CHAPTER 5

NEWTON'S LAWS OF MOTION

Newton's laws of motion are of central importance in classical physics. A large number of principles and results may be derived from Newton's laws. The first two laws relate to the type of motion of a system that results from a given set of forces. These laws may be interpreted in a variety of ways and it is slightly uninteresting and annoying at the outset to go into the technical details of the interpretation. The precise definitions of mass, force and acceleration should be given before we relate them. And these definitions themselves need use of Newton's laws. Thus, these laws turn out to be definitions to some extent. We shall assume that we know how to assign mass to a body, how to assign the magnitude and direction to a force and how to measure the acceleration with respect to a given frame of reference. Some discussions of these aspects were given in the previous chapters. The development here does not follow the historical track these laws have gone through, but are explained to make them simple to apply.

5.1 FIRST LAW OF MOTION

If the (vector) sum of all the forces acting on a particle is zero then and only then the particle remains unaccelerated (i.e., remains at rest or moves with constant velocity).

If the sum of all the forces on a given particle is $\vec{F}$ and its acceleration is $\vec{a}$, the above statement may also be written as

"$\vec{a} = 0$ *if and only if* $\vec{F} = 0$".

Thus, if the sum of the forces acting on a particle is known to be zero, we can be sure that the particle is unaccelerated, or if we know that a particle is unaccelerated, we can be sure that the sum of the forces acting on the particle is zero.

However, the concept of rest, motion or acceleration is meaningful only when a frame of reference is specified. Also the acceleration of the particle is, in general, different when measured from different frames. Is it possible then, that the first law is valid in all frames of reference ?

Let us consider the situation shown in figure (5.1). An elevator cabin falls down after the cable breaks. The cabin and all the bodies fixed in the cabin are accelerated with respect to the earth and the acceleration is about $9.8\ \mathrm{m/s^2}$ in the downward direction.

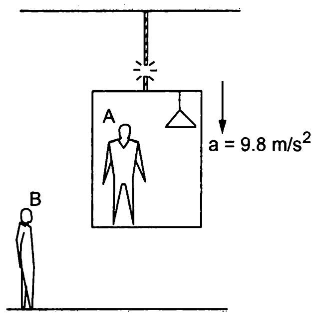


Figure 5.1

Consider the lamp in the cabin. The forces acting on the lamp are (a) the gravitational force W by the earth and (b) the electromagnetic force T (tension) by the rope. The direction of W is downward and the directon of T is upward. The sum is $(W - T)$ downward.

Measure the acceleration of the lamp from the frame of reference of the cabin. The lamp is at rest. The acceleration of the lamp is zero. The person A who measured this acceleration is a learned one and uses Newton's first law to conclude that the sum of the forces acting on the particle is zero, i.e.,

$$W - T = 0 \text{ or, } W = T.$$

Instead, if we measure the acceleration from the ground, the lamp has an acceleration of $9.8\ \mathrm{m/s^2}$. Thus, $a \neq 0$ and hence the person B who measured this acceleration, concludes from Newton's first law that the sum of the forces is not zero. Thus, $W - T \neq 0$ or $W \neq T$. If A measures acceleration and applies the first law he gets $W = T$. If B measures acceleration and applies the same first law, he gets $W \neq T$. Both of them cannot be correct simultaneously as W and T can be either equal or unequal. At least one of the two frames is a bad frame and one should not apply the first law in that frame.

There are some frames of reference in which Newton's first law is valid. Measure acceleration from such a frame and you are allowed to say that "$\vec{a} = 0$ if and only if $\vec{F} = 0$". But there are frames in which Newton's first law is not valid. You may find that even if the sum of the forces is not zero, the acceleration is still zero. Or you may find that the sum of the forces is zero, yet the particle is accelerated. So the validity of Newton's first law depends on the frame of reference from which the observer measures the state of rest, motion and acceleration of the particle.

A frame of reference in which Newton's first law is valid is called an *inertial frame of reference*. A frame in which Newton's first law is not valid is called a *noninertial frame of reference*.

Newton's first law, thus, reduces to a definition of inertial frame. Why do we call it a law then? Suppose after going through this lesson, you keep the book on your table fixed rigidly with the earth (figure 5.2).

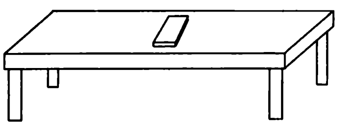


Figure 5.2

The book is at rest with respect to the earth. The acceleration of the book with respect to the earth is zero. The forces on the book are (a) the gravitational force $\vec{W}$ exerted by the earth and (b) the contact force $\vec{\mathcal{N}}$ by the table. Is the sum of $\vec{W}$ and $\vec{\mathcal{N}}$ zero? A very accurate measurement will give the answer "No". The sum of the forces is not zero although the book is at rest. The earth is not strictly an inertial frame. However, the sum is not too different from zero and we can say that the earth is an inertial frame of reference to a good approximation. Thus, for routine affairs, "$\vec{a} = 0$ if and only if $\vec{F} = 0$" is true in the earth frame of reference. This fact was identified and formulated by Newton and is known as Newton's *first law*. If we restrict that all measurements will be made from the earth frame, indeed it becomes a law. If we try to universalise this to different frames, it becomes a definition. We shall assume that unless stated otherwise, we are working from an inertial frame of reference.

Example 5.1

A heavy particle of mass 0·50 kg *is hanging from a string fixed with the roof. Find the force exerted by the string on the particle (Figure* 5.3*). Take* g = 9·8 m/s^2.

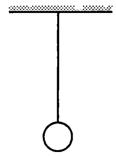


Figure 5.3

Solution : The forces acting on the particle are

(a) pull of the earth, 0·50 kg × 9·8 m/s^2 = 4·9 N, vertically downward

(b) pull of the string, T vertically upward.

The particle is at rest with respect to the earth (which we assume to be an inertial frame). Hence, the sum of the forces should be zero. Therefore, T is 4·9 N acting vertically upward.

Inertial Frames other than Earth

Suppose S is an inertial frame and S' a frame moving uniformly with respect to S. Consider a particle P having acceleration $\vec{a}_{P,S}$ with respect to S and $\vec{a}_{P,S'}$ with respect to S'.

We know that,

$$\vec{a}_{P,S} = \vec{a}_{P,S'} + \vec{a}_{S',S}.$$

As S' moves uniformly with respect to S,

$$\vec{a}_{S',S} = 0.$$

Thus, $$\vec{a}_{P,S} = \vec{a}_{P,S'} \quad \ldots \text{(i)}$$

Now S is an inertial frame. So from definition, $\vec{a}_{P,S} = 0$, if and only if $\vec{F} = 0$ and hence, from (i), $\vec{a}_{P,S'} = 0$ if and only if $\vec{F} = 0$.

Thus, S' is also an inertial frame. We arrive at an important result : *All frames moving uniformly with respect to an inertial frame are themselves inertial.* Thus, a train moving with uniform velocity with respect to the ground, a plane flying with uniform velocity with respect to a highway, etc., are examples of inertial frames. The sum of the forces acting on a suitcase kept on the shelf of a ship sailing smoothly and uniformly on a calm sea is zero.

5.2 SECOND LAW OF MOTION

The acceleration of a particle as measured from an inertial frame is given by the (vector) sum of all the forces acting on the particle divided by its mass.

In symbols : $\vec{a} = \vec{F}/m$ or, $\vec{F} = m\vec{a}$. ... (5.2)

The inertial frame is already defined by the first law of motion. A force $\vec{F}$ acting on a particle of mass m produces an acceleration $\vec{F}/m$ in it with respect to an inertial frame. This is a law of nature. If the force ceases to act at some instant, the acceleration becomes zero at the same instant. In equation (5.2) $\vec{a}$ and $\vec{F}$ are measured at the same instant of time.

5.3 WORKING WITH NEWTON'S FIRST AND SECOND LAW

Newton's laws refer to a particle and relate the forces acting on the particle with its acceleration and its mass. Before attempting to write an equation from Newton's law, we should very clearly understand which particle we are considering. In any practical situation, we deal with extended bodies which are collection of a large number of particles. The laws as stated above may be used even if the object under consideration is an extended body, provided each part of this body has the same acceleration (in magnitude and direction). A systematic algorithm for writing equations from Newton's laws is as follows :

Step 1 : Decide the System

The first step is to decide the system on which the laws of motion are to be applied. The system may be a single particle, a block, a combination of two blocks one kept over the other, two blocks connected by a string, a piece of string etc. The only restriction is that all parts of the system should have identical acceleration.

Consider the situation shown in figure (5.4). The block B does not slip over A, the disc D slides over the string and all parts of the string are tight.

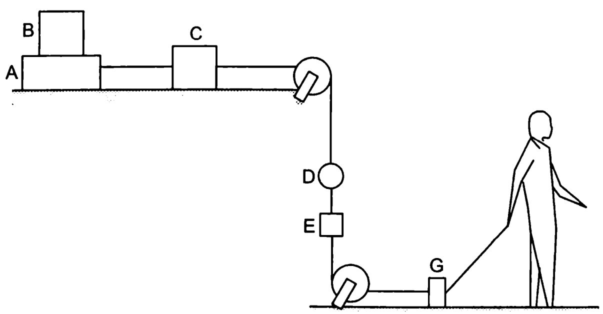


Figure 5.4

A and B move together. C is not in contact with A or B. But as the length of the string between A and C does not change, the distance moved by C in any time interval is same as that by A. The same is true for G. The distance moved by G in any time interval is same as that by A, B or C. The direction of motion is also the same for A, B, C and G. They have identical accelerations. We can take any of these blocks as a system or any combination of the blocks from these as a system. Some of the examples are (A), (B), (A + B), (B + C), (A + B + C), (C + G), (A + C + G), (A + B + C + G) etc. The distance covered by E is also the same as the distance covered by G but their directions are different. E moves in a vertical line whereas G in a horizontal line. (E + G) should not be taken as a system. At least at this stage we are unable to apply Newton's law treating E + G as a single particle. As the disc D slides over the string the distance covered by D is not equal to that by E in the same time interval. We should not treat D + E as a system. *Think carefully.*

Step 2 : Identify the Forces

Once the system is decided, make a list of the forces acting *on* the system due to all the objects other than the system. Any force applied *by* the system should not be included in the list of the forces.

Consider the situation shown in figure (5.5). The boy stands on the floor balancing a heavy load on his head. The load presses the boy, the boy pushes the load upward the boy presses the floor downward, the floor pushes the boy upward, the earth attracts the load downward, the load attracts the earth upward, the boy attracts the earth upward and the earth attracts the boy downward. There are many forces operating in this world. Which of these forces should we include in the list of forces ?

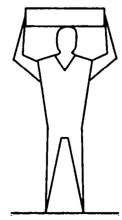


Figure 5.5

We cannot answer this question. Not because we do not know, but because we have not yet specified the system. Which is the body under consideration ? Do not try to identify forces before you have decided the system. Suppose we concentrate on the state of motion of the boy. We should then concentrate on the forces acting *on* the boy. The forces are listed in the upper half of table (5.1). Instead, if we take the load as the system and discuss the equilibrium of the load, the list of the forces will be different. These forces appear in the lower half of table (5.1).

Table 5.1

System	Force exerted by	Magnitude of the force	Direction of the force	Nature of the force
	Earth	W	Downward	Gravitational
Boy	Floor	$\mathcal{N}$	Upward	Electro-magnetic
	Load	$\mathcal{N}_1$	Downward	,,
	Earth	W'	Downward	Gravitational
Load				
	Boy	$\mathcal{N}_1$	Upward	Electro-magnetic

One may furnish as much information as one has about the magnitude and direction of the forces. The contact forces may have directions other than normal to the contact surface if the surfaces are rough. We shall discuss more about it under the heading of friction.

Step 3 : Make a Free Body Diagram

Now, represent the system by a point in a separate diagram and draw vectors representing the forces acting on the system with this point as the common origin. The forces may lie along a line, may be distributed in a plane (coplanar) or may be distributed in the space (non-planar). We shall rarely encounter situations dealing with non-planar forces. For coplanar forces the plane of diagram represents the plane of the forces acting on the system. Indicate the magnitudes and directions of the forces in this diagram. This is called a *free body diagram*. The free body diagram for the example discussed above with the boy as the system and with the load as the system are shown in figure (5.6).

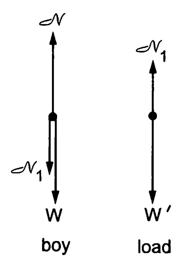


Figure 5.6

Step 4 : Choose Axes and Write Equations

Any three mutually perpendicular directions may be chosen as the X-Y-Z axes. We give below some suggestions for choosing the axes to solve problems.

If the forces are coplanar, only two axes, say X and Y, taken in the plane of forces are needed. Choose the X-axis along the direction in which the system is known to have or is likely to have the acceleration. A direction perpendicular to it may be chosen as the Y-axis. If the system is in equilibrium, any mutually perpendicular directions in the plane of the diagram may be chosen as the axes. Write the components of all the forces along the X-axis and equate their sum to the product of the mass of the system and its acceleration. This gives you one equation. Write the components of the forces along the Y-axis and equate the sum to zero. This gives you another equation. If the forces are collinear, this second equation is not needed.

If necessary you can go to step 1, choose another object as the system, repeat steps 2, 3 and 4 to get more equations. These are called equations of motion. Use mathematical techniques to get the unknown quantities out of these equations. This completes the algorithm.

The magnitudes of acceleration of different objects in a given situation are often related through kinematics. This should be properly foreseen and used together with the equations of motion. For example in figure (5.4) the accelerations of C and E have same magnitudes. Equations of motion for C and for E should use the same variable a for acceleration.

Example 5.2

A block of mass M is pulled on a smooth horizontal table by a string making an angle θ with the horizontal as shown in figure (5.7). If the acceleration of the block is a, find the force applied by the string and by the table on the block.

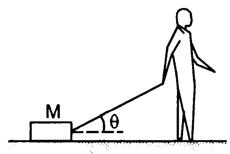


Figure 5.7

Solution : Let us consider the block as the system.

The forces on the block are

(a) pull of the earth, Mg, vertically downward,

(b) contact force by the table, $\mathcal{N}$, vertically upward,

(c) pull of the string, T, along the string.

The free body diagram for the block is shown in figure (5.8).

The acceleration of the block is horizontal and towards the right. Take this direction as the X-axis and vertically upward direction as the Y-axis. We have,

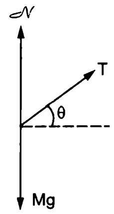


Figure 5.8

component of Mg along the X-axis = 0

component of $\mathcal{N}$ along the X-axis = 0

component of T along the X-axis = $T \cos\theta$.

Hence the total force along the X-axis = $T \cos\theta$.

Using Newton's law, $T \cos\theta = Ma$. ... (i)

Component of Mg along the Y-axis = $-Mg$

component of $\mathcal{N}$ along the Y-axis = $\mathcal{N}$

component of T along the Y-axis = $T \sin\theta$.

Total force along the Y-axis = $\mathcal{N} + T \sin\theta - Mg$.

Using Newton's law, $\mathcal{N} + T \sin\theta - Mg = 0$. ... (ii)

From equation (i), $T = \frac{Ma}{\cos\theta}$ · Putting this in equation (ii)

$\mathcal{N} = Mg - Ma \tan\theta$.

