CHAPTER 8

WORK AND ENERGY

8.1 KINETIC ENERGY

A dancing, running man is said to be more energetic compared to a sleeping snoring man. In physics, a moving particle is said to have more energy than an identical particle at rest. Quantitatively the energy of the moving particle (over and above its energy at rest) is defined by

$$K(v) = \frac{1}{2} mv^2 = \frac{1}{2} m\vec{v} \cdot \vec{v} \qquad \dots (8.1)$$

and is called the *kinetic energy* of the particle. The kinetic energy of a system of particles is the sum of the kinetic energies of all its constituent particles, i.e.,

$$K = \sum_i \frac{1}{2} m_i v_i^2 .$$

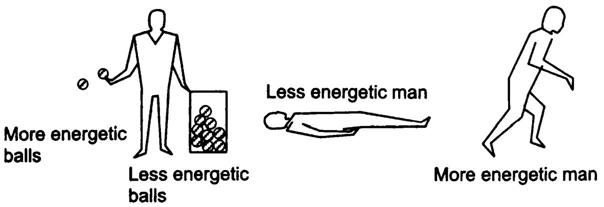


Figure 8.1

The kinetic energy of a particle or a system of particles can increase or decrease or remain constant as time passes.

If no force is applied on the particle, its velocity v remains constant and hence the kinetic energy remains the same. A force is necessary to change the kinetic energy of a particle. If the resultant force acting on a particle is perpendicular to its velocity, the speed of the particle does not change and hence the kinetic energy does not change. Kinetic energy changes only when the speed changes and that happens only when the resultant force has a tangential component. When a particle falls near the earth's surface, the force of gravity is parallel to its velocity. Its kinetic energy increases as time passes. On the other hand, a particle projected upward has the force opposite to the velocity and its kinetic energy decreases.

From the definition of kinetic energy

$$\frac{dK}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt} = F_t v,$$

where F_t is the resultant tangential force. If the resultant force $\vec{F}$ makes an angle θ with the velocity,

$$F_t = F\cos\theta \text{ and } \frac{dK}{dt} = Fv\cos\theta = \vec{F}\cdot\vec{v} = \vec{F}\cdot\frac{d\vec{r}}{dt}$$

or,
$$dK = \vec{F}\cdot d\vec{r}. \qquad \dots (8.2)$$

8.2 WORK AND WORK-ENERGY THEOREM

The quantity $\vec{F}\cdot d\vec{r} = F\,dr\cos\theta$ is called the *work done* by the force $\vec{F}$ on the particle during the small displacement $d\vec{r}$.

The work done on the particle by a force $\vec{F}$ acting on it during a finite displacement is obtained by

$$W = \int \vec{F}\cdot d\vec{r} = \int F\cos\theta\, dr, \qquad \dots (8.3)$$

where the integration is to be performed along the path of the particle. If $\vec{F}$ is the resultant force on the particle we can use equation (8.2) to get

$$W = \int \vec{F}\cdot d\vec{r} = \int dK = K_2 - K_1 .$$

Thus, *the work done on a particle by the resultant force is equal to the change in its kinetic energy.* This is called the *work-energy theorem.*

Let $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots$ be the individual forces acting on a particle. The resultant force is $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 \dots$, and the work done by the resultant force on the particle is

$$\begin{aligned} W &= \int \vec{F}\cdot d\vec{r} \\ &= (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 \,\dots\dots\,)\cdot d\vec{r} \\ &= \int \vec{F}_1\cdot d\vec{r} + \int \vec{F}_2\cdot d\vec{r} + \int \vec{F}_3\cdot d\vec{r} \dots , \end{aligned}$$

where $= \int \vec{F}_1\cdot d\vec{r}$ is the work done on the particle by $\vec{F}_1$ and so on. Thus, *the work done by the resultant force is equal to the sum of the work done by the individual forces.* Note that the work done on a particle by an individual force is not equal to the change in its kinetic energy; the sum of the work done by all the forces acting on the particle (which is equal to the work done by the resultant force) is equal to the change in its kinetic energy.

The rate of doing work is called the *power* delivered. The work done by a force $\vec{F}$ in a small displacement $d\vec{r}$ is $dW = \vec{F} \cdot d\vec{r}$.

Thus, the power delivered by the force is

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}.$$

The SI unit of power is joule/second and is written as "watt". A commonly used unit of power is *horsepower* which is equal to 746 W.

8.3 CALCULATION OF WORK DONE

The work done by a force on a particle during a displacement has been defined as

$$W = \int \vec{F} \cdot d\vec{r}.$$

Constant Force

Suppose, the force is constant (in direction and magnitude) during the displacement. Then $W = \int \vec{F} \cdot d\vec{r} = \vec{F} \cdot \int d\vec{r} = \vec{F} \cdot \vec{r}$, where $\vec{r}$ is the total displacement of the particle during which the work is calculated. If θ be the angle between the constant force $\vec{F}$ and the displacement $\vec{r}$, the work is

$$W = Fr \cos\theta. \qquad \ldots (8.4)$$

In particular, if the displacement is along the force, as is the case with a freely and vertically falling particle, $\theta = 0$ and $W = Fr$.

The force of gravity $(m\vec{g})$ is constant in magnitude and direction if the particle moves near the surface of the earth. Suppose a particle moves from A to B along some curve and that $\vec{AB}$ makes an angle θ with the vertical as shown in figure (8.2). The work done by the force of gravity during the transit from A to B is

$$W = mg\,(AB) \cos\theta = mgh,$$

where h is the height descended by the particle. If a particle ascends a height h, the work done by the force of gravity is $-mgh$.

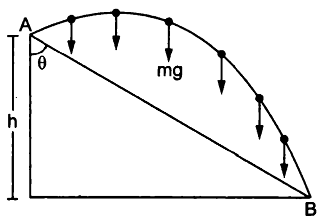


Figure 8.2

If the particle goes from the point A to the point B along some other curve, the work done by the force of gravity is again mgh. We see that the work done by a constant force in going from A to B depends only on the positions of A and B and not on the actual path taken. In case of gravity, the work is weight mg times the height descended. If a particle starts from A and reaches to the same point A after some time, the work done by gravity during this round trip is zero, as the height descended is zero. We shall encounter other forces having this property.

Spring Force

Consider the situation shown in figure (8.3). One end of a spring is attached to a fixed vertical support and the other end to a block which can move on a horizontal table. Let $x = 0$ denote the position of the block when the spring is in its natural length. We shall calculate the work done on the block by the spring-force as the block moves from $x = 0$ to $x = x_1$.

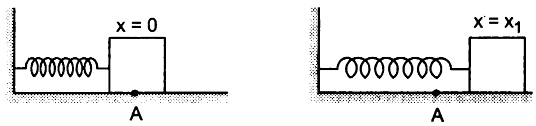


Figure 8.3

The force on the block is k times the elongation of the spring. But the elongation changes as the block moves and so does the force. We cannot take $\vec{F}$ out of the integration $\int \vec{F} \cdot d\vec{r}$. We have to write the work done during a small interval in which the block moves from x to $x + dx$. The force in this interval is kx and the displacement is dx. The force and the displacement are opposite in direction.

So, $\qquad \vec{F} \cdot d\vec{r} = -F\,dx = -kx\,dx$

during this interval. The total work done as the block is displaced from $x = 0$ to $x = x_1$ is

$$W = \int_0^{x_1} -kx\,dx = \left[-\frac{1}{2}kx^2\right]_0^{x_1} = -\frac{1}{2}kx_1^2.$$

If the block moves from $x = x_1$ to $x = x_2$, the limits of integration are x_1 and x_2 and the work done is

$$W = \left(\frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2\right). \qquad \ldots (8.5)$$

Note that if the block is displaced from x_1 to x_2 and brought back to $x = x_1$, the work done by the spring-force is zero. The work done during the return journey is negative of the work during the onward journey. The net work done by the spring-force in a round trip is zero.

Three positions of a spring are shown in figure (8.4). In (i) the spring is in its natural length, in (ii) it is compressed by an amount x and in (iii) it is elongated by an amount x. Work done by the spring-force on the block in various situations is shown in the following table.

Table 8.1

Initial state of the spring	Final state of the spring	x_1	x_2	W
Natural	Compressed	0	$-x$	$-\frac{1}{2}kx^2$
Natural	Elongated	0	x	$-\frac{1}{2}kx^2$
Elongated	Natural	x	0	$\frac{1}{2}kx^2$
Compressed	Natural	$-x$	0	$\frac{1}{2}kx^2$
Elongated	Compressed	x	$-x$	0
Compressed	Elongated	$-x$	x	0

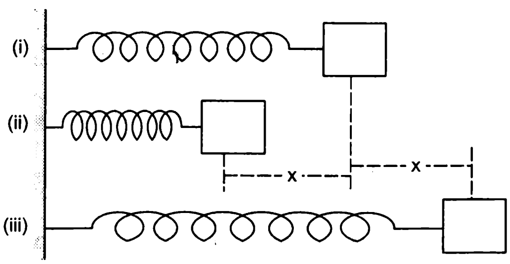


Figure 8.4

Force Perpendicular to Velocity

Suppose $\vec{F} \perp \vec{v}$ for all the time. Then $\vec{F} \cdot d\vec{r} = \vec{F} \cdot \vec{v}\, dt$ is zero in any small interval and the work done by this force is zero.

For example, if a particle is fastened to the end of a string and is whirled in a circular path, the tension is always perpendicular to the velocity of the particle and hence the work done by the tension is zero in circular motion.

Example 8.1

A spring of spring constant 50 N/m *is compressed from its natural position through* 1 cm. *Find the work done by the spring-force on the agency compressing the spring.*

Solution : The magnitude of the work is

$$\frac{1}{2}kx^2 = \frac{1}{2} \times (50 \text{ N/m}) \times (1 \text{ cm})^2$$

$$= (25 \text{ N/m}) \times (1 \times 10^{-2}\text{m})^2 = 2{\cdot}5 \times 10^{-3} \text{ J}.$$

As the compressed spring will push the agency, the force will be opposite to the displacement of the point of application and the work will be negative. Thus, the work done by the spring-force is $-2{\cdot}5$ mJ.

The following three cases occur quite frequently :

(a) The force is perpendicular to the velocity at all the instants. The work done by the force is then zero.

(b) The force is constant (both in magnitude and direction). The work done by the force is $W = Fd\cos\theta$, where F and d are magnitudes of the force and the displacement and θ is the angle between them. The amount of work done depends only on the end positions and not on the intermediate path. The work in a round trip is zero. Force of gravity on the bodies near the earth's surface is an example.

The work done due to the force of gravity on a particle of mass m is mgh, where h is the vertical height 'descended' by the particle.

(c) The force is $F = -kx$ as is the case with an elastic spring. The magnitude of the work done by the force during a displacement x from or to its natural position ($x = 0$) is $\frac{1}{2}kx^2$. The work may be $+\frac{1}{2}kx^2$ or $-\frac{1}{2}kx^2$ depending on whether the force and the displacement are along the same or opposite directions.

Example 8.2

A particle of mass 20 g *is thrown vertically upwards with a speed of* 10 m/s. *Find the work done by the force of gravity during the time the particle goes up.*

Solution : Suppose the particle reaches a maximum height h. As the velocity at the highest point is zero, we have

$$0 = u^2 - 2gh$$

or,
$$h = \frac{u^2}{2g}.$$

The work done by the force of gravity is

$$-mgh = -mg\frac{u^2}{2g} = -\frac{1}{2}mu^2$$

$$= -\frac{1}{2}(0{\cdot}02 \text{ kg}) \times (10 \text{ m/s})^2 = -1{\cdot}0 \text{ J}.$$

8.4 WORK-ENERGY THEOREM FOR A SYSTEM OF PARTICLES

So far we have considered the work done on a single particle. The total work done on a particle equals the change in its kinetic energy. In other words, to change the kinetic energy of a particle we have to apply a force on it and the force must do work on it. Next, consider a system containing more than one particle and suppose the particles exert forces on each other. As a simple example, take a system of two charged particles as shown in figure (8.5) attracting each other (such as a positive and a negative charge).

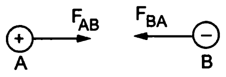


Figure 8.5

Because of mutual attraction, the particles are accelerated towards each other and the kinetic energy of the system increases. We have not applied any external force on the system, yet the kinetic energy has changed. Let us examine this in more detail. The particle B exerts a force $\vec{F}_{AB}$ on A. As A moves towards B, this force does work. The work done by this force is equal to the increase in the kinetic energy of A.

Similarly, A exerts a force $\vec{F}_{BA}$ on B. This force does work on B and this work is equal to the increase in the kinetic energy of B. The work by $\vec{F}_{AB}$ + the work by $\vec{F}_{BA}$ is equal to the increase in the total kinetic energy of the two particles. Note that $\vec{F}_{AB} = -\vec{F}_{BA}$, so that $\vec{F}_{AB} + \vec{F}_{BA} = 0$. But the work by $\vec{F}_{AB}$ + the work by $\vec{F}_{BA} \neq 0$. The two forces are opposite in direction but the displacements are also opposite. Thus, the work done by both the forces are positive and are added. The total work done on different particles of the system by the internal forces may not be zero. The change in the kinetic energy of a system is equal to the work done on the system by the external as well as the internal forces.

8.5 POTENTIAL ENERGY

Consider the example of the two charged particles A and B taken in the previous section. Suppose at some instant t_1 the particles are at positions A, B and are going away from each other with speeds v_1 and v_2 (figure 8.6).

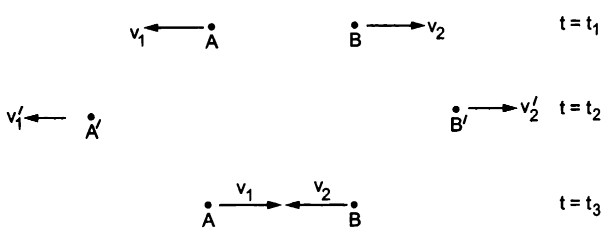


Figure 8.6

The kinetic energy of the system is K_1. We call the positions of the particles at time t_1 as configuration-1. The particle B attracts A and hence the speed v_1 decreases as time passes. Similarly, the speed v_2 of B decreases. Thus, the kinetic energy of the two-particle system decreases as time passes. Suppose at a time t_2, the particles are at A' and B', the speeds have changed to v_1' and v_2' and the kinetic energy becomes K_2. We call the positions of the particles at time t_2 as configuration-2. The kinetic energy of the system is decreased by $K_1 - K_2$.

However, if you wait for some more time, the particles return to the original positions A and B, i.e., in configuration-1. At this time, say t_3, the particles move towards each other with speeds v_1 and v_2. Their kinetic energy is again K_1.

When the particles were in configuration-1 the kinetic energy was K_1. When they reached configuration-2 it decreased to K_2. The kinetic energy has decreased but is not lost for ever. We just have to wait. When the particles return to configuration-1 at time t_3, the kinetic energy again becomes K_1. It seems meaningful and reasonable if we think of yet another kind of energy which depends on the configuration. We call this as the *potential energy* of the system. Some kinetic energy was converted into potential energy when the system passed from configuration-1 to configuration-2. As the system returns to configuration-1, this potential energy is converted back into kinetic energy. The sum of the kinetic energy and the potential energy remains constant.

How do we precisely define the potential energy of a system? Before defining potential energy, let us discuss the idea of conservative and nonconservative forces.

8.6 CONSERVATIVE AND NONCONSERVATIVE FORCES

Let us consider the following two examples.

(1) Suppose a block of mass m rests on a rough horizontal table (figure 8.7). It is dragged horizontally towards right through a distance l and then back to its initial position. Let μ be the friction coefficient between the block and the table. Let us calculate the work done by friction during the round trip.

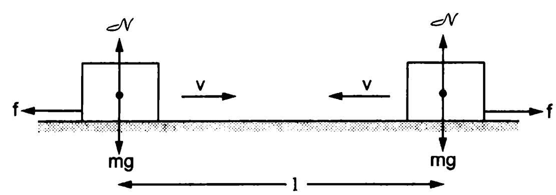


Figure 8.7

The normal force between the table and the block is $\mathcal{N} = mg$ and hence the force of friction is μmg. When the block moves towards right, friction on it is towards left and the work by friction is $(-\mu mgl)$. When the block moves towards left, friction on it is towards right and the work is again $(-\mu mgl)$.

Hence, the total work done by the force of friction in the round trip is $(-2\mu mgl)$.

