CHAPTER 38

ELECTROMAGNETIC INDUCTION

38.1 FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

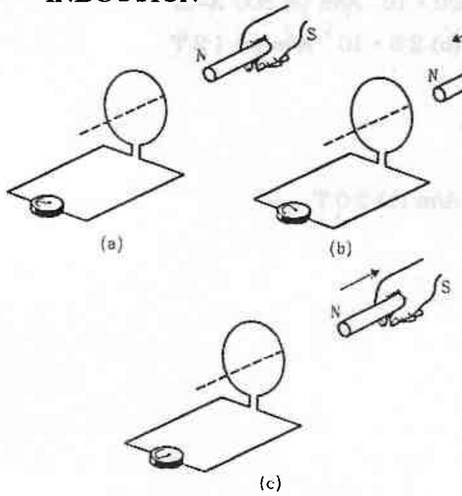


Figure 38.1

Figure (38.1a) shows a bar magnet placed along the axis of a conducting loop containing a galvanometer. There is no current in the loop and correspondingly no deflection in the galvanometer. If we move the magnet towards the loop (figure 38.1b), there is a deflection in the galvanometer showing that there is an electric current in the loop. If the magnet is moved away from the loop (figure 38.1c), again there is a current but the current is in the opposite direction. The current exists as long as the magnet is moving. Faraday studied this behaviour in detail by performing a number of experiments and discovered the following law of nature:

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop. The emf is given by

$$\mathcal{E} = -\frac{d\Phi}{dt} \quad \dots (38.1)$$

where $\Phi = \int \vec{B} \cdot d\vec{S}$ *is the flux of the magnetic field through the area.*

We shall call the quantity Φ *magnetic flux*. The SI unit of magnetic flux is called weber which is equivalent to tesla–metre^2.

The law described by equation (38.1) is called *Faraday's law of electromagnetic induction.* The flux may be changed in a number of ways. One can change the magnitude of the magnetic field $\vec{B}$ at the site of the loop, the area of the loop or the angle between the area-vector $d\vec{S}$ and the magnetic field $\vec{B}$. In any case, as long as the flux keeps changing, the emf is present. The emf so produced drives an electric current through the loop. If the resistance of the loop is R, the current is

$$i = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt}. \quad \dots (38.2)$$

The emf developed by a changing flux is called *induced emf* and the current produced by this emf is called *induced current.*

Direction of Induced Current

The direction of the induced current in a loop may be obtained using equation (38.1) or (38.2). The procedure to decide the direction is as follows:

Put an arrow on the loop to choose the positive sense of current. This choice is arbitrary. Using right-hand thumb rule find the positive direction of the normal to the area bounded by the loop. If the fingers curl along the loop in the positive sense, the thumb represents the positive direction of the normal. Calculate the flux $\Phi = \int \vec{B} \cdot d\vec{S}$ through the area bounded by the loop. If the flux increases with time, $\frac{d\Phi}{dt}$ is positive and $\mathcal{E}$ is negative from equation (38.1). Correspondingly, the current is negative. It is, therefore, in the direction opposite to the arrow put on the loop. If Φ decreases with time, $\frac{d\Phi}{dt}$ is negative, $\mathcal{E}$ is positive and the current is along the arrow.

Putting the numerical values in (i), the energy at $t = 10$ ms is

$$\frac{1}{2} \times (1{\cdot}0 \text{ H}) \times [0{\cdot}12 \text{ A}(1 - 1/e)]^2$$

$$= 2{\cdot}8 \text{ mJ}.$$

28. *An inductance L and a resistance R are connected in series with a battery of emf $\mathcal{E}$. Find the maximum rate at which the energy is stored in the magnetic field.*

Solution :

The energy stored in the magnetic field at time t is

$$U = \frac{1}{2} Li^2 = \frac{1}{2} Li_0^2 (1 - e^{-t/\tau})^2.$$

The rate at which the energy is stored is

$$P = \frac{dU}{dt} = Li_0^2 (1 - e^{-t/\tau})(-e^{-t/\tau})\left(-\frac{1}{\tau}\right)$$

$$= \frac{Li_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}). \quad \text{... (i)}$$

This rate will be maximum when $\frac{dP}{dt} = 0$

or, $\frac{Li_0^2}{\tau}\left[-\frac{1}{\tau} e^{-t/\tau} + \frac{2}{\tau} e^{-2t/\tau}\right] = 0$

or, $e^{-t/\tau} = \frac{1}{2}$.

Putting in (i),

$$P_{max} = \frac{Li_0^2}{\tau}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= \frac{L\mathcal{E}^2}{4R^2(L/R)} = \frac{\mathcal{E}^2}{4R}.$$

29. *Two conducting circular loops of radii R_1 and R_2 are placed in the same plane with their centres coinciding. Find the mutual inductance between them assuming $R_2 << R_1$.*

Solution : Suppose a current i is established in the outer loop. The magnetic field at the centre will be

$$B = \frac{\mu_0 i}{2R_1}.$$

As the radius R_2 of the inner coil is small compared to R_1, the flux of magnetic field through it will be approximately

$$\Phi = \frac{\mu_0 i}{2R_1} \pi R_2^2$$

so that the mutual inductance is

$$M = \frac{\Phi}{i} = \frac{\mu_0 \pi R_2^2}{2R_1}.$$

□

QUESTIONS FOR SHORT ANSWER

1. A metallic loop is placed in a nonuniform magnetic field. Will an emf be induced in the loop ?
2. An inductor is connected to a battery through a switch. Explain why the emf induced in the inductor is much larger when the switch is opened as compared to the emf induced when the switch is closed.
3. The coil of a moving-coil galvanometer keeps on oscillating for a long time if it is deflected and released. If the ends of the coil are connected together, the oscillation stops at once. Explain.
4. A short magnet is moved along the axis of a conducting loop. Show that the loop repels the magnet if the magnet is approaching the loop and attracts the magnet if it is going away from the loop.
5. Two circular loops are placed coaxially but separated by a distance. A battery is suddenly connected to one of the loops establishing a current in it. Will there be a current induced in the other loop ? If yes, when does the current start and when does it end ? Do the loops attract each other or do they repel ?
6. The battery discussed in the previous question is suddenly disconnected. Is a current induced in the other loop ? If yes, when does it start and when does it end ? Do the loops attract each other or repel ?
7. If the magnetic field outside a copper box is suddenly changed, what happens to the magnetic field inside the box ? Such low-resistivity metals are used to form enclosures which shield objects inside them against varying magnetic fields.
8. Metallic (nonferromagnetic) and nonmetallic particles in a solid waste may be separated as follows. The waste is allowed to slide down an incline over permanent magnets. The metallic particles slow down as compared to the nonmetallic ones and hence are separated. Discuss the role of eddy currents in the process.
9. A pivoted aluminium bar falls much more slowly through a small region containing a magnetic field than a similar bar of an insulating material. Explain.
10. A metallic bob A oscillates through the space between the poles of an electromagnet (figure 38-Q1). The oscillations are more quickly damped when the circuit

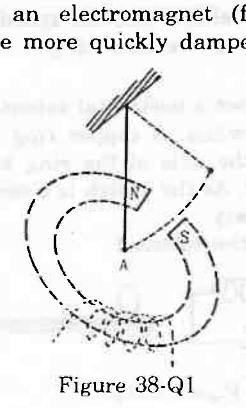


Figure 38-Q1

is on, as compared to the case when the circuit is off. Explain.

11. Two circular loops are placed with their centres separated by a fixed distance. How would you orient the loops to have (a) the largest mutual inductance (b) the smallest mutual inductance?

12. Consider the self-inductance per unit length of a solenoid at its centre and that near its ends. Which of the two is greater?

13. Consider the energy density in a solenoid at its centre and that near its ends. Which of the two is greater?

OBJECTIVE I

1. A rod of length l rotates with a small but uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the centre of the rod and an end is

(a) zero (b) $\frac{1}{8}\omega Bl^2$ (c) $\frac{1}{2}\omega Bl^2$ (d) $B\omega l^2$.

2. A rod of length l rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

(a) zero (b) $\frac{1}{2}Bl\omega^2$ (c) $Bl\omega^2$ (d) $2Bl\omega^2$.

3. Consider the situation shown in figure (38-Q2). If the switch is closed and after some time it is opened again, the closed loop will show
(a) an anticlockwise current-pulse
(b) a clockwise current-pulse
(c) an anticlockwise current-pulse and then a clockwise current-pulse
(d) a clockwise current-pulse and then an anticlockwise current-pulse.

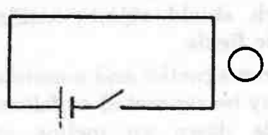


Figure 38-Q2

4. Solve the previous question if the closed loop is completely enclosed in the circuit containing the switch.

5. A bar magnet is released from rest along the axis of a very long, vertical copper tube. After some time the magnet
(a) will stop in the tube
(b) will move with almost contant speed
(c) will move with an acceleration g
(d) will oscillate.

6. Figure (38-Q3) shows a horizontal solenoid connected to a battery and a switch. A copper ring is placed on a frictionless track, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will
(a) remain stationary
(b) move towards the solenoid
(c) move away from the solenoid
(d) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

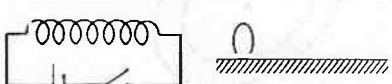


Figure 38-Q3

7. Consider the following statements:
(A) An emf can be induced by moving a conductor in a magnetic field.
(B) An emf can be induced by changing the magnetic field.
(a) Both A and B are true. (b) A is true but B is false.
(c) B is true but A is false. (d) Both A and B are false.

8. Consider the situation shown in figure (38-Q4). The wire AB is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a semicircular wire, the magnitude of the induced current will
(a) increase (b) remain the same (c) decrease
(d) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

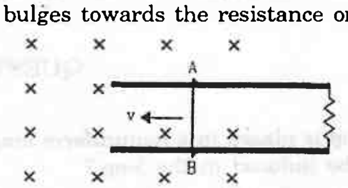


Figure 38-Q4

9. Figure (38-Q5a) shows a conducting loop being pulled out of a magnetic field with a speed v. Which of the four plots shown in figure (38-Q5b) may represent the power delivered by the pulling agent as a function of the speed v?

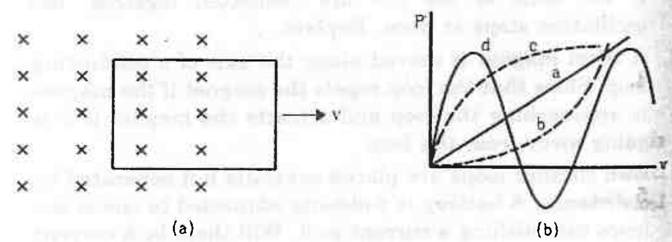


Figure 38-Q5

10. Two circular loops of equal radii are placed coaxially at some separation. The first is cut and a battery is inserted in between to drive a current in it. The current changes slightly because of the variation in resistance with temperature. During this period, the two loops
(a) attract each other (b) repel each other
(c) do not exert any force on each other
(d) attract or repel each other depending on the sense of the current.

11. A small, conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is
(a) clockwise (b) anticlockwise (c) zero
(d) clockwise or anticlockwise depending on whether the resistance is increased or decreased.

12. A conducting square loop of side l and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A uniform and constant magnetic field B exists along the perpendicular to the plane of the loop as shown in figure (38-Q6). The current induced in the loop is
(a) Blv/R clockwise (b) Blv/R anticlockwise
(c) $2Blv/R$ anticlockwise (d) zero.

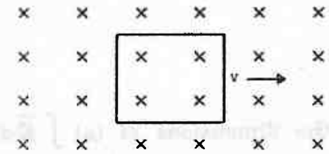


Figure 38-Q6

OBJECTIVE II

1. A bar magnet is moved along the axis of a copper ring placed far away from the magnet. Looking from the side of the magnet, an anticlockwise current is found to be induced in the ring. Which of the following may be true ?
(a) The south pole faces the ring and the magnet moves towards it.
(b) The north pole faces the ring and the magnet moves towards it.
(c) The south pole faces the ring and the magnet moves away from it.
(d) The north pole faces the ring and the magnet moves away from it.

2. A conducting rod is moved with a constant velocity v in a magnetic field. A potential difference appears across the two ends
(a) if $\vec{v} \parallel \vec{l}$ (b) if $\vec{v} \parallel \vec{B}$ (c) if $\vec{l} \parallel \vec{B}$
(d) none of these.

3. A conducting loop is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the loop if
(a) it is translated
(b) it is rotated about its axis
(c) it is rotated about a diameter
(d) it is deformed.

4. A metal sheet is placed in front of a strong magnetic pole. A force is needed to
(a) hold the sheet there if the metal is magnetic
(b) hold the sheet there if the metal is nonmagnetic
(c) move the sheet away from the pole with uniform velocity if the metal is magnetic
(d) move the sheet away from the pole with uniform velocity if the metal is nonmagnetic.
Neglect any effect of paramagnetism, diamagnetism and gravity.

5. A constant current i is maintained in a solenoid. Which of the following quantities will increase if an iron rod is inserted in the solenoid along its axis ?
(a) magnetic field at the centre
(b) magnetic flux linked with the solenoid
(c) self-inductance of the solenoid
(d) rate of Joule heating.

6. Two solenoids have identical geometrical construction but one is made of thick wire and the other of thin wire. Which of the following quantities are different for the two solenoids ?
(a) self-inductance
(b) rate of Joule heating if the same current goes through them
(c) magnetic field energy if the same current goes through them
(d) time constant if one solenoid is connected to one battery and the other is connected to another battery.

7. An LR circuit with a battery is connected at $t = 0$. Which of the following quantities is not zero just after the connection ?
(a) current in the circuit
(b) magnetic field energy in the inductor
(c) power delivered by the battery
(d) emf induced in the inductor.

8. A rod AB moves with a uniform velocity v in a uniform magnetic field as shown in figure (38-Q7).
(a) The rod becomes electrically charged.
(b) The end A becomes positively charged.
(c) The end B becomes positively charged.
(d) The rod becomes hot because of Joule heating.

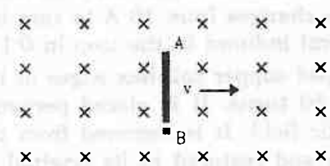


Figure 38-Q7

9. L, C and R represent the physical quantities inductance, capacitance and resistance respectively. Which of the following combinations have dimensions of frequency ?
(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) C/L.

10. The switches in figure (38-Q8a) and (38-Q8b) are closed at $t = 0$ and reopened after a long time at $t = t_0$.

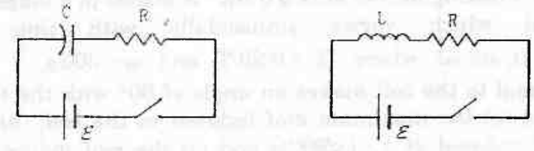


Figure 38-Q8

(a) The charge on C just after $t = 0$ is $\mathcal{E}C$.
(b) The charge on C long after $t = 0$ is $\mathcal{E}C$.
(c) The current in L just before $t = t_0$ is $\mathcal{E}/R$.
(d) The current in L long after $t = t_0$ is $\mathcal{E}/R$.

EXERCISES

1. Calculate the dimensions of (a) $\int \vec{E}\cdot d\vec{l}$. (b) vBl and (c) $\frac{d\Phi_B}{dt}$. The symbols have their usual meanings.
2. The flux of magnetic field through a closed conducting loop changes with time according to the equation, $\Phi = at^2 + bt + c$. (a) Write the SI units of a, b and c. (b) If the magnitudes of a, b and c are 0·20, 0·40 and 0·60 respectively, find the induced emf at $t = 2$ s.
3. (a) The magnetic field in a region varies as shown in figure (38-E1). Calculate the average induced emf in a conducting loop of area $2{\cdot}0 \times 10^{-3}$ m^2 placed perpendicular to the field in each of the 10 ms intervals shown. (b) In which intervals is the emf not constant? Neglect the behaviour near the ends of 10 ms intervals.

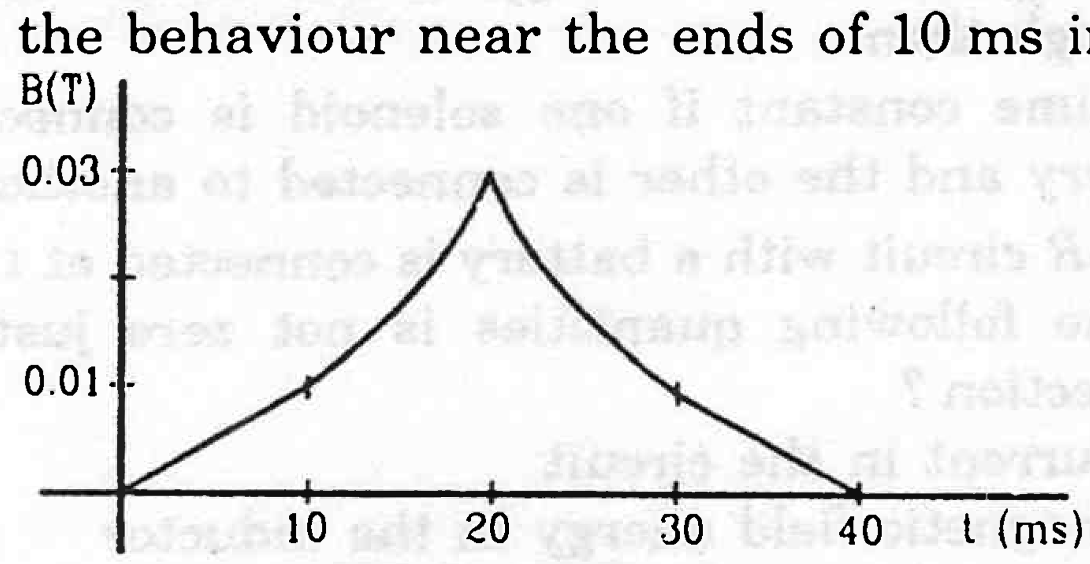


Figure 38-E1

4. A conducting circular loop having a radius of 5·0 cm, is placed perpendicular to a magnetic field of 0·50 T. It is removed from the field in 0·50 s. Find the average emf produced in the loop during this time.
5. A conducting circular loop of area 1 mm^2 is placed coplanarly with a long, straight wire at a distance of 20 cm from it. The straight wire carries an electric current which changes from 10 A to zero in 0·1 s. Find the average emf induced in the loop in 0·1 s.
6. A square-shaped copper coil has edges of length 50 cm and contains 50 turns. It is placed perpendicular to a 1·0 T magnetic field. It is removed from the magnetic field in 0·25 s and restored in its original place in the next 0·25 s. Find the magnitude of the average emf induced in the loop during (a) its removal, (b) its restoration and (c) its motion.
7. Suppose the resistance of the coil in the preivous problem is 25 Ω. Assume that the coil moves with uniform velocity during its removal and restoration. Find the thermal energy developed in the coil during (a) its removal, (b) its restoration and (c) its motion.
8. A conducting loop of area 5·0 cm^2 is placed in a magnetic field which varies sinusoidally with time as $B = B_0 \sin \omega t$ where $B_0 = 0{\cdot}20$ T and $\omega = 300$ s^{-1}. The normal to the coil makes an angle of 60° with the field. Find (a) the maximum emf induced in the coil, (b) the emf induced at $\tau = (\pi/900)$s and (c) the emf induced at $t = (\pi/600)$ s.
9. Figure (38-E2) shows a conducting square loop placed parallel to the pole-faces of a ring magnet. The pole-faces have an area of 1 cm^2 each and the field between the poles is 0·10 T. The wires making the loop are all outside the magnetic field. If the magnet is removed in 1·0 s, what is the average emf induced in the loop?

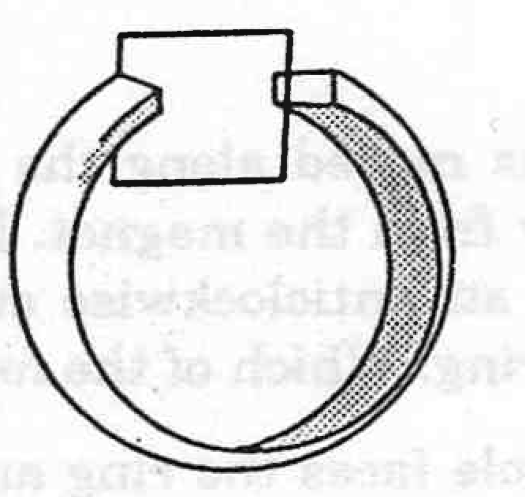


Figure 38-E2

10. A conducting square loop having edges of length 2·0 cm is rotated through 180° about a diagonal in 0·20 s. A magnetic field B exists in the region which is perpendicular to the loop in its initial position. If the average induced emf during the rotation is 20 mV, find the magnitude of the magnetic field.
11. A conducting loop of face-area A and resistance R is placed perpendicular to a magnetic field B. The loop is withdrawn completely from the field. Find the charge which flows through any cross-section of the wire in the process. Note that it is independent of the shape of the loop as well as the way it is withdrawn.
12. A long solenoid of radius 2 cm has 100 turns/cm and carries a current of 5 A. A coil of radius 1 cm having 100 turns and a total resistance of 20 Ω is placed inside the solenoid coaxially. The coil is connected to a galvanometer. If the current in the solenoid is reversed in direction, find the charge flown through the galvanometer.
13. Figure (38-E3) shows a metallic square frame of edge a in a vertical plane. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the figure. Two boys pull the opposite corners of the square to deform it into a rhombus. They start pulling the corners at $t = 0$ and displace the corners at a uniform speed u. (a) Find the induced emf in the frame at the instant when the angles at these corners reduce to 60°. (b) Find the induced current in the frame at this instant if the total resistance of the frame is R. (c) Find the total charge which flows through a side of the frame by the time the square is deformed into a straight line.