5.4 NEWTON'S THIRD LAW OF MOTION

Newton's third law has already been introduced in chapter 4. *"If a body A exerts a force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A."*

Thus, the force exerted by A on B and that by B on A are equal in magnitude but opposite in direction. This law connects the forces exerted by two bodies on one another. The forces connected by the third law act on two different bodies and hence will never appear together in the list of forces at step 2 of applying Newton's first or second law.

For example, suppose a table exerts an upward force $\mathcal{N}$ on a block placed on it. This force should be accounted if we consider the block as the system. The block pushes the table down with an equal force $\mathcal{N}$. But this force acts on the table and should be considered only if we take the table as the system. Thus, only one of the two forces connected by the third law may appear in the equation of motion depending on the system chosen. The force exerted by the earth on a particle of mass M is Mg downward and therefore, by the particle on the earth is Mg upward. These two forces will not cancel each other. The downward force on the particle will cause acceleration of the particle and that on the earth will cause acceleration (how large ?) of the earth.

Newton's third law of motion is not strictly correct when interaction between two bodies separated by a large distance is considered. We come across such deviations when we study electric and magnetic forces.

Working with the Tension in a String

The idea of tension was qualitatively introduced in chapter 4. Suppose a block of mass M is hanging through a string from the ceiling (figure 5.9).

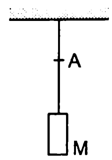


Figure 5.9

Consider a cross-section of the string at A. The cross-section divides the string in two parts, lower part and the upper part. The two parts are in physical contact at the cross-section at A. The lower part of the string will exert an electromagnetic force on the upper part and the upper part will exert an electromagnetic force on the lower part. According to the third law, these two forces will have equal magnitude. The lower part pulls down the upper part with a force T and the upper part pulls up the lower part with equal force T. The common magnitude of the forces exerted by the two parts of the string on each other is called the tension in the string at A. What is the tension in the string at the lower end ? The block and the string are in contact at this end and exert electromagnetic forces on each other. The common magnitude of these forces is the tension in the string at the lower end. What is the tension in the string at the upper end ? At this end, the string and the ceiling meet. The string pulls the ceiling down and the ceiling pulls the string up. The common magnitude of these forces is the tension in the string at the upper end.

Example 5.3

The mass of the part of the string below A in figure (5.9) is m. Find the tension of the string at the lower end and at A.

Solution : To get the tension at the lower end we need the force exerted by the string on the block.

Take the block as the system. The forces on it are

(a) pull of the string, T, upward,

(b) pull of the earth, Mg, downward,

The free body diagram for the block is shown in figure (5.10a). As the acceleration of the block is zero, these forces should add to zero. Hence the tension at the lower end is $T = Mg$.

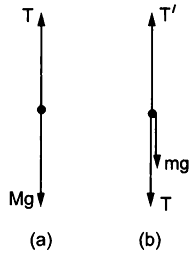


Figure 5.10

To get the tension T at A we need the force exerted by the upper part of the string on the lower part of the string. For this we may write the equation of motion for the lower part of the string. So take the string below A as the system. The forces acting on this part are

(a) T', upward, by the upper part of the string

(b) mg, downward, by the earth

(c) T, downward, by the block.

Note that in (c) we have written T for the force by the block on the string. We have already used the symbol T for the force by the string on the block. We have used Newton's third law here. The force exerted by the block on the string is equal in magnitude to the force exerted by the string on the block.

The free body diagram for this part is shown in figure (5.10b). As the system under consideration (the lower part of the string) is in equilibrium, Newton's first law gives

$$T' = T + mg$$

But $T = Mg$ hence, $T' = (M + m)g$.

Example 5.4

The block shown in figure (5.11) has a mass M and descends with an acceleration a. The mass of the string below the point A is m. Find the tension of the string at the point A and at the lower end.

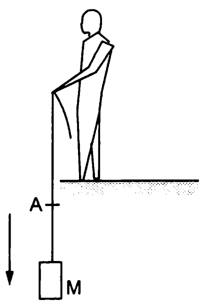


Figure 5.11

Solution : Consider "the block + the part of the string below A" as the system. Let the tension at A be T. The forces acting on this system are

(a) $(M + m)g$, downward, by the earth

(b) T, upward, by the upper part of the string.

The first is gravitational and the second is electromagnetic. We do not have to write the force by the string on the block. This electromagnetic force is by one part of the system on the other part. Only the forces acting on the system by the objects other than the system are to be included.

The system is descending with an acceleration a. Taking the downward direction as the X-axis, the total force along the X-axis is $(M + m)g - T$. Using Newton's law

$$(M + m)g - T = (M + m)a.$$

or, $$T = (M + m)(g - a). \qquad \dots \text{(i)}$$

We have omitted the free body diagram. This you can do if you can draw the free body diagram in your mind and write the equations correctly.

To get the tension T' at the lower end we can put $m = 0$ in (i).

Effectively, we take the point A at the lower end. Thus, we get $T' = M(g - a)$.

Suppose the string in ***Example 5.3*** or ***5.4*** is very light so that we can neglect the mass of the string. Then $T' = T$. The tension is then the same throughout the string. This result is of general nature. The tension at all the points in a string or a spring is the same provided it is assumed massless and no massive particle or body is connected in between.

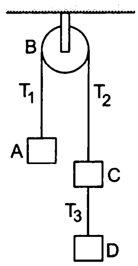


Figure 5.12

If the string in figure (5.12) is light, the tension T_1 of the string is same at all the points between the block A and the pulley B. The tension T_2 is same at all the points between the pulley B and the block C. The tension T_3 is same at all the points between the block C and the block D. The three tensions T_1, T_2 and T_3 may be different from each other. If the pulley B is also light, then $T_1 = T_2$.

5.5 PSEUDO FORCES

In this section we discuss the techniques of solving the motion of a body with respect to a noninertial frame of reference.

Consider the situation shown in figure (5.13). Suppose the frame of reference S' moves with a constant acceleration $\vec{a}_0$ with respect to an inertial frame S. The acceleration of a particle P measured with respect to S' is $\vec{a}_{P,S'} = \vec{a}$ and that with respect to S is $\vec{a}_{P,S}$. The acceleration of S' with respect to S is $\vec{a}_{S',S} = \vec{a}_0$.

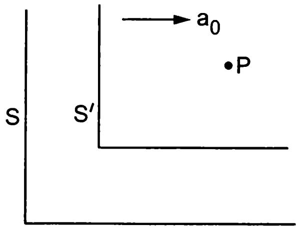


Figure 5.13

Since S' is translating with respect to S we have,

$$\vec{a}_{P,S'} = \vec{a}_{P,S} + \vec{a}_{S,S'} = \vec{a}_{P,S} - \vec{a}_{S',S}$$

or, $$\vec{a} = \vec{a}_{P,S} - \vec{a}_0$$

or, $$m\,\vec{a} = m\,\vec{a}_{P,S} - m\,\vec{a}_0$$

where m is the mass of the particle P. Since S is an inertial frame $m\,\vec{a}_{P,S}$ is equal to the sum of all the forces acting on P. Writing this sum as $\vec{F}$, we get

$$m\,\vec{a} = \vec{F} - m\,\vec{a}_0$$

or, $$\vec{a} = \frac{\vec{F} - m\,\vec{a}_0}{m}. \qquad \ldots (5.3)$$

This equation relates the acceleration of the particle and the forces acting on it. Compare it with equation (5.2) which relates the acceleration and the force when the acceleration is measured with respect to an inertial frame. The acceleration of the frame (with respect to an inertial frame) comes into the equation of a particle. Newton's second law $\vec{a} = \vec{F}/m$ is not valid in such a noninertial frame. An extra term $-m\,\vec{a}_0$ has to be added to the sum of all the forces acting on the particle before writing the equation $\vec{a} = \vec{F}/m$. Note that in this extra term, m is the mass of the particle under consideration and $\vec{a}_0$ is the acceleration of the working frame of reference with respect to some inertial frame.

However, we people spend most of our lifetime on the earth which is an (approximate) inertial frame. We are so familiar with the Newton's laws that we would still like to use the terminology of Newton's laws even when we use a noninertial frame. This can be done if we agree to call $(-m\,\vec{a}_0)$ a *force* acting on the particle. Then while preparing the list of the forces acting on the particle P, we include all the (real) forces acting on P by all other objects and also include an imaginary force $-m\,\vec{a}_0$. Applying Newton's second law will then lead to equation (5.3). Such correction terms $-m\,a_0$ in the list of forces are called *pseudo forces*. This so-called force is to be included in the list only because we are discussing the motion from a noninertial frame and still want to use Newton's second law as "total force equals mass times acceleration". If we work from an inertial frame, the acceleration $\vec{a}_0$ of the frame is zero and no pseudo force is needed. The pseudo forces are also called *inertial forces* although their need arises because of the use of noninertial frames.

Example 5.5

A pendulum is hanging from the ceiling of a car having an acceleration a_0 with respect to the road. Find the angle made by the string with the vertical.

Solution : The situation is shown in figure (5.14a). Suppose the mass of the bob is m and the string makes an angle θ with the vertical. We shall work from the car frame. This frame is noninertial as it has an acceleration $\vec{a}_0$ with respect to an inertial frame (the road). Hence, if we use Newton's second law we shall have to include a pseudo force.

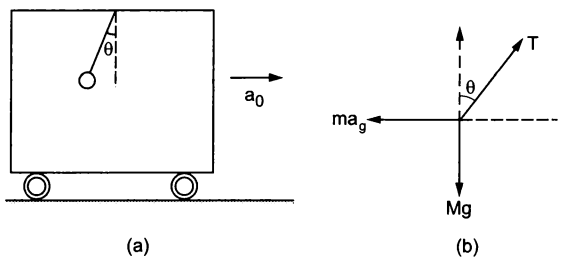


Figure 5.14

Take the bob as the system.

The forces are :

(a) T along the string, by the string
(b) mg downward, by the earth
(c) ma_0 towards left (pseudo force).

The free body diagram is shown in figure (5.14b). As the bob is at rest (remember we are discussing the motion with respect to the car) the force in (a), (b) and (c) should add to zero. Take X-axis along the forward horizontal direction and Y-axis along the upward vertical direction. The components of the forces along the X-axis give

$$T\sin\theta - m\,a_0 = 0 \quad \text{or,} \quad T\sin\theta = m\,a_0 \qquad \ldots \text{(i)}$$

and the components along the Y-axis give

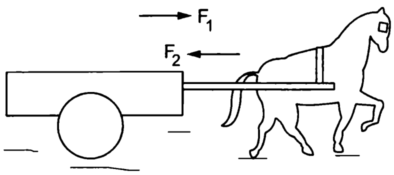
$$T\cos\theta - mg = 0 \quad \text{or,} \quad T\cos\theta = mg. \qquad \ldots \text{(ii)}$$

Dividing (i) by (ii) $\tan\theta = a_0/g$.

Thus, the string makes an angle $\tan^{-1}(a_0/g)$ with the vertical.

5.6 THE HORSE AND THE CART

A good example which illustrates the ideas discussed in this chapter is the motion of a cart pulled by a horse. Suppose the cart is at rest when the driver whips the horse. The horse pulls the cart and the cart accelerates forward. The question posed is as follows. The horse pulls the cart by a force F_1 in the forward direction. From the third law of motion the cart pulls the horse by an equal force $F_2 = F_1$ in the backward direction. The sum of these forces is, therefore, zero (figure 5.15). Why should then the cart accelerate forward ?



F_1 : Force on the cart by the horse

F_2 : Force on the horse by the cart

$F_1 = F_2 = F$

Figure 5.15

Try to locate the mistake in the argument. According to our scheme, we should first decide the system. We can take the horse as the system or the cart as the system or the cart and the horse taken together as the system. Suppose you take the cart as the system. Then the forces on the cart should be listed and the forces on the horse should not enter the discussion. The force on the cart is F_1 in the forward direction and the acceleration of the cart is also in the forward direction. How much is this acceleration? Take the mass of the cart to be M_C. Is the acceleration of the cart $a = F_1/M_C$ in forward direction? Think carefully. We shall return to this question.

Let us now try to understand the motion of the horse. This time we have to consider the forces on the horse. The forward force F_1 by the horse acts on the cart and it should not be taken into account when we discuss the motion of the horse. The force on the horse by the cart is F_2 in the backward direction. Why does the horse go in the forward direction when whipped? The horse exerts a force on the cart in the forward direction and hence the cart is accelerated forward. But the cart exerts an equal force on the horse in the backward direction. Why is the horse not accelerated in backward direction? (Imagine this situation. If the cart is accelerated forward and the horse backward, the horse will sit on the cart kicking out the driver and the passengers.) Where are we wrong? We have not considered *all* the forces acting on the horse. The road pushes the horse by a force P which has a forward component. This force acts on the horse and we must add this force when we discuss the motion of the horse. The horse accelerates forward if the forward component f of the force P exceeds F_2 (Figure 5.16). The acceleration of the horse is $(f - F_2)/M_h$. We should make sure that *all* the forces acting on the system are added. Note that the force of gravity acting on the horse has no forward component.

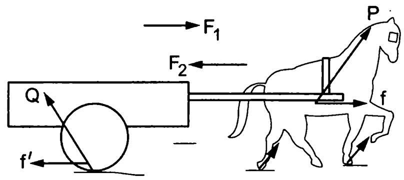


Figure 5.16

Going back to the previous paragraph the acceleration of the cart may not be F_1/M_C. The road exerts a force Q on the cart which may have a backward component f'. The total force on the cart is $F_1 - f'$. The acceleration of the cart is then $a = \dfrac{F_1 - f'}{M_C}$ in the forward direction.

The forces f and f' are self adjustable and they so adjust their values that $\dfrac{F_1 - f'}{M_C} = \dfrac{f - F_2}{M_h}$. The acceleration of the horse and that of the cart are equal in magnitude and direction and hence they move together.

So, once again we remind you that only the forces on the system are to be considered to discuss the motion of the system and all the forces acting on the system are to be considered. Only then apply $\vec{F} = m\vec{a}$.

5.7 INERTIA

A particle is accelerated (in an inertial frame) if and only if a resultant force acts on it. Loosely speaking, the particle does not change its state of rest or of uniform motion along a straight line unless it is forced to do this. This unwillingness of a particle to change its state of rest or of uniform motion along a straight line is called as *inertia*. We can understand the property of inertia in more precise terms as follows. If equal forces are applied on two particles, in general, the acceleration of the particles will be different. The property of a particle to allow a smaller acceleration is called *inertia*. It is clear that larger the mass of the particle, smaller will be the acceleration and hence larger will be the inertia.

Worked Out Examples

1. *A body of mass m is suspended by two strings making angles α and β with the horizontal. Find the tensions in the strings.*

Solution : Take the body of mass m as the system. The forces acting on the system are

(i) mg downwards (by the earth),

(ii) T_1 along the first string (by the first string) and

(iii) T_2 along the second string (by the second string).

