CHAPTER 43

BOHR'S MODEL AND PHYSICS OF THE ATOM

43.1 EARLY ATOMIC MODELS

The idea that all matter is made of very small indivisible particles is very old. It has taken a long time, intelligent reasoning and classic experiments to cover the journey from this idea to the present day atomic models.

We can start our discussion with the mention of English scientist Robert Boyle (1627–1691) who studied the expansion and compression of air. The fact that air can be compressed or expanded, tells that air is made of tiny particles with lot of empty space between the particles. When air is compressed, these particles get closer to each other, reducing the empty space. We mention Robert Boyle here, because, with him atomism entered a new phase, from mere reasoning to experimental observations. The smallest unit of an element, which carries all the properties of the element is called an atom. Experiments on discharge tube, measurement of e/m by Thomson etc. established the existence of negatively charged electrons in the atoms. And then started the search for the structure of the positive charge inside an atom because the matter as a whole is electrically neutral.

Thomson's Model of the Atom

Thomson suggested in 1898 that the atom is a positively charged solid sphere and electrons are embedded in it in sufficient number so as to make the atom electrically neutral. One can compare Thomson's atom to a birthday cake in which cherries are embedded. This model was quite attractive as it could explain several observations available at that time. It could explain why only negatively charged particles are emitted when a metal is heated and never the positively charged particles. It could also explain the formation of ions and ionic compounds of chemistry.

Lenard's Suggestion

Lenard had noted that cathode rays could pass through materials of small thickness almost undeviated. If the atoms were solid spheres, most of the electrons in the cathode rays would hit them and would not be able to go ahead in the forward direction. Lenard, therefore, suggested in 1903 that the atom must have a lot of empty space in it. He proposed that the atom is made of electrons and similar tiny particles carrying positive charge. But then, the question was, why on heating a metal, these tiny positively charged particles were not ejected ?

Rutherford's Model of the Atom

Thomson's model and Lenard's model, both had certain advantages and disadvantages. Thomson's model made the positive charge immovable by assuming it to be spread over the total volume of the atom. On the other hand, electrons were tiny particles and could be ejected on heating a metal. But the almost free passage of cathode rays through an atom was not consistent with Thomson's model. For that, the atom should have a lot of empty space as suggested by Lenard. So, the positive charge should be in the form of tiny particles occupying a very small volume, yet these particles should not be able to come out on heating.

It was Ernest Rutherford who solved the problem by doing a series of experiments from 1906 to 1911 on alpha particle scattering.

In these experiments, a beam of alpha particles was bombarded on a thin gold foil and their deflections were studied (figure 43.1). Most of the alpha particles passed through the gold foil either undeviated or with a small deviation. This was expected because an alpha particle is a heavy particle and will brush aside any tiny particle coming in its way. However, some of the alpha particles were deflected by large angles.

to the third excited state is

$$13{\cdot}6\ \text{eV} - 0{\cdot}85\ \text{eV} = 12{\cdot}75\ \text{eV etc.}$$

Thus, 10·2 eV photons and 12·1 eV photons have large probability of being absorbed from the given range 6·21 eV to 12·42 eV. The corresponding wavelengths are

$$\frac{1242\ \text{eV-nm}}{10{\cdot}2\ \text{eV}} = 122\ \text{nm}$$

and $$\frac{1242\ \text{eV-nm}}{12{\cdot}1\ \text{eV}} = 103\ \text{nm}.$$

These wavelengths will have low intensity in the transmitted beam.

17. *A neutron moving with speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron ≈ mass of hydrogen = $1{\cdot}67 \times 10^{-27}$ kg.*

Solution : Suppose the neutron and the hydrogen atom move at speeds v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way. Using conservation of linear momentum and energy,

$$mv = mv_1 + mv_2 \qquad \text{(i)}$$

and $$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E. \qquad \text{(ii)}$$

From (i), $$v^2 = v_1^2 + v_2^2 + 2v_1v_2.$$

From (ii), $$v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}.$$

Thus, $$2v_1v_2 = \frac{2\Delta E}{m}.$$

Hence, $$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}.$$

As $v_1 - v_2$ must be real,

$$v^2 - \frac{4\Delta E}{m} > 0$$

or, $$\frac{1}{2}mv^2 > 2\Delta E.$$

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10·2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2}mv_m^2 = 2 \times 10{\cdot}2\ \text{eV} = 20{\cdot}4\ \text{eV}.$$

18. *Light corresponding to the transition n = 4 to n = 2 in hydrogen atoms falls on cesium metal (work function = 1·9 eV). Find the maximum kinetic energy of the photoelectrons emitted.*

Solution : The energy of the photons emitted in transition $n = 4$ to $n = 2$ is

$$h\nu = 13{\cdot}6\ \text{eV}\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = 2{\cdot}55\ \text{eV}.$$

The maximum kinetic energy of the photoelectrons is

$$= 2{\cdot}55\ \text{eV} - 1{\cdot}9\ \text{eV} = 0{\cdot}65\ \text{eV}.$$

19. *A small particle of mass m moves in such a way that the potential energy $U = \frac{1}{2}m^2\omega^2r^2$ where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to √n.*

Solution : The force at a distance r is

$$F = -\frac{dU}{dr} = -m^2\omega^2 r. \qquad \dots \text{(i)}$$

Suppose the particle moves along a circle of radius r. The net force on it should be mv^2/r along the radius. Comparing with (i),

$$\frac{mv^2}{r} = m^2\omega^2 r$$

or, $$v = \sqrt{m}\,\omega r. \qquad \dots \text{(ii)}$$

The quantization of angular momentum gives

$$mvr = \frac{nh}{2\pi}$$

or, $$v = \frac{nh}{2\pi mr}. \qquad \dots \text{(iii)}$$

From (ii) and (iii),

$$r = \left(\frac{nh}{2\pi m^{3/2}\omega}\right)^{1/2}$$

Thus, the radius of the nth orbit is proportional to $\sqrt{n}$.

□

QUESTIONS FOR SHORT ANSWER

1. How many wavelengths are emitted by atomic hydrogen in visible range (380 nm–780 nm)? In the range 50 nm to 100 nm?
2. The first excited energy of a He^+ ion is the same as the ground state energy of hydrogen. Is it always true that one of the energies of any hydrogen-like ion will be the same as the ground state energy of a hydrogen atom?
3. Which wavelengths will be emitted by a sample of atomic hydrogen gas (in ground state) if electrons of energy 12·2 eV collide with the atoms of the gas?
4. When white radiation is passed through a sample of hydrogen gas at room temperature, absorption lines are observed in Lyman series only. Explain.

