Solution : The area can be divided into strips by drawing ordinates between $x = 0$ and $x = 6$ at a regular interval of dx. Consider the strip between the ordinates at x and $x + dx$. The height of this strip is $y = x^2$. The area of this strip is $dA = y\,dx = x^2 dx$.

The total area of the shaded part is obtained by summing up these strip-areas with x varying from 0 to 6. Thus,

$$A = \int_0^6 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_0^6 = \frac{216 - 0}{3} = 72.$$

15. *Evaluate* $\int_0^t A \sin \omega t\,dt$ *where A and ω are constants.*

Solution : $\int_0^t A \sin \omega t\,dt$

$$= A\left[\frac{-\cos \omega t}{\omega}\right]_0^t = \frac{A}{\omega}(1 - \cos \omega t).$$

16. *The velocity v and displacement x of a particle executing simple harmonic motion are related as*

$$v\frac{dv}{dx} = -\omega^2 x.$$

At $x = 0$, $v = v_0$. *Find the velocity v when the displacement becomes x.*

Solution : We have

$$v\frac{dv}{dx} = -\omega^2 x$$

or, $$v\,dv = -\omega^2 x\,dx$$

or, $$\int_{v_0}^{v} v\,dv = \int_0^x -\omega^2 x\,dx \qquad \ldots \text{(i)}$$

When summation is made on $-\omega^2 x\,dx$ the quantity to be varied is x. When summation is made on $v\,dv$ the quantity to be varied is v. As x varies from 0 to x the velocity varies from v_0 to v. Therefore, on the left the limits of integration are from v_0 to v and on the right they are from 0 to x. Simplifying (i),

$$\left[\frac{1}{2}v^2\right]_{v_0}^{v} = -\omega^2\left[\frac{x^2}{2}\right]_0^x$$

or, $$\frac{1}{2}(v^2 - v_0^2) = -\omega^2\frac{x^2}{2}$$

or, $$v^2 = v_0^2 - \omega^2 x^2$$

or, $$v = \sqrt{v_0^2 - \omega^2 x^2}.$$

17. *The charge flown through a circuit in the time interval between t and t + dt is given by* $dq = e^{-t/\tau}dt$ *where τ is a constant. Find the total charge flown through the circuit between t − 0 to t = τ.*

Solution : The total charge flown is the sum of all the dq's for t varying from $t = 0$ to $t = t_0$. Thus, the total charge flown is

$$Q = \int_0^\tau e^{-t/\tau}dt$$

$$= \left[\frac{e^{-t/\tau}}{-1/\tau}\right]_0^\tau = \tau\left(1 - \frac{1}{e}\right).$$

18. *Evaluate* (21·6002 + 234 + 2732·10) × 13.

Solution :

21·6002		22
234	→	234
2732·10		2732
		2988

The three numbers are arranged with their decimal points aligned (shown on the left part above). The column just left to the decimals has 4 as the doubtful digit. Thus, all the numbers are rounded to this column. The rounded numbers are shown on the right part above. The required expression is 2988 × 13 = 38844. As 13 has only two significant digits the product should be rounded off after two significant digits. Thus the result is 39000.

□

QUESTIONS FOR SHORT ANSWER

1. Is a vector necessarily changed if it is rotated through an angle ?
2. Is it possible to add two vectors of unequal magnitudes and get zero ? Is it possible to add three vectors of equal magnitudes and get zero ?
3. Does the phrase "direction of zero vector" have physical significance ? Discuss in terms of velocity, force etc.
4. Can you add three unit vectors to get a unit vector ? Does your answer change if two unit vectors are along the co-ordinate axes ?

5. Can we have physical quantities having magnitude and direction which are not vectors ?

6. Which of the following two statements is more appropriate ?

(a) Two forces are added using triangle rule because force is a vector quantity.

(b) Force is a vector quantity because two forces are added using triangle rule.

7. Can you add two vectors representing physical quantities having different dimensions ? Can you multiply two vectors representing physical quantities having different dimensions ?

8. Can a vector have zero component along a line and still have nonzero magnitude ?

9. Let ε_1 and ε_2 be the angles made by $\vec{A}$ and $-\vec{A}$ with th positive X-axis. Show that $\tan\varepsilon_1 = \tan\varepsilon_2$. Thus, givi $\tan\varepsilon$ does not uniquely determine the direction of $\vec{A}$.

