CHAPTER 46

THE NUCLEUS

At the centre of an atom exists the *nucleus* which contains protons and neutrons. The electrons surround this nucleus to form the atom. As discussed earlier, this structure of atom was revealed by the experiments of Rutherford in which a beam of alpha particles was made to strike a thin gold foil. Most of the alpha particles crossed the foil without being appreciably deviated, but there were some alpha particles which suffered large deviation from their original lines of motion. The data suggested that positive charges in an atom are concentrated in a small volume which we call nucleus and this nucleus is responsible for the large deviation of alpha particles. Later on, the existence of protons and neutrons in the nucleus was established. In this chapter, we shall discuss the physics of the nucleus.

46.1 PROPERTIES OF A NUCLEUS

Nuclear Constituents

A nucleus is made of protons and neutrons. A proton has a positive charge of magnitude equal to that of an electron and has a mass of about 1840 times the mass of an electron. A neutron has a mass slightly greater than that of a proton. The masses of a proton and a neutron are

$$m_p = 1{\cdot}6726231 \times 10^{-27}\ \text{kg}$$

and $$m_n = 1{\cdot}6749286 \times 10^{-27}\ \text{kg}.$$

It is customary in nuclear physics and high energy physics to represent mass in energy units according to the conversion formula $E = mc^2$. (Matter can be viewed as a condensed form of energy. Theory of relativity reveals that a mass m is equivalent to an energy E where $E = mc^2$.) For example, the mass of an electron is $m_e = 9{\cdot}1093897 \times 10^{-31}$ kg and the equivalent energy is

$$m_e c^2 = 510{\cdot}99\ \text{keV}.$$

Thus, the mass of an electron is $510{\cdot}99\ \text{keV/c}^2$. Similarly, the mass of a proton is $938{\cdot}27231\ \text{MeV/c}^2$ and the mass of a neutron is $939{\cdot}56563\ \text{MeV/c}^2$. The energy corresponding to the mass of a particle when it is at rest is called its *rest mass energy*.

Another unit which is widely used in describing mass in nuclear physics as well as in atomic physics is *unified atomic mass unit* denoted by the symbol u. It is 1/12 of the mass of a neutral carbon atom in its lowest energy state which contains six protons, six neutrons and six electrons. We have

$$1\ \text{u} = 1{\cdot}6605402 \times 10^{-27}\ \text{kg} = 931{\cdot}478\ \text{MeV/c}^2.$$

Protons and neutrons are fermions and obey the Pauli exclusion principle like electrons. No two protons or two neutrons can have the same quantum state. But one proton and one neutron can exist in the same quantum state. Protons and neutrons are collectively called *nucleons.*

The number of protons in a nucleus is denoted by Z, the number of neutrons by N and the total number of nucleons by A. Thus, $A = Z + N$. The total number of nucleons A is also called the *mass number* of the nucleus. The number of protons Z is called the *atomic number.* A nucleus is symbolically expressed as ${}^{A}_{Z}\text{X}$ in which X is the chemical symbol of the element. Thus, ${}^{4}_{2}\text{He}$ represents helium nucleus which contains 2 protons and a total of 4 nucleons. So it contains 2 neutrons. Similarly, ${}^{238}_{92}\text{U}$ represents a uranium nucleus which contains 92 protons and 146 neutrons.

The distribution of electrons around the nucleus is determined by the number of protons Z and hence the chemical properties of an element are also determined by Z. The nuclei having the same number of protons but different number of neutrons are called *isotopes.* Nuclei with the same neutron number N but different atomic number Z are called *isotones* and the nuclei with the same mass number A are called *isobars.* All nuclei with a given Z and N are collectively called a *nuclide.* Thus, all the ${}^{56}_{26}\text{Fe}$ nuclei taken together is one nuclide and all the ${}^{32}_{16}\text{S}$ nuclei taken together is another nuclide.

or, $$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}.$$

15. *Calculate the energy released when three alpha particles combine to form a* ^{12}C *nucleus. The atomic mass of* $_2^4$He *is* 4·002603 u.

Solution : The mass of a ^{12}C atom is exactly 12 u. The energy released in the reaction $3\left({}_2^4\text{He}\right) \rightarrow {}_6^{12}\text{C}$ is

$$[3\ m({}_2^4\text{He}) - m({}_6^{12}\text{C})]c^2$$

$$= [3 \times 4{\cdot}002603\ \text{u} - 12\ \text{u}]\ (931\ \text{MeV/u}) = 7{\cdot}27\ \text{MeV}.$$

□

QUESTIONS FOR SHORT ANSWER

1. If neutrons exert only attractive force, why don't we have a nucleus containing neutrons alone ?
2. Consider two pairs of neutrons. In each pair, the separation between the neutrons is the same. Can the force between the neutrons have different magnitudes for the two pairs ?
3. A molecule of hydrogen contains two protons and two electrons. The nuclear force between these two protons is always neglected while discussing the behaviour of a hydrogen molecule. Why ?
4. Is it easier to take out a nucleon from carbon or from iron ? From iron or from lead ?
5. Suppose we have 12 protons and 12 neutrons. We can assemble them to form either a ^{24}Mg nucleus or two ^{12}C nuclei. In which of the two cases more energy will be liberated ?
6. What is the difference between cathode rays and beta rays ? When the two are travelling in space, can you make out which is the cathode ray and which is the beta ray ?
7. If the nucleons of a nucleus are separated from each other, the total mass is increased. Where does this mass come from ?
8. In beta decay, an electron (or a positron) is emitted by a nucleus. Does the remaining atom get oppositely charged ?
9. When a boron nucleus ($_5^{10}$B) is bombarded by a neutron, an α-particle is emitted. Which nucleus will be formed as a result ?
10. Does a nucleus lose mass when it suffers gamma decay ?
11. In a typical fission reaction, the nucleus is split into two middle-weight nuclei of unequal masses. Which of the two (heavier or lighter) has greater kinetic energy ? Greater linear momentum ?
12. If three helium nuclei combine to form a carbon nucleus, energy is liberated. Why can't helium nuclei combine on their own and minimise the energy ?

OBJECTIVE I

1. The mass of a neutral carbon atom in ground state is
   (a) exact 12 u
   (b) less than 12 u
   (c) more than 12 u
   (d) depends on the form of carbon such as graphite or charcoal.
2. The mass number of a nucleus is equal to
   (a) the number of neutrons in the nucleus
   (b) the number of protons in the nucleus
   (c) the number of nucleons in the nucleus
   (d) none of them.
3. As compared to ^{12}C atom, ^{14}C atom has
   (a) two extra protons and two extra electrons
   (b) two extra protons but no extra electron
   (c) two extra neutrons and no extra electron
   (d) two extra neutrons and two extra electrons.
4. The mass number of a nucleus is
   (a) always less than its atomic number
   (b) always more than its atomic number
   (c) equal to its atomic number
   (d) sometimes more than and sometimes equal to its atomic number.
5. The graph of $\ln(R/R_0)$ versus $\ln A$ (R = radius of a nucleus and A = its mass number) is
   (a) a straight line
   (b) a parabola
   (c) an ellipse
   (d) none of them.
6. Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of the nuclear force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$
   (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$
   (d) $F_{pp} < F_{pn} = F_{nn}$.
7. Let F_{pp}, F_{pn} and F_{nn} denote the magnitudes of the net force by a proton on a proton, by a proton on a neutron and by a neutron on a neutron respectively. Neglect gravitational force. When the separation is 1 fm,
   (a) $F_{pp} > F_{pn} = F_{nn}$
   (b) $F_{pp} = F_{pn} = F_{nn}$
   (c) $F_{pp} > F_{pn} > F_{nn}$
   (d) $F_{pp} < F_{pn} = F_{nn}$.
8. Two protons are kept at a separation of 10 nm. Let F_n and F_e be the nuclear force and the electromagnetic force between them.
   (a) $F_e = F_n$.
   (b) $F_e >> F_n$.
   (c) $F_e << F_n$.
   (d) F_e and F_n differ only slightly.

9. As the mass number A increases, the binding energy per nucleon in a nucleus
(a) increases (b) decreases (c) remains the same
(d) varies in a way that depends on the actual value of A.

10. Which of the following is a wrong description of binding energy of a nucleus ?
(a) It is the energy required to break a nucleus into its constituent nucleons.
(b) It is the energy made available when free nucleons combine to form a nucleus.
(c) It is the sum of the rest mass energies of its nucleons minus the rest mass energy of the nucleus.
(d) It is the sum of the kinetic energy of all the nucleons in the nucleus.

11. In one average-life,
(a) half the active nuclei decay
(b) less than half the active nuclei decay
(c) more than half the active nuclei decay
(d) all the nuclei decay.

