towards left.
When another shell is fired, then the velocity of the car, with respect to the platform is,
$\Rightarrow$ v\` = $\frac{200}{48}$ m/s towards left
$\therefore$ Velocity of the car w.r.t the earth is $\left(\frac{200}{49} + \frac{200}{48}\right)$ m/s towards left.

28. Two persons each of mass m are standing at the two extremes of a railroad car of mass m resting on a smooth track.
Case – I
Let the velocity of the railroad car w.r.t the earth is V after the jump of the left man.
$\therefore$ 0 = – mu + (M + m) V

$\Rightarrow$ V = $\dfrac{mu}{M+m}$ towards right

Case – II

When the man on the right jumps, the velocity of it w.r.t the car is u.



$\therefore$ **0 = mu – Mv'**

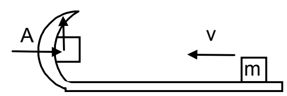
$\Rightarrow$ **v′ =** $\dfrac{mu}{M}$

(V′ is the change is velocity of the platform when platform itself is taken as reference assuming the car to be at rest)

$\therefore$ So, net velocity towards left (i.e. the velocity of the car w.r.t. the earth)

$= \dfrac{mv}{M} - \dfrac{mv}{M+m} = \dfrac{mMu + m^2v - Mmu}{M(M+m)} = \dfrac{m^2v}{M(M+m)}$

29. A small block of mass m which is started with a velocity V on the horizontal part of the bigger block of mass M placed on a horizontal floor.
    Since the small body of mass m is started with a velocity V in the horizontal direction, so the total initial momentum at the initial position in the horizontal direction will remain same as the total final momentum at the point A on the bigger block in the horizontal direction.
    From L.C.K. m:



    mv + M×O = (m + M) v $\Rightarrow$ v′ = $\dfrac{mv}{M+m}$

30. Mass of the boggli = 200kg, V_B = 10 km/hour.
    $\therefore$ Mass of the boy = 2.5kg & V_{Boy} = 4km/hour.
    If we take the boy & boggle as a system then total momentum before the process of sitting will remain constant after the process of sitting.
    $\therefore$ $m_b V_b = m_{boy}V_{boy} = (m_b + m_{boy})$ v
    $\Rightarrow$ 200 × 10 + 25 × 4 = (200 +25) × v
    $\Rightarrow$ v = $\dfrac{2100}{225} = \dfrac{28}{3}$ = 9.3 m/sec

31. Mass of the ball = m_1 = 0.5kg, velocity of the ball = 5m/s
    Mass of the another ball m_2 = 1kg
    Let it's velocity = v′ m/s
    Using law of conservation of momentum,
    0.5 × 5 + 1 × v′ = 0 $\Rightarrow$ v′ = – 2.5
    $\therefore$ Velocity of second ball is 2.5 m/s opposite to the direction of motion of 1^{st} ball.

32. Mass of the man = m_1 = 60kg
    Speed of the man = v_1 = 10m/s
    Mass of the skater = m_2 = 40kg
    let its velocity = v′
    $\therefore$ 60 × 10 + 0 = 100 × v′ $\Rightarrow$ v′ = 6m/s
    loss in K.E. = (1/2)60 ×$(10)^2$ – (1/2)× 100 × 36 = 1200 J

33. Using law of conservation of momentum.
    $m_1u_1 + m_2u_2 = m_1v(t) + m_2v'$
    Where v′ = speed of 2^{nd} particle during collision.
    $\Rightarrow m_1u_1 + m_2u_2 = m_1u_1 + m_1 + (t/\Delta t)(v_1 - u_1) + m_2v'$
    $\Rightarrow \dfrac{m_2u_2}{m^2} - \dfrac{m_1}{m2}\dfrac{t}{\Delta t}(v_1 - u_1)v'$
    $\therefore$ v′ = $u_2 - \dfrac{m_1}{m_2}\dfrac{t}{\Delta t}(v_1 - u)$

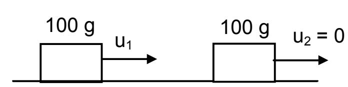
34. Mass of the bullet = m and speed = v
    Mass of the ball = M
    m′ = frictional mass from the ball.

Using law of conservation of momentum,
$mv + 0 = (m' + m)\,v' + (M - m')\,v_1$
where v' = final velocity of the bullet + frictional mass
$\Rightarrow v' = \dfrac{mv - (M + m')V_1}{m + m'}$

35. Mass of 1st ball = m and speed = v
Mass of 2nd ball = m
Let final velocities of 1st and 2nd ball are v_1 and v_2 respectively
Using law of conservation of momentum,
$m(v_1 + v_2) = mv.$
$\Rightarrow v_1 + v_2 = v$ …(1)
Also
$v_1 - v_2 = ev$ …(2)
Given that final K.E. = ¾ Initial K.E.
$\Rightarrow \frac{1}{2} mv_1^2 + \frac{1}{2} mv_2^2 = \frac{3}{4} \times \frac{1}{2} mv^2$
$\Rightarrow v_1^2 + v_2^2 = \frac{3}{4} v^2$
$\Rightarrow \dfrac{(v_1 + v_2)^2 + (v_1 - v_2)^2}{2} = \dfrac{3}{4} v^2$
$\Rightarrow \dfrac{(1 + e^2)v^2}{2} = \dfrac{3}{4} v^2 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

36. Mass of block = 2kg and speed = 2m/s
Mass of 2nd block = 2kg.
Let final velocity of 2nd block = v
using law of conservation of momentum.
$2 \times 2 = (2 + 2)\,v \Rightarrow v = 1$m/s
$\therefore$ Loss in K.E. in inelastic collision
$= (1/2) \times 2 \times (2)^2\, v - (1/2)\,(2 + 2) \times (1)^2 = 4 - 2 = 2$ J
b) Actual loss = $\dfrac{\text{Maximum loss}}{2} = 1$J
$(1/2) \times 2 \times 2^2 - (1/2)\, 2 \times v_1^2 + (1/2) \times 2 \times v_2^2 = 1$
$\Rightarrow 4 - (v_1^2 + v_2^2) = 1$
$\Rightarrow 4 - \dfrac{(1 + e^2) \times 4}{2} = 1$
$\Rightarrow 2(1 + e^2) = 3 \Rightarrow 1 + e^2 = \dfrac{3}{2} \Rightarrow e^2 = \dfrac{1}{2} \Rightarrow e = \dfrac{1}{\sqrt{2}}$

37. Final K.E. = 0.2J
Initial K.E. = $\frac{1}{2} mV_1^2 + 0 = \frac{1}{2} \times 0.1\, u^2 = 0.05\, u^2$
$mv_1 = mv_2' = mu$
Where v_1 and v_2 are final velocities of 1st and 2nd block respectively.



