CHAPTER 44

X-RAYS

44.1 PRODUCTION OF X-RAYS

When highly energetic electrons are made to strike a metal target, electromagnetic radiation comes out. A large part of this radiation has wavelength of the order of 0·1 nm (≈ 1 A) and is known as *X-ray*.

X-ray was discovered by the German physicist W.C. Roentgen in 1895. He found that photographic film wrapped light-tight in black paper became exposed when placed near a cathode-ray tube. He concluded that some invisible radiation was coming from the cathode-ray tube which penetrated the black paper to affect the photographic plate. He named this radiation as X-ray because its nature and properties could not be known at that time. In mathematics, we generally use the symbol x for unknown quantities. However, after some calculation we finally get the value of this unknown x. Similarly, we now know about the nature and properties of X-rays.

A device used to produce X-rays is generally called an *X-ray tube*. Figure (41.1) shows a schematic diagram of such a device. This was originally designed by Coolidge and is known as *Coolidge tube* to produce X-rays.

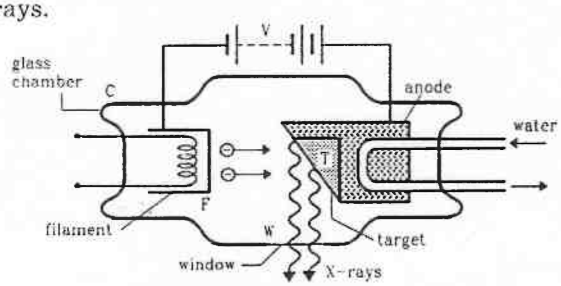


Figure 44.1

A filament F and a metallic target T are fixed in an evacuated glass chamber C. The filament is heated electrically and emits electrons by thermionic emission. A constant potential difference of several kilovolts is maintained between the filament and the target using a DC power supply so that the target is at a higher potential than the filament. The electrons emitted by the filament are, therefore, accelerated by the electric field set up between the filament and the target and hit the target with a very high speed. These electrons are stopped by the target and in the process X-rays are emitted. These X-rays are brought out of the tube through a window W made of thin mica or mylar or some such material which does not absorb X-rays appreciably.

In the process, large amount of heat is developed, and thus an arrangement is provided to cool down the tube continuously by running water.

The exact design of the X-ray tube depends on the type of use for which these X-rays are required.

44.2 CONTINUOUS AND CHARACTERISTIC X-RAYS

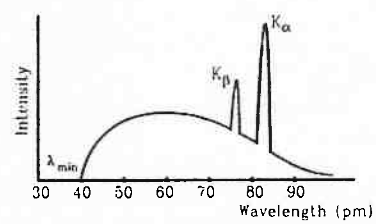


Figure 44.2

If the X-rays coming from a Coolidge tube are examined for the wavelengths present, and the intensity of different wavelength components are measured, we obtain a plot of the nature shown in figure (44.2). We see that there is a minimum wavelength below which no X-ray is emitted. This is called the *cutoff wavelength* or the *threshold wavelength*. The X-rays emitted can be clearly divided in two categories. At certain sharply defined wavelengths, the intensity of X-rays is very large as marked K_α, K_β in figure (44.2). These X-rays are known as *characteristic X-rays*. At other wavelengths the intensity varies gradually and these X-rays are called *continuous X-rays*. Let us examine the origin of these two types of X-rays.

Suppose, the potential difference applied between the target and the filament is V and electrons are

5. *The K_α X-ray of molybdenum has wavelength 71 pm. If the energy of a molybdenum atom with a K electron knocked out is 23·32 keV, what will be the energy of this atom when an L electron is knocked out?*

Solution : K_α X-ray results from the transition of an electron from L shell to K shell. If the energy of the atom with a vacancy in the K shell is E_K and the energy with a vacancy in the L shell is E_L, the energy of the photon emitted is $E_K - E_L$. The energy of the 71 pm photon is

$$E = \frac{hc}{\lambda}$$

$$= \frac{1242 \text{ eV-nm}}{71 \times 10^{-3} \text{ nm}} = 17{\cdot}5 \text{ keV}.$$

Thus, $E_K - E_L = 17{\cdot}5$ keV

or, $E_L = E_K - 17{\cdot}5$ keV

$= 23{\cdot}32 \text{ keV} - 17{\cdot}5 \text{ keV} = 5{\cdot}82$ keV.

6. *Show that the frequency of K_β X-ray of a material equals the sum of the frequencies of K_α and L_α X-rays of the same material.*

Solution :

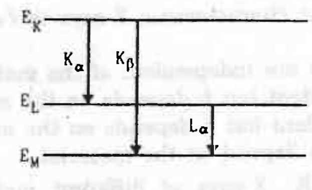


Figure 44-W1

The energy level diagram of an atom with one electron knocked out is shown in figure (44-W1).

