or, $\frac{(t - 5080\text{ s})}{5080\text{ s}} = -3.48 \times 10^{-10}$

or, $(t - 5080\text{ s}) = -1.77 \times 10^{-6}\text{ s}.$

The satellite's clock falls behind by 1.77×10^{-6} s in one revolution.

5. *The radius of our galaxy is about* 3×10^{20} m. *With what speed should a person travel so that he can reach from the centre of the galaxy to its edge in 20 years of his lifetime ?*

Solution : Let the speed of the person be v. As seen by the person, the edge of the galaxy is coming towards him at a speed v. In 20 years (as measured by the person), the edge moves (20 y) v and reaches the person. The radius of the galaxy as measured by the person is, therefore, (20 y)v. The rest length of the radius of the galaxy is 3×10^{20} m. Thus,

$$(20\text{ y})v = (3 \times 10^{20}\text{ m})\sqrt{1 - v^2/c^2}$$

or, $(6.312 \times 10^8\text{ s})^2 v^2 = (9 \times 10^{40}\text{ m}^2)(1 - v^2/c^2).$

Solving this,

$$v = 0.9999996\,c.$$

6. *Find the speed at which the mass of an electron is double of its rest mass.*

Solution : The mass of an electron at speed v is

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

where m_0 is its rest mass. If $m = 2\,m_0$,

$$2 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

or, $1 - \frac{v^2}{c^2} = \frac{1}{4}$

or, $v = \frac{\sqrt{3}}{2}c = 2.598 \times 10^8$ m/s.

7. *Calculate the increase in mass when a body of rest mass* 1 kg *is lifted up through* 1 m *near the earth's surface.*

Solution : The increase in energy = mgh

$= (1\text{ kg})(9.8\text{ m/s}^2)(1\text{ m}) = 9.8\text{ J}.$

The increase in mass $= \frac{9.8\text{ J}}{c^2}$

$= 1.11 \times 10^{-16}$ kg.

8. *A body of rest mass* m_0 *collides perfectly inelastically at a speed of* 0.8*c with another body of equal rest mass kept at rest. Calculate the common speed of the bodies after the collision and the rest mass of the combined body.*

Solution : The linear momentum of the first body

$$= \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \frac{m_0 \times 0.8c}{0.6}$$

$$= \frac{4}{3}m_0 c.$$

This should be the total linear momentum after the collision. If the rest mass of the combined body is M_0 and it moves at speed v',

$$\frac{M_0 v'}{\sqrt{1 - v'^2/c^2}} = \frac{4}{3}m_0 c. \qquad \text{... (i)}$$

The energy before the collison is

$$\frac{m_0}{\sqrt{1 - v^2/c^2}}c^2 + m_0 c^2 = m_0 c^2\left(\frac{1}{0.6} + 1\right)$$

$$= \frac{8}{3}m_0 c^2.$$

The energy after the collision is

$$\frac{M_0 c^2}{\sqrt{1 - v'^2/c^2}}.$$

Thus, $$\frac{M_0 c^2}{\sqrt{1 - v'^2/c^2}} = \frac{8}{3}m_0 c^2. \qquad \text{... (ii)}$$

Dividing (i) by (ii),

$$\frac{v'}{c^2} = \frac{1}{2c} \quad \text{or,} \quad v' = \frac{c}{2}.$$

Putting this value of v' in (ii),

$$M_0 = \frac{8}{3}m_0\sqrt{1 - \frac{1}{4}}$$

or, $M_0 = 2.309\,m_0.$

The rest mass of the combined body is greater than the sum of the rest masses of the individual bodies.

□

QUESTIONS FOR SHORT ANSWER

1. The speed of light in glass is 2.0×10^8 m/s. Does it violate the second postulate of special relativity ?
2. A uniformly moving train passes by a long platform. Consider the events 'engine crossing the beginning of the platform' and 'engine crossing the end of the platform'. Which frame (train frame or the platform frame) is the proper frame for the pair of events ?

3. An object may be regarded to be at rest or in motion depending on the frame of reference chosen to view the object. Because of length contraction it would mean that the same rod may have two different lengths depending on the state of the observer. Is this true ?