$\Rightarrow v_1 + v_2 = u$ …(1)
$(v_1 - v_2) + \ell\,(a_1 - u_2) = 0 \Rightarrow \ell a = v_2 - v_1$ ..(2)
$u_2 = 0,\quad u_1 = u.$
Adding Eq.(1) and Eq.(2)
$2v_2 = (1 + \ell)u \Rightarrow v_2 = (u/2)(1 + \ell)$
$\therefore v_1 = u - \dfrac{u}{2} - \dfrac{u}{2}\ell$
$v_1 = \dfrac{u}{2}(1 - \ell)$
Given $(1/2)mv_1^2 + (1/2)mv_2^2 = 0.2$
$\Rightarrow v_1^2 + v_2^2 = 4$

$\Rightarrow \frac{u^2}{4}(1-\ell)^2 + \frac{u^2}{4}(1+\ell)^2 = 4$ $\quad \Rightarrow \frac{u^2}{2}(1+\ell^2) = 4$ $\quad \Rightarrow u^2 = \frac{8}{1+\ell^2}$

For maximum value of u, denominator should be minimum,

$\Rightarrow \ell = 0$.

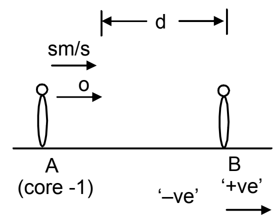
$\Rightarrow u^2 = 8 \Rightarrow u = 2\sqrt{2}$ m/s

For minimum value of u, denominator should be maximum,

$\Rightarrow \ell = 1$

$u^2 = 4 \Rightarrow u = 2$ m/s

38. Two friends A & B (each 40kg) are sitting on a frictionless platform some distance d apart A rolls a ball of mass 4kg on the platform towards B, which B catches. Then B rolls the ball towards A and A catches it. The ball keeps on moving back & forth between A and B. The ball has a fixed velocity 5m/s.



a) Case – I :– Total momentum of the man A & the ball will remain constant

$\therefore 0 = 4 \times 5 - 40 \times v$ $\quad \Rightarrow v = 0.5$ m/s towards left

b) Case – II :– When B catches the ball, the momentum between the B & the ball will remain constant.

$\Rightarrow 4 \times 5 = 44v \Rightarrow v = (20/44)$ m/s

Case – III :– When B throws the ball, then applying L.C.L.M

$\Rightarrow 44 \times (20/44) = -4 \times 5 + 40 \times v$ $\quad \Rightarrow v = 1$m/s (towards right)

Case – IV :– When a Catches the ball, the applying L.C.L.M.

$\Rightarrow -4 \times 5 + (-0.5) \times 40 = -44v$ $\quad \Rightarrow v = \frac{10}{11}$ m/s towards left.

c) Case – V :– When A throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (10/11) = 4 \times 5 - 40 \times V$ $\quad \Rightarrow V = 60/40 = 3/2$ m/s towards left.

Case – VI :– When B receives the ball, then applying L.C.L.M

$\Rightarrow 40 \times 1 + 4 \times 5 = 44 \times v$ $\quad \Rightarrow v = 60/44$ m/s towards right.

Case – VII :– When B throws the ball, then applying L.C.L.M.

$\Rightarrow 44 \times (66/44) = -4 \times 5 + 40 \times V$ $\quad \Rightarrow V = 80/40 = 2$ m/s towards right.

Case – VIII :– When A catches the ball, then applying L.C.L.M

$\Rightarrow -4 \times 5 - 40 \times (3/2) = -44v$ $\quad \Rightarrow v = (80/44) = (20/11)$ m/s towards left.

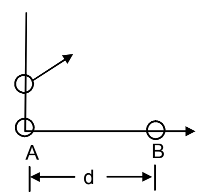
Similarly after 5 round trips

The velocity of A will be (50/11) & velocity of B will be 5 m/s.

d) Since after 6 round trip, the velocity of A is 60/11 i.e.

> 5m/s. So, it can't catch the ball. So it can only roll the ball six.

e) Let the ball & the body A at the initial position be at origin.

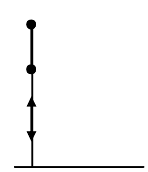


$\therefore X_C = \frac{40 \times 0 + 4 \times 0 + 40 \times d}{40 + 40 + 4} = \frac{10}{11}d$

39. $u = \sqrt{2gh}$ = velocity on the ground when ball approaches the ground.

$\Rightarrow u = \sqrt{2 \times 9.8 \times 2}$

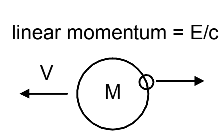
v = velocity of ball when it separates from the ground.



$\vec{v} + \ell\vec{u} = 0$

$\Rightarrow \ell\vec{u} = -\vec{v}$ $\Rightarrow \ell = \frac{\sqrt{2 \times 9.8 \times 1.5}}{\sqrt{2 \times 9.8 \times 2}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

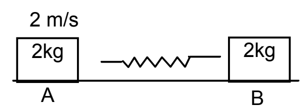
40. K.E. of Nucleus = $(1/2)mv^2 = (1/2)\, m\left(\frac{E}{mc}\right)^2 = \frac{E^2}{2mc^2}$



Energy limited by Gamma photon = E.

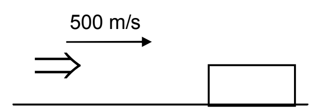
Decrease in internal energy = $E + \frac{E^2}{2mc^2}$

41. Mass of each block M_A and M_B = 2kg.
Initial velocity of the 1st block, (V) = 1m/s
V_A = 1 m/s, V_B = 0m/s
Spring constant of the spring = 100 N/m.
The block A strikes the spring with a velocity 1m/s/
After the collision, it's velocity decreases continuously and at a instant the whole system (Block A + the compound spring + Block B) move together with a common velocity.
Let that velocity be V.
Using conservation of energy, $(1/2)\, M_A V_A^2 + (1/2) M_B V_B^2 = (1/2) M_A v^2 + (1/2) M_B v^2 + (1/2) kx^2$.
$(1/2) \times 2(1)^2 + 0 = (1/2) \times 2\times v^2 + (1/2) \times 2 \times v^2 + (1/2)\, x^2 \times 100$
(Where x = max. compression of spring)
$\Rightarrow 1 = 2v^2 + 50x^2$ …(1)
As there is no external force in the horizontal direction, the momentum should be conserved.
$\Rightarrow M_A V_A + M_B V_B = (M_A + M_B)V$.
$\Rightarrow 2 \times 1 = 4 \times v$
$\Rightarrow V = (1/2)$ m/s. …(2)
Putting in eq.(1)



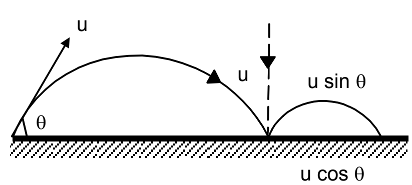
$1 = 2 \times (1/4) + 50x+2+$
$\Rightarrow (1/2) = 50x^2$
$\Rightarrow x^2 = 1/100\text{m}^2$
$\Rightarrow x = (1/10)\text{m} = 0.1\text{m} = 10\text{cm}$.