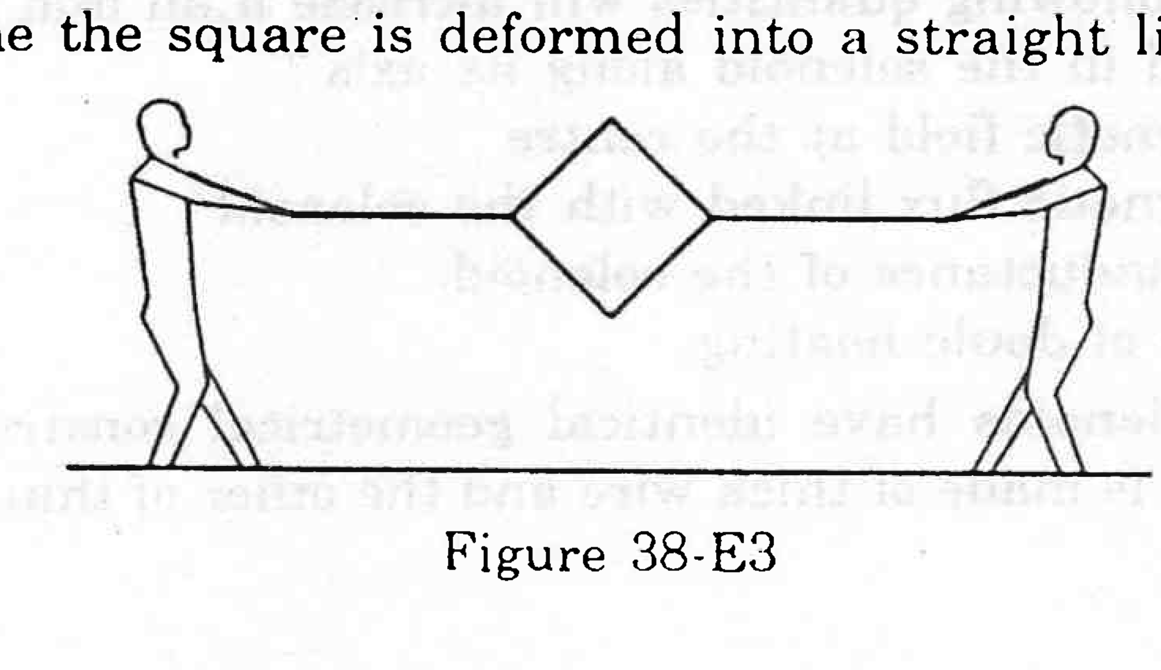


Figure 38-E3

14. The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure 38-E4). As a result, the flux through the coil changes from 0·35 weber to 0·85 weber in an interval of half a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet?

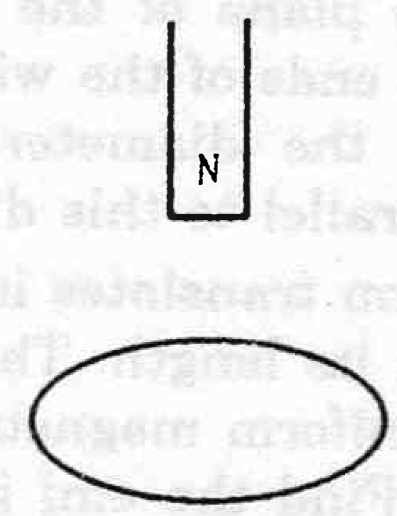


Figure 38-E4

15. A wire-loop confined in a plane is rotated in its own plane with some angular velocity. A uniform magnetic field exists in the region. Find the emf induced in the loop.

16. Figure (38-E5) shows a square loop of side 5 cm being moved towards right at a constant speed of 1 cm/s. The front edge enters the 20 cm wide magnetic field at $t = 0$. Find the emf induced in the loop at (a) $t = 2$ s, (b) $t = 10$ s, (c) $t = 22$ s and (d) $t = 30$ s.

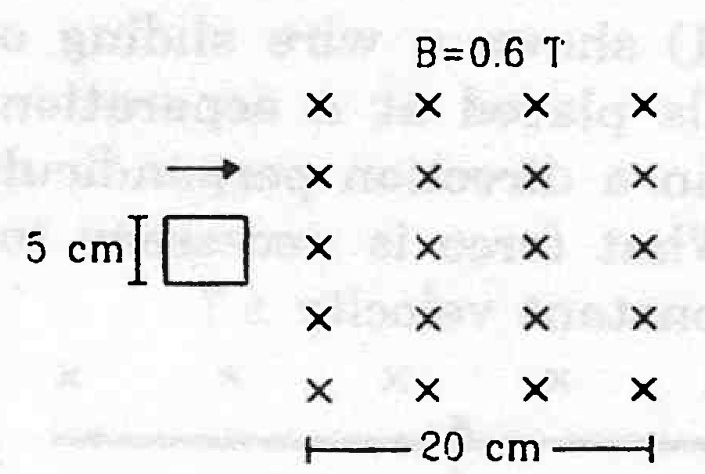


Figure 38-E5

17. Find the total heat produced in the loop of the previous problem during the interval 0 to 30 s if the resistance of the loop is 4·5 mΩ.

18. A uniform magnetic field B exists in a cylindrical region of radius 10 cm as shown in figure (38-E6). A uniform wire of length 80 cm and resistance 4·0 Ω is bent into a square frame and is placed with one side along a diameter of the cylindrical region. If the magnetic field increases at a constant rate of 0·010 T/s, find the current induced in the frame.

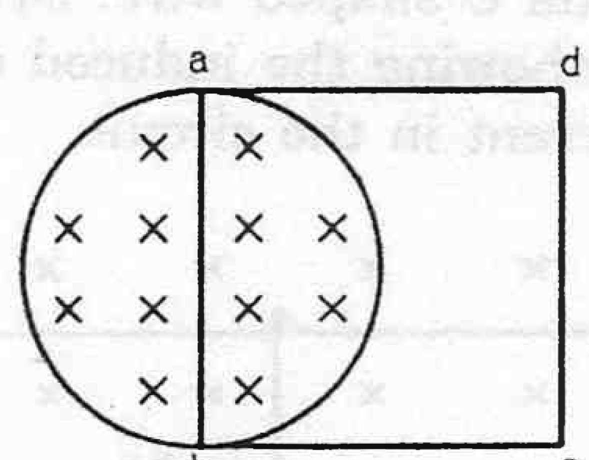


Figure 38-E6

19. The magnetic field in the cylindrical region shown in figure (38-E7) increases at a constant rate of 20·0 mT/s. Each side of the square loop $abcd$ and $defa$ has a length of 1·00 cm and a resistance of 4·00 Ω. Find the current (magnitude and sense) in the wire ad if (a) the switch S_1 is closed but S_2 is open, (b) S_1 is open but S_2 is closed, (c) both S_1 and S_2 are open and (d) both S_1 and S_2 are closed.

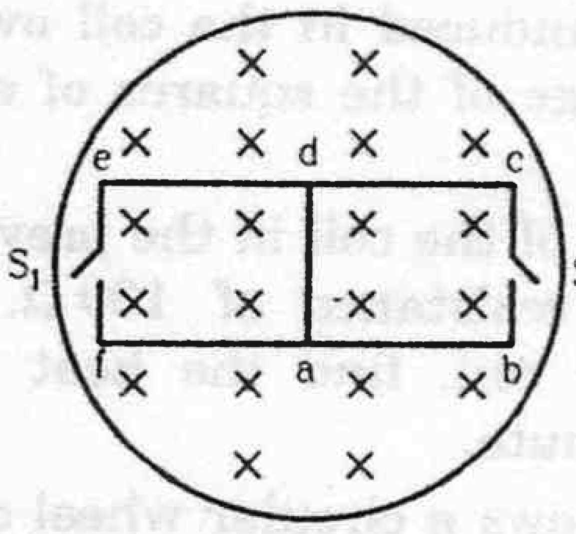


Figure 38-E7

20. Figure (38-E8) shows a circular coil of N turns and radius a, connected to a battery of emf $\mathcal{E}$ through a rheostat. The rheostat has a total length L and resistance R. The resistance of the coil is r. A small circular loop of radius a' and resistance r' is placed coaxially with the coil. The centre of the loop is at a distance x from the centre of coil. the In the beginning, the sliding contact of the rheostat is at the left end and then onwards it is moved towards right at a constant speed v. Find the emf induced in the small circular loop at the instant (a) the contact begins to slide and (b) it has slid through half the length of the rheostat.

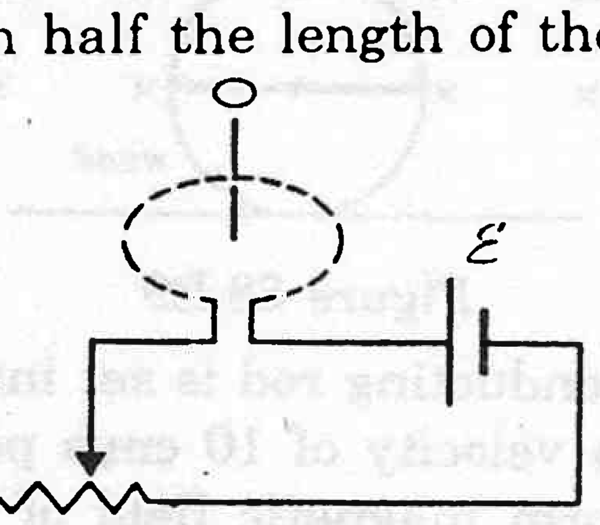


Figure 38-E8

21. A circular coil of radius 2·00 cm has 50 turns. A uniform magnetic field $B = 0·200$ T exists in the space in a direction parallel to the axis of the loop. The coil is now rotated about a diameter through an angle of 60·0°. The operation takes 0·100 s. (a) Find the average emf induced in the coil. (b) If the coil is a closed one (with the two ends joined together) and has a resistance of 4·00 Ω, calculate the net charge crossing a cross-section of the wire of the coil.

22. A closed coil having 100 turns is rotated in a uniform magnetic field $B = 4·0 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 25 cm^2 and its resistance is 4·0 Ω. Find (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field, (b) the average emf in a full turn and (c) the net charge displaced in part (a).

23. A coil of radius 10 cm and resistance 40 Ω has 1000 turns. It is placed with its plane vertical and its axis parallel to the magnetic meridian. The coil is connected to a galvanometer and is rotated about the vertical diameter through an angle of 180°. Find the charge which flows through the galvanometer if the horizontal component of the earth's magnetic field is $B_H = 3·0 \times 10^{-5}$ T.

24. A circular coil of one turn of radius 5·0 cm is rotated about a diameter with a constant angular speed of 80 revolutions per minute. A uniform magnetic field $B = 0·010$ T exists in a direction perpendicular to the axis of rotation. Find (a) the maximum emf induced, (b)

the average emf induced in the coil over a long period and (c) the average of the squares of emf induced over a long period.

25. Suppose the ends of the coil in the previous problem are connected to a resistance of 100 Ω. Neglecting the resistance of the coil, find the heat produced in the circuit in one minute.

26. Figure (38-E9) shows a circular wheel of radius 10·0 cm whose upper half, shown dark in the figure, is made of iron and the lower half of wood. The two junctions are joined by an iron rod. A uniform magnetic field B of magnitude $2{\cdot}00 \times 10^{-4}$ T exists in the space above the central line as suggested by the figure. The wheel is set into pure rolling on the horizontal surface. If it takes 2·00 seconds for the iron part to come down and the wooden part to go up, find the average emf induced during this period.

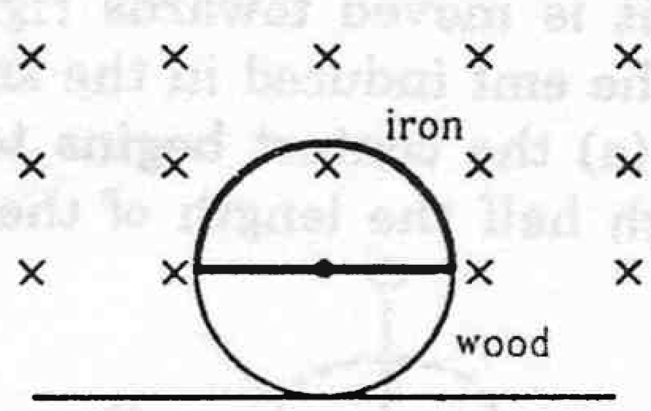


Figure 38-E9

27. A 20 cm long conducting rod is set into pure translation with a uniform velocity of 10 cm/s perpendicular to its length. A uniform magnetic field of magnitude 0·10 T exists in a direction perpendicular to the plane of motion. (a) Find the average magnetic force on the free electrons of the rod. (b) For what electric field inside the rod, the electric force on a free elctron will balance the magnetic force? How is this electric field created? (c) Find the motional emf between the ends of the rod.

28. A metallic metre stick moves with a velocity of 2 m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0·2 T. Find the emf induced between the ends of the stick.

29. A 10 m wide spacecraft moves through the interstellar space at a speed 3×10^{7} m/s. A magnetic field $B = 3 \times 10^{-10}$ T exists in the space in a direction perpendicular to the plane of motion. Treating the spacecraft as a conductor, calculate the emf induced across its width.

30. The two rails of a railway track, insulated from each other and from the ground, are connected to a millivoltmeter. What will be the reading of the millivoltmeter when a train travels on the track at a speed of 180 km/h? The vertical component of earth's magnetic field is $0{\cdot}2 \times 10^{-4}$ T and the rails are separated by 1 m.

31. A right-angled triangle abc, made from a metallic wire, moves at a uniform speed v in its plane as shown in figure (38-E10). A uniform magnetic field B exists in the perpendicular direction. Find the emf induced (a) in the loop abc, (b) in the segment bc, (c) in the segment ac and (d) in the segment ab.

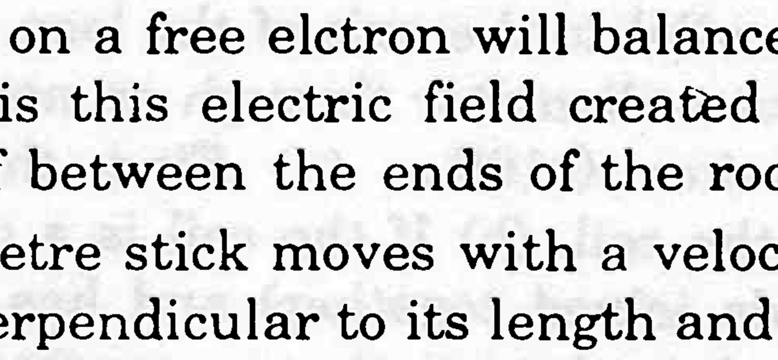


Figure 38-E10

32. A copper wire bent in the shape of a semicircle of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the direction perpendicular to the plane of the wire. Find the emf induced between the ends of the wire if (a) the velocity is perpendicular to the diameter joining free ends, (b) the velocity is parallel to this diameter.

33. A wire of length 10 cm translates in a direction making an angle of 60° with its length. The plane of motion is perpendicular to a uniform magnetic field of 1·0 T that exists in the space. Find the emf induced between the ends of the rod if the speed of translation is 20 cm/s.

34. A circular copper-ring of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring. (a) Between which pair of points is the emf maximum? What is the value of this maximum emf? (b) Between which pair of points is the emf minimum? What is the value of this minimum emf?

35. Figure (38-E11) shows a wire sliding on two parallel, conducting rails placed at a separation l. A magnetic field B exists in a direction perpendicular to the plane of the rails. What force is necessary to keep the wire moving at a constant velocity v?

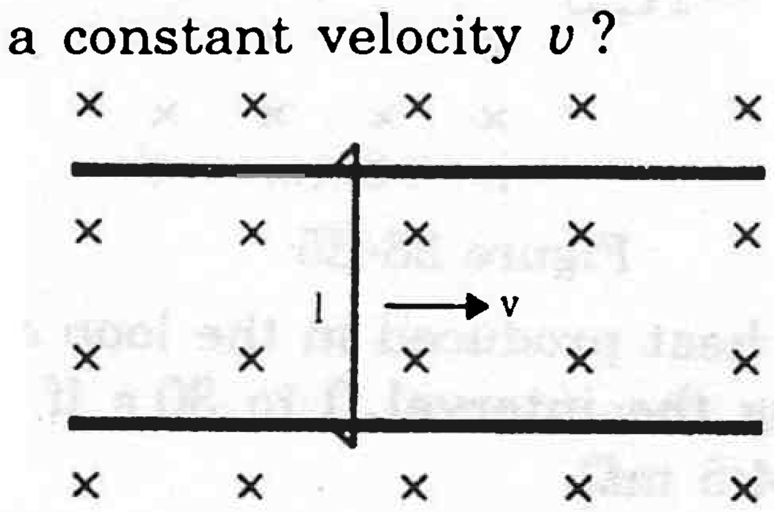


Figure 38-E11

36. Figure (38-E12) shows a long U-shaped wire of width l placed in a perpendicular magnetic field B. A wire of length l is slid on the U-shaped wire with a constant velocity v towards right. The resistance of all the wires is r per unit length. At $t = 0$, the sliding wire is close to the left edge of the U-shaped wire. Draw an equivalent circuit diagram, showing the induced emf as a battery. Calculate the current in the circuit.

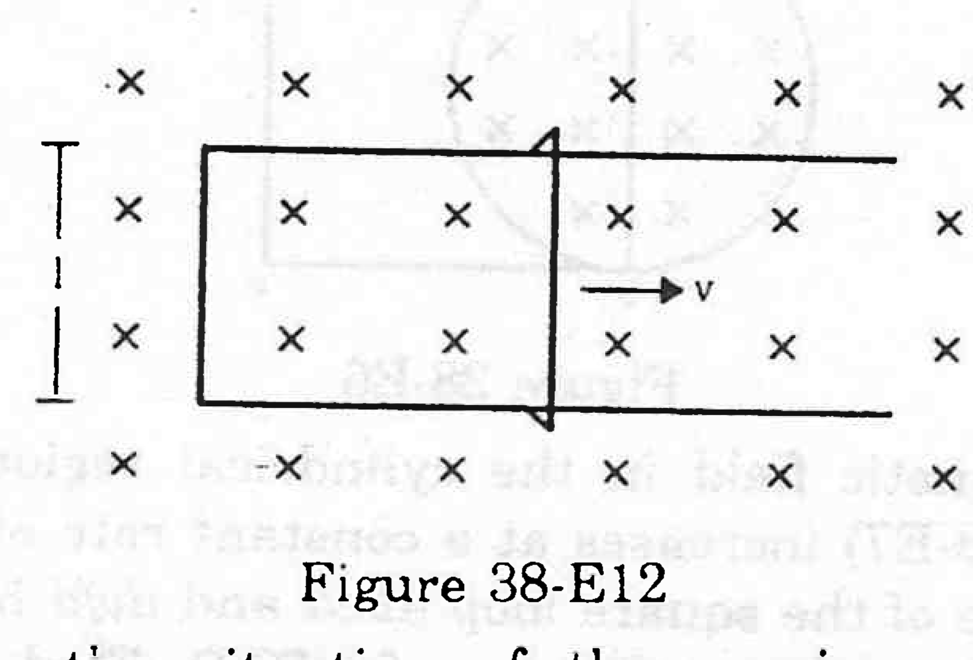


Figure 38-E12

37. Consider the situation of the previous problem. (a) Calculate the force needed to keep the sliding wire moving with a constant velocity v. (b) If the force needed just after $t = 0$ is F_0, find the time at which the force needed will be $F_0/2$.

38. Consider the situation shown in figure (38-E13). The wire PQ has mass m, resistance r and can slide on the smooth, horizontal parallel rails separated by a distance l. The resistance of the rails is negligible. A uniform magnetic field B exists in the rectangular region and a resistance R connects the rails outside the field region. At $t = 0$, the wire PQ is pushed towards right with a speed v_0. Find (a) the current in the loop at an instant when the speed of the wire PQ is v, (b) the acceleration of the wire at this instant, (c) the velocity v as a function of x and (d) the maximum distance the wire will move.

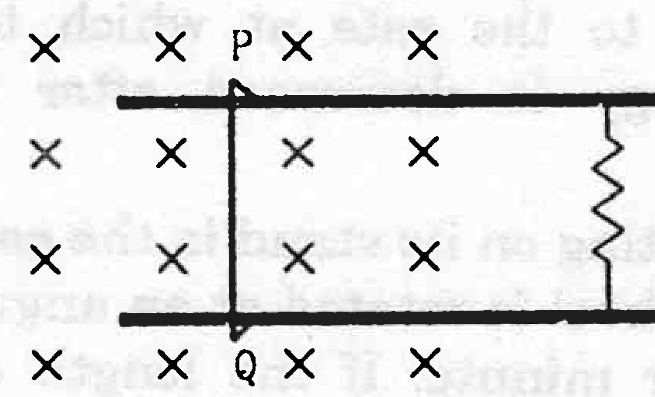


Figure 38-E13

39. A rectangular frame of wire $abcd$ has dimensions 32 cm × 8·0 cm and a total resistance of 2·0 Ω. It is pulled out of a magnetic field $B = 0{\cdot}020$ T by applying a force of $3{\cdot}2 \times 10^{-5}$ N (figure 38-E14). It is found that the frame moves with constant speed. Find (a) this constant speed, (b) the emf induced in the loop, (c) the potential difference between the points a and b and (d) the potential difference between the points c and d.