4. Mass of a particle depends on its speed. Does the attraction of the earth on the particle also depend on the particle's speed ?

5. A person travelling in a fast spaceship measures the distance between the earth and the moon. Is it the same, smaller or larger than the value quoted in this book ?

OBJECTIVE I

1. The magnitude of linear momentum of a particle moving at a relativistic speed v is proportional to
   (a) v (b) $1 - v^2/c^2$
   (c) $\sqrt{1 - v^2/c^2}$ (d) none of these.

2. As the speed of a particle increases, its rest mass
   (a) increases (b) decreases
   (c) remains the same (d) changes.

3. An experimenter measures the length of a rod. Initially the experimenter and the rod are at rest with respect to the lab. Consider the following statements.
   (A) If the rod starts moving parallel to its length but the observer stays at rest, the measured length will be reduced.
   (B) If the rod stays at rest but the observer starts moving parallel to the measured length of the rod, the length will be reduced.
   (a) A is true but B is false. (b) B is true but A is false.
   (c) Both A and B are true. (d) Both A and B are false.

4. An experimenter measures the length of a rod. In the cases listed, all motions are with respect to the lab and parallel to the length of the rod. In which of the cases the measured length will be minimum ?
   (a) The rod and the experimenter move with the same speed v in the same direction.
   (b) The rod and the experimenter move with the same speed v in opposite directions.
   (c) The rod moves at speed v but the experimenter stays at rest.
   (d) The rod stays at rest but the experimenter moves with the speed v.

5. If the speed of a particle moving at a relativistic speed is doubled, its linear momentum will
   (a) become double (b) become more than double
   (c) remain equal (d) become less than double.

6. If a constant force acts on a particle, its acceleration will
   (a) remain constant (b) gradually decrease
   (c) gradually increase (d) be undefined.

7. A charged particle is projected at a very high speed perpendicular to a uniform magnetic field. The particle will
   (a) move along a circle
   (b) move along a curve with increasing radius of curvature
   (c) move along a curve with decreasing radius of curvature
   (d) move along a straight line.

OBJECTIVE II

1. Mark the correct statements:
   (a) Equations of special relativity are not applicable for small speeds.
   (b) Equations of special relativity are applicable for all speeds.
   (c) Nonrelativistic equations give exact result for small speeds.
   (d) Nonrelativistic equations never give exact result.

2. If the speed of a rod moving at a relativistic speed parallel to its length is doubled,
   (a) the length will become half of the original value
   (b) the mass will become double of the original value
   (c) the length will decrease
   (d) the mass will increase.

3. Two events take place simultaneously at points A and B as seen in the lab frame. They also occur simultaneously in a frame moving with respect to the lab in a direction
   (a) parallel to AB (b) perpendicular to AB
   (c) making an angle of 45° with AB
   (d) making an angle of 135° with AB.

4. Which of the following quantities related to an electron has a finite upper limit ?
   (a) mass (b) momentum (c) speed (d) kinetic energy.

5. A rod of rest length L moves at a relativistic speed. Let $L' = L/\gamma$. Its length
   (a) must be equal to L' (b) may be equal to L
   (c) may be more than L' but less than L
   (d) may be more than L.

6. When a rod moves at a relativistic speed v, its mass
   (a) must incrase by a factor of γ
   (b) may remain unchanged
   (c) may increase by a factor other than γ
   (d) may decrease.