5. Balmer series was observed and analysed before the other series. Can you suggest a reason for such an order ?

6. What will be the energy corresponding to the first excited state of a hydrogen atom if the potential energy of the atom is taken to be 10 eV when the electron is widely separated from the proton? Can we still write $E_n = E_1/n^2$? $r_n = a_0 n^2$?

7. The difference in the frequencies of series limit of Lyman series and Balmer series is equal to the frequency of the first line of the Lyman series. Explain.

8. The numerical value of ionization energy in eV equals the ionization potential in volts. Does the equality hold if these quantities are measured in some other units ?

9. We have stimulated emission and spontaneous emission. Do we also have stimulated absorption and spontaneous absorption ?

10. An atom is in its excited state. Does the probability of its coming to ground state depend on whether the radiation is already present or not? If yes, does it also depend on the wavelength of the radiation present ?

OBJECTIVE I

1. The minimum orbital angular momentum of the electron in a hydrogen atom is
(a) h (b) $h/2$ (c) $h/2\pi$ (d) h/λ.

2. Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12·1 eV, 10·2 eV and 1·9 eV. These photons must come from
(a) a single atom (b) two atoms
(c) three atoms (d) either two atoms or three atoms.

3. Suppose, the electron in a hydrogen atom makes transition from $n = 3$ to $n = 2$ in 10^{-8} s. The order of the torque acting on the electron in this period, using the relation between torque and angular momentum as discussed in the chapter on rotational mechanics is
(a) 10^{-34} N–m (b) 10^{-24} N–m
(c) 10^{-42} N–m (d) 10^{-8} N–m.

4. In which of the following transitions will the wavelength be minimum ?
(a) $n = 5$ to $n = 4$ (b) $n = 4$ to $n = 3$
(c) $n = 3$ to $n = 2$ (d) $n = 2$ to $n = 1$.

5. In which of the following systems will the radius of the first orbit ($n = 1$) be minimum ?
(a) hydrogen atom (b) deuterium atom
(c) singly ionized helium (d) doubly ionized lithium.

6. In which of the following systems will the wavelength corresponding to $n = 2$ to $n = 1$ be minimum ?
(a) hydrogen atom (b) deuterium atom
(c) singly ionized helium (d) doubly ionized lithium.

7. Which of the following curves may represent the speed of the electron in a hydrogen atom as a function of the principal quantum number n ?

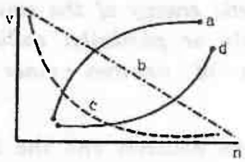


Figure 43-Q1

8. As one considers orbits with higher values of n in a hydrogen atom, the electric potential energy of the atom
(a) decreases (b) increases
(c) remains the same (d) does not increase.

9. The energy of an atom (or ion) in its ground state is – 54·4 eV. It may be
(a) hydrogen (b) deuterium (c) He^+ (d) Li^{++}.

10. The radius of the shortest orbit in a one-electron system is 18 pm. It may be
(a) hydrogen (b) deuterium (c) He^+ (d) Li^{++}.

11. A hydrogen atom in ground state absorbs 10·2 eV of energy. The orbital angular momentum of the electron is increased by
(a) $1·05 \times 10^{-34}$ J–s (b) $2·11 \times 10^{-34}$ J–s
(c) $3·16 \times 10^{-34}$ J–s (d) $4·22 \times 10^{-34}$ J–s.

12. Which of the following parameters are the same for all hydrogen-like atoms and ions in their ground states ?
(a) radius of the orbit (b) speed of the electron
(c) energy of the atom
(d) orbital angular momentum of the electron.

13. In a laser tube, all the photons
(a) have same wavelength (b) have same energy
(c) move in same direction (d) move with same speed.

OBJECTIVE II

1. In a laboratory experiment on emission from atomic hydrogen in a discharge tube, only a small number of lines are observed whereas a large number of lines are present in the hydrogen spectrum of a star. This is because in a laboratory
(a) the amount of hydrogen taken is much smaller than that present in the star
(b) the temperature of hydrogen is much smaller than that of the star
(c) the pressure of hydrogen is much smaller than that of the star
(d) the gravitational pull is much smaller than that in the star.

2. An electron with kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision
(a) must be elastic (b) may be partially elastic
(c) must be completely inelastic
(d) may be completely inelastic.
3. Which of the following products in a hydrogen atom are independent of the principal quantum number n? The symbols have their usual meanings.
(a) vn (b) Er (c) En (d) vr.
4. Let A_n be the area enclosed by the nth orbit in a hydrogen atom. The graph of $\ln(A_n/A_1)$ against $\ln(n)$
(a) will pass through the origin
(b) will be a straight line with slope 4
(c) will be a monotonically increasing nonlinear curve
(d) will be a circle.
5. Ionization energy of a hydrogen-like ion A is greater than that of another hydrogen-like ion B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state
(a) $r_A > r_B$ (b) $u_A > u_B$ (c) $E_A > E_B$ (d) $L_A > L_B$.
6. When a photon stimulates the emission of another photon, the two photons have
(a) same energy (b) same direction
(c) same phase (d) same wavelength.

EXERCISES

Planck constant $h = 6.63 \times 10^{-34}$ J–s $= 4.14 \times 10^{-15}$ eV–s, first Bohr radius of hydrogen $a_0 = 53$ pm. Energy of hydrogen atom in ground state $= -13.6$ eV. Rydberg's constant $= 1.097 \times 10^{7}\ \mathrm{m}^{-1}$.

