CHAPTER 36

PERMANENT MAGNETS

36.1 MAGNETIC POLES AND BAR MAGNETS

We have seen that a small current-loop carrying a current i, produces a magnetic field

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{d^3} \qquad \dots \text{(i)}$$

at an axial point. Here $\vec{\mu} = i\vec{A}$ is the magnetic dipole moment of the current loop. The vector $\vec{A}$ represents the area-vector of the current loop. Also, a current loop placed in a magnetic field $\vec{B}$ experiences a torque

$$\vec{\Gamma} = \vec{\mu} \times \vec{B}. \qquad \dots \text{(ii)}$$

We also know that an electric dipole produces an electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{2\vec{p}}{d^3} \qquad \dots \text{(iii)}$$

at an axial point and it experiences a torque

$$\vec{\Gamma} = \vec{p} \times \vec{E} \qquad \dots \text{(iv)}$$

when placed in an electric field. Equations (i) and (ii) for a current loop are similar in structure to the equations (iii) and (iv) for an electric dipole with $\vec{\mu}$ taking the role of $\vec{p}$ and $\frac{\mu_0}{4\pi}$ taking the role of $\frac{1}{4\pi\varepsilon_0}$. The similarity suggests that the behaviour of a current loop can be described by the following hypothetical model:

(a) There are two types of magnetic charges, positive magnetic charge and negative magnetic charge. A magnetic charge m placed in a magnetic field $\vec{B}$ experiences a force

$$\vec{F} = m\vec{B}. \qquad \dots (36.1)$$

The force on a positive magnetic charge is along the field and the force on a negative magnetic charge is opposite to the field.

(b) A magnetic charge m produces a magnetic field

$$B = \frac{\mu_0}{4\pi} \frac{m}{r^2} \qquad \dots (36.2)$$

at a distance r from it. The field is radially outward if the magnetic charge is positive and is inward if it is negative.

(c) A magnetic dipole is formed when a negative magnetic charge $-m$ and a positive magnetic charge $+m$ are placed at a small separation d. The magnetic dipole moment is $\mu = md$ and its direction is from $-m$ to $+m$. The line joining $-m$ and $+m$ is called the *axis* of the dipole.

(d) A current loop of area A carrying a current i may be replaced by a magnetic dipole of dipole moment $\mu = md = iA$ placed along the axis of the loop. The area-vector $\vec{A}$ points in the direction $-m$ to $+m$.

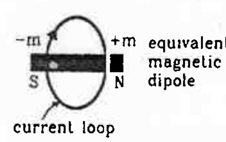


Figure 36.1

The model is very useful in studying magnetic effects and is widely used. It is customary to call a positive magnetic charge a *north pole* and a negative magnetic charge a *south pole*. They are represented by the letters N and S respectively. The quantity m is called *pole strength*. From the equation $md = iA$ or $F = mB$, we can easily see that the unit of pole strength is A–m. We can find the magnetic field due to a magnetic dipole at any point P using equation (36.2) for both the poles.

A solenoid very closely resembles a combination of circular loops placed side by side. If i be the current through it and A be the area of cross-section, the dipole moment of each turn is $\mu = iA$. In our model, each turn may be replaced by a small dipole placed at the centre

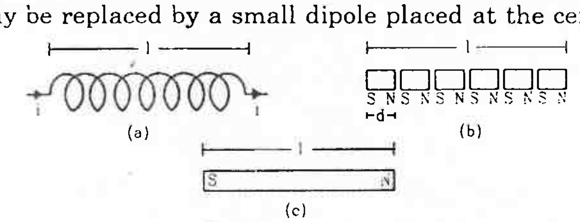


Figure 36.2

$$v' = \frac{1}{2\pi}\sqrt{\frac{M(B_H + B)}{I}}$$

or, $$\frac{v'^2}{v^2} = \frac{B_H + B}{B_H}$$

or, $$\left(\frac{60}{40}\right)^2 = 1 + \frac{B}{B_H}$$

or, $$\frac{B}{B_H} = 1.25$$

or, $$B = 1.25 \times 24\ \mu\text{T} = 30 \times 10^{-6}\ \text{T}.$$

The oscillating magnet is in end-on position of the short magnet. Thus, the field B can be written as

$$B = \frac{\mu_0}{4\pi}\frac{2M'}{d^3}$$

or, $$M' = \frac{2\pi}{\mu_0} B d^3$$

$$= 0.5 \times 10^7 \frac{\text{A}}{\text{T-m}} \times (30 \times 10^{-6}\ \text{T}) \times (20 \times 10^{-2}\ \text{m})^3$$

$$= 1.2\ \text{A-m}^2.$$

19. *A bar magnet of mass* 100 g, *length* 7·0 cm, *width* 1·0 cm *and height* 0·50 cm *takes* $\pi/2$ *seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of* 25 μT. *(a) Find the magnetic moment of the magnet. (b) If the magnet is put in the magnetometer with its* 0·50 cm *edge horizontal, what would be the time period?*