EXERCISES

1. The *guru* of a *yogi* lives in a Himalyan cave, 1000 km away from the house of the yogi. The yogi claims that whenever he thinks about his guru, the guru immediately knows about it. Calculate the minimum possible time interval between the yogi thinking about the guru and the guru knowing about it.
2. A suitcase kept on a shop's rack is measured 50 cm × 25 cm × 10 cm by the shop's owner. A traveller takes this suitcase in a train moving with velocity $0.6c$. If the suitcase is placed with its length along the train's velocity, find the dimensions measured by (a) the traveller and (b) a ground observer.
3. The length of a rod is exactly 1 m when measured at rest. What will be its length when it moves at a speed of (a) 3×10^5 m/s, (b) 3×10^6 m/s and (c) 3×10^7 m/s ?
4. A person standing on a platform finds that a train moving with velocity $0.6c$ takes one second to pass by him. Find (a) the length of the train as seen by the person and (b) the rest length of the train.
5. An aeroplane travels over a rectangular field 100 m × 50 m, parallel to its length. What should be the speed of the plane so that the field becomes square in the plane frame?
6. The rest distance between Patna and Delhi is 1000 km. A nonstop train travels at 360 km/h. (a) What is the distance between Patna and Delhi in the train frame? (b) How much time elapses in the train frame between Patna and Delhi?
7. A person travels by a car at a speed of 180 km/h. It takes exactly 10 hours by his wristwatch to go from the station *A* to the station *B*. (a) What is the rest distance between the two stations? (b) How much time is taken in the road frame by the car to go from the station *A* to the station *B*?
8. A person travels on a spaceship moving at a speed of $5c/13$. (a) Find the time interval calculated by him between the consecutive birthday celebrations of his friend on the earth. (b) Find the time interval calculated by the friend on the earth between the consecutive birthday celebrations of the traveller.
9. According to the station clocks, two babies are born at the same instant, one in Howrah and other in Delhi. (a) Who is elder in the frame of 2301 Up Rajdhani Express going from Howrah to Delhi? (b) Who is elder in the frame of 2302 Dn Rajdhani Express going from Delhi to Howrah.
10. Two babies are born in a moving train, one in the compartment adjacent to the engine and other in the compartment adjacent to the guard. According to the train frame, the babies are born at the same instant of time. Who is elder according to the ground frame?
11. Suppose Swarglok (heaven) is in constant motion at a speed of $0.9999c$ with respect to the earth. According to the earth's frame, how much time passes on the earth before one day passes on Swarglok?
12. If a person lives on the average 100 years in his rest frame, how long does he live in the earth frame if he spends all his life on a spaceship going at 60% of the speed of light.
13. An electric bulb, connected to a make and break power supply, switches off and on every second in its rest frame. What is the frequency of its switching off and on as seen from a spaceship travelling at a speed $0.8c$?
14. A person travelling by a car moving at 100 km/h finds that his wristwatch agrees with the clock on a tower *A*. By what amount will his wristwatch lag or lead the clock on another tower *B*, 1000 km (in the earth's frame) from the tower *A* when the car reaches there?
15. At what speed the volume of an object shrinks to half its rest value?
16. A particular particle created in a nuclear reactor leaves a 1 cm track before decaying. Assuming that the particle moved at $0.995c$, calculate the life of the particle (a) in the lab frame and (b) in the frame of the particle.
17. By what fraction does the mass of a spring change when it is compressed by 1 cm? The mass of the spring is 200 g at its natural length and the spring constant is 500 N/m.
18. Find the increase in mass when 1 kg of water is heated from 0°C to 100°C. Specific heat capacity of water = 4200 J/kg–K.
19. Find the loss in the mass of 1 mole of an ideal monatomic gas kept in a rigid container as it cools down by 10°C. The gas constant $R = 8.3$ J/mol–K.
20. By what fraction does the mass of a boy increase when he starts running at a speed of 12 km/h?
21. A 100 W bulb together with its power supply is suspended from a sensitive balance. Find the change in the mass recorded after the bulb remains on for 1 year.
22. The energy from the sun reaches just outside the earth's atmosphere at a rate of 1400 W/m^2. The distance between the sun and the earth is 1.5×10^{11} m. (a) Calculate the rate at which the sun is losing its mass. (b) How long will the sun last assuming a constant decay at this rate? The present mass of the sun is 2×10^{30} kg.
23. An electron and a positron moving at small speeds collide and annihilate each other. Find the energy of the resulting gamma photon.
24. Find the mass, the kinetic energy and the momentum of an electron moving at $0.8c$.
25. Through what potential difference should an electron be accelerated to give it a speed of (a) $0.6c$, (b) $0.9c$ and (c) $0.99c$?
26. Find the speed of an electron with kinetic energy (a) 1 eV, (b) 10 keV and (c) 10 MeV.
27. What is the kinetic energy of an electron in electronvolts with mass equal to double its rest mass?
28. Find the speed at which the kinetic energy of a particle will differ by 1% from its nonrelativistic value $\frac{1}{2} m_0 v^2$.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (c) 3. (c) 4. (b) 5. (b) 6. (b)
7. (b)

