CHAPTER 6

FRICTION

6.1 FRICTION AS THE COMPONENT OF CONTACT FORCE

When two bodies are kept in contact, electromagnetic forces act between the charged particles at the surfaces of the bodies. As a result, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and hence the contact forces obey Newton's third law.

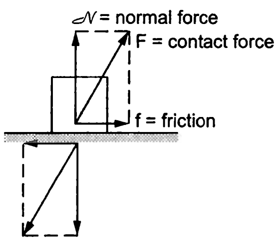


Figure 6.1

The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (Figure 6.1). The perpendicular component is called the *normal contact force* or *normal force* and the parallel component is called *friction.*

Example 6.1

A body of mass 400 g *slides on a rough horizontal surface. If the frictional force is* 3·0 N, *find (a) the angle made by the contact force on the body with the vertical and (b) the magnitude of the contact force. Take* $g = 10 \text{ m/s}^2$.

Solution : Let the contact force on the block by the surface be F which makes an angle θ with the vertical (figure 6.2).

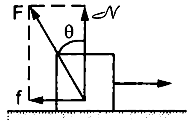


Figure 6.2

The component of F perpendicular to the contact surface is the normal force $\mathcal{N}$ and the component of F parallel to the surface is the friction f. As the surface is horizontal, $\mathcal{N}$ is vertically upward. For vertical equilibrium,

$$\mathcal{N} = Mg = (0{\cdot}400 \text{ kg})\,(10 \text{ m/s}^2) = 4{\cdot}0 \text{ N}.$$

The frictional force is $f = 3{\cdot}0$ N.

(a) $$\tan\theta = \frac{f}{\mathcal{N}} = \frac{3}{4}$$

or, $$\theta = \tan^{-1}(3/4) = 37°.$$

(b) The magnitude of the contact force is

$$F = \sqrt{\mathcal{N}^2 + f^2}$$
$$= \sqrt{(4{\cdot}0 \text{ N})^2 + (3{\cdot}0 \text{ N})^2} = 5{\cdot}0 \text{ N}.$$

Friction can operate between a given pair of solids, between a solid and a fluid or between a pair of fluids. Frictional force exerted by fluids is called *viscous force* and we shall study it in a later chapter. Here we shall study about the frictional forces operating between a pair of solid surfaces.

When two solid bodies slip over each other, the force of friction is called *kinetic friction*. When two bodies do not slip on each other, the force of friction is called *static friction.*

It is difficult to work out a reliable theory of friction starting from the electromagnetic interaction between the particles at the surface. However, a wide range of observations can be summarized in a small number of *laws of friction* which we shall discuss.

6.2 KINETIC FRICTION

When two bodies in contact move with respect to each other, rubbing the surfaces in contact, the friction between them is called kinetic friction. The directions of the frictional forces are such that the relative slipping is opposed by the friction.

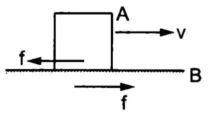


Figure 6.3

Suppose a body A placed in contact with B is moved with respect to it as shown in figure (6.3). The force of friction acting on A due to B will be opposite to the velocity of A with respect to B. In figure (6.3) this force is shown towards left. The force of friction on B due to A is opposite to the velocity of B with respect to A. In figure (6.3) this force is shown towards right. The force of kinetic friction opposes the relative motion. We can formulate the rules for finding the direction and magnitude of kinetic friction as follows :

(a) Direction of Kinetic Friction

The kinetic friction on a body A slipping against another body B is opposite to the velocity of A with respect to B.

It should be carefully noted that the velocity coming into picture is with respect to the body applying the force of friction.

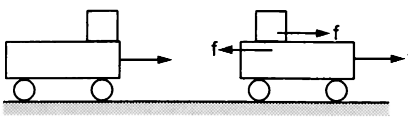


Figure 6.4

As another example, suppose we have a long box having wheels and moving on a horizontal road (figure 6.4). A small block is placed on the box which slips on the box to fall from the rear end. As seen from the road, both the box and the block are moving towards right, of course the velocity of the block is smaller than that of the box. What is the direction of the kinetic friction acting on the block due to the box ? The velocity of the block as seen from the box is towards left. Thus, the friction on the block is towards right. The friction acting on the box due to the block is towards left.

(b) Magnitude of the Kinetic Friction

The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k \mathcal{N} \qquad \dots \quad (6.1)$$

where $\mathcal{N}$ is the normal force. The proportionality constant μ_k is called the *coefficient of kinetic friction* and its value depends on the nature of the two surfaces in contact. If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

According to equation (6.1) the coefficient of kinetic friction does not depend on the speed of the sliding bodies. Once the bodies slip on each other the frictional force is $\mu_k \mathcal{N}$, whatever be the speed. This is approximately true for relative speeds not too large (say for speeds < 10 m/s).

We also see from equation (6.1) that as long as the normal force $\mathcal{N}$ is same, the frictional force is independent of the area of the surface in contact. For example, if a rectangular slab is slid over a table, the frictional force is same whether the slab lies flat on the table or it stands on its face of smaller area (figure 6.5)

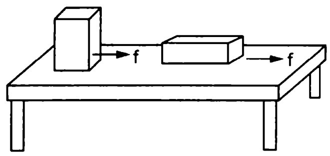


Figure 6.5

Example 6.2

A heavy box of mass 20 kg *is pulled on a horizontal surface by applying a horizontal force. If the coefficient of kinetic friction between the box and the horizontal surface is* 0·25, *find the force of friction exerted by the horizontal surface on the box.*

Solution : The situation is shown in figure (6.6). In the vertical direction there is no acceleration, so

$$\mathcal{N} = Mg.$$

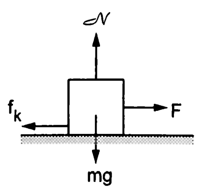


Figure 6.6

As the box slides on the horizontal surface, the surface exerts kinetic friction on the box. The magnitude of the kinetic friction is

$$f_k = \mu_k \mathcal{N} = \mu_k Mg$$

$$= 0{\cdot}25 \times (20 \text{ kg}) \times (9{\cdot}8 \text{ m/s}^2) = 49 \text{ N}.$$

This force acts in the direction opposite to the pull.

6.3 STATIC FRICTION

Frictional forces can also act between two bodies which are in contact but are not sliding with respect to each other. The friction in such cases is called *static friction*. For example, suppose several labourers are trying to push a heavy almirah on the floor to take it out of a room (figure 6.7).

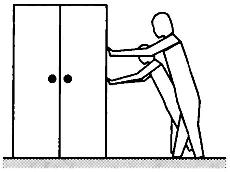


Figure 6.7

The almirah is heavy and even the most sincere effort by them is not able to slide it on the floor even by a millimeter. As the almirah is at rest the resultant force on the almirah should be zero. Thus, something is exerting a force on the almirah in the opposite direction. In this case, it is the floor which exerts a frictional force on the almirah. The labourers push the almirah towards left in figure (6.7) and the floor exerts a frictional force on the almirah towards right. This is an example of static friction.

How strong is this frictional force? Suppose the almirah is pushed with a small force in the beginning and the force is gradually increased. It does not slide until the force applied is greater than a minimum value say F. The force of static friction is equal and opposite to the force exerted by the labourers as long as the almirah is at rest. This means that the magnitude of static friction adjusts its value according to the applied force. As the applied force increases, the frictional force also increases. The static friction is thus, self adjustable. It adjusts its magnitude (and direction) in such a way that together with other forces applied on the body, it maintains 'relative rest' between the two surfaces. However, the frictional force cannot go beyond a maximum. When the applied force exceeds this maximum, friction fails to increase its value and slipping starts. The maximum static friction that a body can exert on the other body in contact with it, is called *limiting friction*. This limiting friction is proportional to the normal contact force between the two bodies. We can write

$$f_{max} = \mu_s \mathcal{N} \qquad \text{... (6.2)}$$

where f_{max} is the maximum possible force of static friction and $\mathcal{N}$ is the normal force. The constant of proportionality is called the *coefficient of static friction* and its value again depends on the material and roughness of the two surfaces in contact. In general, μ_s is slightly greater than μ_k. As long as the normal force is constant, the maximum possible friction does not depend on the area of the surfaces in contact.

Once again we emphasise that $\mu_s \mathcal{N}$ is the **maximum** possible force of static friction that **can** act between the bodies. The actual force of static friction may be smaller than $\mu_s \mathcal{N}$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest. Thus,

$$f_s \le f_{max} = \mu_s \mathcal{N}. \qquad \text{... (6.3)}$$

Example 6.3

A boy (30 kg) sitting on his horse whips it. The horse speeds up at an average acceleration of $2{\cdot}0 \text{ m/s}^2$. *(a) If the boy does not slide back, what is the force of friction exerted by the horse on the boy? (b) If the boy slides back during the acceleration, what can be said about the coefficient of static friction between the horse and the boy. Take* $g = 10 \text{ m/s}^2$.

Solution : (a) The forces acting on the boy are

(i) the weight Mg.
(ii) the normal contact force $\mathcal{N}$ and
(iii) the static friction f_s.

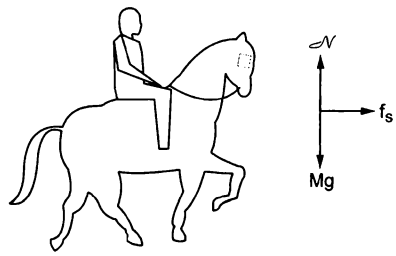


Figure 6.8

As the boy does not slide back, its acceleration a is equal to the acceleration of the horse. As friction is the only horizontal force, it must act along the acceleration and its magnitude is given by Newton's second law

$$f_s = Ma = (30 \text{ kg})(2{\cdot}0 \text{ m/s}^2) = 60 \text{ N}.$$

(b) If the boy slides back, the horse could not exert a friction of 60 N on the boy. The maximum force of static friction that the horse may exert on the boy is

$$f_s = \mu_s \mathcal{N} = \mu_s Mg$$
$$= \mu_s (30 \text{ kg})(10 \text{ m/s}^2) = \mu_s\, 300 \text{ N}$$

where μ_s is the coefficient of static friction. Thus,

$$\mu_s (300 \text{ N}) < 60 \text{ N}$$

or,
$$\mu_s < \frac{60}{300} = 0{\cdot}20.$$

Finding the Direction of Static Friction

The direction of static friction on a body is such that the total force acting on it keeps it at rest with respect to the body in contact. Newton's first or second law can often be used to find the direction of static friction. Figure (6.9) shows a block A placed on another block B which is placed on a horizontal table.

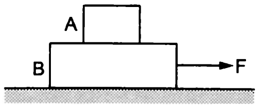


Figure 6.9

The block B is pulled by a force F towards right. Suppose the force is small and the blocks do not move. Let us focus our attention on the upper block. The upper block is at rest with respect to the ground which is assumed to be inertial. Thus, the resultant force on the upper block is zero (Newton's first law). As no other external force acts on the upper block the friction acting on the upper block due to the lower block, must be zero. If the force F is increased, the two blocks move together towards right, with some acceleration. As the upper block accelerates towards right the resultant force on it must be towards right. As friction is the only horizontal force on the upper block it must be towards right.

Notice that it is the friction on the upper block which accelerates it towards right. It is a general misconception that friction always opposes the motion. It is not really true. In many cases friction causes the motion. A vehicle accelerates on the road only because the frictional force on the vehicle due to the road drives it. It is not possible to accelerate a vehicle on a frictionless road. Friction opposes the relative motion between the bodies in contact.

Another way to find the direction of static friction is as follows. For a moment consider the surfaces to be frictionless. In absence of friction the bodies will start slipping against each other. One should then find the direction of friction as opposite to the velocity with respect to the body applying the friction.

6.4 LAWS OF FRICTION

We can summarise the laws of friction between two bodies in contact as follows :

(1) If the bodies slip over each other, the force of friction is given by

$$f_k = \mu_k \mathcal{N}$$

where $\mathcal{N}$ is the normal contact force and μ_k is the coefficient of kinetic friction between the surfaces.

(2) The direction of kinetic friction on a body is opposite to the velocity of this body with respect to the body applying the force of friction.

(3) If the bodies do not slip over each other, the force of friction is given by

$$f_s \leq \mu_s \mathcal{N}$$

where μ_s is the coefficient of static friction between the bodies and $\mathcal{N}$ is the normal force between them. The direction and magnitude of static friction are such that the condition of no slipping between the bodies is ensured.

(4) The frictional force f_k or f_s does not depend on the area of contact as long as the normal force $\mathcal{N}$ is same.

Table (6.1) gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will varry.