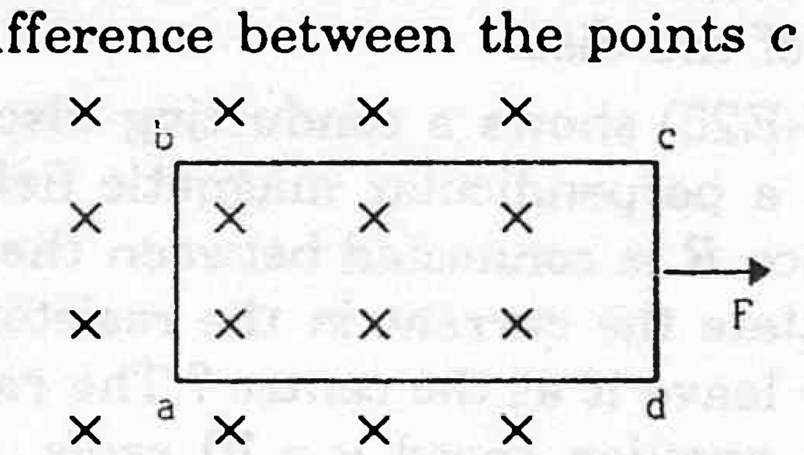


Figure 38-E14

40. Figure (38-E15) shows a metallic wire of resistance 0·20 Ω sliding on a horizontal, U-shaped metallic rail. The separation between the parallel arms is 20 cm. An electric current of 2·0 μA passes through the wire when it is slid at a rate of 20 cm/s. If the horizontal component of the earth's magnetic field is $3{\cdot}0 \times 10^{-5}$ T, calculate the dip at the place.

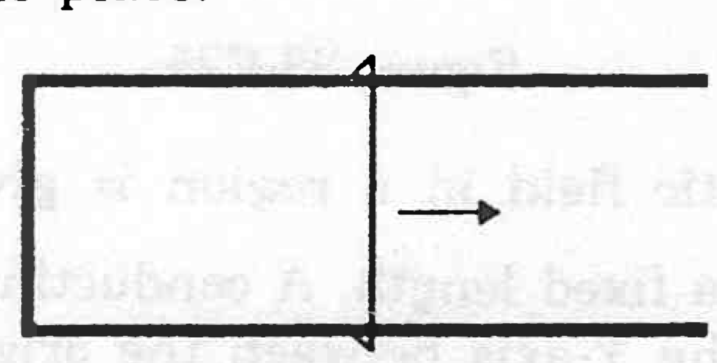


Figure 38-E15

41. A wire ab of length l, mass m and resistance R slides on a smooth, thick pair of metallic rails joined at the bottom as shown in figure (38-E16). The plane of the rails makes an angle θ with the horizontal. A vertical magnetic field B exists in the region. If the wire slides on the rails at a constant speed v, show that

$$B = \sqrt{\frac{mg\,R\sin\theta}{vl^2\cos^2\theta}}.$$

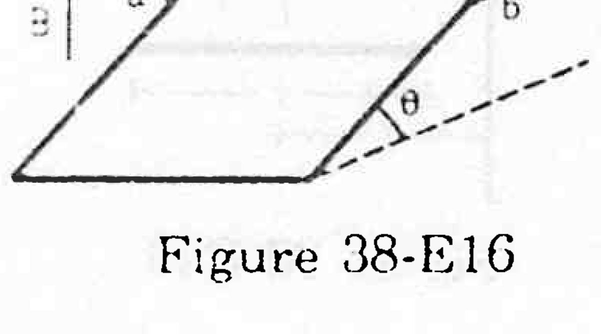


Figure 38-E16

42. Consider the situation shown in figure (38-E17). The wires P_1Q_2 and P_2Q_2 are made to slide on the rails with the same speed 5 cm/s. Find the electric current in the 19 Ω resistor if (a) both the wires move towards right and (b) if P_1Q_1 moves towards left but P_2Q_2 moves towards right.

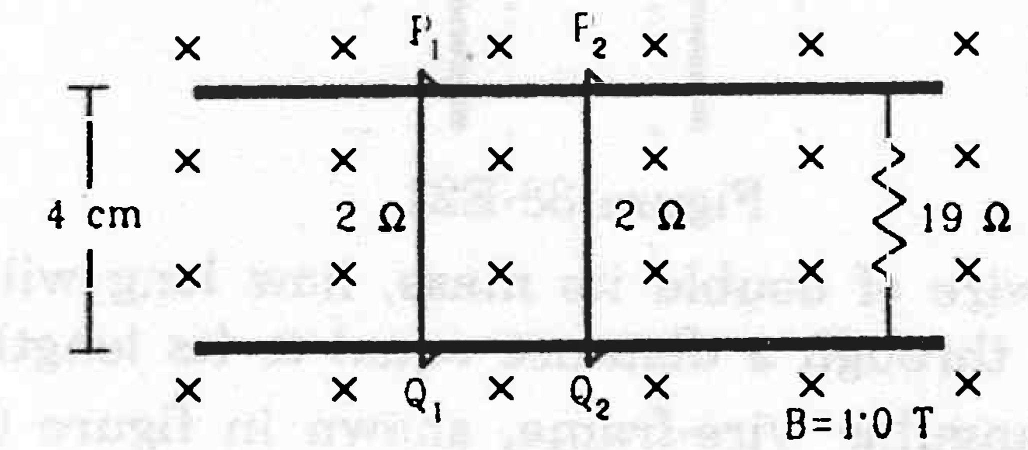


Figure 38-E17

43. Suppose the 19 Ω resistor of the previous problem is disconnected. Find the current through P_2Q_2 in the two situations (a) and (b) of that problem.

44. Consider the situation shown in figure (38-E18). The wire PQ has a negligible resistance and is made to slide on the three rails with a constant speed of 5 cm/s. Find the current in the 10 Ω resistor when the switch S is thrown to (a) the middle rail (b) the bottom rail.

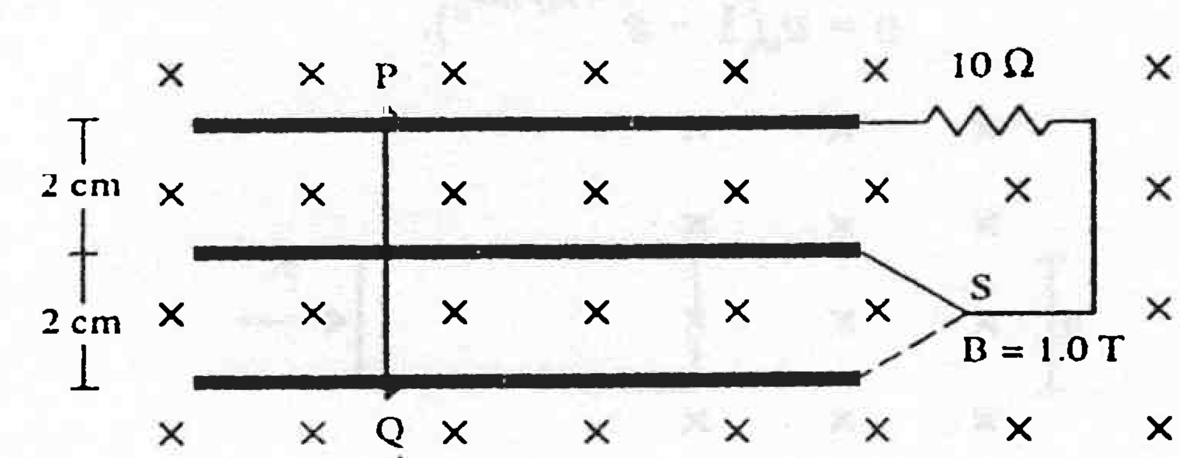


Figure 38-E18

45. The current generator I_g, shown in figure (38-E19), sends a constant current i through the circuit. The wire cd is fixed and ab is made to slide on the smooth, thick rails with a constant velocity v towards right. Each of these wires has resistance r. Find the current through the wire cd.

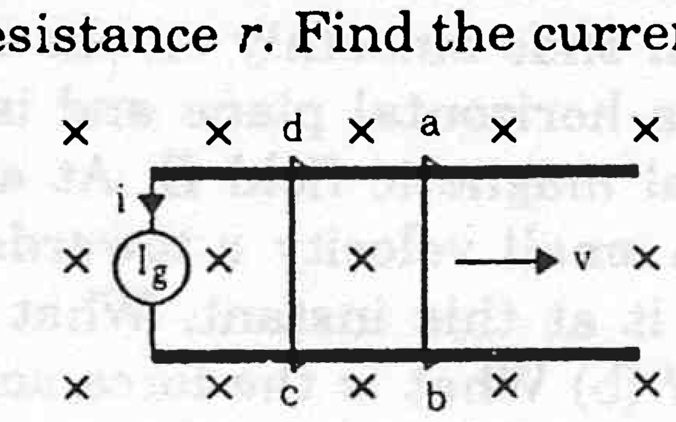


Figure 38-E19

46. The current generator I_g, shown in figure (38-E20), sends a constant current i through the circuit. The wire ab has a length l and mass m and can slide on the smooth, horizontal rails connected to I_g. The enitre system lies in a vertical magnetic field B. Find the velocity of the wire as a function of time.

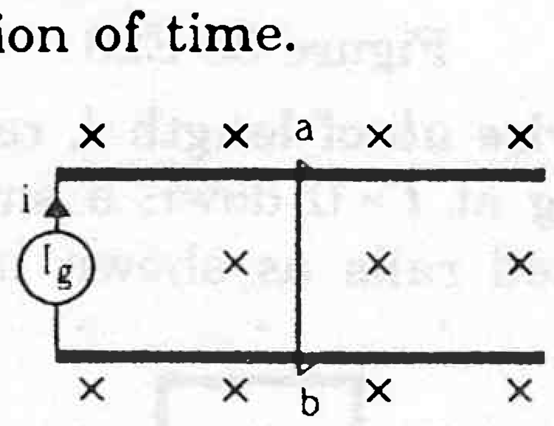


Figure 38-E20

47. The system containing the rails and the wire of the previous problem is kept vertically in a uniform horizontal magnetic field B that is perpendicular to the plane of the rails (figure 38-E21). It is found that the wire stays in equilibrium. If the wire ab is replaced by

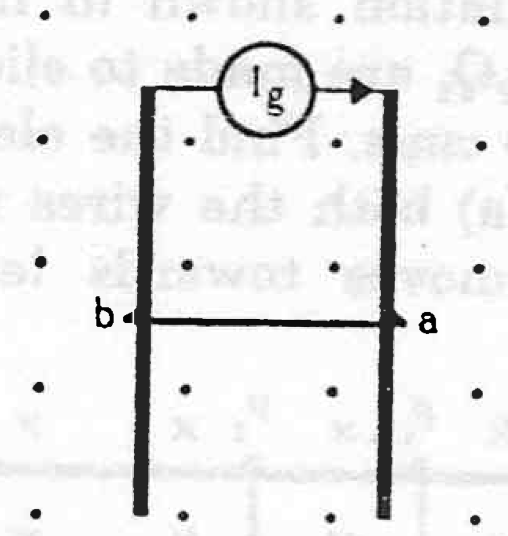


Figure 38-E21

another wire of double its mass, how long will it take in falling through a distance equal to its length ?

48. The rectangular wire-frame, shown in figure (38-E22), has a width d, mass m, resistance R and a large length. A uniform magnetic field B exists to the left of the frame. A constant force F starts pushing the frame into the magnetic field at $t = 0$. (a) Find the acceleration of the frame when its speed has increased to v. (b) Show that after some time the frame will move with a constant velocity till the whole frame enters into the magnetic field. Find this velocity v_0. (c) Show that the velocity at time t is given by

$$v = v_0(1 - e^{-Ft/mv_0}).$$

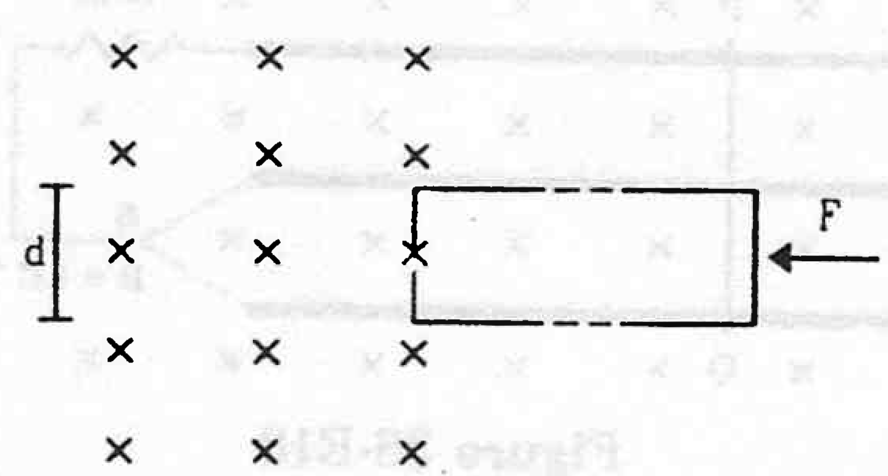


Figure 38-E22

49. Figure (38-E23) shows a smooth pair of thick metallic rails connected across a battery of emf $\mathcal{E}$ having a negligible internal resistance. A wire ab of length l and resistance r can slide smoothly on the rails. The entire system lies in a horizontal plane and is immersed in a uniform vertical magnetic field B. At an instant t, the wire is given a small velocity v towards right. (a) Find the current in it at this instant. What is the direction of the current ? (b) What is the force acting on the wire at this instant ? (c) Show that after some time the wire ab will slide with a constant velocity. Find this velocity.

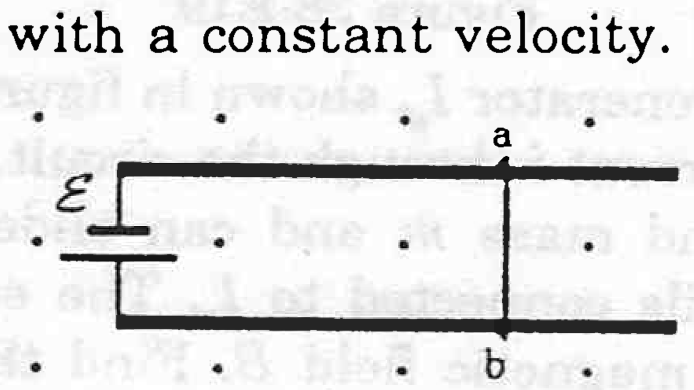


Figure 38-E23

50. A conducting wire ab of length l, resistance r and mass m starts sliding at $t = 0$ down a smooth, vertical, thick pair of connected rails as shown in figure (38-E24). A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the rails. (a) Write the induced emf in the loop at an instant t when the speed of the wire is v. (b) What would be the magnitude and direction of the induced current in the wire ? (c) Find the downward acceleration of the wire at this instant. (d) After sufficient time, the wire starts moving with a constant velocity. Find this velocity v_m. (e) Find the velocity of the wire as a function of time. (f) Find the displacement of the wire as a function of time. (g) Show that the rate of heat developed in the wire is equal to the rate at which the gravitational potential energy is decreased after steady state is reached.

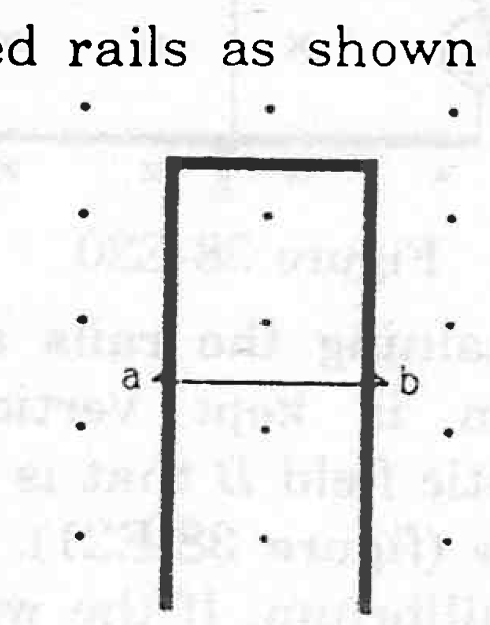


Figure 38-E24

51. A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 100 revolutions per minute. If the length of each spoke is 30·0 cm and the horizontal component of the earth's magnetic field is $2{\cdot}0 \times 10^{-5}$ T, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

52. A conducting disc of radius r rotates with a small but constant angular velocity ω about its axis. A uniform magnetic field B exists parallel to the axis of rotation. Find the motional emf between the centre and the periphery of the disc.

53. Figure (38-E25) shows a conducting disc rotating about its axis in a perpendicular magnetic field B. A resistor of resistance R is connected between the centre and the rim. Calculate the current in the resistor. Does it enter the disc or leave it at the centre ? The radius of the disc is 5·0 cm, angular speed $\omega = 10$ rad/s, $B = 0{\cdot}40$ T and $R = 10\ \Omega$.

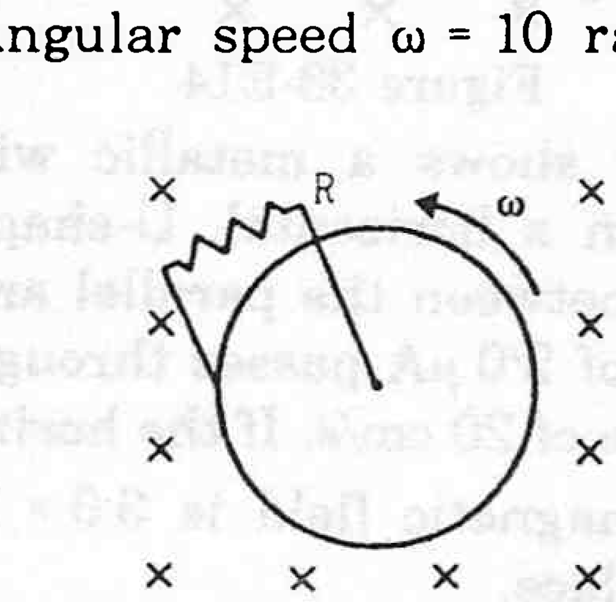


Figure 38-E25

54. The magnetic field in a region is given by $\vec{B} = \vec{k}\dfrac{B_0}{L}y$ where L is a fixed length. A conducting rod of length L lies along the Y-axis between the origin and the point $(0, L, 0)$. If the rod moves with a velocity $v = v_0\,\vec{i}$, find the emf induced between the ends of the rod.

55. Figure (38-E26) shows a straight, long wire carrying a current i and a rod of length l coplanar with the wire and perpendicular to it. The rod moves with a constant velocity v in a direction parallel to the wire. The distance of the wire from the centre of the rod is x. Find the motional emf induced in the rod.

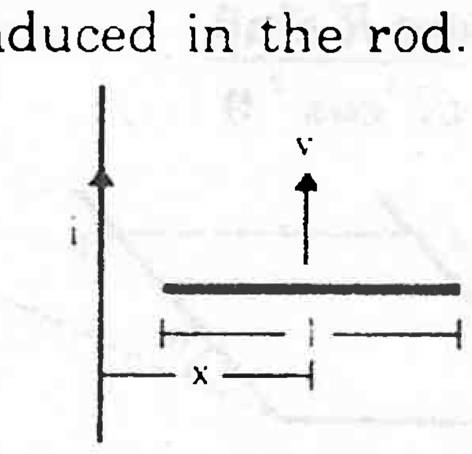


Figure 38-E26

56. Consider a situation similar to that of the previous problem except that the ends of the rod slide on a pair of thick metallic rails laid parallel to the wire. At one end the rails are connected by resistor of resistance R. (a) What force is needed to keep the rod sliding at a constant speed v? (b) In this situation what is the current in the resistance R? (c) Find the rate of heat developed in the resistor. (d) Find the power delivered by the external agent exerting the force on the rod.

57. Figure (38-E27) shows a square frame of wire having a total resistance r placed coplanarly with a long, straight wire. The wire carries a current i given by $i = i_0 \sin \omega t$. Find (a) the flux of the magnetic field through the square frame, (b) the emf induced in the frame and (c) the heat developed in the frame in the time interval 0 to $\frac{20\pi}{\omega}$.

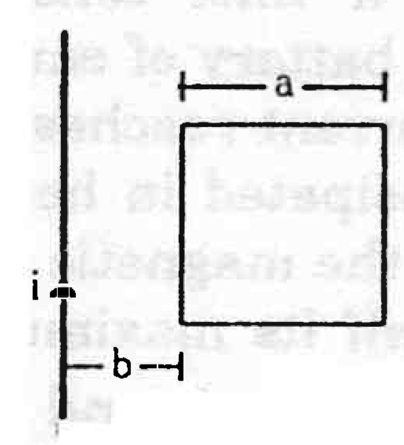


Figure 38-E27

58. A rectangular metallic loop of length l and width b is placed coplanarly with a long wire carrying a current i (figure 38-E29). The loop is moved perpendicular to the wire with a speed v in the plane containing the wire and the loop. Calculate the emf induced in the loop when the rear end of the loop is at a distance a from the wire. Solve by using Faraday's law for the flux through the loop and also by replacing different segments with equivalent batteries.

