6. *A series AC circuit contains an inductor* (20 mH), *a capacitor* (100 μF), *a resistor* (50 Ω) *and an AC source of* 12 V, 50 Hz. *Find the energy dissipated in the circuit in* 1000 s.

Solution **:**

The time period of the source is

$$T = 1/\nu = 20\text{ ms}.$$

The given time 1000 s is much larger than the time period. Hence we can write the average power dissipated as

$$P_{av} = V_{rms}\, i_{rms} \cos\varphi$$

where $\cos\varphi = R/Z$ is the power factor. Thus,

$$P_{av} = V_{rms} \frac{V_{rms}}{Z}\frac{R}{Z} = \frac{R\,V_{rms}^2}{Z^2}$$

$$= \frac{(50\ \Omega)(12\ \text{V})^2}{Z^2}$$

$$= \frac{7200}{Z^2}\ \Omega\text{-V}^2. \quad \text{... (i)}$$

The capacitive reactance $\frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}}\ \Omega$

$$= \frac{100}{\pi}\ \Omega.$$

The inductive reactance $= \omega L$

$$= 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega = 2\pi\ \Omega.$$

The net reactance is $X = \frac{1}{\omega C} - \omega L$

$$= \frac{100}{\pi}\ \Omega - 2\pi\ \Omega \approx 25\text{·}5\ \Omega.$$

Thus,

$$Z^2 = (50\ \Omega)^2 + (25\text{·}5\ \Omega)^2 = 3150\ \Omega^2.$$

From (i), average power $P_{av} = \frac{7200\ \Omega\text{-V}^2}{3150\ \Omega^2} = 2\text{·}286\ \text{W}.$

The energy dissipated in 1000 s $= P_{av} \times 1000$ s

$$\approx 2\text{·}3 \times 10^3\ \text{J}.$$

7. *An inductor of inductance* 100 mH *is connected in series with a resistance, a variable capacitance and an AC source of frequency* 2·0 kHz. *What should be the value of the capacitance so that maximum current may be drawn into the circuit ?*

Solution **:**

This is an *LCR* series circuit. The current will be maximum when the net reactance is zero. For this,

$$\frac{1}{\omega C} = \omega L$$

or, $$C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 \times (2\text{·}0 \times 10^3\ \text{s}^{-1})^2 (0\text{·}1\ \text{H})}$$

$$= 63\ \text{nF}.$$

8. *An inductor coil joined to a* 6 V *battery draws a steady current of* 12 A. *This coil is connected to a capacitor and an AC source of rms voltage* 6 V *in series. If the current in the circuit is in phase with the emf, find the rms current.*

Solution **:**

The resistance of the coil is $R = \frac{6\ \text{V}}{12\ \text{A}} = 0\text{·}5\ \Omega.$

In the AC circuit, the current is in phase with the emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, i.e., $Z = 0\text{·}5\ \Omega$.

The rms current $= \frac{\text{rms voltage}}{Z} = \frac{6\ \text{V}}{0\text{·}5\ \Omega} = 12\ \text{A}.$

□

QUESTIONS FOR SHORT ANSWER

1. What is the reactance of a capacitor connected to a constant DC source ?
2. The voltage and current in a series AC circuit are given by
$$V = V_0 \cos \omega t \text{ and } i = i_0 \sin \omega t.$$
What is the power dissipated in the circuit ?
3. Two alternating currents are given by
$$i_1 = i_0 \sin \omega t \text{ and } i_2 = i_0 \sin\left(\omega t + \frac{\pi}{3}\right).$$
Will the rms values of the currents be equal or different ?
4. Can the peak voltage across the inductor be greater than the peak voltage of the source in an *LCR* circuit ?
5. In a circuit containing a capacitor and an AC source, the current is zero at the instant the source voltage is maximum. Is it consistent with Ohm's law ?
6. An AC source is connected to a capacitor. Will the rms current increase, decrease or remain constant if a dielectric slab is inserted into the capacitor ?
7. When the frequency of the AC source in an *LCR* circuit equals the resonant frequency, the reactance of the circuit is zero. Does it mean that there is no current through the inductor or the capacitor ?
8. When an AC source is connected to a capacitor there is a steady-state current in the circuit. Does it mean that

the charges jump from one plate to the other to complete the circuit ?

9. A current $i_1 = i_0 \sin \omega t$ passes through a resistor of resistance R. How much thermal energy is produced in one time period ? A current $i_2 = -i_0 \sin \omega t$ passes through the resistor. How much thermal energy is produced in one time period ? If i_1 and i_2 both pass through the resistor simultaneously, how much thermal energy is produced ? Is the principle of superposition obeyed in this case ?

10. Is energy produced when a transformer steps up the voltage ?

11. A transformer is designed to convert an AC voltage of 220 V to an AC voltage of 12 V. If the input terminals are connected to a DC voltage of 220 V, the transformer usually burns. Explain.