Energy of K_α X-ray is $E_{K_\alpha} = E_K - E_L$

of K_β X-ray is $E_{K_\beta} = E_K - E_M$,

and of L_α X-ray is $E_{L_\alpha} = E_L - E_M$.

Thus, $E_{K_\beta} = E_{K_\alpha} + E_{L_\alpha}$

or, $h\nu_{K_\beta} = h\nu_{K_\alpha} + h\nu_{L_\alpha}$

or, $\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$.

□

QUESTIONS FOR SHORT ANSWER

1. When a Coolidge tube is operated for some time it becomes hot. Where does the heat come from?
2. In a Coolidge tube, electrons strike the target and stop inside it. Does the target get more and more negatively charged as time passes?
3. Can X-rays be used for photoelectric effect?
4. Can X-rays be polarized?
5. X-ray and visible light travel at the same speed in vacuum. Do they travel at the same speed in glass?
6. Characteristic X-rays may be used to identify the element from which they are coming. Can continuous X-rays be used for this purpose?
7. Is it possible that in a Coolidge tube characterstic L_α X-rays are emitted but not K_α X-rays?
8. Can L_α X-ray of one material have shorter wavelength than K_α X-ray of another?
9. Can a hydrogen atom emit characteristic X-ray?
10. Why is exposure to X-ray injurious to health but exposure to visible light is not, when both are electromagnetic waves?

OBJECTIVE I

1. X-ray beam can be deflected
(a) by an electric field (b) by a magnetic field
(c) by an electric field as well as by a magnetic field
(d) neither by an electric field nor by a magnetic field.

2. Consider a photon of continuous X-ray coming from a Coolidge tube. Its energy comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target.

3. The energy of a photon of characteristic X-ray from a Coolidge tube comes from
(a) the kinetic energy of the striking electron
(b) the kinetic energy of the free electrons of the target
(c) the kinetic energy of the ions of the target
(d) an atomic transition in the target.

4. If the potential difference applied to the tube is doubled and the separation between the filament and the target is also doubled, the cutoff wavelength
(a) will remain unchanged (b) will be doubled

(c) will be halved
(d) will become four times the original.

5. If the current in the circuit for heating the filament is increased, the cutoff wavelength
(a) will increase (b) will decrease
(c) will remain unchanged (d) will change.

6. Moseley's law for characteristic X-rays is $\sqrt{\nu} = a(Z - b)$. In this,
(a) both a and b are independent of the material
(b) a is independent but b depends on the material
(c) b is independent but a depends on the material
(d) both a and b depend on the material.

7. Frequencies of K_α X-rays of different materials are measured. Which one of the graphs in figure (44-Q1) may represent the relation between the frequency ν and the atomic number Z.

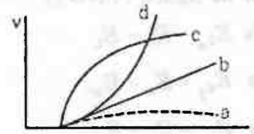


Figure 44-Q1

8. The X-ray beam coming from an X-ray tube
(a) is monochromatic
(b) has all wavelengths smaller than a certain maximum wavelength
(c) has all wavelengths greater than a certain minimum wavelength
(d) has all wavelengths lying between a minimum and a maximum wavelength.

9. One of the following wavelengths is absent and the rest are present in the X-rays coming from a Coolidge tube. Which one is the absent wavelength ?
(a) 25 pm (b) 50 pm (c) 75 pm (d) 100 pm.

10. Figure (44-Q2) shows the intensity–wavelength relations of X-rays coming from two different Coolidge tubes. The solid curve represents the relation for the tube A in which the potential difference between the target and the filament is V_A and the atomic number of the target material is Z_A. These quantities are V_B and Z_B for the other tube. Then,
(a) $V_A > V_B$, $Z_A > Z_B$ (b) $V_A > V_B$, $Z_A < Z_B$
(c) $V_A < V_B$, $Z_A > Z_B$ (d) $V_A < V_B$, $Z_A < Z_B$.

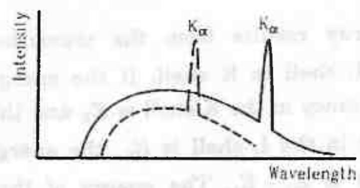


Figure 44-Q2

11. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. If the potential difference between the target and the filament is increased, the fraction of the X-ray passing through the same foil will be
(a) 0% (b) < 50% (c) 50% (d) > 50%.