Table 6.1 : *The Friction Coefficients*

Material	μ_s	Material	μ_s
Steel and steel	0·58	Copper and copper	1·60
Steel and brass	0·35	Teflon and teflon	0·04
Glass and glass	1·00	Rubber tyre on dry concrete road	1·0
Wood and wood	0·35		
Wood and metal	0·40	Rubber tyre on wet concrete road	0·7
Ice and ice	0·10		

Dust, impurities, surface oxidation etc. have a great role in determining the friction coefficient. Suppose we take two blocks of pure copper, clean them carefully to remove any oxide or dust layer at the surfaces, heat them to push out any dissolved gases and keep them in contact with each other in an evacuated chamber at a very low pressure of air. The blocks stick to each other and a large force is needed to slide one over the other. The friction coefficient as defined above, becomes much larger than one. If a small amount of air is allowed to go into the chamber so that some oxidation takes place at the surface, the friction coefficient reduces to usual values.

6.5 UNDERSTANDING FRICTION AT ATOMIC LEVEL

It has already been pointed out that friction appears because of the interaction between the charged particles of the two bodies near the surfaces of contact. Any macroscopic object like a steel plate or a wood piece has irregular surface at atomic scale. A polished steel surface may look plane to naked eyes but if seen under a powerful microscope, its surface is found to be quite irregular. Figure (6.10) shows qualitatively how an apparently plane surface may be at the atomic scale.

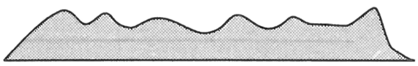


Figure 6.10

When two bodies are kept one over the other, the real area in contact is much smaller than the total surface area of the bodies (figure 6.11) The distance between the particles of the two bodies becomes very small at these actual points of contact and the molecular forces start operating across the surface. Molecular bonds are formed at these contact points. When one of the two bodies is pulled over the other, these bonds are broken, the materials under the bond is deformed and new bonds are formed. The local deformation of the bodies send vibration waves into the bodies. These vibrations finally damp out and the energy appears as the increased random motion of the particles of the bodies. The bodies thus, become heated. A force is, therefore, needed to start the motion or to maintain the motion.

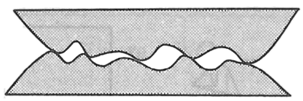


Figure 6.11

6.6 A LABORATORY METHOD TO MEASURE FRICTION COEFFICIENT

(a) Horizontal Table Method

Figure (6.12) shows the apparatus. A wooden plank A is fixed on a wooden frame kept on a table. A frictionless pulley is fixed to one end of the plank. A block B is kept on the plank and is attached to a hanger H by a string which passes over the pulley.

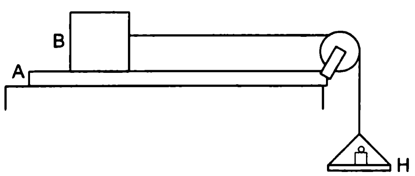


Figure 6.12

The plank is kept in a horizontal position. The friction coefficient between the block B and the plank A can be obtained by using this apparatus.

The weights of the block B and the hanger H are measured. Standard weights are kept on the hanger. The weights are gradually increased and the minimum weight needed to just slide the block is noted.

Suppose the weight of the block is W_1 and the weight of the hanger together with the standard weights is W_2 when the block just starts to slide. The tension in the string is W_2 and same is the force of friction on the block by the plank. Thus, the maximum force of static friction on the block is $f_{max} = W_2$. The normal force on the block by the plank is equal to the weight of the block itself as the block is in vertical equilibrium. Thus, the normal force is $\mathcal{N} = W_1$.

The coefficient of static friction is

$$\mu_s = \frac{f_{max}}{\mathcal{N}} = \frac{W_2}{W_1}.$$

To obtain the coefficient of kinetic friction, the weight on the hanger is slightly reduced and the block is gently pushed with a finger to move it on the plank. The weight on the hanger is so adjusted that once pushed, the block continues to move on the plank with uniform speed. In this case, the tension in the string equals the force of kinetic friction. As the hanger also moves with uniform velocity, the tension equals the weight of the hanger plus the standard weights kept in it. For vertical equilibrium of the block, the normal force on the block equals the weight of the block. Thus, if W_1 is the weight of the block and W_2' is the weight of the hanger plus the standard weights, the coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{\mathcal{N}} = \frac{W_2'}{W_1}.$$

One can put certain standard weights on the block to increase the normal force and repeat the experiment. It can be verified that the force of friction also increases and $f_k / \mathcal{N}$ comes out to be the same as it should be because the nature of the surfaces is same. If the block is kept on the plank on some other face, the area of contact is changed. It can be verified by repeating the above experiment that the force of friction does not depend on the area of contact for a given value of normal contact force.

(b) Inclined Table Method

In this method no pulley is needed. A wooden plank A is fixed on a wooden frame. One end of the plank is fixed to the frame on a hinge and the other end can be moved vertically and can be fixed at the desired position. Thus, the plank can be fixed in an inclined position and the angle of incline can be adjusted. A schematic diagram is shown in figure (6.13).

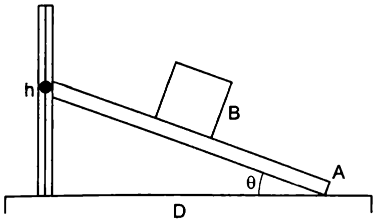


Figure 6.13

Block B is placed on the incline and the angle of the incline is gradually increased. The angle of the incline is so adjusted that the block just starts to slide. The height h and the horizontal distance D between the two ends of the plank are measured. The angle of incline θ satisfies

$$\tan\theta = h/D.$$

Let m be the mass of the block. The forces on the block in case of limiting equilibrium are (figure 6.14)

(i) weight of the block mg,

(ii) the normal contact force $\mathcal{N}$, and

(iii) the force of static friction f_s.

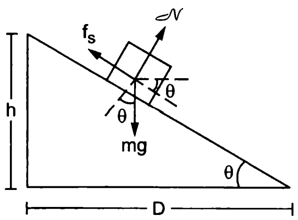


Figure 6.14

Taking components along the incline and applying Newton's first law,

$$f_s = mg \sin\theta.$$

Taking components along the normal to the incline,

$$\mathcal{N} = mg \cos\theta.$$

Thus, the coefficient of static friction between the block and the plank is

$$\mu_s = \frac{f_s}{\mathcal{N}} = \frac{mg \sin\theta}{mg \cos\theta} = \tan\theta = \frac{h}{D} \cdot$$

To obtain the kinetic friction, the inclination is reduced slightly and the block is made to move on the plank by gently pushing it with a finger. The inclination is so adjusted that once started, the block continues with uniform velocity on the plank. The height h' and the distance D' are noted. An identical analysis shows that the force of kinetic friction is

$$f_k = mg \sin\theta$$

and the normal contact force is

$$\mathcal{N} = mg \cos\theta$$

so that the coefficient of kinetic friction between the block and the plank is

$$\mu_k = \frac{f_k}{\mathcal{N}} = \tan\theta = h'/D'.$$

Example 6.4

A wooden block is kept on a polished wooden plank and the inclination of the plank is gradually increased. It is found that the block starts slipping when the plank makes an angle of 18° *with the horizontal. However, once started the block can continue with uniform speed if the inclination is reduced to* 15°. *Find the coefficients of static and kinetic friction between the block and the plank.*

Solution : The coefficient of static friction is

$$\mu_s = \tan 18°$$

and the coefficient of kinetic friction is

$$\mu_k = \tan 15°$$

Rolling Friction

It is quite difficult to pull a heavy iron box on a rough floor. However, if the box is provided with four wheels, also made of iron, it becomes easier to move the box on the same floor.

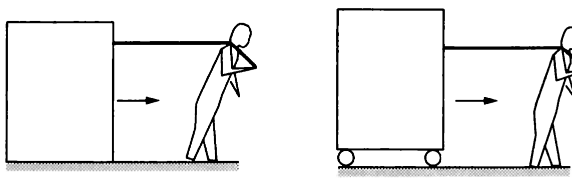


Figure 6.15

The wheel does not slide on the floor rather it rolls on the floor. The surfaces at contact do not rub each other. The velocity of the point of contact of the wheel with respect to the floor remains zero all the time although the centre of the wheel moves forward. The friction in the case of rolling is quite small as compared to kinetic friction. Quite often the rolling friction is negligible in comparison to the static or kinetic friction which may be present simultaneously. To reduce the wear and tear and energy loss against friction, small steel balls are kept between the rotating parts of machines which are known as ball bearings (figure 6.16).

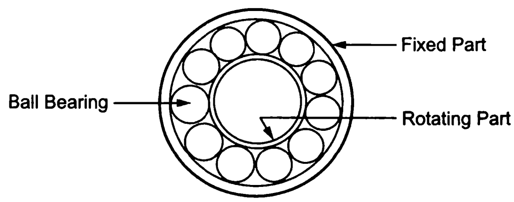


Figure 6.16

As one part moves with respect to the other, the balls roll on the two parts. No kinetic friction is involed and rolling friction being very small causes much less energy loss.

Worked Out Examples

1. *The coefficient of static friction between a block of mass m and an incline is* $\mu_s = 0{\cdot}3$. *(a) What can be the maximum angle* θ *of the incline with the horizontal so that the block does not slip on the plane ? (b) If the incline makes an angle* $\theta/2$ *with the horizontal, find the frictional force on the block.*

Solution : The situation is shown in figure (6-W1).

(a) the forces on the block are

(i) the weight mg downward by the earth,

(ii) the normal contact force $\mathcal{N}$ by the incline, and

(iii) the friction f parallel to the incline up the plane, by the incline.

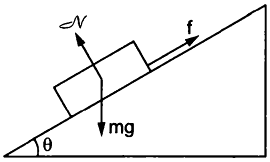


Figure 6-W1

As the block is at rest, these forces should add up to zero. Also, since θ is the maximum angle to prevent slipping, this is a case of limiting equilibrium and so $f = \mu_s \mathcal{N}$.

Taking components prependicular to the incline,

$$\mathcal{N} - mg\cos\theta = 0$$

or, $$\mathcal{N} = mg\cos\theta. \quad \ldots \text{(i)}$$

Taking components parallel to the incline,

$$f - mg\sin\theta = 0$$

or, $$f = mg\sin\theta$$

or, $$\mu_s \mathcal{N} = mg\sin\theta. \quad \ldots \text{(ii)}$$

Dividing (ii) by (i) $\mu_s = \tan\theta$

or, $$\theta = \tan^{-1}\mu_s = \tan^{-1}(0{\cdot}3).$$

(b) If the angle of incline is reduced to $\theta/2$, the equilibrium is not limiting, and hence the force of static friction f is less than $\mu_s \mathcal{N}$. To know the value of f, we proceed as in part (a) and get the equations

$$\mathcal{N} = mg\cos(\theta/2)$$

and $$f = mg\sin(\theta/2).$$

Thus, the force of friction is $mg\sin(\theta/2)$.

2. *A horizontal force of* 20 N *is applied to a block of mass* 4 kg *resting on a rough horizontal table. If the block does not move on the table, how much frictional force the table is applying on the block ? What can be said about the coefficient of static friction between the block and the table ? Take* $g = 10\ \text{m/s}^2$.

Solution : The situation is shown in figure (6-W2). The forces on the block are

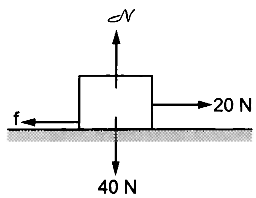


Figure 6-W2

(a) $4\ \text{kg} \times 10\ \text{m/s}^2 = 40\ \text{N}$ downward by the earth,

(b) $\mathcal{N}$ upward by the table,

(c) 20 N towards right by the experimenter and

(d) f towards left by the table (friction).

As the block is at rest, these forces should add up to zero. Taking horizontal and vertical components,

$$f = 20\ \text{N} \quad \text{and} \quad \mathcal{N} = 40\ \text{N}.$$

Thus, the table exerts a frictional (static) force of 20 N on the block in the direction opposite to the applied force. Since it is a case of static friction,

$$f \le \mu_s \mathcal{N}, \quad \text{or,} \quad \mu_s \ge f/\mathcal{N} \quad \text{or,} \quad \mu_s \ge 0{\cdot}5.$$

3. *The coefficient of static friction between the block of* 2 kg *and the table shown in figure (6-W3) is* $\mu_s = 0{\cdot}2$. *What should be the maximum value of m so that the blocks do not move ? Take* $g = 10\ \text{m/s}^2$. *The string and the pulley are light and smooth.*

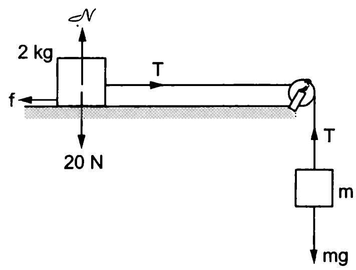


Figure 6-W3

Solution : Consider the equilibrium of the block of mass m. The forces on this block are

(a) mg downward by the earth and

(b) T upward by the string.

Hence, $T - mg = 0$ or, $T = mg$. ... (i)

Now consider the equilibrium of the 2 kg block. The forces on this block are

(a) T towards right by the string,

(b) f towards left (friction) by the table,

(c) 20 N downward (weight) by the earth and

(d) $\mathcal{N}$ upward (normal force) by the table.