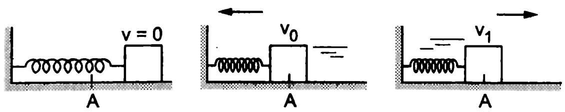


Figure 8.8

(2) Suppose a block connected by a spring is kept on a rough table as shown in figure (8.8). The block is pulled aside and then released. It moves towards the centre A and has some velocity v_0 as it passes through the centre. It goes to the other side of A and then comes back. This time it passes through the centre with somewhat smaller velocity v_1. Compare these two cases in which the block is at A, once going towards left and then towards right. In both the cases the system (table + block + spring) has the same configuration. The spring has the same length. The block is at the same point on the table and the table of course is fixed to the ground. The kinetic energy in the second case is less than the kinetic energy in the first case. This loss in the kinetic energy is a real loss. Every time the block passes through the mean position A, the kinetic energy of the system is smaller and in due course, the block stops on the table. We hold friction as the culprit, because in absence of friction the system regains its kinetic energy as it returns to its original configuration. Remember, work done by friction in a round trip is negative and not zero [example (1) above].

We divide the forces in two categories (a) *conservative forces* and (b) *nonconservative forces*. *If the work done by a force during a round trip of a system is always zero, the force is said to be conservative. Otherwise, it is called nonconservative.*

Conservative force can also be defined as follows :

If the work done by a force depends only on the initial and final states and not on the path taken, it is called a conservative force.

Thus, the force of gravity, Coulomb force and the force of spring are conservative forces, as the work done by these forces are zero in a round trip. The force of friction is nonconservative because the work done by the friction is not zero in a round trip.

8.7 DEFINITION OF POTENTIAL ENERGY AND CONSERVATION OF MECHANICAL ENERGY

We define the *change in potential energy* of a system corresponding to a conservative internal force as

$$U_f - U_i = -W = -\int_i^f \vec{F} \cdot d\vec{r}$$

where W is the work done by the internal force on the system as the system passes from the initial configuration i to the final configuration f.

We don't (or can't) define potential energy corresponding to a nonconservative internal force.

Suppose only conservative internal forces operate between the parts of the system and the potential energy U is defined corresponding to these forces. There are either no external forces or the work done by them is zero. We have

$$U_f - U_i = -W = -(K_f - K_i)$$

or, $$U_f + K_f = U_i + K_i . \qquad \ldots (8.6)$$

The sum of the kinetic energy and the potential energy is called the total *mechanical energy*. We see from equation (8.6) that *the total mechanical energy of a system remains constant if the internal forces are conservative and the external forces do no work.* This is called the *principle of conservation of energy.*

The total mechanical energy $K + U$ is not constant if nonconservative forces, such as friction, act between the parts of the system. We can't apply the principle of conservation of energy in presence of nonconservative forces. The work-energy theorem is still valid even in the presence of nonconservative forces.

Note that only a change in potential energy is defined above. We are free to choose the zero potential energy in any configuration just as we are free to choose the origin in space anywhere we like.

If nonconservative internal forces operate within the system, or external forces do work on the system, the mechanical energy changes as the configuration changes. According to the work-energy theorem, the work done by all the forces equals the change in the kinetic energy. Thus,

$$W_c + W_{nc} + W_{ext} = K_f - K_i$$

where the three terms on the left denote the work done by the conservative internal forces, nonconservative internal forces and the external forces.

As $$W_c = -(U_f - U_i) ,$$

we get

$$W_{nc} + W_{ext} = (K_f + U_f) - (K_i + U_i)$$
$$= E_f - E_i \qquad \ldots (8.7)$$

where $E = K + U$ is the total mechanical energy.

If the internal forces are conservative but external forces also act on the system and they do work, $W_{nc} = 0$ and from (8.7),

$$W_{ext} = E_f - E_i . \qquad \dots (8.8)$$

The work done by the external forces equals the change in the mechanical energy of the system.

Let us summarise the concepts developed so far in this chapter.

(1) Work done on a particle is equal to the change in its kinetic energy.

(2) Work done on a system by all the (external and internal) forces is equal to the change in its kinetic energy.

(3) A force is called conservative if the work done by it during a round trip of a system is always zero. The force of gravitation, Coulomb force, force by a spring etc. are conservative. If the work done by it during a round trip is not zero, the force is nonconservative. Friction is an example of nonconservative force.

(4) The change in the potential energy of a system corresponding to conservative internal forces is equal to negative of the work done by these forces.

(5) If no external forces act (or the work done by them is zero) and the internal forces are conservative, the mechanical energy of the system remains constant. This is known as the principle of conservation of mechanical energy.

(6) If some of the internal forces are nonconservative, the mechanical energy of the system is not constant.

(7) If the internal forces are conservative, the work done by the external forces is equal to the change in mechanical energy.

Example 8.3

Two charged particles A and B repel each other by a force k/r^2, where k is a constant and r is the separation between them. The particle A is clamped to a fixed point in the lab and the particle B which has a mass m, is released from rest with an initial separation r_0 from A. Find the change in the potential energy of the two-particle system as the separation increases to a large value. What will be the speed of the particle B in this situation?

Solution : The situation is shown in figure (8.9). Take $A + B$ as the system. The only external force acting on the system is that needed to hold A fixed. (You can imagine the experiment being conducted in a gravity free region or the particles may be kept and allowed to move on a smooth horizontal surface, so that the normal force balances the force of gravity). This force does no work on the system because it acts on the charge A which does not move. Thus, the external forces do no work and internal forces are conservative. The total mechanical energy must, therefore, remain constant. There are two internal forces; F_{AB} acting on A and F_{BA} acting on B. The force F_{AB} does no work because it acts on A which does not move. The work done by F_{BA} as the particle B is taken away is,

$$W = \int \vec{F} \cdot d\vec{r} = \int_{r_0}^{\infty} \frac{k}{r^2} dr = \frac{k}{r_0} . \qquad \dots \text{(i)}$$

Figure 8.9

The change in the potential energy of the system is

$$U_f - U_i = -W = -\frac{k}{r_0} .$$

As the total mechanical energy is conserved,

$$K_f + U_f = K_i + U_i$$

or, $$K_f = K_i - (U_f - U_i)$$

or, $$\frac{1}{2} mv^2 = \frac{k}{r_0}$$

or, $$v = \sqrt{\frac{2k}{mr_0}} .$$

8.8 CHANGE IN THE POTENTIAL ENERGY IN A RIGID-BODY-MOTION

If the separation between the particles do not change during motion, such as in the case of the motion of a rigid body, the internal forces do no work. This is a consequence of Newton's third law. As an example, consider a system of two particles A and B. Suppose, the particles move in such a way that the line AB translates parallel to itself. The displacement $d\vec{r_A}$ of the particle A is equal to the displacement $d\vec{r_B}$ of the particle B in any short time interval. The net work done by the internal forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ is

$$W = \int (\vec{F}_{AB} \cdot d\vec{r_A} + \vec{F}_{BA} \cdot d\vec{r_B})$$

$$= \int (\vec{F}_{AB} + \vec{F}_{BA}) \cdot d\vec{r_A} = 0.$$

Thus, the work done by $\vec{F}_{AB}$ and $\vec{F}_{BA}$ add up to zero. Even if AB does not translate parallel to itself but rotates, the result is true. The internal forces acting between the particles of a rigid body do no work in its motion and we need not consider the potential energy corresponding to these forces.

The potential energy of a system changes only when the separations between the parts of the system change. In other words, *the potential energy depends only on the separation between the interacting particles.*

8.9 GRAVITATIONAL POTENTIAL ENERGY

Consider a block of mass m kept near the surface of the earth and suppose it is raised through a height h. Consider "the earth + the block" as the system. The gravitational force between the earth and the block is conservative and we can define a potential energy corresponding to this force. The earth is very heavy as compared to the block and so one can neglect its acceleration. Thus, we take our reference frame attached to the earth, it will still be very nearly an inertial frame. The work done by the gravitational force due to the block on the earth is zero in this frame. The force mg on the block does work $(-mgh)$ if the block ascends through a height h and hence the potential energy is increased by mgh. Thus, *if a block of mass m ascends a height h above the earth's surface (h << radius of earth), the potential energy of the "earth + block" system increases by mgh.* If the block descends by a height h, the potential energy decreases by mgh. Since the earth almost remains fixed, it is customary to call the potential energy of the earth-block system as the potential energy of the block only. We then say that the gravitational potential energy of the "block" is increased by an amount mgh when it is raised through a hieght h above the earth's surface.

We have been talking in terms of the changes in gravitational potential energy. We can choose any position of the block and call the gravitational potential energy to be zero in this position. The potential energy at a height h above this position is mgh. The position of the zero potential energy is chosen according to the convenience of the problem.

Example 8.4

A block of mass m slides along a frictionless surface as shown in the figure (8.10). If it is released from rest at A, what is its speed at B?

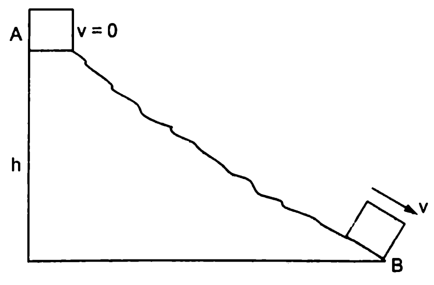


Figure 8.10

Solution : Take the block + the earth as the system. Only the block moves, so only the work done on the block will contribute to the gravitational potential energy. As it descends through a height h between A and B, the potential energy decreases by mgh. The normal contact force $\mathcal{N}$ on the block by the surface does no work as it is perpendicular to its velocity. No external force does any work on the system. Hence,

increase in kinetic energy = decrease in potential energy

or, $$\frac{1}{2}mv^2 = mgh \text{ or, } v = \sqrt{2gh}.$$

Example 8.5

A pendulum bob has a speed 3 m/s *while passing through its lowest position. What is its speed when it makes an angle of* 60° *with the vertical? The length of the pendulum is* 0·5 m. *Take* $g = 10 \text{ m/s}^2$.

Solution : Take the bob + earth as the system. The external force acting on the system is that due to the string. But this force is always perpendicular to the velocity of the bob and so the work done by this force is zero. Hence, the total mechanical energy will remain constant. As is clear from figure (8.11), the height ascended by the bob at an angular displacement θ is $l - l\cos\theta = l(1 - \cos\theta)$. The increase in the potential energy is $mgl(1 - \cos\theta)$. This should be equal to the decrease in the kinetic energy of the system. Again, as the earth does not move in the lab frame, this is the decrease in the kinetic energy of the bob. If the speed at an angular displacement θ is v_1, the decrease in kinetic energy is

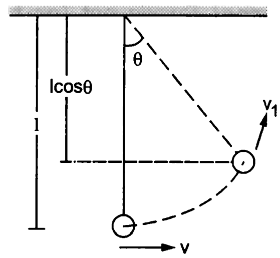


Figure 8.11

$$\frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2,$$

where v_0 is the speed of the block at the lowest position.

Thus, $$\frac{1}{2}mv_0^2 - \frac{1}{2}mv_1^2 = mgl(1 - \cos\theta)$$

or, $$v_1 = \sqrt{v_0^2 - 2gl(1 - \cos\theta)}$$

$$= \sqrt{(9 \text{ m}^2/\text{s}^2) - 2 \times (10 \text{ m/s}^2) \times (0\cdot5 \text{ m})\left(1 - \frac{1}{2}\right)}$$

$$= 2 \text{ m/s}.$$

8.10 POTENTIAL ENERGY OF A COMPRESSED OR EXTENDED SPRING

Consider a massless spring of natural length l, one end of which is fastened to a wall (figure 8.12). The other end is attached to a block which is slowly pulled on a smooth horizontal surface to extend the spring. Take the spring as the system. When it is elongated by a distance x, the tension in it is kx, where k is its spring constant. It pulls the wall towards right and the block towards left by forces of magnitude kx. The forces exerted on the spring are (i) kx towards left by the wall and (ii) kx towards right by the block.

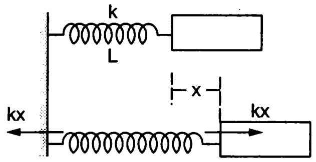


Figure 8.12

How much work has been done on the spring by these two external forces ? The force by the wall does no work as the point of application is fixed. The force by the block does work $\int_0^x kx\,dx = \frac{1}{2}kx^2$. The work is positive as the force is towards right and the particles of the spring, on which this force is acting, also move towards right. Thus, the total external work done on the spring is $\frac{1}{2}kx^2$, when the spring is elongated by an amount x from its natural length. The same is the external work done on the spring if it is compressed by a distance x.

We have seen (equation 8.8) that the external work done on a system is equal to the change in its total mechanical energy. The spring is assumed to be massless and hence its kinetic energy remains zero all the time. Thus, its potential energy has increased by $\frac{1}{2}kx^2$.

We conclude that a stretched or compressed spring has a potential energy $\frac{1}{2}kx^2$ larger than its potential energy at its natural length. The potential energy of the spring corresponds to the internal forces between the particles of the spring when it is stretched or compressed. It is called *elastic potential energy* or the *strain energy* of the spring. Again, the calculation gives only the change in the elastic potential energy of the spring and we are free to choose any length of the spring and call the potential energy zero at that length. It is customary to choose the potential energy of a spring in its natural length to be zero. With this choice *the potential energy of a spring is* $\frac{1}{2}kx^2$, *where* x *is the elongation or the compression of the spring.*

Example 8.6

A block of mass m, attached to a spring of spring constant k, oscillates on a smooth horizontal table. The other end of the spring is fixed to a wall. If it has a speed v when the spring is at its natural length, how far will it move on the table before coming to an instantaneous rest ?

Solution : Consider the block + the spring as the system. The external forces acting on the system are (a) the force of gravity, (b) the normal force by the table and (c) the force by the wall. None of these do any work on this system and hence the total mechanical energy is conserved. If the block moves a distance x before comming to rest, we have,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

or, $$x = v\sqrt{m/k}.$$

Example 8.7

A block of mass m is suspended through a spring of spring constant k and is in equilibrium. A sharp blow gives the block an initial downward velocity v. How far below the equilibrium position, the block comes to an instantaneous rest ?

Solution : Let us consider the block + the spring + the earth as the system. The system has gravitational potential energy corresponding to the force between the block and the earth as well as the elastic potential energy corresponding to the spring-force. The total mechanical energy includes kinetic energy, gravitational potential energy and elastic potential energy.

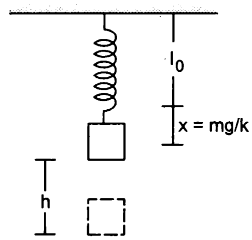


Figure 8.13

When the block is in equilibrium, it is acted upon by two forces, (a) the force of gravity mg and (b) the tension in the spring $T = kx$, where x is the elongation. For equilibrium, $mg = kx$, so that the spring is stretched by a length $x = mg/k$. The potential energy of the spring in this position is

$$\frac{1}{2}k\,(mg/k)^2 = \frac{m^2g^2}{2k}.$$

Take the gravitational potential energy to be zero in this position. The total mechanical energy of the system just after the blow is

$$\frac{1}{2}mv^2 + \frac{m^2g^2}{2k}.$$

The only external force on this system is that due to the ceiling which does no work. Hence, the mechanical energy of this system remains constant. If the block descends through a height h before coming to an instantaneous rest, the elastic potential energy becomes $\frac{1}{2} k\,(mg/k + h)^2$ and the gravitational potential energy $-\,mgh$. The kinetic energy is zero in this state. Thus, we have

$$\frac{1}{2} mv^2 + \frac{m^2 g^2}{2k} = \frac{1}{2} k\,(mg/k + h)^2 - mgh.$$

Solving this we get,

$$h = v\sqrt{m/k}.$$

Compare this with the result obtained in Example (8.6). If we neglect gravity and consider the length of the spring in equilibrium position as the natural length, the answer is same. This simplification is often used while dealing with vertical springs.

8.11 DIFFERENT FORMS OF ENERGY : MASS ENERGY EQUIVALENCE

The kinetic energy and the potential energy of a system, taken together, form mechanical energy. Energy can exist in many other forms. In measuring kinetic energy of an extended body, we use the speed of the body as a whole. Even if we keep the body at rest, the particles in it are continuously moving inside the body. These particles also exert forces on each other and there is a potential energy corresponding to these forces. The total energy corresponding to the internal motion of molecules and their interaction, is called *internal energy* or *thermal energy* of the body. Light and sound are other forms of energy. When a source emits light or sound, it loses energy. Chemical energy is significant if there are chemical reactions.