12. 50% of the X-ray coming from a Coolidge tube is able to pass through a 0·1 mm thick aluminium foil. The potential difference between the target and the filament is increased. The thickness of aluminium foil, which will allow 50% of the X-ray to pass through, will be
(a) zero (b) < 0·1 mm (c) 0·1 mm (d) > 0·1 mm.

13. X-ray from a Coolidge tube is incident on a thin aluminium foil. The intensity of the X-ray transmitted by the foil is found to be I_0. The heating current is increased so as to increase the temperature of the filament. The intensity of the X-ray transmitted by the foil will be
(a) zero (b) $< I_0$ (c) I_0 (d) $> I_0$.

14. Visible light passing through a circular hole forms a diffraction disc of radius 0·1 mm on a screen. If X-ray is passed through the same set-up, the radius of the diffraction disc will be
(a) zero (b) < 0·1 mm (c) 0·1 mm (d) > 0·1 mm.

OBJECTIVE II

1. For harder X-rays,
(a) the wavelength is higher
(b) the intensity is higher
(c) the frequency is higher
(d) the photon energy is higher.

2. Cutoff wavelength of X-rays coming from a Coolidge tube depends on the
(a) target material (b) accelerating voltage
(c) separation between the target and the filament
(d) temperature of the filament.

3. Mark the correct options.
(a) An atom with a vacancy has smaller energy than a neutral atom.
(b) K X-ray is emitted when a hole makes a jump from the K shell to some other shell.
(c) The wavelength of K X-ray is smaller than the wavelength of L X-ray of the same material.
(d) The wavelength of K_α X-ray is smaller than the wavelength of K_β X-ray of the same material.

4. For a given material, the energy and wavelength of characterstic X-rays satisfy
(a) $E(K_\alpha) > E(K_\beta) > E(K_\gamma)$ (b) $E(M_\alpha) > E(L_\alpha) > E(K_\alpha)$
(c) $\lambda(K_\alpha) > \lambda(K_\beta) > \lambda(K_\gamma)$ (d) $\lambda(M_\alpha) > \lambda(L_\alpha) > \lambda(K_\alpha)$.

5. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
(a) the intensity increases
(b) the minimum wavelength increases
(c) the intensity remains unchanged
(d) the minimum wavelength decreases.

6. When an electron strikes the target in a Coolidge tube, its entire kinetic energy

(a) is converted into a photon
(b) may be converted into a photon
(c) is converted into heat
(d) may be converted into heat.

7. X-ray incident on a material
(a) exerts a force on it (b) transfers energy to it
(c) transfers momentum to it
(d) transfers impulse to it.

8. Consider a photon of continuous X-ray and a photon of characteristic X-ray of the same wavelength. Which of the following is/are different for the two photons?
(a) frequency (b) energy
(c) penetrating power (d) method of creation.

EXERCISES

Planck constant $h = 4.14 \times 10^{-15}$ eV-s, speed of light $c = 3 \times 10^{8}$ m/s.

1. Find the energy, the frequency and the momentum of an X-ray photon of wavelength 0·10 nm.
2. Iron emits K_α X-ray of energy 6·4 keV and calcium emits K_α X-ray of energy 3·69 keV. Calculate the times taken by an iron K_α photon and a calcium K_α photon to cross through a distance of 3 km.
3. Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 30 kV.
4. What potential difference should be applied across an X-ray tube to get X-ray of wavelength not less than 0·10 nm? What is the maximum energy of a photon of this X-ray in joule?
5. The X-ray coming from a Coolidge tube has a cutoff wavelength of 80 pm. Find the kinetic energy of the electrons hitting the target.
6. If the operating potential in an X-ray tube is increased by 1%, by what percentage does the cutoff wavelength decrease?
7. The distance between the cathode (filament) and the target in an X-ray tube is 1·5 m. If the cutoff wavelength is 30 pm, find the electric field between the cathode and the target.
8. The short-wavelength limit shifts by 26 pm when the operating voltage in an X-ray tube is increased to 1·5 times the original value. What was the original value of the operating voltage?
9. The electron beam in a colour TV is accelerated through 32 kV and then strikes the screen. What is the wavelength of the most energetic X-ray photon?
10. When 40 kV is applied across an X-ray tube, X-ray is obtained with a maximum frequency of 9.7×10^{18} Hz. Calculate the value of Planck constant from these data.
11. An X-ray tube operates at 40 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelengths emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.
12. The wavelength of K_α X-ray of tungsten is 21·3 pm. It takes 11·3 keV to knock out an electron from the L shell of a tungsten atom. What should be the minimum accelerating voltage across an X-ray tube having tungsten target which allows production of K_α X-ray?
13. The K_β X-ray of argon has a wavelength of 0·36 nm. The minimum energy needed to ionize an argon atom is 16 eV. Find the energy needed to knock out an electron from the K shell of an argon atom.
14. The K_α X-rays of aluminium ($Z = 13$) and zinc ($Z = 30$) have wavelengths 887 pm and 146 pm respectively. Use Moseley's law $\sqrt{\nu} = a(Z - b)$ to find the wavelength of the K_α X-ray of iron ($Z = 26$).
15. A certain element emits K_α X-ray of energy 3·69 keV. Use the data from the previous problem to identify the element.
16. The K_β X-rays from certain elements are given below. Draw a Moseley-type plot of $\sqrt{\nu}$ versus Z for K_β radiation.