12. Can you have an AC series circuit in which there is a phase difference of 180° between the emf and the current ? 120° ?

13. A resistance is connected to an AC source. If a capacitor is included in the series circuit, will the average power absorbed by the resistance increase or decrease ? If an inductor of small inductance is also included in the series circuit, will the average power absorbed increase or decrease further ?

14. Can a hot-wire ammeter be used to measure a direct current having a constant value ? Do we have to change the graduations ?

OBJECTIVE I

1. A capacitor acts as an infinite resistance for
(a) DC (b) AC
(c) DC as well as AC (d) neither AC nor DC.

2. An AC source producing emf
$$\mathcal{E} = \mathcal{E}_0 \left[\cos(100\pi\ s^{-1})t + \cos(500\pi\ s^{-1})t\right]$$
is connected in series with a capacitor and a resistor. The steady-state current in the circuit is found to be
$$i = i_1 \cos\left[(100\pi\ s^{-1})t + \varphi_1\right] + i_2 \cos\left[(500\pi\ s^{-1})t + \phi_2\right]$$
(a) $i_1 > i_2$ (b) $i_1 = i_2$ (c) $i_1 < i_2$
(d) the information is insufficient to find the relation between i_1 and i_2.

3. The peak voltage in a 220 V AC source is
(a) 220 V (b) about 160 V
(c) about 310 V (d) 440 V.

4. An AC source is rated 220 V, 50 Hz. The average voltage is calculated in a time interval of 0·01 s. It
(a) must be zero (b) may be zero
(c) is never zero (d) is $(220/\sqrt{2})$V.

5. The magnetic field energy in an inductor changes from maximum value to minimum value in 5·0 ms when connected to an AC source. The frequency of the source is
(a) 20 Hz (b) 50 Hz (c) 200 Hz (d) 500 Hz.

6. Which of the following plots may represent the reactance of a series LC combination ?

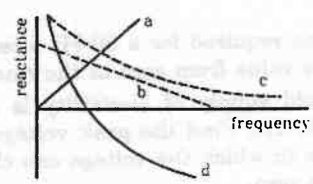

Figure 39-Q1

7. A series AC circuit has a resistance of 4 Ω and a reactance of 3 Ω. The impedance of the circuit is
(a) 5 Ω (b) 7 Ω (c) 12/7 Ω (d) 7/12 Ω.

8. Transformers are used
(a) in DC circuits only (b) in AC circuits only
(c) in both DC and AC circuits
(d) neither in DC nor in AC circuits.

9. An alternating current is given by
$$i = i_1 \cos \omega t + i_2 \sin \omega t.$$
The rms current is given by
(a) $\dfrac{i_1 + i_2}{\sqrt{2}}$ (b) $\dfrac{|i_1 + i_2|}{\sqrt{2}}$ (c) $\sqrt{\dfrac{i_1^2 + i_2^2}{2}}$ (d) $\sqrt{\dfrac{i_1^2 + i_2^2}{\sqrt{2}}}$.

10. An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is
(a) 14 A (b) about 20 A (c) 7 A (d) about 10 A.

11. A constant current of 2·8 A exists in a resistor. The rms current is
(a) 2·8 A (b) about 2 A
(c) 1·4 A (d) undefined for a direct current.

OBJECTIVE II

1. An inductor, a resistor and a capacitor are joined in series with an AC source. As the frequency of the source is slightly increased from a very low value, the reactance
(a) of the inductor increases
(b) of the resistor increases
(c) of the capacitor increases
(d) of the circuit increases.

2. The reactance of a circuit is zero. It is possible that the circuit contains
(a) an inductor and a capacitor
(b) an inductor but no capacitor

(c) a capacitor but no inductor
(d) neither an inductor nor a capacitor.

3. In an AC series circuit, the instantaneous current is zero when the instantaneous voltage is maximum. Connected to the source may be a
(a) pure inductor (b) pure capacitor
(c) pure resistor
(d) combination of an inductor and a capacitor.

4. An inductor-coil having some resistance is connected to an AC source. Which of the following quantities have zero average value over a cycle?
(a) current (b) induced emf in the inductor
(c) Joule heat
(d) magnetic energy stored in the inductor.

5. The AC voltage across a resistance can be measured using
(a) a potentiometer (b) a hot-wire voltmeter
(c) a moving-coil galvanometer
(d) a moving-magnet galvanometer.

6. To convert mechanical energy into electrical energy, one can use
(a) DC dynamo (b) AC dynamo
(c) motor (d) transformer.

7. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source
(a) must be 1000 W (b) may be 1000 W
(c) may be greater than 1000 W
(d) may be less than 1000 W.