Solution : (a) The moment of inertia of the magnet about the axis of rotation is

$$I = \frac{m'}{12}(L^2 + b^2)$$

$$= \frac{100 \times 10^{-3}}{12}[(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2]\ \text{kg-m}^2$$

$$= \frac{25}{6} \times 10^{-5}\ \text{kg-m}^2.$$

We have,

$$T = 2\pi\sqrt{\frac{I}{MB}} \qquad \text{... (i)}$$

or, $$M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25 \times 10^{-5}\ \text{kg/m}^2}{6 \times (25 \times 10^{-6}\ \text{T}) \times \frac{\pi^2}{4}\ \text{s}^2}$$

$$= 27\ \text{A-m}^2.$$

(b) In this case the moment of inertia becomes

$$I' = \frac{m'}{12}(L^2 + b'^2) \text{ where } b' = 0.5\ \text{cm}.$$

The time period would be

$$T' = \sqrt{\frac{I'}{MB}} \qquad \text{... (ii)}$$

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}}$$

$$= \frac{\sqrt{\frac{m'}{12}(L^2 + b'^2)}}{\sqrt{\frac{m'}{12}(L^2 + b^2)}} = \frac{\sqrt{(7\ \text{cm})^2 + (0.5\ \text{cm})^2}}{\sqrt{(7\ \text{cm})^2 + (1.0\ \text{cm})^2}}$$

$$= 0.992$$

or, $$T' = \frac{0.992 \times \pi}{2}\ \text{s} = 0.496\pi\ \text{s}.$$

□

QUESTIONS FOR SHORT ANSWER

1. Can we have a single north pole ? A single south pole ?
2. Do two distinct poles actually exist at two nearby points in a magnetic dipole ?
3. An iron needle is attracted to the ends of a bar magnet but not to the middle region of the magnet. Is the material making up the ends of a bar magnet different from that of the middle region ?
4. Compare the direction of the magnetic field inside a solenoid with that of the field there if the solenoid is replaced by its equivalent combination of north pole and south pole.
5. Sketch the magnetic field lines for a current-carrying circular loop near its centre. Replace the loop by an equivalent magnetic dipole and sketch the magnetic field lines near the centre of the dipole. Identify the difference.
6. The force on a north pole, $\vec{F} = m\vec{B}$, is parallel to the field $\vec{B}$. Does it contradict our earlier knowledge that a magnetic field can exert forces only perpendicular to itself ?
7. Two bar magnets are placed close to each other with their opposite poles facing each other. In absence of other forces, the magnets are pulled towards each other and their kinetic energy increases. Does it contradict our earlier knowledge that magnetic forces cannot do any work and hence cannot increase kinetic energy of a system ?
8. Magnetic scalar potential is defined as

$$U(\vec{r}_2) - U(\vec{r}_1) = -\int_{\vec{r}_1}^{\vec{r}_2} \vec{B} \cdot d\vec{l}.$$

Apply this equation to a closed curve enclosing a long straight wire. The RHS of the above equation is then $-\mu_0 i$ by Ampere's law. We see that $U(\vec{r_2}) \neq U(\vec{r_1})$ even when $\vec{r_2} = \vec{r_1}$. Can we have a magnetic scalar potential in this case ?

9. Can the earth's magnetic field be vertical at a place ? What will happen to a freely suspended magnet at such a place ? What is the value of dip here ?

10. Can the dip at a place be zero ? 90° ?

11. The reduction factor K of a tangent galvanometer is written on the instrument. The manual says that the current is obtained by multiplying this factor to $\tan\theta$. The procedure works well at Bhuwaneshwar. Will the procedure work if the instrument is taken to Nepal ? If there is some error, can it be corrected by correcting the manual or the instrument will have to be taken back to the factory ?

OBJECTIVE I

1. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the axis of the loop is in
(a) end-on position (b) broadside-on position
(c) both (d) none.

2. A circular loop carrying a current is replaced by an equivalent magnetic dipole. A point on the loop is in
(a) end-on position (b) broadside-on position
(c) both (d) none.

3. When a current in a circular loop is equivalently replaced by a magnetic dipole,
(a) the pole strength m of each pole is fixed
(b) the distance d between the poles is fixed
(c) the product md is fixed
(d) none of the above.