For vertical equilibrium of this block,

$$\mathcal{N} = 20 \text{ N}. \qquad \text{... (ii)}$$

As m is the largest mass which can be used without moving the system, the friction is limiting.

Thus, $f = \mu_s \mathcal{N}$. ... (iii)

For horizontal equilibrium of the 2 kg block,

$$f = T. \qquad \text{... (iv)}$$

Using equations (i), (iii) and (iv)

$$\mu_s \mathcal{N} = mg$$

or, $0{\cdot}2 \times 20 \text{ N} = mg$

or, $m = \dfrac{0{\cdot}2 \times 20}{10} \text{ kg} = 0{\cdot}4 \text{ kg}.$

4. *The coefficient of static friction between the two blocks shown in figure (6-W4) is μ and the table is smooth. What maximum horizontal force F can be applied to the block of mass M so that the blocks move together?*

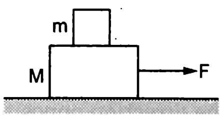


Figure 6-W4

Solution : When the maximum force F is applied, both the blocks move together towards right. The only horizontal force on the upper block of mass m is that due to the friction by the lower block of mass M. Hence this force on m should be towards right. The force of friction on M by m should be towards left by Newton's third law. As we are talking of the maximum possible force F that can be applied, the friction is limiting and hence $f = \mu \mathcal{N}$, where $\mathcal{N}$ is the normal force between the blocks.

Consider the motion of m. The forces on m are (figure 6-W5),

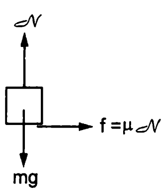


Figure 6-W5

(a) mg downward by the earth (gravity),

(b) $\mathcal{N}$ upward by the block M (normal force) and

(c) $f = \mu \mathcal{N}$ (friction) towards right by the block M.

In the vertical direction, there is no acceleration. This gives

$$\mathcal{N} = mg. \qquad \text{... (i)}$$

In the horizontal direction, let the acceleration be a, then

$$\mu \mathcal{N} = m\,a$$

or, $\mu\, mg = ma$

or, $a = \mu g$. ... (ii)

Next, consider the motion of M (figure 6-W6).

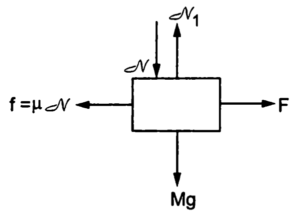


Figure 6-W6

The forces on M are

(a) Mg downward by the earth (gravity),

(b) $\mathcal{N}_1$ upward by the table (normal force),

(c) $\mathcal{N}$ downward by m (normal force),

(d) $f = \mu \mathcal{N}$ (friction) towards left by m and

(e) F (applied force) by the experimenter.

The equation of motion is

$$F - \mu \mathcal{N} = M a$$

or, $F - \mu\, mg = M \mu g$ [Using (i) and (ii)]

or, $F = \mu g\,(M + m)$.

5. *A block slides down an incline of angle 30° with an acceleration $g/4$. Find the kinetic friction coeffcient.*

Solution : Let the mass of the block be m. The forces on the block are (Figure 6-W7),

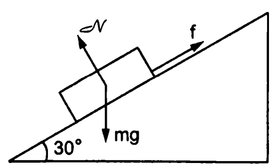


Figure 6-W7

(a) mg downward by the earth (gravity),

(b) $\mathcal{N}$ normal force by the incline and

(c) f up the plane, (friction) by the incline.

Taking components parallel to the incline and writing Newton's second law,

$$mg \sin 30° - f = mg/4$$

or, $f = mg/4$.

There is no acceleration perpendicular to the incline. Hence,

$$\mathcal{N} = mg \cos 30° = mg \cdot \frac{\sqrt{3}}{2}.$$

As the block is slipping on the incline, friction is $f = \mu_k \mathcal{N}$.

So, $$\mu_k = \frac{f}{\mathcal{N}} = \frac{mg}{4\,mg\,\sqrt{3}/2} = \frac{1}{2\sqrt{3}}.$$

6. *A block of mass* 2·5 kg *is kept on a rough horizontal surface. It is found that the block does not slide if a horizontal force less than* 15 N *is applied to it. Also it is found that it takes* 5 seconds *to slide through the first* 10 m *if a horizontal force of* 15 N *is applied and the block is gently pushed to start the motion. Taking* $g = 10\ \text{m/s}^2$, *calculate the coefficients of static and kinetic friction between the block and the surface.*

Solution : The forces acting on the block are shown in figure (6-W8). Here M = 2·5 kg and F = 15 N.

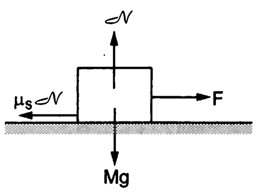


Figure 6-W8

When F = 15 N is applied to the block, the block remains in limiting equilibrium. The force of friction is thus $f = \mu_s \mathcal{N}$. Applying Newton's first law,

$$f = \mu_s \mathcal{N} \text{ and } \mathcal{N} = mg$$

so that $$F = \mu_s Mg$$

or, $$\mu_s = \frac{F}{mg} = \frac{15\ \text{N}}{(2{\cdot}5\ \text{kg})\ (10\ \text{m/s}^2)} = 0{\cdot}60.$$

When the block is gently pushed to start the motion, kinetic friction acts between the block and the surface. Since the block takes 5 second to slide through the first 10 m, the acceleration a is given by

$$10\ \text{m} = \frac{1}{2} a\ (5\ \text{s})^2$$

or, $$a = \frac{20}{25}\ \text{m/s}^2 = 0{\cdot}8\ \text{m/s}^2.$$

The frictional force is

$$f = \mu_k \mathcal{N} = \mu_k Mg.$$

Applying Newton's second law

$$F - \mu_k Mg = Ma$$

or, $$\mu_k = \frac{F - Ma}{Mg}$$

$$= \frac{15\ \text{N} - (2{\cdot}5\ \text{kg})\ (0{\cdot}8\ \text{m/s}^2)}{(2{\cdot}5\ \text{kg})\ (10\ \text{m/s}^2)} = 0{\cdot}52.$$

7. *A block placed on a horizontal surface is being pushed by a force F making an angle* θ *with the vertical. If the friction coefficient is* μ, *how much force is needed to get the block just started. Discuss the situation when* tanθ < μ.

Solution : The situation is shown in figure (6-W9). In the limiting equilibrium the frictional force f will be equal to $\mu \mathcal{N}$. For horizontal equilibrium

$$F \sin\theta = \mu \mathcal{N}$$

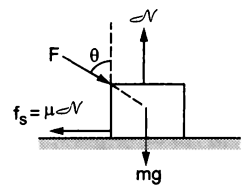


Figure 6-W9

For vertical equilibrium

$$F \cos\theta + mg = \mathcal{N}.$$

Eliminating $\mathcal{N}$ from these equations,

$$F \sin\theta = \mu F \cos\theta + \mu\, mg$$

or, $$F = \frac{\mu\, mg}{\sin\theta - \mu \cos\theta}.$$

If $\tan\theta < \mu$ we have $(\sin\theta - \mu \cos\theta) < 0$ and then F is negative. So for angles less than $\tan^{-1}\mu$, one cannot push the block ahead, however large the force may be.

8. *Find the maximum value of* M/m *in the situation shown in figure (6-W10) so that the system remains at rest. Friction coefficient at both the contacts is* μ. *Discuss the situation when* tanθ < μ.

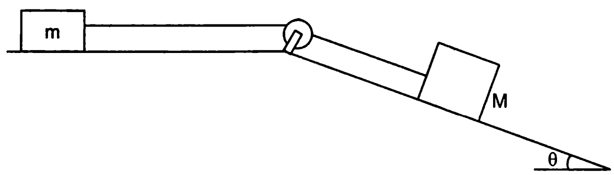


Figure 6-W10

Solution : Figure (6-W11) shows the forces acting on the two blocks. As we are looking for the maximum value of M/m, the equilibrium is limiting. Hence, the frictional forces are equal to μ times the corresponding normal forces.

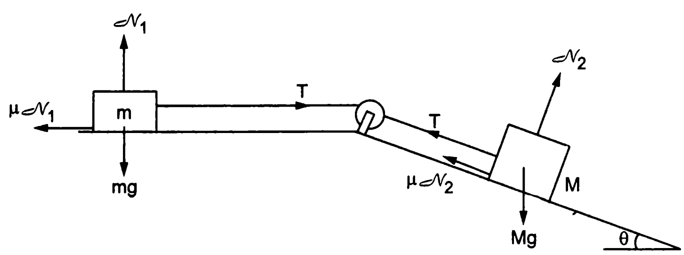


Figure 6-W11

Equilibrium of the block m gives

$T = \mu \mathcal{N}_1$ and $\mathcal{N}_1 = mg$

which gives

$$T = \mu mg. \quad \text{... (i)}$$

Next, consider the equilibrium of the block M. Taking components parallel to the incline

$$T + \mu \mathcal{N}_2 = Mg \sin\theta.$$

Taking components normal to the incline

$$\mathcal{N}_2 = Mg \cos\theta.$$

These give $\quad T = Mg (\sin\theta - \mu \cos\theta). \quad$ ... (ii)

From (i) and (ii), $\mu mg = Mg (\sin\theta - \mu \cos\theta)$

or, $$M/m = \frac{\mu}{\sin\theta - \mu \cos\theta}$$

If $\tan\theta < \mu$, $(\sin\theta - \mu \cos\theta) < 0$ and the system will not slide for any value of M/m.

9. *Consider the situation shown in figure (6-W12). The horizontal surface below the bigger block is smooth. The coefficient of friction between the blocks is μ. Find the minimum and the maximum force F that can be applied in order to keep the smaller block at rest with respect to the bigger block.*

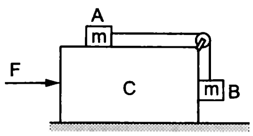


Figure 6-W12

Solution : If no force is applied, the block A will slip on C towards right and the block B will move downward. Suppose the minimum force needed to prevent slipping is F. Taking $A + B + C$ as the system, the only external horizontal force on the system is F. Hence the acceleration of the system is

$$a = \frac{F}{M + 2m}. \quad \text{... (i)}$$

Now take the block A as the system. The forces on A are (figure 6-W13),

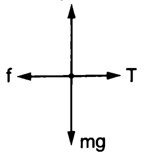


Figure 6-W13

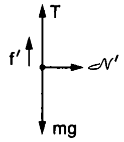


Figure 6-W14

(i) tension T by the string towards right,
(ii) friction f by the block C towards left,
(iii) weight mg downward and
(iv) normal force $\mathcal{N}$ upward.

For vertical equilibrium $\mathcal{N} = mg$.

As the minimum force needed to prevent slipping is applied, the friction is limiting. Thus,

$$f = \mu \mathcal{N} = \mu mg.$$

As the block moves towards right with an acceleration a,

$$T - f = ma$$

or, $$T - \mu mg = ma. \quad \text{... (ii)}$$

Now take the block B as the system. The forces are (figure 6-W14),

(i) tension T upward,
(ii) weight mg downward,
(iii) normal force $\mathcal{N}'$ towards right, and
(iv) friction f' upward.

As the block moves towards right with an acceleration a,

$$\mathcal{N}' = ma.$$

As the friction is limiting, $f' = \mu \mathcal{N}' = \mu ma$.

For vertical equilibrium

$$T + f' = mg$$

or, $$T + \mu ma = mg. \quad \text{... (iii)}$$

Eliminating T from (ii) and (iii)

$$a_{\min} = \frac{1 - \mu}{1 + \mu} g.$$

When a large force is applied the block A slips on C towards left and the block B slips on C in the upward direction. The friction on A is towards right and that on B is downwards. Solving as above, the acceleration in this case is

$$a_{\max} = \frac{1 + \mu}{1 - \mu} g.$$

Thus, a lies between $\frac{1 - \mu}{1 + \mu} g$ and $\frac{1 + \mu}{1 - \mu} g$.

From (i) the force F should be between

$$\frac{1 - \mu}{1 + \mu} (M + 2m) g \text{ and } \frac{1 + \mu}{1 - \mu} (M + 2m) g.$$

10. *Figure (6-W15) shows two blocks connected by a light string placed on the two inclined parts of a triangular structure. The coefficients of static and kinetic friction are 0·28 and 0·25 respectively at each of the surfaces. (a) Find the minimum and maximum values of m for which the system remains at rest. (b) Find the acceleration of either block if m is given the minimum value calculated in the first part and is gently pushed up the incline for a short while.*

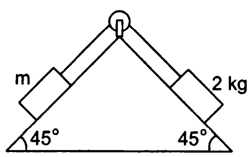


Figure 6-W15

Solution : (a) Take the 2 kg block as the system. The forces on this block are shown in figure (6-W16) with M = 2 kg. It is assumed that m has its minimum value so that the 2 kg block has a tendency to slip down. As the block is in equilibrium, the resultant force should be zero.