OBJECTIVE II

1. (b), (d) 2. (c), (d) 3. (b)
4. (c) 5. (b), (c) 6. (a)

EXERCISES

1. 1/300 s
2. (a) 50 cm × 25 cm × 10 cm (b) 40 cm × 25 cm × 10 cm
3. (a) 0·9999995 m (b) 0·99995 m (c) 0·995 m
4. (a) $1·8 \times 10^8$ m (b) $2·25 \times 10^8$ m
5. 0·866c
6. (a) 56 nm less than 1000 km
   (b) 0·56 ns less than $\frac{500}{3}$ min
7. (a) 25 nm more than 1800 km
   (b) 0·5 ns more than 10 hours
8. $\frac{13}{12}$ y in both cases
9. (a) Delhi baby is elder (b) Howrah baby is elder
10. the baby adjacent to the guard is elder
11. 70·7 days
12. 125 y
13. 0·6 s
14. will lag by 0·154 ns
15. $\frac{\sqrt{3}c}{2}$
16. (a) 33·5 ps (b) 3·35 ps
17. $1·4 \times 10^{-18}$
18. $4·7 \times 10^{-12}$ kg
19. $1·38 \times 10^{15}$ kg
20. $6·17 \times 10^{-17}$
21. $3·5 \times 10^{-8}$ kg
22. (a) $4·4 \times 10^9$ kg/s (b) $1·44 \times 10^{13}$ y
23. 1·02 MeV
24. $15·2 \times 10^{-31}$ kg, $5·5 \times 10^{-14}$ J, $3·65 \times 10^{-22}$ kg-m/s
25. (a) 128 kV (b) 661 kV (c) 3·1 MV
26. (a) $5·92 \times 10^5$ m/s (b) $5·85 \times 10^7$ m/s (c) $2·996 \times 10^8$ m/s
27. 511 keV
28. $3·46 \times 10^7$ m/s

□

THE SPECIAL THEORY OF RELATIVITY
CHAPTER - 47

1. $S = 1000\text{ km} = 10^6\text{ m}$

   The process requires minimum possible time if the velocity is maximum.

   We know that maximum velocity can be that of light i.e. $= 3 \times 10^8$ m/s.

   So, time $= \dfrac{\text{Distance}}{\text{Speed}} = \dfrac{10^6}{3 \times 10^8} = \dfrac{1}{300}$ s.

2. $\ell = 50$ cm, b = 25 cm, h = 10 cm, v = 0.6 c

   a) The observer in the train notices the same value of ℓ, b, h because relativity are in due to difference in frames.

   b) In 2 different frames, the component of length parallel to the velocity undergoes contraction but the perpendicular components remain the same. So length which is parallel to the x-axis changes and breadth and height remain the same.

   $e' = e\sqrt{1 - \dfrac{V^2}{C^2}} = 50\sqrt{1 - \dfrac{(0.6)^2 C^2}{C^2}}$

   $= 50\sqrt{1 - 0.36} = 50 \times 0.8 = 40$ cm.

   The lengths observed are 40 cm × 25 cm × 10 cm.

3. L = 1 m

   a) v 3×10^5 m/s

   $L' = 1\sqrt{1 - \dfrac{9 \times 10^{10}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-6}} = 0.9999995$ m

   b) v = 3 x 10^6 m/s

   $L' = 1\sqrt{1 - \dfrac{9 \times 10^{12}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-4}} = 0.99995$ m.

   c) v = 3×10^7 m/s

   $L' = 1\sqrt{1 - \dfrac{9 \times 10^{14}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-2}} = 0.9949 = 0.995$ m.

4. v = 0.6 cm/sec ; t = 1 sec

   a) length observed by the observer = vt $\Rightarrow 0.6 \times 3 \times 10^6 \Rightarrow 1.8 \times 10^8$ m

   b) $\ell = \ell_0\sqrt{1 - v^2/c^2} \Rightarrow 1.8 \times 10^8 = \ell_0\sqrt{1 - \dfrac{(0.6)^2 C^2}{C^2}}$

   $\ell_0 = \dfrac{1.8 \times 10^8}{0.8} = 2.25 \times 10^8$ m/s.