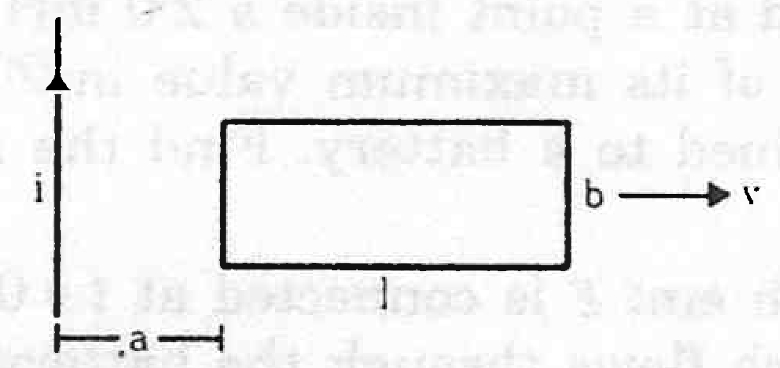


Figure 38-E28

59. Figure (38-E29) shows a conducting circular loop of radius a placed in a uniform, perpendicular magnetic field B. A thick metal rod OA is pivoted at the centre O. The other end of the rod touches the loop at A. The centre O and a fixed point C on the loop are connected by a wire OC of resistance R. A force is applied at the middle point of the rod OA perpendicularly, so that the rod rotates clockwise at a uniform angular velocity ω. Find the force.

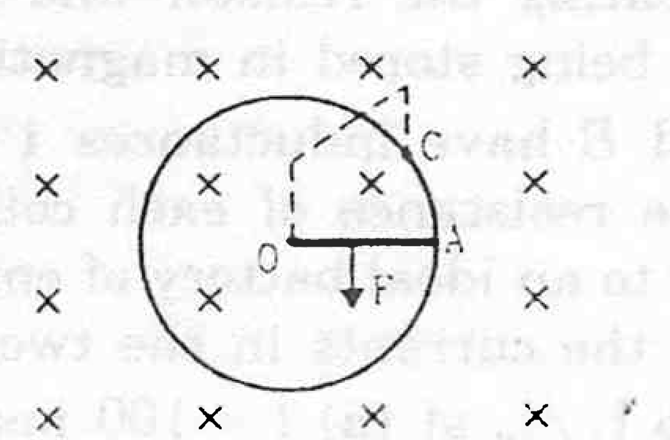


Figure 38-E29

60. Consider the situation shown in the figure of the previous problem. Suppose the wire connecting O and C has zero resistance but the circular loop has a resistance R uniformly distributed along its length. The rod OA is made to rotate with a uniform angular speed ω as shown in the figure. Find the current in the rod when $\angle AOC = 90°$.

61. Consider a variation of the previous problem (figure 38-E29). Suppose the circular loop lies in a vertical plane. The rod has a mass m. The rod and the loop have negligible resistances but the wire connecting O and C has a resistance R. The rod is made to rotate with a uniform angular velocity ω in the clockwise direction by applying a force at the midpoint of OA in a direction perpendicular to it. Find the magnitude of this force when the rod makes an angle θ with the vertical.

62. Figure (38-E30) shows a situation similar to the previous problem. All parameters are the same except that a battery of emf $\mathcal{E}$ and a variable resistance R are connected between O and C. The connecting wires have zero resistance. No external force is applied on the rod (except gravity, forces by the magnetic field and by the pivot). In what way should the resistance R be changed so that the rod may rotate with uniform angular velocity in the clockwise direction? Express your answer in terms of the given quantities and the angle θ made by the rod OA with the horizontal.

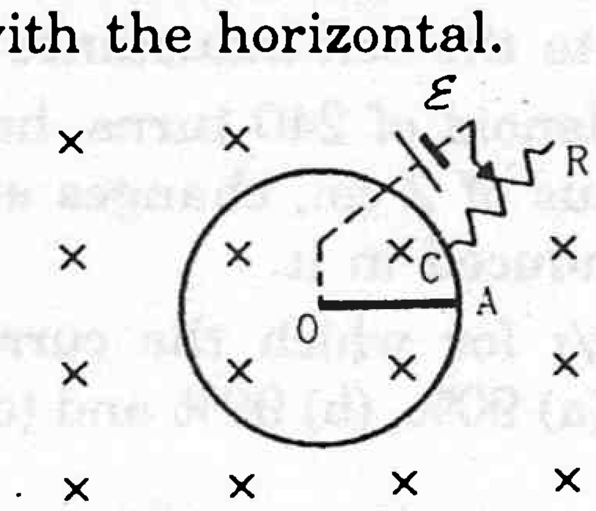


Figure 38-E30

63. A wire of mass m and length l can slide freely on a pair of smooth, vertical rails (figure 38-E31). A magnetic field B exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by a capacitor of capacitance C. Find the acceleration of the wire neglecting any electric resistance.

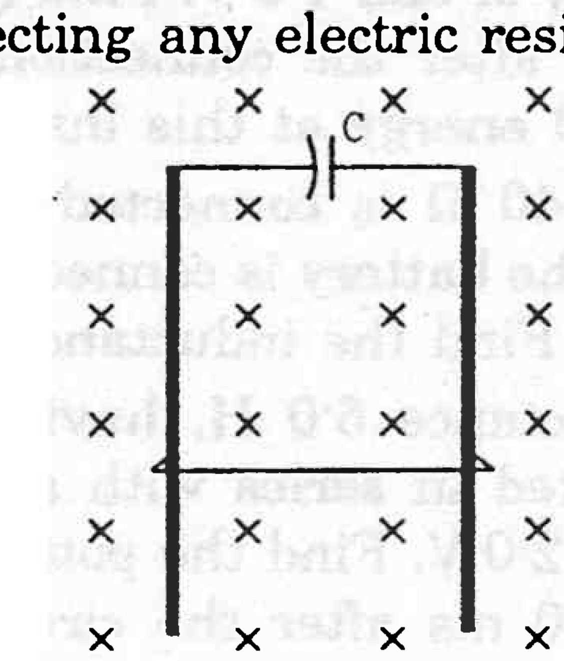


Figure 38-E31

64. A uniform magnetic field B exists in a cylindrical region, shown dotted in figure (38-E32). The magnetic field increases at a constant rate $\frac{dB}{dt}$. Consider a circle of

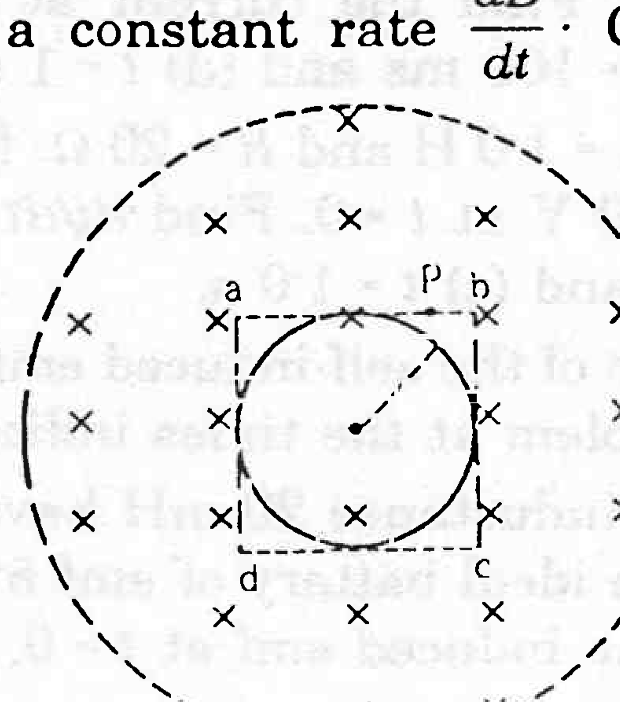


Figure 38-E32

radius r coaxial with the cylindrical region. (a) Find the magnitude of the electric field E at a point on the circumference of the circle. (b) Consider a point P on the side of the square circumscribing the circle. Show that the component of the induced electric field at P along ba is the same as the magnitude found in part (a).

65. The current in an ideal, long solenoid is varied at a uniform rate of 0·01 A/s. The solenoid has 2000 turns/m and its radius is 6·0 cm. (a) Consider a circle of radius 1·0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2·0 seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance 8·0 cm from its axis.

66. An average emf of 20 V is induced in an inductor when the current in it is changed from 2·5 A in one direction to the same value in the opposite direction in 0·1 s. Find the self-inductance of the inductor.

67. A magnetic flux of 8×10^{-4} weber is linked with each turn of a 200-turn coil when there is an electric current of 4 A in it. Calculate the self-inductance of the coil.

68. The current in a solenoid of 240 turns, having a length of 12 cm and a radius of 2 cm, changes at a rate of 0·8 A/s. Find the emf induced in it.

69. Find the value of t/τ for which the current in an LR circuit builds up to (a) 90%, (b) 99% and (c) 99·9% of the steady-state value.

70. An inductor-coil carries a steady-state current of 2·0 A when connected across an ideal battery of emf 4·0 V. If its inductance is 1·0 H, find the time constant of the circuit.

71. A coil having inductance 2·0 H and resistance 20 Ω is connected to a battery of emf 4·0 V. Find (a) the current at the instant 0·20 s after the connection is made and (b) the magnetic field energy at this instant.

72. A coil of resistance 40 Ω is connected across a 4·0 V battery. 0·10 s after the battery is connected, the current in the coil is 63 mA. Find the inductance of the coil.

73. An inductor of inductance 5·0 H, having a negligible resistance, is connected in series with a 100 Ω resistor and a battery of emf 2·0 V. Find the potential difference across the resistor 20 ms after the circuit is switched on.

74. The time constant of an LR circuit is 40 ms. The circuit is connected at $t = 0$ and the steady-state current is found to be 2·0 A. Find the current at (a) $t = 10$ ms (b) $t = 20$ ms, (c) $t = 100$ ms and (d) $t = 1$ s.

75. An LR circuit has $L = 1·0$ H and $R = 20$ Ω. It is connected across an emf of 2·0 V at $t = 0$. Find di/dt at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1·0$ s.

76. What are the values of the self-induced emf in the circuit of the previous problem at the times indicated therein ?

77. An inductor-coil of inductance 20 mH having resistance 10 Ω is joined to an ideal battery of emf 5·0 V. Find the rate of change of the induced emf at $t = 0$, (b) $t = 10$ ms and (c) $t = 1·0$ s.

78. An LR circuit contains an inductor of 500 mH, a resistor of 25·0 Ω and an emf of 5·00 V in series. Find the potential difference across the resistor at t = (a) 20·0 ms, (b) 100 ms and (c) 1·00 s.

79. An inductor-coil of resistance 10 Ω and inductance 120 mH is connected across a battery of emf 6 V and internal resistance 2 Ω. Find the charge which flows through the inductor in (a) 10 ms, (b) 20 ms and (c) 100 ms after the connections are made.

80. An inductor-coil of inductance 17 mH is constructed from a copper wire of length 100 m and cross-sectional area 1 mm^2. Calculate the time constant of the circuit if this inductor is joined across an ideal battery. The resistivity of copper = $1·7 \times 10^{-8}$ Ω–m.

81. An LR circuit having a time constant of 50 ms is connected with an ideal battery of emf $\mathcal{E}$. Find the time elapsed before (a) the current reaches half its maximum value, (b) the power dissipated in heat reaches half its maximum value and (c) the magnetic field energy stored in the circuit reaches half its maximum value.

82. A coil having an inductance L and a resistance R is connected to a battery of emf $\mathcal{E}$. Find the time taken for the magnetic energy stored in the circuit to change from one fourth of the steady-state value to half of the steady-state value.

83. A solenoid having inductance 4·0 H and resistance 10 Ω is connected to a 4·0 V battery at $t = 0$. Find (a) the time constant, (b) the time elapsed before the current reaches 0·63 of its steady-state value, (c) the power delivered by the battery at this instant and (d) the power dissipated in Joule heating at this instant.

84. The magnetic field at a point inside a 2·0 mH inductor-coil becomes 0·80 of its maximum value in 20 μs when the inductor is joined to a battery. Find the resistance of the circuit.

85. An LR circuit with emf $\mathcal{E}$ is connected at $t = 0$. (a) Find the charge Q which flows through the battery during 0 to t. (b) Calculate the work done by the battery during this period. (c) Find the heat developed during this period. (d) Find the magnetic field energy stored in the circuit at time t. (e) Verify that the results in the three parts above are consistent with energy conservation.

86. An inductor of inductance 2·00 H is joined in series with a resistor of resistance 200 Ω and a battery of emf 2·00 V. At $t = 10$ ms, find (a) the current in the circuit, (b) the power delivered by the battery, (c) the power dissipated in heating the resistor and (d) the rate at which energy is being stored in magnetic field.

87. Two coils A and B have inductances 1·0 H and 2·0 H respectively. The resistance of each coil is 10 Ω. Each coil is connected to an ideal battery of emf 2·0 V at $t = 0$. Let i_A and i_B be the currents in the two circuit at time t. Find the ratio i_A/i_B at (a) $t = 100$ ms, (b) $t = 200$ ms and (c) $t = 1$ s.

88. The current in a discharging LR circuit without the battery drops from 2·0 A to 1·0 A in 0·10 s. (a) Find the time constant of the circuit. (b) If the inductance of the circuit is 4·0 H, what is its resistance ?

89. A constant current exists in an inductor-coil connected to a battery. The coil is short-circuited and the battery is removed. Show that the charge flown through the coil after the short-circuiting is the same as that which flows in one time constant before the short-circuiting.

90. Consider the circuit shown in figure (38-E33). (a) Find the current through the battery a long time after the switch S is closed. (b) Suppose the switch is again opened at $t = 0$. What is the time constant of the discharging circuit ? (c) Find the current through the inductor after one time constant.

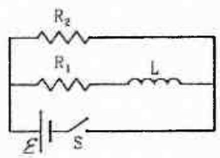


Figure 38-E33

91. A current of 1·0 A is established in a tightly wound solenoid of radius 2 cm having 1000 turns/metre. Find the magnetic energy stored in each metre of the solenoid.

92. Consider a small cube of volume 1 mm^3 at the centre of a circular loop of radius 10 cm carrying a current of 4 A. Find the magnetic energy stored inside the cube.

93. A long wire carries a current of 4·00 A. Find the energy stored in the magnetic field inside a volume of 1·00 mm^3 at a distance of 10·0 cm from the wire.

94. The mutual inductance between two coils is 2·5 H. If the current in one coil is changed at the rate of 1 A/s, what will be the emf induced in the other coil ?

95. Find the mutual inductance between the straight wire and the square loop of figure (38-E27).

96. Find the mutual inductance between the circular coil and the loop shown in figure (38-E8).

97. A solenoid of length 20 cm, area of cross-section 4·0 cm^2 and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area 8·0 cm^2 and length 10 cm. Find the mutual inductance between the solenoids.

98. The current in a long solenoid of radius R and having n turns per unit length is given by $i = i_0 \sin \omega t$. A coil having N turns is wound around it near the centre. Find (a) the induced emf in the coil and (b) the mutual inductance between the solenoid and the coil.

□

ANSWERS

OBJECTIVE I

1. (b)	2. (a)	3. (d)	4. (c)	5. (b)	6. (c)
7. (a)	8. (b)	9. (b)	10. (a)	11. (c)	12. (d)

OBJECTIVE II

1. (b), (c)	2. (d)	3. (c), (d)
4. (a), (c), (d)	5. (a), (b), (c)	6. (b), (d)
7. (d)	8. (b)	9. (a), (b), (c)
10. (b), (c)		

EXERCISE

1. $ML^2I^{-1}T^{-3}$ in each case
2. (a) volt/sec, volt, volt-sec (or weber) (b) 1·2 volt
3. (a) – 2·0 mV, – 4·0 mV, 4·0 mV, 2·0 mV (b)10 ms to 20 ms and 20 ms to 30 ms
4. $7·8 \times 10^{-3}$ V
5. 1×10^{-10} V
6. (a) 50 V (b) 50 V (c) zero
7. (a) 25 J (b) 25 J (c) 50 J
8. (a) 0·015 V (b) $7·5 \times 10^{-3}$ V (c) zero
9. 10 μV
10. 5·0 T
11. BA/R
12. 2×10^{-4} C
13. (a) $2Bav$ (b) $2Bav/R$ (c) $a^2 B/R$
14. $\mathcal{E}$ = 1·0 V, anticlockwise
15. zero
16. (a) 3×10^{-4} V, (b) zero, (c) 3×10^{-4} V and (d) zero
17. 2×10^{-4} J
18. $3·9 \times 10^{-5}$ A
19. (a) $1·25 \times 10^{-7}$ A, a to d (b) $1·25 \times 10^{-7}$ A, d to a, (c) zero (d) zero
20. $\dfrac{\pi\mu_0 N a^2 a'^2 \mathcal{E} R v}{2L(a^2 + x^2)^{3/2}(R' + r)}$ where $R' = R$ for part (a) and $R/2$ for part (b)
21. (a) $6·28 \times 10^{-2}$ V (b) $1·57 \times 10^{-3}$ C
22. (a) $2·0 \times 10^{-3}$ V (b) zero (c) $5·0 \times 10^{-5}$ C
23. $4·7 \times 10^{-5}$ C
24. (a) $6·6 \times 10^{-4}$ V (b) zero (c) $2·2 \times 10^{-7}$ V^2
25. $1·3 \times 10^{-7}$ J
26. $1·57 \times 10^{-6}$ V
27. (a) $1·6 \times 10^{-4}$ N (b) $1·0 \times 10^{-2}$ V/m (c) $2·0 \times 10^{-3}$ V
28. 0·4 V

29. 0·09 V
30. 1 mV
31. (a) zero (b) $vB(bc)$, positive at c (c) zero (d) $vB(bc)$, positive at a
32. (a) $2rvB$ (b) zero
33. 17×10^{-3} V
34. (a) at the ends of the diameter perpendicular to the velocity, $2\,rvB$ (b) at the ends of the diameter parallel to the velocity, zero
35. zero
36. $\dfrac{Blv}{2r(l+vt)}$
37. (a) $\dfrac{B^2l^2v}{2r(l+vt)}$ (b) l/v
38. (a) $\dfrac{Blv}{R+r}$ (b) $\dfrac{B^2l^2v}{m(R+r)}$ towards left (c) $v = v_0 - \dfrac{B^2l^2x}{m(R+r)}$ (d) $\dfrac{mv_0(R+r)}{B^2l^2}$
39. (a) 25 m/s (b) $4·0 \times 10^{-2}$ V (c) $3·6 \times 10^{-2}$ V (d) $4·0 \times 10^{-3}$ V
40. $\tan^{-1}(1/3)$
42. (a) 0·1 mA (b) zero
43. (a) zero (b) 1 mA
44. (a) 0·1 mA (b) 0·2 mA
45. $\dfrac{ir - Blv}{2r}$
46. $ilBt/m$, away from the generator
47. $2\sqrt{l/g}$
48. (a) $\dfrac{RF - vl^2B^2}{mR}$ (b) $\dfrac{RF}{l^2B^2}$
49. (a) $\dfrac{1}{r}(E - vBl)$ from b to a (b) $\dfrac{lB}{r}(E - vBl)$ towards right (c) $\dfrac{E}{Bl}$
50. (a) vBl (b) $\dfrac{vBl}{r}$, b to a (c) $g - \dfrac{B^2l^2}{mr}v$ (d) $\dfrac{mgr}{B^2l^2}$ (e) $v_m(1 - e^{-gt/v_m})$ (f) $v_m t - \dfrac{v_m^2}{g}(1 - e^{-gt/v_m})$
51. $9·4 \times 10^{-6}$ V
52. $\dfrac{1}{2}\omega r^2 B$
53. 0·5 mA, leaves
54. $\dfrac{B_0 v_0 l}{2}$
55. $\dfrac{\mu_0 iv}{2\pi}\ln\left(\dfrac{2x+l}{2x-l}\right)$
56. (a) $\dfrac{v}{R}\left\{\dfrac{\mu_0 i}{2\pi}\ln\dfrac{2x+l}{2x-l}\right\}^2$ (b) $\dfrac{\mu_o iv}{2\pi R}\ln\dfrac{2x+l}{2x-l}$ (c) $\dfrac{1}{R}\left\{\dfrac{\mu_0 iv}{2\pi}\ln\dfrac{2x+l}{2x-l}\right\}^2$ (d) same as (c)
57. (a) $\dfrac{\mu_0 ia}{2\pi}\ln\left(1+\dfrac{a}{b}\right)$ (b) $\dfrac{\mu_0 ai_0\omega\cos\omega t}{2\pi}\ln\left(1+\dfrac{a}{b}\right)$ (c) $\dfrac{5\mu_0^2 a^2 i_0^2 \omega}{2\pi r}\ln^2\left(1+\dfrac{a}{b}\right)$
58. $\dfrac{\mu\, ilvb}{2\pi a(a+l)}$
59. $\dfrac{\omega a^3 B^2}{2R}$ to the right of OA in the figure.
60. $\dfrac{8}{3}\dfrac{\omega a^2 B}{R}$
61. $\dfrac{\omega a^3 B^2}{2R} - mg\sin\theta$
62. $\dfrac{aB(2\mathcal{E} + \omega a^2 B)}{2\,mg\cos\theta}$
63. $\dfrac{mg}{m + CB^2l^2}$
64. (a) $\dfrac{r}{2}\dfrac{dB}{dt}$
65. (a) $1·6 \times 10^{-8}$ weber (b) $1·2 \times 10^{-7}$ V/m (c) $5·6 \times 10^{-[illegible]}$ V/m
66. 0·4 H
67. 4×10^{-2} H
68. 6×10^{-4} V
69. 2·3, 4·6, 6·9
70. 0·50 s
71. (a) 0·17 A (b) 0·03 J
72. 4·0 H
73. 0·66 V
74. (a) 0·44 A (b) 0·79 A (c) 1·8 A and (d) 2·0 A
75. (a) 0·27 A/s (b) 0·036 A/s and (c) $4·1 \times 10^{-9}$ A/s
76. (a) 0·27 V (b) 0·036 V (c) $4·1 \times 10^{-9}$ V
77. (a) $2·5 \times 10^3$ V/s (b) 17 V/s and (c) 0·00 V/s
78. (a) 3·16 V (b) 4·97 V and (c) 5·00 V
79. (a) 1·8 mC (b) 5·7 mC and (c) 45 mC
80. 10 ms
81. (a) 35 ms (b) 61 ms (c) 61 ms
82. $\tau\ln\dfrac{1}{2-\sqrt{2}}$
83. (a) 0·40 s (b) 0·40 s (c) 1·0 W and (d) 0·64 W
84. 160 Ω
85. (a) $\dfrac{\mathcal{E}}{R}\{t - \dfrac{L}{R}(1-x)\}$ (b) $\dfrac{\mathcal{E}^2}{R}\{t - \dfrac{L}{R}(1-x)\}$ (c) $\dfrac{\mathcal{E}^2}{R}\{t - \dfrac{L}{2R}(3 - 4x + x^2)\}$ (d) $\dfrac{L\mathcal{E}^2}{2R^2}(1-x)^2$ where $x = e^{-Rt/L}$
86. (a) 6·3 mA (b) 12·6 mW (c) 8·0 mW and (d) 4·6 mW