12. In a radioactive decay, neither the atomic number nor the mass number changes. Which of the following particles is emitted in the decay ?
(a) proton (b) neutron (c) electron (d) photon.

13. During a negative beta decay,
(a) an atomic electron is ejected
(b) an electron which is already present within the nucleus is ejected
(c) a neutron in the nucleus decays emitting an electron
(d) a proton in the nucleus decays emitting an electron.

14. A freshly prepared radioactive source of half-life 2 h emits radiation of intensity which is 64 times the permissible safe level. The minimum time after which it would be possible to work safely with this source is
(a) 6 h (b) 12 h (c) 24 h (d) 128 h.

15. The decay constant of a radioactive sample is λ. The half-life and the average-life of the sample are respectively
(a) $1/\lambda$ and (In $2/\lambda$) (b) ($\ln 2/\lambda$) and $1/\lambda$
(c) $\lambda(\ln 2)$ and $1/\lambda$ (d) $\lambda/(\ln 2)$ and $1/\lambda$.

16. An α-particle is bombarded on ^{14}N. As a result, a ^{17}O nucleus is formed and a particle is emitted. This particle is a
(a) neutron (b) proton (c) electron (d) positron.

17. Ten grams of ^{57}Co kept in an open container beta-decays with a half-life of 270 days. The weight of the material inside the container after 540 days will be very nearly
(a) 10 g (b) 5 g (c) 2·5 g (d) 1·25 g.

18. Free ^{238}U nuclei kept in a train emit alpha particles. When the train is stationary and a uranium nucleus decays, a passenger measures that the separation between the alpha particle and the recoiling nucleus becomes x in time t after the decay. If a decay takes place when the train is moving at a uniform speed v, the distance between the alpha particle and the recoiling nucleus at a time t after the decay, as measured by the passenger will be
(a) $x + vt$ (b) $x - vt$ (c) x
(d) depends on the direction of the train.

19. During a nuclear fission reaction,
(a) a heavy nucleus breaks into two fragments by itself
(b) a light nucleus bombarded by thermal neutrons breaks up
(c) a heavy nucleus bombarded by thermal neutrons breaks up
(d) two light nuclei combine to give a heavier nucleus and possibly other products.

OBJECTIVE II

1. As the mass number A increases, which of the following quantities related to a nucleus do not change ?
(a) mass (b) volume (c) density (d) binding energy.

2. The heavier nuclei tend to have larger N/Z ratio because
(a) a neutron is heavier than a proton
(b) a neutron is an unstable particle
(c) a neutron does not exert electric repulsion
(d) Coulomb forces have longer range compared to the nuclear forces.

3. A free neutron decays to a proton but a free proton does not decay to a neutron. This is because
(a) neutron is a composite particle made of a proton and an electron whereas proton is a fundamental particle
(b) neutron is an uncharged particle whereas proton is a charged particle
(c) neutron has larger rest mass than the proton
(d) weak forces can operate in a neutron but not in a proton.

4. Consider a sample of a pure beta-active material.
(a) All the beta particles emitted have the same energy.
(b) The beta particles originally exist inside the nucleus and are ejected at the time of beta decay.
(c) The antineutrino emitted in a beta decay has zero mass and hence zero momentum.
(d) The active nucleus changes to one of its isobars after the beta decay.

5. In which of the following decays the element does not change ?
(a) α-decay (b) β^{+}-decay (c) β -decay (d) γ-decay.

6. In which of the following decays the atomic number decreases ?
(a) α-decay (b) β^{+}-decay (c) β -decay (d) γ-decay.

7. Magnetic field does not cause deflection in
(a) α-rays (b) beta-plus rays
(c) beta-minus rays (d) gamma rays.

8. Which of the following are electromagnetic waves ?
(a) α-rays (b) beta-plus rays
(c) beta-minus rays (d) gamma rays.

9. Two lithium nuclei in a lithium vapour at room temperature do not combine to form a carbon nucleus because
(a) a lithium nucleus is more tightly bound than a

carbon nucleus

(b) carbon nucleus is an unstable particle

(c) it is not energetically favourable

(d) Coulomb repulsion does not allow the nuclei to come very close.

10. For nuclei with $A > 100$,

(a) the binding energy of the nucleus decreases on an average as A increases

(b) the binding energy per nucleon decreases on an average as A increases

(c) if the nucleus breaks into two roughly equal parts, energy is released

(d) if two nuclei fuse to form a bigger nucleus, energy is released.

EXERCISES

Mass of proton $m_p = 1{\cdot}007276$ u, Mass of ^1_1H atom $= 1{\cdot}007825$ u, Mass of neutron $m_n = 1{\cdot}008665$ u, Mass of electron $= 0{\cdot}0005486\ \text{u} \approx 511\ \text{keV}/c^2$, $1\ \text{u} = 931\ \text{MeV}/c^2$.

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1. Assume that the mass of a nucleus is approximately given by $M = Am_p$ where A is the mass number. Estimate the density of matter in kg/m^3 inside a nucleus. What is the specific gravity of nuclear matter ?

2. A neutron star has a density equal to that of the nuclear matter. Assuming the star to be spherical, find the radius of a neutron star whose mass is $4{\cdot}0 \times 10^{30}$ kg (twice the mass of the sun).

3. Calculate the mass of an α-particle. Its binding energy is 28·2 MeV.

4. How much energy is released in the following reaction?

$$^7Li + p \rightarrow \alpha + \alpha$$

Atomic mass of $^7Li = 7{\cdot}0160$ u and that of $^4He = 4{\cdot}0026$ u.

5. Find the binding energy per nucleon of $^{197}_{79}Au$ if its atomic mass is 196·96 u.

6. (a) Calculate the energy released if ^{238}U emits an α-particle. (b) Calculate the energy to be supplied to ^{238}U if two protons and two neutrons are to be emitted one by one. The atomic masses of ^{238}U, ^{234}Th and ^4He are 238·0508 u, 234·04363 u and 4·00260 u respectively.

7. Find the energy liberated in the reaction

$$^{223}Ra \rightarrow {}^{209}Pb + {}^{14}C.$$

The atomic masses needed are as follows:

^{223}Ra	^{209}Pb	^{14}C
223·018 u	208·981 u	14·003 u

8. Show that the minimum energy needed to separate a proton from a nucleus with Z protons and N neutrons is

$$\Delta E = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$

where $M_{Z,N}$ = mass of an atom with Z protons and N neutrons in the nucleus and M_H = mass of a hydrogen atom. This energy is known as *proton-separation energy.*

9. Calculate the minimum energy needed to separate a neutron from a nucleus with Z protons and N neutrons in terms of the masses $M_{Z,N}$, $M_{Z,N-1}$ and the mass of the neutron.

10. ^{32}P beta-decays to ^{32}S. Find the sum of the energy of the antineutrino and the kinetic energy of the β-particle. Neglect the recoil of the daughter nucleus. Atomic mass of $^{32}P = 31{\cdot}974$ u and that of $^{32}S = 31{\cdot}972$ u.

11. A free neutron beta-decays to a proton with a half-life of 14 minutes. (a) What is the decay constant ? (b) Find the energy liberated in the process.

12. Complete the following decay schemes.

(a) $^{226}_{88}Ra \rightarrow \alpha +$

(b) $^{19}_{8}O \rightarrow {}^{19}_{9}F +$

(c) $^{25}_{13}Al \rightarrow {}^{25}_{12}Mg +$

13. In the decay $^{64}Cu \rightarrow {}^{64}Ni + e^+ + \nu$, the maximum kinetic energy carried by the positron is found to be 0·650 MeV. (a) What is the energy of the neutrino which was emitted together with a positron of kinetic energy 0·150 MeV ? (b) What is the momentum of this neutrino in kg–m/s ? Use the formula applicable to a photon.

14. Potassium-40 can decay in three modes. It can decay by β^--emission, β^+-emission or electron capture. (a) Write the equations showing the end products. (b) Find the Q-values in each of the three cases. Atomic masses of $^{40}_{18}Ar$, $^{40}_{19}K$ and $^{40}_{20}Ca$ are 39·9624 u, 39·9640 u and 39·9626 u respectively.

15. Lithium ($Z = 3$) has two stable isotopes ^6Li and ^7Li. When neutrons are bombarded on lithium sample, electrons and α-particles are ejected. Write down the nuclear processes taking place.

16. The masses of ^{11}C and ^{11}B are respectively 11·0114 u and 11·0093 u. Find the maximum energy a positron can have in the β^+-decay of ^{11}C to ^{11}B.