5. The rectangular field appears to be a square when the length becomes equal to the breadth i.e. 50 m.

   i.e. L′ = 50 ; L = 100 ; v = ?

   C = 3×10^8 m/s

   We know, $L' = L\sqrt{1 - v^2/c^2}$

   $\Rightarrow 50 = 100\sqrt{1 - v^2/c^2} \Rightarrow v = \sqrt{3/2}C = 0.866$ C.

6. $L_0 = 1000\text{ km} = 10^6$ m

   v = 360 km/h = (360 × 5) / 18 = 100 m/sec.

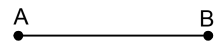
   a) $h' = h_0\sqrt{1 - v^2/c^2} = 10^6\sqrt{1 - \left(\dfrac{100}{3 \times 10^8}\right)^2} = 10^6\sqrt{1 - \dfrac{10^4}{9 \times 10^6}} = 10^9$.

   Solving change in length = 56 nm.

   b) $\Delta t = \Delta L/v$ = 56 nm / 100 m = 0.56 ns.

7. v = 180 km/hr = 50 m/s

t = 10 hours



let the rest dist. be L.

$L' = L\sqrt{1-v^2/c^2} \Rightarrow L' = 10 \times 180 = 1800$ k.m.

$$1800 = L\sqrt{1-\frac{180^2}{(3\times10^5)^2}}$$

or, $1800 \times 1800 = L(1 - 36 \times 10^{-14})$

or, $L = \dfrac{3.24\times10^6}{1-36\times10^{-14}} = 1800 + 25 \times 10^{-12}$

or 25 nm more than 1800 km.

b) Time taken in road frame by Car to cover the dist $= \dfrac{1.8\times10^6 + 25\times10^{-9}}{50}$

$= 0.36 \times 10^5 + 5 \times 10^{-8} = 10$ hours + 0.5 ns.

8. a) u = 5c/13

$$\Delta t = \frac{t}{\sqrt{1-v^2/c^2}} = \frac{1y}{\sqrt{1-\frac{25c^2}{169c^2}}} = \frac{y\times13}{12} = \frac{13}{12}y\ .$$

The time interval between the consecutive birthday celebration is 13/12 y.

b) The fried on the earth also calculates the same speed.

9. The birth timings recorded by the station clocks is proper time interval because it is the ground frame. That of the train is improper as it records the time at two different places. The proper time interval ΔT is less than improper.

i.e. $\Delta T' = v\,\Delta T$

Hence for – (a) up train → Delhi baby is elder (b) down train → Howrah baby is elder.

10. The clocks of a moving frame are out of synchronization. The clock at the rear end leads the one at from by $L_0\ V/C^2$ where L_0 is the rest separation between the clocks, and v is speed of the moving frame.

Thus, the baby adjacent to the guard cell is elder.

11. v = 0.9999 C ; Δt = One day in earth ; $\Delta t'$ = One day in heaven

$$v = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\frac{(0.9999)^2C^2}{C^2}}} = \frac{1}{0.014141782} = 70.712$$

$\Delta t' = v\,\Delta t$;

Hence, $\Delta t'$ = 70.7 days in heaven.

12. t = 100 years ; V = 60/100 K ; C = 0.6 C.

$$\Delta t = \frac{t}{\sqrt{1-V^2/C^2}} = \frac{100y}{\sqrt{1-\frac{(0.6)^2C^2}{C^2}}} = \frac{100y}{0.8} = 125\ y.$$

13. We know

$f' = f\sqrt{1-V^2/C^2}$

f' = apparent frequency ;

f = frequency in rest frame

v = 0.8 C

$$f' = \sqrt{1-\frac{0.64C^2}{C^2}} = \sqrt{0.36} = 0.6\ s^{-1}$$

14. V = 100 km/h, Δt = Proper time interval = 10 hours

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - V^2/C^2}} = \frac{10 \times 3600}{\sqrt{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2}}$$

$$\Delta t' - \Delta t = 10 \times 3600 \left[\frac{1}{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2} - 1\right]$$

By solving we get, $\Delta t' - \Delta t = 0.154$ ns.

$\therefore$ Time will lag by 0.154 ns.