87. (a) 1·6 (b) 1·4 (c) 1·0

88. (a) 0·14 s (b) 28 Ω

90. (a) $\frac{\mathcal{E}(R_1 + R_2)}{R_1 R_2}$ (b) $\frac{L}{R_1 + R_2}$ (c) $\frac{\mathcal{E}}{R_1 e}$

91. $7{\cdot}9 \times 10^{-4}$ J

92. $8\pi \times 10^{-14}$ J

93. $2{\cdot}55 \times 10^{-14}$ J

94. 2·5 V

95. $\frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{a}{b}\right)$

96. $N \frac{\mu_0 \pi a^2 a'^2}{2(a^2 + x^2)^{3/2}}$

97. $2{\cdot}0 \times 10^{-6}$ H

98. (a) $\pi \mu_0 i_0 n N \omega R^2 \cos \omega t$ (b) $\pi \mu_0 n N R^2$

□

ELECTROMAGNETIC INDUCTION
CHAPTER - 38

1. (a) $\int E.dl = MLT^{-3}I^{-1} \times L = ML^2I^{-1}T^{-3}$

   (b) $\vartheta BI = LT^{-1} \times MI^{-1}T^{-2} \times L = ML^2I^{-1}T^{-3}$

   (c) $d\phi_s / dt = MI^{-1}T^{-2} \times L^2 = ML^2I^{-1}T^{-2}$

2. $\phi = at^2 + bt + c$

   (a) a = $\left[\frac{\phi}{t^2}\right] = \left[\frac{\phi / t}{t}\right] = \frac{Volt}{Sec}$

   b = $\left[\frac{\phi}{t}\right]$ = Volt

   c = $[\phi]$ = Weber

   (b) E = $\frac{d\phi}{dt}$ [a = 0.2, b = 0.4, c = 0.6, t = 2s]

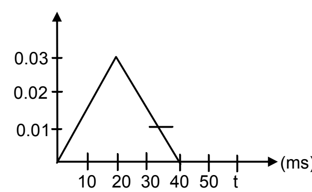
   = 2at + b

   = $2 \times 0.2 \times 2 + 0.4$ = 1.2 volt

3. (a) ϕ_2 = B.A. = $0.01 \times 2 \times 10^{-3} = 2 \times 10^{-5}$.

   $\phi_1 = 0$

   e = $-\frac{d\phi}{dt} = \frac{-2\times10^{-5}}{10\times10^{-3}}$ = – 2 mV



   ϕ_3 = B.A. = $0.03 \times 2 \times 10^{-3} = 6 \times 10^{-5}$

   $d\phi = 4 \times 10^{-5}$

   e = $-\frac{d\phi}{dt}$ = –4 mV

   ϕ_4 = B.A. = $0.01 \times 2 \times 10^{-3} = 2 \times 10^{-5}$

   $d\phi = -4 \times 10^{-5}$

   e = $-\frac{d\phi}{dt}$ = 4 mV

   ϕ_5 = B.A. = 0

   $d\phi = -2 \times 10^{-5}$

   e = $-\frac{d\phi}{dt}$ = 2 mV

   (b) emf is not constant in case of → 10 – 20 ms and 20 – 30 ms as –4 mV and 4 mV.

4. $\phi_1 = BA = 0.5 \times \pi(5 \times 10^{-2})^2 = 5\pi\ 25 \times 10^{-5} = 125 \times 10^{-5}$

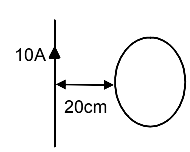
   $\phi_2 = 0$

   E = $\frac{\phi_1 - \phi_2}{t} = \frac{125\pi\times10^{-5}}{5\times10^{-1}}$ = $25\pi \times 10^{-4} = 7.8 \times 10^{-3}$.

5. A = 1 mm^2 ; i = 10 A, d = 20 cm ; dt = 0.1 s

   e = $\frac{d\phi}{dt} = \frac{BA}{dt} = \frac{\mu_0 i}{2\pi d} \times \frac{A}{dt}$

   = $\frac{4\pi\times10^{-7}\times10}{2\pi\times2\times10^{-1}} \times \frac{10^{-6}}{1\times10^{-1}} = 1\times10^{-10}$ V .



6. (a) During removal,

   ϕ_1 = B.A. = $1 \times 50 \times 0.5 \times 0.5 - 25 \times 0.5$ = 12.5 Tesla-m^2

   $\phi_2 = 0,\ \tau = 0.25$

$e = -\frac{d\phi}{dt} = \frac{\phi_2 - \phi_1}{dt} = \frac{12.5}{0.25} = \frac{125 \times 10^{-1}}{25 \times 10^{-2}} = 50V$

(b) During its restoration

$\phi_1 = 0$; $\phi_2 = 12.5$ Tesla-m^2 ; t = 0.25 s

$E = \frac{12.5 - 0}{0.25}$ = 50 V.

(c) During the motion

$\phi_1 = 0$, $\phi_2 = 0$

$E = \frac{d\phi}{dt} = 0$

7. R = 25 Ω

(a) e = 50 V, T = 0.25 s

i = e/R = 2A, H = i^2 RT

= 4 × 25 × 0.25 = 25 J

(b) e = 50 V, T = 0.25 s

i = e/R = 2A, H = i^2 RT = 25 J

(c) Since energy is a scalar quantity

Net thermal energy developed = 25 J + 25 J = 50 J.

8. A = 5 cm^2 = 5×10^{-4} m^2

$B = B_0 \sin \omega t = 0.2 \sin(300\, t)$

θ = 60°

a) Max emf induced in the coil

$E = -\frac{d\phi}{dt} = \frac{d}{dt}(BA\cos\theta)$

$= \frac{d}{dt}(B_0 \sin\omega t \times 5 \times 10^{-4} \times \frac{1}{2})$

$= B_0 \times \frac{5}{2} \times 10^{-4} \frac{d}{dt}(\sin\omega t) = \frac{B_0 5}{2} \times 10^{-4} \cos\omega t \cdot \omega$

$= \frac{0.2 \times 5}{2} \times 300 \times 10^{-4} \times \cos\omega t = 15 \times 10^{-3} \cos\omega t$

$E_{max} = 15 \times 10^{-3} = 0.015$ V

b) Induced emf at t = (π/900) s

$E = 15 \times 10^{-3} \times \cos \omega t$

$= 15 \times 10^{-3} \times \cos (300 \times \pi/900) = 15 \times 10^{-3} \times ½$

$= 0.015/2 = 0.0075 = 7.5 \times 10^{-3}$ V

c) Induced emf at t = π/600 s

$E = 15 \times 10^{-3} \times \cos (300 \times \pi/600)$

$= 15 \times 10^{-3} \times 0 = 0$ V.

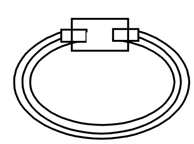
9. $\vec{B}$ = 0.10 T

A = 1 cm^2 = 10^{-4} m^2

T = 1 s

ϕ = B.A. = $10^{-1} \times 10^{-4} = 10^{-5}$

$e = \frac{d\phi}{dt} = \frac{10^{-5}}{1} = 10^{-5}$ = 10 μV



10. E = 20 mV = 20×10^{-3} V

A = $(2 \times 10^{-2})^2 = 4 \times 10^{-4}$

Dt = 0.2 s, θ = 180°

ϕ_1 = BA, ϕ_2 = –BA

$d\phi$ = 2BA

$E = \frac{d\phi}{dt} = \frac{2BA}{dt}$

$\Rightarrow 20 \times 10^{-3} = \frac{2 \times B \times 2 \times 10^{-4}}{2 \times 10^{-1}}$

$\Rightarrow 20 \times 10^{-3} = 4 \times B \times 10^{-3}$

$\Rightarrow B = \frac{20 \times 10^{-3}}{42 \times 10^{-3}} = 5T$

11. Area = A, Resistance = R, B = Magnetic field

ϕ = BA = Ba cos 0° = BA

$e = \frac{d\phi}{dt} = \frac{BA}{1}$; $i = \frac{e}{R} = \frac{BA}{R}$

ϕ = iT = BA/R

12. r = 2 cm = 2×10^{-2} m

n = 100 turns / cm = 10000 turns/m

i = 5 A

B = μ_0 ni

= $4\pi \times 10^{-7} \times 10000 \times 5 = 20\pi \times 10^{-3} = 62.8 \times 10^{-3}$ T

n_2 = 100 turns

R = 20 Ω

r = 1 cm = 10^{-2} m

Flux linking per turn of the second coil = $B\pi r^2 = B\pi \times 10^{-4}$

ϕ_1 = Total flux linking = $Bn_2 \pi r^2 = 100 \times \pi \times 10^{-4} \times 20\pi \times 10^{-3}$

When current is reversed.

$\phi_2 = -\phi_1$

$d\phi = \phi_2 - \phi_1 = 2 \times 100 \times \pi \times 10^{-4} \times 20\pi \times 10^{-3}$

$E = -\frac{d\phi}{dt} = \frac{4\pi^2 \times 10^{-4}}{dt}$

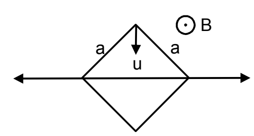
$I = \frac{E}{R} = \frac{4\pi^2 \times 10^{-4}}{dt \times 20}$

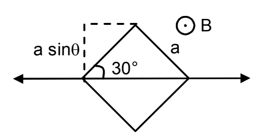
$q = Idt = \frac{4\pi^2 \times 10^{-4}}{dt \times 20} \times dt = 2 \times 10^{-4}$ C.

13. Speed = u

Magnetic field = B

Side = a





a) The perpendicular component i.e. a sinθ is to be taken which is $\perp$r to velocity.

So, l = a sin θ 30° = a/2.

Net 'a' charge = 4 × a/2 = 2a

So, induced emf = BϑI = 2auB

b) Current = $\frac{E}{R} = \frac{2auB}{R}$

14. ϕ_1 = 0.35 weber, ϕ_2 = 0.85 weber

D$\phi = \phi_2 - \phi_1$ = (0.85 – 0.35) weber = 0.5 weber

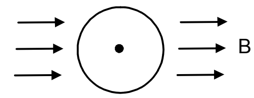
dt = 0.5 sec

$E = \frac{d\phi}{dt'} = \frac{0.5}{0.5}$ = 1 v.

The induced current is anticlockwise as seen from above.

15. i = v(B × l)

= v B l $\cos\theta$



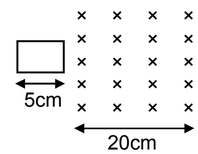
θ is angle between normal to plane and $\vec{B}$ = 90°.

= v B l cos 90° = 0.

16. u = 1 cm/′, B = 0.6 T

a) At t = 2 sec, distance moved = 2 × 1 cm/s = 2 cm

$E = \frac{d\phi}{dt} = \frac{0.6 \times (2 \times 5 - 0) \times 10^{-4}}{2}$ = 3 ×10^{-4} V



b) At t = 10 sec

distance moved = 10 × 1 = 10 cm

The flux linked does not change with time

$\therefore$ E = 0

c) At t = 22 sec

distance = 22 × 1 = 22 cm

The loop is moving out of the field and 2 cm outside.

$E = \frac{d\phi}{dt} = B \times \frac{dA}{dt}$

$= \frac{0.6 \times (2 \times 5 \times 10^{-4})}{2}$ = 3 × 10^{-4} V

d) At t = 30 sec

The loop is total outside and flux linked = 0

$\therefore$ E = 0.

17. As heat produced is a scalar prop.

So, net heat produced = $H_a + H_b + H_c + H_d$

R = 4.5 mΩ = 4.5 ×10^{-3} Ω

a) e = 3 × 10^{-4} V

$i = \frac{e}{R} = \frac{3 \times 10^{-4}}{4.5 \times 10^{-3}} = 6.7 \times 10^{-2}$ Amp.

$H_a = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$

$H_b = H_d = 0$ [since emf is induced for 5 sec]

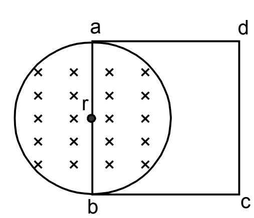
$H_c = (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5$

So Total heat = $H_a + H_c$

$= 2 \times (6.7 \times 10^{-2})^2 \times 4.5 \times 10^{-3} \times 5 = 2 \times 10^{-4}$ J.

18. r = 10 cm, R = 4 Ω

$\frac{dB}{dt} = 0.010\ T/'$, $\frac{d\phi}{dt} = \frac{dB}{dt} A$



$E = \frac{d\phi}{dt} = \frac{dB}{dt} \times A = 0.01\left(\frac{\pi \times r^2}{2}\right)$

$= \frac{0.01 \times 3.14 \times 0.01}{2} = \frac{3.14}{2} \times 10^{-4}$ = 1.57 × 10^{-4}

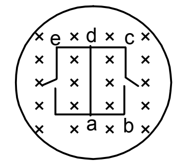
$i = \frac{E}{R} = \frac{1.57 \times 10^{-4}}{4}$ = 0.39 × 10^{-4} = 3.9 × 10^{-5}A

19. a) S_1 closed S_2 open

net R = 4 × 4 = 16 Ω

$e = \frac{d\phi}{dt} = A\frac{dB}{dt} = 10^{-4} \times 2 \times 10^{-2}$ = 2 × 10^{-6} V

i through ad = $\frac{e}{R} = \frac{2 \times 10^{-6}}{16} = 1.25 \times 10^{-7}$ A along ad



b) R = 16 Ω

$e = A \times \frac{dB}{dt}$ = 2 × 0^{-5} V

$i = \frac{2 \times 10^{-6}}{16}$ = 1.25 × 10^{-7} A along d a

c) Since both S_1 and S_2 are open, no current is passed as circuit is open i.e. i = 0

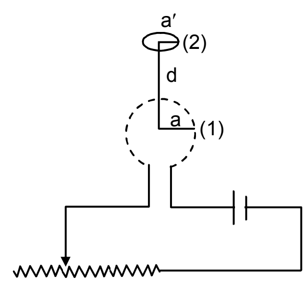
d) Since both S_1 and S_2 are closed, the circuit forms a balanced wheat stone bridge and no current will flow along ad i.e. i = 0.

20. Magnetic field due to the coil (1) at the center of (2) is $B = \frac{\mu_0 Nia^2}{2(a^2+x^2)^{3/2}}$

Flux linked with the second,

$= B.A_{(2)} = \frac{\mu_0 Nia^2}{2(a^2+x^2)^{3/2}}\pi a'^2$

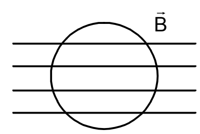
E.m.f. induced $\frac{d\phi}{dt} = \frac{\mu_0 Na^2a'^2\pi}{2(a^2+x^2)^{3/2}}\frac{di}{dt}$



$= \frac{\mu_0 N\pi a^2a'^2}{2(a^2+x^2)^{3/2}}\frac{d}{dt}\frac{E}{((R/L)x+r)}$

$= \frac{\mu_0 N\pi a^2a'^2}{2(a^2+x^2)^{3/2}}E\frac{-1.R/L.v}{((R/L)x+r)^2}$

b) $= \frac{\mu_0 N\pi a^2a'^2}{2(a^2+x^2)^{3/2}}\frac{ERV}{L(R/2+r)^2}$ (for x = L/2, R/L x = R/2)



a) For x = L

$E = \frac{\mu_0 N\pi a^2a'^2RvE}{2(a^2+x^2)^{3/2}(R+r)^2}$

21. N = 50, $\vec{B}$ = 0.200 T ; r = 2.00 cm = 0.02 m

θ = 60°, t = 0.100 s

a) $e = \frac{Nd\phi}{dt} = \frac{N \times B.A}{T} = \frac{NBA\cos 60°}{T}$

$= \frac{50 \times 2 \times 10^{-1} \times \pi \times (0.02)^2}{0.1}$ = 5 × 4 × 10^{-3} × π

= 2π × 10^{-2} V = 6.28 × 10^{-2} V

b) $i = \frac{e}{R} = \frac{6.28 \times 10^{-2}}{4}$ = 1.57 × 10^{-2} A

Q = it = 1.57 × 10^{-2} × 10^{-1} = 1.57 × 10^{-3} C

22. n = 100 turns, B = 4 × 10^{-4} T

A = 25 cm^2 = 25 × 10^{-4} m^2

a) When the coil is perpendicular to the field

φ = nBA

When coil goes through half a turn

φ = BA cos 18° = 0 – nBA

dφ = 2nBA

The coil undergoes 300 rev, in 1 min

$300 \times 2\pi$ rad/min = $10\,\pi$ rad/sec

10π rad is swept in 1 sec.

π/π rad is swept $1/10\pi \times \pi = 1/10$ sec

$$E = \frac{d\phi}{dt} = \frac{2nBA}{dt} = \frac{2\times 100\times 4\times 10^{-4}\times 25\times 10^{-4}}{1/10} = 2 \times 10^{-3}\text{ V}$$

b) $\phi_1 = nBA$, $\phi_2 = nBA$ ($\theta = 360°$)

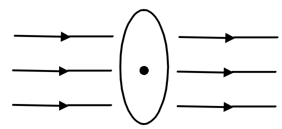
$d\phi = 0$

c) $i = \frac{E}{R} = \frac{2\times 10^{-3}}{4} = \frac{1}{2}\times 10^{-3}$

$= 0.5 \times 10^{-3} = 5 \times 10^{-4}$

$q = idt = 5 \times 10^{-4} \times 1/10 = 5 \times 10^{-5}$ C.