42. Mass of bullet m = 0.02kg.
Initial velocity of bullet V_1 = 500m/s
Mass of block, M = 10kg.
Initial velocity of block u_2 = 0.
Final velocity of bullet = 100 m/s = v.



Let the final velocity of block when the bullet emerges out, if block = v′.
$mv_1 + Mu_2 = mv + Mv'$
$\Rightarrow 0.02 \times 500 = 0.02 \times 100 + 10 \times v'$
$\Rightarrow v' = 0.8$m/s
After moving a distance 0.2 m it stops.
$\Rightarrow$ change in K.E. = Work done
$\Rightarrow 0 - (1/2) \times 10\times (0.8)^2 = -\mu \times 10 \times 10 \times 0.2 \Rightarrow \mu = 0.16$

43. The projected velocity = u.
The angle of projection = θ.
When the projectile hits the ground for the 1st time, the velocity would be the same i.e. u.
Here the component of velocity parallel to ground, u cos θ should remain constant. But the vertical component of the projectile undergoes a change after the collision.
$\Rightarrow e = \dfrac{u \sin\theta}{v} \Rightarrow v = eu \sin \theta$.



Now for the 2nd projectile motion,
U = velocity of projection = $\sqrt{(u\cos\theta)^2 + (eu\sin\theta)^2}$
and Angle of projection = $\alpha = \tan^{-1}\left(\dfrac{eu\sin\theta}{a\cos\theta}\right) = \tan^{-1}(e \tan \theta)$
or $\tan \alpha = e \tan \theta$ …(2)
Because, $y = x \tan \alpha - \dfrac{gx^2 \sec^2 \alpha}{2u^2}$ …(3)
Here, y = 0, $\tan \alpha = e \tan \theta$, $\sec^2 \alpha = 1 + e^2 \tan^2 \theta$
And $u^2 = u^2 \cos^2 \theta + e^2 \sin^2 \theta$
Putting the above values in the equation (3),

$x\ e\ \tan\theta = \dfrac{gx^2(1+e^2\tan^2\theta)}{2u^2(\cos^2\theta+e^2\sin^2\theta)}$

$\Rightarrow x = \dfrac{2eu^2\tan\theta(\cos^2\theta+e^2\sin^2\theta)}{g(1+e^2\tan^2\theta)}$

$\Rightarrow x = \dfrac{2eu^2\tan\theta-\cos^2\theta}{g} = \dfrac{eu^2\sin 2\theta}{g}$

$\Rightarrow$ So, from the starting point O, it will fall at a distance

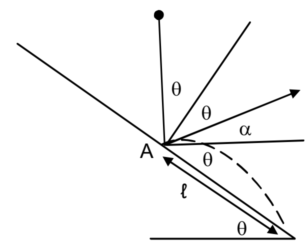
$= \dfrac{u^2\sin 2\theta}{g}+\dfrac{eu^2\sin 2\theta}{g} = \dfrac{u^2\sin 2\theta}{g}(1+e)$

44. Angle inclination of the plane = θ
M the body falls through a height of h,

The striking velocity of the projectile with the indined plane v = $\sqrt{2gh}$

Now, the projectile makes on angle (90° – 2θ)

Velocity of projection = u = $\sqrt{2gh}$



Let AB = L.
So, x = $\ell\cos\theta$, y = – $\ell\sin\theta$
From equation of trajectory,

$y = x\tan\alpha - \dfrac{gx^2\sec^2\alpha}{2u^2}$

$-\ell\sin\theta = \ell\cos\theta\ .\ \tan(90°-2\theta) - \dfrac{g\times\ell^2\cos^2\theta\sec^2(90^\circ-2\theta)}{2\times 2gh}$

$\Rightarrow -\ell\sin\theta = \ell\cos\theta\ .\ \cot 2\theta - \dfrac{g\ell^2\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4gh}$

So, $\dfrac{\ell\cos^2\theta\,\mathrm{cosec}^2 2\theta}{4h} = \sin\theta + \cos\theta\cot 2\theta$

$\Rightarrow \ell = \dfrac{4h}{\cos^2\theta\,\mathrm{cosec}^2 2\theta}(\sin\theta+\cos\theta\cot 2\theta) = \dfrac{4h\times\sin^2 2\theta}{\cos^2\theta}\left(\sin\theta+\cos\theta\times\dfrac{\cos 2\theta}{\sin 2\theta}\right)$

$= \dfrac{4h\times 4\sin^2\theta\cos^2\theta}{\cos^2\theta}\left(\dfrac{\sin\theta\times\sin 2\theta+\cos\theta\cos 2\theta}{\sin 2\theta}\right) = 16\,h\sin^2\theta\times\dfrac{\cos\theta}{2\sin\theta\cos\theta} = 8h\sin\theta$

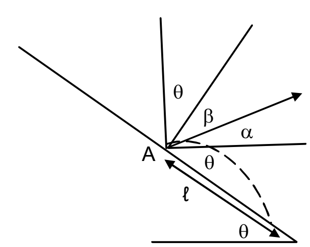
45. h = 5m, θ = 45°, e = (3/4)

Here the velocity with which it would strike = v = $\sqrt{2g\times 5}$ = 10m/sec

After collision, let it make an angle β with horizontal. The horizontal component of velocity 10 cos 45° will remain unchanged and the velocity in the perpendicular direction to the plane after wllisine.

$\Rightarrow V_y$ = e × 10 sin 45°

$= (3/4)\times 10\times\dfrac{1}{\sqrt{2}} = (3.75)\sqrt{2}$ m/sec



$V_x = 10\cos 45° = 5\sqrt{2}$ m/sec

So, $u = \sqrt{V_x^2+V_y^2} = \sqrt{50+28.125} = \sqrt{78.125} = 8.83$ m/sec

Angle of reflection from the wall $\beta = \tan^{-1}\left(\dfrac{3.75\sqrt{2}}{5\sqrt{2}}\right) = \tan^{-1}\left(\dfrac{3}{4}\right) = 37°$

$\Rightarrow$ Angle of projection $\alpha = 90 - (\theta+\beta) = 90 - (45°+37°) = 8°$

Let the distance where it falls = L

$\Rightarrow$ x = L cos θ, y = – L sin θ

Angle of projection (α) = –8°

Using equation of trajectory, $y = x \tan\alpha - \dfrac{gx^2 \sec^2\alpha}{2u^2}$

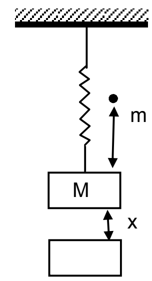
$\Rightarrow -\ell \sin\theta = \ell\cos\theta \times \tan 8° - \dfrac{g}{2} \times \dfrac{\ell \cos^2\theta \sec^2 8°}{u^2}$

$\Rightarrow -\sin 45° = \cos 45° - \tan 8° - \dfrac{10\cos^2 45° \sec 8°}{(8.83)^2}(\ell)$

Solving the above equation we get,

$\ell = 18.5$ m.