1. The Bohr radius is given by $a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$. Verify that the RHS has dimensions of length.
2. Find the wavelength of the radiation emitted by hydrogen in the transitions (a) $n = 3$ to $n = 2$, (b) $n = 5$ to $n = 4$ and (c) $n = 10$ to $n = 9$.
3. Calculate the smallest wavelength of radiation that may be emitted by (a) hydrogen, (b) He^{+} and (c) Li^{++}.
4. Evaluate Rydberg constant by putting the values of the fundamental constants in its expression.
5. Find the binding energy of a hydrogen atom in the state $n = 2$.
6. Find the radius and energy of a He^{+} ion in the states (a) $n = 1$, (b) $n = 4$ and (c) $n = 10$.
7. A hydrogen atom emits ultraviolet radiation of wavelength 102.5 nm. What are the quantum numbers of the states involved in the transition?
8. (a) Find the first excitation potential of He^{+} ion. (b) Find the ionization potential of Li^{++} ion.
9. A group of hydrogen atoms are prepared in $n = 4$ states. List the wavelengths that are emitted as the atoms make transitions and return to $n = 2$ states.
10. A positive ion having just one electron ejects it if a photon of wavelength 228 Å or less is absorbed by it. Identify the ion.
11. Find the maximum Coulomb force that can act on the electron due to the nucleus in a hydrogen atom.
12. A hydrogen atom in a state having a binding energy of 0.85 eV makes transition to a state with excitation energy 10.2 eV. (a) Identify the quantum numbers n of the upper and the lower energy states involved in the transition. (b) Find the wavelength of the emitted radiation.
13. Whenever a photon is emitted by hydrogen in Balmer series, it is followed by another photon in Lyman series. What wavelength does this latter photon correspond to?
14. A hydrogen atom in state $n = 6$ makes two successive transitions and reaches the ground state. In the first transition a photon of 1.13 eV is emitted. (a) Find the energy of the photon emitted in the second transition. (b) What is the value of n in the intermediate state?
15. What is the energy of a hydrogen atom in the first excited state if the potential energy is taken to be zero in the ground state?
16. A hot gas emits radiation of wavelengths 46.0 nm, 82.8 nm and 103.5 nm only. Assume that the atoms have only two excited states and the difference between consecutive energy levels decreases as energy is increased. Taking the energy of the highest energy state to be zero, find the energies of the ground state and the first excited state.
17. A gas of hydrogen-like ions is prepared in a particular excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A.
18. Find the maximum angular speed of the electron of a hydrogen atom in a stationary orbit.
19. A spectroscopic instrument can resolve two nearby wavelengths λ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda$ is smaller than 8000. This is used to study the spectral lines of the Balmer series of hydrogen. Approximately how many lines will be resolved by the instrument?
20. Suppose, in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n changes by 2. (a) Find the smallest wavelength emitted by hydrogen. (b) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm).
21. According to Maxwell's theory of electrodynamics, an electron going in a circle should emit radiation of frequency equal to its frequency of revolution. What

should be the wavelength of the radiation emitted by a hydrogen atom in ground state if this rule is followed ?

22. The average kinetic energy of molecules in a gas at temperature T is 1·5 kT. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its atoms. Will hydrogen remain in molecular form at this temperature ? Take $k = 8{\cdot}62 \times 10^{-5}$ eV/K.

23. Find the temperature at which the average thermal kinetic energy is equal to the energy needed to take a hydrogen atom from its ground state to $n = 3$ state. Hydrogen can now emit red light of wavelength 653·1 nm. Because of Maxwellian distribution of speeds, a hydrogen sample emits red light at temperatures much lower than that obtained from this problem. Assume that hydrogen molecules dissociate into atoms.

24. Average lifetime of a hydrogen atom excited to $n = 2$ state is 10^{-8} s. Find the number of revolutions made by the electron on the average before it jumps to the ground state.

25. Calculate the magnetic dipole moment corresponding to the motion of the electron in the ground state of a hydrogen atom.

26. Show that the ratio of the magnetic dipole moment to the angular momentum ($l = mvr$) is a universal constant for hydrogen-like atoms and ions. Find its value.

27. A beam of light having wavelengths distributed uniformly between 450 nm to 550 nm passes through a sample of hydrogen gas. Which wavelength will have the least intensity in the transmitted beam ?

28. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms falls on helium ions in $n = 1$ and $n = 2$ states. What are the possible transitions of helium ions as they absorb energy from the radiation ?

29. A hydrogen atom in ground state absorbs a photon of ultraviolet radiation of wavelength 50 nm. Assuming that the entire photon energy is taken up by the electron, with what kinetic energy will the electron be ejected ?

30. A parallel beam of light of wavelength 100 nm passes through a sample of atomic hydrogen gas in ground state. (a) Assume that when a photon supplies some of its energy to a hydrogen atom, the rest of the energy appears as another photon moving in the same direction as the incident photon. Neglecting the light emitted by the excited hydrogen atoms in the direction of the incident beam, what wavelengths may be observed in the transmitted beam ? (b) A radiation detector is placed near the gas to detect radiation coming perpendicular to the incident beam. Find the wavelengths of radiation that may be detected by the detector.

31. A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface ($\Phi = 1{\cdot}9$ eV). These photoelectrons are made to collide with hydrogen atoms in ground state. Find the maximum value of λ for which (a) hydrogen atoms may be ionized, (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

32. Electrons are emitted from an electron gun at almost zero velocity and are accelerated by an electric field E through a distance of 1·0 m. The electrons are now scattered by an atomic hydrogen sample in ground state. What should be the minimum value of E so that red light of wavelength 656·3 nm may be emitted by the hydrogen ?

33. A neutron having kinetic energy 12·5 eV collides with a hydrogen atom at rest. Nelgect the difference in mass between the neutron and the hydrogen atom and assume that the neutron does not leave its line of motion. Find the possible kinetic energies of the neutron after the event.

34. A hydrogen atom moving at speed v collides with another hydrogen atom kept at rest. Find the minimum value of v for which one of the atoms may get ionized. The mass of a hydrogen atom $= 1{\cdot}67 \times 10^{-27}$ kg.

35. A neutron moving with a speed v strikes a hydrogen atom in ground state moving towards it with the same speed. Find the minimum speed of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron ≈ mass of hydrogen $= 1{\cdot}67 \times 10^{-27}$ kg.

36. When a photon is emitted by a hydrogen atom, the photon carries a momentum with it. (a) Calculate the momentum carried by the photon when a hydrogen atom emits light of wavelength 656·3 nm. (b) With what speed does the atom recoil during this transition ? Take the mass of the hydrogen atom $= 1{\cdot}67 \times 10^{-27}$ kg. (c) Find the kinetic energy of recoil of the atom.

37. When a photon is emitted from an atom, the atom recoils. The kinetic energy of recoil and the energy of the photon come from the difference in energies between the states involved in the transition. Suppose, a hydrogen atom changes its state from $n = 3$ to $n = 2$. Calculate the fractional change in the wavelength of light emitted, due to the recoil.

38. The light emitted in the transition $n = 3$ to $n = 2$ in hydrogen is called H_α light. Find the maximum work function a metal can have so that H_α light can emit photoelectrons from it.

39. Light from Balmer series of hydrogen is able to eject photoelectrons from a metal. What can be the maximum work function of the metal ?

40. Radiation from hydrogen discharge tube falls on a cesium plate. Find the maximum possible kinetic energy of the photoelectrons. Work function of cesium is 1·9 eV.

41. A filter transmits only the radiation of wavelength greater than 440 nm. Radiation from a hydrogen-discharge tube goes through such a filter and is incident on a metal of work function 2·0 eV. Find the stopping potential which can stop the photoelectrons.