10. Is the vector sum of the unit vectors $\vec{i}$ and $\vec{j}$ a un vector ? If no, can you multiply this sum by a scal number to get a unit vector ?

11. Let $\vec{A} = 3\vec{i} + 4\vec{j}$. Write four vectors $\vec{B}$ such that $\vec{A} \neq \vec{B}$ b $A = B$.

12. Can you have $\vec{A} \times \vec{B} = \vec{A} \cdot \vec{B}$ with $A \neq 0$ and $B \neq 0$? Wh if one of the two vectors is zero ?

13. If $\vec{A} \times \vec{B} = 0$, can you say that (a) $\vec{A} = \vec{B}$, (b) $\vec{A} \neq \vec{B}$?

14. Let $\vec{A} = 5\vec{i} - 4\vec{j}$ and $\vec{B} = -7{\cdot}5\,\vec{i} + 6\vec{j}$. Do we ha $\vec{B} = k\vec{A}$? Can we say $\frac{\vec{B}}{\vec{A}} = k$?

OBJECTIVE I

1. A vector is not changed if
   (a) it is rotated through an arbitrary angle
   (b) it is multiplied by an arbitrary scalar
   (c) it is cross multiplied by a unit vector
   (d) it is slid parallel to itself.

2. Which of the sets given below may represent the magnitudes of three vectors adding to zero ?
   (a) 2, 4, 8 (b) 4, 8, 16 (c) 1, 2, 1 (d) 0·5, 1, 2.

3. The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$,
   (a) $\alpha < \beta$ (b) $\alpha < \beta$ if $A < B$
   (c) $\alpha < \beta$ if $A > B$ (d) $\alpha < \beta$ if $A = B$.

4. The component of a vector is
   (a) always less than its magnitude
   (b) always greater than its magnitude
   (c) always equal to its magnitude
   (d) none of these.

5. A vector $\vec{A}$ points vertically upward and $\vec{B}$ point towards north. The vector product $\vec{A} \times \vec{B}$ is
   (a) along west (b) along east
   (c) zero (d) vertically downward.

6. The radius of a circle is stated as 2·12 cm. Its area shoul be written as
   (a) 14 cm^2 (b) 14·1 cm^2 (c) 14·11 cm^2 (d) 14·1124 cm^2

OBJECTIVE II

1. A situation may be described by using different sets of co-ordinate axes having different orientations. Which of the following do not depend on the orientation of the axes ?
   (a) the value of a scalar (b) component of a vector
   (c) a vector (d) the magnitude of a vector.

2. Let $\vec{C} = \vec{A} + \vec{B}$.
   (a) $|\vec{C}|$ is always greater than $|\vec{A}|$
   (b) It is possible to have $|\vec{C}| < |\vec{A}|$ and $|\vec{C}| < |\vec{B}|$
   (c) C is always equal to $A + B$
   (d) C is never equal to $A + B$.

3. Let the angle between two nonzero vectors $\vec{A}$ and $\vec{B}$ be 120° and its resultant be $\vec{C}$.
   (a) C must be equal to $|A - B|$
   (b) C must be less than $|A - B|$
   (c) C must be greater than $|A - B|$
   (d) C may be equal to $|A - B|$.

4. The x-component of the resultant of several vectors
   (a) is equal to the sum of the x-components of the vector
   (b) may be smaller than the sum of the magnitudes o the vectors
   (c) may be greater than the sum of the magnitudes o the vectors
   (d) may be equal to the sum of the magnitudes of the vectors.

5. The magnitude of the vector product of two vectors $\vec{A}$ and $\vec{B}$ may be
   (a) greater than AB (b) equal to AB
   (c) less than AB (d) equal to zero.

EXERCISES

1. A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the X-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

2. Let $\vec{A}$ and $\vec{B}$ be the two vectors of magnitude 10 unit each. If they are inclined to the X-axis at angles 30° and 60° respectively, find the resultant.

3. Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the X-axis at angles 45°, 135° and 315° respectively.

4. Let $\vec{a} = 4\vec{i} + 3\vec{j}$ and $\vec{b} = 3\vec{i} + 4\vec{j}$. (a) Find the magnitudes of (a) $\vec{a}$, (b) $\vec{b}$, (c) $\vec{a} + \vec{b}$ and (d) $\vec{a} - \vec{b}$.