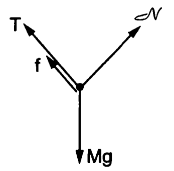


Figure 6-W16

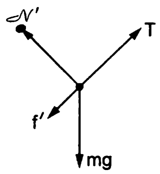


Figure 6-W17

Taking components ⊥ to the incline

$$\mathcal{N} = Mg\cos 45° = Mg/\sqrt{2}.$$

Taking components ∥ to the incline

$$T + f = Mg\sin 45° = Mg/\sqrt{2}$$

or, $$T = Mg/\sqrt{2} - f.$$

As it is a case of limiting equilibrium,

$$f = \mu_s \mathcal{N}$$

or, $$T = \frac{Mg}{\sqrt{2}} - \mu_s\frac{Mg}{\sqrt{2}} = \frac{Mg}{\sqrt{2}}(1-\mu_s). \quad \dots \text{(i)}$$

Now consider the other block as the system. The forces acting on this block are shown in figure (6-W17).

Taking components ⊥ to the incline,

$$\mathcal{N}' = mg\cos 45° = mg/\sqrt{2}.$$

Taking components ∥ to the incline

$$T = mg\sin 45° + f' = \frac{mg}{\sqrt{2}} + f'.$$

As it is the case of limiting equilibrium

$$f' = \mu_s\mathcal{N}' = \mu_s\frac{mg}{\sqrt{2}}.$$

Thus, $$T = \frac{mg}{\sqrt{2}}(1+\mu_s). \quad \dots \text{(ii)}$$

From (i) and (ii)

$$m(1+\mu_s) = M(1-\mu_s) \quad \dots \text{(iii)}$$

or, $$m = \frac{(1-\mu_s)}{(1+\mu_s)}M = \frac{1-0{\cdot}28}{1+0{\cdot}28}\times 2\text{ kg}$$
$$= \frac{9}{8}\text{ kg}.$$

When maximum possible value of m is supplied, the directions of friction are reversed because m has the tendency to slip down and 2 kg block to slip up. Thus, the maximum value of m can be obtained from (iii) by putting $\mu = -0{\cdot}28$. Thus, the maximum value of m is

$$m = \frac{1+0{\cdot}28}{1-0{\cdot}28}\times 2\text{ kg}$$
$$= \frac{32}{9}\text{ kg}.$$

(b) If $m = 9/8$ kg and the system is gently pushed, kinetic friction will operate. Thus,

$$f = \mu_k\cdot\frac{Mg}{\sqrt{2}} \quad\text{and}\quad f' = \frac{\mu_k mg}{\sqrt{2}},$$

where $\mu_k = 0{\cdot}25$. If the acceleration is a, Newton's second law for M gives (figure 6-W16).

$$Mg\sin 45° - T - f = Ma$$

or, $$\frac{Mg}{\sqrt{2}} - T - \frac{\mu_k Mg}{\sqrt{2}} = Ma. \quad \dots \text{(iv)}$$

Applying Newton's second law m (figure 6-W17),

$$T - mg\sin 45° - f' = ma$$

or, $$T - \frac{mg}{\sqrt{2}} - \frac{\mu_k mg}{\sqrt{2}} = ma. \quad \dots \text{(v)}$$

Adding (iv) and (v)

$$\frac{Mg}{\sqrt{2}}(1-\mu_k) - \frac{mg}{\sqrt{2}}(1+\mu_k) = (M+m)a$$

or, $$a = \frac{M(1-\mu_k) - m(1+\mu_k)}{\sqrt{2}(M+m)}g$$
$$= \frac{2\times 0{\cdot}75 - 9/8\times 1{\cdot}25}{\sqrt{2}(2+9/8)}g$$
$$= 0{\cdot}31\text{ m/s}^2.$$

□

QUESTIONS FOR SHORT ANSWER

1. For most of the surfaces used in daily life, the friction coefficient is less than 1. Is it always necessary that the friction coefficient is less than 1 ?
2. Why is it easier to push a heavy block from behind than to press it on the top and push ?
3. What is the average friction force when a person has a usual 1 km walk ?
4. Why is it difficult to walk on solid ice ?
5. Can you accelerate a car on a frictionless horizontal road by putting more petrol in the engine ? Can you stop a car going on a frictionless horizontal road by applying brakes ?
6. Spring fitted doors close by themselves when released. You want to keep the door open for a long time, say for an hour. If you put a half kg stone in front of the open door, it does not help. The stone slides with the door and the door gets closed. However, if you sandwitch a

20 g piece of wood in the small gap between the door and the floor, the door stays open. Explain why a much lighter piece of wood is able to keep the door open while the heavy stone fails.

7. A classroom demonstration of Newton's first law is as follows : A glass is covered with a plastic card and a coin is placed on the card. The card is given a quick strike and the coin falls in the glass. (a) Should the friction coefficient between the card and the coin be small or large ? (b) Should the coin be light or heavy ? (c) Why does the experiment fail if the card is gently pushed ?

8. Can a tug of war be ever won on a frictionless surface ?

9. Why do tyres have a better grip of the road while going on a level road than while going on an incline ?

10. You are standing with your bag in your hands, on the ice in the middle of a pond. The ice is so slippery that it can offer no friction. How can you come out of the ice ?

11. When two surfaces are polished, the friction coefficient between them decreases. But the friction coefficient increases and becomes very large if the surfaces are made highly smooth. Explain.

OBJECTIVE I

1. In a situation the contact force by a rough horizontal surface on a body placed on it has constant magnitude. If the angle between this force and the vertical is decreased, the frictional force between the surface and the body will
(a) increase (b) decrease
(c) remain the same (d) may increase or decrease.

2. While walking on ice, one should take small steps to avoid slipping. This is because smaller steps ensure
(a) larger friction (b) smaller friction
(c) larger normal force (d) smaller normal force.

3. A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force but the body is not moving. The force by the surface on A is F, where
(a) $F = Mg$ (b) $F = \mu Mg$
(c) $Mg \le F \le Mg\sqrt{1+\mu^2}$ (d) $Mg \ge F \ge Mg\sqrt{1-\mu^2}$.

4. A scooter starting from rest moves with a constant acceleration for a time Δt_1, then with a constant velocity for the next Δt_2 and finally with a constant deceleration for the next Δt_3 to come to rest. A 500 N man sitting on the scooter behind the driver manages to stay at rest with respect to the scooter without touching any other part. The force exerted by the seat on the man is
(a) 500 N throughout the journey
(b) less than 500 N throughout the journey
(c) more than 500 N throughout the journey
(d) > 500 N for time Δt_1 and Δt_3 and 500 N for Δt_2.

5. Consider the situation shown in figure (6-Q1). The wall is smooth but the surfaces of A and B in contact are rough. The friction on B due to A in equilibrium

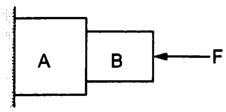


Figure 6-Q1

(a) is upward (b) is downward (c) is zero
(d) the system cannot remain in equilibrium.

6. Suppose all the surfaces in the previous problem are rough. The direction of friction on B due to A
(a) is upward (b) is downward (c) is zero
(d) depends on the masses of A and B.

7. Two cars of unequal masses use similar tyres. If they are moving at the same initial speed, the minimum stopping distance
(a) is smaller for the heavier car
(b) is smaller for the lighter car
(c) is same for both cars
(d) depends on the volume of the car.

8. In order to stop a car in shortest distance on a horizontal road, one should
(a) apply the brakes very hard so that the wheels stop rotating
(b) apply the brakes hard enough to just prevent slipping
(c) pump the brakes (press and release)
(d) shut the engine off and not apply brakes.

9. A block A kept on an inclined surface just begins to slide if the inclination is 30°. The block is replaced by another block B and it is found that it just begins to slide if the inclination is 40°.
(a) mass of A > mass of B (b) mass of A < mass of B
(c) mass of A = mass of B (d) all the three are possible.

10. A boy of mass M is applying a horizontal force to slide a box of mass M' on a rough horizontal surface. The coefficient of friction between the shoes of the boy and the floor is μ and that between the box and the floor is μ'. In which of the following cases it is certainly not possible to slide the box ?
(a) $\mu < \mu'$, $M < M'$ (b) $\mu > \mu'$, $M < M'$
(c) $\mu < \mu'$, $M > M'$ (d) $\mu > \mu'$, $M > M'$.

OBJECTIVE II

1. Let F, F_N and f denote the magnitudes of the contact force, normal force and the friction exerted by one surface on the other kept in contact. If none of these is zero,
(a) $F > F_N$ (b) $F > f$ (c) $F_N > f$ (d) $F_N - f < F < F_N + f$.

2. The contact force exerted by a body A on another body B is equal to the normal force between the bodies. We conclude that
(a) the surfaces must be frictionless
(b) the force of friction between the bodies is zero
(c) the magnitude of normal force equals that of friction
(d) the bodies may be rough but they don't slip on each other.

3. Mark the correct statements about the friction between two bodies.
(a) Static friction is always greater than the kinetic friction.
(b) Coefficient of static friction is always greater than the coefficient of kinetic friction.
(c) Limiting friction is always greater than the kinetic friction.
(d) Limiting friction is never less than static friction.

4. A block is placed on a rough floor and a horizontal force F is applied on it. The force of friction f by the floor on the block is measured for different values of F and a graph is plotted between them.
(a) The graph is a straight line of slope 45°.
(b) The graph is a straight line parallel to the F-axis.
(c) The graph is a straight line of slope 45° for small F and a straight line parallel to the F-axis for large F.
(d) There is a small kink on the graph.

5. Consider a vehicle going on a horizontal road towards east. Neglect any force by the air. The frictional forces on the vehicle by the road
(a) is towards east if the vehicle is accelerating
(b) is zero if the vehicle is moving with a uniform velocity
(c) must be towards east
(d) must be towards west.

EXERCISES

1. A body slipping on a rough horizontal plane moves with a deceleration of 4·0 m/s^2. What is the coefficient of kinetic friction between the block and the plane ?

2. A block is projected along a rough horizontal road with a speed of 10 m/s. If the coefficient of kinetic friction is 0·10, how far will it travel before coming to rest ?

3. A block of mass m is kept on a horizontal table. If the static friction coefficient is μ, find the frictional force acting on the block.

4. A block slides down an inclined surface of inclination 30° with the horizontal. Starting from rest it covers 8 m in the first two seconds. Find the coefficient of kinetic friction between the two.

5. Suppose the block of the previous problem is pushed down the incline with a force of 4 N. How far will the block move in the first two seconds after starting from rest ? The mass of the block is 4 kg.

6. A body of mass 2 kg is lying on a rough inclined plane of inclination 30°. Find the magnitude of the force parallel to the incline needed to make the block move (a) up the incline (b) down the incline. Coefficient of static friction = 0·2.

7. Repeat part (a) of problem 6 if the push is applied horizontally and not parallel to the incline.

8. In a children-park an inclined plane is constructed with an angle of incline 45° in the middle part (figure 6-E1). Find the acceleration of a boy sliding on it if the friction coefficient between the cloth of the boy and the incline is 0·6 and $g = 10\ m/s^2$.

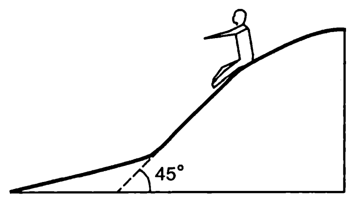


Figure 6-E1

9. A body starts slipping down an incline and moves half meter in half second. How long will it take to move the next half meter ?

10. The angle between the resultant contact force and the normal force exerted by a body on the other is called the *angle of friction.* Show that, if λ be the angle of friction and μ the coefficient of static friction, $\lambda \le \tan^{-1}\mu$.

11. Consider the situation shown in figure (6-E2). Calculate (a) the acceleration of the 1·0 kg blocks, (b) the tension in the string connecting the 1·0 kg blocks and (c) the tension in the string attached to 0·50 kg.

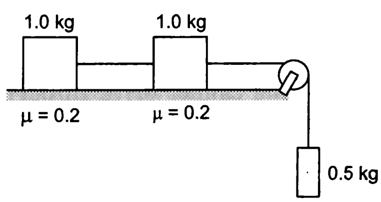


Figure 6-E2

12. If the tension in the string in figure (6-E3) is 16 N and the acceleration of each block is $0{\cdot}5\ m/s^2$, find the friction coefficients at the two contacts with the blocks.

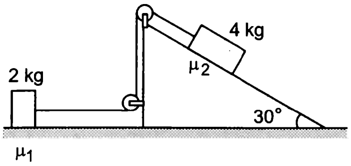


Figure 6-E3

13. The friction coefficient between the table and the block shown in figure (6-E4) is 0·2. Find the tensions in the two strings.

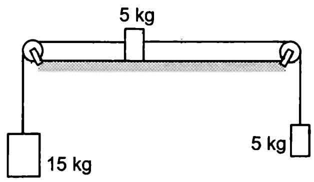


Figure 6-E4

14. The friction coefficient between a road and the tyre of a vehicle is 4/3. Find the maximum incline the road may have so that once hard brakes are applied and the wheel starts skidding, the vehicle going down at a speed of 36 km/hr is stopped within 5 m.