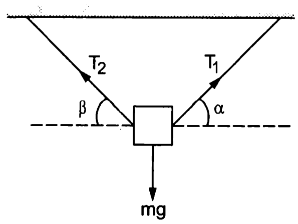


Figure 5-W1

These forces are shown in figure (5-W1). As the body is in equilibrium, these forces must add to zero. Taking horizontal components,

$$T_1 \cos\alpha - T_2 \cos\beta + mg \cos\frac{\pi}{2} = 0$$

or, $$T_1 \cos\alpha = T_2 \cos\beta. \quad \text{... (i)}$$

Taking vertical components,

$$T_1 \sin\alpha + T_2 \sin\beta - mg = 0. \quad \text{... (ii)}$$

Eliminating T_2 from (i) and (ii),

$$T_1 \sin\alpha + T_1 \frac{\cos\alpha}{\cos\beta} \sin\beta = mg$$

or, $$T_1 = \frac{mg}{\sin\alpha + \frac{\cos\alpha}{\cos\beta}\sin\beta} = \frac{mg\cos\beta}{\sin(\alpha+\beta)}.$$

From (i), $$T_2 = \frac{mg\cos\alpha}{\sin(\alpha+\beta)}.$$

2. *Two bodies of masses m_1 and m_2 are connected by a light string going over a smooth light pulley at the end of an incline. The mass m_1 lies on the incline and m_2 hangs vertically. The system is at rest. Find the angle of the incline and the force exerted by the incline on the body of mass m_1 (figure 5-W2).*

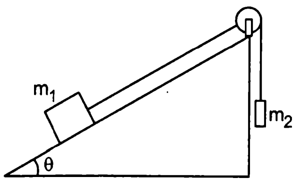


Figure 5-W2

Solution : Figure (5-W3) shows the situation with the forces on m_1 and m_2 shown. Take the body of mass m_2 as the system. The forces acting on it are

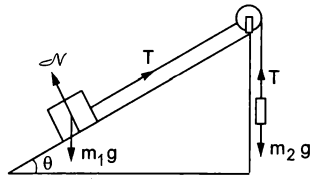


Figure 5-W3

(i) $m_2 g$ vertically downward (by the earth),

(ii) T vertically upward (by the string).

As the system is at rest, these forces should add to zero. This gives $$T = m_2 g. \quad \text{... (i)}$$

Next, consider the body of mass m_1 as the system. The forces acting on this system are

(i) $m_1 g$ vertically downward (by the earth),

(ii) T along the string up the incline (by the string),

(iii) $\mathcal{N}$ normal to the incline (by the incline).

As the string and the pulley are all light and smooth, the tension in the string is uniform everywhere. Hence, same T is used for the equations of m_1 and m_2. As the system is in equilibrium, these forces should add to zero.

Taking components parallel to the incline,

$$T = m_1 g \cos\left(\frac{\pi}{2} - \theta\right) = m_1 g \sin\theta. \quad \text{... (ii)}$$

Taking components along the normal to the incline,

$$\mathcal{N} = m_1 g \cos\theta. \quad \text{... (iii)}$$

Eliminating T from (i) and (ii),

$$m_2 g = m_1 g \sin\theta$$

or, $$\sin\theta = m_2/m_1$$

giving $$\theta = \sin^{-1}(m_2/m_1).$$

From (iii) $$\mathcal{N} = m_1 g\sqrt{1-(m_2/m_1)^2}.$$

3. *A bullet moving at* 250 m/s *penetrates* 5 cm *into a tree limb before coming to rest. Assuming that the force exerted by the tree limb is uniform, find its magnitude. Mass of the bullet is* 10 g.

Solution : The tree limb exerts a force on the bullet in the direction opposite to its velocity. This force causes deceleration and hence the velocity decreases from 250 m/s to zero in 5 cm. We have to find the force exerted by the tree limb on the bullet. If a be the deceleration of the bullet, we have,

$$u = 250 \text{ m/s}, \ v = 0, \ x = 5 \text{ cm} = 0{\cdot}05 \text{ m}$$

giving, $$a = \frac{(250 \text{ m/s})^2 - 0^2}{2 \times 0{\cdot}05 \text{ m}} = 625000 \text{ m/s}^2.$$

The force on the bullet is $F = ma = 6250$ N.

4. *The force on a particle of mass 10 g is* $(\vec{i}\,10 + \vec{j}\,5)$ N. *If it starts from rest what would be its position at time* $t = 5$ s ?

Solution : We have $F_x = 10$ N giving

$$a_x = \frac{F_x}{m} = \frac{10 \text{ N}}{0{\cdot}01 \text{ kg}} = 1000 \text{ m/s}^2.$$

As this is a case of constant acceleration in x-direction,

$$x = u_x t + \frac{1}{2} a_x t^2 = \frac{1}{2} \times 1000 \text{ m/s}^2 \times (5 \text{ s})^2$$
$$= 12500 \text{ m}$$

Similarly, $a_y = \frac{F_y}{m} = \frac{5 \text{ N}}{0{\cdot}01 \text{ kg}} = 500 \text{ m/s}^2$

and $y = 6250$ m.

Thus, the position of the particle at $t = 5$ s is,

$$\vec{r} = (\vec{i}\,12500 + \vec{j}\,6250) \text{ m}.$$

5. *With what acceleration 'a' should the box of figure (5-W4) descend so that the block of mass M exerts a force* $Mg/4$ *on the floor of the box ?*

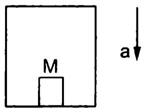


Figure 5-W4

Solution : The block is at rest with respect to the box which is accelerated with respect to the ground. Hence, the acceleration of the block with respect to the ground is 'a' downward. The forces on the block are

(i) Mg downward (by the earth) and
(ii) $\mathcal{N}$ upward (by the floor).

The equation of motion of the block is, therefore

$$Mg - \mathcal{N} = Ma.$$

If $\mathcal{N} = Mg/4$, the above equation gives $a = 3g/4$. The block and hence the box should descend with an acceleration $3g/4$.

6. *A block 'A' of mass m is tied to a fixed point C on a horizontal table through a string passing round a massless smooth pulley B (figure 5-W5). A force F is applied by the experimenter to the pulley. Show that if the pulley is displaced by a distance x, the block will be displaced by 2x. Find the acceleration of the block and the pulley.*

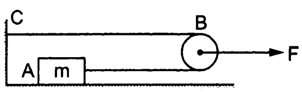


Figure 5-W5

Solution : Suppose the pulley is displaced to B' and the block to A' (figure 5-W6). The length of the string is $CB + BA$ and is also equal to $CB + BB' + B'B + BA'$. Hence, $CB + BA' + A'A = CB + BB' + B'B + BA'$

or, $A'A = 2\,BB'$.

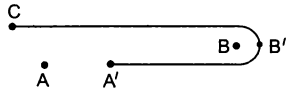


Figure 5-W6

The displacement of A is, therefore, twice the displacement of B in any given time interval. Diffrentiating twice, we find that the acceleration of A is twice the acceleration of B.

To find the acceleration of the block we will need the tension in the string. That can be obtained by considering the pulley as the system.

The forces acting on the pulley are

(i) F towards right by the experimenter,
(ii) T towards left by the portion BC of the string and
(iii) T towards left by the portion BA of the string.

The vertical forces, if any, add to zero as there is no vertical motion.

As the mass of the pulley is zero, the equation of motion is

$F - 2T = 0$ giving $T = F/2$.

Now consider the block as the system. The only horizontal force acting on the block is the tension T towards right. The acceleration of the block is, therefore, $a = T/m = \frac{F}{2m}$. The acceleration of the pulley is $a/2 = \frac{F}{4m}$.

7. *A smooth ring A of mass m can slide on a fixed horizontal rod. A string tied to the ring passes over a fixed pulley B and carries a block C of mass M* ($= 2m$) *as shown in figure (5-W7). At an instant the string between the ring and the pulley makes an angle θ with the rod. (a) Show that, if the ring slides with a speed v, the block descends with speed v* cos θ. *(b) With what acceleration will the ring start moving if the system is released from rest with* $\theta = 30°$?

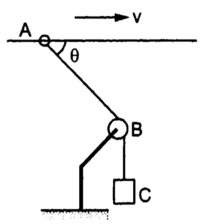


Figure 5-W7

Solution : (a) Suppose in a small time interval Δt the ring is displaced from A to A' (figure 5-W8) and the block from C to C'. Drop a perpendicular $A'P$ from A' to AB. For small displacement $A'B \approx PB$. Since the length of the string is constant, we have

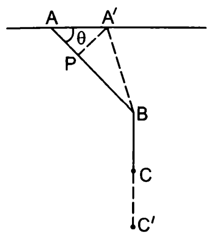


Figure 5-W8

$$AB + BC = A'B + BC'$$

or, $$AP + PB + BC = A'B + BC'$$

or, $$AP = BC' - BC = CC' \quad (\text{as } A'B \approx PB)$$

or, $$AA' \cos\theta = CC'$$

or, $$\frac{AA' \cos\theta}{\Delta t} = \frac{CC'}{\Delta t}$$

or, (velocity of the ring) cos θ = (velocity of the block).

(b) If the initial acceleration of the ring is a, that of the block will be $a \cos\theta$. Let T be the tension in the string at this instant. Consider the block as the system. The forces acting on the block are

(i) Mg downward due to the earth, and

(ii) T upward due to the string.

The equation of motion of the block is

$$Mg - T = Ma \cos\theta. \quad \dots \text{(i)}$$

Now consider the ring as the system. The forces on the ring are

(i) Mg downward due to gravity,

(ii) $\mathcal{N}$ upward due to the rod,

(iii) T along the string due to the string.

Taking components along the rod, the equation of motion of the ring is

$$T \cos\theta = ma. \quad \dots \text{(ii)}$$

From (i) and (ii)

$$Mg - \frac{ma}{\cos\theta} = M\,a \cos\theta$$

or, $$a = \frac{M\,g \cos\theta}{m + M \cos^2\theta}.$$

Putting $\theta = 30°$, $M = 2\,m$ and $g = 9{\cdot}8 \text{ m/s}^2$; therefore

$$a = 6{\cdot}78 \text{ m/s}^2.$$

8. *A light rope fixed at one end of a wooden clamp on the ground passes over a tree branch and hangs on the other side (figure 5-W9). It makes an angle of* 30° *with the ground. A man weighing* (60 kg) *wants to climb up the rope. The wooden clamp can come out of the ground if an upward force greater than* 360 N *is applied to it. Find the maximum acceleration in the upward direction with which the man can climb safely. Neglect friction at the tree branch. Take* $g = 10 \text{ m/s}^2$.

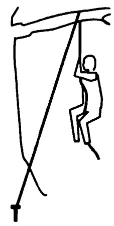


Figure 5-W9

Solution : Let T be the tension in the rope. The upward force on the clamp is $T \sin 30° = T/2$. The maximum tension that will not detach the clamp from the ground is, therefore, given by

$$\frac{T}{2} = 360 \text{ N}$$

or, $$T = 720 \text{ N}.$$

If the acceleration of the man in the upward direction is a, the equation of motion of the man is

$$T - 600 \text{ N} = (60 \text{ kg})\, a$$

The maximum acceleration of the man for safe climbing is, therefore

$$a = \frac{720 \text{ N} - 600 \text{ N}}{60 \text{ kg}} = 2 \text{ m/s}^2.$$

9. *Three blocks of masses* m_1, m_2 *and* m_3 *are connected as shown in the figure (5–W10). All the surfaces are frictionless and the string and the pulleys are light. Find the acceleration of* m_1.

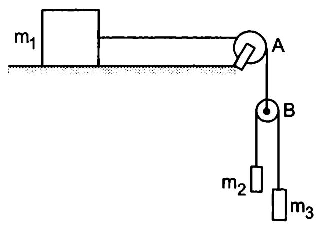


Figure 5-W10

Solution : Suppose the acceleration of m_1 is a_0 towards right. That will also be the downward acceleration of the pulley B because the string connecting m_1 and B is constant in length. Also the string connecting m_2 and m_3 has a constant length. This implies that the decrease in the separation between m_2 and B equals the increase in the separation between m_3 and B. So, the upward acceleration of m_2 with respect to B equals the downward acceleration of m_3 with respect to B. Let this acceleration be a.

The acceleration of m_2 with respect to the ground $= a_0 - a$ (downward) and the acceleration of m_3 with respect to the ground $= a_0 + a$ (downward).

These accelerations will be used in Newton's laws. Let the tension be T in the upper string and T' in the lower string. Consider the motion of the pulley B.

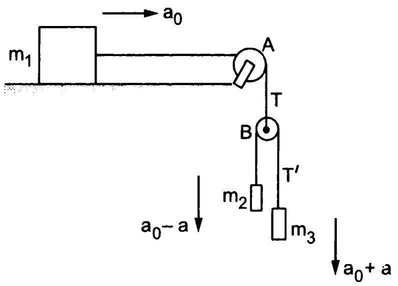


Figure 5-W11

The forces on this light pulley are
(a) T upwards by the upper string and
(b) $2T'$ downwards by the lower string.
As the mass of the pulley is negligible,

$$2T' - T = 0$$

giving

$$T' = T/2. \quad \text{... (i)}$$

Motion of m_1 :

The acceleration is a_0 in the horizontal direction. The forces on m_1 are

(a) T by the string (horizontal).

(b) $m_1 g$ by the earth (vertically downwards) and

(c) $\mathscr{N}$ by the table (vertically upwards).

In the horizontal direction, the equation is

$$T = m_1 a_0. \quad \text{... (ii)}$$

Motion of m_2 : acceleration is $a_0 - a$ in the downward direction. The forces on m_2 are

(a) $m_2 g$ downward by the earth and

(b) $T' = T/2$ upward by the string.

Thus, $$m_2 g - T/2 = m_2(a_0 - a) \quad \text{... (iii)}$$

Motion of m_3 : The acceleration is $(a_0 + a)$ downward. The forces on m_3 are

(a) $m_3 g$ downward by the earth and

(b) $T' = T/2$ upward by the string. Thus,

$$m_3 g - T/2 = m_3(a_0 + a). \quad \text{... (iv)}$$

We want to calculate a_0, so we shall eliminate T and a from (ii), (iii) and (iv).

Putting T from (ii) in (iii) and (iv),

$$a_0 - a = \frac{m_2 g - m_1 a_0/2}{m_2} = g - \frac{m_1 a_0}{2m_2}$$

and $$a_0 + a = \frac{m_3 g - m_1 a_0/2}{m_3} = g - \frac{m_1 a_0}{2m_3}.$$

Adding, $$2a_0 = 2g - \frac{m_1 a_0}{2}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0 = g - \frac{m_1 a_0}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)$$

or, $$a_0\left[1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)\right] = g$$

or, $$a_0 = \frac{g}{1 + \frac{m_1}{4}\left(\frac{1}{m_2} + \frac{1}{m_3}\right)}.$$

10. *A particle slides down a smooth inclined plane of elevation θ, fixed in an elevator going up with an acceleration a_0 (figure 5-W12). The base of the incline has a length L. Find the time taken by the particle to reach the bottom.*

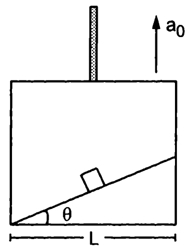


Figure 5-W12

Solution : Let us work in the elevator frame. A pseudo force ma_0 in the downward direction is to be applied on the particle of mass m together with the real forces. Thus, the forces on m are (figure 5-W13)

(i) $\mathscr{N}$ normal force,
(ii) mg downward (by the earth),
(iii) ma_0 downward (pseudo).