Einstein's special theory of relativity shows that a material particle itself is a form of energy. Thus, about $8{\cdot}18 \times 10^{-14}$ J of energy may be converted to form an electron and equal amount of energy may be obtained by destroying an electron. The ralation between the mass of a particle m and its equivalent energy E is given as

$$E = mc^2,$$

where $c = 3 \times 10^8$ m/s is the speed of light in vacuum.

When all forms of energy are taken into account, we arrive at the generalised law of conservation of energy.

Energy can never be created or destroyed, it can only be changed from one form into another.

Worked Out Examples

1. *A porter lifts a suitcase weighing* 20 kg *from the platform and puts it on his head* 2·0 m *above the platform. Calculate the work done by the porter on the suitcase.*

Solution : The kinetic energy of the suitcase was zero when it was at the platform and it again became zero when it was put on the head. The change in kinetic energy is zero and hence the total work done on the suitcase is zero. Two forces act on the suitcase, one due to gravity and the other due to the porter. Thus, the work done by the porter is negative of the work done by gravity. As the suitcase is lifted up, the work done by gravity is

$$W = -\,mgh$$
$$= -\,(20\text{ kg})\,(9{\cdot}8\text{ m/s}^2)\,(2\text{ m}) = -\,392\text{ J}$$

The work done by the porter is 392 J $\approx$ 390 J.

2. *An elevator weighing* 500 kg *is to be lifted up at a constant velocity of* 0·20 m/s. *What would be the minimum horsepower of the motor to be used ?*

Solution : As the elevator is going up with a uniform velocity, the total work done on it is zero in any time interval. The work done by the motor is, therefore, equal to the work done by the force of gravity in that interval (in magnitude). The rate of doing work, i.e., the power delivered is

$$P = F\,v = mgv$$
$$= (500\text{ kg})\,(9{\cdot}8\text{ m/s}^2)\,(0{\cdot}2\text{ m/s}) = 980\text{ W}$$

Assuming no loss against friction etc., in the motor, the minimum horsepower of the motor is

$$P = 980\text{ W} = \frac{980}{746}\text{ hp} = 1{\cdot}3\text{ hp}.$$

3. *A block of mass* 2·0 kg *is pulled up on a smooth incline of angle* 30° *with the horizontal. If the block moves with an acceleration of* 1·0 m/s², *find the power delivered by the pulling force at a time* 4·0 s *after the motion starts. What is the average power delivered during the* 4·0 s *after the motion starts ?*

Solution : The forces acting on the block are shown in figure (8-W1). Resolving the forces parallel to the incline, we get

$$F - mg\sin\theta = ma$$

or, $$F = mg\sin\theta + ma$$
$$= (2{\cdot}0\text{ kg})\,[(9{\cdot}8\text{ m/s}^2)\,(1/2) + 1{\cdot}0\text{ m/s}^2] = 11{\cdot}8\text{ N}.$$

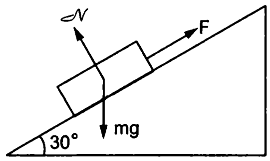


Figure 8-W1

The velocity at $t = 4{\cdot}0$ s is

$$v = at = (1{\cdot}0 \text{ m/s}^2)\,(4{\cdot}0 \text{ s}) = 4{\cdot}0 \text{ m/s}.$$

The power delivered by the force at $t = 4{\cdot}0$ s is

$$P = \vec{F} \cdot \vec{v} = (11{\cdot}8 \text{ N})\,(4{\cdot}0 \text{ m/s}) \approx 47 \text{ W}.$$

The displacement during the first four seconds is

$$x = \frac{1}{2}at^2 = \frac{1}{2}(1{\cdot}0 \text{ m/s}^2)\,(16 \text{ s}^2) = 8{\cdot}0 \text{ m}.$$

The work done in these four seconds is, therefore,

$$W = \vec{F} \cdot \vec{d} = (11{\cdot}8 \text{ N})\,(8{\cdot}0 \text{ m}) = 94{\cdot}4 \text{ J}.$$

$$\text{The average power delivered} = \frac{94{\cdot}4 \text{ J}}{4{\cdot}0 \text{ s}}$$

$$= 23{\cdot}6 \text{ W} \approx 24 \text{ W}.$$

4. *A force $F = (10 + 0{\cdot}50\,x)$ acts on a particle in the x direction, where F is in newton and x in meter. Find the work done by this force during a displacement from $x = 0$ to $x = 2{\cdot}0$ m.*

Solution : As the force is variable, we shall find the work done in a small displacement x to $x + dx$ and then integrate it to find the total work. The work done in this small displacement is

$$dW = \vec{F} \cdot \vec{dx} = (10 + 0{\cdot}5\,x)\,dx.$$

$$\text{Thus, } W = \int_0^{2\cdot0} (10 + 0{\cdot}50\,x)\,dx$$

$$= \left[10\,x + 0{\cdot}50\,\frac{x^2}{2}\right]_0^{2\cdot0} = 21 \text{ J}.$$

5. *A body dropped from a height H reaches the ground with a speed of $1{\cdot}2\sqrt{gH}$. Calculate the work done by air-friction.*

Solution : The forces acting on the body are the force of gravity and the air-friction. By work-energy theorem, the total work done on the body is

$$W = \frac{1}{2}m(1{\cdot}2\sqrt{gH})^2 - 0 = 0{\cdot}72\,mgH.$$

The work done by the force of gravity is mgH. Hence, the work done by the air-friction is

$$0{\cdot}72\,mgH - mgH = -\,0{\cdot}28\,mgH.$$

6. *A block of mass M is pulled along a horizontal surface by applying a force at an angle θ with the horizontal. The friction coefficient between the block and the surface is μ. If the block travels at a uniform velocity, find the work done by this applied force during a displacement d of the block.*

Solution : Forces on the block are

(i) its weight Mg,
(ii) the normal force $\mathcal{N}$,
(iii) the applied force F and
(iv) the kinetic friction $\mu\mathcal{N}$.

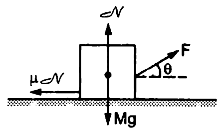


Figure 8-W2

The forces are shown in figure (8-W2). As the block moves with a uniform velocity, the forces add up to zero. Taking horizontal and vertical components,

$$F\cos\theta = \mu\mathcal{N}$$

$$\text{and} \qquad F\sin\theta + \mathcal{N} = Mg.$$

Eliminating $\mathcal{N}$ from these equations,

$$F\cos\theta = \mu\,(Mg - F\sin\theta)$$

$$\text{or,} \qquad F = \frac{\mu\,Mg}{\cos\theta + \mu\sin\theta}.$$

The work done by this force during a displacement d is

$$W = F\,d\cos\theta = \frac{\mu\,Mgd\cos\theta}{\cos\theta + \mu\sin\theta}.$$

7. *Two cylindrical vessels of equal cross-sectional area A contain water upto heights h_1 and h_2. The vessels are interconnected so that the levels in them become equal. Calculate the work done by the force of gravity during the process. The density of water is ρ.*

Solution : Since the total volume of the water is constant, the height in each vessel after interconnection will be $(h_1 + h_2)/2$. The level in the left vessel shown in the figure, drops from A to C and that in the right vessel rises from B to D. Effectively, the water in the part AC has dropped down to DB.

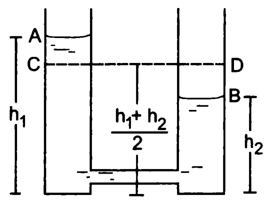


Figure 8-W3

The mass of this volume of water is

$$m = \rho A\left(h_1 - \frac{h_1 + h_2}{2}\right)$$

$$= \rho A \left(\frac{h_1 - h_2}{2} \right).$$

The height descended by this water is $AC = (h_1 - h_2)/2$. The work done by the force of gravity during this process is, therefore,

$$= \rho A \left(\frac{h_1 - h_2}{2} \right)^2 g.$$

8. *What minimum horizontal speed should be given to the bob of a simple pendulum of length l so that it describes a complete circle ?*

Solution : Suppose the bob is given a horizontal speed v_0 at the bottom and it describes a complete vertical circle. Let its speed at the highest point be v. Taking the gravitational potential energy to be zero at the bottom, the conservation of energy gives,

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + 2mgl$$

or, $$mv^2 = mv_0^2 - 4\,mgl. \qquad \text{... (i)}$$

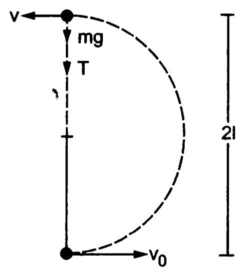


Figure 8-W4

The forces acting on the bob at the highest point are mg due to the gravity and T due to the tension in the string. The resultant force towards the centre is, therefore, $mg + T$. As the bob is moving in a circle, its acceleration towards the centre is v^2/l. Applying Newton's second law and using (i),

$$mg + T = m\frac{v^2}{l} = \frac{1}{l}(mv_0^2 - 4mgl)$$

or, $$mv_0^2 = 5\,mgl + T\,l.$$

Now, for v_0 to be minimum, T should be minimum. As the minimum value of T can be zero, for minimum speed,

$$mv_0^2 = 5\,mgl \quad \text{or,} \quad v_0 = \sqrt{5\,gl}.$$

9. *A uniform chain of length l and mass m overhangs a smooth table with its two third part lying on the table. Find the kinetic energy of the chain as it completely slips off the table.*

Solution : Let us take the zero of potential energy at the table. Consider a part dx of the chain at a depth x below the surface of the table. The mass of this part is $dm = m/l \;\; dx$ and hence its potential energy is $-(m/l\; dx)gx$.

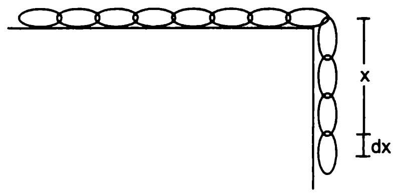


Figure 8-W5

The potential energy of the $l/3$ of the chain that overhangs is $U_1 = \int_0^{l/3} -\frac{m}{l} gx\, dx$

$$= -\left[\frac{m}{l} g \left(\frac{x^2}{2} \right) \right]_0^{l/3} = -\frac{1}{18} mgl.$$

This is also the potential energy of the full chain in the initial position because the part lying on the table has zero potential energy. The potential energy of the chain when it completely slips off the table is

$$U_2 = \int_0^{l} -\frac{m}{l} gx\, dx = -\frac{1}{2} mgl.$$

The loss in potential energy $= \left(-\frac{1}{18} mgl \right) - \left(-\frac{1}{2} mgl \right)$

$$= \frac{4}{9} mgl.$$

This should be equal to the gain in the kinetic energy. But the initial kinetic enegry is zero. Hence, the kinetic energy of the chain as it completely slips off the table is $\frac{4}{9} mgl$.

10. *A block of mass m is pushed against a spring of spring constant k fixed at one end to a wall. The block can slide on a frictionless table as shown in figure (8-W6). The natural length of the spring is L_0 and it is compressed to half its natural length when the block is released. Find the velocity of the block as a function of its distance x from the wall.*

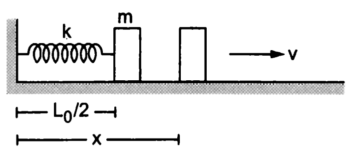


Figure 8-W6

Solution : When the block is released, the spring pushes it towards right. The velocity of the block increases till the spring acquires its natural length. Thereafter, the block loses contact with the spring and moves with constant velocity.

Initially, the compression of the spring is $L_0/2$. When the distance of the block from the wall becomes x, where

$x < L_0$, the compression is $(L_0 - x)$. Using the principle of conservation of energy,

$$\frac{1}{2} k \left(\frac{L_0}{2} \right)^2 = \frac{1}{2} k \, (L_0 - x)^2 + \frac{1}{2} mv^2.$$

Solving this,

$$v = \sqrt{\frac{k}{m}} \left[\frac{L_0^2}{4} - (L_0 - x)^2 \right]^{1/2}$$

When the spring acquires its natural length, $x = L_0$ and $v = \sqrt{\frac{k}{m}} \frac{L_0}{2}$. Thereafter, the block continues with this velocity.

11. *A particle is placed at the point A of a frictionless track ABC as shown in figure (8-W7). It is pushed slightly towards right. Find its speed when it reaches the point B. Take* $g = 10 \text{ m/s}^2$.

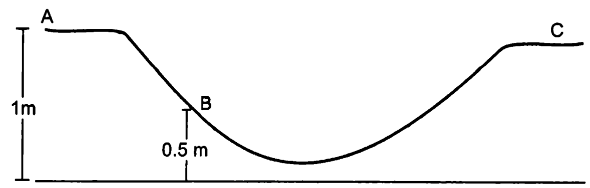


Figure 8-W7

Solution : Let us take the gravitational potential energy to be zero at the horizontal surface shown in the figure. The potential energies of the particle at A and B are

$$U_A = Mg \text{ (1 m)}$$

and $$U_B = Mg \text{ (0·5 m)}.$$

The kinetic energy at the point A is zero. As the track is frictionless, no energy is lost. The normal force on the particle does no work. Applying the principle of conservation of energy,

$$U_A + K_A = U_B + K_B$$

or, $$Mg(1 \text{ m}) = Mg(0·5 \text{ m}) + \frac{1}{2} M v_B^2$$

or, $$\frac{1}{2} v_B^2 = g(1 \text{ m} - 0·5 \text{ m})$$

$$= (10 \text{ m/s}^2) \times 0·5 \text{ m}$$

$$= 5 \text{ m}^2/\text{s}^2$$

or, $$v_B = \sqrt{10} \text{ m/s}.$$

12. *Figure (8-W8) shows a smooth curved track terminating in a smooth horizontal part. A spring of spring constant* 400 N/m *is attached at one end to a wedge fixed rigidly with the horizontal part. A* 40 g *mass is released from rest at a height* of 4·9 m *on the curved track. Find the maximum compression of the spring.*

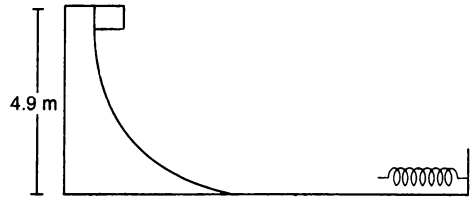


Figure 8-W8

Solution : At the instant of maximum compression the speed of the 40 g mass reduces to zero. Taking the gravitational potential energy to be zero at the horizontal part, the conservation of energy shows,

$$mgh = \frac{1}{2} kx^2$$

where m = 0·04 kg, h = 4·9 m, k = 400 N/m and x is the maximum compression.