46. Mass of block



Block of the particle = m = 120gm = 0.12kg.

In the equilibrium condition, the spring is stretched by a distance x = 1.00 cm = 0.01m.

$\Rightarrow 0.2 \times g = K.\ x.$

$\Rightarrow 2 = K \times 0.01 \Rightarrow K = 200$ N/m.

The velocity with which the particle m will strike M is given by u

$= \sqrt{2 \times 10 \times 0.45} = \sqrt{9} = 3$ m/sec.

So, after the collision, the velocity of the particle and the block is

$V = \dfrac{0.12 \times 3}{0.32} = \dfrac{9}{8}$ m/sec.

Let the spring be stretched through an extra deflection of δ.

$0 - (1/2) \times 0.32 \times (81/64) = 0.32 \times 10 \times \delta - (\ 1/2\ \times 200 \times (\delta + 0.1)^2 - (1/2) \times 200 \times (0.01)^2$

Solving the above equation we get

$\delta = 0.045 = 4.5$cm

47. Mass of bullet = 25g = 0.025kg.

Mass of pendulum = 5kg.

The vertical displacement h = 10cm = 0.1m

Let it strike the pendulum with a velocity u.

Let the final velocity be v.

$\Rightarrow mu = (M + m)v.$

$\Rightarrow v = \dfrac{m}{(M+m)}u = \dfrac{0.025}{5.025} \times u = \dfrac{u}{201}$

Using conservation of energy.

$0 - (1/2)(M + m).\ V^2 = -(M + m)\ g \times h \Rightarrow \dfrac{u^2}{(201)^2} = 2 \times 10 \times 0.1 = 2$

$\Rightarrow u = 201 \times \sqrt{2} = 280$ m/sec.

48. Mass of bullet = M = 20gm = 0.02kg.

Mass of wooden block M = 500gm = 0.5kg

Velocity of the bullet with which it strikes u = 300 m/sec.

Let the bullet emerges out with velocity V and the velocity of block = V′

As per law of conservation of momentum.

$mu = Mv' + mv$ ….(1)

Again applying work – energy principle for the block after the collision,

$0 - (1/2)\ M \times V'^2 = -Mgh$ (where h = 0.2m)

$\Rightarrow V'^2 = 2gh$

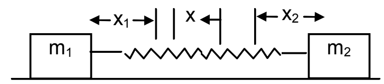
$V' = \sqrt{2gh} = \sqrt{20 \times 0.2} = 2$m/sec

Substituting the value of V′ in the equation (1), we get\

$0.02 \times 300 = 0.5 \times 2 + 0.2 \times v$

$\Rightarrow V = \dfrac{6.1}{0.02} = 250$m/sec.

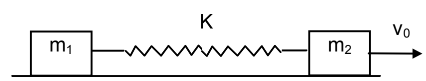
49. Mass of the two blocks are m_1 , m_2.
Initially the spring is stretched by x_0
Spring constant K.
For the blocks to come to rest again,
Let the distance travelled by m_1 & m_2
Be x_1 and x_2 towards right and left respectively.
As o external forc acts in horizontal direction,



$m_1x_1 = m_2x_2$ …(1)
Again, the energy would be conserved in the spring.
$\Rightarrow$ (1/2) k × x^2 = (1/2) k $(x_1 + x_2 - x_0)^2$
$\Rightarrow x_o = x_1 + x_2 - x_0$
$\Rightarrow x_1 + x_2 = 2x_0$ …(2)
$\Rightarrow x_1 = 2x_0 - x_2$ similarly $x_1 = \left(\frac{2m_2}{m_1+m_2}\right)x_0$

$\Rightarrow m_1(2x_0 - x_2) = m_2x_2$ $\Rightarrow 2m_1x_0 - m_1x_2 = m_2x_2$ $\Rightarrow x_2 = \left(\frac{2m_1}{m_1+m_2}\right)x_0$

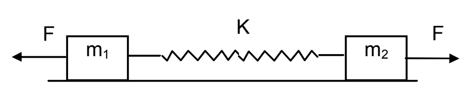
50. a) $\therefore$ Velocity of centre of mass = $\frac{m_2 \times v_0 + m_1 \times 0}{m_1+m_2} = \frac{m_2v_0}{m_1+m_2}$
b) The spring will attain maximum elongation when both velocity of two blocks will attain the velocity of centre of mass.
d) $x \rightarrow$ maximum elongation of spring.
Change of kinetic energy = Potential stored in spring.



$\Rightarrow$ (1/2) $m_2 v_0^2$ – (1/2) $(m_1 + m_2)$ $\left(\frac{m_2v_0}{m_1+m_2}\right)^2$ = (1/2) kx^2
$\Rightarrow m_2 v_0^2 \left(1 - \frac{m_2}{m_1+m_2}\right) = kx^2$ $\Rightarrow x = \left(\frac{m_1m_2}{m_1+m_2}\right)^{1/2} \times v_0$

51. If both the blocks are pulled by some force, they suddenly move with some acceleration and instantaneously stop at same position where the elongation of spring is maximum.
$\therefore$ Let $x_1, x_2 \rightarrow$ extension by block m_1 and m_2
Total work done = $Fx_1 + Fx_2$ …(1)
$\therefore$ Increase the potential energy of spring = (1/2) K $(x_1+ x_2)^2$ …(2)
Equating (1) and (2)
$F(x_1 + x_2)$ = (1/2) K $(x_1+ x_2)^2$ $\Rightarrow (x_1+ x_2) = \frac{2F}{K}$
Since the net external force on the two blocks is zero thus same force act on opposite direction.
$\therefore m_1x_1 = m_2x_2$ …(3)
And $(x_1+ x_2) = \frac{2F}{K}$

$\therefore x_2 = \frac{m_1}{m_2} \times 1$



Substituting $\frac{m_1}{m_2}$ × 1 + x_1 = $\frac{2F}{K}$

$\Rightarrow x_1\left(1+\frac{m_1}{m_2}\right) = \frac{2F}{K}$ $\Rightarrow x_1 = \frac{2F}{K}\frac{m_2}{m_1+m_2}$

Similarly $x_2 = \frac{2F}{K}\frac{m_1}{m_1+m_2}$

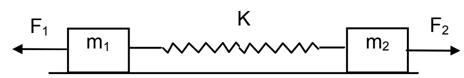
52. Acceleration of mass $m_1 = \dfrac{F_1 - F_2}{m_1 + m_2}$

Similarly Acceleration of mass $m_2 = \dfrac{F_2 - F_1}{m_1 + m_2}$

Due to F_1 and F_2 block of mass m_1 and m_2 will experience different acceleration and experience an inertia force.