17. ^{228}Th emits an alpha particle to reduce to ^{224}Ra. Calculate the kinetic energy of the alpha particle emitted in the following decay:

$$^{228}Th \rightarrow {}^{224}Ra^* + \alpha$$

$$^{224}Ra^* \rightarrow {}^{224}Ra + \gamma\ (217\ \text{keV}).$$

Atomic mass of ^{228}Th is 228·028726 u, that of ^{224}Ra is 224·020196 u and that of ^4_2He is 4·00260 u.

18. Calculate the maximum kinetic energy of the beta particle emitted in the following decay scheme:

$$^{12}N \rightarrow {}^{12}C^* + e^+ + \nu$$

$$^{12}C^* \rightarrow {}^{12}C + \gamma\ (4{\cdot}43\ \text{MeV}).$$

The atomic mass of ^{12}N is 12·018613 u.

19. The decay constant of $^{197}_{80}Hg$ (electron capture to $^{197}_{79}Au$) is $1{\cdot}8 \times 10^{-6}\ \text{s}^{-1}$. (a) What is the half-life ? (b) What is the average-life ? (c) How much time will it take to convert 25% of this isotope of mercury into gold ?

20. The half-life of ^{198}Au is 2·7 days. (a) Find the activity of a sample containing 1·00 µg of ^{198}Au. (b) What will be the activity after 7 days? Take the atomic weight of ^{198}Au to be 198 g/mol.

21. Radioactive ^{131}I has a half-life of 8·0 days. A sample containing ^{131}I has activity 20 µCi at $t = 0$. (a) What is its activity at $t = 4·0$ days? (b) What is its decay constant at $t = 4·0$ days?

22. The decay constant of ^{238}U is $4·9 \times 10^{-18}\ s^{-1}$. (a) What is the average-life of ^{238}U? (b) What is the half-life of ^{238}U? (c) By what factor does the activity of a ^{238}U sample decrease in 9×10^{9} years?

23. A certain sample of a radioactive material decays at the rate of 500 per second at a certain time. The count rate falls to 200 per second after 50 minutes. (a) What is the decay constant of the sample? (b) What is its half-life?

24. The count rate from a radioactive sample falls from $4·0 \times 10^{6}$ per second to $1·0 \times 10^{6}$ per second in 20 hours. What will be the count rate 100 hours after the beginning?

25. The half-life of ^{226}Ra is 1602 y. Calculate the activity of 0·1 g of $RaCl_2$ in which all the radium is in the form of ^{226}Ra. Taken atomic weight of Ra to be 226 g/mol and that of Cl to be 35·5 g/mol.

26. The half-life of a radioisotope is 10 h. Find the total number of disintegrations in the tenth hour measured from a time when the activity was 1 Ci.

27. The selling rate of a radioactive isotope is decided by its activity. What will be the second-hand rate of a one month old ^{32}P($t_{1/2} = 14·3$ days) source if it was originally purchased for 800 rupees?

28. ^{57}Co decays to ^{57}Fe by β^{+}-emission. The resulting ^{57}Fe is in its excited state and comes to the ground state by emitting γ-rays. The half-life of β^{+}-decay is 270 days and that of the γ-emission is 10^{-8} s. A sample of ^{57}Co gives $5·0 \times 10^{9}$ gamma rays per second. How much time will elapse before the emission rate of gamma rays drops to $2·5 \times 10^{9}$ per second?

29. Carbon ($Z = 6$) with mass number 11 decays to boron ($Z = 5$). (a) Is it a β^{+}-decay or a β^{-}-decay? (b) The half-life of the decay scheme is 20·3 minutes. How much time will elapse before a mixture of 90% carbon-11 and 10% boron-11 (by the number of atoms) converts itself into a mixture of 10% carbon-11 and 90% boron-11?

30. 4×10^{23} tritium atoms are contained in a vessel. The half-life of decay of tritium nuclei is 12·3 y. Find (a) the activity of the sample, (b) the number of decays in the next 10 hours (c) the number of decays in the next 6·15 y.

31. A point source emitting alpha particles is placed at a distance of 1 m from a counter which records any alpha particle falling on its 1 cm^{2} window. If the source contains $6·0 \times 10^{16}$ active nuclei and the counter records a rate of 50000 counts/second, find the decay constant. Assume that the source emits alpha particles uniformly in all directions and the alpha particles fall nearly normally on the window.

32. ^{238}U decays to ^{206}Pb with a half-life of $4·47 \times 10^{9}$ y. This happens in a number of steps. Can you justify a single half-life for this chain of processes? A sample of rock is found to contain 2·00 mg of ^{238}U and 0·600 mg of ^{206}Pb. Assuming that all the lead has come from uranium, find the life of the rock.

33. When charcoal is prepared from a living tree, it shows a disintegration rate of 15·3 disintegrations of ^{14}C per gram per minute. A sample from an ancient piece of charcoal shows ^{14}C activity to be 12·3 disintegrations per gram per minute. How old is this sample? Half-life of ^{14}C is 5730 y.

34. Natural water contains a small amount of tritium (^{3}H). This isotope beta-decays with a half-life of 12·5 years. A mountaineer while climbing towards a difficult peak finds debris of some earlier unsuccessful attempt. Among other things he finds a sealed bottle of whisky. On return he analyses the whisky and finds that it contains only 1·5 per cent of the ^{3}H radioactivity as compared to a recently purchased bottle marked '8 years old'. Estimate the time of that unsuccessful attempt.

35. The count rate of nuclear radiation coming from a radioactive sample containing ^{128}I varies with time as follows.

Time t (minute):	0	25	50	75	100
Count rate $R(10^{9}\ s^{-1})$:	30	16	8·0	3·8	2·0

(a) Plot $\ln(R_0/R)$ against t. (b) From the slope of the best straight line through the points, find the decay constant λ. (c) Calculate the half-life $t_{1/2}$.

36. The half-life of ^{40}K is $1·30 \times 10^{9}$ y. A sample of 1·00 g of pure KCl gives 160 counts/s. Calculate the relative abundance of ^{40}K (fraction of ^{40}K present) in natural potassium.

37. $^{197}_{80}$Hg decays to $^{197}_{79}$Au through electron capture with a decay constant of 0·257 per day. (a) What other particle or particles are emitted in the decay? (b) Assume that the electron is captured from the K shell. Use Moseley's law $\sqrt{\nu} = a(Z - b)$ with $a = 4·95 \times 10^{7}\ s^{-1/2}$ and $b = 1$ to find the wavelength of the K_{α} X-ray emitted following the electron capture.

38. A radioactive isotope is being produced at a constant rate $dN/dt = R$ in an experiment. The isotope has a half-life $t_{1/2}$. Show that after a time $t >> t_{1/2}$, the number of active nuclei will become constant. Find the value of this constant.

39. Consider the situation of the previous problem. Suppose the production of the radioactive isotope starts at $t = 0$. Find the number of active nuclei at time t.

40. In an agricultural experiment, a solution containing 1 mole of a radioactive material ($t_{1/2} = 14·3$ days) was injected into the roots of a plant. The plant was allowed 70 hours to settle down and then activity was measured in its fruit. If the activity measured was 1 µCi, what per cent of activity is transmitted from the root to the fruit in steady state?

41. A vessel of volume 125 cm^{3} contains tritium (^{3}H, $t_{1/2} = 12·3$ y) at 500 kPa and 300 K. Calculate the activity of the gas.

42. ${}^{212}_{83}\mathrm{Bi}$ can disintegrate either by emitting an α-particle or by emitting a β^{-}-particle. (a) Write the two equations showing the products of the decays. (b) The probabilities of disintegration by α- and β-decays are in the ratio 7/13. The overall half-life of ${}^{212}\mathrm{Bi}$ is one hour. If 1 g of pure ${}^{212}\mathrm{Bi}$ is taken at 12·00 noon, what will be the composition of this sample at 1 p.m. the same day?

43. A sample contains a mixture of ${}^{108}\mathrm{Ag}$ and ${}^{110}\mathrm{Ag}$ isotopes each having an activity of $8{\cdot}0 \times 10^{8}$ disintegrations per second. ${}^{110}\mathrm{Ag}$ is known to have larger half-life than ${}^{108}\mathrm{Ag}$. The activity A is measured as a function of time and the following data are obtained.

Time (s)	Activity (A) (10^{8} disintegrations/s)	Time (s)	Activity (A) (10^{8} disintegrations/s)
20	11·799	200	3·0828
40	9·1680	300	1·8899
60	7·4492	400	1·1671
80	6·2684	500	0 7212
100	5 4115		

(a) Plot $\ln(A/A_0)$ versus time. (b) See that for large values of time, the plot is nearly linear. Deduce the half-life of ${}^{110}\mathrm{Ag}$ from this portion of the plot. (c) Use the half-life of ${}^{110}\mathrm{Ag}$ to calculate the activity corresponding to ${}^{108}\mathrm{Ag}$ in the first 50 s. (d) Plot $\ln(A/A_0)$ versus time for ${}^{108}\mathrm{Ag}$ for the first 50 s. (e) Find the half-life of ${}^{108}\mathrm{Ag}$.