Thus, $$x = \sqrt{\frac{2mgh}{k}}$$

$$= \sqrt{\frac{2 \times (0·04 \text{ kg}) \times (9·8 \text{ m/s}^2) \times (4·9 \text{ m})}{(400 \text{ N/m})}}$$

$$= 9·8 \text{ cm}.$$

13. *Figure (8-W9) shows a loop-the-loop track of radius R. A car (without engine) starts from a platform at a distance h above the top of the loop and goes around the loop without falling off the track. Find the minimum value of h for a successful looping. Neglect friction.*

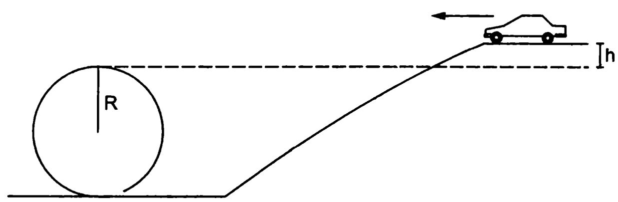


Figure 8-W9

Solution : Suppose the speed of the car at the topmost point of the loop is v. Taking the gravitational potential energy to be zero at the platform and assuming that the car starts with a negligible speed, the conservation of energy shows,

$$0 = -mgh + \frac{1}{2} mv^2$$

or, $$mv^2 = 2\,mgh, \qquad \text{... (i)}$$

where m is the mass of the car. The car moving in a circle must have radial acceleration v^2/R at this instant. The forces on the car are, mg due to gravity and $\mathcal{N}$ due to the contact with the track. Both these forces are in radial direction at the top of the loop. Thus, from Newton's Law

$$mg + \mathcal{N} = \frac{mv^2}{R}$$

or, $$mg + \mathcal{N} = 2\, mgh/R.$$

For h to be minimum, $\mathcal{N}$ should assume the minimum value which can be zero. Thus,

$$2\, mg \frac{h_{\min}}{R} = mg \quad \text{or,} \quad h_{\min} = R/2.$$

14. *A heavy particle is suspended by a string of length l. The particle is given a horizontal velocity v_0. The string becomes slack at some angle and the particle proceeds on a parabola. Find the value of v_0 if the particle passes through the point of suspension.*

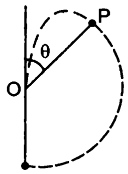


Figure 8-W10

Solution : Suppose the string becomes slack when the particle reaches the point P (figure 8-W10). Suppose the string OP makes an angle θ with the upward vertical. The only force acting on the particle at P is its weight mg. The radial component of the force is $mg\cos\theta$. As the particle moves on the circle upto P,

$$mg \cos\theta = m\left(\frac{v^2}{l}\right)$$

or, $$v^2 = gl\cos\theta \qquad \text{... (i)}$$

where v is its speed at P. Using conservation of energy,

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 + mgl\,(1 + \cos\theta)$$

or, $$v^2 = v_0^2 - 2gl(1 + \cos\theta). \qquad \text{... (ii)}$$

From (i) and (ii), $v_0^2 - 2\,g\,l\,(1 + \cos\theta) = g\,l\cos\theta$

or, $$v_0^2 = gl\,(2 + 3\cos\theta). \qquad \text{... (iii)}$$

Now onwards the particle goes in a parabola under the action of gravity. As it passes through the point of suspension O, the equations for horizontal and vertical motions give,

$$l\sin\theta = (v\cos\theta)\,t$$

and $$-l\cos\theta = (v\sin\theta)\,t - \frac{1}{2} g t^2$$

or, $$-l\cos\theta = (v\sin\theta)\left(\frac{l\sin\theta}{v\cos\theta}\right) - \frac{1}{2} g \left(\frac{l\sin\theta}{v\cos\theta}\right)^2$$

or, $$-\cos^2\theta = \sin^2\theta - \frac{1}{2} g \frac{l\sin^2\theta}{v^2\cos\theta}$$

or, $$-\cos^2\theta = 1 - \cos^2\theta - \frac{1}{2}\frac{gl\sin^2\theta}{gl\cos^2\theta} \quad \text{[From (i)]}$$

or, $$1 = \frac{1}{2}\tan^2\theta$$

or, $$\tan\theta = \sqrt{2}.$$

From (iii), $v_0 = [gl\,(2 + \sqrt{3})]^{1/2}$.

□

QUESTIONS FOR SHORT ANSWER

1. When you lift a box from the floor and put it on an almirah the potential energy of the box increases, but there is no change in its kinetic energy. Is it a violation of conservation of energy ?

2. A particle is released from the top of an incline of height h. Does the kinetic energy of the particle at the bottom of the incline depend on the angle of incline ? Do you need any more information to answer this question in Yes or No ?

3. Can the work by kinetic friction on an object be positive ? Zero ?

4. Can static friction do nonzero work on an object ? If yes, give an example. If no, give reason.

5. Can normal force do a nonzero work on an object. If yes, give an example. If no, give reason.

6. Can kinetic energy of a system be increased without applying any external force on the system ?

7. Is work-energy theorem valid in noninertial frames ?

8. A heavy box is kept on a smooth inclined plane and is pushed up by a force F acting parallel to the plane. Does the work done by the force F as the box goes from A to B depend on how fast the box was moving at A and B ? Does the work by the force of gravity depend on this ?

9. One person says that the potential energy of a particular book kept in an almirah is 20 J and the other says it is 30 J. Is one of them necessarily wrong ?

10. A book is lifted from the floor and is kept in an almirah. One person says that the potential energy of the book is increased by 20 J and the other says it is increased by 30 J. Is one of them necessarily wrong ?

11. In one of the exercises to strengthen the wrist and fingers, a person squeezes and releases a soft rubber ball. Is the work done on the ball positive, negative or zero during compression ? During expansion ?

12. In tug of war, the team that exerts a larger tangential force on the ground wins. Consider the period in which a team is dragging the opposite team by applying a larger tangential force on the ground. List which of the

following works are positive, which are negative and which are zero?
(a) work by the winning team on the losing team
(b) work by the losing team on the winning team
(c) work by the ground on the winning team
(d) work by the ground on the losing team
(e) total external work on the two teams.

13. When an apple falls from a tree what happens to its gravitational potential energy just as it reaches the ground? After it strikes the ground?

14. When you push your bicycle up on an incline the potential energy of the bicyle and yourself increases. Where does this energy come from?

15. The magnetic force on a charged particle is always perpendicular to its velocity. Can the magnetic force change the velocity of the particle? Speed of the particle?

16. A ball is given a speed v on a rough horizontal surface. The ball travels through a distance l on the surface and stops. (a) What are the initial and final kinetic energies of the ball? (b) What is the work done by the kinetic friction?

17. Consider the situation of the previous question from a frame moving with a speed v_0 parallel to the initial velocity of the block. (a) What are the initial and final kinetic energies? (b) What is the work done by the kinetic friction?

OBJECTIVE I

1. A heavy stone is thrown from a cliff of height h with a speed v. The stone will hit the ground with maximum speed if it is thrown
(a) vertically downward (b) vertically upward
(c) horizontally
(d) the speed does not depend on the initial direction.

2. Two springs A and $B(k_A = 2k_B)$ are stretched by applying forces of equal magnitudes at the four ends. If the energy stored in A is E, that in B is
(a) $E/2$ (b) $2E$ (c) E (d) $E/4$.

3. Two equal masses are attached to the two ends of a spring of spring constant k. The masses are pulled out symmetrically to stretch the spring by a length x over its natural length. The work done by the spring on each mass is
(a) $\frac{1}{2}kx^2$ (b) $-\frac{1}{2}kx^2$ (c) $\frac{1}{4}kx^2$ (d) $-\frac{1}{4}kx^2$.

4. The negative of the work done by the conservative internal forces on a system equals the change in
(a) total energy (b) kinetic energy
(c) potential energy (d) none of these.

5. The work done by the external forces on a system equals the change in
(a) total energy (b) kinetic energy
(c) potential energy (d) none of these.

6. The work done by all the forces (external and internal) on a system equals the change in
(a) total energy (b) kinetic energy
(c) potential energy (d) none of these.

7. __________ of a two particle system depends only on the separation between the two particles. The most appropriate choice for the blank space in the above sentence is
(a) Kinetic energy (b) Total mechanical energy
(c) Potential energy (d) Total energy.

8. A small block of mass m is kept on a rough inclined surface of inclination θ fixed in an elevator. The elevator goes up with a uniform velocity v and the block does not slide on the wedge. The work done by the force of friction on the block in time t will be
(a) zero (b) $mgvt\cos^2\theta$
(c) $mgvt\sin^2\theta$ (d) $mgvt\sin 2\theta$.

9. A block of mass m slides down a smooth vertical circular track. During the motion, the block is in
(a) vertical equilibrium (b) horizontal equilibrium
(c) radial equilibrium (d) none of these.

10. A particle is rotated in a vertical circle by connecting it to a string of length l and keeping the other end of the string fixed. The minimum speed of the particle when the string is horizontal for which the particle will complete the circle is
(a) $\sqrt{gl}$ (b) $\sqrt{2gl}$ (c) $\sqrt{3gl}$ (d) $\sqrt{5gl}$.

OBJECTIVE II

1. A heavy stone is thrown from a cliff of height h in a given direction. The speed with which it hits the ground
(a) must depend on the speed of projection
(b) must be larger than the speed of projection
(c) must be independent of the speed of projection
(d) may be smaller than the speed of projection.

2. The total work done on a particle is equal to the change in its kinetic energy
(a) always

(b) only if the forces acting on it are conservative
(c) only if gravitational force alone acts on it
(d) only if elastic force alone acts on it.

3. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that
(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a circular path.

4. Consider two observers moving with respect to each other at a speed v along a straight line. They observe a block of mass m moving a distance l on a rough surface. The following quantities will be same as observed by the two observers
(a) kinetic energy of the block at time t
(b) work done by friction
(c) total work done on the block
(d) acceleration of the block.

5. You lift a suitcase from the floor and keep it on a table. The work done by you on the suitcase does not depend on
(a) the path taken by the suitcase
(b) the time taken by you in doing so
(c) the weight of the suitcase
(d) your weight.

6. No work is done by a force on an object if
(a) the force is always perpendicular to its velocity
(b) the force is always perpendicular to its acceleration
(c) the object is stationary but the point of application of the force moves on the object
(d) the object moves in such a way that the point of application of the force remains fixed.

7. A particle of mass m is attached to a light string of length l, the other end of which is fixed. Initially the string is kept horizontal and the particle is given an upward velocity v. The particle is just able to complete a circle.
(a) The string becomes slack when the particle reaches its highest point.
(b) The velocity of the particle becomes zero at the highest point.
(c) The kinetic energy of the ball in initial position was $\frac{1}{2}mv^2 = mgl$.
(d) The particle again passes through the initial position.

8. The kinetic energy of a particle continuously increases with time.
(a) The resultant force on the particle must be parallel to the velocity at all instants.
(b) The resultant force on the particle must be at an angle less than 90° all the time.
(c) Its height above the ground level must continuously decrease.
(d) The magnitude of its linear momentum is increasing continuously.

9. One end of a light spring of spring constant k is fixed to a wall and the other end is tied to a block placed on a smooth horizontal surface. In a displacement, the work done by the spring is $\frac{1}{2}kx^2$. The possible cases are
(a) the spring was initially compressed by a distance x and was finally in its natural length
(b) it was initially streched by a distance x and finally was in its natural length
(c) it was initially in its natural length and finally in a compressed position
(d) it was initially in its natural length and finally in a stretched position.

10. A block of mass M is hanging over a smooth and light pulley through a light string. The other end of the string is pulled by a constant force F. The kinetic energy of the block increases by 20 J in 1 s.
(a) The tension in the string is Mg.
(b) The tension in the string is F.
(c) The work done by the tension on the block is 20 J in the above 1 s.
(d) The work done by the force of gravity is –20 J in the above 1 s.

EXERCISES

1. The mass of cyclist together with the bike is 90 kg. Calculate the increase in kinetic energy if the speed increases from 6·0 km/h to 12 km/h.

2. A block of mass 2·00 kg moving at a speed of 10·0 m/s accelerates at 3·00 m/s^2 for 5·00 s. Compute its final kinetic energy.

3. A box is pushed through 4·0 m across a floor offering 100 N resistance. How much work is done by the resisting force ?

4. A block of mass 5·0 kg slides down an incline of inclination 30° and length 10 m. Find the work done by the force of gravity.

5. A constant force of 2·50 N accelerates a stationary particle of mass 15 g through a displacement of 2·50 m. Find the work done and the average power delivered.

6. A particle moves from a point $\vec{r}_1 = (2\text{ m})\vec{i} + (3\text{ m})\vec{j}$ to another point $\vec{r}_2 = (3\text{ m})\vec{i} + (2\text{ m})\vec{j}$ during which a certain force $\vec{F} = (5\text{ N})\vec{i} + (5\text{ N})\vec{j}$ acts on it. Find the work done by the force on the particle during the displacement.

7. A man moves on a straight horizontal road with a block of mass 2 kg in his hand. If he covers a distance of 40 m with an acceleration of 0·5 m/s^2, find the work done by the man on the block during the motion.

8. A force $F = a + bx$ acts on a particle in the x-direction, where a and b are constants. Find the work done by this force during a displacement from $x = 0$ to $x = d$.

9. A block of mass 250 g slides down an incline of inclination 37° with a uniform speed. Find the work done against the friction as the block slides through 1·0 m.

10. A block of mass m is kept over another block of mass M and the system rests on a horizontal surface (figure 8-E1). A constant horizontal force F acting on the lower block produces an acceleration $\frac{F}{2(m+M)}$ in the system, the two blocks always move together. (a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface. (b) Find the frictional force acting on the smaller block. (c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement d of the system.

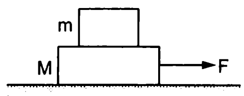


Figure 8-E1

11. A box weighing 2000 N is to be slowly slid through 20 m on a straight track having friction coefficient 0·2 with the box. (a) Find the work done by the person pulling the box with a chain at an angle θ with the horizontal. (b) Find the work when the person has chosen a value of θ which ensures him the minimum magnitude of the force.

12. A block of weight 100 N is slowly slid up on a smooth incline of inclination 37° by a person. Calculate the work done by the person in moving the block through a distance of 2·0 m, if the driving force is (a) parallel to the incline and (b) in the horizontal direction.

13. Find the average frictional force needed to stop a car weighing 500 kg in a distance of 25 m if the initial speed is 72 km/h.

14. Find the average force needed to accelerate a car weighing 500 kg from rest to 72 km/h in a distance of 25 m.

15. A particle of mass m moves on a straight line with its velocity varying with the distance travelled according to the equation $v = a\sqrt{x}$, where a is a constant. Find the total work done by all the forces during a displacement from $x = 0$ to $x = d$.

16. A block of mass 2·0 kg kept at rest on an inclined plane of inclination 37° is pulled up the plane by applying a constant force of 20 N parallel to the incline. The force acts for one second. (a) Show that the work done by the applied force does not exceed 40 J. (b) Find the work done by the force of gravity in that one second if the work done by the applied force is 40 J. (c) Find the kinetic energy of the block at the instant the force ceases to act. Take g = 10 m/s^2.

17. A block of mass 2·0 kg is pushed down an inclined plane of inclination 37° with a force of 20 N acting parallel to the incline. It is found that the block moves on the incline with an acceleration of 10 m/s^2. If the block started from rest, find the work done (a) by the applied force in the first second, (b) by the weight of the block in the first second and (c) by the frictional force acting on the block in the first second. Take g = 10 m/s^2.

18. A 250 g block slides on a rough horizontal table. Find the work done by the frictional force in bringing the block to rest if it is initially moving at a speed of 40 cm/s. If the friction coefficient between the table and the block is 0·1, how far does the block move before coming to rest ?

19. Water falling from a 50 m high fall is to be used for generating electric energy. If $1{\cdot}8 \times 10^5$ kg of water falls per hour and half the gravitational potential energy can be converted into electric energy, how many 100 W lamps can be lit ?

20. A person is painting his house walls. He stands on a ladder with a bucket containing paint in one hand and a brush in other. Suddenly the bucket slips from his hand and falls down on the floor. If the bucket with the paint had a mass of 6·0 kg and was at a height of 2·0 m at the time it slipped, how much gravitational potential energy is lost together with the paint ?

21. A projectile is fired from the top of a 40 m high cliff with an initial speed of 50 m/s at an unknown angle. Find its speed when it hits the ground.

22. The 200 m free style women's swimming gold medal at Seol Olympic 1988 went to Heike Friendrich of East Germany when she set a new Olympic record of 1 minute and 57·56 seconds. Assume that she covered most of the distance with a uniform speed and had to exert 460 W to maintain her speed. Calculate the average force of resistance offered by the water during the swim.

23. The US athlete Florence Griffith-Joyner won the 100 m sprint gold medal at Seol Olympic 1988 setting a new Olympic record of 10·54 s. Assume that she achieved her maximum speed in a very short-time and then ran the race with that speed till she crossed the line. Take her mass to be 50 kg. (a) Calculate the kinetic energy of Griffith-Joyner at her full speed. (b) Assuming that the track, the wind etc. offered an average resistance of one tenth of her weight, calculate the work done by the resistance during the run. (c) What power Griffith-Joyner had to exert to maintain uniform speed ?

24. A water pump lifts water from a level 10 m below the ground. Water is pumped at a rate of 30 kg/minute with negligible velocity. Calculate the minimum horsepower the engine should have to do this.

25. An unruly demonstrator lifts a stone of mass 200 g from the ground and throws it at his opponent. At the time of projection, the stone is 150 cm above the ground and has a speed of 3·00 m/s. Calculate the work done by the demonstrator during the process. If it takes one second for the demonstrator to lift the stone and throw, what horsepower does he use ?

26. In a factory it is desired to lift 2000 kg of metal through a distance of 12 m in 1 minute. Find the minimum horsepower of the engine to be used.

27. A scooter company gives the following specifications about its product.

Weight of the scooter — 95 kg
Maximum speed — 60 km/h
Maximum engine power — 3·5 hp
Pick up time to get the maximum speed — 5 s
Check the validity of these specifications.

28. A block of mass 30·0 kg is being brought down by a chain. If the block acquires a speed of 40·0 cm/s in dropping down 2·00 m, find the work done by the chain during the process.

29. The heavier block in an Atwood machine has a mass twice that of the lighter one. The tension in the string is 16·0 N when the system is set into motion. Find the decrease in the gravitational potential energy during the first second after the system is released from rest.