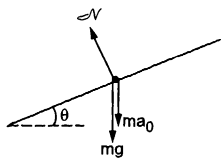


Figure 5-W13

Let a be the acceleration of the particle with respect to the incline. Taking components of the forces parallel to the incline and applying Newton's law,

$$mg\sin\theta + ma_0\sin\theta = ma$$

or, $$a = (g + a_0)\sin\theta.$$

This is the acceleration with respect to the elevator. In this frame, the distance travelled by the particle is $L/\cos\theta$. Hence,

$$\frac{L}{\cos\theta} = \frac{1}{2}(g + a_0)\sin\theta . t^2$$

or, $$t = \left[\frac{2L}{(g + a_0)\sin\theta\cos\theta}\right]^{1/2}$$

11. *All the surfaces shown in figure (5-W14) are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.*

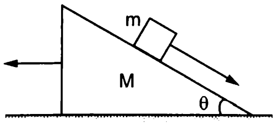


Figure 5-W14

Solution : Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism.

The forces on the block are (figure 5-W15a)

(i) $\mathcal{N}$ normal force,

(ii) mg downward (gravity),

(iii) ma_0 forward (psuedo).

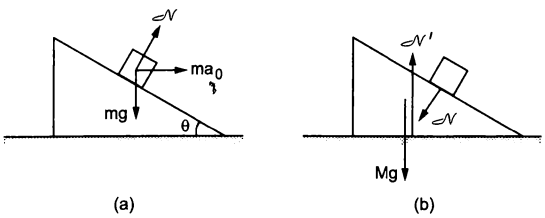


Figure 5-W15

The block slides down the plane. Components of the forces parallel to the incline give

$$ma_0 \cos\theta + mg \sin\theta = ma$$

or, $$a = a_0 \cos\theta + g \sin\theta. \quad \text{... (i)}$$

Components of the force perpendicular to the incline give

$$\mathcal{N} + ma_0 \sin\theta = mg \cos\theta. \quad \text{... (ii)}$$

Now consider the motion of the prism from the lab frame. No pseudo force is needed as the frame used is inertial. The forces are (figure 5-W15b)

(i) Mg downward,

(ii) $\mathcal{N}$ normal to the incline (by the block),

(iii) $\mathcal{N}'$ upward (by the horizontal surface).

Horizontal components give,

$$\mathcal{N} \sin\theta = Ma_0 \quad \text{or,} \quad \mathcal{N} = Ma_0/\sin\theta. \quad \text{... (iii)}$$

Putting in (ii)

$$\frac{Ma_0}{\sin\theta} + ma_0 \sin\theta = mg \cos\theta$$

or, $$a_0 = \frac{m\,g \sin\theta \cos\theta}{M + m \sin^2\theta}.$$

From (i), $$a = \frac{m\,g \sin\theta \cos^2\theta}{M + m \sin^2\theta} + g \sin\theta$$

$$= \frac{(M+m)\,g \sin\theta}{M + m \sin^2\theta}.$$

□

QUESTIONS FOR SHORT ANSWER

1. The apparent weight of an object increases in an elevator while accelerating upward. A moongphaliwala sells his moongphali using a beam balance in an elevator. Will he gain more if the elevator is accelerating up ?

2. A boy puts a heavy box of mass M on his head and jumps down from the top of a multistoried building to the ground. How much is the force exerted by the box on his head during his free fall ? Does the force greatly increase during the period he balances himself after striking the ground ?

3. A person drops a coin. Describe the path of the coin as seen by the person if he is in (a) a car moving at constant velocity and (b) in a freely falling elevator.

4. Is it possible for a particle to describe a curved path if no force acts on it ? Does your answer depend on the frame of reference chosen to view the particle ?

5. You are riding in a car. The driver suddenly applies the brakes and you are pushed forward. Who pushed you forward ?

6. It is sometimes heard that inertial frame of reference is only an ideal concept and no such inertial frame actually exists. Comment.

7. An object is placed far away from all the objects that can exert force on it. A frame of reference is constructed by taking the origin and axes fixed in this object. Will the frame be necessarily inertial ?

8. Figure (5-Q1) shows a light spring balance connected to two blocks of mass 20 kg each. The graduations in the balance measure the tension in the spring. (a) What is the reading of the balance? (b) Will the reading change if the balance is heavy, say 2·0 kg ? (c) What will happen if the spring is light but the blocks have unequal masses ?

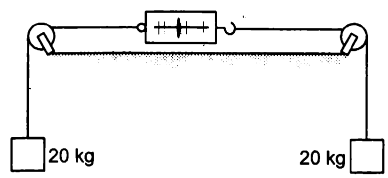


Figure 5-Q1

9. The acceleration of a particle is zero as measured from an inertial frame of reference. Can we conclude that no force acts on the particle ?

10. Suppose you are running fast in a field when you suddendly find a snake in front of you. You stop quickly. Which force is responsible for your deceleration ?

11. If you jump barefooted on a hard surface, your legs get injured. But they are not injured if you jump on a soft surface like sand or pillow. Explain.

12. According to Newton's third law each team pulls the opposite team with equal force in a tug of war. Why then one team wins and the other loses ?

13. A spy jumps from an airplane with his parachute. The spy accelerates downward for some time when the parachute opens. The acceleration is suddenly checked and the spy slowly falls on the ground. Explain the action of parachute in checking the acceleration.

14. Consider a book lying on a table. The weight of the book and the normal force by the table on the book are equal in magnitude and opposite in direction. Is this an example of Newton's third law ?

15. Two blocks of unequal masses are tied by a spring. The blocks are pulled stretching the spring slightly and the system is released on a frictionless horizontal platform. Are the forces due to the spring on the two blocks equal and opposite ? If yes, is it an example of Newton's third law ?

16. When a train starts, the head of a standing passenger seems to be pushed backward. Analyse the situation from the ground frame. Does it really go backward ? Coming back to the train frame, how do you explain the backward movement of the head on the basis of Newton's laws ?

17. A plumb bob is hung from the ceiling of a train compartment. If the train moves with an acceleration 'a' along a straight horizontal track, the string supporting the bob makes an angle $\tan^{-1}(a/g)$ with the normal to the ceiling. Suppose the train moves on an inclined straight track with uniform velocity. If the angle of incline is $\tan^{-1}(a/g)$, the string again makes the same angle with the normal to the ceiling. Can a person sitting inside the compartment tell by looking at the plumb line whether the train is accelerated on a horizontal straight track or it is going on an incline ? If yes, how ? If no, suggest a method to do so.

OBJECTIVE I

1. A body of weight w_1 is suspended from the ceiling of a room through a chain of weight w_2. The ceiling pulls the chain by a force

(a) w_1 (b) w_2 (c) $w_1 + w_2$ (d) $\frac{w_1 + w_2}{2}$.

2. When a horse pulls a cart, the force that helps the horse to move forward is the force exerted by

(a) the cart on the horse (b) the ground on the horse
(c) the ground on the cart (d) the horse on the ground.

3. A car accelerates on a horizontal road due to the force exerted by

(a) the engine of the car (b) the driver of the car
(c) the earth (d) the road.

4. A block of mass 10 kg is suspended through two light spring balances as shown in figure (5-Q2).

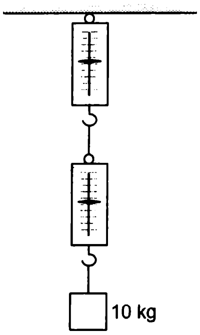


Figure 5-Q2

(a) Both the scales will read 10 kg.
(b) Both the scales will read 5 kg.
(c) The upper scale will read 10 kg and the lower zero.
(d) The readings may be anything but their sum will be 10 kg.

5. A block of mass m is placed on a smooth inclined plane of inclination θ with the horizontal. The force exerted by the plane on the block has a magnitude

(a) mg (b) $mg/\cos\theta$ (c) $mg\cos\theta$ (d) $mg\tan\theta$.

6. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has a magnitude

(a) mg (b) $mg/\cos\theta$ (c) $mg\cos\theta$ (d) $mg\tan\theta$.

7. Neglect the effect of rotation of the earth. Suppose the earth suddenly stops attracting objects placed near its surface. A person standing on the surface of the earth will

(a) fly up (b) slip along the surface
(c) fly along a tangent to the earth's surface
(d) remain standing.

8. Three rigid rods are joined to form an equilateral triangle ABC of side 1 m. Three particles carrying charges 20 μC each are attached to the vertices of the triangle. The whole system is at rest in an inertial frame. The resultant force on the charged particle at A has the magnitude

(a) zero (b) 3·6 N (c) $3{\cdot}6\sqrt{3}$ N (d) 7·2 N.

9. A force F_1 acts on a particle so as to accelerate it from rest to a velocity v. The force F_1 is then replaced by F_2 which decelerates it to rest.
(a) F_1 must be equal to F_2 (b) F_1 may be equal to F_2
(c) F_1 must be unequal to F_2 (d) none of these.

10. Two objects A and B are thrown upward simultaneously with the same speed. The mass of A is greater than the mass of B. Suppose the air exerts a constant and equal force of resistance on the two bodies.
(a) The two bodies will reach the same height.
(b) A will go higher than B.
(c) B will go higher than A.
(d) Any of the above three may happen depending on the speed with which the objects are thrown.

11. A smooth wedge A is fitted in a chamber hanging from a fixed ceiling near the earth's surface. A block B placed at the top of the wedge takes a time T to slide down the length of the wedge. If the block is placed at the top of the wedge and the cable supporting the chamber is broken at the same instant, the block will
(a) take a time longer than T to slide down the wedge
(b) take a time shorter than T to slide down the wedge
(c) remain at the top of the wedge
(d) jump off the wedge.

12. In an imaginary atmosphere, the air exerts a small force F on any particle in the direction of the particle's motion. A particle of mass m projected upward takes a time t_1 in reaching the maximum height and t_2 in the return journey to the original point. Then
(a) $t_1 < t_2$ (b) $t_1 > t_2$ (c) $t_1 = t_2$ (d) the relation between t_1 and t_2 depends on the mass of the particle.

13. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in a time t_1 if the elevator is stationary and in time t_2 if it is moving uniformly. Then
(a) $t_1 = t_2$ (b) $t_1 < t_2$ (c) $t_1 > t_2$ (d) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

14. A free ^{238}U nucleus kept in a train emits an alpha particle. When the train is stationary, a nucleus decays and a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x at time t after the decay. If the decay takes place while the train is moving at a uniform velocity v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay as measured by the passenger is
(a) $x + vt$ (b) $x - vt$ (c) x
(d) depends on the direction of the train.

OBJECTIVE II

1. A reference frame attached to the earth
(a) is an inertial frame by definition
(b) cannot be an inertial frame because the earth is revolving around the sun
(c) is an inertial frame because Newton's laws are applicable in this frame
(d) cannot be an inertial frame because the earth is rotating about its axis.

2. A particle stays at rest as seen in a frame. We can conclude that
(a) the frame is inertial
(b) resultant force on the particle is zero
(c) the frame may be inertial but the resultant force on the particle is zero
(d) the frame may be noninertial but there is a nonzero resultant force.

3. A particle is found to be at rest when seen from a frame S_1 and moving with a constant velocity when seen from another frame S_2. Mark out the possible options.
(a) Both the frames are inertial.
(b) Both the frames are noninertial.
(c) S_1 is inertial and S_2 is noninertial.
(d) S_1 is noninertial and S_2 is inertial.

4. Figure (5-Q3) shows the displacement of a particle going along the X-axis as a function of time. The force acting on the particle is zero in the region
(a) AB (b) BC (c) CD (d) DE.

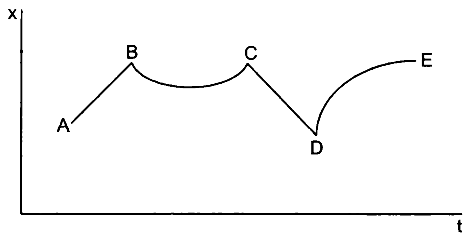


Figure 5-Q3

5. Figure (5-Q4) shows a heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m. At $t = 0$, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.

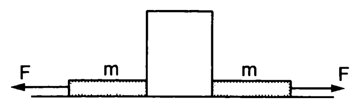


Figure 5-Q4

(a) $F_1 = F_2 = F$ for $t < 0$
(b) $F_1 = F_2 = F + mg$ for $t < 0$
(c) $F_1 = F$, $F_2 = F$ for $t > 0$
(d) $F_1 < F$, $F_2 = F$ for $t > 0$.

6. The force exerted by the floor of an elevator on the foot of a person standing there is more than the weight of the person if the elevator is
(a) going up and slowing down
(b) going up and speeding up
(c) going down and slowing down
(d) going down and speeding up.

7. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be
(a) going up with increasing speed
(b) going down with increasing speed
(c) going up with uniform speed
(d) going down with uniform speed.

8. A particle is observed from two frames S_1 and S_2. The frame S_2 moves with respect to S_1 with an acceleration a. Let F_1 and F_2 be the pseudo forces on the particle when seen from S_1 and S_2 respectively. Which of the following are not possible ?
(a) $F_1 = 0,\ F_2 \neq 0$
(b) $F_1 \neq 0,\ F_2 = 0$
(c) $F_1 \neq 0,\ F_2 \neq 0$
(d) $F_1 = 0,\ F_2 = 0$.

9. A person says that he measured the acceleration of a particle to be nonzero while no force was acting on the particle.
(a) He is a liar.
(b) His clock might have run slow.
(c) His meter scale might have been longer than the standard.
(d) He might have used noninertial frame.

EXERCISES

1. A block of mass 2 kg placed on a long frictionless horizontal table is pulled horizontally by a constant force *F*. It is found to move 10 m in the first two seconds. Find the magnitude of *F*.

2. A car moving at 40 km/h is to be stopped by applying brakes in the next 4·0 m. If the car weighs 2000 kg, what average force must be applied on it ?

3. In a TV picture tube electrons are ejected from the cathode with negligible speed and reach a velocity of 5×10^{6} m/s in travelling one centimeter. Assuming straight line motion, find the constant force exerted on the electron. The mass of the electron is $9{\cdot}1 \times 10^{-31}$ kg.

4. A block of mass 0·2 kg is suspended from the ceiling by a light string. A second block of mass 0·3 kg is suspended from the first block through another string. Find the tensions in the two strings. Take $g = 10\ \text{m/s}^2$.

5. Two blocks of equal mass m are tied to each other through a light string. One of the blocks is pulled along the line joining them with a constant force *F*. Find the tension in the string joining the blocks.

6. A particle of mass 50 g moves on a straight line. The variation of speed with time is shown in figure (5-E1). Find the force acting on the particle at $t = 2$, 4 and 6 seconds.

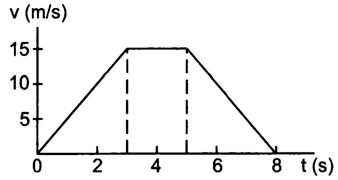


Figure 5-E1

7. Two blocks *A* and *B* of mass m_A and m_B respectively are kept in contact on a frictionless table. The experimenter pushes the block *A* from behind so that the blocks accelerate. If the block *A* exerts a force *F* on the block *B*, what is the force exerted by the experimenter on *A* ?

8. Raindrops of radius 1 mm and mass 4 mg are falling with a speed of 30 m/s on the head of a bald person. The drops splash on the head and come to rest. Assuming equivalently that the drops cover a distance equal to their radii on the head, estimate the force exerted by each drop on the head.