44. A human body excretes (removes by waste discharge, sweating etc.) certain materials by a law similar to radioactivity. If technitium is injected in some form in a human body, the body excretes half the amount in 24 hours. A patient is given an injection containing ${}^{99}\mathrm{Tc}$. This isotope is radioactive with a half-life of 6 hours. The activity from the body just after the injection is 6 μCi. How much time will elapse before the activity falls to 3 μCi?

45. A charged capacitor of capacitance C is discharged through a resistance R. A radioactive sample decays with an average-life τ. Find the value of R for which the ratio of the electrostatic field energy stored in the capacitor to the activity of the radioactive sample remains constant in time.

46. Radioactive isotopes are produced in a nuclear physics experiment at a constant rate $dN/dt = R$. An inductor of inductance 100 mH, a resistor of resistance 100 Ω and a battery are connected to form a series circuit. The circuit is switched on at the instant the production of radioactive isotope starts. It is found that i/N remains constant in time where i is the current in the circuit at time t and N is the number of active nuclei at time t. Find the half-life of the isotope.

47. Calculate the energy released by 1 g of natural uranium assuming 200 MeV is released in each fission event and that the fissionable isotope ${}^{235}\mathrm{U}$ has an abundance of 0·7% by weight in natural uranium.

48. A uranium reactor develops thermal energy at a rate of 300 MW. Calculate the amount of ${}^{235}\mathrm{U}$ being consumed every second. Average energy released per fission is 200 MeV.

49. A town has a population of 1 million. The average electric power needed per person is 300 W. A reactor is to be designed to supply power to this town. The efficiency with which thermal power is converted into electric power is aimed at 25%. (a) Assuming 200 MeV of thermal energy to come from each fission event on an average, find the number of events that should take place every day. (b) Assuming the fission to take place largely through ${}^{235}\mathrm{U}$, at what rate will the amount of ${}^{235}\mathrm{U}$ decrease? Express your answer in kg/day. (c) Assuming that uranium enriched to 3% in ${}^{235}\mathrm{U}$ will be used, how much uranium is needed per month (30 days)?

50. Calculate the Q-values of the following fusion reactions:

(a) ${}^{2}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H}$

(b) ${}^{2}_{1}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}_{2}\mathrm{He} + n$

(c) ${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + n.$

Atomic masses are $m({}^{2}_{1}\mathrm{H}) = 2{\cdot}014102$ u, $m({}^{3}\mathrm{H}) = 3{\cdot}016049$ u, $m({}^{3}\mathrm{He}) = 3{\cdot}016029$ u, $m({}^{4}\mathrm{He}) = 4{\cdot}002603$ u.

51. Consider the fusion in helium plasma. Find the temperature at which the average thermal energy 1·5 kT equals the Coulomb potential energy at 2 fm.

52. Calculate the Q-value of the fusion reaction

$${}^{4}\mathrm{He} + {}^{4}\mathrm{He} = {}^{8}\mathrm{Be}.$$

Is such a fusion energetically favourable? Atomic mass of ${}^{8}\mathrm{Be}$ is 8·0053 u and that of ${}^{4}\mathrm{He}$ is 4·0026 u.

53. Calculate the energy that can be obtained from 1 kg of water through the fusion reaction

$${}^{2}\mathrm{H} + {}^{2}\mathrm{H} \rightarrow {}^{3}\mathrm{H} + p.$$

Assume that $1{\cdot}5 \times 10^{-2}$ % of natural water is heavy water D_2O (by number of molecules) and all the deuterium is used for fusion.

□

ANSWERS

OBJECTIVE I

1. (a) 2. (c) 3. (c) 4. (d) 5. (a) 6. (b)
7. (d) 8. (b) 9. (d) 10. (d) 11. (c) 12. (d)
13. (c) 14. (b) 15. (b) 16. (b) 17. (a) 18. (c)
19. (c)

OBJECTIVE II

1. (c) 2. (c), (d) 3. (c)
4. (d) 5. (d) 6. (a), (b)
7. (d) 8. (d) 9. (d)
10. (b), (c)

EXERCISES

1. 3×10^{17} kg/m^3, 3×10^{14}
2. 15 km
3. 4·0016 u
4. 17·34 MeV
5. 7·94 MeV
6. (a) 4·255 MeV (b) 24·03 MeV
7. 31·65 MeV
9. $(M_{Z,N-1} + m_n - M_{Z,N})c^2$
10. 1·86 MeV
11. (a) $8{\cdot}25 \times 10^{-4}$ s^{-1} (b) 782 keV
12. (a) ${}^{222}_{86}\text{Rn}$ (b) $e^- + \bar{\nu}$ (c) $e^+ + \nu$
13. (a) 500 keV (b) $2{\cdot}67 \times 10^{-22}$ kg–m/s
14. (a) ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{20}\text{Ca} + e^- + \bar{\nu}$, ${}^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar} + e^+ + \nu$,
    ${}^{40}_{19}\text{K} + e^- \rightarrow {}^{40}_{18}\text{Ar} + \nu$
    (b) 1·3034 MeV, 0·4676 MeV, 1·490 MeV
15. ${}^{6}_{3}\text{Li} + n \rightarrow {}^{7}_{3}\text{Li}$, ${}^{7}_{3}\text{Li} + n \rightarrow {}^{8}_{3}\text{Li} \rightarrow {}^{8}_{4}\text{Be} + e^- + \bar{\nu}$,
    ${}^{8}_{4}\text{Be} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$
16. 933·6 keV
17. 5·304 MeV
18. 11·88 MeV
19. (a) 64 min (b) 92 min (c) 1600 s
20. (a) 0·244 Ci (b) 0·040 Ci
21. (a) 14 μCi (b) $1{\cdot}4 \times 10^{-6}$ s^{-1}
22. $6{\cdot}49 \times 10^9$ y (b) $4{\cdot}5 \times 10^9$ y (c) 4
23. $3{\cdot}05 \times 10^{-4}$ s (b) 38 min
24. $3{\cdot}9 \times 10^3$ per second
25. $2{\cdot}8 \times 10^9$ disintegrations/s
26. $6{\cdot}91 \times 10^{13}$
27. 187 rupees
28. 270 days
29. (a) β^+ (b) 64 min
30. (a) $7{\cdot}146 \times 10^{14}$ disintegrations/s
    (b) $2{\cdot}57 \times 10^{19}$ (c) $1{\cdot}17 \times 10^{23}$
31. $1{\cdot}05 \times 10^{-7}$ s^{-1}
32. $1{\cdot}92 \times 10^9$ y
33. 1800 y
34. about 83 years ago
35. (b) 0·028 min^{-1} approx. (c) 25 min approx.
36. 0·12%
37. (a) neutrino (b) 20 pm
38. $\dfrac{Rt_{1/2}}{0\,693}$
39. $\dfrac{R}{\lambda}(1 - e^{-\lambda t})$
40. $1{\cdot}26 \times 10^{-11}$ %
41. 724 Ci
42. (a) ${}^{212}_{83}\text{Bi} \rightarrow {}^{208}_{81}\text{Tl} + \alpha$, ${}^{212}_{83}\text{Bi} \rightarrow {}^{212}_{83}\text{Bi} \rightarrow {}^{212}_{84}\text{Po} + e^- + \bar{\nu}$
    (b) 0·50 g–Bi, 0·175 g–Tl, 0·325 g–Po
43. the half-life of ${}^{110}\text{Ag} = 24{\cdot}4$ s and of ${}^{108}\text{Ag} = 144$ s
44. 4·8 hours
45. $2\tau/C$
46. $6{\cdot}93 \times 10^{-4}$ s
47. $5{\cdot}7 \times 10^8$ J
48. 3·7 mg
49. (a) $3{\cdot}24 \times 10^{24}$ (b) 1·264 kg/day (c) 1263 kg
50. (a) 4·05 MeV (b) 3·25 MeV (c) 17·57 MeV
51. $2{\cdot}23 \times 10^{10}$ K
52. – 93·1 keV, no
53. 3200 MJ

□

THE NUCLEUS
CHAPTER - 46

1. $M = Am_p$, $f = M/V$, $m_p = 1.007276$ u

   $R = R_0A^{1/3} = 1.1 \times 10^{-15}\ A^{1/3}$, $u = 1.6605402 \times 10^{-27}$ kg

   $= \frac{A \times 1.007276 \times 1.6605402 \times 10^{-27}}{4/3 \times 3.14 \times R^3} = 0.300159 \times 10^{18} = 3 \times 10^{17}$ kg/m^3.