Element	Ne	P	Ca	Mn	Zn	Br
Energy (keV)	0·858	2·14	4·02	6·51	9·57	13·3.

17. Use Moseley's law with $b = 1$ to find the frequency of the K_α X-ray of La($Z = 57$) if the frequency of the K_α X-ray of Cu($Z = 29$) is known to be 1.88×10^{18} Hz.
18. The K_α and K_β X-rays of molybdenum have wavelengths 0·71 Å and 0·63 Å respectively. Find the wavelength of L_α X-ray of molybdenum.
19. The wavelengths of K_α and L_α X-rays of a material are 21·3 pm and 141 pm respectively. Find the wavelength of K_β X-ray of the material.
20. The energy of a silver atom with a vacancy in K shell is 25·31 keV, in L shell is 3·56 keV and in M shell is 0·530 keV higher than the energy of the atom with no vacancy. Find the frequency of K_α, K_β and L_α X-rays of silver.
21. Find the maximum potential difference which may be applied across an X-ray tube with tungsten target without emitting any characteristic K or L X-ray. The energy levels of the tungsten atom with an electron knocked out are as follows.

Cell containing vacancy	K	L	M
Energy in keV	69.5	11.3	2.3

22. The electric current in an X-ray tube (from the target to the filament) operating at 40 kV is 10 mA. Assume that on an average, 1% of the total kinetic energy of the electrons hitting the target are converted into X-rays. (a) What is the total power emitted as X-rays and (b) how much heat is produced in the target every second?
23. Heat at the rate of 200 W is produced in an X-ray tube operating at 20 kV. Find the current in the circuit. Assume that only a small fraction of the kinetic energy of electrons is converted into X-rays.

24. Continuous X-rays are made to strike a tissue paper soaked with polluted water. The incoming X-rays excite the atoms of the sample by knocking out the electrons from the inner shells. Characteristic X-rays are subsequently emitted. The emitted X-rays are analysed and the intensity is plotted against the wavelength (figure 44-E1). Assuming that only K_α intensities are detected, list the elements present in the sample from the plot. Use Moseley's equation

$$\nu = (25 \times 10^{14}\ \text{Hz})(Z - 1)^2.$$

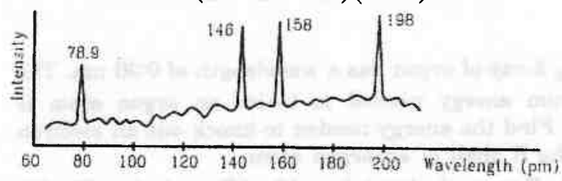


Figure 44-E1

25. A free atom of iron emits K_α X-rays of energy 6·4 keV. Calculate the recoil kinetic energy of the atom. Mass of an iron atom = $9·3 \times 10^{-26}$ kg.

26. The stopping potential in a photoelectric experiment is linearly related to the inverse of the wavelength ($1/\lambda$) of the light falling on the cathode. The potential difference applied across an X-ray tube is linearly related to the inverse of the cutoff wavelength ($1/\lambda$) of the X-ray emitted. Show that the slopes of the lines in the two cases are equal and find its value.

27. Suppose a monochromatic X-ray beam of wavelength 100 pm is sent through a Young's double slit and the interference pattern is observed on a photographic plate placed 40 cm away from the slit. What should be the separation between the slits so that the successive maxima on the screen are separated by a distance of 0·1 mm ?