30. The two blocks in an Atwood machine have masses 2·0 kg and 3·0 kg. Find the work done by gravity during the fourth second after the system is released from rest.

31. Consider the situation shown in figure (8-E2). The system is released from rest and the block of mass 1·0 kg is found to have a speed 0·3 m/s after it has descended through a distance of 1 m. Find the coefficient of kinetic friction between the block and the table.

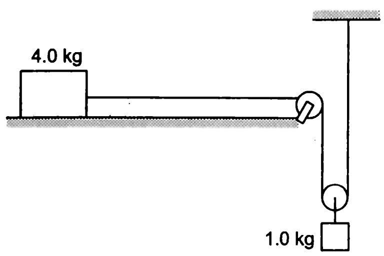


Figure 8-E2

32. A block of mass 100 g is moved with a speed of 5·0 m/s at the highest point in a closed circular tube of radius 10 cm kept in a vertical plane. The cross-section of the tube is such that the block just fits in it. The block makes several oscillations inside the tube and finally stops at the lowest point. Find the work done by the tube on the block during the process.

33. A car weighing 1400 kg is moving at a speed of 54 km/h up a hill when the motor stops. If it is just able to reach the destination which is at a height of 10 m above the point, calculate the work done against friction (negative of the work done by the friction).

34. A small block of mass 200 g is kept at the top of a frictionless incline which is 10 m long and 3·2 m high. How much work was required (a) to lift the block from the ground and put it at the top, (b) to slide the block up the incline ? What will be the speed of the block when it reaches the ground, if (c) it falls off the incline and drops vertically on the ground (d) it slides down the incline ? Take $g = 10 \text{ m/s}^2$.

35. In a children's park, there is a slide which has a total length of 10 m and a height of 8·0 m (figure 8-E3). Vertical ladder are provided to reach the top. A boy weighing 200 N climbs up the ladder to the top of the slide and slides down to the ground. The average friction offered by the slide is three tenth of his weight. Find (a) the work done by the ladder on the boy as he goes up, (b) the work done by the slide on the boy as he comes down. Neglect any work done by forces inside the body of the boy.

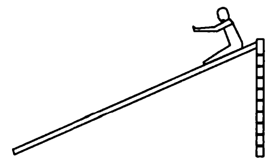


Figure 8-E3

36. Figure (8-E4) shows a particle sliding on a frictionless track which terminates in a straight horizontal section. If the particle starts slipping from the point A, how far away from the track will the particle hit the ground ?

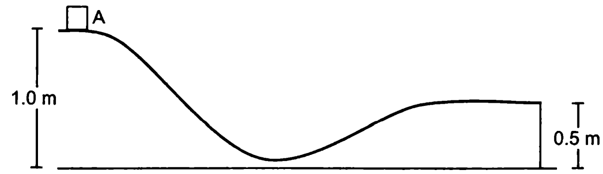


Figure 8-E4

37. A block weighing 10 N travels down a smooth curved track AB joined to a rough horizontal surface (figure 8-E5). The rough surface has a friction coefficient of 0·20 with the block. If the block starts slipping on the track from a point 1·0 m above the horizontal surface, how far will it move on the rough surface ?

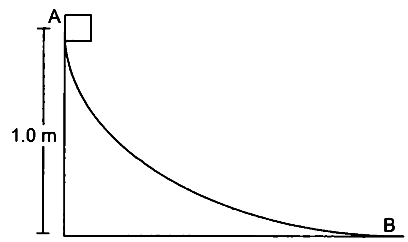


Figure 8-E5

38. A uniform chain of mass m and length l overhangs a table with its two third part on the table. Find the work to be done by a person to put the hanging part back on the table.

39. A uniform chain of length L and mass M overhangs a horizontal table with its two third part on the table. The friction coefficient between the table and the chain is μ. Find the work done by the friction during the period the chain slips off the table.

40. A block of mass 1 kg is placed at the point A of a rough track shown in figure (8-E6). If slightly pushed towards right, it stops at the point B of the track. Calculate the work done by the frictional force on the block during its transit from A to B.

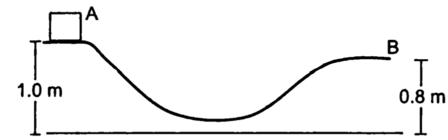


Figure 8-E6

41. A block of mass 5·0 kg is suspended from the end of a vertical spring which is stretched by 10 cm under the load of the block. The block is given a sharp impulse from below so that it acquires an upward speed of 2·0 m/s. How high will it rise ? Take g = 10 m/s^2.

42. A block of mass 250 g is kept on a vertical spring of spring constant 100 N/m fixed from below. The spring is now compressed to have a length 10 cm shorter than its natural length and the system is released from this position. How high does the block rise? Take g = 10 m/s^2.

43. Figure (8-E7) shows a spring fixed at the bottom end of an incline of inclination 37°. A small block of mass 2 kg starts slipping down the incline from a point 4·8 m away from the spring. The block compresses the spring by 20 cm, stops momentarily and then rebounds through a distance of 1 m up the incline. Find (a) the friction coefficient between the plane and the block and (b) the spring constant of the spring. Take g = 10 m/s^2.

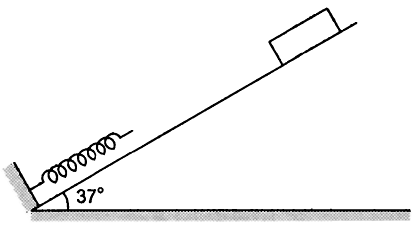


Figure 8-E7

44. A block of mass m moving at a speed v compresses a spring through a distance x before its speed is halved. Find the spring constant of the spring.

45. Consider the situation shown in figure (8-E8). Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring.

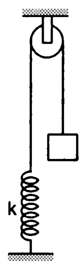


Figure 8-E8

46. A block of mass m is attached to two unstretched springs of spring constants k_1 and k_2 as shown in figure (8-E9). The block is displaced towards right through a distance x and is released. Find the speed of the block as it passes through the mean position shown.

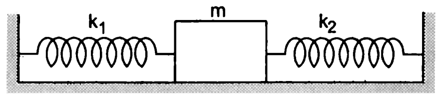


Figure 8-E9

47. A block of mass m sliding on a smooth horizontal surface with a velocity $\vec{v}$ meets a long horizontal spring fixed at one end and having spring constant k as shown in figure (8-E10). Find the maximum compression of the spring. Will the velocity of the block be the same as $\vec{v}$ when it comes back to the original position shown ?

Figure 8-E10

48. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5·0 cm (figure 8-E11). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring ?

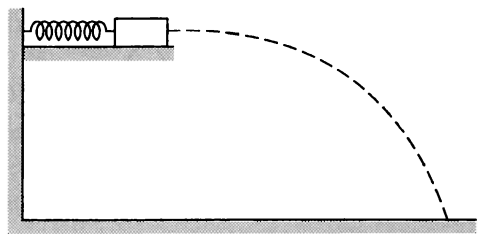


Figure 8-E11

49. A small heavy block is attached to the lower end of a light rod of length l which can be rotated about its clamped upper end. What minimum horizontal velocity should the block be given so that it moves in a complete vertical circle ?

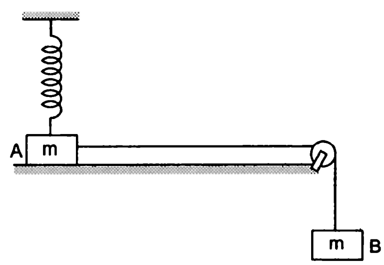


Figure 8-E12

50. Figure (8-E12) shows two blocks A and B, each having a mass of 320 g connected by a light string passing over a smooth light pulley. The horizontal surface on which the block A can slide is smooth. The block A is attached to a spring of spring constant 40 N/m whose other end is fixed to a support 40 cm above the horizontal surface. Initially, the spring is vertical and unstretched when the system is released to move. Find the velocity of the block A at the instant it breaks off the surface below it. Take $g = 10\ m/s^2$.

51. One end of a spring of natural length h and spring constant k is fixed at the ground and the other is fitted with a smooth ring of mass m which is allowed to slide on a horizontal rod fixed at a height h (figure 8-E13). Initially, the spring makes an angle of 37° with the vertical when the system is released from rest. Find the speed of the ring when the spring becomes vertical.

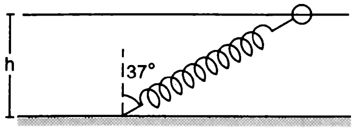


Figure 8-E13

52. Figure (8-E14) shows a light rod of length l rigidly attached to a small heavy block at one end and a hook at the other end. The system is released from rest with the rod in a horizontal position. There is a fixed smooth ring at a depth h below the initial position of the hook and the hook gets into the ring as it reaches there. What should be the minimum value of h so that the block moves in a complete circle about the ring?

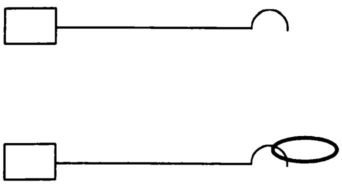


Figure 8-E14

53. The bob of a pendulum at rest is given a sharp hit to impart a horizontal velocity $\sqrt{10\,gl}$, where l is the length of the pendulum. Find the tension in the string when (a) the string is horizontal, (b) the bob is at its highest point and (c) the string makes an angle of 60° with the upward vertical.

54. A simple pendulum consists of a 50 cm long string connected to a 100 g ball. The ball is pulled aside so that the string makes an angle of 37° with the vertical and is then released. Find the tension in the string when the bob is at its lowest position.

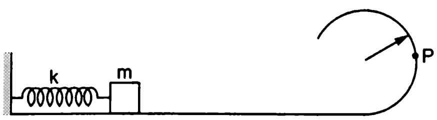


Figure 8-E15

55. Figure (8-E15) shows a smooth track, a part of which is a circle of radius R. A block of mass m is pushed against a spring of spring constant k fixed at the left end and is then released. Find the initial compression of the spring so that the block presses the track with a force mg when it reaches the point P, where the radius of the track is horizontal.

56. The bob of a stationary pendulum is given a sharp hit to impart it a horizontal speed of $\sqrt{3\,gl}$. Find the angle rotated by the string before it becomes slack.

57. A heavy particle is suspended by a 1·5 m long string. It is given a horizontal velocity of $\sqrt{57}$ m/s. (a) Find the angle made by the string with the upward vertical, when it becomes slack. (b) Find the speed of the particle at this instant. (c) Find the maximum height reached by the particle over the point of suspension. Take $g = 10\ m/s^2$.

58. A simple pendulum of length L having a bob of mass m is deflected from its rest position by an angle θ and released (figure 8-E16). The string hits a peg which is fixed at a distance x below the point of suspension and the bob starts going in a circle centred at the peg. (a) Assuming that initially the bob has a height less than the peg, show that the maximum height reached by the bob equals its initial height. (b) If the pendulum is released with $\theta = 90°$ and $x = L/2$ find the maximum height reached by the bob above its lowest position before the string becomes slack. (c) Find the minimum value of x/L for which the bob goes in a complete circle about the peg when the pendulum is released from $\theta = 90°$.

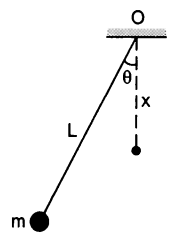


Figure 8-E16

59. A particle slides on the surface of a fixed smooth sphere starting from the topmost point. Find the angle rotated by the radius through the particle, when it leaves contact with the sphere.

60. A particle of mass m is kept on a fixed, smooth sphere of radius R at a position, where the radius through the particle makes an angle of 30° with the vertical. The particle is released from this position. (a) What is the force exerted by the sphere on the particle just after the release? (b) Find the distance travelled by the particle before it leaves contact with the sphere.

61. A particle of mass m is kept on the top of a smooth sphere of radius R. It is given a sharp impulse which imparts it a horizontal speed v. (a) Find the normal force between the sphere and the particle just after the impulse. (b) What should be the minimum value of v for which the particle does not slip on the sphere? (c) Assuming the velocity v to be half the minimum calculated in part, (d) find the angle made by the radius through the particle with the vertical when it leaves the sphere.

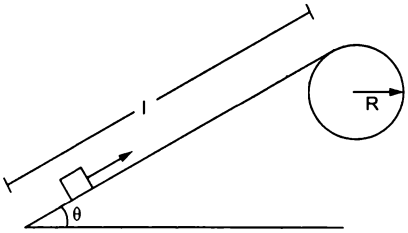


Figure 8-E17

62. Figure (8-E17) shows a smooth track which consists of a straight inclined part of length l joining smoothly with the circular part. A particle of mass m is projected up the incline from its bottom. (a) Find the minimum projection-speed v_0 for which the particle reaches the top of the track. (b) Assuming that the projection-speed is $2v_0$ and that the block does not lose contact with the track before reaching its top, find the force acting on it when it reaches the top. (c) Assuming that the projection-speed is only slightly greater than v_0, where will the block lose contact with the track ?

63. A chain of length l and mass m lies on the surface of a smooth sphere of radius $R > l$ with one end tied to the top of the sphere. (a) Find the gravitational potential energy of the chain with reference level at the centre of the sphere. (b) Suppose the chain is released and slides down the sphere. Find the kinetic energy of the chain, when it has slid through an angle θ. (c) Find the tangential acceleration $\frac{dv}{dt}$ of the chain when the chain starts sliding down.

64. A smooth sphere of radius R is made to translate in a straight line with a constant acceleration a. A particle kept on the top of the sphere is released from there at zero velocity with respect to the sphere. Find the speed of the particle with respect to the sphere as a function of the angle θ it slides.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (b) 3. (d) 4. (c) 5. (a) 6. (b)
7. (c) 8. (c) 9. (d) 10. (c)

OBJECTIVE II

1. (a), (b) 2. (a) 3. (c), (d)
4. (d) 5. (a), (b), (d) 6. (a), (c), (d)
7. (a), (d) 8. (b), (d) 9. (a), (b)
10. (b)

EXERCISES

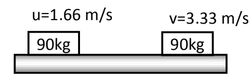
1. 375 J
2. 625 J
3. 400 J
4. 245 J
5. 6·25 J, 36·1 W
6. zero
7. 40 J
8. $\left(a + \frac{1}{2} bd\right) d$
9. 1·5 J
10. (a) $\frac{F}{2(M+m)g}$ (b) $\frac{mF}{2(M+m)}$ (c) $\frac{mFd}{2(M+m)}$
11. (a) $\frac{40000 \text{ J}}{5+\tan\theta}$ (b) 7690 J
12. (a) 120 J (b) 120 J
13. 4000 N
14. 4000 N
15. $ma^2d/2$
16. (b) – 24 J (c) 16 J
17. (a) 100 J (b) 60 J (c) – 60 J
18. – 0·02 J, 8·2 cm
19. 122
20. 118 J
21. 58 m/s
22. 270 N
23. (a) 2250 J (b) – 4900 J (c) 465 W
24. $6 \cdot 6 \times 10^{-2}$ hp
25. 3·84 J, $5 \cdot 14 \times 10^{-3}$ hp
26. 5·3 hp
27. Seems to be somewhat overclaimed.
28. – 586 J
29. 19·6 J
30. 67 J
31. 0·12
32. – 1·45 J
33. 20300 J
34. (a) 6·4 J (b) 6·4 J (c) 8·0 m/s (d) 8·0 m/s
35. (a) zero (b) – 600 J (c) 1600 J
36. At a horizontal distance of 1 m from the end of the track.
37. 5·0 m

38. $mgl/18$
39. $-2\mu MgL/9$
40. -2 J
41. 20 cm
42. 20 cm
43. (a) 0·5 (b) 1000 N/m
44. $\frac{3mv^2}{4x^2}$
45. $2\,mg/k$
46. $\sqrt{\frac{k_1+k_2}{m}}\,x$
47. $v\sqrt{m/k}$, No
48. At a horizontal distance of 1 m from the free end of the spring.
49. $2\sqrt{gl}$
50. 1·5 m/s
51. $\frac{h}{4}\sqrt{k/m}$
52. l
53. (a) 8 mg (b) 5 mg (c) 6·5 mg
54. 1·4 N
55. $\sqrt{\frac{3\,mgR}{k}}$
56. $\cos^{-1}(-1/3)$
57. (a) 53° (b) 3·0 m/s (c) 1·2 m
58. (b) $5L/6$ above the lowest point (c) 0·6
59. $\cos^{-1}(2/3)$
60. $\sqrt{3}\,mg/2$ (b) 0·43 R
61. (a) $mg-\frac{mv^2}{R}$ (b) $\sqrt{Rg}$ (c) $\cos^{-1}(3/4)$
62. (a) $\sqrt{2g\,[R(1-\cos\theta)+l\sin\theta]}$ (b) $6\,mg\left(1-\cos\theta+\frac{l}{R}\sin\theta\right)$
    (c) The radius through the particle makes an angle $\cos^{-1}(2/3)$ with the vertical.
63. (a) $\frac{mR^2g}{l}\sin(l/R)$
    (b) $\frac{mR^2g}{l}\left[\sin\left(\frac{l}{R}\right)+\sin\theta-\sin\left(\theta+\frac{l}{R}\right)\right]$
    (c) $\frac{Rg}{l}[1-\cos(l/R)]$
64. $[2R(a\sin\theta+g-g\cos\theta)]^{1/2}$

□

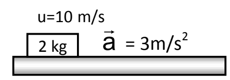
SOLUTIONS TO CONCEPTS
CHAPTER – 8

1. $M = m_c + m_b = 90kg$



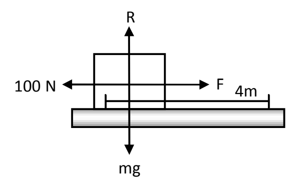
$u = 6$ km/h = 1.666 m/sec
$v = 12$ km/h = 3.333 m/sec
Increase in K.E. = $\frac{1}{2} Mv^2 - \frac{1}{2} Mu^2$
$= \frac{1}{2}\, 90 \times (3.333)^2 - \frac{1}{2} \times 90 \times (1.66)^2 = 494.5 - 124.6 = 374.8 \approx 375$ J

2. $m_b = 2$ kg.
$u = 10$ m/sec
$a = 3$ m/aec^2



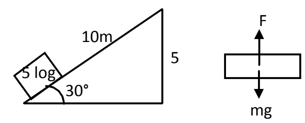
$t = 5$ sec
$v = u + at = 10 + 3$ I $5 = 25$ m/sec.
$\therefore$ F.K.E = $\frac{1}{2} mv^2 = \frac{1}{2} \times 2 \times 625 = 625$ J.