$\therefore$ Net force on $m_1 = F_1 - m_1 a$

$= F_1 - m_1 \times \dfrac{F_1 - F_2}{m_1 + m_2} = \dfrac{m_1F_1 + m_2F_1 - m_1F_1 + F_2m_1}{m_1 + m_2} = \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$



Similarly Net force on $m_2 = F_2 - m_2 a$

$= F_2 - m_2 \times \dfrac{F_2 - F_1}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2 - m_2F_2 + F_1m_2}{m_1 + m_2} = \dfrac{m_1F_2 + m_2F_2}{m_1 + m_2}$

$\therefore$ If m_1 displaces by a distance x_1 and x_2 by m_2 the maximum extension of the spring is $x_1 + m_2$.

$\therefore$ Work done by the blocks = energy stored in the spring.,

$\Rightarrow \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_1 + \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2} \times x_2 = (1/2)\, K\, (x_1 + x_2)^2$

$\Rightarrow x_1 + x_2 = \dfrac{2}{K} \dfrac{m_2F_1 + m_1F_2}{m_1 + m_2}$

53. Mass of the man (M_m) is 50 kg.

Mass of the pillow (M_p) is 5 kg.

When the pillow is pushed by the man, the pillow will go down while the man goes up. It becomes the external force on the system which is zero.

$\Rightarrow$ acceleration of centre of mass is zero

$\Rightarrow$ velocity of centre of mass is constant

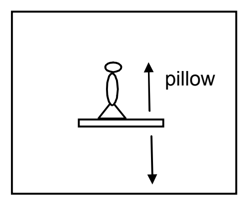
$\therefore$ As the initial velocity of the system is zero.

$\therefore M_m \times V_m = M_p \times V_p$ …(1)

Given the velocity of pillow is 80 ft/s.

Which is relative velocity of pillow w.r.t. man.

$\vec{V}_{p/m} = \vec{V}_p - \vec{V}_m = V_p - (-V_m) = V_p + V_m \Rightarrow V_p = V_{p/m} - V_m$



Putting in equation (1)

$M_m \times V_m = M_p\, (V_{p/m} - V_m)$

$\Rightarrow 50 \times V_m = 5 \times (8 - V_m)$

$\Rightarrow 10 \times V_m = 8 - V_m \Rightarrow V_m = \dfrac{8}{11} = 0.727\text{m/s}$

$\therefore$ Absolute velocity of pillow = 8 – 0.727 = 7.2 ft/sec.

$\therefore$ Time taken to reach the floor $= \dfrac{S}{v} = \dfrac{8}{7.2} = 1.1$ sec.

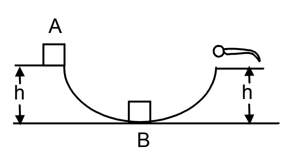
As the mass of wall >>> then pillow

The velocity of block before the collision = velocity after the collision.

$\Rightarrow$ Times of ascent = 1.11 sec.

$\therefore$ Total time taken = 1.11 + 1.11 = 2.22 sec.

54. Let the velocity of A = u_1.



Let the final velocity when reaching at B becomes collision = v_1.

$\therefore (1/2)\, m v_1^2 - (1/2) m u_1^2 = mgh$

$\Rightarrow v_1^2 - u_1^2 = 2\,gh \qquad \Rightarrow v_1 = \sqrt{2gh - u_1^2}$ …(1)

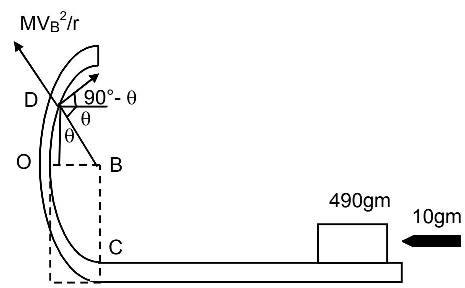
When the block B reached at the upper man's head, the velocity of B is just zero.

For B, block

$\therefore (1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = mgh \qquad \Rightarrow v = \sqrt{2gh}$

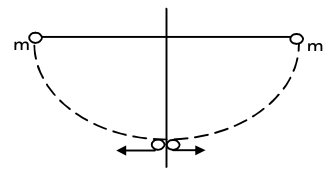
$\therefore$ Before collision velocity of $u_A = v_1,$ $u_B = 0.$
After collision velocity of $v_A = v$ (say) $v_B = \sqrt{2gh}$
Since it is an elastic collision the momentum and K.E. should be coserved.
$\therefore m \times v_1 + 2m \times 0 = m \times v + 2m \times \sqrt{2gh}$
$\Rightarrow v_1 - v = 2\sqrt{2gh}$
Also, $(1/2) \times m \times v_1^2 + (1/2)$ I $2m \times 0^2 = (1/2) \times m \times v^2 + (1/2) \times 2m \times \left(\sqrt{2gh}\right)^2$
$\Rightarrow v_1^2 - v^2 = 2 \times \sqrt{2gh} \times \sqrt{2gh}$ …(2)
Dividing (1) by (2)
$\frac{(v_1+v)(v_1-v)}{(v_1+v)} = \frac{2\times\sqrt{2gh}\times\sqrt{2gh}}{2\times\sqrt{2gh}} \Rightarrow v_1 + v = \sqrt{2gh}$ …(3)
Adding (1) and (3)
$2v_1 = 3\sqrt{2gh} \Rightarrow v_1 = \left(\frac{3}{2}\right)\sqrt{2gh}$
But $v_1 = \sqrt{2gh+u^2} = \left(\frac{3}{2}\right)\sqrt{2gh}$
$\Rightarrow 2gh + u^2 = \frac{9}{4}\times 2gh$
$\Rightarrow u = 2.5\sqrt{2gh}$
So the block will travel with a velocity greater than $2.5\sqrt{2gh}$ so awake the man by B.

