CHAPTER 40

ELECTROMAGNETIC WAVES

40.1 INTRODUCTION

We have seen that in certain situations light may be described as a wave. The wave equation for light propagating in x-direction in vacuum may be written as

$$E = E_0 \sin \omega(t - x/c)$$

where E is the sinusoidally varying electric field at the position x at time t. The constant c is the speed of light in vacuum. The electric field E is in the Y–Z plane, that is, perpendicular to the direction of propagation.

There is also a sinusoidally varying magnetic field associated with the electric field when light propagates. This magnetic field is perpendicular to the direction of propagation as well as to the electric field E. It is given by

$$B = B_0 \sin \omega(t - x/c).$$

Such a combination of mutually perpendicular electric and magnetic fields is referred to as an *electromagnetic wave* in vacuum. The theory of electromagnetic wave was mainly developed by Maxwell around 1864. We give a brief discussion of this theory.

40.2 MAXWELL'S DISPLACEMENT CURRENT

We have stated Ampere's law as

$$\oint \vec{B}\, d\vec{l} = \mu_0 i \qquad \dots (40.1)$$

where i is the electric current crossing a surface bounded by a closed curve and the line integral of $\vec{B}$ (circulation) is calculated along that closed curve. This equation is valid only when the electric field at the surface does not change with time. This law tells us that an electric current produces magnetic field and gives a method to calculate the field.

Ampere's law in this form is not valid if the electric field at the surface varies with time. As an example, consider a parallel-plate capacitor with circular plates, being charged by a battery (figure 40.1). If we place a compass needle in the space between the plates, the needle, in general, deflects. This shows that there is a magnetic field in this region. Figure (40.1) also shows a closed curve γ which lies completely in the region between the plates. The plane surface S bounded by this curve is also parallel to the plates and lies completely inside the region between the plates.

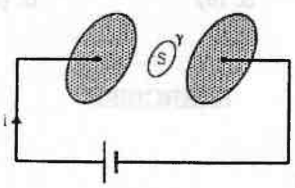


Figure 40.1

During the charging process, there is an electric current through the connecting wires. Charge is accummulated on the plates and the electric field at the points on the surface S changes. It is found that there is a magnetic field at the points on the curve γ and the circulation

$$\oint \vec{B} \cdot d\vec{l}$$

has a nonzero value. As no charge crosses the surface S, the electric current i through the surface is zero. Hence,

$$\oint \vec{B}\, d\vec{l} \neq \mu_0 i. \qquad \dots \text{(i)}$$

Now, Ampere's law (40.1) can be deduced from Biot–Savart law. We can calculate the magnetic field due to each current element from Biot–Savart law and then its circulation along the closed curve γ. The circulation of the magnetic field due to these current elements must satisfy equation (40.1). If we denote this magnetic field by $\vec{B}'$,

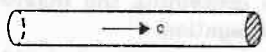
$$\oint \vec{B}' \cdot d\vec{l} = 0. \qquad \dots \text{(ii)}$$

This shows that the actual magnetic field $\vec{B}$ is different from the field $\vec{B}'$ produced by the electric currents only. So, there must be some other source of magnetic field. This other source is nothing but the

$$= (1.6 \times 10^{-19}\ \text{C}) \times (2.0 \times 10^{7}\ \text{m/s}) \times (10^{-6}\ \text{T})$$
$$= 3.2 \times 10^{-18}\ \text{N}.$$

4. *Find the energy stored in a* 60 cm *length of a laser beam operating at* 4 mW.

Solution :



60 cm

Figure 40-W1

The time taken by the electromagnetic wave to move through a distance of 60 cm is $t = \frac{60\ \text{cm}}{c} = 2 \times 10^{-9}$ s. The energy contained in the 60 cm length passes through a cross-section of the beam in 2×10^{-9} s (figure 40-W1). But the energy passing through any cross-section in 2×10^{-9} s is

$$U = (4\ \text{mW}) \times (2 \times 10^{-9}\ \text{s})$$
$$= (4 \times 10^{-3}\ \text{J/s}) \times (2 \times 10^{-9}\ \text{s})$$
$$= 8 \times 10^{-12}\ \text{J}.$$

This is the energy contained in 60 cm length.