3. F = 100 N



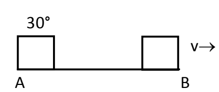
$S = 4m,\ \theta = 0°$
$\omega = \vec{F}.\vec{S} = 100 \times 4 = 400$ J

4. m = 5 kg
$\theta = 30°$
S = 10 m
F = mg



So, work done by the force of gravity
$\omega = mgh = 5 \times 9.8 \times 5 = 245$ J

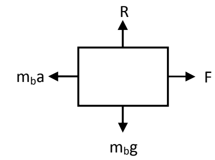
5. F= 2.50N, S = 2.5m, m =15g = 0.015kg.



So, $w = F \times S \Rightarrow a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3}$ m/s^2
$= F \times S \cos 0°$ (acting along the same line)
$= 2.5 \times 2.5 = 6.25$J
Let the velocity of the body at b = U. Applying work-energy principle $\frac{1}{2} mv^2 - 0 = 6.25$
$\Rightarrow V = \sqrt{\frac{6.25 \times 2}{0.015}} = 28.86$ m/sec.
So, time taken to travel from A to B.
$\Rightarrow t = \frac{v-u}{a} = \frac{28.86 \times 3}{500}$
$\therefore$ Average power $= \frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3} = 36.1$

6. Given
$\vec{r}_1 = 2\hat{i} + 3\hat{j}$
$r_2 = 3\hat{i} + 2\hat{j}$
So, displacement vector is given by,
$\vec{r} = \vec{r}_1 - \vec{r}_2 \Rightarrow \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$

So, work done = $\vec{F}\times\vec{s}$ = 5 × 1 + 5(-1) = 0



7. m_b = 2kg, s = 40m, a = 0.5m/sec^2

So, force applied by the man on the box

F = $m_b a$ = 2 × (0.5) = 1 N

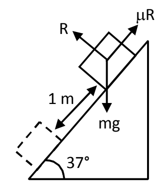
ω = FS = 1 × 40 = 40 J

8. Given that F= a + bx

Where a and b are constants.

So, work done by this force during this force during the displacement x = 0 and x = d is given by

W = $\int_0^d F\,dx = \int_0^d (a+bx)dx$ = ax + (bx^2/2) = [a + ½ bd] d



9. m_b = 250g = .250 kg

θ = 37°, S = 1m.

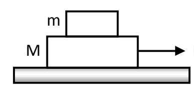
Frictional force f = μR

mg sin θ = μ R ..(1)

mg cos θ ..(2)

so, work done against μR = μRS cos 0° = mg sin θ S = 0.250 × 9.8 × 0.60 × 1 = 1.5 J

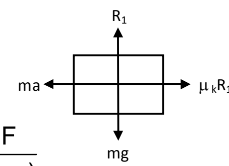
10. a = $\dfrac{F}{2(M+m)}$ (given)



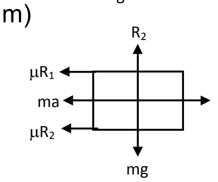
a) from fig (1)

ma = μ_k R_1 and R_1 = mg

$\Rightarrow \mu = \dfrac{ma}{R_1} = \dfrac{F}{2(M+m)g}$



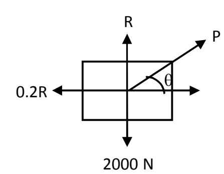
b) Frictional force acting on the smaller block f = μR = $\dfrac{F}{2(M+m)g}\times mg = \dfrac{m\times F}{2(M+m)}$



c) Work done w = fs s = d

$$w = \frac{mF}{2(M+m)}\times d = \frac{mFd}{2(M+m)}$$

11. Weight = 2000 N, S = 20m, μ = 0.2



a) R + Psinθ - 2000 = 0 ..(1)

P cosθ - 0.2 R =0 ..(2)

From (1) and (2) P cosθ – 0.2 (2000 – P sinθ)=0

P = $\dfrac{400}{\cos\theta + 0.2\sin\theta}$ ..(3)

So, work done by the person, W = PS cosθ = $\dfrac{8000\cos\theta}{\cos\theta+0.2\sin\theta} = \dfrac{8000}{1+0.2\sin\theta} = \dfrac{40000}{5+\tan\theta}$

b) For minimum magnitude of force from equn(1)

d/dθ (cos θ + 0.2 sinθ) = 0 $\Rightarrow$ tan θ = 0.2

putting the value in equn (3)

W = $\dfrac{40000}{5+\tan\theta} = \dfrac{40000}{(5.2)}$ = 7690 J

12. w = 100 N, θ = 37°, s = 2m

Force F= mg sin 37° = 100 × 0.60 = 60 N

So, work done, when the force is parallel to incline.

w = Fs cos θ = 60 × 2 × cos θ = 120 J

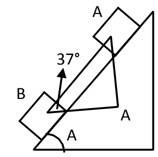
In ΔABC AB= 2m

CB = 37°

so, h = C = 1m

$\therefore$ work done when the force in horizontal direction

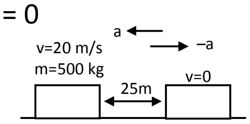
W = mgh = 100 × 1.2 = 120 J

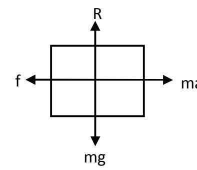


13. m = 500 kg, s = 25m, u = 72km/h= 20 m/s, v = 0

$(-a) = \dfrac{v^2 - u^2}{2S} \Rightarrow a = \dfrac{400}{50} = 8\text{m/sec}^2$

Frictional force f = ma = 500 × 8 = 4000 N

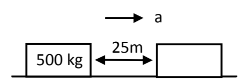


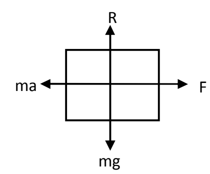


14. m = 500 kg, u = 0, v = 72 km/h = 20m/s

$a = \dfrac{v^2 - u^2}{2s} = \dfrac{400}{50} = 8\text{m/sec}^2$

force needed to accelerate the car F = ma = 500 × 8 = 4000 N





15. Given, v = a$\sqrt{x}$ (uniformly accelerated motion)

displacement s = d – 0 = d

putting x = 0, v_1= 0

putting x = d, v_2= a$\sqrt{d}$

$a = \dfrac{{v_2}^2 - {u_2}^2}{2s} = \dfrac{a^2 d}{2d} = \dfrac{a^2}{2}$

force f = ma = $\dfrac{ma^2}{2}$

work done w = FS cos θ = $\dfrac{ma^2}{2} \times d = \dfrac{ma^2 d}{2}$

16. a) m = 2kg, θ = 37°, F = 20 N

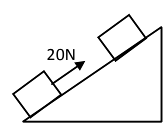
From the free body diagram

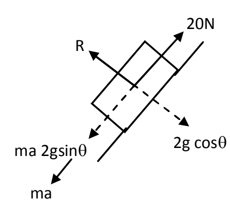
F = (2g sin θ) + ma $\Rightarrow$ a = (20 – 20 sinθ)/s = 4m/sec^2

S = ut + ½ at^2 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done w = Fs = 20 × 2 = 40 J





b) If W = 40 J

$S = \dfrac{W}{F} = \dfrac{40}{20}$

h = 2 sin 37° = 1.2 m

So, work done W = –mgh = – 20 × 1.2 = –24 J

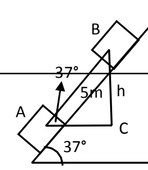
c) v = u + at = 4 × 10 = 40 m/sec

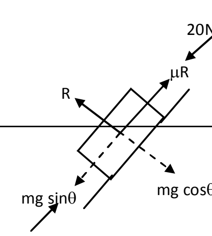
So, K.E. = ½ mv^2 = ½ × 2 × 16 = 16 J

17. m = 2kg, θ = 37°, F = 20 N, a = 10 m/sec^2

a) t = 1sec

So, s= ut + ½ at^2 = 5m





Work done by the applied force w = FS cos 0° = 20 × 5 = 100 J

b) BC (h) = 5 sin 37° = 3m

So, work done by the weight W = mgh = 2 × 10 × 3 = 60 J

c) So, frictional force f = mg sinθ

work done by the frictional forces w = fs cos0° = (mg sinθ) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 25o g = 0.250kg,

u = 40 cm/sec = 0.4m/sec

μ = 0.1, v=0

Here, μ R = ma {where, a = deceleration}

$a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98$ m/sec^2

$S = \frac{v^2 - u^2}{2a} = 0.082$m = 8.2 cm

Again, work done against friction is given by

– w = μ RS cos θ

= 0.1 × 2.5 × 0.082 × 1 (θ = 0°) = 0.02 J

$\Rightarrow$ W = – 0.02 J

19. h = 50m, m = 1.8 × 10^5 kg/hr, P = 100 watt,

P.E. = mgh = 1.8 × 10^5 × 9.8 × 50 = 882 × 10^5 J/hr

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441 × 10^5 J/hr

So, power in watt (J/sec) is given by = $\frac{441 \times 10^5}{3600}$

$\therefore$ number of 100 W lamps, that can be lit $\frac{441 \times 10^5}{3600 \times 100} = 122.5 \approx 122$

20. m = 6kg, h = 2m

P.E. at a height '2m' = mgh = 6 × (9.8) × 2 = 117.6 J

P.E. at floor = 0

Loss in P.E. = 117.6 – 0 = 117. 6 J $\approx$ 118 J

21. h = 40m, u = 50 m/sec

Let the speed be 'v' when it strikes the ground.

Applying law of conservation of energy

mgh + ½ mu^2 = ½ mv^2

$\Rightarrow$ 10 × 40 + (1/2) × 2500 = ½ v^2 $\Rightarrow$ v^2 = 3300 $\Rightarrow$ v = 57.4 m/sec $\approx$58 m/sec

22. t = 1 min 57.56 sec = 11.56 sec, p= 400 W, s =200 m

$p = \frac{w}{t}$, Work w = pt = 460 × 117.56 J

Again, W = FS = $\frac{460 \times 117.56}{200} = 270.3$ N $\approx$ 270 N

23. S = 100 m, t = 10.54 sec, m = 50 kg

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

a) Speed v = S/t = 9.487 e/s

So, K.E. = ½ mv^2 = 2250 J

b) Weight = mg = 490 J

given R = mg /10 = 49 J

so, work done against resistance W_F = – RS = – 49 × 100 = – 4900 J

c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

$P = \frac{W}{t} = 4900 / 10.54 = 465$ W

24. h = 10 m

flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power $P = \frac{mgh}{t} = (0.5) \times 9.8 \times 10 = 49$ W

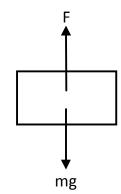
So, horse power (h.p) P/746 = 49/746 = 6.6×10^{-2}hp

25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 sec

Total work done = ½ mv^2 + mgh = (1/2) × (0.2) ×9 + (0.2) × (9.8) × (1.5) = 3.84 J

h.p. used = $\frac{3.84}{746} = 5.14 \times 10^{-3}$

26. m = 200 kg, s = 12m, t = 1 min = 60 sec



So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work]

= 2000 × 10 × 12 = 240000 J

So, power $p = \frac{W}{t} = \frac{240000}{60} = 4000$ watt

h.p = $\frac{4000}{746}$ = 5.3 hp.

27. The specification given by the company are

U = 0, m = 95 kg, P_m = 3.5 hp

V_m = 60 km/h = 50/3 m/sec t_m = 5 sec

So, the maximum acceleration that can be produced is given by,

$a = \frac{(50/3) - 0}{5} = \frac{10}{3}$

So, the driving force is given by

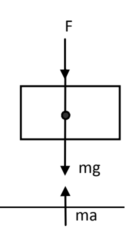
$F = ma = 95 \times \frac{10}{3} = \frac{950}{3}$ N

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

$v = \frac{p}{F} \Rightarrow v = \frac{3.5 \times 746 \times 5}{950}$ = 8.2 m/sec.

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30kg, v = 40 cm/sec = 0.4 m/sec s = 2m



From the free body diagram, the force given by the chain is,

F = (ma – mg) = m(a – g) [where a = acceleration of the block]

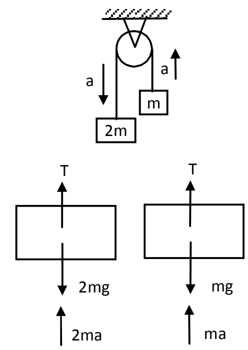
$a = \frac{(v2\ \ u2)}{2s} = \frac{0.16}{0.4} = 0.04$ m/sec^2

So, work done W = Fs cos θ = m(a –g) s cos θ

$\Rightarrow$ W = 30 (0.04 – 9.8) × 2 $\Rightarrow$ W = –585.5 $\Rightarrow$ W = –586 J.

So, W = – 586 J

29. Given, T = 19 N



From the freebody diagrams,

T – 2 mg + 2 ma = 0 …(i)

T – mg – ma = 0 …(ii)

From, Equation (i) & (ii) T = 4ma $\Rightarrow$ a = $\frac{T}{4m}$ $\Rightarrow$ A = $\frac{16}{4m} = \frac{4}{m}$ m/s^2.