55. Mass of block = 490 gm.
Mass of bullet = 10 gm.
Since the bullet embedded inside the block, it is an plastic collision.
Initial velocity of bullet $v_1 = 50\sqrt{7}$ m/s.
Velocity of the block is $v_2 = 0$.
Let Final velocity of both = v.
$\therefore 10 \times 10^{-3} \times 50 \times \sqrt{7} + 10^{-3} \times 190$ I $0 = (490 + 10) \times 10^{-3} \times V_A$
$\Rightarrow V_A = \sqrt{7}$ m/s.
When the block losses the contact at 'D' the component mg will act on it.
$\frac{m(V_B)^2}{r} = mg \sin\theta \Rightarrow (V_B)^2 = gr \sin\theta$ …(1)



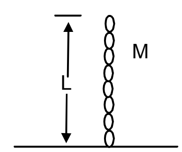
Puttin work energy principle
$(1/2)\ m \times (V_B)^2 - (1/2) \times m \times (V_A)^2 = -mg\ (0.2 + 0.2 \sin\theta)$
$\Rightarrow (1/2) \times gr \sin\theta - (1/2) \times \left(\sqrt{7}\right)^2 = -mg\ (0.2 + 0.2 \sin\theta)$
$\Rightarrow 3.5 - (1/2) \times 9.8 \times 0.2 \times \sin\theta = 9.8 \times 0.2\ (1 + \sin\theta)$
$\Rightarrow 3.5 - 0.98 \sin\theta = 1.96 + 1.96 \sin\theta$
$\Rightarrow \sin\theta = (1/2) \Rightarrow \theta = 30°$
$\therefore$ Angle of projection = 90° - 30° = 60°.
$\therefore$ time of reaching the ground $= \sqrt{\frac{2h}{g}}$
$= \sqrt{\frac{2\times(0.2+0.2\times\sin 30°)}{9.8}} = 0.247$ sec.
$\therefore$ Distance travelled in horizontal direction.
$s = V\cos\theta \times t = \sqrt{gr\sin\theta}\times t = \sqrt{9.8\times 2\times(1/2)}\times 0.247 = 0.196$m
$\therefore$ Total distance = (0.2 – 0.2 cos 30°) + 0.196 = 0.22m.

56. Let the velocity of m reaching at lower end = V_1
From work energy principle.
$\therefore$ (1/2) × m × V_1^2 – (1/2) × m × 0^2 = mg ℓ
$\Rightarrow v_1 = \sqrt{2g\ell}$.
Similarly velocity of heavy block will be $v_2 = \sqrt{2gh}$.
$\therefore v_1 = V_2 = u$(say)
Let the final velocity of m and 2m v_1 and v_2 respectively.
According to law of conservation of momentum.
$m \times x_1 + 2m \times V_2 = mv_1 + 2mv_2$
$\Rightarrow m \times u - 2\,m\,u = mv_1 + 2mv_2$
$\Rightarrow v_1 + 2v_2 = -u$ …(1)
Again, $v_1 - v_2 = -(V_1 - V_2)$
$\Rightarrow v_1 - v_2 = -[u - (-v)] = -2V$ …(2)
Subtracting.



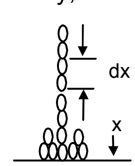
$3v_2 = u \Rightarrow v_2 = \frac{u}{3} = \frac{\sqrt{2g\ell}}{3}$
Substituting in (2)
$v_1 - v_2 = -2u \Rightarrow v_1 = -2u + v_2 = -2u + \frac{u}{3} = -\frac{5}{3}u = -\frac{5}{3} \times \sqrt{2g\ell} = -\frac{\sqrt{50g\ell}}{3}$
b) Putting the work energy principle
(1/2) × 2m × 0^2 – (1/2) × 2m × $(v_2)^2$ = – 2m × g × h
[h $\rightarrow$ height gone by heavy ball]
$\Rightarrow (1/2)\ \frac{2g}{9} = \ell \times h$ $\Rightarrow h = \frac{\ell}{9}$
Similarly, (1/2) × m × 0^2 – (1/2) × m × v_1^2 = m × g × h_2
[height reached by small ball]
$\Rightarrow (1/2) \times \frac{50g\ell}{9} = g \times h_2$ $\Rightarrow h_2 = \frac{25\ell}{9}$
Someh_2 is more than 2ℓ, the velocity at height point will not be zero. And the 'm' will rise by a distance 2ℓ.

57. Let us consider a small element at a distance 'x' from the floor of length 'dy' .
So, $dm = \frac{M}{L}dx$
So, the velocity with which the element will strike the floor is, $v = \sqrt{2gx}$



(Initial position)

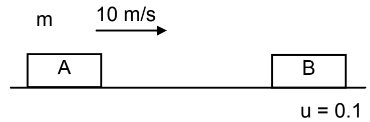
$\therefore$ So, the momentum transferred to the floor is,
$M = (dm)v = \frac{M}{L} \times dx \times \sqrt{2gx}$ [because the element comes to rest]
So, the force exerted on the floor change in momentum is given by,
$F_1 = \frac{dM}{dt} = \frac{M}{L} \times \frac{dx}{dt} \times \sqrt{2gx}$
Because, $v = \frac{dx}{dt} = \sqrt{2gx}$ (for the chain element)
$F_1 = \frac{M}{L} \times \sqrt{2gx} \times \sqrt{2gx} = \frac{M}{L} \times 2gx = \frac{2Mgx}{L}$
Again, the force exerted due to 'x' length of the chain on the floor due to its own weight is given by,
$W = \frac{M}{L}(x) \times g = \frac{Mgx}{L}$
So, the total forced exerted is given by,
$F = F_1 + W = \frac{2Mgx}{L} + \frac{Mgx}{L} = \frac{3Mgx}{L}$



58. $V_1 = 10$ m/s $V_2 = 0$

V_1, $v_2 \rightarrow$ velocity of ACB after collision.

a) If the edlision is perfectly elastic.



$mV_1 + mV_2 = mv_1 + mv_2$

$\Rightarrow 10 + 0 = v_1 + v_2$

$\Rightarrow v_1 + v_2 = 10$ …(1)

Again, $v_1 - v_2 = -(u_1 - v_2) = -(10 - 0) = -10$ …(2)

Subtracting (2) from (1)

$2v_2 = 20 \Rightarrow v_2 = 10$ m/s.

The deacceleration of B = μg

Putting work energy principle

$\therefore (1/2) \times m \times 0^2 - (1/2) \times m \times v_2^2 = -m \times a \times h$

$\Rightarrow -(1/2) \times 10^2 = -\mu\, g \times h$ $\Rightarrow h = \dfrac{100}{2 \times 0.1 \times 10} = 50\text{m}$

b) If the collision perfectly in elastic.

$m \times u_1 + m \times u_2 = (m + m) \times v$

$\Rightarrow m \times 10 + m \times 0 = 2m \times v$ $\Rightarrow v = \dfrac{10}{2} = 5$ m/s.

The two blocks will move together sticking to each other.

$\therefore$ Putting work energy principle.

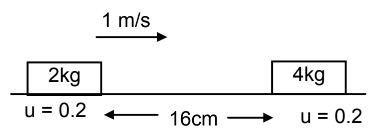
$(1/2) \times 2m \times 0^2 - (1/2) \times 2m \times v^2 = 2m \times \mu\, g \times s$

$\Rightarrow \dfrac{5^2}{0.1 \times 10 \times 2} = s$ $\Rightarrow s = 12.5$ m.