5. *Find the amplitude of the electric field in a parallel beam of light of intensity* 2.0 W/m^2.

Solution :

The intensity of a plane electromagnetic wave is

$$I = u_{av}\, c = \frac{1}{2}\, \varepsilon_0 E_0^2\, c$$

or, $$E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

$$= \sqrt{\frac{2 \times (2.0\ \text{W/m}^2)}{\left(8.85 \times 10^{-12}\ \frac{\text{C}^2}{\text{N-m}^2}\right) \times (3 \times 10^{8}\ \text{m/s})}}$$

$$= 38.8\ \text{N/C}.$$

□

QUESTIONS FOR SHORT ANSWER

1. In a microwave oven, the food is kept in a plastic container and the microwave is directed towards the food. The food is cooked without melting or igniting the plastic container. Explain.
2. A metal rod is placed along the axis of a solenoid carrying a high-frequency alternating current. It is found that the rod gets heated. Explain why the rod gets heated.
3. Can an electromagnetic wave be deflected by an electric field ? By a magnetic field ?
4. A wire carries an alternating current $i = i_0 \sin \omega t$. Is there an electric field in the vicinity of the wire ?
5. A capacitor is connected to an alternating-current source. Is there a magnetic field between the plates ?
6. Can an electromagnetic wave be polarized ?
7. A plane electromagnetic wave is passing through a region. Consider the quantities (a) electric field, (b) magnetic field, (c) electrical energy in a small volume and (d) magnetic energy in a small volume. Construct pairs of the quantities that oscillate with equal frequencies.

OBJECTIVE I

1. A magnetic field can be produced by
   (a) a moving charge (b) a changing electric field
   (c) none of them (d) both of them.
2. A compass needle is placed in the gap of a parallel plate capacitor. The capacitor is connected to a battery through a resistance. The compass needle
   (a) does not deflect
   (b) deflects for a very short time and then comes back to the original position
   (c) deflects and remains deflected as long as the battery is connected
   (d) deflects and gradually comes to the original position in a time which is large compared to the time constant.
3. Dimensions of $1/(\mu_0 \varepsilon_0)$ is
   (a) L/T (b) T/L (c) L^2/T^2 (d) T^2/L^2.
4. Electromagnetic waves are produced by
   (a) a static charge (b) a moving charge
   (c) an accelerating charge (d) chargeless particles.
5. An electromagnetic wave going through vacuum is described by
   $$E = E_0 \sin(kx - \omega t);\ B = B_0 \sin(kx - \omega t).$$
   Then
   (a) $E_0\, k = B_0\, \omega$ (b) $E_0\, B_0 = \omega k$
   (c) $E_0\, \omega = B_0\, k$ (d) none of these.

6. An electric field $\vec{E}$ and a magnetic field $\vec{B}$ exist in a region. The fields are not perpendicular to each other.
(a) This is not possible.
(b) No electromagnetic wave is passing through the region.
(c) An electromagnetic wave may be passing through the region.
(d) An electromagnetic wave is certainly passing through the region.
7. Consider the following two statements regarding a linearly polarized, plane electromagnetic wave:
(A) The electric field and the magnetic field have equal average values.
(B) The electric energy and the magnetic energy have equal average values.
(a) Both A and B are true. (b) A is false but B is true.
(c) B is false but A is true. (d) Both A and B are false.
8. A free electron is placed in the path of a plane electromagnetic wave. The electron will start moving
(a) along the electric field
(b) along the magnetic field
(c) along the direction of propagation of the wave
(d) in a plane containing the magnetic field and the direction of propagation.
9. A plane electromagnetic wave is incident on a material surface. The wave delivers momentum p and energy E.
(a) $p = 0,\ E \neq 0$. (b) $p \neq 0,\ E = 0$.
(c) $p \neq 0,\ E \neq 0$. (d) $p = 0,\ E = 0$.