42. The earth revolves round the sun due to gravitational attraction. Suppose that the sun and the earth are point particles with their existing masses and that Bohr's quantization rule for angular momentum is valid in the case of gravitation. (a) Calculate the minimum radius the earth can have for its orbit. (b) What is the value of the principal quantum number n for the present

radius ? Mass of the earth = $6\cdot0 \times 10^{24}$ kg, mass of the sun = $2\cdot0 \times 10^{30}$ kg, earth–sun distance = $1\cdot5 \times 10^{11}$ m.

43. Consider a neutron and an electron bound to each other due to gravitational force. Assuming Bohr's quantization rule for angular momentum to be valid in this case, derive an expression for the energy of the neutron–electron system.

44. A uniform magnetic field B exists in a region. An electron projected perpendicular to the field goes in a circle. Assuming Bohr's quantization rule for angular momentum, calculate (a) the smallest possible radius of the electron (b) the radius of the nth orbit and (c) the minimum possible speed of the electron.

45. Suppose in an imaginary world the angular momentum is quantized to be even integral multiples of $h/2\pi$. What is the longest possible wavelength emitted by hydrogen atoms in visible range in such a world according to Bohr's model ?

46. Consider an excited hydrogen atom in state n moving with a velocity $v(v << c)$. It emits a photon in the direction of its motion and changes its state to a lower state m. Apply momentum and energy conservation principles to calculate the frequency ν of the emitted radiation. Compare this with the frequency ν_0 emitted if the atom were at rest.

□

ANSWERS

OBJECTIVE I

1. (c)	2. (d)	3. (b)	4. (d)	5. (d)	6. (d)
7. (c)	8. (b)	9. (c)	10. (d)	11. (a)	12. (d)
13. (d)					

OBJECTIVE II

1. (b)	2. (a)	3. (a), (b)	4. (a), (b)
5. (b)	6. all.		

EXERCISES

2. (a) 654 nm (b) 4050 nm (c) 38860 nm
3. (a) 91 nm (b) 23 nm (c) 10 nm
4. $1\cdot097 \times 10^{7}$ m^{-1}
5. 3·4 eV
6. (a) 0·265 A, − 54·4 eV (b) 4·24 A, − 3·4 eV (c) 26·5 A, − 0·544 eV
7. 1 and 3
8. (a) 40·8 V (b) 122·4 V
9. 487 nm, 654 nm, 1910 nm
10. He^{+}
11. $8\cdot2 \times 10^{-8}$ N
12. (a) 4, 2 (b) 487 nm
13. 122 nm
14. 12·1 eV, 3
15. 23·8 eV
16. − 27 eV, − 12 eV
17. He^{+}, 4
18. $4\cdot1 \times 10^{16}$ rad/s
19. 38
20. (a) 103 nm (b) 487 nm
21. 45·7 nm
22. $1\cdot05 \times 10^{5}$ K
23. $9\cdot4 \times 10^{4}$ K
24. $8\cdot2 \times 10^{6}$
25. $9\cdot2 \times 10^{-24}$ A–m^{2}
26. $\frac{e}{2m} = 8\cdot8 \times 10^{10}$ C/kg
27. 487 nm
28. $n = 2$ to $n = 3$ and $n = 2$ to $n = 4$
29. 11·24 eV
30. (a) 100 nm, 560 nm, 3880 nm (b) 103 nm, 121 nm, 654 nm
31. (a) 80 nm (b) 102 nm (c) 89 nm
32. 12·1 V/m
33. zero
34. $7\cdot2 \times 10^{4}$ m/s
35. $3\cdot13 \times 10^{4}$ m/s
36. (a) $1\cdot0 \times 10^{27}$ kg–m/s (b) 0 6 m/s (c) $1\cdot9 \times 10^{-9}$ eV
37. 10^{-9}
38. 1·9 eV
39. 3·4 eV
40. 11·7 eV
41. 0·55 V
42. (a) $2\cdot3 \times 10^{-138}$ m (b) $2\cdot5 \times 10^{74}$

43. $-\dfrac{2\pi^2 G^2 m_n^2 m_e^3}{2h^2 n^2}$

44. (a) $\sqrt{\dfrac{h}{2\pi eB}}$ (b) $\sqrt{\dfrac{nh}{2\pi eB}}$ (c) $\sqrt{\dfrac{heB}{2\pi m^2}}$

45. 487 nm

46. $\nu = \nu_0 \left(1 + \dfrac{v}{c}\right)$

□

BOHR'S THEORY AND PHYSICS OF ATOM
CHAPTER 43

1. $a_0 = \dfrac{\varepsilon_0 h^2}{\pi m e^2} = \dfrac{A^2T^2(ML^2T^{-1})^2}{L^2MLT^{-2}M(AT)^2} = \dfrac{M^2L^4T^{-2}}{M^2L^3T^{-2}} = L$

   $\therefore a_0$ has dimensions of length.

2. We know, $\bar{\lambda} = 1/\lambda$ = $1.1 \times 10^7 \times (1/n_1^2 - 1/n_2^2)$

   a) $n_1 = 2$, $n_2 = 3$

   or, $1/\lambda = 1.1 \times 10^7 \times (1/4 - 1/9)$

   or, $\lambda = \dfrac{36}{5 \times 1.1 \times 10^7}$ = 6.54×10^{-7} = 654 nm

   b) $n_1 = 4$, $n_2 = 5$

   $\bar{\lambda} = 1/\lambda$ = 1.1×10^7 (1/16 – 1/25)

   or, $\lambda = \dfrac{400}{1.1 \times 10^7 \times 9}$ = 40.404×10^{-7} m = 4040.4 nm

   for R = 1.097×10^7, λ = 4050 nm

   c) $n_1 = 9$, $n_2 = 10$

   $1/\lambda = 1.1 \times 10^7$ (1/81 – 1/100)

   or, $\lambda = \dfrac{8100}{19 \times 1.1 \times 10^7}$ = 387.5598×10^{-7} = 38755.9 nm

   for R = 1.097×10^7; λ = 38861.9 nm

3. Small wave length is emitted i.e. longest energy

   $n_1 = 1$, $n_2 = \infty$

   a) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \dfrac{1}{\lambda} = 1.1 \times 10^7 \left(\dfrac{1}{1} - \dfrac{1}{\infty}\right)$

   $\Rightarrow \lambda = \dfrac{1}{1.1 \times 10^7} = \dfrac{1}{1.1} \times 10^{-7}$ = 0.909×10^{-7} = 90.9×10^{-8} = 91 nm.