9. A particle of mass 0·3 kg is subjected to a force $F = -k\,x$ with k = 15 N/m. What will be its initial acceleration if it is released from a point x = 20 cm ?

10. Both the springs shown in figure (5-E2) are unstretched. If the block is displaced by a distance x and released, what will be the initial acceleration?

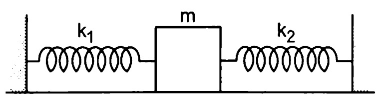


Figure 5-E2

11. A small block *B* is placed on another block *A* of mass 5 kg and length 20 cm. Initially the block B is near the right end of block *A* (figure 5-E3). A constant horizontal force of 10 N is applied to the block *A*. All the surfaces are assumed frictionless. Find the time elapsed before the block *B* separates from *A*.

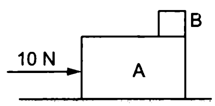


Figure 5-E3

12. A man has fallen into a ditch of width d and two of his friends are slowly pulling him out using a light rope and two fixed pulleys as shown in figure (5-E4). Show that the force (assumed equal for both the friends) exerted by each friend on the road increases as the man moves up. Find the force when the man is at a depth h.

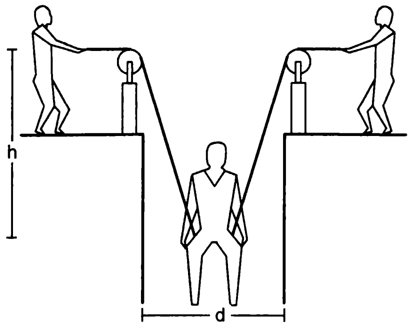


Figure 5-E4

13. The elevator shown in figure (5-E5) is descending with an acceleration of $2\ m/s^2$. The mass of the block A is 0·5 kg. What force is exerted by the block A on the block B ?

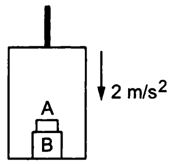


Figure 5-E5

14. A pendulum bob of mass 50 g is suspended from the ceiling of an elevator. Find the tension in the string if the elevator (a) goes up with acceleration $1{\cdot}2\ m/s^2$, (b) goes up with deceleration $1{\cdot}2\ m/s^2$, (c) goes up with uniform velocity, (d) goes down with acceleration $1{\cdot}2\ m/s^2$, (e) goes down with deceleration $1{\cdot}2\ m/s^2$ and (f) goes down with uniform velocity.

15. A person is standing on a weighing machine placed on the floor of an elevator. The elevator starts going up with some acceleration, moves with uniform velocity for a while and finally decelerates to stop. The maximum and the minimum weights recorded are 72 kg and 60 kg. Assuming that the magnitudes of the acceleration and the deceleration are the same, find (a) the true weight of the person and (b) the magnitude of the acceleration. Take $g = 9{\cdot}9\ m/s^2$.

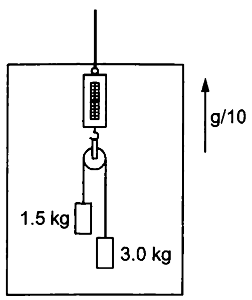


Figure 5-E6

16. Find the reading of the spring balance shown in figure (5-E6). The elevator is going up with an acceleration of $g/10$, the pulley and the string are light and the pulley is smooth.

17. A block of 2 kg is suspended from the ceiling through a massless spring of spring constant $k = 100$ N/m. What is the elongation of the spring ? If another 1 kg is added to the block, what would be the further elongation ?

18. Suppose the ceiling in the previous problem is that of an elevator which is going up with an acceleration of $2{\cdot}0\ m/s^2$. Find the elongations.

19. The force of buoyancy exerted by the atmosphere on a balloon is B in the upward direction and remains constant. The force of air resistance on the balloon acts opposite to the direction of velocity and is proportional to it. The balloon carries a mass M and is found to fall down near the earth's surface with a constant velocity v. How much mass should be removed from the balloon so that it may rise with a constant velocity v ?

20. An empty plastic box of mass m is found to accelerate up at the rate of $g/6$ when placed deep inside water. How much sand should be put inside the box so that it may accelerate down at the rate of $g/6$?

21. A force $\vec{F} = \vec{v} \times \vec{A}$ is exerted on a particle in addition to the force of gravity, where $\vec{v}$ is the velocity of the particle and $\vec{A}$ is a constant vector in the horizontal direction. With what minimum speed a particle of mass m be projected so that it continues to move undeflected with a constant velocity ?

22. In a simple Atwood machine, two unequal masses m_1 and m_2 are connected by a string going over a clamped light smooth pulley. In a typical arrangement (figure 5-E7) $m_1 = 300$ g and $m_2 = 600$ g. The system is released from rest. (a) Find the distance travelled by the first block in the first two seconds. (b) Find the tension in the string. (c) Find the force exerted by the clamp on the pulley.

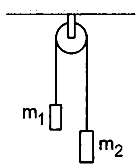


Figure 5-E7

23. Consider the Atwood machine of the previous problem. The larger mass is stopped for a moment 2·0 s after the system is set into motion. Find the time elapsed before the string is tight again.

24. Figure (5-E8) shows a uniform rod of length 30 cm having a mass of 3·0 kg. The strings shown in the figure are pulled by constant forces of 20 N and 32 N. Find the force exerted by the 20 cm part of the rod on the 10 cm part. All the surfaces are smooth and the strings and the pulleys are light.

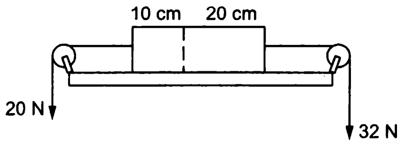


Figure 5-E8

25. Consider the situation shown in figure (5-E9). All the surfaces are frictionless and the string and the pulley are light. Find the magnitude of the acceleration of the two blocks.

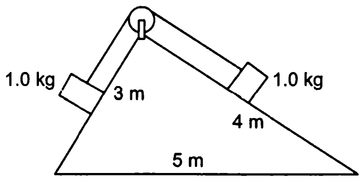


Figure 5-E9

26 A constant force $F = m_2 g/2$ is applied on the block of mass m_1 as shown in figure (5-E10). The string and the pulley are light and the surface of the table is smooth. Find the acceleration of m_1.

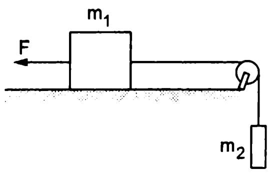


Figure 5-E10

27. In figure (5-E11) $m_1 = 5$ kg, $m_2 = 2$ kg and $F = 1$ N. Find the acceleration of either block. Describe the motion of m_1 if the string breaks but F continues to act.

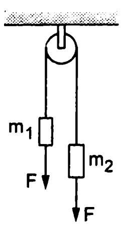


Figure 5-E11

28. Let $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg in figure (5-E12). Find the accelerations of m_1, m_2 and m_3. The string from the upper pulley to m_1 is 20 cm when the system is released from rest. How long will it take before m_1 strikes the pulley?

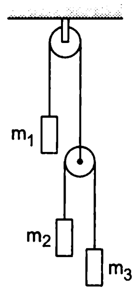


Figure 5-E12

29. In the previous problem, suppose $m_2 = 2{\cdot}0$ kg and $m_3 = 3{\cdot}0$ kg. What should be the mass m so that it remains at rest?

30. Calculate the tension in the string shown in figure (5-E13). The pulley and the string are light and all surfaces are frictionless. Take $g = 10$ m/s^2.

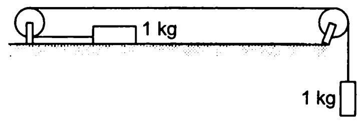


Figure 5-E13

31. Consider the situation shown in figure (5-E14). Both the pulleys and the string are light and all the surfaces are frictionless. (a) Find the acceleration of the mass M. (b) Find the tension in the string. (c) Calculate the force exerted by the clamp on the pulley A in the figure.

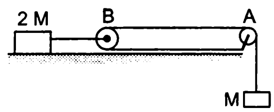


Figure 5-E14

32. Find the acceleration of the block of mass M in the situation shown in figure (5-E15). All the surfaces are frictionless and the pulleys and the string are light.

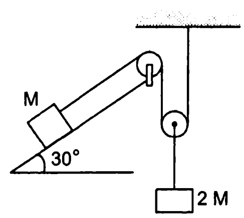


Figure 5-E15

33. Find the mass M of the hanging block in figure (5-E16) which will prevent the smaller block from slipping over the triangular block. All the surfaces are frictionless and the strings and the pulleys are light.

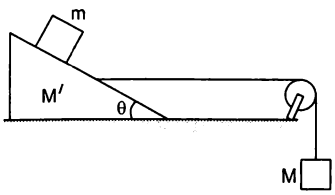


Figure 5-E16

34. Find the acceleration of the blocks A and B in the three situations shown in figure (5-E17).

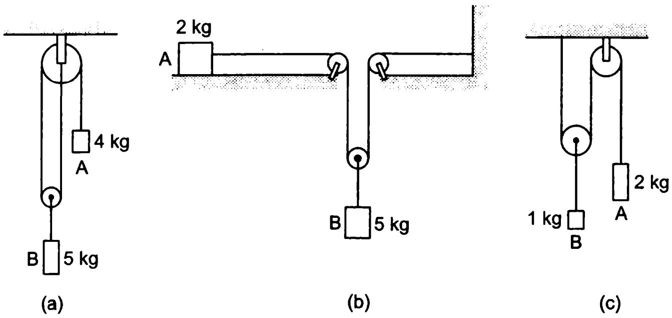


Figure 5-E17

35. Find the acceleration of the 500 g block in figure (5-E18).

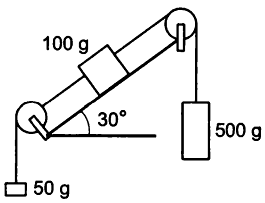


Figure 5-E18

36. A monkey of mass 15 kg is climbing on a rope with one end fixed to the ceiling. If it wishes to go up with an acceleration of 1 m/s ^{2}, how much force should it apply to the rope? If the rope is 5 m long and the monkey starts from rest, how much time will it take to reach the ceiling?

37. A monkey is climbing on a rope that goes over a smooth light pulley and supports a block of equal mass at the other end (figure 5-E19). Show that whatever force the monkey exerts on the rope, the monkey and the block move in the same direction with equal acceleration. If initially both were at rest, their separation will not change as time passes.

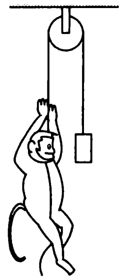


Figure 5-E19

38. The monkey B shown in figure (5-E20) is holding on to the tail of the monkey A which is climbing up a rope. The masses of the monkeys A and B are 5 kg and 2 kg respectively. If A can tolerate a tension of 30 N in its tail, what force should it apply on the rope in order to carry the monkey B with it? Take $g = 10$ m/s ^{2}.

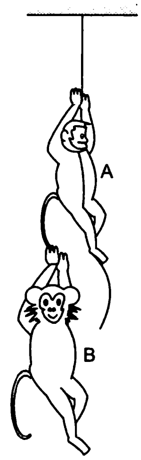


Figure 5-E20

39. Figure (5-E21) shows a man of mass 60 kg standing on a light weighing machine kept in a box of mass 30 kg. The box is hanging from a pulley fixed to the ceiling through a light rope, the other end of which is held by the man himself. If the man manages to keep the box at rest, what is the weight shown by the machine? What force should he exert on the rope to get his correct weight on the machine?

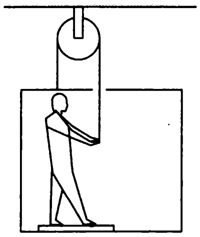


Figure 5-E21

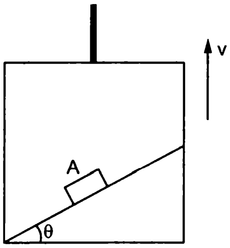


Figure 5-E22

40. A block A can slide on a frictionless incline of angle θ and length l, kept inside an elevator going up with uniform velocity v (figure 5-E22). Find the time taken by the block to slide down the length of the incline if it is released from the top of the incline.

41. A car is speeding up on a horizontal road with an acceleration a. Consider the following situations in the car. (i) A ball is suspended from the ceiling through a string and is maintaining a constant angle with the vertical. Find this angle. (ii) A block is kept on a smooth incline and does not slip on the incline. Find the angle of the incline with the horizontal.

42. A block is kept on the floor of an elevator at rest. The elevator starts descending with an acceleration of 12 m/s^2. Find the displacement of the block during the first 0·2 s after the start. Take $g = 10 \text{ m/s}^2$.

□

ANSWERS

OBJECTIVE I

1. (c)	2. (b)	3. (d)	4. (a)	5. (c)	6. (b)
7. (d)	8. (a)	9. (b)	10. (b)	11. (c)	12. (b)
13. (a)	14. (c)				

OBJECTIVE II

1. (b), (d)	2. (c), (d)	3. (a), (b)
4. (a), (c)	5. (a)	6. (b), (c)
7. (c), (d)	8. (d)	9. (d)

EXERCISES

1. 10 N
2. $3·1 \times 10^4$ N
3. $1·1 \times 10^{-15}$ N
4. 5 N and 3 N
5. $F/2$
6. 0·25 N along the motion, zero and 0·25 N opposite to the motion.
7. $F\left(1 + \dfrac{m_A}{m_B}\right)$
8. 1·8 N
9. 10 m/s^2
10. $(k_1 + k_2)\dfrac{x}{m}$ opposite to the displacement.
11. 0·45 s.
12. $\dfrac{mg}{4h}\sqrt{d^2 + 4h^2}$
13. 4 N
14. (a) 0·55 N (b) 0·43 N (c) 0·49 N (d) 0·43 N (e) 0·55 N (f) 0·49 N
15. 66 kg and $0·9 \text{ m/s}^2$
16. 4·4 kg
17. 0·2 m, 0·1 m
18. 0·24 m, 0·12 m
19. $2\left(M - \dfrac{B}{g}\right)$
20. $2m/5$
21. mg/A
22. (a) 6·5 m (b) 3·9 N (c) 7·8 N
23. 2/3 s
24. 24 N
25. $g/10$
26. $\dfrac{m_2 g}{2(m_1 + m_2)}$ towards right
27. $4·3 \text{ m/s}^2$, moves downward with acceleration $g + 0·2 \text{ m/s}^2$

28. $\frac{19}{29}g\,(up)$, $\frac{17}{29}g$ (down), $\frac{21}{29}g$ (down), 0·25 s
29. 4·8 kg
30. 5 N
31. (a) $2g/3$ (b) $Mg/3$
(c) $\sqrt{2}\,Mg/3$ at an angle of 45° with the horizontal
32. $g/3$ up the plane
33. $\frac{M' + m}{\cot\theta - 1}$
34. (a) $\frac{2}{7}g$ downward, $\frac{g}{7}$ upward
(b) $\frac{10}{13}g$ forward, $\frac{5}{13}g$ downward
(c) $\frac{2}{3}g$ downward, $\frac{g}{3}$ upward
35. $\frac{8}{13}g$ downward
36. 165 N, $\sqrt{10}$ s
38. between 70 N and 105 N
39. 15 kg, 1800 N
40. $\sqrt{\frac{2\,l}{g\sin\theta}}$
41. $\tan^{-1}(a/g)$ in each case
42. 20 cm

□

SOLUTIONS TO CONCEPTS
CHAPTER – 5

1. m = 2kg
   S = 10m
   Let, acceleration = a, Initial velocity u = 0.
   $S = ut + 1/2\ at^2$
   $\Rightarrow 10 = ½\ a\ (2^2) \Rightarrow 10 = 2a \Rightarrow a = 5\ m/s^2$
   Force: F = ma = 2 × 5 = 10N (Ans)

2. u = 40 km/hr = $\frac{40000}{3600}$ = 11.11 m/s.
   m = 2000 kg ; v = 0 ; s = 4m
   acceleration 'a' = $\frac{v^2 - u^2}{2s} = \frac{0^2 - (11.11)^2}{2 \times 4} = -\frac{123.43}{8} = -15.42\ m/s^2$ (deceleration)
   So, braking force = F = ma = 2000 × 15.42 = 30840 = $3.08\ 10^4$ N (Ans)

3. Initial velocity u = 0 (negligible)
   $v = 5 \times 10^6$ m/s.
   $s = 1cm = 1 \times 10^{-2}m$.
   acceleration a = $\frac{v^2 - u^2}{2s} = \frac{(5 \times 10^6)^2 - 0}{2 \times 1 \times 10^{-2}} = \frac{25 \times 10^{12}}{2 \times 10^{-2}} = 12.5 \times 10^{14} ms^{-2}$
   $F = ma = 9.1 \times 10^{-31} \times 12.5 \times 10^{14} = 113.75 \times 10^{-17} = 1.1 \times 10^{-15}$ N.