15. The friction coefficient between an athelete's shoes and the ground is 0·90. Suppose a superman wears these shoes and races for 50 m. There is no upper limit on his capacity of running at high speeds. (a) Find the minimum time that he will have to take in completing the 50 m starting from rest. (b) Suppose he takes exactly this minimum time to complete the 50 m, what minimum time will he take to stop?

16. A car is going at a speed of 21·6 km/hr when it encounters a 12·8 m long slope of angle 30° (figure 6-E5). The friction coefficient between the road and the tyre is $1/2\sqrt{3}$. Show that no matter how hard the driver applies the brakes, the car will reach the bottom with a speed greater than 36 km/hr. Take $g = 10\ m/s^2$.

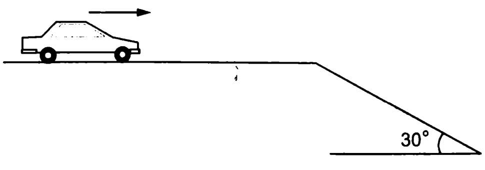


Figure 6-E5

17. A car starts from rest on a half kilometer long bridge. The coefficient of friction between the tyre and the road is 1·0. Show that one cannot drive through the bridge in less than 10 s.

18. Figure (6-E6) shows two blocks in contact sliding down an inclined surface of inclination 30°. The friction coefficient between the block of mass 2·0 kg and the incline is μ_1, and that between the block of mass 4·0 kg and the incline is μ_2. Calculate the acceleration of the 2·0 kg block if (a) $\mu_1 = 0{\cdot}20$ and $\mu_2 = 0{\cdot}30$, (b) $\mu_1 = 0{\cdot}30$ and $\mu_2 = 0{\cdot}20$. Take $g = 10\ m/s^2$.

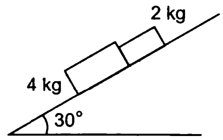


Figure 6-E6

19. Two masses M_1 and M_2 are connected by a light rod and the system is slipping down a rough incline of angle θ with the horizontal. The friction coefficient at both the contacts is μ. Find the acceleration of the system and the force by the rod on one of the blocks.

20. A block of mass M is kept on a rough horizontal surface. The coefficient of static friction between the block and the surface is μ. The block is to be pulled by applying a force to it. What minimum force is needed to slide the block? In which direction should this force act?

21. The friction coefficient between the board and the floor shown in figure (6-E7) is μ. Find the maximum force that the man can exert on the rope so that the board does not slip on the floor.

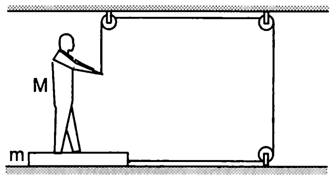


Figure 6-E7

22. A 2 kg block is placed over a 4 kg block and both are placed on a smooth horizontal surface. The coefficient of friction between the blocks is 0·20. Find the acceleration of the two blocks if a horizontal force of 12 N is applied to (a) the upper block, (b) the lower block. Take $g = 10\ m/s^2$.

23. Find the accelerations a_1, a_2, a_3 of the three blocks shown in figure (6-E8) if a horizontal force of 10 N is applied on (a) 2 kg block, (b) 3 kg block, (c) 7 kg block. Take $g = 10\ m/s^2$.

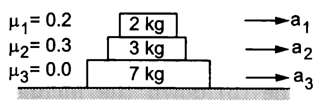


Figure 6-E8

24. The friction coefficient between the two blocks shown in figure (6-E9) is μ but the floor is smooth. (a) What maximum horizontal force F can be applied without disturbing the equilibrium of the system? (b) Suppose the horizontal force applied is double of that found in part (a). Find the accelerations of the two masses.

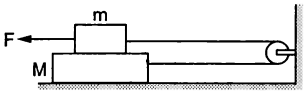


Figure 6-E9

25. Suppose the entire system of the previous question is kept inside an elevator which is coming down with an acceleration $a < g$. Repeat parts (a) and (b).

26. Consider the situation shown in figure (6-E9). Suppose a small electric field E exists in the space in the vertically upward direction and the upper block carries a positive charge Q on its top surface. The friction coefficient between the two blocks is μ but the floor is smooth. What maximum horizontal force F can be applied without disturbing the equilibrium?
[Hint : The force on a charge Q by the electric field E is $F = QE$ in the direction of E.]

27. A block of mass m slips on a rough horizontal table under the action of a horizontal force applied to it. The coefficient of friction between the block and the table is μ. The table does not move on the floor. Find the total frictional force applied by the floor on the legs of the table. Do you need the friction coefficient between the table and the floor or the mass of the table?

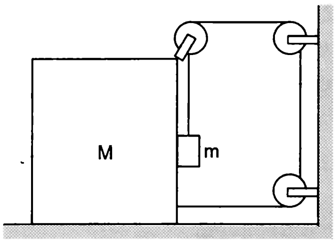


Figure 6-E10

28. Find the acceleration of the block of mass M in the situation of figure (6-E10). The coefficient of friction between the two blocks is μ_1 and that between the bigger block and the ground is μ_2.

29. A block of mass 2 kg is pushed against a rough vertical wall with a force of 40 N, coefficient of static friction being 0·5. Another horizontal force of 15 N, is applied on the block in a direction parallel to the wall. Will the block move? If yes, in which direction? If no, find the frictional force exerted by the wall on the block.

30. A person (40 kg) is managing to be at rest between two vertical walls by pressing one wall A by his hands and feet and the other wall B by his back (figure 6-E11). Assume that the friction coefficient between his body and the walls is 0·8 and that limiting friction acts at all the contacts. (a) Show that the person pushes the two walls with equal force. (b) Find the normal force exerted by either wall on the person. Take $g = 10 \text{ m/s}^2$.

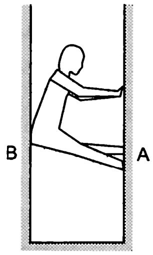


Figure 6-E11

31. Figure (6-E12) shows a small block of mass m kept at the left end of a larger block of mass M and length l. The system can slide on a horizontal road. The system is started towards right with an initial velocity v. The friction coefficient between the road and the bigger block is μ and that between the block is $\mu/2$. Find the time elapsed before the smaller blocks separates from the bigger block.

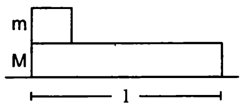


Figure 6-E12

□

ANSWERS

OBJECTIVE I

1. (b) 2. (b) 3. (c) 4. (d) 5. (d) 6. (a)
7. (c) 8. (b) 9. (d) 10. (a)

OBJECTIVE II

1. (a), (b), (d) 2. (b). (d) 3. (b), (c), (d)
4. (c), (d) 5. (a), (b)

EXERCISES

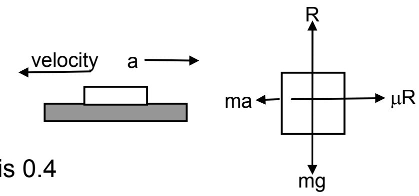
1. 0·4
2. 50 m
3. zero
4. 0·11
5. 10 m
6. (a) 13 N (b) zero

7. 17·5 N
8. $2\sqrt{2}$ m/s^2
9. 0·21 s
11. (a) 0·4 m/s^2 (b) 2·4 N (c) 4·8 N
12. $\mu_1 = 0{\cdot}75$, $\mu_2 = 0{\cdot}06$
13. 96 N in the left string and 68 N in the right
14. 16°
15. (a) $\frac{10}{3}$ s (b) $\frac{10}{3}$ s
18. 2·7 m/s^2, 2·4 m/s^2
19. $a = g(\sin\theta - \mu\cos\theta)$, zero
20. $\frac{\mu mg}{\sqrt{1+\mu^2}}$ at an angle $\tan^{-1}\mu$ with the horizontal
21. $\frac{\mu(M+m)g}{1+\mu}$
22. (a) upper block 4 m/s^2, lower block 1 m/s^2
    (b) both blocks 2 m/s^2
23. (a) $a_1 = 3$ m/s^2, $a_2 = a_3 = 0{\cdot}4$ m/s^2
    (b) $a_1 = a_2 = a_3 = \frac{5}{6}$ m/s^2 (c) same as (b)
24. (a) $2\mu mg$ (b) $\frac{2\mu mg}{M+m}$ in opposite directions
25. (a) $2\mu m(g-a)$ (b) $\frac{2\mu m(g-a)}{m+M}$
26. $2\mu(mg - QE)$
27. μmg
28. $\frac{[2m - \mu_2(M+m)]g}{M+m[5+2(\mu_1-\mu_2)]}$
29. it will move at an angle of 53° with the 15 N force
30. (b) 250 N
31. $\sqrt{\frac{4Ml}{(M+m)\mu g}}$

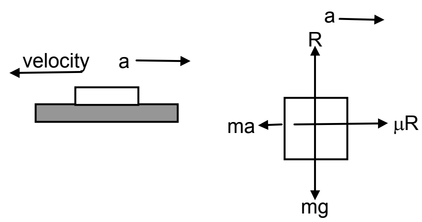
□

SOLUTIONS TO CONCEPTS
CHAPTER 6

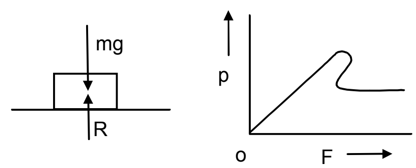
1. Let m = mass of the block
   From the freebody diagram,
   $R - mg = 0 \Rightarrow R = mg$ ...(1)
   Again $ma - \mu R = 0 \Rightarrow ma = \mu R = \mu mg$ (from (1))
   $\Rightarrow a = \mu g \Rightarrow 4 = \mu g \Rightarrow \mu = 4/g = 4/10 = 0.4$
   The co-efficient of kinetic friction between the block and the plane is 0.4



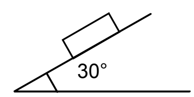
2. Due to friction the body will decelerate
   Let the deceleration be 'a'
   $R - mg = 0 \Rightarrow R = mg$ ...(1)
   $ma - \mu R = 0 \Rightarrow ma = \mu R = \mu mg$ (from (1))
   $\Rightarrow a = \mu g = 0.1 \times 10 = 1 m/s^2.$
   Initial velocity u = 10 m/s
   Final velocity v = 0 m/s
   $a = -1 m/s^2$ (deceleration)
   $$S = \frac{v^2 - u^2}{2a} = \frac{0 - 10^2}{2(-1)} = \frac{100}{2} = 50m$$
   It will travel 50m before coming to rest.

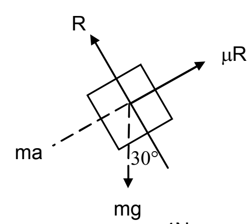


3. Body is kept on the horizontal table.
   If no force is applied, no frictional force will be there
   $f \rightarrow$ frictional force
   $F \rightarrow$ Applied force
   From grap it can be seen that when applied force is zero, frictional force is zero.

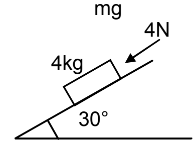


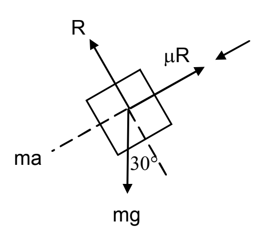
4. From the free body diagram,
   $R - mg\cos\theta = 0 \Rightarrow R = mg\cos\theta$ ..(1)
   For the block
   U = 0, s = 8m, t = 2sec.
   $\therefore s = ut + ½ at^2 \Rightarrow 8 = 0 + ½ a 2^2 \Rightarrow a = 4 m/s^2$
   Again, $\mu R + ma - mg\sin\theta = 0$
   $\Rightarrow \mu mg\cos\theta + ma - mg\sin\theta = 0$ [from (1)]
   $\Rightarrow m(\mu g\cos\theta + a - g\sin\theta) = 0$
   $\Rightarrow \mu \times 10 \times \cos 30° = g\sin 30° - a$
   $\Rightarrow \mu \times 10 \times \sqrt{(3/3)} = 10 \times (1/2) - 4$
   $\Rightarrow (5/\sqrt{3})\mu = 1 \Rightarrow \mu = 1/(5/\sqrt{3}) = 0.11$
   $\therefore$ Co-efficient of kinetic friction between the two is 0.11.





5. From the free body diagram
   $4 - 4a - \mu R + 4g\sin 30° = 0$ …(1)
   $R - 4g\cos 30° = 0$ ...(2)
   $\Rightarrow R = 4g\cos 30°$
   Putting the values of R is & in equn. (1)
   $4 - 4a - 0.11 \times 4g\cos 30° + 4g\sin 30° = 0$
   $\Rightarrow 4 - 4a - 0.11 \times 4 \times 10 \times (\sqrt{3}/2) + 4 \times 10 \times (1/2) = 0$
   $\Rightarrow 4 - 4a - 3.81 + 20 = 0 \Rightarrow a \approx 5 m/s^2$
   For the block u =0, t = 2sec, $a = 5 m/s^2$
   Distance $s = ut + ½ at^2 \Rightarrow s = 0 + (1/2) 5 \times 2^2 = 10m$
   The block will move 10m.