4. Let r be the distance of a point on the axis of a bar magnet from its centre. The magnetic field at such a point is proportional to
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) none of these.

5. Let r be the distance of a point on the axis of a magnetic dipole from its centre. The magnetic field at such a point is proportional to
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) none of these.

6. Two short magnets of equal dipole moments M are fastened perpendicularly at their centres (figure 36-Q1). The magnitude of the magnetic field at a distance d from the centre on the bisector of the right angle is
(a) $\frac{\mu_0}{4\pi}\frac{M}{d^3}$ (b) $\frac{\mu_0}{4\pi}\frac{\sqrt{2}M}{d^3}$ (c) $\frac{\mu_0}{4\pi}\frac{2\sqrt{2}M}{d^3}$ (d) $\frac{\mu_0}{4\pi}\frac{2M}{d^3}$

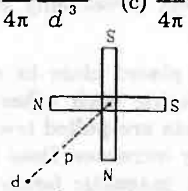


Figure 36-Q1

7. Magnetic meridian is
(a) a point (b) a line along north-south
(c) a horizontal plane (d) a vertical plane.

8. A compass needle which is allowed to move in a horizontal plane is taken to a geomagnetic pole. It
(a) will stay in north-south direction only
(b) will stay in east-west direction only
(c) will become rigid showing no movement
(d) will stay in any position.

9. A dip circle is taken to geomagnetic equator. The needle is allowed to move in a vertical plane perpendicular to the magnetic meridian. The needle will stay
(a) in horizontal direction only
(b) in vertical direction only
(c) in any direction except vertical and horizontal
(d) in any direction it is released.

10. Which of the following four graphs may best represent the current–deflection relation in a tangent galvanometer ?

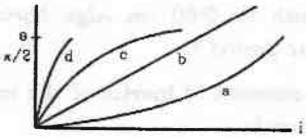


Figure 36-Q2

11. A tangent galvanometer is connected directly to an ideal battery. If the number of turns in the coil is doubled, the deflection will
(a) increase (b) decrease
(c) remain unchanged (d) either increase or decrease.

12. If the current is doubled, the deflection is also doubled in
(a) a tangent galvanometer
(b) a moving-coil galvanometer
(c) both (d) none.

13. A very long bar magnet is placed with its north pole coinciding with the centre of a circular loop carrying an electric current i. The magnetic field due to the magnet at a point on the periphery of the wire is B. The radius of the loop is a. The force on the wire is
(a) very nearly $2\pi aiB$ perpendicular to the plane of the wire
(b) $2\pi aiB$ in the plane of the wire
(c) πaiB along the magnet (d) zero.

OBJECTIVE II

1. Pick the correct options.
(a) Magnetic field is produced by electric charges only.
(b) Magnetic poles are only mathematical assumptions having no real existence.
(c) A north pole is equivalent to a clockwise current and a south pole is equivalent to an anticlockwise current.
(d) A bar magnet is equivalent to a long, straight current.

2. A horizontal circular loop carries a current that looks clockwise when viewed from above. It is replaced by an equivalent magnetic dipole consisting of a south pole S and a north pole N.
(a) The line SN should be along a diameter of the loop.
(b) The line SN should be perpendicular to the plane of the loop.
(c) The south pole should be below the loop.
(d) The north pole should be below the loop.

3. Consider a magnetic dipole kept in the north-south direction. Let P_1, P_2, Q_1, Q_2 be four points at the same distance from the dipole towards north, south, east and west of the dipole respectively. The directions of the magnetic field due to the dipole are the same at
(a) P_1 and P_2 (b) Q_1 and Q_2
(c) P_1 and Q_1 (d) P_2 and Q_2.

4. Consider the situation of the previous problem. The directions of the magnetic field due to the dipole are opposite at
(a) P_1 and P_2 (b) Q_1 and Q_2
(c) P_1 and Q_1 (d) P_2 and Q_2

5. To measure the magnetic moment of a bar magnet, one may use
(a) a tangent galvanometer
(b) a deflection galvanometer if the earth's horizontal field is known
(c) an oscillation magnetometer if the earth's horizontal field is known
(d) both deflection and oscillation magnetometer if the earth's horizontal field is not known.

EXERCISES

1. A long bar magnet has a pole strength of 10 A–m. Find the magnetic field at a point on the axis of the magnet at a distance of 5 cm from the north pole of the magnet.

2. Two long bar magnets are placed with their axes coinciding in such a way that the north pole of the first magnet is 2·0 cm from the south pole of the second. If both the magnets have a pole strength of 10 A–m, find the force exerted by one magnet on the other.