4.

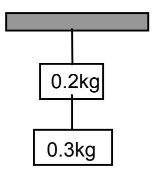


fig 1

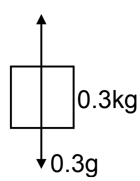


fig 2

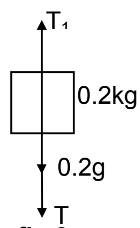


fig 3

$g = 10m/s^2$ $\quad$ $T - 0.3g = 0 \Rightarrow T = 0.3g = 0.3 \times 10 = 3$ N
$T_1 - (0.2g + T) = 0 \Rightarrow T_1 = 0.2g + T = 0.2 \times 10 + 3 = 5N$
$\therefore$ Tension in the two strings are 5N & 3N respectively.

5.

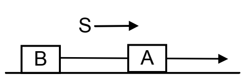


Fig 1

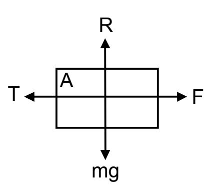


Fig 2

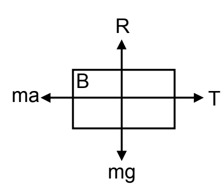
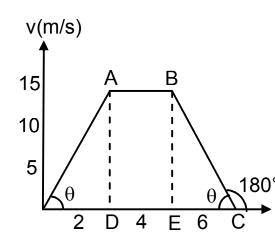


Fig 3

T + ma – F = 0 $\quad$ T – ma = 0 ⇒ T = ma …………(i)
⇒ F= T + ma ⇒ F= T + T $\quad$ from (i)
⇒ 2T = F ⇒ T = F / 2

6. $m = 50g = 5 \times 10^{-2}$ kg
   As shown in the figure,
   Slope of OA = Tanθ $\frac{AD}{OD} = \frac{15}{3} = 5\ m/s^2$
   So, at t = 2sec acceleration is $5m/s^2$
   Force = $ma = 5 \times 10^{-2} \times 5 = 0.25N$ along the motion



At t = 4 sec slope of AB = 0, acceleration = 0 [tan 0° = 0]

$\therefore$ Force = 0

At t = 6 sec, acceleration = slope of BC.

In ΔBEC = $\tan\theta = \dfrac{BE}{EC} = \dfrac{15}{3} = 5$.

Slope of BC = $\tan(180° - \theta) = -\tan\theta = -5\ m/s^2$ (deceleration)

Force = ma = $5 \times 10^{-2}\ 5 = 0.25$ N. Opposite to the motion.

7. Let, F $\rightarrow$ contact force between m_A & m_B.

And, f $\rightarrow$ force exerted by experimenter.

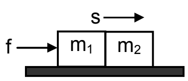


Fig 1

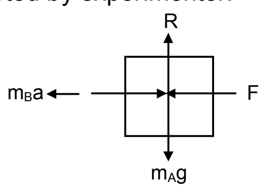


Fig 2

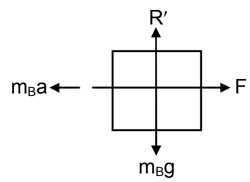


Fig 3

$F + m_A a - f = 0$ $\qquad\qquad$ $m_B a - f = 0$

$\Rightarrow F = f - m_A a$ ....(i) $\qquad\qquad$ $\Rightarrow F = m_B a$ ....(ii)

From eqn (i) and eqn (ii)

$\Rightarrow f - m_A a = m_B a \Rightarrow f = m_B a + m_A a \Rightarrow f = a(m_A + m_B)$.

$\Rightarrow f = \dfrac{F}{m_B}(m_B + m_A) = F\left(1 + \dfrac{m_A}{m_B}\right)$ [because a = F/m_B]

$\therefore$ The force exerted by the experimenter is $F\left(1 + \dfrac{m_A}{m_B}\right)$

8. r = 1mm = 10^{-3}

'm' = 4mg = 4×10^{-6}kg

s = 10^{-3}m.

v = 0

u = 30 m/s.

So, $a = \dfrac{v^2 - u^2}{2s} = \dfrac{-30 \times 30}{2 \times 10^{-3}} = -4.5 \times 10^5\ m/s^2$ (decelerating)

Taking magnitude only deceleration is $4.5 \times 10^5\ m/s^2$

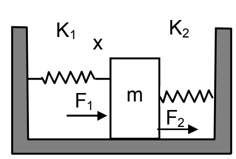
So, force F = $4 \times 10^{-6} \times 4.5 \times 10^5 = 1.8$ N

9. x = 20 cm = 0.2m, k = 15 N/m, m = 0.3kg.

Acceleration $a = \dfrac{F}{m} = \dfrac{-kx}{x} = \dfrac{-15(0.2)}{0.3} = -\dfrac{3}{0.3} = -10 m/s^2$ (deceleration)

So, the acceleration is 10 m/s^2 opposite to the direction of motion

10. Let, the block m towards left through displacement x.



$F_1 = k_1 x$ (compressed)

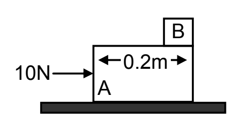
$F_2 = k_2 x$ (expanded)

They are in same direction.

Resultant $F = F_1 + F_2 \Rightarrow F = k_1 x + k_2 x \Rightarrow F = x(k_1 + k_2)$

So, a = acceleration = $\dfrac{F}{m} = \dfrac{x(k_1 + k_2)}{m}$ opposite to the displacement.

11. m = 5 kg of block A.



ma = 10 N

$\Rightarrow$ a 10/5 = 2 m/s^2.

As there is no friction between A & B, when the block A moves, Block B remains at rest in its position.

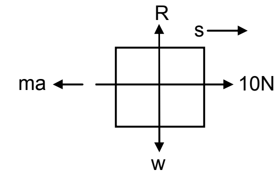
Initial velocity of A = u = 0.

Distance to cover so that B separate out s = 0.2 m.

Acceleration a = 2 m/s^2

$\therefore$ s= ut + ½ at^2

$\Rightarrow$ 0.2 = 0 + (½) ×2 × t^2 $\Rightarrow$ t^2 = 0.2 $\Rightarrow$ t = 0.44 sec $\Rightarrow$ t= 0.45 sec.



12. a) at any depth let the ropes make angle θ with the vertical

From the free body diagram

F cos θ + F cos θ – mg = 0

$\Rightarrow$ 2F cos θ = mg $\Rightarrow$ F = $\dfrac{mg}{2\cos\theta}$

As the man moves up. θ increases i.e. cos θ decreases. Thus F increases.

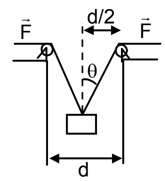


Fig-1

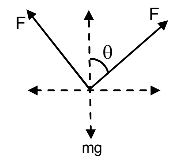
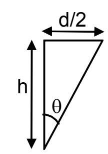


Fig-2

b) When the man is at depth h

$$\cos\theta = \frac{h}{\sqrt{(d/2)^2 + h^2}}$$

$$\text{Force} = \frac{mg}{\dfrac{h}{\sqrt{\dfrac{d^2}{4} + h^2}}} = \frac{mg}{4h}\sqrt{d^2 + 4h^2}$$



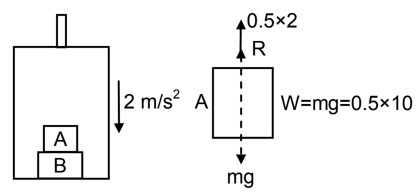
13. From the free body diagram

$\therefore$ R + 0.5 × 2 – w = 0

$\Rightarrow$ R = w – 0.5 × 2

= 0.5 (10 – 2) = 4N.

So, the force exerted by the block A on the block B, is 4N.



14. a) The tension in the string is found out for the different conditions from the free body diagram as shown below.

T – (W + 0.06 × 1.2) = 0

$\Rightarrow$ T = 0.05 × 9.8 + 0.05 × 1.2

= 0.55 N.

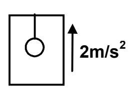


Fig-1

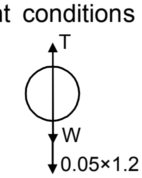


Fig-2

b) $\therefore$ T + 0.05 × 1.2 – 0.05 × 9.8 = 0

$\Rightarrow$ T = 0.05 × 9.8 – 0.05 × 1.2

= 0.43 N.

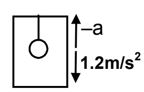


Fig-3

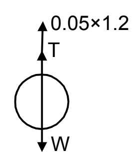


Fig-4

c) When the elevator makes uniform motion

T – W = 0

$\Rightarrow$ T = W = 0.05 × 9.8

= 0.49 N

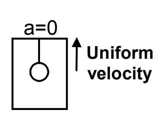


Fig-5

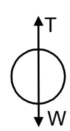


Fig-6

d) T + 0.05 × 1.2 – W = 0

$\Rightarrow$ T = W – 0.05 × 1.2

= 0.43 N.

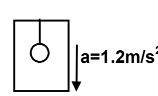


Fig-7

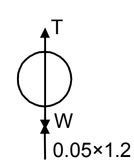


Fig-8

e) T – (W + 0.05 × 1.2) = 0

$\Rightarrow$ T = W + 0.05 × 1.2

= 0.55 N

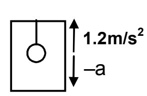


Fig-9

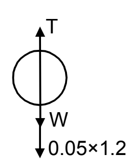


Fig-10

f) When the elevator goes down with uniform velocity acceleration = 0

$T - W = 0$

$\Rightarrow T = W = 0.05 \times 9.8$

$= 0.49$ N.

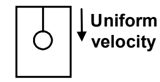


Fig-11

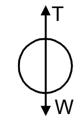


Fig-12

15. When the elevator is accelerating upwards, maximum weight will be recorded.

$R - (W + ma) = 0$

$\Rightarrow R = W + ma = m(g + a)$ max.wt.

When decelerating upwards, maximum weight will be recorded.

$R + ma - W = 0$

$\Rightarrow R = W - ma = m(g - a)$

So, $m(g + a) = 72 \times 9.9$ …(1)

$m(g - a) = 60 \times 9.9$ …(2)

Now, $mg + ma = 72 \times 9.9 \Rightarrow mg - ma = 60 \times 9.9$

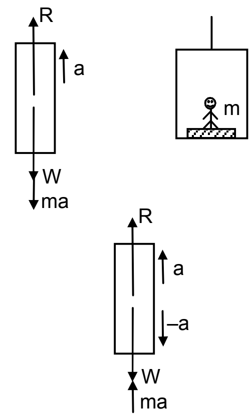
$\Rightarrow 2mg = 1306.8$

$\Rightarrow m = \dfrac{1306.8}{2 \times 9.9} = 66$ Kg

So, the true weight of the man is 66 kg.

Again, to find the acceleration, $mg + ma = 72 \times 9.9$

$\Rightarrow a = \dfrac{72 \times 9.9 - 66 \times 9.9}{66} = \dfrac{9.9}{11} = 0.9\ \text{m/s}^2$.



16. Let the acceleration of the 3 kg mass relative to the elevator is 'a' in the downward direction.

As, shown in the free body diagram

$T - 1.5\,g - 1.5(g/10) - 1.5\,a = 0$ from figure (1)

and, $T - 3g - 3(g/10) + 3a = 0$ from figure (2)

$\Rightarrow T = 1.5\,g + 1.5(g/10) + 1.5a$ … (i)

And $T = 3g + 3(g/10) - 3a$ … (ii)

Equation (i) × 2 $\Rightarrow 3g + 3(g/10) + 3a = 2T$

Equation (ii) × 1 $\Rightarrow 3g + 3(g/10) - 3a = T$

Subtracting the above two equations we get, $T = 6a$

Subtracting $T = 6a$ in equation (ii)

$6a = 3g + 3(g/10) - 3a$.

$\Rightarrow 9a = \dfrac{33g}{10} \Rightarrow a = \dfrac{(9.8)33}{10} = 32.34$

$\Rightarrow a = 3.59 \ \therefore\ T = 6a = 6 \times 3.59 = 21.55$

$T^1 = 2T = 2 \times 21.55 = 43.1$ N cut is T_1 shown in spring.

Mass $= \dfrac{wt}{g} = \dfrac{43.1}{9.8} = 4.39 = 4.4$ kg

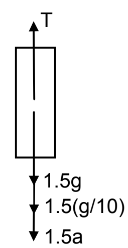


Fig-1

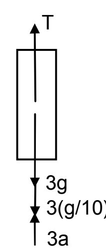


Fig-2

17. Given, m = 2 kg, k = 100 N/m

From the free body diagram, $kl - 2g = 0 \Rightarrow kl = 2g$

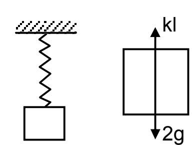
$\Rightarrow l = \dfrac{2g}{k} = \dfrac{2 \times 9.8}{100} = \dfrac{19.6}{100} = 0.196 = 0.2$ m

Suppose further elongation when 1 kg block is added be x,

Then $k(1 + x) = 3g$

$\Rightarrow kx = 3g - 2g = g = 9.8$ N

$\Rightarrow x = \dfrac{9.8}{100} = 0.098 = 0.1$ m



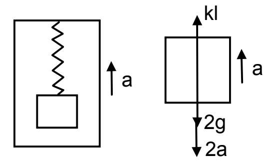
18. $a = 2\ m/s^2$

$kl - (2g + 2a) = 0$

$\Rightarrow kl = 2g + 2a$

$= 2 \times 9.8 + 2 \times 2 = 19.6 + 4$

$\Rightarrow l = \dfrac{23.6}{100} = 0.236\ m = 0.24\ m$

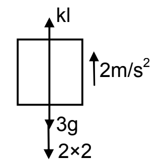


When 1 kg body is added total mass (2 + 1)kg = 3kg.

elongation be l_1

$kl_1 = 3g + 3a = 3 \times 9.8 + 6$

$\Rightarrow l_1 = \dfrac{33.4}{100} = 0.0334 = 0.36$



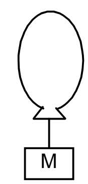
Further elongation = $l_1 - l = 0.36 - 0.12$ m.