   b) $\dfrac{1}{\lambda} = z^2 R\left(\dfrac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \dfrac{1}{1.1 \times 10^{-7} z^2} = \dfrac{91 \text{ nm}}{4}$ = 23 nm

   c) $\dfrac{1}{\lambda} = z^2 R\left(\dfrac{1}{n_1^2 - n_2^2}\right)$

   $\Rightarrow \lambda = \dfrac{91 \text{ nm}}{z^2} = \dfrac{91}{9}$ = 10 nm

4. Rydberg's constant = $\dfrac{me^4}{8h^3C\varepsilon_0^2}$

   $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ c, $h = 6.63 \times 10^{-34}$ J-S, $C = 3 \times 10^8$ m/s, $\varepsilon_0 = 8.85 \times 10^{-12}$

   or, $R = \dfrac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (6.63 \times 10^{-34})^3 \times 3 \times 10^8 \times (8.85 \times 10^{-12})^2}$ = 1.097×10^7 m^{-1}

5. $n_1 = 2$, $n_2 = \infty$

   $E = \dfrac{-13.6}{n_1^2} - \dfrac{-13.6}{n_2^2} = 13.6\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

   = 13.6 ($1/\infty$ – 1/4) = –13.6/4 = –3.4 eV

6. a) n = 1, r = $\frac{\varepsilon_0 h^2 n^2}{\pi m Z e^2} = \frac{0.53 n^2}{Z}$ A°

   = $\frac{0.53 \times 1}{2}$ = 0.265 A°

   ε = $\frac{-13.6 z^2}{n^2} = \frac{-13.6 \times 4}{1}$ = –54.4 eV

   b) n = 4, r = $\frac{0.53 \times 16}{2}$ = 4.24 A

   ε = $\frac{-13.6 \times 4}{164}$ = –3.4 eV

   c) n = 10, r = $\frac{0.53 \times 100}{2}$ = 26.5 A

   ε = $\frac{-13.6 \times 4}{100}$ = –0.544 A

7. As the light emitted lies in ultraviolet range the line lies in hyman series.

   $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

   $\Rightarrow \frac{1}{102.5 \times 10^{-9}}$ = 1.1 × 10^7 ($1/1^2 - 1/n_2^2$)

   $\Rightarrow \frac{10^9}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2) \Rightarrow \frac{10^2}{102.5} = 1.1 \times 10^7 (1 - 1/n_2^2)$

   $\Rightarrow 1 - \frac{1}{n_2^2} = \frac{100}{102.5 \times 1.1} \Rightarrow \frac{1}{n_2^2} = \frac{1 - 100}{102.5 \times 1.1}$

   $\Rightarrow$ n_2 = 2.97 = 3.

8. a) First excitation potential of
   He^+ = 10.2 × z^2 = 10.2 × 4 = 40.8 V

   b) Ionization potential of L_1^{++}
   = 13.6 V × z^2 = 13.6 × 9 = 122.4 V

9. $n_1 = 4 \rightarrow n_2 = 2$

   $n_1 = 4 \rightarrow 3 \rightarrow 2$

   $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{4}\right)$

   $\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1-4}{16}\right) \Rightarrow \frac{1.097 \times 10^7 \times 3}{16}$

   $\Rightarrow \lambda$ = $\frac{16 \times 10^{-7}}{3 \times 1.097}$ = 4.8617 × 10^{-7}

   = 1.861 × 10^{-9} = 487 nm

   n_1 = 4 and n_2 = 3

   $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{9}\right)$

   $\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{9-16}{144}\right) \Rightarrow \frac{1.097 \times 10^7 \times 7}{144}$

   $\Rightarrow \lambda$ = $\frac{144}{7 \times 1.097 \times 10^7}$ = 1875 nm

   $n_1 = 3 \rightarrow n_2 = 2$

   $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{4}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{4-9}{36}\right) \Rightarrow \frac{1.097 \times 10^7 \times 5}{66}$

$\Rightarrow \lambda = \frac{36 \times 10^{-7}}{5 \times 1.097} = 656$ nm

10. λ = 228 A°

E = $\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{228 \times 10^{-10}} = 0.0872 \times 10^{-16}$

The transition takes place form n = 1 to n = 2

Now, ex. $13.6 \times 3/4 \times z^2 = 0.0872 \times 10^{-16}$

$\Rightarrow z^2 = \frac{0.0872 \times 10^{-16} \times 4}{13.6 \times 3 \times 1.6 \times 10^{-19}} = 5.3$

z = $\sqrt{5.3}$ = 2.3

The ion may be Helium.

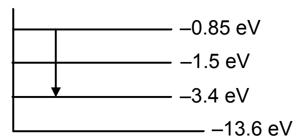
11. F = $\frac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

[Smallest dist. Between the electron and nucleus in the radius of first Bohrs orbit]

= $\frac{(1.6 \times 10^{-19}) \times (1.6 \times 10^{-19}) \times 9 \times 10^9}{(0.53 \times 10^{-10})^2} = 82.02 \times 10^{-9} = 8.202 \times 10^{-8} = 8.2 \times 10^{-8}$ N

12. a) From the energy data we see that the H atom transists from binding energy of 0.85 ev to exitation energy of 10.2 ev = Binding Energy of –3.4 ev.

So, n = 4 to n = 2



b) We know = $1/\lambda = 1.097 \times 10^7$ (1/4 – 1/16)

$\Rightarrow \lambda = \frac{16}{1.097 \times 3 \times 10^7} = 4.8617 \times 10^{-7}$ = 487 nm.

13. The second wavelength is from Balmer to hyman i.e. from n = 2 to n = 1

$n_1 = 2$ to $n_2 = 1$

$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{1^2}\right) \Rightarrow 1.097 \times 10^7 \left(\frac{1}{4} - 1\right)$

$\Rightarrow \lambda = \frac{4}{1.097 \times 3} \times 10^{-7}$

= $1.215 \times 10^{-7} = 121.5 \times 10^{-9}$ = 122 nm.

14. Energy at n = 6, E = $\frac{-13.6}{36}$ = –0.3777777

Energy in groundstate = –13.6 eV

Energy emitted in Second transition = –13.6 –(0.37777 + 1.13)

= –12.09 = 12.1 eV

b) Energy in the intermediate state = 1.13 ev + 0.0377777

= 1.507777 = $\frac{13.6 \times z^2}{n^2} = \frac{13.6}{n^2}$

or, n = $\sqrt{\frac{13.6}{1.507}}$ = 3.03 = 3 = n.

15. The potential energy of a hydrogen atom is zero in ground state.

An electron is board to the nucleus with energy 13.6 ev.,

Show we have to give energy of 13.6 ev. To cancel that energy.

Then additional 10.2 ev. is required to attain first excited state.