6. To make the block move up the incline, the force should be equal and opposite to the net force acting down the incline = μ R + 2 g sin 30°

   = 0.2 × (9.8) $\sqrt{3}$ + 2 I 9.8 × (1/2) [from (1)]

   = 3.39 + 9.8 = 13N

   With this minimum force the body move up the incline with a constant velocity as net force on it is zero.

   b) Net force acting down the incline is given by,

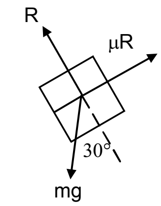
   F = 2 g sin 30° – μR

   = 2 × 9.8 × (1/2) – 3.39 = 6.41N

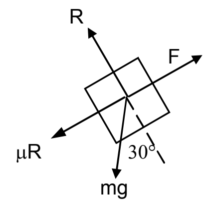
   Due to F = 6.41N the body will move down the incline with acceleration.

   No external force is required.

   $\therefore$ Force required is zero.



(body moving down)



(body moving us)

7. From the free body diagram

   g = 10m/s^2, m = 2kg, θ = 30°, μ = 0.2

   R – mg cos θ - F sin θ = 0

   $\Rightarrow$ R = mg cos θ + F sin θ ...(1)

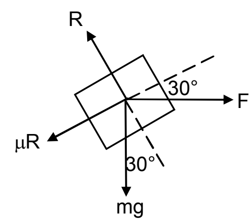
   And mg sin θ + μR – F cos θ = 0

   $\Rightarrow$ mg sin θ + μ(mg cos θ + F sin θ) – F cos θ = 0

   $\Rightarrow$ mg sin θ + μ mg cos θ + μ F sin θ – F cos θ = 0

   $$\Rightarrow F = \frac{(mg\sin\theta - \mu mg\cos\theta)}{(\mu\sin\theta - \cos\theta)}$$

   $$\Rightarrow F = \frac{2\times10\times(1/2)+0.2\times2\times10\times(\sqrt{3}/2)}{0.2\times(1/2)-(\sqrt{3}/2)} = \frac{13.464}{0.76} = 17.7N \approx 17.5N$$



8. m $\rightarrow$ mass of child

   R – mg cos 45° = 0

   $\Rightarrow$ R = mg cos 45° = mg /v^2 ...(1)

   Net force acting on the boy due to which it slides down is mg sin 45° - μR

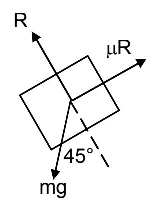
   = mg sin 45° - μ mg cos 45°

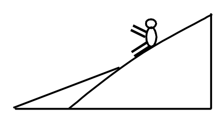
   = m × 10 (1/ $\sqrt{2}$) – 0.6 × m × 10 × (1/ $\sqrt{2}$)

   = m [(5/ $\sqrt{2}$) – 0.6 × (5 / $\sqrt{2}$)]

   = m(2 $\sqrt{2}$)

   $$\text{acceleration} = \frac{\text{Force}}{\text{mass}} = \frac{m(2\sqrt{2})}{m} = 2\sqrt{2}\ m/s^2$$





9. Suppose, the body is accelerating down with acceleration ‘a’.

   From the free body diagram

   R – mg cos θ = 0

   $\Rightarrow$ R = mg cos θ ...(1)

   ma + mg sin θ – μ R = 0

   $$\Rightarrow a = \frac{mg(\sin\theta - \mu\cos\theta)}{m} = g\ (\sin\theta - \mu\cos\theta)$$

   For the first half mt. u = 0, s = 0.5m, t = 0.5 sec.

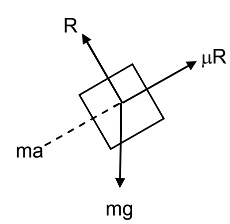
   So, v = u + at = 0 + (0.5)4 = 2 m/s

   S = ut + ½ at^2 $\Rightarrow$ 0.5 = 0 + ½ a (0/5)^2 $\Rightarrow$ a = 4m/s^2 ...(2)

   For the next half metre

   u\` = 2m/s, a = 4m/s^2, s= 0.5.

   $\Rightarrow$ 0.5 = 2t + (1/2) 4 t^2 $\Rightarrow$ 2 t^2 + 2 t – 0.5 =0



$\Rightarrow 4t^2 + 4t - 1 = 0$

$\therefore = \frac{-4 \pm \sqrt{16+16}}{2\times 4} = \frac{1.656}{8} = 0.207sec$

Time taken to cover next half meter is 0.21sec.

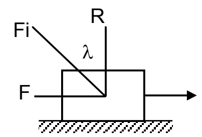
10. $f \rightarrow$ applied force

$F_i \rightarrow$ contact force

$F \rightarrow$ frictional force

$R \rightarrow$ normal reaction

$\mu = \tan\lambda = F/R$

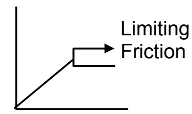


When $F = \mu R$, F is the limiting friction (max friction). When applied force increase, force of friction increase upto limiting friction (μR)

Before reaching limiting friction

$F < \mu R$

$\therefore \tan\lambda = \frac{F}{R} \le \frac{\mu R}{R} \Rightarrow \tan\lambda \le \mu \Rightarrow \lambda \le \tan^{-1}\mu$



11. From the free body diagram

$T + 0.5a - 0.5g = 0$ ...(1)

$\mu R + 1a + T_1 - T = 0$ ...(2)

$\mu R + 1a - T_1 = 0$

$\mu R + 1a = T_1$ ...(3)

From (2) & (3) $\Rightarrow \mu R + a = T - T_1$

$\therefore T - T_1 = T_1$

$\Rightarrow T = 2T_1$

Equation (2) becomes $\mu R + a + T_1 - 2T_1 = 0$

$\Rightarrow \mu R + a - T_1 = 0$

$\Rightarrow T_1 = \mu R + a = 0.2g + a$ ...(4)

Equation (1) becomes $2T_1 + 0/5a - 0.5g = 0$

$\Rightarrow T_1 = \frac{0.5g - 0.5a}{2} = 0.25g - 0.25a$ ...(5)

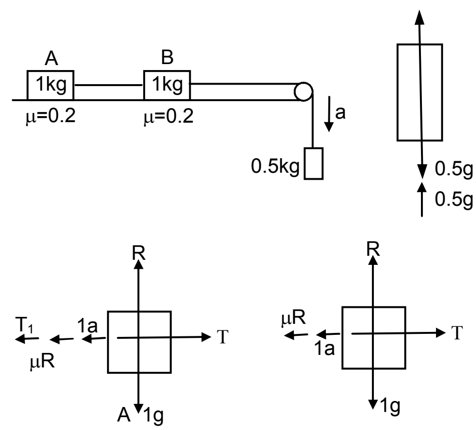
From (4) & (5) $0.2g + a = 0.25g - 0.25a$

$\Rightarrow a = \frac{0.05}{1.25} \times 10 = 0.04 \text{ I } 10 = 0.4m/s^2$

a) Accln of 1kg blocks each is $0.4m/s^2$

b) Tension $T_1 = 0.2g + a + 0.4 = 2.4N$

c) $T = 0.5g - 0.5a = 0.5 \times 10 - 0.5 \times 0.4 = 4.8N$



12. From the free body diagram

$\mu_1 R + 1 - 16 = 0$

$\Rightarrow \mu_1 (2g) + (-15) = 0$

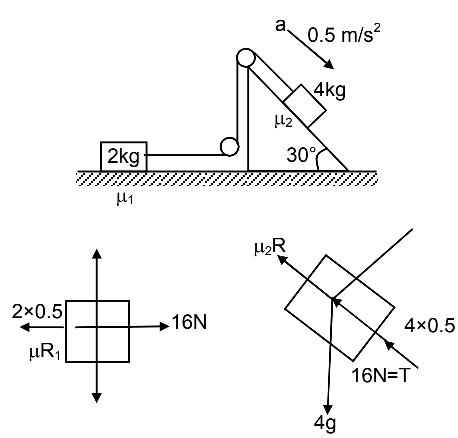
$\Rightarrow \mu_1 = 15/20 = 0.75$

$\mu_2 R_1 + 4 \times 0.5 + 16 - 4g \sin 30° = 0$

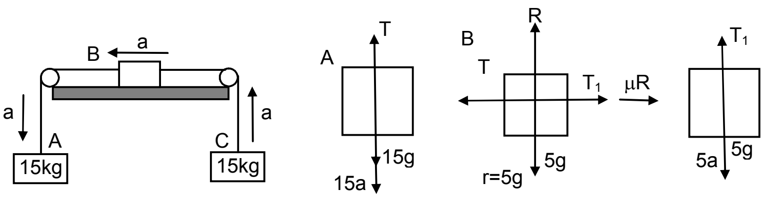
$\Rightarrow \mu_2 (20\sqrt{3}) + 2 + 16 - 20 = 0$

$\Rightarrow \mu_2 = \frac{2}{20\sqrt{3}} = \frac{1}{17.32} = 0.057 \approx 0.06$

$\therefore$ Co-efficient of friction $\mu_1 = 0.75$ & $\mu_2 = 0.06$



13.



From the free body diagram

$T + 15a - 15g = 0$

$\Rightarrow T = 15g - 15a$ ...(i)

$T - (T_1 + 5a + \mu R) = 0$

$\Rightarrow T - (5g + 5a + 5a + \mu R) = 0$

$\Rightarrow T = 5g + 10a + \mu R$ …(ii)

$T_1 - 5g - 5a = 0$

$\Rightarrow T_1 = 5g + 5a$ …(iii)

From (i) & (ii) $15g - 15a = 5g + 10a + 0.2\,(5g)$

$\Rightarrow 25a = 90 \Rightarrow a = 3.6\text{m/s}^2$

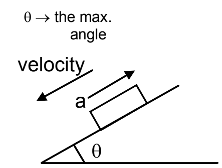
Equation (ii) $\Rightarrow T = 5 \times 10 + 10 \times 3.6 + 0.2 \times 5 \times 10$

$\Rightarrow$ 96N in the left string

Equation (iii) $T_1 = 5g + 5a = 5 \times 10 + 5 \times 3.6 = 68$N in the right string.

14. $s = 5\text{m}$, $\mu = 4/3$, $g = 10\text{m/s}^2$

$u = 36\text{km/h} = 10\text{m/s}$, $v = 0$,



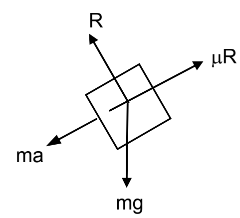
$$a = \frac{v^2 - u^2}{2s} = \frac{0 - 10^2}{2 \times 5} = -10\text{m/s}^2$$

From the freebody diagrams,

$R - mg\cos\theta = 0$; $g = 10\text{m/s}^2$

$\Rightarrow R = mg\cos\theta$ ….(i) ; $\mu = 4/3$.

Again, $ma + mg\sin\theta - \mu R = 0$



$\Rightarrow ma + mg\sin\theta - \mu\, mg\cos\theta = 0$

$\Rightarrow a + g\sin\theta - mg\cos\theta = 0$

$\Rightarrow 10 + 10\sin\theta - (4/3) \times 10\cos\theta = 0$

$\Rightarrow 30 + 30\sin\theta - 40\cos\theta = 0$

$\Rightarrow 3 + 3\sin\theta - 4\cos\theta = 0$

$\Rightarrow 4\cos\theta - 3\sin\theta = 3$

$\Rightarrow 4\sqrt{1 - \sin^2\theta} = 3 + 3\sin\theta$

$\Rightarrow 16\,(1 - \sin^2\theta) = 9 + 9\sin^2\theta + 18\sin\theta$

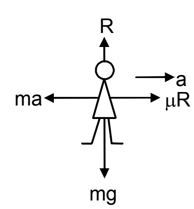
$$\sin\theta = \frac{-18 \pm \sqrt{18^2 - 4(25)(-7)}}{2 \times 25} = \frac{-18 \pm 32}{50} = \frac{14}{50} = 0.28 \text{ [Taking +ve sign only]}$$

$\Rightarrow \theta = \sin^{-1}(0.28) = 16°$

Maximum incline is $\theta = 16°$

15. to reach in minimum time, he has to move with maximum possible acceleration.