23. r = 10 cm = 0.1 m



$R = 40\ \Omega$, $N = 1000$

$\theta = 180°$, $B_H = 3 \times 10^{-5}$ T

$\phi = N(B.A) = NBA \cos 180°$ or $= -NBA$

$= 1000 \times 3 \times 10^{-5} \times \pi \times 1 \times 10^{-2} = 3\pi \times 10^{-4}$ where

$d\phi = 2NBA = 6\pi \times 10^{-4}$ weber

$$e = \frac{d\phi}{dt} = \frac{6\pi\times 10^{-4}\text{ V}}{dt}$$

$$i = \frac{6\pi\times 10^{-4}}{40dt} = \frac{4.71\times 10^{-5}}{dt}$$

$$Q = \frac{4.71\times 10^{-5}\times dt}{dt} = 4.71 \times 10^{-5}\text{ C.}$$

24. $\text{emf} = \frac{d\phi}{dt} = \frac{dB.A\cos\theta}{dt}$

$= B\,A \sin\theta\,\omega = -BA\,\omega \sin\theta$

(dq/dt = the rate of change of angle between arc vector and B = ω)

a) emf maximum = $BA\omega = 0.010 \times 25 \times 10^{-4} \times 80 \times \frac{2\pi\times\pi}{6}$

$= 0.66 \times 10^{-3} = 6.66 \times 10^{-4}$ volt.

b) Since the induced emf changes its direction every time,

so for the average emf = 0

25. $H = \int_0^t i^2 R dt = \int_0^t \frac{B^2A^2\omega^2}{R^2}\sin \omega t\, R\, dt$

$$= \frac{B^2A^2\omega^2}{2R^2}\int_0^t (1-\cos 2\omega t)dt$$

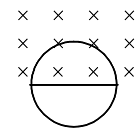
$$= \frac{B^2A^2\omega^2}{2R}\left(t - \frac{\sin 2\omega t}{2\omega}\right)_0^{1\text{ minute}}$$

$$= \frac{B^2A^2\omega^2}{2R}\left(60 - \frac{\sin 2\times 8 - \times 2\pi/60\times 60}{2\times 80\times 2\pi/60}\right)$$

$$= \frac{60}{200}\times \pi^2 r^4 \times B^2 \times \left(80^4 \times \frac{2\pi}{60}\right)^2$$

$$= \frac{60}{200}\times 10 \times \frac{64}{9}\times 10\times 625\times 10^{-8}\times 10^{-4} = \frac{625\times 6\times 64}{9\times 2}\times 10^{-11} = 1.33 \times 10^{-7}\text{ J.}$$

26. $\phi_1 = BA$, $\phi_2 = 0$



$= \dfrac{2 \times 10^{-4} \times \pi(0.1)^2}{2} = \pi \times 10^{-5}$

$E = \dfrac{d\phi}{dt} = \dfrac{\pi \times 10^{-6}}{2} = 1.57 \times 10^{-6}$ V

27. l = 20 cm = 0.2 m

v = 10 cm/s = 0.1 m/s

B = 0.10 T

a) $F = q\,v\,B = 1.6 \times 10^{-19} \times 1 \times 10^{-1} \times 1 \times 10^{-1} = 1.6 \times 10^{-21}$ N

b) qE = qvB

$\Rightarrow E = 1 \times 10^{-1} \times 1 \times 10^{-1} = 1 \times 10^{-2}$ V/m

This is created due to the induced emf.

c) Motional emf = $Bv\ell$

$= 0.1 \times 0.1 \times 0.2 = 2 \times 10^{-3}$ V

28. ℓ = 1 m, B = 0.2 T, v = 2 m/s, e = $B\ell v$

$= 0.2 \times 1 \times 2 = 0.4$ V

29. ℓ = 10 m, v = 3×10^{7} m/s, B = 3×10^{-10} T

Motional emf = $Bv\ell$

$= 3 \times 10^{-10} \times 3 \times 10^{7} \times 10 = 9 \times 10^{-3} = 0.09$ V

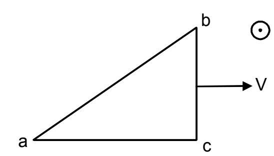
30. v = 180 km/h = 50 m/s

B = 0.2×10^{-4} T, L = 1 m

$E = Bv\ell = 0.2\ \mathrm{I}\ 10^{-4} \times 50 = 10^{-3}$ V

$\therefore$ The voltmeter will record 1 mv.

31. a) Zero as the components of ab are exactly opposite to that of bc. So they cancel each other. Because velocity should be perpendicular to the length.



b) e = Bv × ℓ

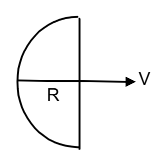
= Bv (bc) +ve at C

c) e = 0 as the velocity is not perpendicular to the length.

d) e = Bv (bc) positive at ‘a’.

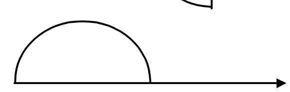
i.e. the component of ‘ab’ along the perpendicular direction.

32. a) Component of length moving perpendicular to V is 2R



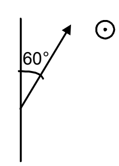
$\therefore$ E = B v 2R

b) Component of length perpendicular to velocity = 0



$\therefore$ E = 0

33. ℓ = 10 cm = 0.1 m ;



θ = 60° ; B = 1T

V = 20 cm/s = 0.2 m/s

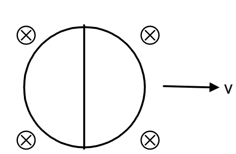
$E = Bv\ell \sin 60°$

[As we have to take that component of length vector which is $\perp$r to the velocity vector]

$= 1 \times 0.2 \times 0.1 \times \sqrt{3}/2$

$= 1.732 \times 10^{-2} = 17.32 \times 10^{-3}$ V.

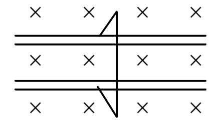
34. a) The e.m.f. is highest between diameter $\perp$r to the velocity. Because here length $\perp$r to velocity is highest.



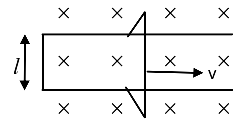
E_{max} = VB2R

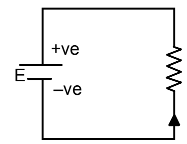
b) The length perpendicular to velocity is lowest as the diameter is parallel to the velocity E_{min} = 0.

35. $F_{magnetic} = i\ell B$
This force produces an acceleration of the wire.
But since the velocity is given to be constant.
Hence net force acting on the wire must be zero.

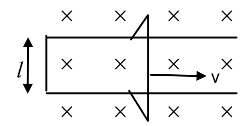


36. $E = Bv\ell$
Resistance = r × total length
$= r \times 2(\ell + vt) = 24(\ell + vt)$
$i = \dfrac{Bv\ell}{2r(\ell + vt)}$





37. $e = Bv\ell$
$i = \dfrac{e}{R} = \dfrac{Bv\ell}{2r(\ell + vt)}$



a) $F = i\ell B = \dfrac{Bv\ell}{2r(\ell + vt)} \times \ell B = \dfrac{B^2\ell^2 v}{2r(\ell + vt)}$

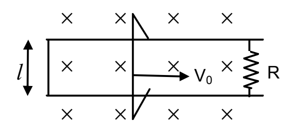
b) Just after t = 0

$F_0 = i\,\ell\,B = \ell B\left(\dfrac{\ell Bv}{2r\ell}\right) = \dfrac{\ell B^2 v}{2r}$

$\dfrac{F_0}{2} = \dfrac{\ell B^2 v}{4r} = \dfrac{\ell^2 B^2 v}{2r(\ell + vt)}$

$\Rightarrow$ $2\ell = \ell + vt$
$\Rightarrow$ $T = \ell/v$

38. a) When the speed is V
Emf = $B\ell v$
Resistance = r + r
Current = $\dfrac{B\ell v}{r + R}$



b) Force acting on the wire = $i\ell B$
$= \dfrac{B\ell v \ell B}{R + r} = \dfrac{B^2\ell^2 v}{R + r}$

Acceleration on the wire = $\dfrac{B^2\ell^2 v}{m(R + r)}$

c) $v = v_0 + at = v_0 - \dfrac{B^2\ell^2 v}{m(R + r)}t$ [force is opposite to velocity]

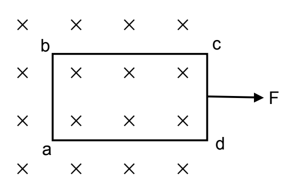
$= v_0 - \dfrac{B^2\ell^2 x}{m(R + r)}$

d) $a = v\dfrac{dv}{dx} = \dfrac{B^2\ell^2 v}{m(R + r)}$

$\Rightarrow dx = \dfrac{dv\,m(R + r)}{B^2\ell^2}$

$\Rightarrow x = \dfrac{m(R + r)v_0}{B^2\ell^2}$

39. R = 2.0 Ω , B = 0.020 T, l = 32 cm = 0.32 m
B = 8 cm = 0.08 m
a) $F = i\ell B = 3.2 \times 10^{-5}$ N
$= \dfrac{B^2\ell^2 v}{R} = 3.2 \times 10^5$



$\Rightarrow \dfrac{(0.020)^2 \times (0.08)^2 \times v}{2} = 3.2 \times 10^{-5}$

$\Rightarrow v = \dfrac{3.2 \times 10^{-5} \times 2}{6.4 \times 10^{-3} \times 4 \times 10^{-4}} = 25$ m/s

b) Emf $E = vB\ell = 25 \times 0.02 \times 0.08 = 4 \times 10^{-2}$ V

c) Resistance per unit length = $\dfrac{2}{0.8}$

Resistance of part ad/cb = $\dfrac{2 \times 0.72}{0.8} = 1.8\ \Omega$

$V_{ab} = iR = \dfrac{B\ell v}{2} \times 1.8 = \dfrac{0.02 \times 0.08 \times 25 \times 1.8}{2} = 0.036\ V = 3.6 \times 10^{-2}$ V

d) Resistance of cd = $\dfrac{2 \times 0.08}{0.8} = 0.2\ \Omega$

$V = iR = \dfrac{0.02 \times 0.08 \times 25 \times 0.2}{2} = 4 \times 10^{-3}$ V

40. $\ell = 20$ cm $= 20 \times 10^{-2}$ m

$v = 20$ cm/s $= 20 \times 10^{-2}$ m/s

$B_H = 3 \times 10^{-5}$ T

$i = 2\ \mu A = 2 \times 10^{-6}$ A

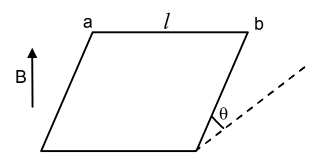
$R = 0.2\ \Omega$

$i = \dfrac{B_v \ell v}{R}$

$\Rightarrow B_v = \dfrac{iR}{\ell v} = \dfrac{2 \times 10^{-6} \times 2 \times 10^{-1}}{20 \times 10^{-2} \times 20 \times 10^{-2}} = 1 \times 10^{-5}$ Tesla

$\tan \delta = \dfrac{B_v}{B_H} = \dfrac{1 \times 10^{-5}}{3 \times 10^{-5}} = \dfrac{1}{3} \Rightarrow \delta(\text{dip}) = \tan^{-1}(1/3)$

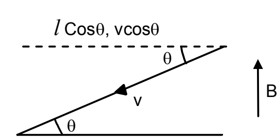
41. $I = \dfrac{B\ell v}{R} = \dfrac{B \times \ell \cos\theta \times v \cos\theta}{R}$



$= \dfrac{B\ell v}{R} \cos^2\theta$

$F = i\ell B = \dfrac{B\ell v \cos^2\theta \times \ell B}{R}$

Now, $F = mg \sin\theta$ [Force due to gravity which pulls downwards]

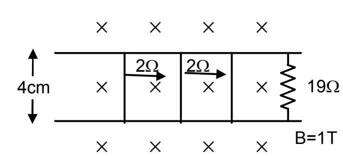


Now, $\dfrac{B^2 \ell^2 v \cos^2\theta}{R} = mg \sin\theta$

$\Rightarrow B = \sqrt{\dfrac{Rmg \sin\theta}{\ell^2 v \cos^2\theta}}$

42. a) The wires constitute 2 parallel emf.

$\therefore$ Net emf $= B\ell v = 1 \times 4 \times 10^{-2} \times 5 \times 10^{-2} = 20 \times 10^{-4}$



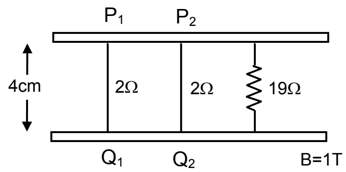
Net resistance = $\dfrac{2 \times 2}{2+2} + 19 = 20\ \Omega$

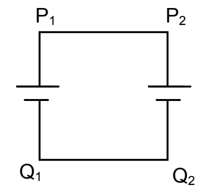
Net current = $\dfrac{20 \times 10^{-4}}{20} = 0.1$ mA.

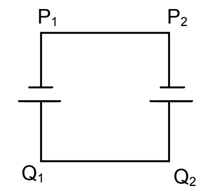
b) When both the wires move towards opposite directions then not emf = 0

$\therefore$ Net current = 0

43.







a) No current will pass as circuit is incomplete.

b) As circuit is complete

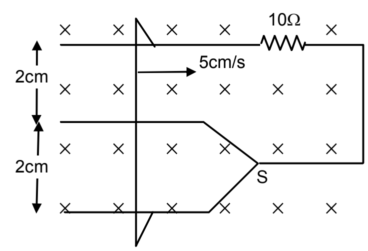
$V_{P_2Q_2} = B \ell v$

$= 1 \times 0.04 \times 0.05 = 2 \times 10^{-3}$ V

$R = 2\Omega$

$i = \dfrac{2 \times 10^{-3}}{2} = 1 \times 10^{-3}$ A = 1 mA.

44. B = 1 T, $V = 5 \text{ I } 10^{-2}$ m/′, R = 10 Ω



a) When the switch is thrown to the middle rail

$E = Bv\ell$

$= 1 \times 5 \times 10^{-2} \times 2 \times 10^{-2} = 10^{-3}$

Current in the 10 Ω resistor = E/R

$= \dfrac{10^{-3}}{10} = 10^{-4} = 0.1$ mA

b) The switch is thrown to the lower rail

$E = Bv\ell$

$= 1 \times 5 \times 10^{-2} \times 2 \times 10^{-2} = 20 \times 10^{-4}$

Current $= \dfrac{20 \times 10^{-4}}{10} = 2 \times 10^{-4} = 0.2$ mA

45. Initial current passing = i

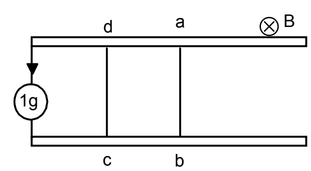
Hence initial emf = ir

Emf due to motion of ab = $B\ell v$

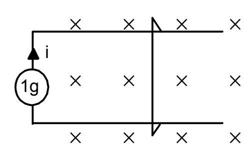
Net emf = $ir - B\ell v$

Net resistance = 2r

Hence current passing $= \dfrac{ir - B\ell v}{2r}$



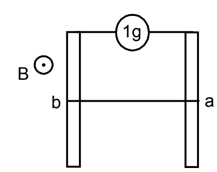
46. Force on the wire = $i\ell B$



Acceleration $= \dfrac{i\ell B}{m}$

Velocity $= \dfrac{i\ell Bt}{m}$

47. Given $B\ell v = mg$ …(1)



When wire is replaced we have

$2\,mg - B\ell v = 2\,ma$ [where a → acceleration]

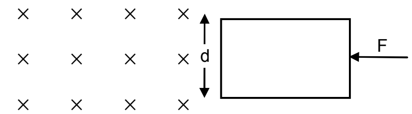
$\Rightarrow a = \dfrac{2mg - B\ell v}{2m}$

Now, $s = ut + \dfrac{1}{2}at^2$

$\Rightarrow \ell = \dfrac{1}{2} \times \dfrac{2mg - B\ell v}{2m} \times t^2$ [$\therefore$ s = ℓ]

$\Rightarrow t = \sqrt{\dfrac{4ml}{2mg - B\ell v}} = \sqrt{\dfrac{4ml}{2mg - mg}} = \sqrt{2\ell / g}$. [from (1)]

48. a) emf developed = Bdv (when it attains a speed v)



Current = $\dfrac{Bdv}{R}$

Force = $\dfrac{Bd^2v^2}{R}$

This force opposes the given force

Net F = $F - \dfrac{Bd^2v^2}{R} = RF - \dfrac{Bd^2v^2}{R}$

Net acceleration = $\dfrac{RF - B^2d^2v}{mR}$

b) Velocity becomes constant when acceleration is 0.

$\dfrac{F}{m} - \dfrac{B^2d^2v_0}{mR} = 0$

$\Rightarrow \dfrac{F}{m} = \dfrac{B^2d^2v_0}{mR}$

$\Rightarrow V_0 = \dfrac{FR}{B^2d^2}$

c) Velocity at line t

a = $-\dfrac{dv}{dt}$

$\Rightarrow \int_0^v \dfrac{dv}{RF - l^2B^2v} = \int_0^t \dfrac{dt}{mR}$

$\Rightarrow \left[l_n[RF - l^2B^2v]\dfrac{1}{-l^2B^2} \right]_0^v \quad \left[\dfrac{t}{Rm} \right]_0^t$

$\Rightarrow \left[l_n(RF - l^2B^2v) \right]_0^v = \dfrac{-tl^2B^2}{Rm}$

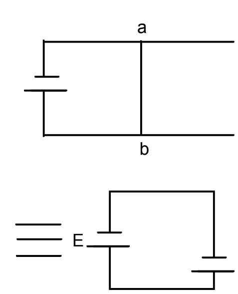
$\Rightarrow l_n(RF - l^2B^2v) - \ln(RF) = \dfrac{-t^2B^2t}{Rm}$

$\Rightarrow 1 - \dfrac{l^2B^2v}{RF} = e^{\frac{-l^2B^2t}{Rm}}$

$\Rightarrow \dfrac{l^2B^2v}{RF} = 1 - e^{\frac{-l^2B^2t}{Rm}}$

$\Rightarrow v = \dfrac{FR}{l^2B^2}\left(1 - e^{\frac{-l^2B^2v_0t}{Rv_0m}} \right) = v_0(1 - e^{-Fv_0m})$

49. Net emf = E – Bvℓ



I = $\dfrac{E - Bv\ell}{r}$ from b to a

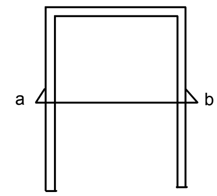
F = I ℓ B

= $\left(\dfrac{E - Bv\ell}{r} \right)\ell B$ = $\dfrac{\ell B}{r}(E - Bv\ell)$ towards right.

After some time when E = Bvℓ,

Then the wire moves constant velocity v

Hence v = E / Bℓ.



50. a) When the speed of wire is V
emf developed = B ℓ V

b) Induced current is the wire = $\frac{B\ell v}{R}$ (from b to a)

c) Down ward acceleration of the wire

$= \frac{mg - F}{m}$ due to the current

$= mg - i\ell B/m = g - \frac{B^2\ell^2 V}{Rm}$

d) Let the wire start moving with constant velocity. Then acceleration = 0

$\frac{B^2\ell^2 v}{Rm} m = g$

$\Rightarrow V_m = \frac{gRm}{B^2\ell^2}$

e) $\frac{dV}{dt} = a$

$\Rightarrow \frac{dV}{dt} = \frac{mg - B^2\ell^2 v/R}{m}$

$\Rightarrow \frac{dv}{\frac{mg - B^2\ell^2 v/R}{m}} = dt$

$\Rightarrow \int_0^v \frac{mdv}{mg - \frac{B^2\ell^2 v}{R}} = \int_0^t dt$

$\Rightarrow \frac{m}{\frac{-B^2\ell^2}{R}}\left(\log(mg - \frac{B^2\ell^2 v}{R}\right)_0^v = t$

$\Rightarrow \frac{-mR}{B^2\ell^2} = \log\left[\log\left(mg - \frac{B^2\ell^2 v}{R}\right) - \log(mg)\right] = t$

$\Rightarrow \log\left[\frac{mg - \frac{B^2\ell^2 v}{R}}{mg}\right] = \frac{-tB^2\ell^2}{mR}$

$\Rightarrow \log\left[1 - \frac{B^2\ell^2 v}{Rmg}\right] = \frac{-tB^2\ell^2}{mR}$

$\Rightarrow 1 - \frac{B^2\ell^2 v}{Rmg} = e^{\frac{-tB^2\ell^2}{mR}}$

$\Rightarrow (1 - e^{-B^2\ell^2/mR}) = \frac{B^2\ell^2 v}{Rmg}$

$\Rightarrow v = \frac{Rmg}{B^2\ell^2}\left(1 - e^{-B^2\ell^2/mR}\right)$

$\Rightarrow v = v_m(1 - e^{-gt/Vm})$ $\left[v_m = \frac{Rmg}{B^2\ell^2}\right]$

f) $\frac{ds}{dt} = v \Rightarrow ds = v\ dt$

$\Rightarrow s = vm \int_0^t (1 - e^{-gt/vm})dt$

$= V_m\left(t - \frac{V_m}{g}e^{-gt/vm}\right) = \left(V_m t + \frac{V_m^2}{g}e^{-gt/vm}\right) - \frac{V_m^2}{g}$

$= V_m t - \frac{V_m^2}{g}\left(1 - e^{-gt/vm}\right)$

g) $\frac{d}{dt}mgs = mg\frac{ds}{dt} = mgV_m(1 - e^{-gt/vm)}$

$\frac{d_H}{dt} = i^2R = R\left(\frac{\ell BV}{R}\right)^2 = \frac{\ell^2B^2v^2}{R}$

$\Rightarrow \frac{\ell^2B^2}{R}V_m^2(1 - e^{-gt/vm})^2$

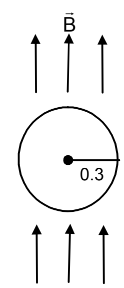
After steady state i.e. $T \to \infty$

$\frac{d}{dt}mgs = mgV_m$

$\frac{d_H}{dt} = \frac{\ell^2B^2}{R}V_m^2 = \frac{\ell^2B^2}{R}V_m\frac{mgR}{\ell^2B^2} = mgV_m$

Hence after steady state $\frac{d_H}{dt} = \frac{d}{dt}mgs$

51. $\ell = 0.3$ m, $\vec{B} = 2.0 \times 10^{-5}$ T, $\omega = 100$ rpm



v = $\frac{100}{60} \times 2\pi = \frac{10}{3}\pi$ rad/s

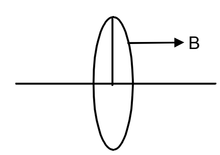
v = $\frac{\ell}{2} \times \omega = \frac{0.3}{2} \times \frac{10}{3}\pi$

Emf = e = $B\ell v$

= $2.0 \times 10^{-5} \times 0.3 \times \frac{0.33}{2} \times \frac{10}{3}\pi$

= $3\pi \times 10^{-6}$ V = $3 \times 3.14 \times 10^{-6}$ V = 9.42×10^{-6} V.