OBJECTIVE II

1. An electromagnetic wave going through vacuum is described by
$$E = E_0 \sin(kx - \omega t).$$
Which of the following is/are independent of the wavelength ?
(a) k (b) ω (c) k/ω (d) $k\omega$.
2. Displacement current goes through the gap between the plates of a capacitor when the charge of the capacitor
(a) increases (b) decreases
(c) does not change (d) is zero.
3. Speed of electromagnetic waves is the same
(a) for all wavelengths (b) in all media
(c) for all intensities (d) for all frequencies.
4. Which of the following have zero average value in a plane electromagnetic wave ?
(a) electric field (b) magnetic field
(c) electric energy (d) magnetic energy.
5. The energy contained in a small volume through which an electromagnetic wave is passing oscillates with
(a) zero frequency (b) the frequency of the wave
(c) half the frequency of the wave
(d) double the frequency of the wave.

EXERCISES

1. Show that the dimensions of the displacement current $\varepsilon_0 \frac{d\varphi_E}{dt}$ are that of an electric current.
2. A point charge is moving along a straight line with a constant velocity v. Consider a small area A perpendicular to the direction of motion of the charge (figure 40-E1). Calculate the displacement current through the area when its distance from the charge is x. The value of x is not large so that the electric field at any instant is essentially given by Coulomb's law.

Figure 40-E1

3. A parallel-plate capacitor having plate-area A and plate separation d is joined to a battery of emf $\mathcal{E}$ and internal resistance R at $t = 0$. Consider a plane surface of area $A/2$, parallel to the plates and situated symmetrically between them. Find the displacement current through this surface as a function of time.
4. Consider the situation of the previous problem. Define displacement resistance $R_d = V/i_d$ of the space between the plates where V is the potential difference between the plates and i_d is the displacement current. Show that R_d varies with time as
$$R_d = R(e^{t/\tau} - 1).$$
5. Using $B = \mu_0 H$ find the ratio E_0/H_0 for a plane electromagnetic wave propagating through vacuum. Show that it has the dimensions of electric resistance. This ratio is a universal constant called the *impedance of free space.*
6. The sunlight reaching the earth has maximum electric field of 810 V/m. What is the maximum magnetic field in this light ?
7. The magnetic field in a plane electromagnetic wave is given by
$$B = (200\ \mu T) \sin[(4.0 \times 10^{15}\ s^{-1})(t - x/c)].$$
Find the maximum electric field and the average energy density corresponding to the electric field.
8. A laser beam has intensity $2.5 \times 10^{14}\ W/m^2$. Find the amplitudes of electric and magnetic fields in the beam.
9. The intensity of the sunlight reaching the earth is $1380\ W/m^2$. Assume this light to be a plane, monochromatic wave. Find the amplitudes of electric and magnetic fields in this wave.

□

ANSWERS

OBJECTIVE I

1. (d) 2. (d) 3. (c) 4. (c) 5. (a) 6. (c)
7. (a) 8. (a) 9. (c)

OBJECTIVE II

1. (c) 2. (a), (b) 3. (c), 4. (a), (b)
5. (d)

EXERCISES

2. $\frac{q A v}{2 \pi x^3}$

3. $\frac{\mathcal{E}}{2R} e^{-\frac{t d}{\varepsilon A R}}$

5. 377 Ω

6. 2·7 μT

7. 6×10^4 N/C, 0·016 J/m^3

8. $4·3 \times 10^8$ N/C, 1·44 T

9. $1·02 \times 10^3$ N/C, $3·40 \times 10^{-6}$ T

□

ELECTROMAGNETIC WAVES
CHAPTER - 40

1. $\frac{\epsilon_0 \, d\phi_E}{dt} = \frac{\epsilon_0 \, EA}{dt \; 4\pi \, \varepsilon_0 r^2}$

$= \frac{M^{-1}L^{-3}T^4A^2}{M^{-1}L^{-3}A^2} \times \frac{A^1T^1}{L^2} \times \frac{L^2}{T} = A^1$

= (Current) (proved).

2. $E = \frac{Kq}{x^2}$, [from coulomb's law]

$\phi_E = EA = \frac{KqA}{x^2}$

$I_d = \epsilon_0 \frac{d\phi E}{dt} = \epsilon_0 \frac{d}{dt} \frac{kqA}{x^2} = \epsilon_0 \, KqA = \frac{d}{dt} x^{-2}$

$= \epsilon_0 \times \frac{1}{4\pi \, \epsilon_0} \times q \times A \times -2 \times x^{-3} \times \frac{dx}{dt} = \frac{qAv}{2\pi x^3}$.