   'f' in CGS = Specific gravity = 3×10^{14}.

2. $f = \frac{M}{v} \Rightarrow V = \frac{M}{f} = \frac{4\times10^{30}}{2.4\times10^{17}} = \frac{1}{0.6}\times10^{13} = \frac{1}{6}\times10^{14}$

   $V = 4/3\ \pi R^3$.

   $\Rightarrow\ \frac{1}{6}\times10^{14} = 4/3\ \pi \times R^3 \Rightarrow R^3 = \frac{1}{6}\times\frac{3}{4}\times\frac{1}{\pi}\times10^{14}$

   $\Rightarrow\ R^3 = \frac{1}{8}\times\frac{100}{\pi}\times10^{12}$

   $\therefore R = ½ \times 10^4 \times 3.17 = 1.585 \times 10^4$ m = 15 km.

3. Let the mass of 'α' particle be xu.

   'α' particle contains 2 protons and 2 neutrons.

   $\therefore$ Binding energy = $(2 \times 1.007825\ u \times 1 \times 1.00866\ u - xu)C^2 = 28.2$ MeV (given).

   $\therefore$ x = 4.0016 u.

4. $Li^7 + p \rightarrow l + \alpha + E$; $Li^7 = 7.016u$

   $\alpha = {}^4He = 4.0026u$; $p = 1.007276$ u

   $E = Li^7 + P - 2\alpha = (7.016 + 1.007276)u - (2 \times 4.0026)u = 0.018076$ u.

   $\Rightarrow 0.018076 \times 931 = 16.828 = 16.83$ MeV.

5. $B = (Zm_p + Nm_n - M)C^2$

   Z = 79 ; N = 118 ; $m_p = 1.007276u$; M = 196.96 u ; $m_n = 1.008665u$

   $B\ = [(79 \times 1.007276 + 118 \times 1.008665)u - Mu]c^2$

   $= 198.597274 \times 931 - 196.96 \times 931 = 1524.302094$

   so, Binding Energy per nucleon = 1524.3 / 197 = 7.737.

6. a) $U^{238}\ {}_2He^4 + Th^{234}$

   $E = [M_u - (N_{HC} + M_{Th})]u = 238.0508 - (234.04363 + 4.00260)]u = 4.25487$ Mev = 4.255 Mev.

   b) $E = U^{238} - [Th^{234} + 2n'_0 + 2p'_1]$

   $= \{238.0508 - [234.64363 + 2(1.008665) + 2(1.007276)]\}u$

   $= 0.024712u = 23.0068 = 23.007$ MeV.

7. ${}^{223}R_a = 223.018$ u ; ${}^{209}Pb = 208.981$ u ; ${}^{14}C = 14.003$ u.

   ${}^{223}R_a \rightarrow {}^{209}Pb + {}^{14}C$

   Δm = mass ${}^{223}R_a$ – mass $({}^{209}Pb + {}^{14}C)$

   $\Rightarrow\ = 223.018 - (208.981 + 14.003) = 0.034$.

   Energy = $\Delta M \times u = 0.034 \times 931 = 31.65$ Me.

8. $E_{Z.N.} \rightarrow E_{Z-1}, N + P_1 \Rightarrow E_{Z.N.} \rightarrow E_{Z-1}, N + {}_1H^1$ [As hydrogen has no neutrons but protons only]

   $\Delta E = (M_{Z-1,\,N} + N_H - M_{Z,N})c^2$

9. $E_2N = E_{Z,N-1} + {}^1_0n$.

   Energy released = (Initial Mass of nucleus – Final mass of nucleus)$c^2 = (M_{Z.N-1} + M_0 - M_{ZN})c^2$.

10. $P^{32} \rightarrow S^{32} + {}_0\overline{v}^0 + {}_1\beta^0$

    Energy of antineutrino and β-particle

    $= (31.974 - 31.972)u = 0.002\ u = 0.002 \times 931 = 1.862$ MeV = 1.86.

11. $In \rightarrow P + e^-$

    We know : Half life = 0.6931 / λ (Where λ = decay constant).

    Or $\lambda = 0.6931 / 14\times60 = 8.25 \times 10^{-4}$ S [As half life = 14 min = 14 × 60 sec].

    Energy = $[M_n - (M_P + M_e)]u = [(M_{nu} - M_{pu}) - M_{pu}]c^2 = [0.00189u - 511\ KeV/c^2]$

    $= [1293159\ ev/c^2 - 511000\ ev/c^2]c^2 = 782159$ eV = 782 Kev.

12. $^{226}_{58}\text{Ra} \to ^{4}_{2}\alpha + ^{222}_{26}\text{Rn}$

$^{19}_{8}\text{O} \to ^{19}_{9}\text{F} + ^{0}_{n}\beta + ^{0}_{0}\overline{v}$

$^{13}_{25}\text{Al} \to ^{25}_{12}\text{MG} + ^{0}_{-1}e + ^{0}_{0}\overline{v}$

13. $^{64}\text{Cu} \to ^{64}\text{Ni} + e^{-} + v$

Emission of nutrino is along with a positron emission.

a) Energy of positron = 0.650 MeV.

Energy of Nutrino = 0.650 – KE of given position = 0.650 – 0.150 = 0.5 MeV = 500 Kev.

b) Momentum of Nutrino = $\dfrac{500 \times 1.6 \times 10^{-19}}{3 \times 10^{8}} \times 10^{3}$ J = 2.67×10^{-22} kg m/s.

14. a) $_{19}K^{40} \to _{20}Ca^{40} + _{-1}e^{0} + _{0}\overline{v}^{0}$

$_{19}K^{40} \to _{18}Ar^{40} + _{-1}e^{0} + _{0}\overline{v}^{0}$

$_{19}K^{40} + _{-1}e^{0} \to _{18}Ar^{40}$

$_{19}K^{40} \to _{20}Ca^{40} + _{-1}e^{0} + _{0}v^{0}$.

b) Q = [Mass of reactants – Mass of products]c^2

= [39.964u – 39.9626u] = [39.964u – 39.9626]uc^2 = (39.964 – 39.9626) 931 Mev = 1.3034 Mev.

$_{19}K^{40} \to _{18}Ar^{40} + _{-1}e^{0} + _{0}\overline{v}^{0}$

Q = (39.9640 – 39.9624)uc^2 = 1.4890 = 1.49 Mev.

$_{19}K^{40} + _{-1}e^{0} \to _{18}Ar^{40}$

Q_{value} = (39.964 – 39.9624)uc^2.

15. $^{6}_{3}\text{Li} + n \to ^{7}_{3}\text{Li}$; $^{7}_{3}\text{Li} + r \to ^{8}_{3}\text{Li}$

$^{8}_{3}\text{Li} \to ^{8}_{4}\text{Be} + e^{-} + v^{-}$

$^{8}_{4}\text{Be} \to ^{4}_{2}\text{He} + ^{4}_{2}\text{He}$

16. $''C \to ''B + \beta^{+} + v$

mass of C″ = 11.014u ; mass of B″ = 11.0093u

Energy liberated = (11.014 – 11.0093)u = 29.5127 Mev.

For maximum K.E. of the positron energy of v may be assumed as 0.

∴ Maximum K.E. of the positron is 29.5127 Mev.

17. Mass $^{238\text{Th}}$ = 228.028726 u ; ^{224}Ra = 224.020196 u ; α = $^{4}_{2}$He → 4.00260u

$^{238}\text{Th} \to ^{224}\text{Ra}^{*} + \alpha$

$^{224}\text{Ra}^{*} \to ^{224}\text{Ra} + v(217\ \text{Kev})$

Now, Mass of ^{224}Ra* = 224.020196 × 931 + 0.217 Mev = 208563.0195 Mev.

KE of α = E $^{226\text{Th}}$ – E(^{224}Ra* + α)

= 228.028726 × 931 – [208563.0195 + 4.00260 × 931] = 5.30383 Mev= 5.304 Mev.

18. $^{12}\text{N} \to ^{12}\text{C}^{*} + e^{+} + v$

$^{12}\text{C}^{*} \to ^{12}\text{C} + v(4.43\ \text{Mev})$

Net reaction : $^{12}\text{N} \to ^{12}\text{C} + e^{+} + v + v(4.43\ \text{Mev})$

Energy of (e^{+} + v) = N^{12} – (c^{12} + v)

= 12.018613u – (12)u – 4.43 = 0.018613 u – 4.43 = 17.328 – 4.43 = 12.89 Mev.