Let, the maximum acceleration is ‘a’



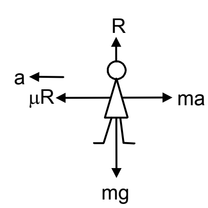
$\therefore ma - \mu R = 0 \Rightarrow ma = \mu\, mg$

$\Rightarrow a = \mu\, g = 0.9 \times 10 = 9\text{m/s}^2$

a) Initial velocity $u = 0$, $t = ?$

$a = 9\text{m/s}^2$, $s = 50\text{m}$

$$s = ut + \tfrac{1}{2} at^2 \Rightarrow 50 = 0 + (1/2)\, 9\, t^2 \Rightarrow t = \sqrt{\frac{100}{9}} = \frac{10}{3} \text{ sec.}$$



b) After overing 50m, velocity of the athelete is

$V = u + at = 0 + 9 \times (10/3) = 30\text{m/s}$

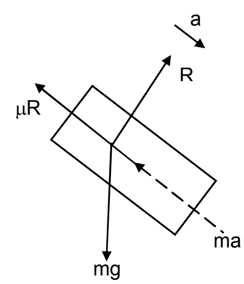
He has to stop in minimum time. So deceleration ia $-a = -9\text{m/s}^2$ (max)

$$\begin{bmatrix} R = ma \\ ma = \mu R(\text{max frictional force}) \\ \Rightarrow a = \mu g = 9m/s^2(\text{Deceleration}) \end{bmatrix}$$

u^1 = 30m/s, v^1 = 0

t = $\frac{v^1 - u^1}{a}$ = $\frac{0-30}{-a}$ = $\frac{-30}{-a}$ = $\frac{10}{3}$ sec.

16. Hardest brake means maximum force of friction is developed between car's type & road.



Max frictional force = μR

From the free body diagram

R – mg cos θ =0

⇒ R = mg cos θ ...(i)

and μR + ma – mg sin) = 0 …(ii)

⇒ μmg cos θ + ma – mg sin θ = 0

⇒ μg cos θ + a – 10 × (1/2) = 0

⇒ a = 5 – {1 – (2 $\sqrt{3}$)} × 10 ($\sqrt{3}/2$) = 2.5 m/s^2

When, hardest brake is applied the car move with acceleration 2.5m/s^2

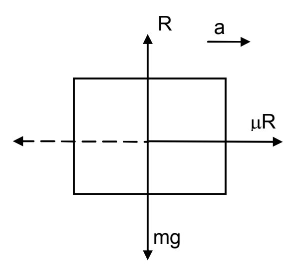
S = 12.8m, u = 6m/s

S0, velocity at the end of incline

V = $\sqrt{u^2 + 2as}$ = $\sqrt{6^2 + 2(2.5)(12.8)}$ = $\sqrt{36+64}$ = 10m/s = 36km/h

Hence how hard the driver applies the brakes, that car reaches the bottom with least velocity 36km/h.

17. Let, , a maximum acceleration produced in car.



∴ ma = μR [For more acceleration, the tyres will slip]

⇒ ma = μ mg ⇒ a = μg = 1 × 10 = 10m/s^2

For crossing the bridge in minimum time, it has to travel with maximum acceleration

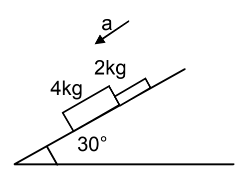
u = 0, s = 500m, a = 10m/s^2

s = ut + ½ at^2

⇒ 500 = 0 + (1/2) 10 t^2 ⇒ t = 10 sec.

If acceleration is less than 10m/s^2, time will be more than 10sec. So one can't drive through the bridge in less than 10sec.

18. From the free body diagram



R = 4g cos 30° = 4 × 10 × $\sqrt{3}/2$ = 20 $\sqrt{3}$ ...(i)

μ_2 R + 4a – P – 4g sin 30° = 0 ⇒ 0.3 (40) cos 30° + 4a – P – 40 sin 20° = 0 …(ii)

P + 2a + μ_1 R_1 – 2g sin 30° = 0 …(iii)

R_1 = 2g cos 30° = 2 × 10 × $\sqrt{3}/2$ = 10 $\sqrt{3}$ ...(iv)

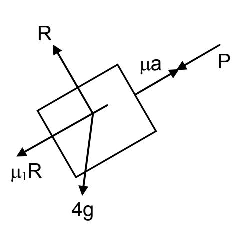
Equn. (ii) 6 $\sqrt{3}$ + 4a – P – 20 = 0

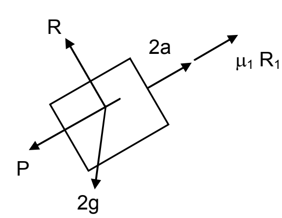
Equn (iv) P + 2a + 2 $\sqrt{3}$ – 10 = 0

From Equn (ii) & (iv) 6 $\sqrt{3}$ + 6a – 30 + 2 $\sqrt{3}$ = 0

⇒ 6a = 30 – 8 $\sqrt{3}$ = 30 – 13.85 = 16.15

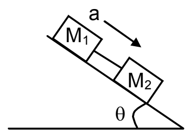
⇒ a = $\frac{16.15}{6}$ = 2.69 = 2.7m/s^2

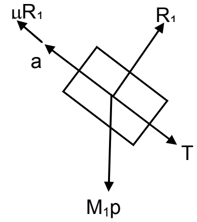


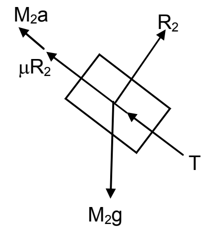


b) can be solved. In this case, the 4 kg block will travel with more acceleration because, coefficient of friction is less than that of 2kg. So, they will move separately. Drawing the free body diagram of 2kg mass only, it can be found that, a = 2.4m/s^2.

19. From the free body diagram







$R_1 = M_1 g \cos\theta$ ...(i)

$R_2 = M_2 g \cos\theta$ ...(ii)

$T + M_1 g \sin\theta - m_1 a - \mu R_1 = 0$ ...(iii)

$T - M_2 - M_2 a + \mu R_2 = 0$ ...(iv)

Equn (iii) $\Rightarrow$ $T + M_1 g \sin\theta - M_1 a - \mu M_1 g \cos\theta = 0$

Equn (iv) $\Rightarrow$ $T - M_2 g \sin\theta + M_2 a + \mu M_2 g \cos\theta = 0$ ...(v)

Equn (iv) & (v) $\Rightarrow$ $g \sin\theta (M_1 + M_2) - a(M_1 + M_2) - \mu g \cos\theta (M_1 + M_2) = 0$

$\Rightarrow a (M_1 + M_2) = g \sin\theta (M_1 + M_2) - \mu g \cos\theta (M_1 + M_2)$

$\Rightarrow a = g(\sin\theta - \mu \cos\theta)$

$\therefore$ The blocks (system has acceleration $g(\sin\theta - \mu\cos\theta)$

The force exerted by the rod on one of the blocks is tension.

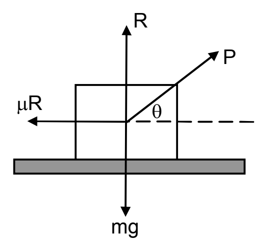
Tension $T = -M_1 g \sin\theta + M_1 a + \mu M_1 g \sin\theta$

$\Rightarrow T = -M_1 g \sin\theta + M_1(g \sin\theta - \mu g \cos\theta) + \mu M_1 g \cos\theta$

$\Rightarrow T = 0$

20. Let 'p' be the force applied to at an angle θ

From the free body diagram



$R + P \sin\theta - mg = 0$

$\Rightarrow R = -P \sin\theta + mg$ ...(i)

$\mu R - p \cos\theta$ ...(ii)

Equn. (i) is $\mu(mg - P \sin\theta) - P\cos\theta = 0$

$\Rightarrow \mu\, mg = \mu\, \rho \sin\theta - P \cos\theta \Rightarrow \rho = \dfrac{\mu mg}{\mu \sin\theta + \cos\theta}$

Applied force P should be minimum, when $\mu \sin\theta + \cos\theta$ is maximum.

Again, $\mu \sin\theta + \cos\theta$ is maximum when its derivative is zero.

$\therefore d/d\theta\, (\mu \sin\theta + \cos\theta) = 0$

$\Rightarrow \mu \cos\theta - \sin\theta = 0 \Rightarrow \theta = \tan^{-1}\mu$

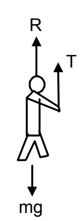
So, $P = \dfrac{\mu mg}{\mu\sin\theta + \cos\theta} = \dfrac{\mu mg/\cos\theta}{\dfrac{\mu\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\cos\theta}} = \dfrac{\mu mg\sec\theta}{1+\mu\tan\theta} = \dfrac{\mu mg\sec\theta}{1+\tan^2\theta}$

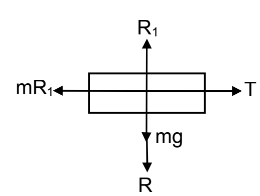
$= \dfrac{\mu mg}{\sec\theta} = \dfrac{\mu mg}{\sqrt{(1+\tan^2\theta}} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

Minimum force is $\dfrac{\mu mg}{\sqrt{1+\mu^2}}$ at an angle $\theta = \tan^{-1}\mu$.

21. Let, the max force exerted by the man is T.

From the free body diagram





$R + T - Mg = 0$

$\Rightarrow R = Mg - T$ ...(i)

$R_1 - R - mg = 0$

$\Rightarrow R_1 = R + mg$ ...(ii)

And $T - \mu R_1 = 0$

$\Rightarrow T - \mu (R + mg) = 0$ [From equn. (ii)]

$\Rightarrow T - \mu R - \mu mg = 0$

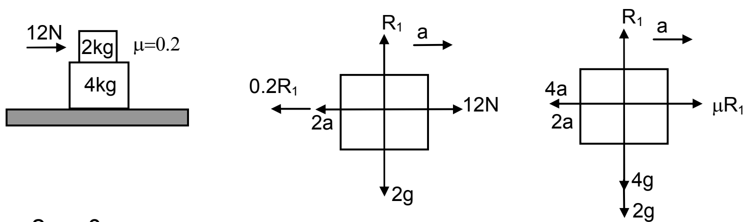
$\Rightarrow T - \mu (Mg + T) - \mu mg = 0$ [from (i)]

$\Rightarrow T (1 + \mu) = \mu Mg + \mu mg$

$\Rightarrow T = \dfrac{\mu(M+m)g}{1+\mu}$

Maximum force exerted by man is $\dfrac{\mu(M+m)g}{1+\mu}$

22.



$R_1 - 2g = 0$

$\Rightarrow R_1 = 2 \times 10 = 20$

$2a + 0.2 R_1 - 12 = 0$

$\Rightarrow 2a + 0.2(20) = 12$

$\Rightarrow 2a = 12 - 4 = 8$

$\Rightarrow a = 4m/s^2$

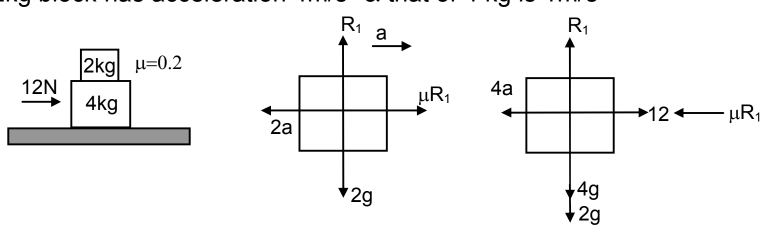
$4a_1 - \mu R_1 = 0$

$\Rightarrow 4a_1 = \mu R_1 = 0.2 (20)$

$\Rightarrow 4a_1 = 4$

$\Rightarrow a_1 = 1m/s^2$

2kg block has acceleration $4m/s^2$ & that of 4 kg is $1m/s^2$



(ii) $R_1 = 2g = 20$

$Ma - \mu R_1 = 0$

$\Rightarrow 2a = 0.2 (20) = 4$

$\Rightarrow a = 2m/s^2$

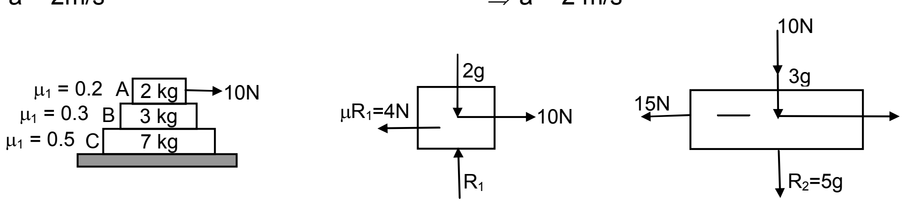
$4a + 0.2 \times 2 \times 10 - 12 = 0$

$\Rightarrow 4a + 4 = 12$

$\Rightarrow 4a = 8$

$\Rightarrow a = 2 m/s^2$

23.



a) When the 10N force applied on 2kg block, it experiences maximum frictional force

$\mu R_1 = \mu \times 2kg = (0.2) \times 20 = 4N$ from the 3kg block.

So, the 2kg block experiences a net force of 10 – 4 = 6N

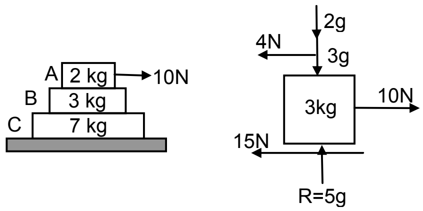
So, $a_1 = 6/2 = 3 m/s^2$

But for the 3kg block, (fig-3) the frictional force from 2kg block (4N) becomes the driving force and the maximum frictional force between 3kg and 7 kg block is

$\mu_2 R_2 = (0.3) \times 5kg = 15N$

So, the 3kg block cannot move relative to the 7kg block. The 3kg block and 7kg block both will have same acceleration ($a_2 = a_3$) which will be due to the 4N force because there is no friction from the floor.