Total energy of an atom in the first excited state is = 13.6 ev. + 10.2 ev. = 23.8 ev.

16. Energy in ground state is the energy acquired in the transition of 2nd excited state to ground state.
As 2nd excited state is taken as zero level.

$$E = \frac{hc}{\lambda_1} = \frac{4.14\times10^{-15}\times3\times10^{8}}{46\times10^{-9}} = \frac{1242}{46} = 27 \text{ ev.}$$

Again energy in the first excited state

$$E = \frac{hc}{\lambda_{II}} = \frac{4.14\times10^{-15}\times3\times10^{8}}{103.5} = 12 \text{ ev.}$$

17. a) The gas emits 6 wavelengths, let it be in nth excited state.

$\Rightarrow \frac{n(n-1)}{2} = 6 \Rightarrow n = 4 \;\; \therefore$ The gas is in 4th excited state.

b) Total no.of wavelengths in the transition is 6. We have $\frac{n(n-1)}{2} = 6 \Rightarrow n = 4$.

18. a) We know, $m \upsilon r = \frac{nh}{2\pi} \Rightarrow mr^2 w = \frac{nh}{2\pi} \Rightarrow w = \frac{hn}{2\pi\times m\times r^2}$

$$= \frac{1\times6.63\times10^{-34}}{2\times3.14\times9.1\times10^{-31}\times(0.53)^2\times10^{-20}} = 0.413\times10^{17} \text{ rad/s} = 4.13\times10^{17} \text{ rad/s.}$$

19. The range of Balmer series is 656.3 nm to 365 nm. It can resolve λ and $\lambda + \Delta\lambda$ if $\lambda/\Delta\lambda = 8000$.

$\therefore$ No.of wavelengths in the range $= \frac{656.3-365}{8000} = 36$

Total no.of lines 36 + 2 = 38 [extra two is for first and last wavelength]

20. a) $n_1 = 1$, $n_2 = 3$, $E = 13.6\,(1/1 - 1/9) = 13.6\times8/9 = hc/\lambda$

or, $\frac{13.6\times8}{9} = \frac{4.14\times10^{-15}\times3\times10^{8}}{\lambda} \Rightarrow \lambda = \frac{4.14\times3\times10^{-7}}{13.6\times8} = 1.027\times10^{-7} = 103$ nm.

b) As 'n' changes by 2, we may consider n = 2 to n = 4

then $E = 13.6\times(1/4 - 1/16) = 2.55$ ev and $2.55 = \frac{1242}{\lambda}$ or $\lambda = 487$ nm.

21. Frequency of the revolution in the ground state is $\frac{V_0}{2\pi r_0}$

[r_0 = radius of ground state, V_0 = velocity in the ground state]

$\therefore$Frequency of radiation emitted is $\frac{V_0}{2\pi r_0} = f$

$\therefore C = f\lambda \Rightarrow \lambda = C/f = \frac{C2\pi r_0}{V_0}$

$\therefore \lambda = \frac{C2\pi r_0}{V_0} = 45.686$ nm = 45.7 nm.

22. KE = 3/2 KT = 1.5 KT, $K = 8.62\times10^{-5}$ eV/k, Binding Energy $= -13.6\,(1/\infty - 1/1) = 13.6$ eV.
According to the question, 1.5 KT = 13.6

$\Rightarrow 1.5\times8.62\times10^{-5}\times T = 13.6$

$\Rightarrow T = \frac{13.6}{1.5\times8.62\times10^{-5}} = 1.05\times10^{5}$ K

No, because the molecule exists an H_2^+ which is impossible.

23. $K = 8.62\times10^{-5}$ eV/k
K.E. of H_2 molecules = 3/2 KT
Energy released, when atom goes from ground state to no = 3

$\Rightarrow 13.6\,(1/1 - 1/9) \Rightarrow 3/2\ KT = 13.6(1/1 - 1/9)$

$\Rightarrow 3/2\times8.62\times10^{-5}\ T = \frac{13.6\times8}{9}$

$\Rightarrow T = 0.9349\times10^{5} = 9.349\times10^{4} = 9.4\times10^{4}$ K.

24. n = 2, T = 10^{-8} s

Frequency = $\dfrac{me^4}{4\varepsilon_0^2 n^3 h^3}$

So, time period = 1/f = $\dfrac{4\varepsilon o^2 n^3 h^3}{me^4} \Rightarrow \dfrac{4 \times (8.85)^2 \times 2^3 \times (6.63)^3}{9.1 \times (1.6)^4} \times \dfrac{10^{-24} - 10^{-102}}{10^{-76}}$

= $12247.735 \times 10^{-19}$ sec.

No.of revolutions = $\dfrac{10^{-8}}{12247.735 \times 10^{-19}}$ = 8.16×10^5

= 8.2×10^6 revolution.

25. Dipole moment (μ)

= n i A = 1 $\times$ q/t A = qfA

= $e \times \dfrac{me^4}{4\varepsilon_0^2 h^3 n^3} \times (\pi r_0^2 n^2) = \dfrac{me^5 \times (\pi r_0^2 n^2)}{4\varepsilon_0^2 h^3 n^3}$

= $\dfrac{(9.1 \times 10^{-31})(1.6 \times 10^{-19})^5 \times \pi \times (0.53)^2 \times 10^{-20} \times 1}{4 \times (8.85 \times 10^{-12})^2 (6.64 \times 10^{-34})^3 (1)^3}$

= $0.0009176 \times 10^{-20} = 9.176 \times 10^{-24}$ A - m^2.

26. Magnetic Dipole moment = n i A = $\dfrac{e \times me^4 \times \pi r_n^2 n^2}{4\varepsilon_0^2 h^3 n^3}$

Angular momentum = mvr = $\dfrac{nh}{2\pi}$

Since the ratio of magnetic dipole moment and angular momentum is independent of Z.
Hence it is an universal constant.

Ratio = $\dfrac{e^5 \times m \times \pi r_0^2 n^2}{24\varepsilon_0 h^3 n^3} \times \dfrac{2\pi}{nh} \Rightarrow \dfrac{(1.6 \times 10^{-19})^5 \times (9.1 \times 10^{-31}) \times (3.14)^2 \times (0.53 \times 10^{-10})^2}{2 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^4 \times 1^2}$

= 8.73×10^{10} C/kg.

27. The energies associated with 450 nm radiation = $\dfrac{1242}{450}$ = 2.76 ev

Energy associated with 550 nm radiation = $\dfrac{1242}{550}$ = 2.258 = 2.26 ev.