19. Let, the air resistance force is F and Buoyant force is B.

Given that

$F_a \propto v$, where $v \to$ velocity

$\Rightarrow F_a = kv$, where $k \to$ proportionality constant.



When the balloon is moving downward,

$B + kv = mg$ …(i)

$\Rightarrow M = \dfrac{B + kv}{g}$

For the balloon to rise with a constant velocity v, (upward)

let the mass be m

Here, $B - (mg + kv) = 0$ …(ii)

$\Rightarrow B = mg + kv$

$\Rightarrow m = \dfrac{B - kw}{g}$

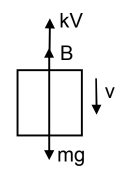


Fig-1

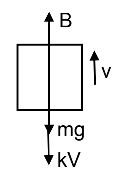


Fig-2

So, amount of mass that should be removed = M – m.

$= \dfrac{B + kv}{g} - \dfrac{B - kv}{g} = \dfrac{B + kv - B + kv}{g} = \dfrac{2kv}{g} = \dfrac{2(Mg - B)}{G} = 2\{M - (B/g)\}$

20. When the box is accelerating upward,

$U - mg - m(g/6) = 0$

$\Rightarrow U = mg + mg/6 = m\{g + (g/6)\} = 7\ mg/7$ …(i)

$\Rightarrow m = 6U/7g.$

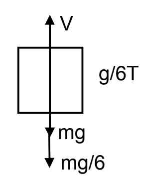


Fig-1

When it is accelerating downward, let the required mass be M.

$U - Mg + Mg/6 = 0$

$\Rightarrow\ U = \dfrac{6Mg - Mg}{6} = \dfrac{5Mg}{6} \Rightarrow M = \dfrac{6U}{5g}$

Mass to be added = $M - m = \dfrac{6U}{5g} - \dfrac{6U}{7g} = \dfrac{6U}{g}\left(\dfrac{1}{5} - \dfrac{1}{7}\right)$

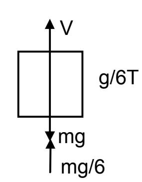


Fig-2

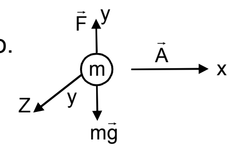
$= \dfrac{6U}{g}\left(\dfrac{2}{35}\right) = \dfrac{12}{35}\left(\dfrac{U}{g}\right)$

$= \dfrac{12}{35}\left(\dfrac{7mg}{6} \times \dfrac{1}{g}\right)$ from (i)

$= 2/5$ m.

$\therefore$ The mass to be added is 2m/5.

21. Given that, $\vec{F} = \vec{u} \times \vec{A}$ and $\overrightarrow{mg}$ act on the particle.



For the particle to move undeflected with constant velocity, net force should be zero.

$\therefore \ (\vec{u} \times \vec{A}) + \overrightarrow{mg} = 0$

$\therefore \ (\vec{u} \times \vec{A}) = -\overrightarrow{mg}$

Because, $(\vec{u} \times \vec{A})$ is perpendicular to the plane containing $\vec{u}$ and $\vec{A}$, $\vec{u}$ should be in the xz-plane.

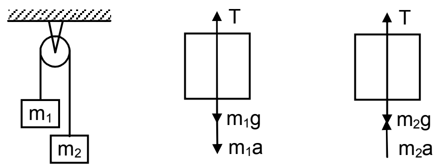
Again, u A sin θ = mg

$\therefore$ u = $\dfrac{mg}{A \sin\theta}$

u will be minimum, when sin θ = 1 $\Rightarrow$ θ = 90°

$\therefore$ u_{min} = $\dfrac{mg}{A}$ along Z-axis.

22.



m_1 = 0.3 kg, m_2 = 0.6 kg

$T - (m_1g + m_1a) = 0$ …(i) $\Rightarrow T = m_1g + m_1a$

$T + m_2a - m_2g = 0$ …(ii) $\Rightarrow T = m_2g - m_2a$

From equation (i) and equation (ii)

$m_1g + m_1a + m_2a - m_2g = 0$, from (i)

$\Rightarrow a(m_1 + m_2) = g(m_2 - m_1)$

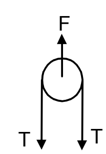
$\Rightarrow a = f\left(\dfrac{m_2 - m_1}{m_1 + m_2}\right) = 9.8\left(\dfrac{0.6 - 0.3}{0.6 + 0.3}\right) = 3.266 \text{ ms}^{-2}$.

a) t = 2 sec acceleration = 3.266 ms^{-2}

Initial velocity u = 0

So, distance travelled by the body is,

$S = ut + 1/2\, at^2 \Rightarrow 0 + ½(3.266)\, 2^2 = 6.5$ m



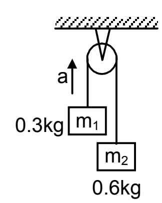
b) From (i) $T = m_1(g + a) = 0.3\,(9.8 + 3.26) = 3.9$ N

c) The force exerted by the clamp on the pully is given by

F – 2T = 0

F = 2T = 2 × 3.9 = 7.8 N.

23. a = 3.26 m/s^2



T = 3.9 N

After 2 sec mass m_1 the velocity

V = u + at = 0 + 3.26 × 2 = 6.52 m/s upward.

At this time m_2 is moving 6.52 m/s downward.

At time 2 sec, m_2 stops for a moment. But m_1 is moving upward with velocity 6.52 m/s.

It will continue to move till final velocity (at highest point) because zero.

Here, v = 0 ; u = 6.52

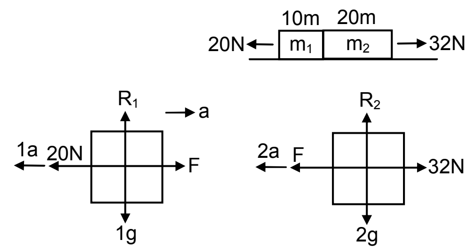
A = –g = –9.8 m/s^2 [moving up ward m_1]

$V = u + at \Rightarrow 0 = 6.52 + (-9.8)t$

$\Rightarrow t = 6.52/9.8 = 0.66 = 2/3$ sec.

During this period 2/3 sec, m_2 mass also starts moving downward. So the string becomes tight again after a time of 2/3 sec.

24. Mass per unit length 3/30 kg/cm = 0.10 kg/cm.
Mass of 10 cm part = m_1 = 1 kg
Mass of 20 cm part = m_2 = 2 kg.
Let, F = contact force between them.
From the free body diagram



F – 20 – 10 = 0 …(i)
And, 32 – F – 2a = 0 …(ii)
From eqa (i) and (ii) 3a – 12 = 0 $\Rightarrow$ a = 12/ 3 = 4 m/s^2
Contact force F = 20 + 1a = 20 + 1 × 4 = 24 N.

25.

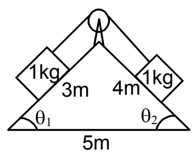


Fig-1

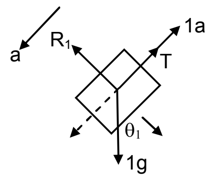


Fig-2

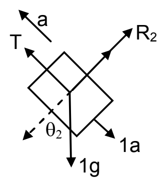


Fig-3

Sin θ_1 = 4/5
sin θ_2 = 3/5

g sin θ_1 – (a + T) = 0
$\Rightarrow$ g sing θ_1 = a + T …(i)
$\Rightarrow$ T + a – g sin $\theta_1 = 0$

T – g sin θ_2 – a = 0
$\Rightarrow$T = g sin θ_2 + a …(ii)

From eqn (i) and (ii), g sin θ_2 + a + a – g sin θ_1 = 0

$\Rightarrow$ 2a = g sin θ_1 – g sin θ_2 = g $\left(\frac{4}{5}-\frac{3}{5}\right)$ = g / 5

$\Rightarrow$ a = $\frac{g}{5}\times\frac{1}{2}=\frac{g}{10}$

26.

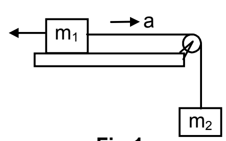


Fig-1

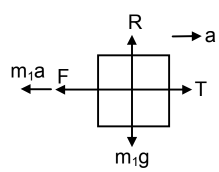


Fig-2

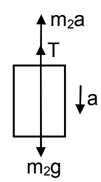


Fig-3

From the above Free body diagram
M_1a + F – T = 0 $\Rightarrow$ T = m_1a + F …(i)

From the above Free body diagram
m_2a + T – m_2g =0 ….(ii)
$\Rightarrow$ m_2a + m_1a + F – m_2g = 0 (from (i))
$\Rightarrow$ a(m_1 + m_2) + $m_2g/2$ – m_2g = 0 {because f = $m^2g/2$}
$\Rightarrow$ a(m_1 + m_2) – m_2g =0
$\Rightarrow$ a(m_1 + m_2) = $m_2g/2$ $\Rightarrow$ a = $\frac{m_2 g}{2(m_1 = m_2)}$

Acceleration of mass m_1 is $\frac{m_2 g}{2(m_1 = m_2)}$ towards right.

27.

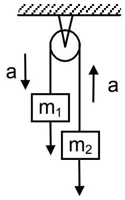


Fig-1

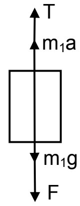


Fig-2

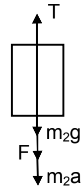


Fig-3

From the above free body diagram
T + m_1a – m(m_1g + F) = 0

From the free body diagram
T – (m_2g + F + m_2a)=0

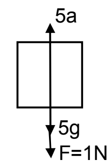
$\Rightarrow$ T = $m_1g + F - m_1a \Rightarrow T = 5g + 1 - 5a$ …(i)

$\Rightarrow T = m_2g + F + m_2a \Rightarrow T = 2g + 1 + 2a$ …(ii)

From the eqn (i) and eqn (ii)

$5g + 1 - 5a = 2g + 1 + 2a \Rightarrow 3g - 7a = 0 \Rightarrow 7a = 3g$

$\Rightarrow a = \dfrac{3g}{7} = \dfrac{29.4}{7} = 4.2\ m/s^2$ [g= 9.8m/s^2]



a) acceleration of block is 4.2 m/s^2

b) After the string breaks m_1 move downward with force F acting down ward.

$m_1a = F + m_1g = (1 + 5g) = 5(g + 0.2)$ Force = 1N, acceleration = 1/5= 0.2m/s.

So, acceleration = $\dfrac{\text{Force}}{\text{mass}} = \dfrac{5(g+0.2)}{5}$ = (g + 0.2) m/s^2

28.

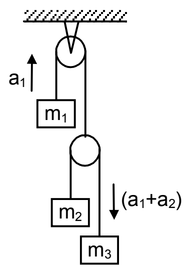


Fig-1

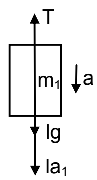


Fig-2

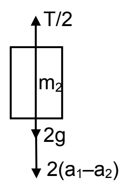


Fig-3

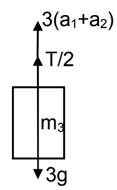


Fig-4

Let the block m+1+ moves upward with acceleration a, and the two blocks m_2 an m_3 have relative acceleration a_2 due to the difference of weight between them. So, the actual acceleration at the blocks m_1, m_2 and m_3 will be a_1.

$(a_1 - a_2)$ and $(a_1 + a_2)$ as shown

$T = 1g - 1a_2 = 0$ ...(i) from fig (2)

$T/2 - 2g - 2(a_1 - a_2) = 0$ ...(ii) from fig (3)

$T/2 - 3g - 3(a_1 + a_2) = 0$ ...(iii) from fig (4)

From eqn (i) and eqn (ii), eliminating T we get, $1g + 1a_2 = 4g + 4(a_1 + a_2) \Rightarrow 5a_2 - 4a_1 = 3g$ (iv)

From eqn (ii) and eqn (iii), we get $2g + 2(a_1 - a_2) = 3g - 3(a_1 - a_2) \Rightarrow 5a_1 + a_2 =$ (v)

Solving (iv) and (v) $a_1 = \dfrac{2g}{29}$ and $a_2 = g - 5a_1 = g - \dfrac{10g}{29} = \dfrac{19g}{29}$

So, $a_1 - a_2 = \dfrac{2g}{29} - \dfrac{19g}{29} = -\dfrac{17g}{29}$

$a_1 + a_2 = \dfrac{2g}{29} + \dfrac{19g}{29} = \dfrac{21g}{29}$ So, acceleration of m_1, m_2, m_3 ae $\dfrac{19g}{29}$(up) $\dfrac{17g}{29}$ (doan) $\dfrac{21g}{29}$(down) respectively.

Again, for m_1, u = 0, s= 20cm=0.2m and $a_2 = \dfrac{19}{29}g$ [g = 10m/s^2]

$\therefore S = ut + ½ at^2 = 0.2 = \dfrac{1}{2} \times \dfrac{19}{29} gt^2 \Rightarrow t = 0.25$sec.

29.

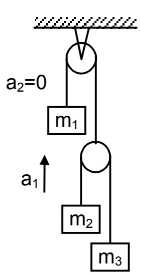


Fig-1

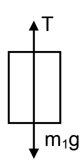


Fig-2

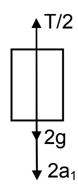


Fig-3

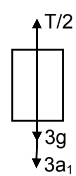


Fig-4

m_1 should be at rest.

$T - m_1g = 0$ $\Rightarrow T = m_1g$ …(i)

$T/2 - 2g - 2a_1 = 0$ $\Rightarrow T - 4g - 4a_1 = 0$ …(ii)

$T/2 - 3g - 3a_1 = 0$ $\Rightarrow T = 6g - 6a_1$ …(iii)

From eqn (ii) & (iii) we get

$3T - 12g = 12g - 2T \Rightarrow T = 24g/5 = 408g.$

Putting yhe value of T eqn (i) we get, $m_1 = 4.8$kg.

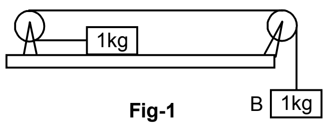
30. 

Fig-1

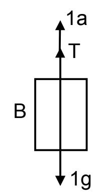


Fig-2

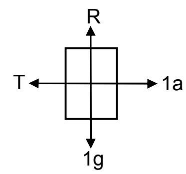


Fig-3

$T + 1a = 1g$ …(i) $T - 1a = 0 \Rightarrow T = 1a$ (ii)

From eqn (i) and (ii), we get

$1a + 1a = 1g \Rightarrow 2a = g \Rightarrow a = \frac{g}{2} = \frac{10}{2} = 5\text{m/s}^2$

From (ii) $T = 1a = 5$N.

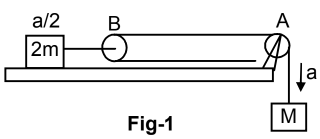
31. 

Fig-1

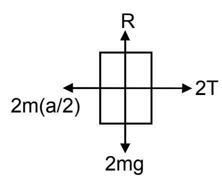


Fig-2

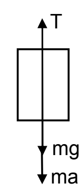


Fig-3

$Ma - 2T = 0$

$\Rightarrow Ma = 2T \Rightarrow T = Ma/2.$

$T + Ma - Mg = 0$

$\Rightarrow Ma/2 + ma = Mg.$ (because $T = Ma/2$)

$\Rightarrow 3\ Ma = 2\ Mg \Rightarrow a = 2g/3$

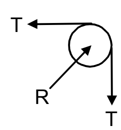
a) acceleration of mass M is 2g/3.