Maximum energy of electron (assuming 0 energy for v) = 12.89 Mev.

19. a) $t_{1/2} = 0.693 / \lambda$ [λ → Decay constant]

⇒ $t_{1/2}$ = 3820 sec = 64 min.

b) Average life = $t_{1/2}$ / 0.693 = 92 min.

c) $0.75 = 1\ e^{-\lambda t} \Rightarrow \ln 0.75 = -\lambda t \Rightarrow t = \ln 0.75 / -0.00018$ = 1598.23 sec.

20. a) 198 grams of Ag contains → N_0 atoms.

1 μg of Ag contains → $N_0/198 \times 1$ μg = $\dfrac{6 \times 10^{23} \times 1 \times 10^{-6}}{198}$ atoms

Activity = $\lambda N = \frac{0.963}{t_{1/2}} \times N = \frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7}$ disintegrations/day.

$= \frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7 \times 3600 \times 24}$ disintegration/sec = $\frac{0.693 \times 6 \times 10^{17}}{198 \times 2.7 \times 36 \times 24 \times 3.7 \times 10^{10}}$ curie = 0.244 Curie.

b) $A = \frac{A_0}{2t_{1/2}} = \frac{0.244}{2 \times \frac{7}{2.7}}$ = 0.0405 = 0.040 Curie.

21. $t_{1/2}$ = 8.0 days ; $A_0 = 20\ \mu$ CI

a) t = 4.0 days ; $\lambda = 0.693/8$

$A = A_0 e^{-\lambda t} = 20 \times 10^{-6} \times e^{(-0.693/8) \times 4} = 1.41 \times 10^{-5}$ Ci = 14 μ Ci

b) $\lambda = \frac{0.693}{8 \times 24 \times 3600} = 1.0026 \times 10^{-6}$.

22. $\lambda = 4.9 \times 10^{-18}\ s^{-1}$

a) Avg. life of $^{238}U = \frac{1}{\lambda} = \frac{1}{4.9 \times 10^{-18}} = \frac{1}{4.9} \times 10^{-18}$ sec.

$= 6.47 \times 10^{3}$ years.

b) Half life of uranium = $\frac{0.693}{\lambda} = \frac{0.693}{4.9 \times 10^{-18}} = 4.5 \times 10^{9}$ years.

c) $A = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow \frac{A_0}{A} = 2^{t/t_{1/2}} = 2^2 = 4$.

23. A = 200, A_0 = 500, t = 50 min

$A = A_0\ e^{-\lambda t}$ or $200 = 500 \times e^{-50 \times 60 \times \lambda}$

$\Rightarrow \lambda = 3.05 \times 10^{-4}$ s.

b) $t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.000305}$ = 2272.13 sec = 38 min.

24. $A_0 = 4 \times 10^{5}$ disintegration / sec

$A' = 1 \times 10^{6}$ dis/sec ; t = 20 hours.

$A' = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow 2^{t/t_{1/2}} = \frac{A_0}{A'} \Rightarrow 2^{t/t_{1/2}} = 4$

$\Rightarrow t/t_{1/2} = 2 \Rightarrow t^{1/2} = t/2$ = 20 hours / 2 = 10 hours.

$A'' = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow A'' = \frac{4 \times 10^{6}}{2^{100/10}} = 0.00390625 \times 10^{6} = 3.9 \times 10^{3}$ dintegrations/sec.

25. $t_{1/2}$ = 1602 Y ; Ra = 226 g/mole ; Cl = 35.5 g/mole.

1 mole $RaCl_2$ = 226 + 71 = 297 g

297g = 1 mole of Ra.

0.1 g = $\frac{1}{297} \times 0.1$ mole of Ra = $\frac{0.1 \times 6.023 \times 10^{23}}{297} = 0.02027 \times 10^{22}$

$\lambda = 0.693 / t_{1/2} = 1.371 \times 10^{-11}$.

Activity = $\lambda N = 1.371 \times 10^{-11} \times 2.027 \times 10^{20} = 2.779 \times 10^{9} = 2.8 \times 10^{9}$ disintegrations/second.

26. $t_{1/2}$ = 10 hours, A_0 = 1 ci

Activity after 9 hours = $A_0\ e^{-\lambda t} = 1 \times e^{\frac{-0.693}{10} \times 9}$ = 0.5359 = 0.536 Ci.

No. of atoms left after 9th hour, $A_9 = \lambda N_9$

$\Rightarrow N_9 = \frac{A_9}{\lambda} = \frac{0.536 \times 10 \times 3.7 \times 10^{10} \times 3600}{0.693} = 28.6176 \times 10^{10} \times 3600 = 103.023 \times 10^{13}$.

Activity after 10 hours = $A_0\ e^{-\lambda t} = 1 \times e^{\frac{-0.693}{10} \times 9}$ = 0.5 Ci.

No. of atoms left after 10th hour

$A_{10} = \lambda N_{10}$

$\Rightarrow$ $N_{10} = \dfrac{A_{10}}{\lambda} = \dfrac{0.5 \times 3.7 \times 10^{10} \times 3600}{0.693/10}$ = $26.37 \times 10^{10} \times 3600 = 96.103 \times 10^{13}$.

No.of disintegrations = $(103.023 - 96.103) \times 10^{13} = 6.92 \times 10^{13}$.

27. $t_{1/2}$ = 14.3 days ; t = 30 days = 1 month
As, the selling rate is decided by the activity, hence A_0 = 800 disintegration/sec.
We know, $A = A_0e^{-\lambda t}$ [$\lambda = 0.693/14.3$]
$A = 800 \times 0.233669 = 186.935$ = 187 rupees.

28. According to the question, the emission rate of γ rays will drop to half when the β+ decays to half of its original amount. And for this the sample would take 270 days.
$\therefore$ The required time is 270 days.

29. a) $P \rightarrow n + e^{+} + v$ Hence it is a β^{+} decay.
b) Let the total no. of atoms be 100 N_0.

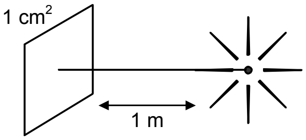
	Carbon	Boron
Initially	$90\ N_0$	$10\ N_0$
Finally	$10\ N_0$	$90\ N_0$

Now, $10\ N_0 = 90\ N_0\ e^{-\lambda t} \Rightarrow 1/9 = e^{\frac{-0.693}{20.3} \times t}$ [because $t_{1/2}$ = 20.3 min]

$\Rightarrow$ $\ln \dfrac{1}{9} = \dfrac{-0.693}{20.3} t \Rightarrow t = \dfrac{2.1972 \times 20.3}{0.693}$ = 64.36 = 64 min.

30. $N = 4 \times 10^{23}$; $t_{1/2}$ = 12.3 years.
a) Activity = $\dfrac{dN}{dt} = \lambda n = \dfrac{0.693}{t_{1/2}} N = \dfrac{0.693}{12.3} \times 4 \times 10^{23}$ dis/year.
$= 7.146 \times 10^{14}$ dis/sec.
b) $\dfrac{dN}{dt} = 7.146 \times 10^{14}$
No.of decays in next 10 hours = $7.146 \times 10^{14} \times 10 \times 36.. = 257.256 \times 10^{17} = 2.57 \times 10^{19}$.
c) $N = N_0\ e^{-\lambda t} = 4 \times 10^{23} \times e^{\frac{-0.693}{20.3} \times 6.16}$ = 2.82×10^{23} = No.of atoms remained
No. of atoms disintegrated = $(4 - 2.82) \times 10^{23} = 1.18 \times 10^{23}$.

31. Counts received per cm^2 = 50000 Counts/sec.
$N = N_3o$ of active nucleic = 6×10^{16}
Total counts radiated from the source = Total surface area × 50000 counts/cm^2
$= 4 \times 3.14 \times 1 \times 10^4 \times 5 \times 10^4 = 6.28 \times 10^9$ Counts = dN/dt



We know, $\dfrac{dN}{dt} = \lambda N$
Or $\lambda = \dfrac{6.28 \times 10^9}{6 \times 10^{16}}$ = $1.0467 \times 10^{-7} = 1.05 \times 10^{-7}\ s^{-1}$.

32. Half life period can be a single for all the process. It is the time taken for 1/2 of the uranium to convert to lead.