52. V at a distance r/2



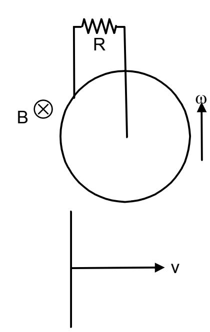
From the centre = $\frac{r\omega}{2}$

E = $B\ell v \Rightarrow$ E = B × r × $\frac{r\omega}{2} = \frac{1}{2}Br^2\omega$

53. B = 0.40 T, ω = 10 rad/′, r = 10Ω

r = 5 cm = 0.05 m

Considering a rod of length 0.05 m affixed at the centre and rotating with the same ω.



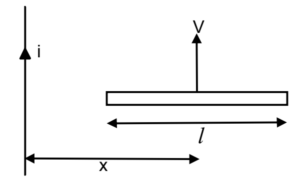
v = $\frac{\ell}{2} \times \omega = \frac{0.05}{2} \times 10$

e = $B\ell v$ = $0.40 \times \frac{0.05}{2} \times 10 \times 0.05 = 5 \times 10^{-3}$ V

I = $\frac{e}{R} = \frac{5 \times 10^{-3}}{10}$ = 0.5 mA

It leaves from the centre.

54. $\vec{B} = \frac{B_0}{L} y\hat{K}$



L = Length of rod on y-axis

$V = V_0 \hat{i}$

Considering a small length by of the rod

$dE = B\ V\ dy$

$\Rightarrow dE = \frac{B_0}{L} y \times V_0 \times dy$

$\Rightarrow dE = \frac{B_0 V_0}{L} ydy$

$\Rightarrow E = \frac{B_0 V_0}{L} \int_0^L ydy$

$= \frac{B_0 V_0}{L}\left[\frac{y^2}{2}\right]_0^L = \frac{B_0 V_0}{L}\frac{L^2}{2} = \frac{1}{2} B_0 V_0 L$

55. In this case $\vec{B}$ varies

Hence considering a small element at centre of rod of length dx at a dist x from the wire.

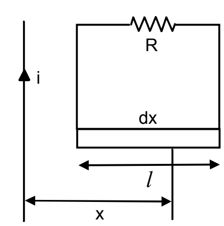
$\vec{B} = \frac{\mu_0 i}{2\pi x}$

So, $de = \frac{\mu_0 i}{2\pi x} \times vxdx$

$e = \int_0^e de = \frac{\mu_0 iv}{2\pi} = \int_{x-t/2}^{x+t/2} \frac{dx}{x} = \frac{\mu_0 iv}{2\pi}$ [ln (x + ℓ/2) – ℓn(x - ℓ/2)]

$= \frac{\mu_0 iv}{2\pi} \ln\left[\frac{x+\ell/2}{x-\ell/2}\right] = \frac{\mu_0 iv}{2x} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

56. a) emf produced due to the current carrying wire = $\frac{\mu_0 iv}{2\pi} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$



Let current produced in the rod = i′ = $\frac{\mu_0 iv}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

Force on the wire considering a small portion dx at a distance x

dF = i′ B ℓ

$\Rightarrow dF = \frac{\mu_0 iv}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \times \frac{\mu_0 i}{2\pi x} \times dx$

$\Rightarrow dF = \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \frac{dx}{x}$

$\Rightarrow F = \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \int_{x-t/2}^{x+t/2} \frac{dx}{x}$

$= \left(\frac{\mu_0 i}{2\pi}\right)^2 \frac{v}{R} \ln\left(\frac{2x+\ell}{2x-\ell}\right) \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

$= \frac{v}{R}\left[\frac{\mu_0 i}{2\pi} \ln\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2$

b) Current = $\frac{\mu_0 \ln}{2\pi R} \ln\left(\frac{2x+\ell}{2x-\ell}\right)$

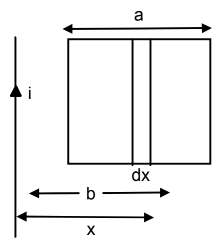
c) Rate of heat developed = i^2R

$$= \left[\frac{\mu_0 iv}{2\pi R}\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2 R = \frac{1}{R}\left[\frac{\mu_0 iv}{2\pi}\ln\left(\frac{2x+\ell}{2x-\ell}\right)^2\right]$$

d) Power developed in rate of heat developed = i^2R

$$= \frac{1}{R}\left[\frac{\mu_0 iv}{2\pi}\ln\left(\frac{2x+\ell}{2x-\ell}\right)\right]^2$$

57. Considering an element dx at a dist x from the wire. We have



a) ϕ = B.A.

$$d\phi = \frac{\mu_0 i \times adx}{2\pi x}$$

$$\phi = \int_0^a d\phi = \frac{\mu_0 ia}{2\pi}\int_b^{a+b}\frac{dx}{x} = \frac{\mu_0 ia}{2\pi}\ln\{1 + a/b\}$$

b) $$e = \frac{d\phi}{dt} = \frac{d}{dt}\frac{\mu_0 ia}{2\pi}\ln[1 + a/b]$$

$$= \frac{\mu_0 a}{2\pi}\ln[1 + a/n]\frac{d}{dt}(i_0 \sin\omega t)$$

$$= \frac{\mu_0 a i_0 \omega\cos\omega t}{2\pi}\ln[1 + a/b]$$

c) $$i = \frac{e}{r} = \frac{\mu_0 a i_0 \omega\cos\omega t}{2\pi r}\ln[1 + a/b]$$

$H = i^2rt$

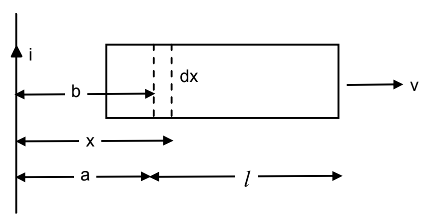
$$= \left[\frac{\mu_0 a i_0 \omega\cos\omega t}{2\pi r}\ln(1 + a/b)\right]^2 \times r \times t$$

$$= \frac{\mu_0^2 \times a^2 \times i_0^2 \times \omega^2}{4\pi \times r^2}\ln^2[1 + a/b] \times r \times \frac{20\pi}{\omega}$$

$$= \frac{5\mu_0^2 a^2 i_0^2 \omega}{2\pi r}\ln^2[1 + a/b] \quad [\therefore\ t = \frac{20\pi}{\omega}]$$

58. a) Using Faraday'' law

Consider a unit length dx at a distance x



$$B = \frac{\mu_0 i}{2\pi x}$$

Area of strip = b dx

$$d\phi = \frac{\mu_0 i}{2\pi x}dx$$

$$\Rightarrow \phi = \int_a^{a+l}\frac{\mu_0 i}{2\pi x}bdx$$

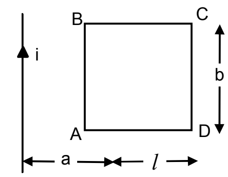
$$= \frac{\mu_0 i}{2\pi}b\int_a^{a+l}\left(\frac{dx}{x}\right) = \frac{\mu_0 ib}{2\pi}\log\left(\frac{a+l}{a}\right)$$

$$\text{Emf} = \frac{d\phi}{dt} = \frac{d}{dt}\left[\frac{\mu_0 ib}{2\pi}\log\left(\frac{a+l}{a}\right)\right]$$

$$= \frac{\mu_0 ib}{2\pi}\frac{a}{a+l}\left(\frac{va-(a+l)v}{a^2}\right) \text{ (where da/dt = V)}$$

$= \frac{\mu_0 ib}{2\pi}\frac{a}{a+l}\frac{vl}{a^2} = \frac{\mu_0 ibvl}{2\pi(a+l)a}$

The velocity of AB and CD creates the emf. since the emf due to AD and BC are equal and opposite to each other.



$B_{AB} = \frac{\mu_0 i}{2\pi a}$ $\Rightarrow$ E.m.f. AB = $\frac{\mu_0 i}{2\pi a}bv$

Length b, velocity v.

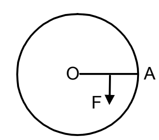
$B_{CD} = \frac{\mu_0 i}{2\pi(a+l)}$

$\Rightarrow$ E.m.f. CD = $\frac{\mu_0 ibv}{2\pi(a+l)}$

Length b, velocity v.

Net emf = $\frac{\mu_0 i}{2\pi a}bv - \frac{\mu_0 ibv}{2\pi(a+l)} = \frac{\mu_0 ibvl}{2\pi a(a+l)}$

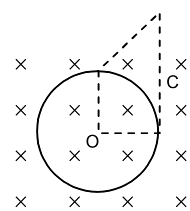
59. e = Bvl = $\frac{B \times a \times \omega \times a}{2}$

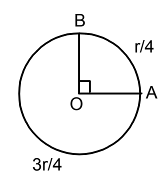


$i = \frac{Ba^2\omega}{2R}$

$F = i\ell B = \frac{Ba^2\omega}{2R} \times a \times B = \frac{B^2a^3\omega}{2R}$ towards right of OA.

60. The 2 resistances r/4 and 3r/4 are in parallel.





$R' = \frac{r/4 \times 3r/4}{r} = \frac{3r}{16}$

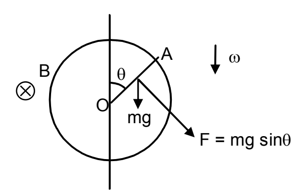
$e = BV\ell$

$= B \times \frac{a}{2}\omega \times a = \frac{Ba^2\omega}{2}$

$i = \frac{e}{R'} = \frac{Ba^2\omega}{2R'} = \frac{Ba^2\omega}{2 \times 3r/16}$

$= \frac{Ba^2\omega 16}{2 \times 3r} = \frac{8}{3}\frac{Ba^2\omega}{r}$

61. We know

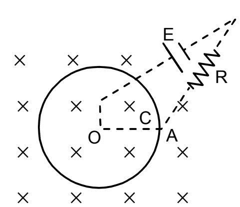


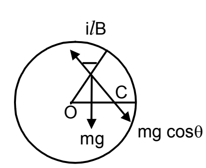
$F = \frac{B^2a^2\omega}{2R} = i\ell B$

Component of mg along F = mg sin θ.

Net force = $\frac{B^2a^3\omega}{2R} - mg\sin\theta$.

62. emf = $\frac{1}{2}B\omega a^2$ [from previous problem]





Current = $\frac{e+E}{R} = \frac{1/2 \times B\omega a^2 + E}{R} = \frac{B\omega a^2 + 2E}{2R}$

$\Rightarrow$ mg cos θ = $i\ell B$ [Net force acting on the rod is O]

$\Rightarrow$ mg cos θ = $\frac{B\omega a^2 + 2E}{2R} a \times B$

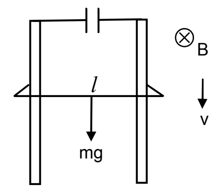
$\Rightarrow R = \frac{(B\omega a^2 + 2E)aB}{2mg\cos\theta}$.

63. Let the rod has a velocity v at any instant,

Then, at the point,

$e = B\ell v$

Now, $q = c \times \text{potential} = ce = CB\ell v$



Current $I = \frac{dq}{dt} = \frac{d}{dt} CBlv$

$= CBl\frac{dv}{dt} = CBla$ (where $a \to$ acceleration)

From figure, force due to magnetic field and gravity are opposite to each other.

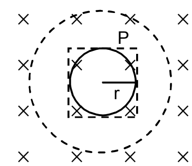
So, $mg - I\ell B = ma$

$\Rightarrow mg - CB\ell a \times \ell B = ma$ $\Rightarrow ma + CB^2\ell^2 a = mg$

$\Rightarrow a(m + CB^2\ell^2) = mg$ $\Rightarrow a = \frac{mg}{m + CB^2\ell 2}$

64. a) Work done per unit test charge

$= \phi E.\, dl$ (E = electric field)



$\phi E.\, dl = e$

$\Rightarrow E\phi\, dl = \frac{d\phi}{dt}$ $\Rightarrow E\, 2\pi r = \frac{dB}{dt} \times A$

$\Rightarrow E\, 2\pi r = \pi r^2 \frac{dB}{dt}$

$\Rightarrow E = \frac{\pi r^2}{2\pi}\frac{dB}{dt} = \frac{r}{2}\frac{dB}{dt}$

b) When the square is considered,

$\phi E\, dl = e$

$\Rightarrow E \times 2r \times 4 = \frac{dB}{dt}(2r)^2$

$\Rightarrow E = \frac{dB}{dt}\frac{4r^2}{8r}$ $\Rightarrow E = \frac{r}{2}\frac{dB}{dt}$

$\therefore$ The electric field at the point p has the same value as (a).

65. $\frac{di}{dt} = 0.01$ A/s

For 2s $\frac{di}{dt} = 0.02$ A/s

n = 2000 turn/m, R = 6.0 cm = 0.06 m

r = 1 cm = 0.01 m

a) $\phi = BA$

$\Rightarrow \frac{d\phi}{dt} = \mu_0 nA\frac{di}{dt}$

$= 4\pi \times 10^{-7} \times 2 \times 10^3 \times \pi \times 1 \times 10^{-4} \times 2 \times 10^{-2}$ $[A = \pi \times 1 \times 10^{-4}]$

$= 16\pi^2 \times 10^{-10}\ \omega$

$= 157.91 \times 10^{-10}\ \omega$

$= 1.6 \times 10^{-8}\ \omega$

or, $\frac{d\phi}{dt}$ for 1 s = 0.785 ω.

b) $\int E.dl = \frac{d\phi}{dt}$

$\Rightarrow E\phi dl = \dfrac{d\phi}{dt} \Rightarrow E = \dfrac{0.785 \times 10^{-8}}{2\pi \times 10^{-2}} = 1.2 \times 10^{-7}$ V/m

c) $\dfrac{d\phi}{dt} = \mu_0 n \dfrac{di}{dt} A = 4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times (0.06)^2$

$E\phi dl = \dfrac{d\phi}{dt}$

$\Rightarrow E = \dfrac{d\phi / dt}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 2000 \times 0.01 \times \pi \times (0.06)^2}{\pi \times 8 \times 10^{-2}} = 5.64 \times 10^{-7}$ V/m

66. V = 20 V

$dI = I_2 - I_1 = 2.5 - (-2.5) = 5A$

dt = 0.1 s

$V = L\dfrac{dI}{dt}$

$\Rightarrow 20 = L(5/0.1) \Rightarrow 20 = L \times 50$

$\Rightarrow L = 20 / 50 = 4/10 = 0.4$ Henry.

67. $\dfrac{d\phi}{dt} = 8 \times 10^{-4}$ weber

n = 200, I = 4A, $E = -nL \dfrac{dI}{dt}$

or, $\dfrac{-d\phi}{dt} = \dfrac{-LdI}{dt}$

or, $L = n\dfrac{d\phi}{dt} = 200 \times 8 \times 10^{-4} = 2 \times 10^{-2}$ H.

68. $E = \dfrac{\mu_0 N^2 A}{\ell} \dfrac{dI}{dt}$

$= \dfrac{4\pi \times 10^{-7} \times (240)^2 \times \pi (2 \times 10^{-2})^2}{12 \times 10^{-2}} \times 0.8$

$= \dfrac{4\pi \times (24)^2 \times \pi \times 4 \times 8}{12} \times 10^{-8}$

$= 60577.3824 \times 10^{-8} = 6 \times 10^{-4}$ V.

69. We know $i = i_0 (1 - e^{-t/r})$

a) $\dfrac{90}{100} i_0 = i_0 (1 - e^{-t/r})$

$\Rightarrow 0.9 = 1 - e^{-t/r}$

$\Rightarrow e^{-t/r} = 0.1$

Taking ℓn from both sides

$\ell n\, e^{-t/r} = \ell n\, 0.1 \Rightarrow -t = -2.3 \Rightarrow t/r = 2.3$

b) $\dfrac{99}{100} i_0 = i_0 (1 - e^{-t/r})$

$\Rightarrow e^{-t/r} = 0.01$

$\ell n e^{-t/r} = \ell n\, 0.01$

or, $-t/r = -4.6$ or $t/r = 4.6$

c) $\dfrac{99.9}{100} i_0 = i_0 (1 - e^{-t/r})$

$e^{-t/r} = 0.001$

$\Rightarrow \ln e^{-t/r} = \ln 0.001 \Rightarrow e^{-t/r} = -6.9 \Rightarrow t/r = 6.9.$

70. i = 2A, E = 4V, L = 1H

R = $\frac{E}{i} = \frac{4}{2} = 2$

i = $\frac{L}{R} = \frac{1}{2} = 0.5$

71. L = 2.0 H, R = 20 Ω, emf = 4.0 V, t = 0.20 S

i_0 = $\frac{e}{R} = \frac{4}{20}$, τ = $\frac{L}{R} = \frac{2}{20} = 0.1$

a) i = i_0 $(1 - e^{-t/\tau})$ = $\frac{4}{20}\left(1 - e^{-0.2/0.1}\right)$

= 0.17 A

b) $\frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times (0.17)^2$ = 0.0289 = 0.03 J.

72. R = 40 Ω, E = 4V, t = 0.1, i = 63 mA

i = i_0 – $(1 - e^{tR/2})$

$\Rightarrow$ $63 \times 10^{-3} = 4/40\ (1 - e^{-0.1 \times 40/L})$

$\Rightarrow$ $63 \times 10^{-3} = 10^{-1}\ (1 - e^{-4/L})$

$\Rightarrow$ $63 \times 10^{-2} = (1 - e^{-4/L})$

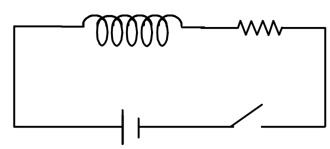
$\Rightarrow$ $1 - 0.63 = e^{-4/L} \Rightarrow e^{-4/L} = 0.37$

$\Rightarrow$ $-4/L = \ln(0.37) = -0.994$

$\Rightarrow$ L = $\frac{-4}{-0.994}$ = 4.024 H = 4 H.

73. L = 5.0 H, R = 100 Ω, emf = 2.0 V

t = 20 ms = 20×10^{-3} s = 2×10^{-2} s



i_0 = $\frac{2}{100}$ now i = i_0 $(1 - e^{-t/\tau})$

τ = $\frac{L}{R} = \frac{5}{100}$ $\Rightarrow$ i = $\frac{2}{100}\left(1 - e^{\frac{-2\times10^{-2}\times100}{5}}\right)$

$\Rightarrow$ i = $\frac{2}{100}(1 - e^{-2/5})$

$\Rightarrow$ 0.00659 = 0.0066.

V = iR = 0.0066 × 100 = 0.66 V.

74. τ = 40 ms

i_0 = 2 A

a) t = 10 ms

i = i_0 $(1 - e^{-t/\tau})$ = $2(1 - e^{-10/40})$ = $2(1 - e^{-1/4})$

= 2(1 – 0.7788) = $2(0.2211)^A$ = 0.4422 A = 0.44 A

b) t = 20 ms

i = i_0 $(1 - e^{-t/\tau})$ = $2(1 - e^{-20/40})$ = $2(1 - e^{-1/2})$

= 2(1 – 0.606) = 0.7869 A = 0.79 A

c) t = 100 ms

i = i_0 $(1 - e^{-t/\tau})$ = $2(1 - e^{-100/40})$ = $2(1 - e^{-10/4})$

= 2(1 – 0.082) = 1.835 A =1.8 A

d) t = 1 s

i = i_0 $(1 - e^{-t/\tau})$ = $2(1 - e^{-1/40\times10^{-3}})$ = $2(1 - e^{-10/40})$

= $2(1 - e^{-25})$ = 2 × 1 = 2 A

75. L = 1.0 H, R = 20 Ω , emf = 2.0 V

$\tau = \frac{L}{R} = \frac{1}{20} = 0.05$

$i_0 = \frac{e}{R} = \frac{2}{20} = 0.1$ A

$i = i_0 (1 - e^{-t}) = i_0 - i_0 e^{-t}$

$\Rightarrow \frac{di}{dt} = \frac{di_0}{dt}(i_0 x - 1/\tau \times e^{-t/\tau}) = i_0 / \tau\ e^{-t/\tau}$.