59. Let velocity of 2kg block on reaching the 4kg block before collision $= u_1$.

Given, $V_2 = 0$ (velocity of 4kg block).

$\therefore$ From work energy principle,



$(1/2)\, m \times u_1^2 - (1/2)\, m \times 1^2 = -m \times ug \times s$

$\Rightarrow \dfrac{u_1^2 - 1}{2} = -2 \times 5$ $\Rightarrow -16 = \dfrac{u_1^2 - 1}{4}$

$\Rightarrow 64 \times 10^{-2} = u_1^2 - 1$ $\Rightarrow u_1 = 6$m/s

Since it is a perfectly elastic collision.

Let V_1, $V_2 \rightarrow$ velocity of 2kg & 4kg block after collision.

$m_1V_1 + m_2V_2 = m_1v_1 + m_2v_2$

$\Rightarrow 2 \times 0.6 + 4 \times 0 = 2v_1 + 4\, v_2$ $\Rightarrow v_1 + 2v_2 = 0.6$ …(1)

Again, $V_1 - V_2 = -(u_1 - u_2) = -(0.6 - 0) = -0.6$ …(2)

Subtracting (2) from (1)

$3v_2 = 1.2$ $\Rightarrow v_2 = 0.4$ m/s.

$\therefore v_1 = -0.6 + 0.4 = -0.2$ m/s

$\therefore$ Putting work energy principle for 1st 2kg block when come to rest.

$(1/2) \times 2 \times 0^2 - (1/2) \times 2 \times (0.2)^2 = -2 \times 0.2 \times 10 \times s$

$\Rightarrow (1/2) \times 2 \times 0.2 \times 0.2 = 2 \times 0.2 \times 10 \times s$ $\Rightarrow S_1 = 1$cm.

Putting work energy principle for 4kg block.

$(1/2) \times 4 \times 0^2 - (1/2) \times 4 \times (0.4)^2 = -4 \times 0.2 \times 10 \times s$

$\Rightarrow 2 \times 0.4 \times 0.4 = 4 \times 0.2 \times 10 \times s$ $\Rightarrow S_2 = 4$ cm.

Distance between 2kg & 4kg block = $S_1 + S_2 = 1 + 4 = 5$ cm.

60. The block 'm' will slide down the inclined plane of mass M with acceleration a_1 g sin α (relative) to the inclined plane.

The horizontal component of a_1 will be, $a_x = g \sin\alpha \cos\alpha$, for which the block M will accelerate towards left. Let, the acceleration be a_2.

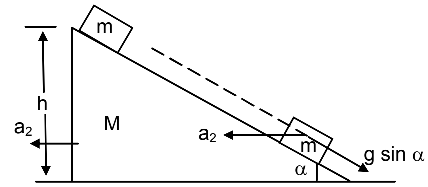
According to the concept of centre of mass, (in the horizontal direction external force is zero).

$ma_x = (M + m)\, a_2$

$\Rightarrow a_2 = \frac{ma_x}{M+m} = \frac{mg\sin\alpha\cos\alpha}{M+m}$ …(1)

So, the absolute (Resultant) acceleration of 'm' on the block 'M' along the direction of the incline will be,

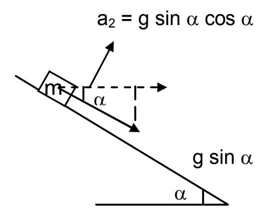
$a = g \sin\alpha - a_2 \cos\alpha$



$= g\sin\alpha - \frac{mg\sin\alpha\cos^2\alpha}{M+m} = g\sin\alpha\left[1-\frac{m\cos^2\alpha}{M+m}\right]$

$= g\sin\alpha\left[\frac{M+m-m\cos^2\alpha}{M+m}\right]$

So, $a = g\sin\alpha\left[\frac{M+m\sin^2\alpha}{M+m}\right]$ …(2)



Let, the time taken by the block 'm' to reach the bottom end be 't'.

Now, $S = ut + (1/2)\,at^2$

$\Rightarrow \frac{h}{\sin\alpha} = (1/2)\,at^2 \qquad \Rightarrow t = \sqrt{\frac{2}{a\sin\alpha}}$

So, the velocity of the bigger block after time 't' will be.

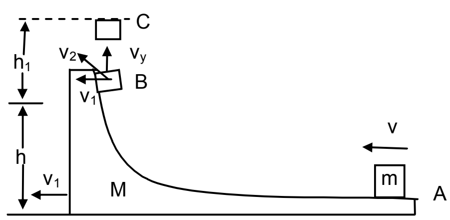
$V_m = u + a_2t = \frac{mg\sin\alpha\cos\alpha}{M+m}\sqrt{\frac{2h}{a\sin\alpha}} = \sqrt{\frac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2a\sin\alpha}}$

Now, subtracting the value of a from equation (2) we get,

$V_M = \left[\frac{2m^2g^2h\sin^2\alpha\cos^2\alpha}{(M+m)^2\sin\alpha}\times\frac{(M+m)}{g\sin\alpha(M+m\sin^2\alpha)}\right]^{1/2}$

or $V_M = \left[\frac{2m^2g^2h\cos^2\alpha}{(M+m)(M+m\sin^2\alpha)}\right]^{1/2}$

61.



The mass 'm' is given a velocity 'v' over the larger mass M.

a) When the smaller block is travelling on the vertical part, let the velocity of the bigger block be v_1 towards left.

From law of conservation of momentum, (in the horizontal direction)

$mv = (M + m)\,v_1$

$\Rightarrow v_1 = \frac{mv}{M+m}$

b) When the smaller block breaks off, let its resultant velocity is v_2.

From law of conservation of energy,

$(1/2)\,mv^2 = (1/2)\,Mv_1^2 + (1/2)\,mv_2^2 + mgh$

$\Rightarrow v_2^2 = v^2 - \frac{M}{m}v_1^2 - 2gh$ ..(1)

$\Rightarrow v_2^2 = v^2\left[1-\frac{M}{m}\times\frac{m^2}{(M+m)^2}\right] - 2gh$

$\Rightarrow v_2 = \left[\frac{(m^2+Mm+m^2)}{(M+m)^2}v^2 - 2gh\right]^{1/2}$

e) Now, the vertical component of the velocity v_2 of mass 'm' is given by,

$v_y^2 = v_2^2 - v_1^2$

$= \frac{(M^2 + Mm + m^2)}{(M+m)^2}v^2 - 2gh - \frac{m^2v^2}{(M+m)^2}$

$[\therefore\ v_1 = \frac{mv}{M+v}]$

$\Rightarrow v_y^2 = \frac{M^2 + Mm + m^2 - m^2}{(M+m)^2}v^2 - 2gh$

$\Rightarrow v_y^2 = \frac{Mv^2}{(M+m)} - 2gh$ …(2)

To find the maximum height (from the ground), let us assume the body rises to a height 'h', over and above 'h'.