No. of atoms of U^{238} = $\dfrac{6 \times 10^{23} \times 2 \times 10^{-3}}{238}$ = $\dfrac{12}{238} \times 10^{20}$ = 0.05042×10^{20}

No. of atoms in Pb = $\dfrac{6 \times 10^{23} \times 0.6 \times 10^{-3}}{206} = \dfrac{3.6}{206} \times 10^{20}$

Initially total no. of uranium atoms = $\left(\dfrac{12}{235} + \dfrac{3.6}{206}\right) \times 10^{20}$ = 0.06789

$N = N_0\ e^{-\lambda t} \Rightarrow N = N_0\ e^{\frac{-0.693}{t/t_{1/2}}} \Rightarrow 0.05042 = 0.06789\ e^{\frac{-0.693}{4.47 \times 10^9}}$

$\Rightarrow$ $\log\left(\dfrac{0.05042}{0.06789}\right) = \dfrac{-0.693t}{4.47 \times 10^9}$

$\Rightarrow t = 1.92 \times 10^9$ years.

33. $A_0 = 15.3$; $A = 12.3$; $t_{1/2} = 5730$ year

$$\lambda = \frac{0.6931}{T_{1/2}} = \frac{0.6931}{5730}\text{yr}^{-1}$$

Let the time passed be t,

We know $A = A_0e^{-\lambda t} - \frac{0.6931}{5730} \times t \Rightarrow 12.3 = 15.3 \times e.$

$\Rightarrow$ t = 1804.3 years.

34. The activity when the bottle was manufactured = A_0

Activity after 8 years = $A_0e^{\frac{-0.693}{12.5}\times 8}$

Let the time of the mountaineering = t years from the present

$A = A_0e^{\frac{-0.693}{12.5}\times t}$; A = Activity of the bottle found on the mountain.

A = (Activity of the bottle manufactured 8 years before) × 1.5%

$$\Rightarrow A_0e^{\frac{-0.693}{12.5}} = A_0e^{\frac{-0.693}{12.5}\times 8} \times 0.015$$

$$\Rightarrow \frac{-0.693}{12.5}t = \frac{-0.693\times 8}{12.5} + \ln[0.015]$$

$\Rightarrow 0.05544\,t = 0.44352 + 4.1997 \Rightarrow t = 83.75$ years.

35. a) Here we should take R_0 at time is $t_0 = 30 \times 10^9\ s^{-1}$

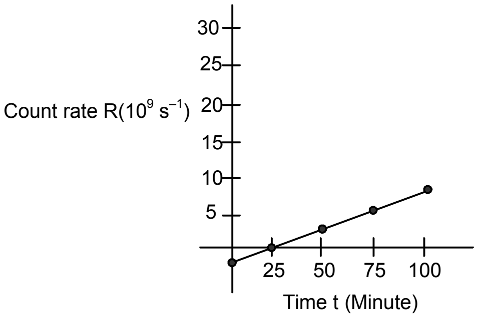
i) $\ln(R_0/R_1) = \ln\left(\frac{30\times10^9}{30\times10^9}\right) = 0$

ii) $\ln(R_0/R_2) = \ln\left(\frac{30\times10^9}{16\times10^9}\right) = 0.63$

iii) $\ln(R_0/R_3) = \ln\left(\frac{30\times10^9}{8\times10^9}\right) = 1.35$

iv) $\ln(R_0/R_4) = \ln\left(\frac{30\times10^9}{3.8\times10^9}\right) = 2.06$

v) $\ln(R_0/R_5) = \ln\left(\frac{30\times10^9}{2\times10^9}\right) = 2.7$



b) ∴ The decay constant $\lambda = 0.028\ \text{min}^{-1}$

c) ∴ The half life period = $t_{1/2}$.

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.028} = 25 \text{ min.}$$

36. Given : Half life period $t_{1/2} = 1.30 \times 10^9$ year , A = 160 count/s = $1.30 \times 10^9 \times 365 \times 86400$

$$\therefore A = \lambda N \Rightarrow 160 = \frac{0.693}{t_{1/2}}N$$

$$\Rightarrow N = \frac{160\times1.30\times365\times86400\times10^9}{0.693} = 9.5 \times 10^{18}$$

∴ 6.023×10^{23} No. of present in 40 grams.

$6.023 \times 10^{23} = 40\text{ g} \Rightarrow 1 = \frac{40}{6.023\times10^{23}}$

∴ 9.5×10^{18} present in = $\frac{40\times9.5\times10^{18}}{6.023\times10^{23}} = 6.309 \times 10^{-4} = 0.00063.$

∴ The relative abundance at 40 k in natural potassium = $(2 \times 0.00063 \times 100)\% = 0.12\%$.

37. a) P + e → n + v neutrino [a → 4.95×10^{7} $s^{-1/2}$; b → 1]

b) $\sqrt{f} = a(z - b)$

$\Rightarrow \sqrt{c/\lambda} = 4.95 \times 10^{7} (79 - 1) = 4.95 \times 10^{7} \times 78 \Rightarrow C/\lambda = (4.95 \times 78)^{2} \times 10^{14}$

$\Rightarrow \lambda = \dfrac{3 \times 10^{8}}{14903.2 \times 10^{14}} = 2 \times 10^{-5} \times 10^{-6} = 2 \times 10^{-4}$ m = 20 pm.

38. Given : Half life period = $t_{1/2}$, Rate of radio active decay = $\dfrac{dN}{dt} = R \Rightarrow R = \dfrac{dN}{dt}$

Given after time t >> $t_{1/2}$, the number of active nuclei will become constant.

i.e. $(dN/dt)_{present} = R = (dN/dt)_{decay}$

$\therefore R = (dN/dt)_{decay}$

$\Rightarrow$ R = λN [where, λ = Radioactive decay constant, N = constant number]

$\Rightarrow R = \dfrac{0.693}{t_{1/2}}(N) \Rightarrow Rt_{1/2} = 0.693\ N \Rightarrow N = \dfrac{Rt_{1/2}}{0.693}$.

39. Let N_0 = No. of radioactive particle present at time t = 0

N = No. of radio active particle present at time t.

$\therefore N = N_0\, e^{-\lambda t}$ [λ - Radioactive decay constant]

∴ The no.of particles decay = $N_0 - N = N_0 - N_0 e^{-\lambda t} = N_0 (1 - e^{-\lambda t})$

We know, $A_0 = \lambda N_0$; $R = \lambda N_0$; $N_0 = R/\lambda$

From the above equation

$N = N_0 (1 - e^{-\lambda t}) = \dfrac{R}{\lambda}(1 - e^{-\lambda t})$ (substituting the value of N_0)

40. n = 1 mole = 6×10^{23} atoms, $t_{1/2}$ = 14.3 days

t = 70 hours, dN/dt in root after time t = λN

$N = No\, e^{-\lambda t} = 6 \times 10^{23} \times e^{\frac{-0.693 \times 70}{14.3 \times 24}} = 6 \times 10^{23} \times 0.868 = 5.209 \times 10^{23}$.

$5.209 \times 10^{23} \times \dfrac{-0.693}{14.3 \times 24} = \dfrac{0.0105 \times 10^{23}}{3600}$ dis/hour.

$= 2.9 \times 10^{-6} \times 10^{23}$ dis/sec = 2.9×10^{17} dis/sec.

Fraction of activity transmitted = $\left(\dfrac{1\mu ci}{2.9 \times 10^{17}}\right) \times 100\%$

$\Rightarrow \left(\dfrac{1 \times 3.7 \times 10^{8}}{2.9 \times 10^{11}} \times 100\right)\% = 1.275 \times 10^{-11}\ \%$.

41. V = 125 cm^3 = 0.125 L, P = 500 K pa = 5 atm.

T = 300 K, $t_{1/2}$ = 12.3 years = 3.82×10^{8} sec. Activity = λ × N

$N = n \times 6.023 \times 10^{23} = \dfrac{5 \times 0.125}{8.2 \times 10^{-2} \times 3 \times 10^{2}} \times 6.023 \times 10^{23} = 1.5 \times 10^{22}$ atoms.

$\lambda = \dfrac{0.693}{3.82 \times 10^{8}} = 0.1814 \times 10^{-8} = 1.81 \times 10^{-9}\ s^{-1}$

Activity = $\lambda N = 1.81 \times 10^{-9} \times 1.5 \times 10^{22} = 2.7 \times 10^{3}$ disintegration/sec

$= \dfrac{2.7 \times 10^{13}}{3.7 \times 10^{10}}$ Ci = 729 Ci.

42. ${}^{212}_{83}Bi \rightarrow {}^{208}_{81}Ti + {}^{4}_{2}He(\alpha)$

${}^{212}_{83}Bi \rightarrow {}^{212}_{84}Bi \rightarrow {}^{212}_{84}P_0 + e^{-}$

$t_{1/2}$ = 1 h. Time elapsed = 1 hour

at t = 0 Bi^{212} Present = 1 g

∴ at t = 1 Bi^{212} Present = 0.5 g

Probability α-decay and β-decay are in ratio 7/13.