3. $E = \frac{Q}{\epsilon_0 \, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0 \, A} \frac{A}{2} = \frac{Q}{\epsilon_0 \, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0 \, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

$= \frac{1}{2} \frac{d}{dt}(EC\,e^{-t/RC}) = \frac{1}{2} EC - \frac{1}{RC} e^{-t/RC} = \frac{-E}{2R} e^{\frac{-td}{RE_0\lambda}}$

4. $E = \frac{Q}{\epsilon_0 \, A}$ (Electric field)

$\phi = E.A. = \frac{Q}{\epsilon_0 \, A} \frac{A}{2} = \frac{Q}{\epsilon_0 \, 2}$

$i_0 = \epsilon_0 \frac{d\phi_E}{dt} = \epsilon_0 \frac{d}{dt}\left(\frac{Q}{\epsilon_0 \, 2}\right) = \frac{1}{2}\left(\frac{dQ}{dt}\right)$

5. $B = \mu_0 H$

$\Rightarrow H = \frac{B}{\mu_0}$

$\frac{E_0}{H_0} = \frac{B_0 / (\mu_0 \, \epsilon_0 \, C)}{B_0 / \mu_0} = \frac{1}{\epsilon_0 \, C}$

$= \frac{1}{8.85 \times 10^{-12} \times 3 \times 10^8}$ = 376.6 Ω = 377 Ω.

Dimension $\frac{1}{\epsilon_0 \, C} = \frac{1}{[LT^{-1}][M^{-1}L^{-3}T^4A^2]} = \frac{1}{M^{-1}L^{-2}T^3A^2}$ = $M^1L^2T^{-3}A^{-2}$ = [R].

6. E_0 = 810 V/m, B_0 = ?

We know, $B_0 = \mu_0 \, \epsilon_0 \, C \, E_0$

Putting the values,

$B_0 = 4\pi \times 10^{-7} \times 8.85 \times 10^{-12} \times 3 \times 10^8 \times 810$

$= 27010.9 \times 10^{-10} = 2.7 \times 10^{-6}$ T = 2.7 μT.

7. B = (200 μT) Sin [(4 $\times$ 10^{15} 5^{-1}) (t – x/C)]

   a) B_0 = 200 μT

   $E_0 = C \times B_0 = 200 \times 10^{-6} \times 3 \times 10^8 = 6 \times 10^4$

   b) Average energy density = $\frac{1}{2\mu_0}B_0^2 = \frac{(200\times10^{-6})^2}{2\times4\pi\times10^{-7}} = \frac{4\times10^{-8}}{8\pi\times10^{-7}} = \frac{1}{20\pi}$ = 0.0159 = 0.016.

8. $I = 2.5 \times 10^{14}$ W/m^2

   We know, $I = \frac{1}{2}\in_0 E_0^2 C$

   $\Rightarrow E_0^2 = \frac{2I}{\in_0 C}$ or $E_0 = \sqrt{\frac{2I}{\in_0 C}}$

   $E_0 = \sqrt{\frac{2\times2.5\times10^{14}}{8.85\times10^{-12}\times3\times10^8}} = 0.4339 \times 10^9 = 4.33 \times 10^8$ N/c.

   $B_0 = \mu_0 \in_0 C E_0$

   $= 4 \times 3.14 \times 10^{-7} \times 8.854 \times 10^{-12} \times 3 \times 10^8 \times 4.33 \times 10^8 = 1.44$ T.

9. Intensity of wave = $\frac{1}{2}\in_0 E_0^2 C$

   $\in_0 = 8.85 \times 10^{-12}$; E_0 = ? ; $C = 3 \times 10^8$, $I = 1380$ W/m^2

   $1380 = 1/2 \times 8.85 \times 10^{-12} \times E_0^2 \times 3 \times 10^8$

   $\Rightarrow E_0^2 = \frac{2\times1380}{8.85\times3\times10^{-4}} = 103.95 \times 10^4$

   $\Rightarrow E_0 = 10.195 \times 10^2 = 1.02 \times 10^3$

   $E_0 = B_0C$

   $\Rightarrow B_0 = E_0/C = \frac{1.02\times10^3}{3\times10^8} = 3.398 \times 10^{-5} = 3.4 \times 10^{-5}$ T.

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