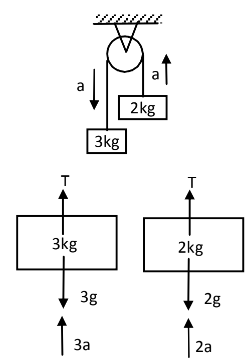
Now, S = ut + ½ at^2

$\Rightarrow$ S = $\frac{1}{2} \times \frac{4}{m} \times 1$ $\Rightarrow$ S = $\frac{2}{m}$ m [because u=0]

Net mass = 2m – m = m

Decrease in P.E. = mgh $\Rightarrow$P.E. = m × g × $\frac{2}{m}$ $\Rightarrow$ P.E. = 9.8 × 2 $\Rightarrow$P.E. = 19.6 J

30. Given, m_1 = 3 kg, m_2 = 2kg, t = during 4th second



From the freebody diagram

T – 3g + 3a = 0 ..(i)

T – 2g – 2a = 0 ..(ii)

Equation (i) & (ii), we get 3g – 3a = 2g + 2a $\Rightarrow$ a = $\frac{g}{5}$ m/sec^2

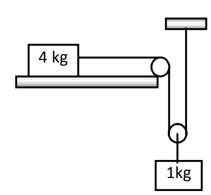
Distance travelled in 4th sec is given by

$$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2\times 4-1) = \frac{7g}{10} = \frac{7\times 9.8}{10} \text{ m}$$

Net mass 'm' = $m_1 - m_2$ = 3 – 2 = 1kg

So, decrease in P.E. = mgh = 1 × 9.8 × $\frac{7}{10}$ × 9.8 = 67.2 = 67 J

31. m_1 = 4kg, m_2 = 1kg, V_2 = 0.3m/sec V_1 = 2 × (0.3) = 0.6 m/sec



($v_1 = 2x_2$ m this system)

h = 1m = height descent by 1kg block

s = 2 × 1 = 2m distance travelled by 4kg block

u = 0

Applying change in K.E. = work done (for the system)

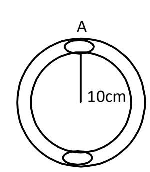
[(1/2)$m_1v_1^2$ + (1/2) $m_2v_m^2$] –0 = (–μR)S + m_2g [R = 4g = 40 N]

$\Rightarrow$ ½ × 4 × (0.36) × ½ ×1 × (0.09) = – μ × 40 × 2 + 1 × 40 × 1

$\Rightarrow$ 0.72 + 0.045 = – 80μ + 10

$\Rightarrow$ μ = $\frac{9.235}{80}$ = 0.12

32. Given, m = 100g = 0.1kg, v = 5m/sec, r = 10cm



Work done by the block = total energy at A – total energy at B

(1/2 mv^2 + mgh) – 0

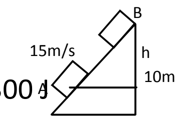
$\Rightarrow$ W = ½ mv^2 + mgh – 0 = ½ × (0.1) × 25 + (0.1) × 10 × (0.2) [h = 2r = 0.2m]

$\Rightarrow$ W = 1.25 – 0.2 $\Rightarrow$ W = 1.45 J

So, the work done by the tube on the body is

W_t = –1.45 J

33. m = 1400kg, v = 54km/h = 15m/sec, h = 10m

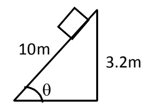


Work done = (total K.E.) – total P.E.

= 0 + ½ mv^2 – mgh = ½ × 1400 × $(15)^2$ – 1400 × 9.8 × 10 = 157500 – 137200 = 20300 J

So, work done against friction, W_t = 20300 J

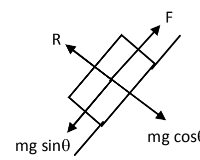
34. m = 200g = 0.2kg, s = 10m, h = 3.2m, g = 10 m/sec^2

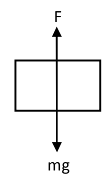


a) Work done W = mgh = 0.2 × 10 × 3.2 = 6.4 J

b) Work done to slide the block up the incline

w = (mg sin θ) = (0.2) × 10 × $\frac{3.2}{10}$ × 10 = 6.4 J





c) Let, the velocity be v when falls on the ground vertically,

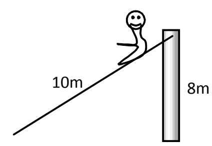
½ mv^2 – 0 = 6.4J $\Rightarrow$ v = 8 m/s

d) Let V be the velocity when reaches the ground by liding

½ mV^2 – 0 = 6.4 J $\Rightarrow$ V = 8m/sec

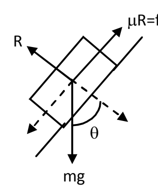
35. ℓ = 10m, h = 8m, mg = 200N

f = 200 × $\frac{3}{10}$ = 60N



a) Work done by the ladder on the boy is zero when the boy is going up because the work is done by the boy himself.

b) Work done against frictional force, W = μRS = *f* ℓ = (–60) × 10 = – 600 J



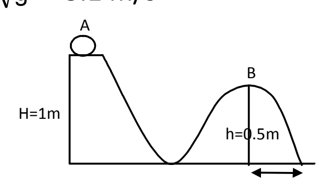
c) Work done by the forces inside the boy is

W_b = (mg sinθ) × 10 = 200 × $\frac{8}{10}$ × 10 = 1600 J

36. H = 1m, h = 0.5m

Applying law of conservation of Energy for point A & B

mgH = ½ mv^2 + mgh $\Rightarrow$ g = (1/2) v^2 + 0.5g $\Rightarrow$ v^2 2(g – 0.59) = g $\Rightarrow$ v = $\sqrt{g}$ = 3.1 m/s



After point B the body exhibits projectile motion for which

θ = 0°, v = – 0.5

So, –0.5 = (u sinθ) t - (1/2) gt^2 $\Rightarrow$ 0.5 = 4.9 t^2 $\Rightarrow$ t = 0.31 sec.

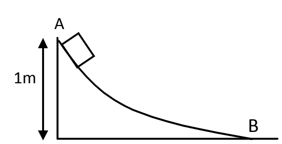
So, x = (4 cos θ) t = 3.1 × 3.1 = 1m.

So, the particle will hit the ground at a horizontal distance in from B.

37. mg = 10N, μ = 0.2, H = 1m, u = v = 0

change in P.E. = work done.

Increase in K.E.



$\Rightarrow$ w = mgh = 10 × 1 = 10 J

Again, on the horizontal surface the fictional force

F = μR = μmg = 0.2 × 10 = 2 N

So, the K.E. is used to overcome friction

$\Rightarrow$ S = $\frac{W}{F}$ = $\frac{10J}{2N}$ = 5m

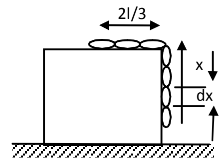
38. Let 'dx' be the length of an element at a distance × from the table

mass of 'dx' length = (m/ℓ) dx

Work done to put dx part back on the table

W = (m/ℓ) dx g(x)

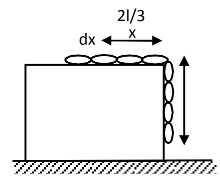
So, total work done to put $\ell/3$ part back on the table



$$W = \int_0^{1/3}(m/\ell)gx\ dx \Rightarrow w = (m/\ell)\ g\left[\frac{x^2}{2}\right]_0^{\frac{\ell}{3}} = \frac{mg\ell^2}{18\ell} = \frac{mg\ell}{18}$$

39. Let, x length of chain is on the table at a particular instant.

So, work done by frictional force on a small element 'dx'



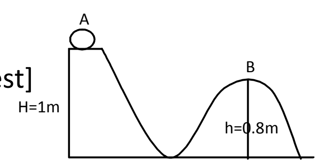
$$dW_f = \mu Rx = \mu\left(\frac{M}{L}dx\right)gx \qquad \left[\text{where } dx = \frac{M}{L}dx\right]$$

Total work don by friction,

$$W_f = \int_{2L/3}^{0}\mu\frac{M}{L}gx\ dx$$

$$\therefore\ W_f = \mu\frac{m}{L}g\left[\frac{x^2}{2}\right]_{2L/3}^{0} = \mu\frac{M}{L}\left[\frac{4L^2}{18}\right] = 2\mu Mg\ L/9$$

40. Given, m = 1kg, H = 1m, h = 0.8m



Here, work done by friction = change in P.E. [as the body comes to rest]

$\Rightarrow W_f = mgh - mgH$

= mg (h – H)

= 1 × 10 (0.8 – 1) = – 2J

41. m = 5kg, x = 10cm = 0.1m, v = 2m/sec,

h =? G = 10m/sec^2

$$S0,\ k = \frac{mg}{x} = \frac{50}{0.1} = 500\ N/m$$

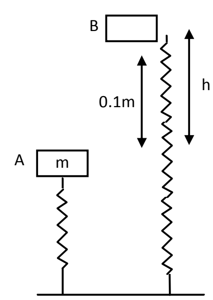
Total energy just after the blow E = ½ mv^2 + ½ kx^2 ...(i)

Total energy a a height h = ½ k (h – x)^2 + mgh ...(ii)

½ mv^2 + ½ kx^2 = ½ k (h – x)^2 + mgh

On, solving we can get,

H = 0.2 m = 20 cm



42. m = 250 g = 0.250 kg,

k = 100 N/m, m = 10 cm = 0.1m

g = 10 m/sec^2

Applying law of conservation of energy

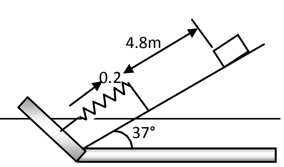
$$½\ kx^2 = mgh \Rightarrow h = \frac{1}{2}\left(\frac{kx^2}{mg}\right) = \frac{100\times(0.1)^2}{2\times0.25\times10} = 0.2\ m = 20\ cm$$

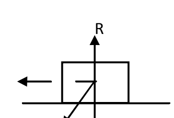
43. m = 2kg, s_1 = 4.8m, x = 20cm = 0.2m, s_2 = 1m,

sin 37° = 0.60 = 3/5, θ = 37°, cos 37° = .79 = 0.8 = 4/5

g = 10m/sec^2

Applying work – Energy principle for downward motion of the body





$0 - 0 = mg \sin 37° \times 5 - \mu R \times 5 - ½\, kx^2$

$\Rightarrow 20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\, k\,(0.2)^2 = 0$

$\Rightarrow 60 - 80\mu - 0.02k = 0 \Rightarrow 80\mu + 0.02k = 60$ ...(i)

Similarly, for the upward motion of the body the equation is

$0 - 0 = (-mg \sin 37°) \times 1 - \mu\, R \times 1 + ½\, k\,(0.2)^2$

$\Rightarrow -20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\, k\,(0.2)^2 = 0$

$\Rightarrow -12 - 16\mu + 0.02\, K = 0$ ..(ii)

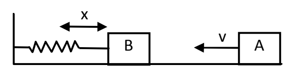
Adding equation (i) & equation (ii), we get $96\,\mu = 48$

$\Rightarrow \mu = 0.5$

Now putting the value of μ in equation (i) K = 1000N/m

44. Let the velocity of the body at A be v

So, the velocity of the body at B is v/2

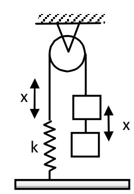


Energy at point A = Energy at point B

So, $½\, mv_A^2 = ½\, mv_B^2 + ½\, kx^{2+}$

$\Rightarrow ½\, kx^2 = ½\, mv_A^2 - ½\, mv_B^2 \Rightarrow kx^2 = m\,(v_A^{2+\,-}\, v_B^2) \Rightarrow kx^2 = m\left(v^2 - \frac{v^2}{4}\right) \Rightarrow k = \frac{3mv^2}{3x^2}$

45. Mass of the body = m



Let the elongation be x

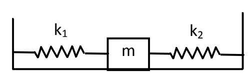
So, $½\, kx^2 = mgx$

$\Rightarrow x = 2mg / k$

46. The body is displaced x towards right

Let the velocity of the body be v at its mean position

Applying law of conservation of energy

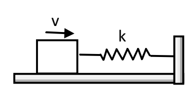


$½\, mv^2 = ½\, k_1x^2 + ½\, k_2x^2 \Rightarrow mv^2 = x^2\,(k_1 + k_2) \Rightarrow v^2 = \frac{x^2(k_1 + k_2)}{m}$

$\Rightarrow v = x\sqrt{\frac{k_1 + k_2}{m}}$

47. Let the compression be x

According to law of conservation of energy

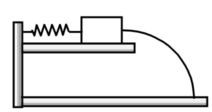


$½\, mv^2 = ½\, kx^2 \Rightarrow x^2 = mv^2 / k \Rightarrow x = v\sqrt{(m/k)}$

b) No. It will be in the opposite direction and magnitude will be less due to loss in spring.

48. m = 100g = 0.1kg, x = 5cm = 0.05m, k = 100N/m

when the body leaves the spring, let the velocity be v



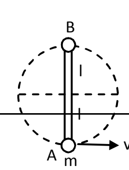
$½\, mv^2 = ½\, kx^2 \Rightarrow v = x\sqrt{k/m} = 0.05 \times \sqrt{\frac{100}{0.1}} = 1.58\text{m/sec}$

For the projectile motion, $\theta = 0°$, Y = –2

Now, $y = (u \sin \theta)t - ½\, gt^2$

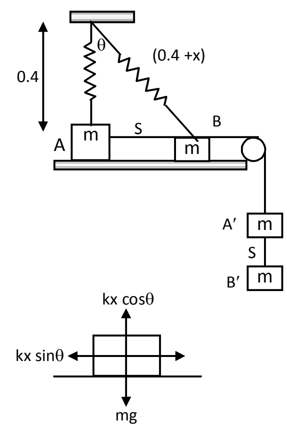
$\Rightarrow -2 = (-1/2) \times 9.8 \times t^2 \Rightarrow t = 0.63$ sec.

So, $x = (u \cos \theta)\, t \Rightarrow 1.58 \times 0.63 = 1\text{m}$

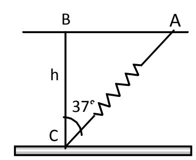


49. Let the velocity of the body at A is 'V' for minimum velocity given at A velocity of the body at point B is zero.
   Applying law of conservation of energy at A & B
   $\frac{1}{2} mv^2 = mg\,(2\ell) \Rightarrow v = \sqrt{(4g\ell)} = 2\sqrt{g\ell}$

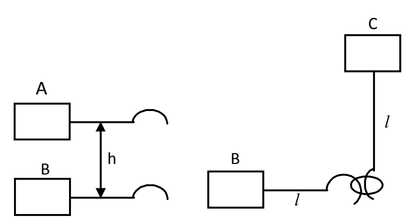
50. m = 320g = 0.32kg
   k = 40N/m
   h = 40cm = 0.4m
   $g = 10\ m/s^2$
   From the free body diagram,
   $kx \cos\theta = mg$
   (when the block breaks off R = 0)
   $\Rightarrow \cos\theta = mg/kx$
   So, $\frac{0.4}{0.4+x} = \frac{3.2}{40 \times x} \Rightarrow 16x = 3.2x + 1.28 \Rightarrow x = 0.1$ m
   S0, s = AB = $\sqrt{(h+x)^2 - h^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3$ m
   Let the velocity of the body at B be v
   Charge in K.E. = work done (for the system)
   $(1/2\ mv^2 + \frac{1}{2}\ mv^2) = -1/2\ kx^2 + mgs$
   $\Rightarrow (0.32) \times v^2 = -(1/2) \times 40 \times (0.1)^2 + 0.32 \times 10 \times (0.3) \Rightarrow v = 1.5$ m/s.



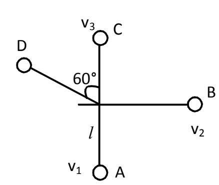
51. $\theta = 37°$; l = h = natural length
   Let the velocity when the spring is vertical be 'v'.
   Cos 37° = BC/AC = 0.8 = 4/5
   Ac = (h + x) = 5h/4 (because BC = h)
   So, x = (5h/4) – h = h/4
   Applying work energy principle $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$
   $\Rightarrow v = x\sqrt{(k/m)} = \frac{h}{4}\sqrt{\frac{k}{m}}$

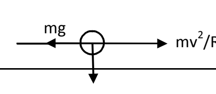


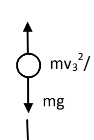
52. The minimum velocity required to cross the height point c = $\sqrt{2gl}$
   Let the rod released from a height h.
   Total energy at A = total energy at B
   mgh = 1/2 mv^2 ; mgh = 1/2 m (2gl)
   [Because v = required velocity at B such that the block makes a complete circle. [Refer Q – 49]
   So, h = l.



53. a) Let the velocity at B be v_2
   $1/2\ mv_1^2 = 1/2\ mv_2^2 + mgl$
   $\Rightarrow 1/2\ m\,(10\ gl) = 1/2\ mv_2^2 + mgl$
   $v_2^2 = 8\ gl$
   So, the tension in the string at horizontal position







$T = \frac{mv^2}{R} = \frac{m8gl}{l} = 8\ mg$

b) Let the velocity at C be V_3

$1/2\ mv_1^2 = 1/2\ mv_3^2 + mg\ (2l)$

$\Rightarrow 1/2\ m\ (\log l) = 1/2\ mv_3^2 + 2mgl$

$\Rightarrow v_3^2 = 6\ mgl$

So, the tension in the string is given by

$T_c = \frac{mv^2}{l} - mg = \frac{6\ glm}{l}\ mg = 5\ mg$

c) Let the velocity at point D be v_4

Again, $1/2\ mv_1^2 = 1/2\ mv_4^2 + mgh$

$1/2 \times m \times (10\ gl) = 1.2\ mv_4^2 + mgl\ (1 + \cos 60°)$

$\Rightarrow v_4^2 = 7\ gl$

So, the tension in the string is

$T_D = (mv^2/l) - mg \cos 60°$

$= m(7\ gl)/l - l - 0.5\ mg \Rightarrow 7\ mg - 0.5\ mg = 6.5\ mg.$

54. From the figure, $\cos\theta = AC/AB$

$\Rightarrow AC = AB\cos\theta \Rightarrow (0.5) \times (0.8) = 0.4.$

So, $CD = (0.5) - (0.4) = (0.1)$ m

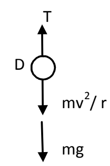
Energy at D = energy at B

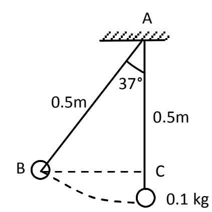
$1/2\ mv^2 = mg\ (CD)$

$v^2 = 2 \times 10 \times (0.1) = 2$

So, the tension is given by,

$T = \frac{mv^2}{r} + mg = (0.1)\left(\frac{2}{0.5} + 10\right) = 1.4\ N.$





55. Given, N = mg

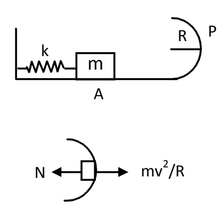
As shown in the figure, $mv^2 / R = mg$

$\Rightarrow v^2 = gR$ …(1)

Total energy at point A = energy at P

$1/2\ kx^2 = \frac{mgR + 2mgR}{2}$ [because $v^2 = gR$]

$\Rightarrow x^2 = 3mgR/k \Rightarrow x = \sqrt{(3mgR)/k}$.