EXERCISES

1. Find the time required for a 50 Hz alternating current to change its value from zero to the rms value.
2. The household supply of electricity is at 220 V (rms value) and 50 Hz. Find the peak voltage and the least possible time in which the voltage can change from the rms value to zero.
3. A bulb rated 60 W at 220 V is connected across a household supply of alternating voltage of 220 V. Calculate the maximum instantaneous current through the filament.
4. An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and gives normal brightness, what would be the peak voltage of the source?
5. The peak power consumed by a resistive coil when connected to an AC source is 80 W. Find the energy consumed by the coil in 100 seconds which is many times larger than the time period of the source.
6. The dielectric strength of air is $3{\cdot}0 \times 10^6$ V/m. A parallel-plate air-capacitor has area 20 cm^2 and plate separation 0·10 mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.
7. The current in a discharging LR circuit is given by $i = i_0 e^{-t/\tau}$ where τ is the time constant of the circuit. Calculate the rms current for the period $t = 0$ to $t = \tau$.
8. A capacitor of capacitance 10 μF is connected to an oscillator giving an output voltage $\mathcal{E} = (10\ \text{V})\sin \omega t$. Find the peak currents in the circuit for $\omega = 10\ s^{-1}$, $100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.
9. A coil of inductance 5·0 mH and negligible resistance is connected to the oscillator of the previous problem. Find the peak currents in the circuit for $\omega = 100\ s^{-1}$, $500\ s^{-1}$, $1000\ s^{-1}$.
10. A coil has a resistance of 10 Ω and an inductance of 0·4 henry. It is connected to an AC source of 6·5 V, $\frac{30}{\pi}$ Hz. Find the average power consumed in the circuit.
11. A resistor of resistance 100 Ω is connected to an AC source $\mathcal{E} = (12\ \text{V})\sin(250\,\pi\ s^{-1})t$. Find the energy dissipated as heat during $t = 0$ to $t = 1{\cdot}0$ ms.
12. In a series RC circuit with an AC source, $R = 300\ \Omega$, $C = 25\ \mu F$, $\mathcal{E}_0 = 50$ V and $\nu = 50/\pi$ Hz. Find the peak current and the average power dissipated in the circuit.
13. An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage?
14. In a series LCR circuit with an AC source, $R = 300\ \Omega$, $C = 20\ \mu F$, $L = 1{\cdot}0$ henry, $\mathcal{E}_0 = 50$ V and $\nu = 50/\pi$ Hz. Find (a) the rms current in the circuit and (b) the rms potential differences across the capacitor, the resistor and the inductor. Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.
15. Consider the situation of the previous problem. Find the average electric field energy stored in the capacitor and the average magnetic field energy stored in the coil.
16. An inductance of 2·0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20 V with adjustable frequency. (a) What frequency should be chosen to maximise the current in the circuit? (b) What is the value of this maximum current?
17. An inductor-coil, a capacitor and an AC source of rms voltage 24 V are connected in series. When the frequency of the source is varied, a maximum rms current of 6·0 A is observed. If this inductor coil is connected to a battery of emf 12 V and internal resistance 4·0 Ω, what will be the current?
18. Figure (39-E1) shows a typical circuit for low-pass filter. An AC input $V_i = 10$ mV is applied at the left end and

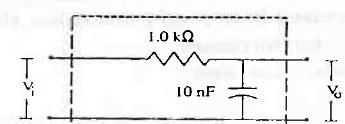

Figure 39-E1

the output V_0 is received at the right end. Find the output voltages for v = 10 kHz, 100kHz, 1·0 MHz and 10·0 MHz. Note that as the frequency is increased the output decreases and hence the name low-pass filter.

19. A transformer has 50 turns in the primary and 100 in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary ?

□

ANSWERS

OBJECTIVE I

1. (a) 2. (c) 3. (c) 4. (b) 5. (b) 6. (d)
7. (a) 8. (b) 9. (c) 10. (d) 11. (a)

OBJECTIVE II

1. (a) 2. (a), (d) 3. (a), (b), (d)
4. (a), (b) 5. (b) 6. (a), (b)
7. (b), (d)

EXERCISES

1. 2·5 ms
2. 311 V, 2·5 ms
3. 0·39 A
4. 17 volts
5. 4·0 kJ
6. 210 V
7. $\frac{i}{e}\sqrt{(e^2 - 1)/2}$
8. 1·0 × 10$^{-[illegible]}$ A, 0·01 A, 0·05 A, 0·1 A
9. 20 A, 4·0 A, 0·20 A
10. 5/8 W
11. 2·61 × 10$^{-[illegible]}$ J
12. 0·10 A, 1·5 W
13. 1·2 H
14. (a) 0·10 A (b) 50 V, 30 V, 10 V
15. 25 mJ, 5 mJ
16. (a) 27 Hz (b) 2 mA
17. 1·5 A
18. 8·5 mV, 1·6 mV, 0·16 mV, 16 μV
19. zero.

□