15. Let the volume (initial) be V.

$V' = V/2$

So, $V/2 = v\sqrt{1 - V^2/C^2}$

$\Rightarrow C/2 = \sqrt{C^2 - V^2} \Rightarrow C^2/4 = C^2 - V^2$

$\Rightarrow V^2 = C^2 - \frac{C^2}{4} = \frac{3}{4}C^2 \Rightarrow V = \frac{\sqrt{3}}{2}C$.

16. d = 1 cm, v = 0.995 C

a) time in Laboratory frame = $\frac{d}{v} = \frac{1 \times 10^{-2}}{0.995C}$

$= \frac{1 \times 10^{-2}}{0.995 \times 3 \times 10^8} = 33.5 \times 10^{-12} = 33.5$ PS

b) In the frame of the particle

$t' = \frac{t}{\sqrt{1 - V^2/C^2}} = \frac{33.5 \times 10^{-12}}{\sqrt{1 - (0.995)^2}} = 335.41$ PS.

17. x = 1 cm = 1×10^{-2} m ; K = 500 N/m, m = 200 g

Energy stored = ½ Kx^2 = ½ $\times 500 \times 10^{-4} = 0.025$ J

Increase in mass = $\frac{0.025}{C^2} = \frac{0.025}{9 \times 10^{16}}$

Fractional Change of max = $\frac{0.025}{9 \times 10^{16}} \times \frac{1}{2 \times 10^{-1}} = 0.01388 \times 10^{-16} = 1.4 \times 10^{-8}$.

18. Q = MS $\Delta\theta \Rightarrow 1 \times 4200 (100 - 0) = 420000$ J.

$E = (\Delta m)C^2$

$\Rightarrow \Delta m = \frac{E}{C^2} = \frac{Q}{C^2} = \frac{420000}{(3 \times 10^8)^2}$

$= 4.66 \times 10^{-12} = 4.7 \times 10^{-12}$ kg.

19. Energy possessed by a monoatomic gas = 3/2 nRdt.

Now dT = 10, n = 1 mole, R = 8.3 J/mol-K.

E = 3/2 $\times$ t $\times$ 8.3 $\times$ 10

Loss in mass = $\frac{1.5 \times 8.3 \times 10}{C^2} = \frac{124.5}{9 \times 10^{15}}$

$= 1383 \times 10^{-16} = 1.38 \times 10^{-15}$ Kg.

20. Let initial mass be m

½ mv^2 = E

$\Rightarrow E = \frac{1}{2}m\left(\frac{12 \times 5}{18}\right)^2 = \frac{m \times 50}{9}$

$\Delta m = E/C^2$

$\Rightarrow$ $\Delta m = \dfrac{m \times 50}{9 \times 9 \times 10^{16}}$ $\Rightarrow$ $\dfrac{\Delta m}{m} = \dfrac{50}{81 \times 10^{16}}$

$\Rightarrow$ $0.617 \times 10^{-16} = 6.17 \times 10^{-17}$.

21. Given : Bulb is 100 Watt = 100 J/s

So, 100 J in expended per 1 sec.

Hence total energy expended in 1 year = $100 \times 3600 \times 24 \times 365 = 3153600000$ J

Change in mass recorded = $\dfrac{\text{Total energy}}{C^2} = \dfrac{315360000}{9 \times 10^{16}}$

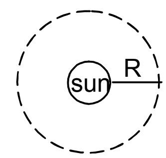
$= 3.504 \times 10^8 \times 10^{-16}$ kg $= 3.5 \times 10^{-8}$ Kg.

22. $I = 1400$ w/m^2

Power = 1400 w/m$^2 \times A$

$= (1400 \times 4\pi R^2)$w $= 1400 \times 4\pi \times (1.5 \times 10^{11})^2$

$= 1400 \times 4\pi \times (1/5)^2 \times 10^{22}$



a) $\dfrac{E}{t} = \dfrac{\Delta m C^2}{t} = \dfrac{\Delta m}{t} = \dfrac{E/t}{C^2}$

$C^2 = \dfrac{1400 \times 4\pi \times 2.25 \times 10^{22}}{9 \times 10^{16}} = 1696 \times 10^{66} = 4.396 \times 10^9 = 4.4 \times 10^9$.

b) 4.4×10^9 Kg disintegrates in 1 sec.