The light comes under visible range
Thus, $n_1 = 2$, $n_2 = 3, 4, 5, \ldots\ldots$
$E_2 - E_3 = 13.6\ (1/2^2 - 1/3^2) = 1.9$ ev
$E_2 - E_4 = 13.6\ (1/4 - 1/16) = 2.55$ ev
$E_2 - E_5 = 13.6\ (1/4 - 1/25) = 2.856$ ev
Only $E_2 - E_4$ comes in the range of energy provided. So the wavelength corresponding to that energy will be absorbed.

$\lambda = \dfrac{1242}{2.55}$ = 487.05 nm = 487 nm

487 nm wavelength will be absorbed.

28. From transitions n =2 to n =1.
E = 13.6 (1/1 – 1/4) = 13.6 $\times$ 3/4 = 10.2 eV
Let in check the transitions possible on He. n = 1 to 2
$E_1 = 4 \times 13.6\ (1 - 1/4) = 40.8$ eV [E_1 > E hence it is not possible]
n = 1 to n = 3
$E_2 = 4 \times 13.6\ (1 - 1/9) = 48.3$ eV [E_2 > E hence impossible]
Similarly n = 1 to n = 4 is also not possible.
n = 2 to n = 3
$E_3 = 4 \times 13.6\ (1/4 - 1/9) = 7.56$ eV

n = 2 to n = 4
$E_4 = 4 \times 13.6\ (1/4 - 1/16) = 10.2$ eV
As, $E_3 < E$ and $E_4 = E$
Hence E_3 and E_4 can be possible.

29. $\lambda = 50$ nm
Work function = Energy required to remove the electron from $n_1 = 1$ to $n_2 = \infty$.
$E = 13.6\ (1/1 - 1/\infty) = 13.6$
$\frac{hc}{\lambda} - 13.6 = KE$
$\Rightarrow \frac{1242}{50} - 13.6 = KE \Rightarrow KE = 24.84 - 13.6 = 11.24$ eV.

30. $\lambda = 100$ nm
$E = \frac{hc}{\lambda} = \frac{1242}{100} = 12.42$ eV
a) The possible transitions may be E_1 to E_2
E_1 to E_2, energy absorbed = 10.2 eV
Energy left = 12.42 – 10.2 = 2.22 eV
$2.22\text{ eV} = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = 559.45 = 560$ nm
E_1 to E_3, Energy absorbed = 12.1 eV
Energy left = 12.42 – 12.1 = 0.32 eV
$0.32 = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = \frac{1242}{0.32} = 3881.2 = 3881$ nm
E_3 to E_4, Energy absorbed = 0.65
Energy left = 12.42 – 0.65 = 11.77 eV
$11.77 = \frac{hc}{\lambda} = \frac{1242}{\lambda}$ or $\lambda = \frac{1242}{11.77} = 105.52$
b) The energy absorbed by the H atom is now radiated perpendicular to the incident beam.
$\rightarrow 10.2 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{10.2} = 121.76$ nm
$\rightarrow 12.1 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{12.1} = 102.64$ nm
$\rightarrow 0.65 = \frac{hc}{\lambda}$ or $\lambda = \frac{1242}{0.65} = 1910.76$ nm

31. $\phi = 1.9$ eV
a) The hydrogen is ionized
$n_1 = 1$, $n_2 = \infty$
Energy required for ionization = $13.6\ (1/n_1^2 - 1/n_2^2) = 13.6$
$\frac{hc}{\lambda} - 1.9 = 13.6 \Rightarrow \lambda = 80.1\text{ nm} = 80$ nm.
b) For the electron to be excited from $n_1 = 1$ to $n_2 = 2$
$E = 13.6\ (1/n_1^2 - 1/n_2^2) = 13.6(1 - ¼) = \frac{13.6 \times 3}{4}$
$\frac{hc}{\lambda} - 1.9 = \frac{13.6 \times 3}{4} \Rightarrow \lambda = 1242 / 12.1 = 102.64 = 102$ nm.

32. The given wavelength in Balmer series.
The first line, which requires minimum energy is from $n_1 = 3$ to $n_2 = 2$.
$\therefore$ The energy should be equal to the energy required for transition from ground state to n = 3.
i.e. E = 13.6 [1 – (1/9)] = 12.09 eV
$\therefore$ Minimum value of electric field = 12.09 v/m = 12.1 v/m

33. In one dimensional elastic collision of two bodies of equal masses.
The initial velocities of bodies are interchanged after collision.
$\therefore$ Velocity of the neutron after collision is zero.
Hence, it has zero energy.

34. The hydrogen atoms after collision move with speeds v_1 and v_2.
$mv = mv_1 + mv_2$ …(1)
$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ …(2)
From (1) $v^2 = (v_1 + v_2)^2 = v_1^2 + v_2^2 + 2v_1v_2$
From (2) $v^2 = v_1^2 + v_2^2 + 2\Delta E/m$
$= 2v_1v_2 = \frac{2\Delta E}{m}$ …(3)
$(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2$
$\Rightarrow (v_1 - v_2) = v^2 - 4\Delta E/m$
For minimum value of 'v'
$v_1 = v_2 \Rightarrow v^2 - (4\Delta E/m) = 0$
$\Rightarrow v^2 = \frac{4\Delta E}{m} = \frac{4 \times 13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}$
$\Rightarrow v = \sqrt{\frac{4 \times 13.6 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = 7.2 \times 10^4$ m/s.

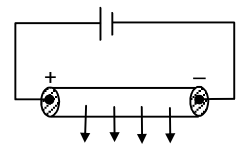
35. Energy of the neutron is $\frac{1}{2} mv^2$.
The condition for inelastic collision is $\Rightarrow \frac{1}{2} mv^2 > 2\Delta E$
$\Rightarrow \Delta E = \frac{1}{4} mv^2$
ΔE is the energy absorbed.
Energy required for first excited state is 10.2 ev.
$\therefore \Delta E < 10.2$ ev
$\therefore 10.2 \text{ ev} < \frac{1}{4} mv^2 \Rightarrow V_{min} = \sqrt{\frac{4 \times 10.2}{m}}$ ev
$\Rightarrow v = \sqrt{\frac{10.2 \times 1.6 \times 10^{-19} \times 4}{1.67 \times 10^{-27}}} = 6 \times 10^4$ m/sec.

36. a) $\lambda = 656.3$ nm
Momentum $P = E/C = \frac{hc}{\lambda} \times \frac{1}{c} = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{656.3 \times 10^{-9}} = 0.01 \times 10^{-25} = 1 \times 10^{-27}$ kg-m/s
b) $1 \times 10^{-27} = 1.67 \times 10^{-27} \times v$
$\Rightarrow v = 1/1.67 = 0.598 = 0.6$ m/s
c) KE of atom $= \frac{1}{2} \times 1.67 \times 10^{-27} \times (0.6)^2 = \frac{0.3006 \times 10^{-27}}{1.6 \times 10^{-19}}$ ev $= 1.9 \times 10^{-9}$ ev.