$\therefore a_2 = a_3 = 4/10 = 0.4m/s^2$



b) When the 10N force is applied to the 3kg block, it can experience maximum frictional force of 15 + 4 = 19N from the 2kg block & 7kg block.

So, it can not move with respect to them.

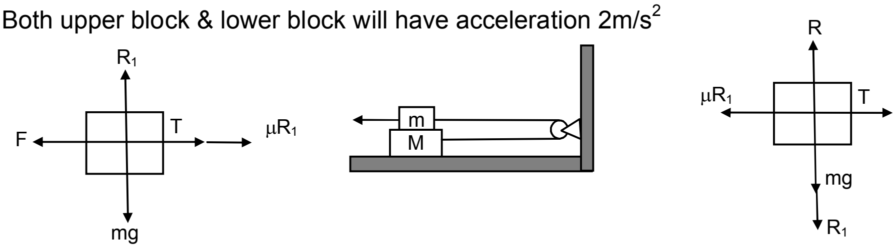
As the floor is frictionless, all the three bodies will move together

$\therefore a_1 = a_2 = a_3 = 10/12 = (5/6)\text{m/s}^2$

c) Similarly, it can be proved that when the 10N force is applied to the 7kg block, all the three blocks will move together.

Again $a_1 = a_2 = a_3 = (5/6)\text{m/s}^2$

24. Both upper block & lower block will have acceleration 2m/s^2



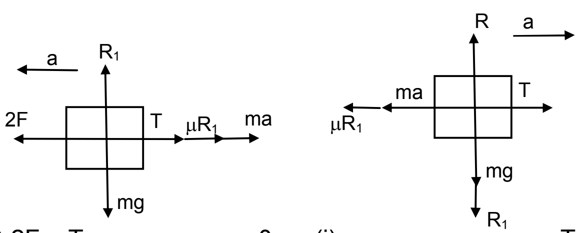
$R_1 = mg$ ...(i)

$F - \mu R_1 - T = 0 \Rightarrow F - \mu mg - T = 0$ ...(ii)

$\therefore F = \mu\, mg + \mu\, mg = 2\,\mu\, mg$ [putting $T = \mu\, mg$]

$T - \mu R_1 = 0$

$\Rightarrow T = \mu mg$



b) $2F - T - \mu\, mg - ma = 0$ …(i)

$T - Ma - \mu\, mg = 0$ [$\therefore R_1 = mg$]

$\Rightarrow T = Ma + \mu\, mg$

Putting value of T in (i)

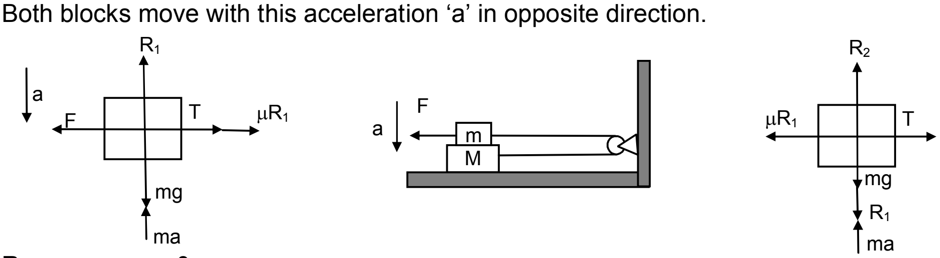
$2f - Ma - \mu mg - \mu\, mg - ma = 0$

$\Rightarrow 2(2\mu mg) - 2\,\mu\, mg = a(M + m)$ [Putting $F = 2\,\mu mg$]

$\Rightarrow 4\mu\, mg - 2\,\mu\, mg = a\,(M + m)$ $\Rightarrow a = \dfrac{2\mu mg}{M+m}$

Both blocks move with this acceleration ‘a’ in opposite direction.

25.



$T = mR_1$
$= m\,(mg - ma)$

$R_1 + ma - mg = 0$

$\Rightarrow R_1 = m(g-a) = mg - ma$ ...(i)

$T - \mu\, R_1 = 0 \Rightarrow T = m\,(mg - ma)$ ...(ii)

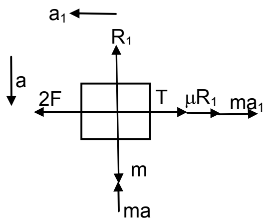
Again, $F - T - \mu\, R_1 = 0$

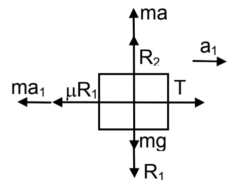
$\Rightarrow$ F – {μ(mg –ma)} – u(mg – ma) = 0

$\Rightarrow$ F – μ mg + μ ma – μ mg + μ ma = 0

$\Rightarrow$ F = 2 μ mg – 2μ ma $\Rightarrow$ F = 2μ m(g–a)

b) Acceleration of the block be a_1





$R_1 = mg - ma$ ...(i)

$2F - T - \mu R_1 - ma_1 = 0$

$\Rightarrow 2F - t - \mu mg + \mu a - ma_1 = 0$ ...(ii)

$T - \mu R_1 - M a_1 = 0$

$\Rightarrow T = \mu R_1 + M a_1$

$\Rightarrow T = \mu (mg - ma) + Ma_1$

$\Rightarrow T = \mu mg - \mu ma + M a_1$

Subtracting values of F & T, we get

$2(2\mu m(g - a)) - 2(\mu mg - \mu ma + Ma_1) - \mu mg + \mu ma - \mu a_1 = 0$

$\Rightarrow 4\mu mg - 4 \mu ma - 2 \mu mg + 2\mu ma = ma_1 + M a_1$ $\Rightarrow a_1 = \dfrac{2\mu m(g-a)}{M+m}$

Both blocks move with this acceleration but in opposite directions.

26. $R_1 + QE - mg = 0$

$R_1 = mg - QE$ ...(i)

$F - T - \mu R_1 = 0$

$\Rightarrow F - T \ \mu(mg - QE) = 0$

$\Rightarrow F - T - \mu mg + \mu QE = 0$ …(2)

$T - \mu R_1 = 0$

$\Rightarrow T = \mu R_1 = \mu (mg - QE) = \mu mg - \mu QE$

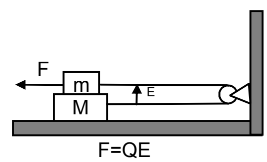
Now equation (ii) is $F - mg + \mu QE - \mu mg + \mu QE = 0$

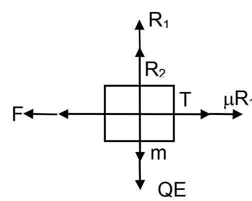
$\Rightarrow F - 2 \mu mg + 2\mu QE = 0$

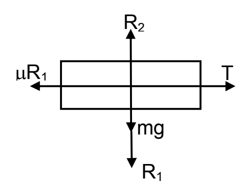
$\Rightarrow F = 2\mu mg - 2\mu QE$

$\Rightarrow F = 2\mu(mg - QE)$

Maximum horizontal force that can be applied is $2\mu(mg - QE)$.







27. Because the block slips on the table, maximum frictional force acts on it.

From the free body diagram

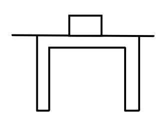
R = mg

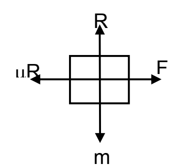
$\therefore F - \mu R = 0 \Rightarrow F = \mu R = \mu mg$

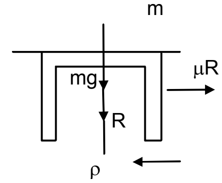
But the table is at rest. So, frictional force at the legs of the table is not μR_1. Let be f, so form the free body diagram.

$f_o - \mu R = 0 \Rightarrow f_o = \mu R = \mu mg$.

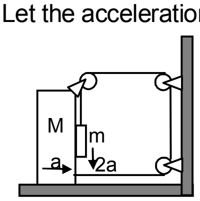
Total frictional force on table by floor is μ mg.

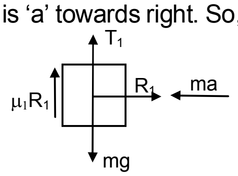




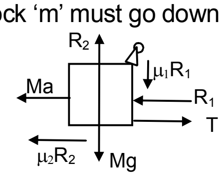


28. Let the acceleration of block M is 'a' towards right. So, the block 'm' must go down with an acceleration '2a'.





(FBD-1)



(FBD-2)

As the block 'm' is in contact with the block 'M', it will also have acceleration 'a' towards right. So, it will experience two inertia forces as shown in the free body diagram-1.

From free body diagram -1

$R_1 - ma = 0 \Rightarrow R_1 = ma$ ...(i)

Again, $2ma + T - mg + \mu_1 R_1 = 0$

$\Rightarrow T = mg - (2 - \mu_1)ma$ …(ii)

From free body diagram-2

$T + \mu_1 R_1 + mg - R_2 = 0$

$\Rightarrow R_2 = T + \mu_1\, ma + Mg$ [Putting the value of R_1 from (i)]

$= (mg - 2ma - \mu_1\, ma) + \mu_1\, ma + Mg$ [Putting the value of T from (ii)]

$\therefore R_2 = Mg + mg - 2ma$ …(iii)

Again, form the free body diagram -2

$T + T - R - Ma - \mu_2 R_2 = 0$

$\Rightarrow 2T - MA - mA - \mu_2\, (Mg + mg - 2ma) = 0$ [Putting the values of R_1 and R_2 from (i) and (iii)]

$\Rightarrow 2T = (M + m) + \mu_2(Mg + mg - 2ma)$ ...(iv)

From equation (ii) and (iv)

$2T = 2\, mg - 2(2 + \mu_1)mg = (M + m)a + \mu_2(Mg + mg - 2ma)$

$\Rightarrow 2mg - \mu_2(M + m)g = a\, (M + m - 2\mu_2 m + 4m + 2\mu_1 m)$

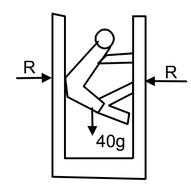
$\Rightarrow a = \dfrac{[2m - \mu_2(M+m)]g}{M + m[5 + 2(\mu_1 - \mu_2)]}$

29. Net force = *(202 + (15)2 – (0.5) × 40 = 25 – 20 = 5N

$\therefore \tan\theta = 20/15 = 4/3 \Rightarrow \mu = \tan^{-1}(4/3) = 53°$

So, the block will move at an angle 53 ° with an 15N force

30. a) Mass of man = 50kg. $g = 10\ m/s^2$



Frictional force developed between hands, legs & back side with the wall the wt of man. So he remains in equilibrium.

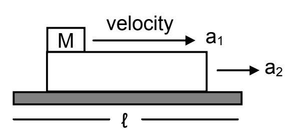
He gives equal force on both the walls so gets equal reaction R from both the walls. If he applies unequal forces R should be different he can't rest between the walls. Frictional force $2\mu R$ balance his wt.

From the free body diagram

$\mu R + \mu R = 40g \Rightarrow 2\,\mu R = 40 \times 10 \Rightarrow R = \dfrac{40 \times 10}{2 \times 0.8} = 250N$

b) The normal force is 250 N.

31. Let a_1 and a_2 be the accelerations of ma and M respectively.



Here, $a_1 > a_2$ so that m moves on M

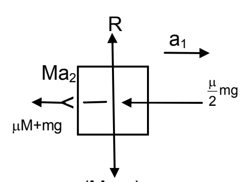
Suppose, after time 't' m separate from M.

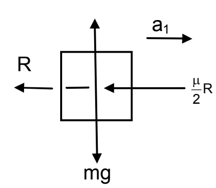
In this time, m covers $vt + ½\, a_1 t^2$ and $S_M = vt + ½\, a_2 t^2$

For 'm' to m to 'm' separate from M. $vt + ½\, a_1 t^2 = vt + ½\, a_2 t^2 + \ell$ ...(1)

Again from free body diagram

$Ma_1 + \mu/2\, R = 0$





$\Rightarrow ma_1 = -(\mu/2)\, mg = -(\mu/2)m \times 10 \Rightarrow a_1 = -5\mu$

Again,

$Ma_2 + \mu\, (M + m)g - (\mu/2)mg = 0$

$\Rightarrow 2Ma_2 + 2\mu\, (M + m)g - \mu\, mg = 0$

$\Rightarrow 2\, M\, a_2 = \mu\, mg - 2\mu Mg - 2\,\mu mg$

$\Rightarrow a_2\ \dfrac{-\mu mg - 2\mu Mg}{2M}$

Putting values of a_1 & a_2 in equation (1) we can find that

$T = \sqrt{\left(\dfrac{4ml}{(M+m)\mu g}\right)}$

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