2×10^{30} Kg disintegrates in $\dfrac{2 \times 10^{30}}{4.4 \times 10^9}$ sec.

$= \left(\dfrac{1 \times 10^{21}}{2.2 \times 365 \times 24 \times 3600}\right) = 1.44 \times 10^{-8} \times 10^{21}$ y $= 1.44 \times 10^{13}$ y.

23. Mass of Electron = Mass of positron = 9.1×10^{-31} Kg

Both are oppositely charged and they annihilate each other.

Hence, $\Delta m = m + m = 2 \times 9.1 \times 10^{-31}$ Kg

Energy of the resulting γ particle = $\Delta m\, C^2$

$= 2 \times 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J $= \dfrac{2 \times 9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ ev

$= 102.37 \times 10^4$ ev $= 1.02 \times 10^6$ ev $= 1.02$ Mev.

24. $m_e = 9.1 \times 10^{-31}$, $v_0 = 0.8$ C

a) $m' = \dfrac{Me}{\sqrt{1 - V^2/C^2}} = \dfrac{9.1 \times 10^{-31}}{\sqrt{1 - 0.64C^2/C^2}} = \dfrac{9.1 \times 10^{-31}}{0.6}$

$= 15.16 \times 10^{-31}$ Kg $= 15.2 \times 10^{-31}$ Kg.

b) K.E. of the electron : $m'C^2 - m_eC^2 = (m' - m_e)\, C^2$

$= (15.2 \times 10^{-31} - 9.1 \times 10^{-31})(3 \times 10^8)^2 = (15.2 \times 9.1) \times 9 \times 10^{-31} \times 10^{18}$ J

$= 54.6 \times 10^{-15}$ J $= 5.46 \times 10^{-14}$ J $= 5.5 \times 10^{-14}$ J.

c) Momentum of the given electron = Apparent mass $\times$ given velocity

$= 15.2 \times 10^{-31} - 0.8 \times 3 \times 10^8$ m/s $= 36.48 \times 10^{-23}$ kg m/s

$= 3.65 \times 10^{-22}$ kg m/s

25. a) $ev - m_0C^2 = \dfrac{m_0 C^2}{2\sqrt{1 - \dfrac{V^2}{C^2}}}$ $\Rightarrow$ $ev - 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$= \dfrac{9.1 \times 9 \times 10^{-31} \times 10^{16}}{2\sqrt{1 - \dfrac{0.36C^2}{C^2}}}$ $\Rightarrow$ $eV - 9.1 \times 9 \times 10^{-15}$

$= \dfrac{9.1 \times 9 \times 10^{-15}}{2 \times 0.8} \Rightarrow eV - 9.1 \times 9 \times 10^{-15} = \dfrac{9.1 \times 9 \times 10^{-15}}{1.6}$

$\Rightarrow eV = \left(\dfrac{9.1 \times 9}{1.6} + 9.1 \times 9\right) \times 10^{-15} = eV\left(\dfrac{81.9}{1.6} + 81.9\right) \times 10^{-15}$

$eV = 133.0875 \times 10^{-15} \Rightarrow V = 83.179 \times 10^{4} = 831$ KV.

b) $eV - m_0C^2 = \dfrac{m_0C^2}{2\sqrt{1 - \dfrac{V^2}{C^2}}} \Rightarrow eV - 9.1 \times 9 \times 10^{-19} \times 9 \times 10^{16} = \dfrac{9.1 \times 9 \times 10^{-15}}{2\sqrt{1 - \dfrac{0.81C^2}{C^2}}}$

$\Rightarrow eV - 81.9 \times 10^{-15} = \dfrac{9.1 \times 9 \times 10^{-15}}{2 \times 0.435}$

$\Rightarrow eV = 12.237 \times 10^{-15}$

$\Rightarrow V = \dfrac{12.237 \times 10^{-15}}{1.6 \times 10^{-19}} = 76.48$ kV.