□

ANSWERS

OBJECTIVE I

1. (d)	2. (a)	3. (d)	4. (c)	5. (c)	6. (a)
7. (d)	8. (c)	9. (a)	10. (b)	11. (d)	12. (d)
13. (d)	14. (b)				

OBJECTIVE II

1. (c), (d)	2. (b)	3. (b), (c)
4. (c), (d)	5. (c), (d)	6. (b), (d)
7. all	8. (d)	

EXERCISES

1. 12·4 keV, 3×10^{18} Hz, $6·62 \times 10^{-24}$ kg–m/s
2. 10 μs by both
3. 41·4 pm
4. 12·4 kV, $2·0 \times 10^{-15}$ J
5. 15·5 keV
6. approximately 1%
7. 27·7 kV/m
8. 15·9 kV
9. 38·8 pm
10. $4·12 \times 10^{-15}$ eV–s
11. 44·3 pm, 148 pm, 493 pm
12. 69·5 kV
13. 3·47 keV
14. 198 pm
15. calcium
17. $7·52 \times 10^{18}$ Hz
18. 5·64 Å
19. 18·5 pm
20. $5·25 \times 10^{18}$ Hz, $5·98 \times 10^{18}$ Hz, $7·32 \times 10^{17}$ Hz
21. less than 11·3 kV
22. (a) 4 W (b) 396 J
23. 10 mA
24. Zr, Zn, Cu, Fe
25. $3·9 \times 10^{-4}$ eV
26. $\frac{hc}{e} = 1·242 \times 10^{-6}$ V–m
27. 4×10^{-7} m

□

X - RAYS
CHAPTER 44

1. λ = 0.1 nm

a) Energy = $\frac{hc}{\lambda} = \frac{1242 \text{ ev.nm}}{0.1\text{ nm}}$

= 12420 ev = 12.42 Kev = 12.4 kev.

b) Frequency = $\frac{C}{\lambda} = \frac{3\times10^8}{0.1\times10^{-9}} = \frac{3\times10^8}{10^{-10}} = 3\times10^{18}\text{Hz}$

c) Momentum = E/C = $\frac{12.4\times10^3\times1.6\times10^{-19}}{3\times10^8}$ = 6.613×10^{-24} kg-m/s = 6.62×10^{-24} kg-m/s.

2. Distance = 3 km = 3×10^3 m

C = 3×10^8 m/s

t = $\frac{\text{Dist}}{\text{Speed}} = \frac{3\times10^3}{3\times10^8} = 10^{-5}$ sec.

$\Rightarrow 10 \times 10^{-8}$ sec = 10 μs in both case.

3. V = 30 KV

$\lambda = \frac{hc}{E} = \frac{hc}{eV} = \frac{1242 \text{ ev} - \text{nm}}{e\times30\times10^3}$ = 414×10^{-4} nm = 41.4 Pm.

4. λ = 0.10 nm = 10^{-10} m ; h = 6.63×10^{-34} J-s

C = 3×10^8 m/s ; e = 1.6×10^{-19} C

$\lambda_{min} = \frac{hc}{eV}$ or V = $\frac{hc}{e\lambda}$

= $\frac{6.63\times10^{-34}\times3\times10^8}{1.6\times10^{-19}\times10^{-10}}$ = 12.43×10^3 V = 12.4 KV.

Max. Energy = $\frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^8}{10^{-10}}$ = $19.89 \times 10^{-18} = 1.989 \times 10^{-15} = 2 \times 10^{-15}$ J.

5. λ = 80 pm, E = $\frac{hc}{\lambda} = \frac{1242}{80\times10^{-3}}$ = 15.525×10^3 eV = 15.5 KeV

6. We know $\lambda = \frac{hc}{V}$

Now $\lambda = \frac{hc}{1.01V} = \frac{\lambda}{1.01}$

$\lambda - \lambda' = \frac{0.01}{1.01}\lambda$.

% change of wave length = $\frac{0.01\times\lambda}{1.01\times\lambda}\times100 = \frac{1}{1.01}$ = 0.9900 = 1%.

7. d = 1.5 m, λ = 30 pm = 30×10^{-3} nm

E = $\frac{hc}{\lambda} = \frac{1242}{30\times10^{-3}}$ = 41.4×10^3 eV

Electric field = $\frac{V}{d} = \frac{41.4\times10^3}{1.5}$ = 27.6×10^3 V/m = 27.6 KV/m.

8. Given $\lambda' = \lambda - 26$ pm, V′ = 1.5 V

Now, $\lambda = \frac{hc}{ev}$, $\lambda' = \frac{hc}{ev'}$

or $\lambda V = \lambda' V'$

$\Rightarrow \lambda V = (\lambda - 26 \times 10^{-12}) \times 1.5$ V

$\Rightarrow \lambda = 1.5\,\lambda - 1.5 \times 26 \times 10^{-12}$

$\Rightarrow \lambda = \dfrac{39 \times 10^{-12}}{0.5} = 78 \times 10^{-12}$ m

$V = \dfrac{hc}{e\lambda} = \dfrac{6.63 \times 3 \times 10^{-34} \times 10^{8}}{1.6 \times 10^{-19} \times 78 \times 10^{-12}} = 0.15937 \times 10^{5} = 15.93 \times 10^{3}$ V = 15.93 KV.