5. Refer to figure (2-E1). Find (a) the magnitude, (b) x and y components and (c) the angle with the X-axis of the resultant of $\vec{OA}$, $\vec{BC}$ and $\vec{DE}$.

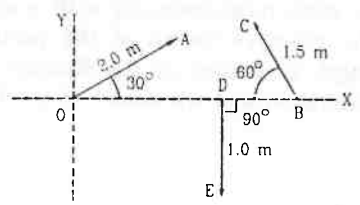

Figure 2-E1

6. Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

7. A spy report about a suspected car reads as follows. "The car moved 2·00 km towards east, made a perpendicular left turn, ran for 500 m, made a perpendicular right turn, ran for 4·00 km and stopped". Find the displacement of the car.

8. A carrom board (4 ft × 4 ft square) has the queen at the centre. The queen, hit by the striker moves to the front edge, rebounds and goes in the hole behind the striking line. Find the magnitude of displacement of the queen (a) from the centre to the front edge, (b) from the front edge to the hole and (c) from the centre to the hole.

9. A mosquito net over a 7 ft × 4 ft bed is 3 ft high. The net has a hole at one corner of the bed through which a mosquito enters the net. It flies and sits at the diagonally opposite upper corner of the net. (a) Find the magnitude of the displacement of the mosquito. (b) Taking the hole as the origin, the length of the bed as the X-axis, its width as the Y-axis, and vertically up as the Z-axis, write the components of the displacement vector.

10. Suppose $\vec{a}$ is a vector of magnitude 4·5 unit due north. What is the vector (a) $3\vec{a}$, (b) $-4\vec{a}$?

11. Two vectors have magnitudes 2 m and 3 m. The angle between them is 60°. Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product.

12. Let $A_1 A_2 A_3 A_4 A_5 A_6 A_1$ be a regular hexagon. Write the x-components of the vectors represented by the six sides taken in order. Use the fact that the resultant of these six vectors is zero, to prove that

$$\cos 0 + \cos\pi/3 + \cos 2\pi/3 + \cos 3\pi/3 + \cos 4\pi/3 + \cos 5\pi/3 = 0.$$

Use the known cosine values to verify the result.

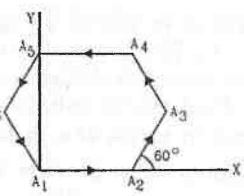

Figure 2-E2

13. Let $\vec{a} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{b} = 3\vec{i} + 4\vec{j} + 5\vec{k}$. Find the angle between them.

14. Prove that $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$.

15. If $\vec{A} = 2\vec{i} + 3\vec{j} + 4\vec{k}$ and $\vec{B} = 4\vec{i} + 3\vec{j} + 2\vec{k}$, find $\vec{A} \times \vec{B}$.

16. If $\vec{A}$, $\vec{B}$, $\vec{C}$ are mutually perpendicular, show that $\vec{C} \times (\vec{A} \times \vec{B}) = 0$. Is the converse true ?

17. A particle moves on a given straight line with a constant speed v. At a certain time it is at a point P on its straight line path. O is a fixed point. Show that $\vec{OP} \times \vec{v}$ is independent of the position P.

18. The force on a charged particle due to electric and magnetic fields is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. Suppose $\vec{E}$ is along the X-axis and $\vec{B}$ along the Y-axis. In what direction and with what minimum speed v should a positively charged particle be sent so that the net force on it is zero ?

19. Give an example for which $\vec{A} \cdot \vec{B} = \vec{C} \cdot \vec{B}$ but $\vec{A} \neq \vec{C}$.

20. Draw a graph from the following data. Draw tangents at x = 2, 4, 6 and 8. Find the slopes of these tangents. Verify that the curve drawn is $y = 2x^2$ and the slope of tangent is $\tan\theta = \frac{dy}{dx} = 4x$.

x	1	2	3	4	5	6	7	8	9	10
y	2	8	18	32	50	72	98	128	162	200

21. A curve is represented by $y = \sin x$. If x is changed from $\frac{\pi}{3}$ to $\frac{\pi}{3} + \frac{\pi}{100}$, find approximately the change in y.

22. The electric current in a charging R–C circuit is given by $i = i_0 e^{-t/RC}$ where i_0, R and C are constant parameters of the circuit and t is time. Find the rate of change of current at (a) $t = 0$, (b) $t = RC$, (c) $t = 10\,RC$.