37. Difference in energy in the transition from n = 3 to n = 2 is 1.89 ev.
Let recoil energy be E.
$\frac{1}{2} m_e [V_2^2 - V_3^2] + E = 1.89 \text{ ev} \Rightarrow 1.89 \times 1.6 \times 10^{-19}$ J
$\therefore \frac{1}{2} \times 9.1 \times 10^{-31} \left[\left(\frac{2187}{2}\right)^2 - \left(\frac{2187}{3}\right)^2\right] + E = 3.024 \times 10^{-19}$ J
$\Rightarrow E = 3.024 \times 10^{-19} - 3.0225 \times 10^{-25}$

38. $n_1 = 2$, $n_2 = 3$
Energy possessed by H_α light
$= 13.6\ (1/n_1^2 - 1/n_2^2) = 13.6 \times (1/4 - 1/9) = 1.89$ eV.
For $H\alpha$ light to be able to emit photoelectrons from a metal the work function must be greater than or equal to 1.89 ev.

39. The maximum energy liberated by the Balmer Series is $n_1 = 2$, $n_2 = \infty$
$E = 13.6(1/n_1^2 - 1/n_2^2) = 13.6 \times 1/4 = 3.4$ eV
3.4 ev is the maximum work function of the metal.

40. Wocs = 1.9 eV

The radiations coming from the hydrogen discharge tube consist of photons of energy = 13.6 eV.
Maximum KE of photoelectrons emitted
= Energy of Photons – Work function of metal.
= 13.6 eV – 1.9 eV = 11.7 eV

41. $\lambda = 440$ nm, e = Charge of an electron, $\phi = 2$ eV, V_0 = stopping potential.
We have, $\frac{hc}{\lambda} - \phi = eV_0 \Rightarrow \frac{4.14 \times 10^{-15} \times 3 \times 10^8}{440 \times 10^{-9}} - 2eV = eV_0$
$\Rightarrow eV_0 = 0.823$ eV $\Rightarrow V_0 = 0.823$ volts.

42. Mass of Earth = Me = 6.0×10^{24} kg
Mass of Sun = Ms = 2.0×10^{30} kg
Earth – Sun dist = 1.5×10^{11} m

$mvr = \frac{nh}{2\pi}$ or, $m^2 v^2 r^2 = \frac{n^2h^2}{4\pi^2}$ …(1)

$\frac{GMeMs}{r^2} = \frac{Mev^2}{r}$ or $v^2 = GMs/r$ …(2)

Dividing (1) and (2)

We get $Me^2r = \frac{n^2h^2}{4\pi^2GMs}$

for n = 1

$r = \sqrt{\frac{h^2}{4\pi^2GMsMe^2}} = 2.29 \times 10^{-138}$ m $= 2.3 \times 10^{-138}$ m.

b) $n^2 = \frac{Me^2 \times r \times 4 \times \pi^2 \times G \times Ms}{h^2} = 2.5 \times 10^{74}$.

43. $m_eVr = \frac{nh}{z\pi}$ …(1)

$\frac{GM_nM_e}{r^2} = \frac{m_eV^2}{r} \Rightarrow \frac{GM_n}{r} = \nu^2$ …(2)

Squaring (2) and dividing it with (1)

$\frac{m_e^2v^2r^2}{\nu^2} = \frac{n^2h^2r}{4\pi^2Gm_n} \Rightarrow me^2r = \frac{n^2h^2r}{4\pi^2Gm_n} \Rightarrow r = \frac{n^2h^2r}{4\pi^2Gm_nme^2}$

$\Rightarrow \nu = \frac{nh}{2\pi rm_e}$ from (1)

$\Rightarrow \nu = \frac{nh4\pi^2GM_nM_e^2}{2\pi M_en^2h^2} = \frac{2\pi GM_nM_e}{nh}$

$KE = \frac{1}{2}m_eV^2 = \frac{1}{2}m_e\frac{(2\pi GM_nM_e)^2}{nh} = \frac{4\pi^2G^2M_n^2M_e^3}{2n^2h^2}$

$PE = \frac{-GM_nM_e}{r} = \frac{-GM_nM_e 4\pi^2GM_nM_e^2}{n^2h^2} = \frac{-4\pi^2G^2M_n^2M_e^3}{n^2h^2}$

Total energy = KE + PE $= \frac{2\pi^2G^2M_n^2M_e^3}{2n^2h^2}$

44. According to Bohr's quantization rule

$mvr = \frac{nh}{2\pi}$

'r' is less when 'n' has least value i.e. 1

or, $mv = \frac{nh}{2\pi R}$ …(1)

Again, $r = \frac{mv}{qB}$, or, $mv = rqB$ …(2)

From (1) and (2)

$rqB = \frac{nh}{2\pi r}$ [q = e]

$\Rightarrow r^2 = \frac{nh}{2\pi eB} \Rightarrow r = \sqrt{h/2\pi\ eB}$ [here n = 1]

b) For the radius of nth orbit, $r = \sqrt{\frac{nh}{2\pi eB}}$.

c) $mvr = \frac{nh}{2\pi}$, $r = \frac{mv}{qB}$

Substituting the value of 'r' in (1)

$mv \times \frac{mv}{qB} = \frac{nh}{2\pi}$

$\Rightarrow m^2v^2 = \frac{nheB}{2\pi}$ [n = 1, q = e]

$\Rightarrow v^2 = \frac{heB}{2\pi m^2} \Rightarrow$ or $v = \sqrt{\frac{heB}{2\pi m^2}}$.

45. even quantum numbers are allowed

$n_1 = 2$, $n_2 = 4 \rightarrow$ For minimum energy or for longest possible wavelength.

$E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 2.55$

$\Rightarrow 2.55 = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{hc}{2.55} = \frac{1242}{2.55} = 487.05$ nm = 487 nm

46. Velocity of hydrogen atom in state 'n' = u

Also the velocity of photon = u

But u << C

Here the photon is emitted as a wave.

So its velocity is same as that of hydrogen atom i.e. u.

$\therefore$ According to Doppler's effect

frequency $v = v_0\left(\frac{1+u/c}{1-u/c}\right)$

as u <<< C $1 - \frac{u}{c} = q$

$\therefore v = v_0\left(\frac{1+u/c}{1}\right) = v_0\left(1+\frac{u}{c}\right) \Rightarrow v = v_0\left(1+\frac{u}{c}\right)$

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