∴ Tl remained = 0.175 g

∴ P_0 remained = 0.325 g

43. Activities of sample containing ^{108}Ag and ^{110}Ag isotopes = 8.0×10^8 disintegration/sec.

a) Here we take $A = 8 \times 10^8$ dis./sec

$\therefore$ i) $\ln (A_1/A_{0_1}) = \ln (11.794/8) = 0.389$

ii) $\ln (A_2/A_{0_2}) = \ln(9.1680/8) = 0.1362$

iii) $\ln (A_3/A_{0_3}) = \ln(7.4492/8) = -0.072$

iv) $\ln (A_4/A_{0_4}) = \ln(6.2684/8) = -0.244$

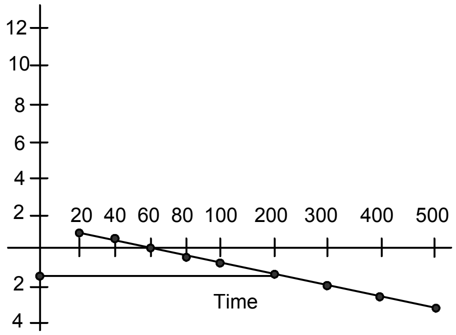
v) $\ln(5.4115/8) = -0.391$

vi) $\ln(3.0828/8) = -0.954$

vii) $\ln(1.8899/8) = -1.443$

viii) $\ln(1.167/8) = -1.93$

ix) $\ln(0.7212/8) = -2.406$



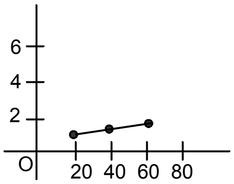
b) The half life of 110 Ag from this part of the plot is 24.4 s.

c) Half life of ^{110}Ag = 24.4 s.

$\therefore$ decay constant $\lambda = 0.693/24.4 = 0.0284 \Rightarrow t = 50$ sec,

The activity $A = A_0 e^{-\lambda t} = 8 \times 10^8 \times e^{-0.0284 \times 50} = 1.93 \times 10^8$

d)



e) The half life period of ^{108}Ag from the graph is 144 s.

44. $t_{1/2} = 24$ h

$$\therefore t_{1/2} = \frac{t_1 t_2}{t_1 + t_2} = \frac{24 \times 6}{24 + 6} = 4.8 \text{ h.}$$

$A_0 = 6$ rci ; $A = 3$ rci

$$\therefore A = \frac{A_0}{2^{t/t_{1/2}}} \Rightarrow 3 \text{ rci} = \frac{6 \text{ rci}}{2^{t/4.8\text{h}}} \Rightarrow \frac{t}{24.8\text{h}} = 2 \Rightarrow t = 4.8 \text{ h.}$$

45. $Q = qe^{-t/CR}$; $A = A_0 e^{-\lambda t}$

$$\frac{\text{Energy}}{\text{Activity}} = \frac{1q^2 \times e^{-2t/cR}}{2\,CA_0 e^{-\lambda t}}$$

Since the term is independent of time, so their coefficients can be equated,

So, $\frac{2t}{CR} = \lambda t$ or, $\lambda = \frac{2}{CR}$ or, $\frac{1}{\tau} = \frac{2}{CR}$ or, $R = 2\frac{\tau}{C}$ (Proved)

46. $R = 100\ \Omega$; $L = 100$ mH

After time t, $i = i_0\,(1 - e^{-t/Lr})$ $N = N_0\,(e^{-\lambda t})$

$\frac{i}{N} = \frac{i_0(1 - e^{-tR/L})}{N_0 e^{-\lambda t}}$ i/N is constant i.e. independent of time.

Coefficients of t are equal $-R/L = -\lambda \Rightarrow R/L = 0.693/t_{1/2}$

$= t_{1/2} = 0.693 \times 10^{-3} = 6.93 \times 10^{-4}$ sec.

47. 1 g of 'I' contain 0.007 g U^{235} So, 235 g contains 6.023×10^{23} atoms.

So, 0.7 g contains $\frac{6.023 \times 10^{23}}{235} \times 0.007$ atom

1 atom given 200 Mev. So, 0.7 g contains $\frac{6.023 \times 10^{23} \times 0.007 \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{235}$ J $= 5.74 \times 10^{-8}$ J.

48. Let n atoms disintegrate per second

Total energy emitted/sec = $(n \times 200 \times 10^6 \times 1.6 \times 10^{-19})$ J = Power

300 MW = 300×10^6 Watt = Power

$300 \times 10^{6} = n \times 200 \times 10^{6} \times 1.6 \times 10^{-19}$

$\Rightarrow n = \dfrac{3}{2 \times 1.6} \times 10^{19} = \dfrac{3}{3.2} \times 10^{19}$

6×10^{23} atoms are present in 238 grams

$\dfrac{3}{3.2} \times 10^{19}$ atoms are present in $\dfrac{238 \times 3 \times 10^{19}}{6 \times 10^{23} \times 3.2}$ = 3.7×10^{-4} g = 3.7 mg.

49. a) Energy radiated per fission = 2×10^{8} ev

Usable energy = $2 \times 10^{8} \times 25/100 = 5 \times 10^{7}$ ev = $5 \times 1.6 \times 10^{-12}$

Total energy needed = $300 \times 10^{8} = 3 \times 10^{8}$ J/s

No. of fission per second = $\dfrac{3 \times 10^{8}}{5 \times 1.6 \times 10^{-12}} = 0.375 \times 10^{20}$

No. of fission per day = $0.375 \times 10^{20} \times 3600 \times 24 = 3.24 \times 10^{24}$ fissions.

b) From ‘a’ No. of atoms disintegrated per day = 3.24×10^{24}

We have, 6.023×10^{23} atoms for 235 g

for 3.24×10^{24} atom = $\dfrac{235}{6.023 \times 10^{23}} \times 3.24 \times 10^{24}$ g = 1264 g/day = 1.264 kg/day.

50. a) ${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{1}^{3}H + {}_{1}^{1}H$

Q value = $2M({}_{1}^{2}H) = [M({}_{1}^{3}H) + M({}_{1}^{3}H)]$

= [2 × 2.014102 – (3.016049 + 1.007825)]u = 4.0275 Mev = 4.05 Mev.

b) ${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}H + n$

Q value = $2[M({}_{1}^{2}H) - M({}_{2}^{3}He) + M_{n}]$

= [2 × 2.014102 – (3.016049 + 1.008665)]u = 3.26 Mev = 3.25 Mev.

c) ${}_{1}^{2}H + {}_{1}^{3}H \rightarrow {}_{2}^{4}H + n$

Q value = $[M({}_{1}^{2}H) + M({}_{1}^{3}He) - M({}_{2}^{4}He) + M_{n}]$

= (2.014102 + 3.016049) – (4.002603 + 1.008665)]u = 17.58 Mev = 17.57 Mev.

51. PE = $\dfrac{Kq_{1}q_{2}}{r} = \dfrac{9 \times 10^{9} \times (2 \times 1.6 \times 10^{-19})^{2}}{r}$ …(1)

$1.5\text{ KT} = 1.5 \times 1.38 \times 10^{-23} \times T$ …(2)

Equating (1) and (2) $1.5 \times 1.38 \times 10^{-23} \times T = \dfrac{9 \times 10^{9} \times 10.24 \times 10^{-38}}{2 \times 10^{-15}}$

$\Rightarrow T = \dfrac{9 \times 10^{9} \times 10.24 \times 10^{-38}}{2 \times 10^{-15} \times 1.5 \times 1.38 \times 10^{-23}} = 22.26087 \times 10^{9}\text{ K} = 2.23 \times 10^{10}\text{ K}.$

52. ${}^{4}H + {}^{4}H \rightarrow {}^{8}Be$

$M({}^{2}H) \rightarrow 4.0026$ u

$M({}^{8}Be) \rightarrow 8.0053$ u

Q value = $[2\,M({}^{2}H) - M({}^{8}Be)]$ = (2 × 4.0026 – 8.0053) u

= –0.0001 u = –0.0931 Mev = –93.1 Kev.

53. In 18 g of N_0 of molecule = 6.023×10^{23}

In 100 g of N_0 of molecule = $\dfrac{6.023 \times 10^{26}}{18} = 3.346 \times 10^{25}$

∴ % of Deuterium = $3.346 \times 10^{26} \times 99.985$

Energy of Deuterium = 30.4486×10^{25} = (4.028204 – 3.016044) × 93

= 942.32 ev = 1507×10^{5} J = 1507 mJ

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