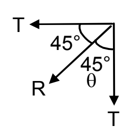
b) Tension $T = \frac{Ma}{2} = \frac{M}{2} = \frac{2g}{3} = \frac{Mg}{3}$

c) Let, R^1 = resultant of tensions = force exerted by the clamp on the pulley

$R^1 = \sqrt{T^2 + T^2} = \sqrt{2}T$

$\therefore R = \sqrt{2}T = \sqrt{2}\frac{Mg}{3} = \frac{\sqrt{2}Mg}{3}$





Again, $\tan\theta = \frac{T}{T} = 1 \Rightarrow \theta = 45°.$

So, it is $\frac{\sqrt{2}Mg}{3}$ at an angle of 45° with horizontal.

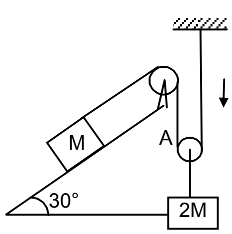
32. 

Fig-1

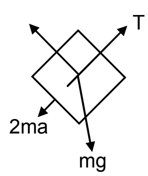


Fig-2

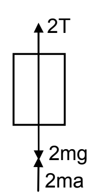


Fig-3

$2Ma + Mg\sin\theta - T = 0$

$\Rightarrow T = 2Ma + Mg\sin\theta$ …(i)

$2T + 2Ma - 2Mg = 0$

$\Rightarrow 2(2Ma + Mg\sin\theta) + 2Ma - 2Mg = 0$ [From (i)]

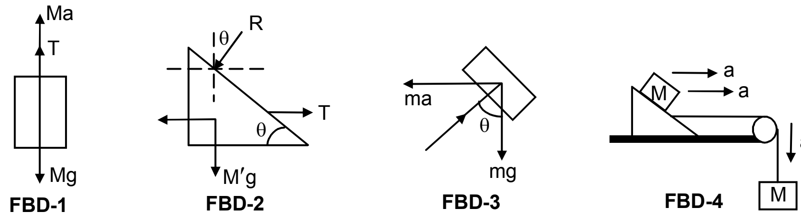
$\Rightarrow 4Ma + 2Mg\sin\theta + 2Ma - 2Mg = 0$

$\Rightarrow 6Ma + 2Mg\sin 30° - 2Mg = 0$

$\Rightarrow 6Ma = Mg \Rightarrow a = g/6$.

Acceleration of mass M is 2a = s × g/6 = g/3 up the plane.

33.



FBD-1 FBD-2 FBD-3 FBD-4

As the block ‘m’ does not slinover M′, ct will have same acceleration as that of M′

From the freebody diagrams.

$T + Ma - Mg = 0$ ...(i) (From FBD – 1)

$T - M'a - R\sin\theta = 0$ ...(ii) (From FBD -2)

$R\sin\theta - ma = 0$ ...(iii) (From FBD -3)

$R\cos\theta - mg = 0$ ...(iv) (From FBD -4)

Eliminating T, R and a from the above equation, we get $M = \dfrac{M' + m}{\cot\theta - 1}$

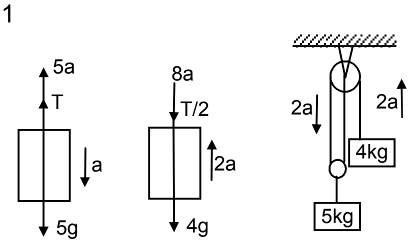
34. a) $5a + T - 5g = 0 \Rightarrow T = 5g - 5a$ ...(i) (From FBD-1)

Again (1/2) – 4g – 8a = 0 $\Rightarrow$ T = 8g – 16a ...(ii) (from FBD-2)

From equn (i) and (ii), we get

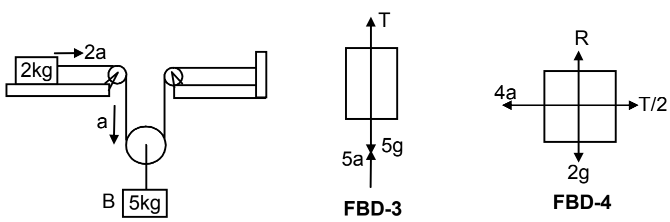
$5g - 5a = 8g + 16a \Rightarrow 21a = -3g \Rightarrow a = -1/7g$

So, acceleration of 5 kg mass is g/7 upward and that of 4 kg mass is 2a = 2g/7 (downward).



FBD-1 FBD-2 FBD-3

b)



FBD-3 FBD-4

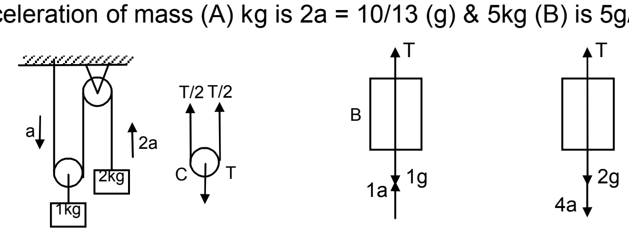
$4a - t/2 = 0 \Rightarrow 8a - T = 0 \Rightarrow T = 8a$ … (ii) [From FBD -4]

Again, $T + 5a - 5g = 0 \Rightarrow 8a + 5a - 5g = 0$

$\Rightarrow 13a - 5g = 0 \Rightarrow a = 5g/13$ downward. (from FBD -3)

Acceleration of mass (A) kg is 2a = 10/13 (g) & 5kg (B) is 5g/13.

c)



FBD-5 FBD-6

$T + 1a - 1g = 0 \Rightarrow T = 1g - 1a$ …(i) [From FBD – 5]

Again, $\dfrac{T}{2} - 2g - 4a = 0 \Rightarrow T - 4g - 8a = 0$ …(ii) [From FBD -6]

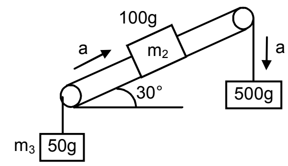
$\Rightarrow 1g - 1a - 4g - 8a = 0$ [From (i)]

$\Rightarrow$ a = –(g/3) downward.

Acceleration of mass 1kg(b) is g/3 (up)

Acceleration of mass 2kg(A) is 2g/3 (downward).

35. m_1 = 100g = 0.1kg



m_2 = 500g = 0.5kg

m_3 = 50g = 0.05kg.

T + 0.5a – 0.5g = 0 ...(i)

T_1 – 0.5a – 0.05g = a ...(ii)

T_1 + 0.1a – T + 0.05g = 0 ...(iii)

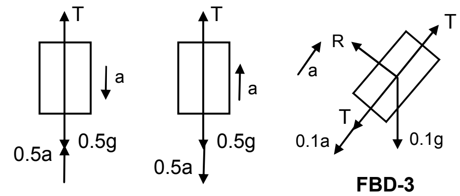
From equn (ii) T_1 = 0.05g + 0.05a ...(iv)

From equn (i) T_1 = 0.5g – 0.5a ...(v)

Equn (iii) becomes T_1 + 0.1a – T + 0.05g = 0

$\Rightarrow$ 0.05g + 0.05a + 0.1a – 0.5g + 0.5a + 0.05g = 0 [From (iv) and (v)]

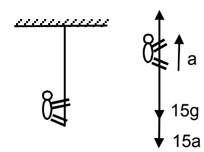
$\Rightarrow 0.65a = 0.4g \Rightarrow a = \dfrac{0.4}{0.65} = \dfrac{40}{65}g = \dfrac{8}{13}g$ downward



FBD-1 FBD-2 FBD-3

Acceleration of 500gm block is 8g/13g downward.

36. m = 15 kg of monkey. a = 1 m/s^2.



From the free body diagram

$\therefore$ T – [15g + 15(1)] = 0 $\Rightarrow$ T = 15 (10 + 1) $\Rightarrow$ T = 15 × 11 $\Rightarrow$ T = 165 N.

The monkey should apply 165N force to the rope.

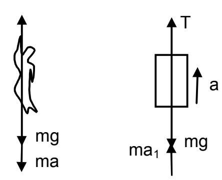
Initial velocity u = 0 ; acceleration a = 1m/s^2 ; s = 5m.

$\therefore$ s = ut + ½ at^2

5 = 0 + (1/2)1 t^2 $\Rightarrow$ t^2 = 5 × 2 $\Rightarrow$ t = $\sqrt{10}$ sec.

Time required is $\sqrt{10}$ sec.

37. Suppose the monkey accelerates upward with acceleration 'a' & the block, accelerate downward with acceleration a_1. Let Force exerted by monkey is equal to 'T'



From the free body diagram of monkey

$\therefore$ T – mg – ma = 0 ...(i)

$\Rightarrow$ T = mg + ma.

Again, from the FBD of the block,

T = ma_1 – mg = 0.

$\Rightarrow$ mg + ma + ma_1 – mg = 0 [From (i)] $\Rightarrow$ ma = –ma_1 $\Rightarrow$ a = a_1.

Acceleration '–a' downward i.e. 'a' upward.

$\therefore$The block & the monkey move in the same direction with equal acceleration.

If initially they are rest (no force is exertied by monkey) no motion of monkey of block occurs as they have same weight (same mass). Their separation will not change as time passes.

38. Suppose A move upward with acceleration a, such that in the tail of A maximum tension 30N produced.

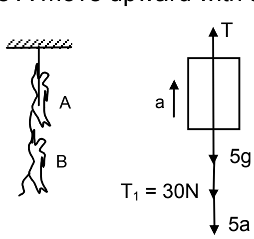


Fig-2

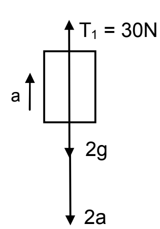


Fig-3

T – 5g – 30 – 5a = 0 ...(i)

$\Rightarrow$ T = 50 + 30 +(5 × 5) = 105 N (max)

30 – 2g – 2a = 0 ...(ii)

$\Rightarrow$ 30 – 20 – 2a = 0 $\Rightarrow$ a = 5 m/s^2

So, A can apply a maximum force of 105 N in the rope to carry the monkey B with it.

For minimum force there is no acceleration of monkey 'A' and B. $\Rightarrow a = 0$

Now equation (ii) is $T'_1 - 2g = 0 \Rightarrow T'_1 = 20$ N (wt. of monkey B)

Equation (i) is $T - 5g - 20 = 0$ [As $T'_1 = 20$ N]

$\Rightarrow T = 5g + 20 = 50 + 20 = 70$ N.

$\therefore$ The monkey A should apply force between 70 N and 105 N to carry the monkey B with it.

39. (i) Given, Mass of man = 60 kg.

Let R′ = apparent weight of man in this case.

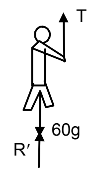
Now, $R' + T - 60g = 0$ [From FBD of man]

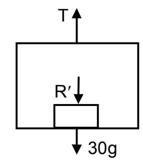
$\Rightarrow T = 60g - R'$ ...(i)

$T - R' - 30g = 0$ ...(ii) [From FBD of box]

$\Rightarrow 60g - R' - R' - 30g = 0$ [From (i)]

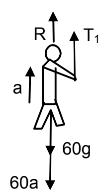
$\Rightarrow R' = 15g$ The weight shown by the machine is 15kg.

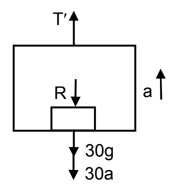




(ii) To get his correct weight suppose the applied force is 'T' and so, acclerates upward with 'a'.

In this case, given that correct weight = R = 60g, where g = acc^n due to gravity





From the FBD of the man

$T^1 + R - 60g - 60a = 0$

$\Rightarrow T^1 - 60a = 0$ [$\therefore R = 60g$]

$\Rightarrow T^1 = 60a$ ...(i)

From the FBD of the box

$T^1 - R - 30g - 30a = 0$

$\Rightarrow T^1 - 60g - 30g - 30a = 0$

$\Rightarrow T^1 - 30a = 90g = 900$

$\Rightarrow T^1 = 30a - 900$ ...(ii)

From eqn (i) and eqn (ii) we get $T^1 = 2T^1 - 1800 \Rightarrow T^1 = 1800$N.

$\therefore$ So, he should exert 1800 N force on the rope to get correct reading.

40. The driving force on the block which n the body to move sown the plane is $F = mg \sin\theta$,

So, acceleration = $g \sin\theta$

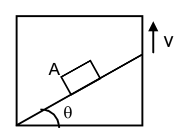
Initial velocity of block u = 0.

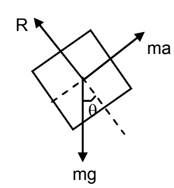
$s = \ell$, $a = g \sin\theta$

Now, $S = ut + \frac{1}{2} at^2$

$\Rightarrow \ell = 0 + \frac{1}{2}(g \sin\theta)\, t^2 \Rightarrow g^2 = \dfrac{2\ell}{g \sin\theta} \Rightarrow t = \sqrt{\dfrac{2\ell}{g\sin\theta}}$

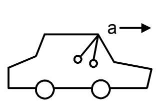
Time taken is $\sqrt{\dfrac{2\ell}{g \sin\theta}}$

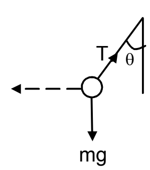




41. Suppose pendulum makes θ angle with the vertical. Let, m = mass of the pendulum.

From the free body diagram





$T \cos\theta - mg = 0$

$\Rightarrow T \cos\theta = mg$

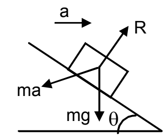
$\Rightarrow T = \dfrac{mg}{\cos\theta}$ ...(i)

$ma - T \sin\theta = 0$

$\Rightarrow ma = T \sin\theta$

$\Rightarrow t = \dfrac{ma}{\sin\theta}$ ...(ii)

From (i) & (ii) $\frac{mg}{\cos\theta} = \frac{ma}{\sin\theta} \Rightarrow \tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\frac{a}{g}$

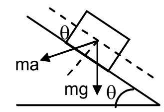


The angle is $\text{Tan}^{-1}(a/g)$ with vertical.

(ii) m $\rightarrow$ mass of block.

Suppose the angle of incline is ‘θ’

From the diagram

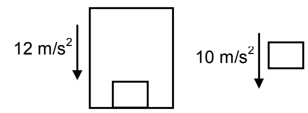


ma cos θ – mg sin θ = 0 $\Rightarrow$ ma cos θ = mg sin $\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{a}{g}$

$\Rightarrow \tan\theta = a/g \Rightarrow \theta = \tan^{-1}(a/g)$.

42. Because, the elevator is moving downward with an acceleration 12 m/s^2 (>g), the bodygets separated. So, body moves with acceleration g = 10 m/s^2 [freely falling body] and the elevator move with acceleration 12 m/s^2

Now, the block has acceleration = g = 10 m/s^2



u = 0

t = 0.2 sec

So, the distance travelled by the block is given by.

$\therefore$ s = ut + ½ at^2

= 0 + (½) 10 (0.2)^2 = 5 × 0.04 = 0.2 m = 20 cm.

The displacement of body is 20 cm during first 0.2 sec.

* * * *