23. The electric current in a discharging R–C circuit is given by $i = i_0 e^{-t/RC}$ where i_0, R and C are constant parameters and t is time. Let i_0 = 2·00 A, R = 6·00 ×

10^5 Ω and $C = 0·500$ μF. (a) Find the current at t = 0·3 s. (b) Find the rate of change of current at t = 0·3 s. (c) Find approximately the current at t = 0·31 s.

24. Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the X-axis with the ordinates at $x = 5$ and $x = 10$.

25. Find the area enclosed by the curve $y = \sin x$ and the X-axis between $x = 0$ and $x = \pi$.

26. Find the area bounded by the curve $y = e^{-x}$, the X-axis and the Y-axis.

27. A rod of length L is placed along the X-axis between $x = 0$ and $x = L$. The linear density (mass/length) ρ of the rod varies with the distance x from the origin as $\rho = a + bx$. (a) Find the SI units of a and b. (b) Find the mass of the rod in terms of a, b and L.

28. The momentum p of a particle changes with time t according to the relation $\frac{dp}{dt} = (10 \text{ N}) + (2 \text{ N/s})t$. If the momentum is zero at $t = 0$, what will the momentum be at t = 10 s?

29. The changes in a function y and the independent variable x are related as $\frac{dy}{dx} = x^2$. Find y as a function of x.

30. Write the number of significant digits in (a) 100 (b) 100·1, (c) 100·10, (d) 0·001001.

31. A metre scale is graduated at every millimetre. Ho many significant digits will be there in a lengt measurement with this scale?

32. Round the following numbers to 2 significant digit (a) 3472, (b) 84·16, (c) 2·55 and (d) 28·5.

33. The length and the radius of a cylinder measured wi a slide callipers are found to be 4·54 cm and 1·75 c respectively. Calculate the volume of the cylinder.

34. The thickness of a glass plate is measured to b 2·17 mm, 2·17 mm and 2·18 mm at three differe places. Find the average thickness of the plate from th data.

35. The length of the string of a simple pendulum i measured with a metre scale to be 90·0 cm. The radiu of the bob plus the length of the hook is calculated t be 2·13 cm using measurements with a slide calliper What is the effective length of the pendulum? (Th effective length is defined as the distance between th point of suspension and the centre of the bob.)

□

ANSWER

OBJECTIVE I

1. (d) 2. (c) 3. (c) 4. (d) 5. (a) 6. (b)

OBJECTIVE II

1. (a), (c), (d) 2. (b) 3. (c) 4. (a), (b), (d)
5. (b), (c), (d)

EXERCISES

1. 5 m at 73° with X-axis
2. 20 cos15° unit at 45° with X-axis
3. 100 unit at 45° with X-axis
4. (a) 5 (b) 5 (c) $7\sqrt{2}$ (d) $\sqrt{2}$
5. (a) 1·6 m (b) 0·98 m and 1·3 m respectively (c) $\tan^{-1}(1·32)$
6. (a) 180° (b) 90° (c) 0
7. 6·02 km, $\tan^{-1}\frac{1}{12}$.
8. (a) $\frac{2}{3}\sqrt{10}$ ft (b) $\frac{4}{3}\sqrt{10}$ ft (c) $2\sqrt{2}$ ft
9. (a) $\sqrt{74}$ ft (b) 7 ft, 4 ft, 3 ft
10. (a) 13·5 unit due north (b) 18 unit due south
11. (a) 3 m^2 (b) $3\sqrt{3}$ m^2
13. $\cos^{-1}(38/\sqrt{1450})$
15. $-6\vec{i} + 12\vec{j} - 6\vec{k}$
16. no
18. along Z-axis with speed E/B
21. 0·0157
22. (a) $\frac{-i_0}{RC}$ (b) $\frac{-i_0}{RCe}$ (c) $\frac{-i_0}{RCe^{10}}$
23. (a) $\frac{2·00}{e}$ A (b) $\frac{20}{3e}$ A/s (c) $\frac{5·8}{3e}$ A
24. 1135
25. 2
26. 1
27. (a) kg/m, kg/m^2 (b) $aL + bL^2/2$
28. 200 kg–m/s
29. $y = \frac{x^3}{3} + C$
30. (a) 4 (b) 4 (c) 5 (d) 4
31. 1, 2, 3 or 4
32. (a) 3500 (b) 84 (c) 2·6 (d) 28
33. 43·7 cm^3
34. 2·17 mm
35. 92·1 cm

□