56. $V = \sqrt{3gl}$

$1/2\ mv^2 - 1/2\ mu^2 = -mgh$

$v^2 = u^2 - 2g(l + l\cos\theta)$

$\Rightarrow v^2 = 3gl - 2gl\ (1 + \cos\theta)$ …(1)

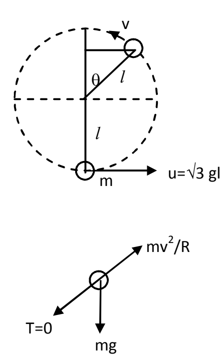
Again,

$mv^2/l = mg\cos\theta$

$v^2 = lg\cos\theta$

From equation (1) and (2), we get

$3gl - 2gl - 2gl\cos\theta = gl\cos\theta$



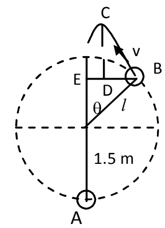
$3 \cos\theta = 1 \Rightarrow \cos\theta = 1/3$

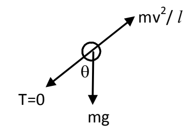
$\theta = \cos^{-1}(1/3)$

So, angle rotated before the string becomes slack

$= 180° - \cos^{-1}(1/3) = \cos^{-1}(-1/3)$

57. $l = 1.5$ m; $u = \sqrt{57}$ m/sec.





a) $mg \cos\theta = mv^2 / l$

$v^2 = lg \cos\theta$ ...(1)

change in K.E. = work done

$1/2\ mv^2 - 1/2\ mu^2 = mgh$

$\Rightarrow v^2 - 57 = -2 \times 1.5\ g\ (1 + \cos\theta)$...(2)

$\Rightarrow v^2 = 57 - 3g(1 + \cos\theta)$

Putting the value of v from equation (1)

$15 \cos\theta = 57 - 3g\ (1 + \cos\theta) \Rightarrow 15 \cos\theta = 57 - 30 - 30 \cos\theta$

$\Rightarrow 45 \cos\theta = 27 \Rightarrow \cos\theta = 3/5.$

$\Rightarrow \theta = \cos^{-1}(3/5) = 53°$

b) $v = \sqrt{57 - 3g(1+\cos\theta)}$ from equation (2)

$= \sqrt{9} = 3$ m/sec.

c) As the string becomes slack at point B, the particle will start making projectile motion.

$$H = OE + DC = 1.5 \cos\theta + \frac{u^2 \sin^2\theta}{2g}$$

$$= (1.5) \times (3/5) + \frac{9 \times (0.8)^2}{2 \times 10} = 1.2 \text{ m.}$$

58.

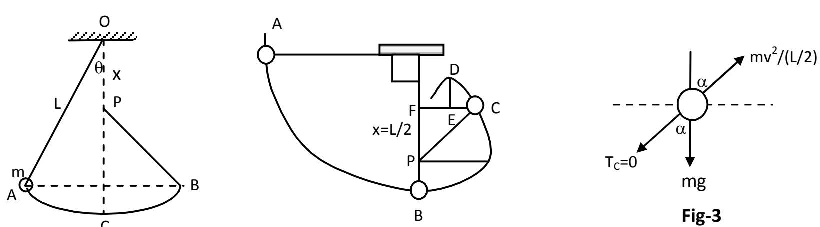


Fig-1 Fig-2 Fig-3

a) When the bob has an initial height less than the peg and then released from rest (figure 1), let body travels from A to B.

Since, Total energy at A = Total energy at B

$\therefore (K.E)_A = (PE)_A = (KE)_B + (PE)_B$

$\Rightarrow \quad (PE)_A = (PE)_B \quad$ [because, $(KE)_A = (KE)_B = 0$]

So, the maximum height reached by the bob is equal to initial height.

b) When the pendulum is released with $\theta = 90°$ and $x = L/2$, (figure 2) the path of the particle is shown in the figure 2.

At point C, the string will become slack and so the particle will start making projectile motion. (Refer Q.No. 56)

$(1/2)mv_c^2 - 0 = mg\ (L/2)\ (1 - \cos\alpha)$

because, distance between A nd C in the vertical direction is L/2 (1 – cos α)

$\Rightarrow v_c^2 = gL(1 - \cos\theta)$ ..(1)

Again, form the freebody diagram (fig – 3)

$\frac{mv_c^2}{L/2} = mg\cos\alpha$ {because $T_c = 0$}

So, $V_C^2 = \frac{gL}{2}\cos\alpha$ ..(2)

From Eqn.(1) and equn (2),

$gL(1 - \cos\alpha) = \frac{gL}{2}\cos\alpha$

$\Rightarrow 1 - \cos\alpha = 1/2\cos\alpha$

$\Rightarrow 3/2\cos\alpha = 1 \Rightarrow \cos\alpha = 2/3$ ..(3)

To find highest position C, before the string becomes slack

$BF = \frac{L}{2} + \frac{L}{2}\cos\theta = \frac{L}{2} + \frac{L}{2}\times\frac{2}{3} = L\left(\frac{1}{2} + \frac{1}{3}\right)$

So, BF = (5L/6)

c) If the particle has to complete a vertical circle, at the point C.

$\frac{mv_c^2}{(L - x)} = mg$

$\Rightarrow v_c^2 = g(L - x)$ ..(1)

Again, applying energy principle between A and C,

$1/2\,mv_c^2 - 0 = mg(OC)$

$\Rightarrow 1/2\,v_c^2 = mg[L - 2(L - x)] = mg(2x - L)$

$\Rightarrow v_c^2 = 2g(2x - L)$ ..(2)

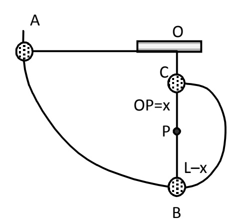
From equn. (1) and equn (2)

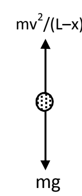
$g(L - x) = 2g(2x - L)$

$\Rightarrow L - x = 4x - 2L$

$\Rightarrow 5x = 3L$

$\therefore \frac{x}{L} = \frac{3}{5} = 0.6$

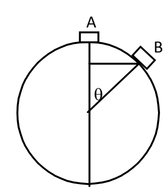




So, the rates (x/L) should be 0.6

59. Let the velocity be v when the body leaves the surface.

From the freebody diagram,

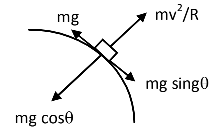


$\frac{mv^2}{R} = mg\cos\theta$ [Because normal reaction]

$v^2 = Rg\cos\theta$ ..(1)

Again, form work-energy principle,

Change in K.E. = work done



$\Rightarrow 1/2\,mv^2 - 0 = mg(R - R\cos\theta)$

$\Rightarrow v^2 = 2gR(1 - \cos\theta)$ ..(2)

From (1) and (2)

$Rg\cos\theta = 2gR(1 - \cos\theta)$

$3gR \cos\theta = 2\, gR$

$\text{Cos}\,\theta = 2/3$

$\theta = \cos^{-1}(2/3)$

60. a) When the particle is released from rest (fig–1), the centrifugal force is zero.

N force is zero = mg cos θ

$= mg \cos 30° = \dfrac{\sqrt{3}mg}{2}$

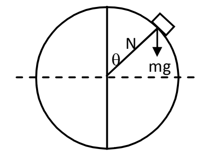


Fig-1

b) When the particle leaves contact with the surface (fig–2), N = 0.

So, $\dfrac{mv^2}{R}$ mg cos θ

$\Rightarrow v^2 = Rg \cos\theta$ ..(1)

Again, ½ $mv^2 = mgR(\cos 30° - \cos\theta)$

$\Rightarrow v^2 = 2Rg\left(\dfrac{\sqrt{3}}{2} - \cos\theta\right)$ ..(2)

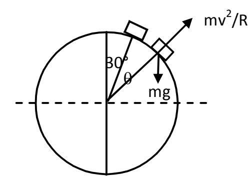


Fig-2

From equn. (1) and equn. (2)

$Rg \cos\theta = \sqrt{3}\, Rg - 2Rg \cos\theta$

$\Rightarrow 3 \cos\theta = \sqrt{3}$

$\Rightarrow \cos\theta = \dfrac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

So, the distance travelled by the particle before leaving contact,

$\ell = R(\theta - \pi/6)$ [because $30° = \pi/6$]

putting the value of θ, we get $\ell = 0.43R$

61. a) Radius =R

horizontal speed = v

From the free body diagram, (fig–1)

N = Normal force $= mg - \dfrac{mv^2}{R}$

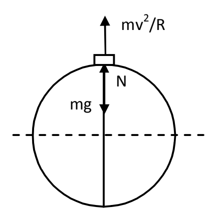


Fig-1

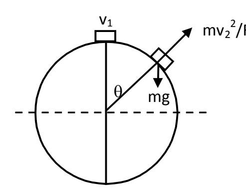


Fig-2

b) When the particle is given maximum velocity so that the centrifugal force balances the weight, the particle does not slip on the sphere.

$\dfrac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

c) If the body is given velocity v_1

$v_1 = \sqrt{gR}/2$

$v_1^2 - gR/4$

Let the velocity be v_2 when it leaves contact with the surface, (fig–2)

So, $\dfrac{mv^2}{R} = mg \cos\theta$

$\Rightarrow v_2^2 = Rg \cos\theta$ ..(1)

Again, $1/2\ mv_2^2 - 1/2\ mv_1^2 = mgR(1 - \cos\theta)$

$\Rightarrow v_2^2 = v_1^2 + 2gR(1 - \cos\theta)$ ..(2)

From equn. (1) and equn (2)

$Rg\cos\theta = (Rg/4) + 2gR(1-\cos\theta)$

$\Rightarrow \cos\theta = (1/4) + 2 - 2\cos\theta$

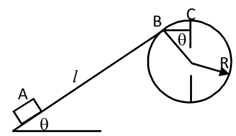
$\Rightarrow 3\cos\theta = 9/4$

$\Rightarrow \cos\theta = 3/4$

$\Rightarrow \theta = \cos^{-1}(3/4)$

62. a) Net force on the particle between A & B, $F = mg\sin\theta$

work done to reach B, $W = FS = mg\sin\theta\,\ell$



Again, work done to reach B to C = mgh = $mg R(1-\cos\theta)$

So, Total workdone = $mg[\ell\sin\theta + R(1-\cos\theta)]$

Now, change in K.E. = work done

$\Rightarrow 1/2\, mv_O^2 = mg[\ell\sin\theta + R(1-\cos\theta)]$

$\Rightarrow v_o = \sqrt{2g(R(1-\cos\theta)+\ell\sin\theta)}$

b) When the block is projected at a speed $2v_o$.

Let the velocity at C will be V_c.

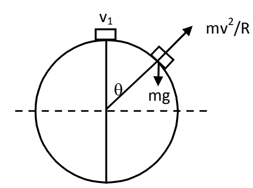
Applying energy principle,

$1/2\, mv_c^2 - 1/2\, m(2v_o)^2 = -mg[\ell\sin\theta + R(1-\cos\theta)]$

$\Rightarrow v_c^2 = 4v_o - 2g[\ell\sin\theta + R(1-\cos\theta)]$

$4.2g[\ell\sin\theta + R(1-\cos\theta)] - 2g[\ell\sin\theta + R(1-\cos\theta)$

$= 6g[\ell\sin\theta + R(1-\cos\theta)]$



So, force acting on the body,

$\Rightarrow N = \dfrac{mv_c^2}{R} = 6mg[(\ell/R)\sin\theta + 1 - \cos\theta]$

c) Let the block loose contact after making an angle θ

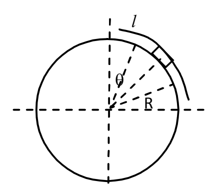
$\dfrac{mv^2}{R} = mg\cos\theta \Rightarrow v^2 = Rg\cos\theta$ ..(1)

Again, $1/2\, mv^2 = mg(R - R\cos\theta) \Rightarrow v^2 = 2gR(1-\cos\theta)$ ..(2)........(?)

From (1) and (2) $\cos\theta = 2/3 \Rightarrow \theta = \cos^{-1}(2/3)$

63. Let us consider a small element which makes angle ‘dθ’ at the centre.

$\therefore dm = (m/\ell)R\,d\theta$



a) Gravitational potential energy of ‘dm’ with respect to centre of the sphere

= (dm)g R cos θ

= $(mg/\ell)R\cos\theta\, d\theta$

So, Total G.P.E. = $\int_0^{\ell/r} \dfrac{mgR^2}{\ell}\cos\theta\, d\theta$ [α = (ℓ/R)](angle subtended by the chain at the centre)..........

$= \dfrac{mR^2g}{\ell}[\sin\theta](\ell/R) = \dfrac{mRg}{\ell}\sin(\ell/R)$

b) When the chain is released from rest and slides down through an angle θ, the K.E. of the chain is given

K.E. = Change in potential energy.

$= \frac{mR^2g}{\ell} \sin(\ell/R) - m \int \frac{gR^2}{\ell} \cos\theta \, d\theta$..........?

$= \frac{mR^2g}{\ell} [\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

c) Since, K.E. = 1/2 mv^2 = $\frac{mR^2g}{\ell} [\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

Taking derivative of both sides with respect to 't'

$(1/2) \times 2v \times \frac{dv}{dt} = \frac{R^2g}{\ell} [\cos\theta \times \frac{d\theta}{dt} - \cos(\theta + \ell/R) \frac{d\theta}{dt}]$

$\therefore (R \frac{d\theta}{dt}) \frac{dv}{dt} = \frac{R^2g}{\ell} \times \frac{d\theta}{dt} [\cos\theta - \cos(\theta + (\ell/R))]$

When the chain starts sliding down, $\theta = 0$.

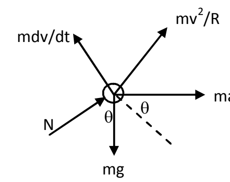
So, $\frac{dv}{dt} = \frac{Rg}{\ell} [1 - \cos(\ell/R)]$

64. Let the sphere move towards left with an acceleration 'a

Let m = mass of the particle

The particle 'm' will also experience the inertia due to acceleration 'a' as it is on the sphere. It will also experience the tangential inertia force (m (dv/dt)) and centrifugal force (mv^2/R).

$m\frac{dv}{dt} = ma \cos\theta + mg \sin\theta \Rightarrow mv \frac{dv}{dt} = ma \cos\theta \left(R\frac{d\theta}{dt}\right) + mg \sin\theta \left(R\frac{d\theta}{dt}\right)$

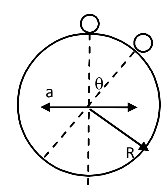


Because, $v = R\frac{d\theta}{dt}$

$\Rightarrow v\,dv = a R \cos\theta \, d\theta + gR \sin\theta \, d\theta$

Integrating both sides we get,

$\frac{v^2}{2} = a R \sin\theta - gR \cos\theta + C$



Given that, at $\theta = 0$, $v = 0$, So, C = gR

So, $\frac{v^2}{2} = a R \sin\theta - g R \cos\theta + g R$

$\therefore v^2 = 2R (a \sin\theta + g - g\cos\theta) \Rightarrow v = [2R (a \sin\theta + g - g\cos\theta)]^{1/2}$

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