So,

a) t = 100 ms $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-0.1/0.05} = 0.27$ A

b) t = 200 ms $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-0.2/0.05} = 0.0366$ A

c) t = 1 s $\Rightarrow \frac{di}{dt} = \frac{0.1}{0.05} \times e^{-1/0.05} = 4 \times 10^{-9}$ A

76. a) For first case at t = 100 ms

$\frac{di}{dt} = 0.27$

Induced emf = $L\frac{di}{dt} = 1 \times 0.27 = 0.27$ V

b) For the second case at t = 200 ms

$\frac{di}{dt} = 0.036$

Induced emf = $L\frac{di}{dt} = 1 \times 0.036 = 0.036$ V

c) For the third case at t = 1 s

$\frac{di}{dt} = 4.1 \times 10^{-9}$ V

Induced emf = $L\frac{di}{dt} = 4.1 \times 10^{-9}$ V

77. L = 20 mH; e = 5.0 V, R = 10 Ω

$\tau = \frac{L}{R} = \frac{20 \times 10^{-3}}{10}$, $i_0 = \frac{5}{10}$

$i = i_0(1 - e^{-t/\tau})^2$

$\Rightarrow i = i_0 - i_0 e^{-t/\tau^2}$

$\Rightarrow iR = i_0R - i_0R\, e^{-t/\tau^2}$

a) $10 \times \frac{di}{dt} = \frac{d}{dt} i_0 R + 10 \times \frac{5}{10} \times \frac{10}{20 \times 10^{-3}} \times e^{-0 \times 10/2 \times 10^{-2}}$

$= \frac{5}{2} \times 10^{-3} \times 1 = \frac{5000}{2} = 2500 = 2.5 \times 10^{-3}$ V/s.

b) $\frac{Rdi}{dt} = R \times i_0 \times \frac{1}{\tau} \times e^{-t/\tau}$

t = 10 ms = 10 × 10^{-3} s

$\frac{dE}{dt} = 10 \times \frac{5}{10} \times \frac{10}{20 \times 10^{-3}} \times e^{-0.01 \times 10/2 \times 10^{-2}}$

= 16.844 = 17 V/′

c) For t = 1 s

$$\frac{dE}{dt} = \frac{Rdi}{dt} = \frac{5}{2}\, 10^3 \times e^{10/2\times10^{-2}} = 0.00 \text{ V/s.}$$

78. L = 500 mH, R = 25 Ω, E = 5 V

a) t = 20 ms

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-20\times10^{-3}\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-1})$

$= \frac{1}{5}(1 - 0.3678) = 0.1264$

Potential difference iR = 0.1264 × 25 = 3.1606 V = 3.16 V.

b) t = 100 ms

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-100\times10^{-3}\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-5})$

$= \frac{1}{5}(1 - 0.0067) = 0.19864$

Potential difference = iR = 0.19864 × 25 = 4.9665 = 4.97 V.

c) t = 1 sec

$i = i_0\,(1 - e^{-tR/L}) = \frac{E}{R}(1 - E^{-tR/L})$

$= \frac{5}{25}\left(1 - e^{-1\times25/100\times10^{-3}}\right) = \frac{1}{5}(1 - e^{-50})$

$= \frac{1}{5}\times 1 = 1/5 \text{ A}$

Potential difference = iR = (1/5 × 25) V = 5 V.

79. L = 120 mH = 0.120 H

R = 10 Ω , emf = 6, r = 2

$i = i_0\,(1 - e^{-t/\tau})$

Now, dQ = idt

$= i_0\,(1 - e^{-t/\tau})\,dt$

$Q = \int dQ = \int_0^1 i_0(1 - e^{-t/\tau})dt$

$= i_0\left[\int_0^t dt - \int_0^1 e^{-t/\tau}dt\right] = i_0\left[t - (-\tau)\int_0^t e^{-t/\tau}dt\right]$

$= i_0[t + \tau(e^{-t/\tau - 1})] = i_0[t + \tau e^{-t/\tau}\tau]$

Now, $i_0 = \frac{6}{10+2} = \frac{6}{12} = 0.5$ A

$\tau = \frac{L}{R} = \frac{0.120}{12} = 0.01$

a) t = 0.01 s

So, Q $= 0.5[0.01 + 0.01\, e^{-0.01/0.01} - 0.01]$

$= 0.00183 = 1.8 \times 10^{-3}$ C = 1.8 mC

b) t = 20 ms = 2 × 10^{-2} ′ = 0.02 s

So, Q = 0.5[0.02 + 0.01 $e^{-0.02/0.01}$ – 0.01]

= 0.005676 = 5.6 × 10^{-3} C = 5.6 mC

c) t = 100 ms = 0.1 s

So, Q = 0.5[0.1 + 0.01 $e^{-0.1/0.01}$ – 0.01]

= 0.045 C = 45 mC

80. L = 17 mH, ℓ = 100 m, A = 1 mm^2 = 1 × 10^{-6} m^2, f_{cu} = 1.7 × 10^{-8} Ω-m

$R = \frac{f_{cu}\ell}{A} = \frac{1.7 \times 10^{-8} \times 100}{1 \times 10^{-6}} = 1.7\ \Omega$

$i = \frac{L}{R} = \frac{0.17 \times 10^{-8}}{1.7} = 10^{-2}$ sec = 10 m sec.

81. τ = L/R = 50 ms = 0.05 ′

a) $\frac{i_0}{2} = i_0(1 - e^{-t/0.06})$

$\Rightarrow \frac{1}{2} = 1 - e^{-t/0.05} = e^{-t/0.05} = \frac{1}{2}$

$\Rightarrow$ ℓn $e^{-t/0.05}$ = ℓ$n^{1/2}$

$\Rightarrow$ t = 0.05 × 0.693 = 0.3465 ′ = 34.6 ms = 35 ms.

b) $P = i^2R = \frac{E^2}{R}(1 - E^{-t.R/L})^2$

Maximum power = $\frac{E^2}{R}$

So, $\frac{E^2}{2R} = \frac{E^2}{R}(1 - e^{-tR/L})^2$

$\Rightarrow 1 - e^{-tR/L} = \frac{1}{\sqrt{2}} = 0.707$

$\Rightarrow e^{-tR/L} = 0.293$

$\Rightarrow \frac{tR}{L} = -\ln 0.293 = 1.2275$

$\Rightarrow$ t = 50 × 1.2275 ms = 61.2 ms.

82. Maximum current = $\frac{E}{R}$

In steady state magnetic field energy stored = $\frac{1}{2}L\frac{E^2}{R^2}$

The fourth of steady state energy = $\frac{1}{8}L\frac{E^2}{R^2}$

One half of steady energy = $\frac{1}{4}L\frac{E^2}{R^2}$

$\frac{1}{8}L\frac{E^2}{R^2} = \frac{1}{2}L\frac{E^2}{R^2}(1 - e^{-t_1R/L})^2$

$\Rightarrow 1 - e^{t_1R/L} = \frac{1}{2}$

$\Rightarrow e^{t_1R/L} = \frac{1}{2} \Rightarrow t_1\frac{R}{L} = \ell n\, 2 \Rightarrow t_1 = \tau \ell n 2$

Again $\frac{1}{4}L\frac{E^2}{R^2} = \frac{1}{2}L\frac{E^2}{R^2}(1 - e^{-t_2R/L})^2$

$\Rightarrow\ e^{t_2R/L} = \frac{\sqrt{2}-1}{\sqrt{2}} = \frac{2-\sqrt{2}}{2}$

$\Rightarrow\ t_2 = \tau\left[\ell n\left(\frac{1}{2-\sqrt{2}}\right) + \ell n2\right]$

So, $t_2 - t_1 = \tau \ell n \frac{1}{2-\sqrt{2}}$

83. L = 4.0 H, R = 10 Ω, E = 4 V

a) Time constant = $\tau = \frac{L}{R} = \frac{4}{10}$ = 0.4 s.

b) $i = 0.63\ i_0$

Now, $0.63\ i_0 = i_0\ (1 - e^{-t/\tau})$

$\Rightarrow e^{-t/\tau} = 1 - 0.63 = 0.37$

$\Rightarrow\ \ell ne^{-t/\tau} = \ln 0.37$

$\Rightarrow -t/\tau = -0.9942$

$\Rightarrow t = 0.9942 \times 0.4 = 0.3977 = 0.40$ s.

c) $i = i_0\ (1 - e^{-t/\tau})$

$\Rightarrow\ \frac{4}{10}(1 - e^{-0.4/0.4}) = 0.4 \times 0.6321 = 0.2528$ A.

Power delivered = VI

$= 4 \times 0.2528 = 1.01 = 1\ \omega$.

d) Power dissipated in Joule heating $= I^2R$

$= (0.2528)^2 \times 10 = 0.639 = 0.64\ \omega$.

84. $i = i_0(1 - e^{-t/\tau})$

$\Rightarrow\ \mu_0 ni = \mu_0 n\ i_0(1 - e^{-t/\tau})$ $\qquad\Rightarrow\qquad B = B_0\ (1 - e^{-IR/L})$

$\Rightarrow\ 0.8\ B_0 = B_0\ (1 - e^{-20\times10^{-5}\times R/2\times10^{-3}})$ $\qquad\Rightarrow\qquad 0.8 = (1 - e^{-R/100})$

$\Rightarrow\ e^{-R/100} = 0.2$ $\qquad\Rightarrow\qquad \ell n(e^{-R/100}) = \ell n(0.2)$

$\Rightarrow\ -R/100 = -1.609$ $\qquad\Rightarrow\qquad R = 16.9 = 160\ \Omega$.

85. Emf = E LR circuit

a) $dq = idt$

$= i_0\ (1 - e^{-t/\tau})dt$

$= i_0\ (1 - e^{-IR.L})dt$ $\qquad [\therefore\ \tau = L/R]$

$Q = \int_0^t dq = i_0\left[\int_0^t dt - \int_0^t e^{-tR/L}dt\right]$

$= i_0\ [t - (-L/R)\ (e^{-IR/L})\ t_0]$

$= i_0\ [t - L/R\ (1 - e^{-IR/L})]$

$Q = E/R\ [t - L/R\ (1 - e^{-IR/L})]$

b) Similarly as we know work done = VI = EI

$= E\ i_0\ [t - L/R\ (1 - e^{-IR/L})]$

$= \frac{E^2}{R}\ [t - L/R\ (1 - e^{-IR/L})]$

c) $H = \int_0^t i^2R \cdot dt = \frac{E^2}{R^2} \cdot R \cdot \int_0^t (1 - e^{-tR/L})^2 \cdot dt$

$= \frac{E^2}{R}\int_0^t (1 + e^{(-2+B)/L} - 2e^{-tR/L}) \cdot dt$

$= \frac{E^2}{R}\left(t - \frac{L}{2R}e^{-2tR/L} + \frac{L}{R}2 \cdot e^{-tR/L}\right)_0^t$

$= \frac{E^2}{R}\left(t - \frac{L}{2R}e^{-2tR/L} + \frac{2L}{R} \cdot e^{-tR/L}\right) - \left(-\frac{L}{2R} + \frac{2L}{R}\right)$

$= \frac{E^2}{R}\left[\left(t - \frac{L}{2R}x^2 + \frac{2L}{R} \cdot x\right) - \frac{3}{2}\frac{L}{R}\right]$

$= \frac{E^2}{2}\left(t - \frac{L}{2R}(x^2 - 4x + 3)\right)$

d) $E = \frac{1}{2}Li^2$

$= \frac{1}{2}L \cdot \frac{E^2}{R^2} \cdot (1 - e^{-tR/L})^2$ $[x = e^{-tR/L}]$

$= \frac{LE^2}{2R^2}(1 - x)^2$

e) Total energy used as heat as stored in magnetic field

$= \frac{E^2}{R}T - \frac{E^2}{R} \cdot \frac{L}{2R}x^2 + \frac{E^2}{R}\frac{L}{r} \cdot 4x^2 - \frac{3L}{2R} \cdot \frac{E^2}{R} + \frac{LE^2}{2R^2} + \frac{LE^2}{2R^2}x^2 - \frac{LE^2}{R^2}x$

$= \frac{E^2}{R}t + \frac{E^2L}{R^2}x - \frac{LE^2}{R^2}$

$= \frac{E^2}{R}\left(t - \frac{L}{R}(1 - x)\right)$

= Energy drawn from battery.

(Hence conservation of energy holds good).

86. $L = 2H$, $R = 200\ \Omega$, $E = 2$ V, $t = 10$ ms

a) $\ell = \ell_0 (1 - e^{-t/\tau})$

$= \frac{2}{200}\left(1 - e^{-10 \times 10^{-3} \times 200/2}\right)$

$= 0.01 (1 - e^{-1}) = 0.01 (1 - 0.3678)$

$= 0.01 \times 0.632 = 6.3$ A.

b) Power delivered by the battery

= VI

$= EI_0 (1 - e^{-t/\tau}) = \frac{E^2}{R}(1 - e^{-t/\tau})$

$= \frac{2 \times 2}{200}(1 - e^{-10 \times 10^{-3} \times 200/2}) = 0.02 (1 - e^{-1}) = 0.1264 = 12$ mw.

c) Power dissepited in heating the resistor = I^2R

$= [i_0(1 - e^{-t/\tau})]^2R$

$= (6.3 \text{ mA})^2 \times 200 = 6.3 \times 6.3 \times 200 \times 10^{-6}$

$= 79.38 \times 10^{-4} = 7.938 \times 10^{-3} = 8$ mA.

d) Rate at which energy is stored in the magnetic field

d/dt (1/2 LI^2]

$= \frac{LI_0^2}{\tau}(e^{-t/\tau} - e^{-2t/\tau}) = \frac{2 \times 10^{-4}}{10^{-2}}(e^{-1} - e^{-2})$

$= 2 \times 10^{-2}\ (0.2325) = 0.465 \times 10^{-2}$

$= 4.6 \times 10^{-3} = 4.6$ mW.

87. $L_A = 1.0$ H ; $L_B = 2.0$ H ; $R = 10\ \Omega$

a) $t = 0.1$ s, $\tau_A = 0.1$, $\tau_B = L/R = 0.2$

$i_A = i_0(1 - e^{-t/\tau})$

$= \frac{2}{10}\left(1 - e^{\frac{-0.1\times10}{1}}\right) = 0.2\ (1 - e^{-1}) = 0.126424111$

$i_B = i_0(1 - e^{-t/\tau})$

$= \frac{2}{10}\left(1 - e^{\frac{-0.1\times10}{2}}\right) = 0.2\ (1 - e^{-1/2}) = 0.078693$

$$\frac{i_A}{i_B} = \frac{0.12642411}{0.78693} = 1.6$$

b) $t = 200$ ms $= 0.2$ s

$i_A = i_0(1 - e^{-t/\tau})$

$= 0.2(1 - e^{-0.2\times10/1}) = 0.2 \times 0.864664716 = 0.172932943$

$i_B = 0.2(1 - e^{-0.2\times10/2}) = 0.2 \times 0.632120 = 0.126424111$

$$\frac{i_A}{i_B} = \frac{0.172932943}{0.126424111} = 1.36 = 1.4$$

c) $t = 1$ s

$i_A = 0.2(1 - e^{-1\times10/1}) = 0.2 \times 0.9999546 = 0.19999092$

$i_B = 0.2(1 - e^{-1\times10/2}) = 0.2 \times 0.99326 = 0.19865241$

$$\frac{i_A}{i_B} = \frac{0.19999092}{0.19865241} = 1.0$$

88. a) For discharging circuit

$i = i_0\ e^{-t/\tau}$

$\Rightarrow 1 = 2\ e^{-0.1/\tau}$

$\Rightarrow (1/2) = e^{-0.1/\tau}$

$\Rightarrow \ell n\ (1/2) = \ell n\ (e^{-0.1/\tau})$

$\Rightarrow -0.693 = -0.1/\tau$

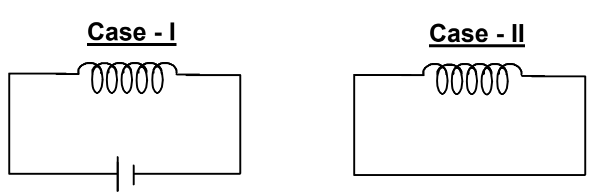
$\Rightarrow \tau = 0.1/0.693 = 0.144 = 0.14.$

b) $L = 4$ H, $i = L/R$

$\Rightarrow 0.14 = 4/R$

$\Rightarrow R = 4 / 0.14 = 28.57 = 28\ \Omega.$

89.



In this case there is no resistor in the circuit.

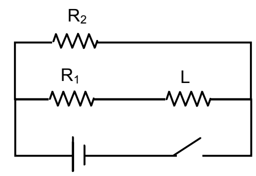
So, the energy stored due to the inductor before and after removal of battery remains same. i.e.

$$V_1 = V_2 = \frac{1}{2}Li^2$$

So, the current will also remain same.

Thus charge flowing through the conductor is the same.

90. a) The inductor does not work in DC. When the switch is closed the current charges so at first inductor works. But after a long time the current flowing is constant.



Thus effect of inductance vanishes.

$$i = \frac{E}{R_{net}} = \frac{E}{\frac{R_1R_2}{R_1+R_2}} = \frac{E(R_1+R_2)}{R_1R_2}$$

b) When the switch is opened the resistors are in series.

$$\tau = \frac{L}{R_{net}} = \frac{L}{R_1+R_2}.$$

91. i = 1.0 A, r = 2 cm, n = 1000 turn/m

Magnetic energy stored = $\frac{B^2V}{2\mu_0}$

Where B $\rightarrow$ Magnetic field, V $\rightarrow$ Volume of Solenoid.

$$= \frac{\mu_0 n^2 i^2}{2\mu_0} \times \pi r^2 h$$

$$= \frac{4\pi \times 10^{-7} \times 10^6 \times 1 \times \pi \times 4 \times 10^{-4} \times 1}{2} \qquad [h = 1\text{ m}]$$

$= 8\pi^2 \times 10^{-5}$

$= 78.956 \times 10^{-5} = 7.9 \times 10^{-4}$ J.

92. Energy density = $\frac{B^2}{2\mu_0}$

Total energy stored = $\frac{B^2V}{2\mu_0} = \frac{(\mu_0 i/2r)^2}{2\mu_0} V = \frac{\mu_0 i^2}{4r^2 \times 2} V$

$$= \frac{4\pi \times 10^{-7} \times 4^2 \times 1 \times 10^{-9}}{4 \times (10^{-1})^2 \times 2} = 8\pi \times 10^{-14} \text{ J.}$$

93. I = 4.00 A, V = 1 mm^3,

d = 10 cm = 0.1 m

$$\vec{B} = \frac{\mu_0 i}{2\pi r}$$

Now magnetic energy stored = $\frac{B^2}{2\mu_0} V$

$$= \frac{\mu_0^2 i^2}{4\pi r^2} \times \frac{1}{2\mu_0} \times V = \frac{4\pi \times 10^{-7} \times 16 \times 1 \times 1 \times 10^{-9}}{4 \times 1 \times 10^{-2} \times 2}$$

$$= \frac{8}{\pi} \times 10^{-14} \text{ J}$$

$= 2.55 \times 10^{-14}$ J

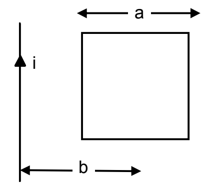
94. M = 2.5 H

$$\frac{dI}{dt} = \frac{\ell A}{s}$$

$$E = -\mu \frac{dI}{dt}$$

$\Rightarrow$ E = 2.5 $\times$ 1 = 2.5 V

95. We know



$$\frac{d\phi}{dt} = E = M \times \frac{di}{dt}$$

From the question,

$$\frac{di}{dt} = \frac{d}{dt}(i_0 \sin\omega t) = i_0 \omega \cos\omega t$$

$$\frac{d\phi}{dt} = E = \frac{\mu_0 a i_0 \omega \cos\omega t}{2\pi} \ell n[1 + a/b]$$

Now, E = $M \times \dfrac{di}{dt}$

or, $\dfrac{\mu_0 a i_0 \omega \cos\omega t}{2\pi} \ell n[1 + a/b] = M \times i_0 \omega \cos\omega t$

$\Rightarrow$ $M = \dfrac{\mu_0 a}{2\pi} \ell n[1 + a/b]$

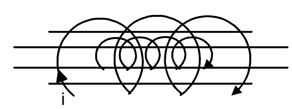
96. emf induced = $\dfrac{\pi\mu_0 N a^2 a'^2 ERV}{2L(a^2 + x^2)^{3/2}(R/Lx + r)^2}$

$\dfrac{dI}{dt} = \dfrac{ERV}{L\left(\dfrac{Rx}{L} + r\right)^2}$ (from question 20)

μ = $\dfrac{E}{di/dt} = \dfrac{N\mu_0 \pi a^2 a'^2}{2(a^2 + x^2)^{3/2}}$.

97. **Solenoid I :**

a_1 = 4 cm^2 ; n_1 = 4000/0.2 m ; ℓ_1 = 20 cm = 0.20 m



Solenoid II :

a_2 = 8 cm^2 ; n_2 = 2000/0.1 m ; ℓ_2 = 10 cm = 0.10 m

B = $\mu_0 n_2 i$ let the current through outer solenoid be i.

ϕ = n_1B.A = n_1 n_2 μ_0 i × a_1

= $2000 \times \dfrac{2000}{0.1} \times 4\pi \times 10^{-7} \times i \times 4 \times 10^{-4}$

E = $\dfrac{d\phi}{dt} = 64\pi \times 10^{-4} \times \dfrac{di}{dt}$

Now M = $\dfrac{E}{di/dt}$ = $64\pi \times 10^{-4}$ H = 2×10^{-2} H. [As E = Mdi/dt]

98. a) B = Flux produced due to first coil

= μ_0 n i

Flux ϕ linked with the second

= μ_0 n i × NA = μ_0 n i N π R^2

Emf developed

= $\dfrac{dI}{dt} = \dfrac{dt}{dt}(\mu_0 n i N \pi R^2)$

= $\mu_0 n N \pi R^2 \dfrac{di}{dt} = \mu_0 n N \pi R^2 i_0 \omega \cos\omega t$.

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