9. V = 32 KV = 32×10^{3} V
   When accelerated through 32 KV
   E = 32×10^{3} eV
   $\lambda = \dfrac{hc}{E} = \dfrac{1242}{32 \times 10^{3}} = 38.8 \times 10^{-3}$ nm = 38.8 pm.

10. $\lambda = \dfrac{hc}{eV}$; V = 40 kV, f = 9.7×10^{18} Hz
    or, $\dfrac{h}{c} = \dfrac{h}{eV}$; or, $\dfrac{i}{f} = \dfrac{h}{eV}$; or $h = \dfrac{eV}{f} V - s$
    $= \dfrac{eV}{f} V - s = \dfrac{40 \times 10^{3}}{9.7 \times 10^{18}} = 4.12 \times 10^{-15}$ eV-s.

11. V = 40 KV = 40×10^{3} V
    Energy = 40×10^{3} eV
    Energy utilized = $\dfrac{70}{100} \times 40 \times 10^{3} = 28 \times 10^{3}$ eV
    $\lambda = \dfrac{hc}{E} = \dfrac{1242 - \text{ev nm}}{28 \times 10^{3}\ \text{ev}} \Rightarrow 44.35 \times 10^{-3}$ nm = 44.35 pm.
    For other wavelengths,
    E = 70% (left over energy) = $\dfrac{70}{100} \times (40 - 28)10^{3} = 84 \times 10^{2}$.
    $\lambda' = \dfrac{hc}{E} = \dfrac{1242}{8.4 \times 10^{3}} = 147.86 \times 10^{-3}$ nm = 147.86 pm = 148 pm.
    For third wavelength,
    $E = \dfrac{70}{100} = (12 - 8.4) \times 10^{3} = 7 \times 3.6 \times 10^{2} = 25.2 \times 10^{2}$
    $\lambda' = \dfrac{hc}{E} = \dfrac{1242}{25.2 \times 10^{2}} = 49.2857 \times 10^{-2}$ nm = 493 pm.

12. $K_{\lambda} = 21.3 \times 10^{-12}$ pm,   Now, $E_K - E_L = \dfrac{1242}{21.3 \times 10^{-3}} = 58.309$ kev
    E_L = 11.3 kev, E_K = 58.309 + 11.3 = 69.609 kev
    Now, Ve = 69.609 KeV,   or V = 69.609 KV.

13. λ = 0.36 nm
    $E = \dfrac{1242}{0.36} = 3450$ eV ($E_M - E_K$)
    Energy needed to ionize an organ atom = 16 eV
    Energy needed to knock out an electron from K-shell
    = (3450 + 16) eV = 3466 eV = 3.466 KeV.

14. λ_1 = 887 pm
    $v = \dfrac{C}{\lambda} = \dfrac{3 \times 10^{8}}{887 \times 10^{-12}} = 3.382 \times 10^{7} = 33.82 \times 10^{16} = 5.815 \times 10^{8}$
    λ_2 = 146 pm
    $v = \dfrac{3 \times 10^{8}}{146 \times 10^{-12}} = 0.02054 \times 10^{20} = 2.054 \times 10^{18} = 1.4331 \times 10^{9}$.

We know, $\sqrt{v} = a(z - b)$

$$\Rightarrow \frac{\sqrt{5.815 \times 10^8} = a(13 - b)}{\sqrt{1.4331 \times 10^9} = a(30 - b)}$$

$$\Rightarrow \frac{13 - b}{30 - b} = \frac{5.815 \times 10^{-1}}{1.4331} = 0.4057.$$

$\Rightarrow 30 \times 0.4057 - 0.4057\ b = 13 - b$

$\Rightarrow 12.171 - 0.4.57\ b + b = 13$

$$\Rightarrow b = \frac{0.829}{0.5943} = 1.39491$$

$$\Rightarrow a = \frac{5.815 \times 10^8}{11.33} = 0.51323 \times 10^8 = 5 \times 10^7.$$

For 'Fe',

$\sqrt{v} = 5 \times 10^7\ (26 - 1.39) = 5 \times 24.61 \times 10^7 = 123.05 \times 10^7$

$c/\lambda = 15141.3 \times 10^{14}$

$$= \lambda = \frac{3 \times 10^8}{15141.3 \times 10^{14}} = 0.000198 \times 10^{-6}\ m = 198 \times 10^{-12} = 198\ pm.$$