$V = 0.99\ C = eV - m_0C^2 = \dfrac{m_0C^2}{2\sqrt{1 - \dfrac{V^2}{C^2}}}$

$\Rightarrow eV = \dfrac{m_0C^2}{2\sqrt{1 - \dfrac{V^2}{C^2}}} + m_0C^2 = \dfrac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{2\sqrt{1 - (0.99)^2}} + 9.1 \times 10^{-31} \times 9 \times 10^{16}$

$\Rightarrow eV = 372.18 \times 10^{-15} \Rightarrow V = \dfrac{372.18 \times 20^{-15}}{1.6 \times 10^{-19}} = 272.6 \times 10^{4}$

$\Rightarrow V = 2.726 \times 10^{6} = 2.7$ MeV.

26. a) $\dfrac{m_0C^2}{\sqrt{1 - \dfrac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-19}$

$\Rightarrow m_0C^2\left(\dfrac{1}{\sqrt{1 - V^2/C^2}} - 1\right) = 1.6 \times 10^{-19}$

$\Rightarrow \dfrac{1}{\sqrt{1 - V^2/C^2}} - 1 = \dfrac{1.6 \times 10^{-19}}{9.1 \times 10^{-31} \times 9 \times 10^{16}}$

$\Rightarrow V = C \times 0.001937231 = 3 \times 0.001967231 \times 0^{8} = 5.92 \times 10^{5}$ m/s.

b) $\dfrac{m_0C^2}{\sqrt{1 - \dfrac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-19} \times 10 \times 10^{3}$

$\Rightarrow m_0C^2\left(\dfrac{1}{\sqrt{1 - V^2/C^2}} - 1\right) = 1.6 \times 10^{-15}$

$\Rightarrow \dfrac{1}{\sqrt{1 - V^2/C^2}} - 1 = \dfrac{1.6 \times 10^{-15}}{9.1 \times 9 \times 10^{15}}$

$\Rightarrow V = 0.584475285 \times 10^{8} = 5.85 \times 10^{7}$ m/s.

c) K.E. = 10 Mev = 10×10^{6} eV = $10^{7} \times 1.6 \times 10^{-19}$ J = 1.6×10^{-12} J

$\Rightarrow \dfrac{m_0C^2}{\sqrt{1 - \dfrac{V^2}{C^2}}} - m_0C^2 = 1.6 \times 10^{-12}$ J

$\Rightarrow V^2 = 8..999991359 \times 10^{16} \Rightarrow V = 2.999987038 \times 10^{8}$.

27. $\Delta m = m - m_0 = 2m_0 - m_0 = m_0$

Energy $E = m_0c^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16}$ J

E in e.v. = $\dfrac{9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}}$ = 51.18×10^4 ev = 511 Kev.

28. $$\frac{\left(\dfrac{m_0C^2}{\sqrt{1-\dfrac{V^2}{C^2}}} - m_0C^2\right) - \dfrac{1}{2}mv^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \left[\frac{m_0C^2(1 + \dfrac{v^2}{2C^2} + \dfrac{1}{2}\times\dfrac{3}{4}\dfrac{V^2}{C^2} + \dfrac{1}{2}\times\dfrac{3}{4}\times\dfrac{5}{6}\dfrac{V^6}{C^6}) - m_0C^2}{\dfrac{1}{2}m_0v^2}\right] - \frac{1}{2}mv^2 = 0.1$$

$$\Rightarrow \frac{\dfrac{1}{2}m_0v^2 + \dfrac{3}{8}m_0\dfrac{V^4}{C^2} + \dfrac{15}{96}m_0\dfrac{V^4}{C^2} - \dfrac{1}{2}m_0v^2}{\dfrac{1}{2}m_0v^2} = 0.01$$

$$\Rightarrow \frac{3}{4}\frac{V^4}{C^2} + \frac{15}{96 \times 2}\frac{V^4}{C^4} = 0.01$$

Neglecting the v^4 term as it is very small

$$\Rightarrow \frac{3}{4}\frac{V^2}{C^2} = 0.01 \Rightarrow \frac{V^2}{C^2} = 0.04 / 3$$

$$\Rightarrow V/C = 0.2/\sqrt{3} = V = 0.2/\sqrt{3}\,C = \frac{0.2}{1.732} \times 3 \times 10^8$$

$= 0.346 \times 10^8$ m/s $= 3.46 \times 10^7$ m/s.

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