Now, $(1/2)mv_y^2 = mgh_1 \Rightarrow h_1 = \frac{v_y^2}{2g}$ …(3)

So, Total height $= h + h_1 = h + \frac{v_y^2}{2g} = h + \frac{mv^2}{(M+m)2g} - h$

[from equation (2) and (3)]

$\Rightarrow H = \frac{mv^2}{(M+m)2g}$

d) Because, the smaller mass has also got a horizontal component of velocity 'v_1' at the time it breaks off from 'M' (which has a velocity v_1), the block 'm' will again land on the block 'M' (bigger one).

Let us find out the time of flight of block 'm' after it breaks off.

During the upward motion (BC),

$0 = v_y - gt_1$

$\Rightarrow t_1 = \frac{v_y}{g} = \frac{1}{g}\left[\frac{Mv^2}{(M+m)} - 2gh\right]^{1/2}$ …(4) [from equation (2)]

So, the time for which the smaller block was in its flight is given by,

$T = 2t_1 = \frac{2}{g}\left[\frac{Mv^2 - 2(M+m)gh}{(M+m)}\right]^{1/2}$

So, the distance travelled by the bigger block during this time is,

$S = v_1T = \frac{mv}{M+m} \times \frac{2}{g}\frac{[Mv^2 - 2(M+m)gh]^{1/2}}{(M+m)^{1/2}}$

or $S = \frac{2mv[Mv^2 - 2(M+m)gh]^{1/2}}{g(M+m)^{3/2}}$

62. Given h < < < R.

G_{mass} = 6 I 10^{24} kg.

$M_b = 3 \times 10^{24}$ kg.

Let $V_e \rightarrow$ Velocity of earth

$V_b \rightarrow$ velocity of the block.

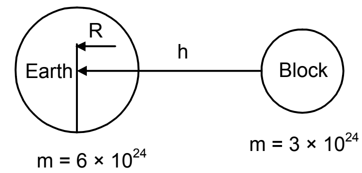
The two blocks are attracted by gravitational force of attraction. The gravitation potential energy stored will be the K.E. of two blocks.

$\overline{G}^{pim}\left[\frac{1}{R+(h/2)} - \frac{1}{R+h}\right] = (1/2)\ m_e \times v_e^2 + (1/2)\ m_b \times v_b^2$

Again as the an internal force acts.

$M_eV_e = m_bV_b$ $\Rightarrow V_e = \frac{m_bV_b}{M_e}$ …(2)

Putting in equation (1)



$G_{me} \times m_b \left[\dfrac{2}{2R+h} - \dfrac{1}{R+h}\right]$

$= (1/2) \times M_e \times \dfrac{{m_b}^2{v_b}^2}{{M_e}^2} \times {v_e}^2 + (1/2)\, M_b \times {V_b}^2$

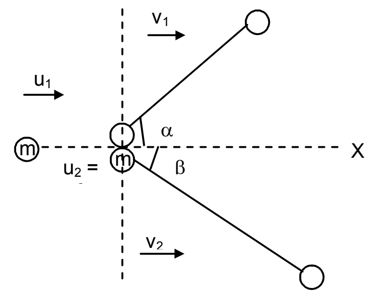
$= (1/2) \times m_b \times {V_b}^2 \left(\dfrac{M_b}{M_e} + 1\right)$

$\Rightarrow GM\left[\dfrac{2R+2h-2R-h}{(2R+h)(R+h)}\right] = (1/2) \times {V_b}^2 \times \left(\dfrac{3\times10^{24}}{6\times10^{24}}+1\right) \qquad \Rightarrow \left[\dfrac{GM\times h}{2R^2+3Rh+h^2}\right] = (1/2) \times {V_b}^2 \times (3/2)$

As h < < < R, if can be neglected

$\Rightarrow \dfrac{GM\times h}{2R^2} = (1/2) \times {V_b}^2 \times (3/2) \qquad \Rightarrow V_b = \sqrt{\dfrac{2gh}{3}}$

63. Since it is not an head on collision, the two bodies move in different dimensions. Let V_1, $V_2 \rightarrow$ velocities of the bodies vector collision. Since, the collision is elastic. Applying law of conservation of momentum on X-direction.



$mu_1 + mxo = mv_1 \cos\alpha + mv_2 \cos\beta$

$\Rightarrow v_1 \cos a + v_2 \cos b = u_1 \quad \ldots(1)$

Putting law of conservation of momentum in y direction.

$0 = mv_1 \sin\alpha - mv_2 \sin\beta$

$\Rightarrow v_1 \sin\alpha = v_2 \sin\beta \qquad \ldots(2)$

Again $½\, m\, {u_1}^2 + 0 = ½\, m\, {v_1}^2 + ½\, m \times {v_2}^2$

$\Rightarrow {u_1}^2 = {v_1}^2 + {v_2}^2 \qquad \ldots(3)$

Squaring equation(1)

${u_1}^2 = {v_1}^2 \cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1v_2 \cos\alpha \cos\beta$

Equating (1) & (3)

${v_1}^2 + {v_2}^2 = {v_1}^2\cos^2\alpha + {v_2}^2 \cos^2\beta + 2\, v_1v_2 \cos\alpha\cos\beta$

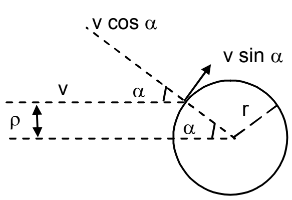
$\Rightarrow {v_1}^2 \sin^2\alpha + {v_2}^2 \sin^2\beta = 2\, v_1v_2 \cos\alpha\cos\beta$

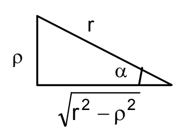
$\Rightarrow 2{v_1}^2 \sin^2\alpha = 2 \times v_1 \times \dfrac{v_1 \sin\alpha}{\sin\beta} \times \cos\alpha\cos\beta$

$\Rightarrow \sin\alpha\sin\beta = \cos\alpha\cos\beta \qquad \Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta = 0$

$\Rightarrow \cos(\alpha+\beta) = 0 = \cos 90° \qquad \Rightarrow (\alpha+\beta) = 90°$

64.





Let the mass of both the particle and the spherical body be ‘m’. The particle velocity ‘v’ has two components, v cos α normal to the sphere and v sin α tangential to the sphere.

After the collision, they will exchange their velocities. So, the spherical body will have a velocity v cos α and the particle will not have any component of velocity in this direction.

[The collision will due to the component v cos α in the normal direction. But, the tangential velocity, of the particle v sin α will be unaffected]

So, velocity of the sphere = v cos α = $\dfrac{v}{r}\sqrt{r^2-\rho^2}$ [from (fig-2)]

And velocity of the particle = v sin α = $\dfrac{v\rho}{r}$

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