15. E = 3.69 kev = 3690 eV

$$\lambda = \frac{hc}{E} = \frac{1242}{3690} = 0.33658\ nm$$

$\sqrt{c/\lambda} = a(z - b)$; $a = 5 \times 10^7\ \sqrt{Hz}$, b = 1.37 (from previous problem)

$$\sqrt{\frac{3 \times 10^8}{0.34 \times 10^{-9}}} = 5 \times 10^7 (Z - 1.37) \Rightarrow \sqrt{8.82 \times 10^{17}} = 5 \times 10^7 (Z - 1.37)$$

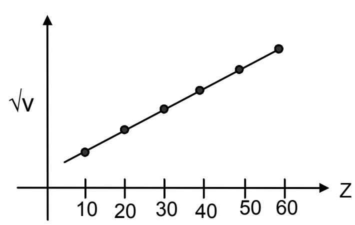
$\Rightarrow 9.39 \times 10^8 = 5 \times 10^7\ (Z - 1.37) \Rightarrow 93.9 / 5 = Z - 1.37$

$\Rightarrow Z = 20.15 = 20$

$\therefore$ The element is calcium.

16. K_B radiation is when the e jumps from

n = 3 to n = 1 (here n is principal quantum no)



$$\Delta E = h\nu = Rhc\ (z - h)^2 \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\Rightarrow \sqrt{v} = \sqrt{\frac{9RC}{8}}(z - h)$$

$\therefore \sqrt{v} \propto z$

Second method :

We can directly get value of v by `

hv = Energy

$$\Rightarrow v = \frac{\text{Energy(in kev)}}{h}$$

This we have to find out $\sqrt{v}$ and draw the same graph as above.

17. b = 1

For $\propto$ a (57)

$\sqrt{v} = a\ (Z - b)$

$\Rightarrow \sqrt{v} = a\ (57 - 1) = a \times 56$ …(1)

For Cu(29)

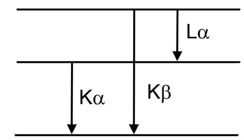
$\sqrt{1.88 \times 10^{78}} = a(29 - 1) = 28\ a$ …(2)

dividing (1) and (2)

$$\sqrt{\frac{v}{1.88 \times 10^{18}}} = \frac{a \times 56}{a \times 28} = 2.$$

$\Rightarrow v = 1.88 \times 10^{18}(2)^2 = 4 \times 1.88 \times 10^{18} = 7.52 \times 10^8$ Hz.

18. $K_\alpha = E_K - E_L$,,,(1) $\lambda K_\alpha = 0.71$ A°



$K_\beta = E_K - E_M$,,,(2) $\lambda K_\beta = 0.63$ A°

$L_\alpha = E_L - E_M$,,,(3)

Subtracting (2) from (1)

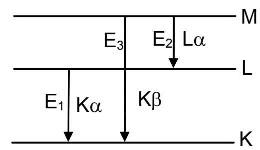
$K_\alpha - K_\beta = E_M - E_L = -L_\alpha$

or, $L_\alpha = K_\beta - K_\alpha = \dfrac{3 \times 10^8}{0.63 \times 10^{-10}} - \dfrac{3 \times 10^8}{0.71 \times 10^{-10}}$

$= 4.761 \times 10^{18} - 4.225 \times 10^{18} = 0.536 \times 10^{18}$ Hz.

Again $\lambda = \dfrac{3 \times 10^8}{0.536 \times 10^{18}} = 5.6 \times 10^{-10} = 5.6$ A°.

19. $E_1 = \dfrac{1242}{21.3 \times 10^{-3}} = 58.309 \times 10^3$ ev



$E_2 = \dfrac{1242}{141 \times 10^{-3}} = 8.8085 \times 10^3$ ev

$E_3 = E_1 + E_2 \Rightarrow (58.309 + 8.809)$ ev $= 67.118 \times 10^3$ ev

$\lambda = \dfrac{hc}{E_3} = \dfrac{1242}{67.118 \times 10^3} = 18.5 \times 10^{-3}$ nm = 18.5 pm.

20. $E_K = 25.31$ KeV, $E_L = 3.56$ KeV, $E_M = 0.530$ KeV

$K_\alpha = E_K - K_L = hv$

$\Rightarrow v = \dfrac{E_K - E_L}{h} = \dfrac{25.31 - 3.56}{4.14 \times 10^{-15}} \times 10^3 = 5.25 \times 10^{15}$ Hz

$K_\beta = E_K - K_M = hv$

$\Rightarrow v = \dfrac{E_K - E_M}{h} = \dfrac{25.31 - 0.53}{4.14 \times 10^{-15}} \times 10^3 = 5.985 \times 10^{18}$ Hz.

21. Let for, k series emission the potential required = v

$\therefore$ Energy of electrons = ev

This amount of energy ev = energy of L shell

The maximum potential difference that can be applied without emitting any electron is 11.3 ev.

22. V = 40 KV, i = 10 mA

1% of T_{KE} (Total Kinetic Energy) = X ray

i = ne or $n = \dfrac{10^{-2}}{1.6 \times 10^{-19}} = 0.625 \times 10^{17}$ no.of electrons.

KE of one electron = $eV = 1.6 \times 10^{-19} \times 40 \times 10^3 = 6.4 \times 10^{-15}$ J

$T_{KE} = 0.625 \times 6.4 \times 10^{17} \times 10^{-15} = 4 \times 10^2$ J.

a) Power emitted in X-ray = $4 \times 10^2 \times (-1/100) = 4$w

b) Heat produced in target per second = 400 – 4 = 396 J.

23. Heat produced/sec = 200 w

$\Rightarrow \dfrac{neV}{t} = 200 \Rightarrow (ne/t)V = 200$

$\Rightarrow$ i = 200 /V = 10 mA.

24. Given : $v = (25 \times 10^{14}$ Hz$)(Z - 1)^2$

Or $C/\lambda = 25 \times 10^{14} (Z - 1)^2$

a) $\dfrac{3 \times 10^8}{78.9 \times 10^{-12} \times 25 \times 10^{14}} = (Z-1)^2$

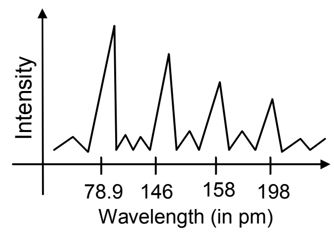
or, $(Z - 1)^2 = 0.001520 \times 10^6 = 1520$

$\Rightarrow$ Z – 1 = 38.98 or Z = 39.98 = 40. It is (Zr)

b) $\frac{3\times10^{8}}{146\times10^{-12}\times25\times10^{14}}=(Z-1)^{2}$

or, $(Z - 1)^2 = 0.0008219 \times 10^6$

$\Rightarrow$ Z – 1 = 28.669 or Z = 29.669 = 30. It is (Zn).



c) $\frac{3\times10^{8}}{158\times10^{-12}\times25\times10^{14}}=(Z-1)^{2}$

or, $(Z - 1)^2 = 0.0007594 \times 10^6$

$\Rightarrow$ Z – 1 = 27.5589 or Z = 28.5589 = 29. It is (Cu).

d) $\frac{3\times10^{8}}{198\times10^{-12}\times25\times10^{14}}=(Z-1)^{2}$

or, $(Z - 1)^2 = 0.000606 \times 10^6$

$\Rightarrow$ Z – 1 = 24.6182 or Z = 25.6182 = 26. It is (Fe).

25. Here energy of photon = E

E = 6.4 KeV = 6.4×10^3 ev

Momentum of Photon = E/C = $\frac{6.4\times10^{3}}{3\times10^{8}}$ = 3.41×10^{-24} m/sec.

According to collision theory of momentum of photon = momentum of atom

$\therefore$ Momentum of Atom = P = 3.41×10^{-24} m/sec

$\therefore$ Recoil K.E. of atom = $P^2 / 2m$

$\Rightarrow \frac{(3.41\times10^{-24})^{2}\,eV}{(2)(9.3\times10^{-26}\times1.6\times10^{-19})}=3.9$ eV [1 Joule = 1.6×10^{-19} ev]

26. $V_0 \rightarrow$ Stopping Potential, $\lambda \rightarrow$ Wavelength, $eV_0 = hv - hv_0$

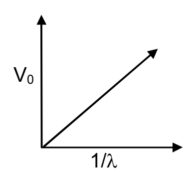
$eV_0 = hc/\lambda \Rightarrow V_0\lambda = hc/e$

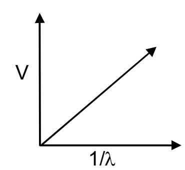
V $\rightarrow$ Potential difference across X-ray tube, $\lambda \rightarrow$ Cut of wavelength

λ = hc / eV or $V\lambda$ = hc / e

Slopes are same i.e. $V_0\lambda = V\lambda$

$\frac{hc}{e}=\frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}}$ = 1.242×10^{-6} Vm





27. λ = 10 pm = 100×10^{-12} m

D = 40 cm = 40×10^{-2} m

β = 0.1 mm = 0.1×10^{-3} m

$\beta=\frac{\lambda D}{d}$

$\Rightarrow$ d = $\frac{\lambda D}{\beta}=\frac{100\times10^{-12}\times40\times10^{-2}}{10^{-3}\times0.1}$ = 4×10^{-7} m.

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