3. A uniform magnetic field of $0{\cdot}20 \times 10^{-3}$ T exists in the space. Find the change in the magnetic scalar potential as one moves through 50 cm along the field.

4. Figure (36-E1) shows some of the equipotential surfaces of the magnetic scalar potential. Find the magnetic field B at a point in the region.

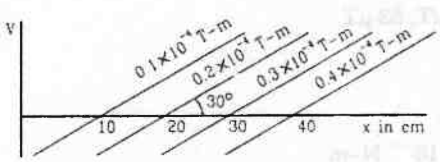


Figure 36·E1

5. The magnetic field at a point, 10 cm away from a magnetic dipole, is found to be $2{\cdot}0 \times 10^{-4}$ T. Find the magnetic moment of the dipole if the point is (a) in end-on position of the dipole and (b) in broadside-on position of the dipole.

6. Show that the magnetic field at a point due to a magnetic dipole is perpendicular to the magnetic axis if the line joining the point with the centre of the dipole makes an angle of $\tan^{-1}(\sqrt{2})$ with the magnetic axis.

7. A bar magnet has a length of 8 cm. The magnetic field at a point at a distance 3 cm from the centre in the broadside-on position is found to be 4×10^{-6} T. Find the pole strength of the magnet.

8. A magnetic dipole of magnetic moment 1·44 A–m^2 is placed horizontally with the north pole pointing towards north. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.

9. A magnetic dipole of magnetic moment 0·72 A–m^2 is placed horizontally with the north pole pointing towards south. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.

10. A magnetic dipole of magnetic moment $0{\cdot}72\sqrt{2}$ A–m^2 is placed horizontally with the north pole pointing towards east. Find the position of the neutral point if the horizontal component of the earth's magnetic field is 18 μT.

11. The magnetic moment of the assumed dipole at the earth's centre is $8{\cdot}0 \times 10^{22}$ A–m^2. Calculate the magnetic field B at the geomagnetic poles of the earth. Radius of the earth is 6400 km.

12. If the earth's magnetic field has a magnitude $3{\cdot}4 \times 10^{-5}$ T at the magnetic equator of the earth, what would be its value at the earth's geomagnetic poles ?

13. The magnetic field due to the earth has a horizontal component of 26 μT at a place where the dip is 60°. Find the vertical component and the magnitude of the field.

14. A magnetic needle is free to rotate in a vertical plane which makes an angle of 60° with the magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}(2/\sqrt{3})$ with the horizontal, what would be the dip at that place ?

15. The needle of a dip circle shows an apparent dip of 45° in a particular position and 53° when the circle is rotated through 90°. Find the true dip.

16. A tangent galvanometer shows a deflection of 45° when 10 mA of current is passed through it. If the horizontal component of the earth's magnetic field is $B_H = 3{\cdot}6 \times 10^{-5}$ T and radius of the coil is 10 cm, find the number of turns in the coil.

17. A moving-coil galvanometer has a 50-turn coil of size 2 cm × 2 cm. It is suspended between the magnetic poles producing a magnetic field of 0·5 T. Find the torque on the coil due to the magnetic field when a current of 20 mA passes through it.

18. A short magnet produces a deflection of 37° in a deflection magnetometer in Tan-*A* position when placed at a separation of 10 cm from the needle. Find the ratio of the magnetic moment of the magnet to the earth's horizontal magnetic field.

19. The magnetometer of the previous problem is used with the same magnet in Tan-*B* position. Where should the magnet be placed to produce a 37° deflection of the needle ?

20. A deflection magnetometer is placed with its arms in north-south direction. How and where should a short magnet having $M/B_H = 40$ A–m^2/T be placed so that the needle can stay in any position ?

21. A bar magnet takes $\pi/10$ second to complete one oscillation in an oscillation magnetometer. The moment of inertia of the magnet about the axis of rotation is $1{\cdot}2 \times 10^{-4}$ kg–m^2 and the earth's horizontal magnetic field is 30 μT. Find the magnetic moment of the magnet.

22. The combination of two bar magnets makes 10 oscillations per second in an oscillation magnetometer when like poles are tied together and 2 oscillations per second when unlike poles are tied together. Find the ratio of the magnetic moments of the magnets. Neglect any induced magnetism.

23. A short magnet oscillates in an oscillation magnetometer with a time period of 0·10 s where the earth's horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire placed 20 cm east of the magnet. Find the new time period.

24. A bar magnet makes 40 oscillations per minute in an oscillation magnetometer. An identical magnet is demagnetized completely and is placed over the magnet in the magnetometer. Find the time taken for 40 oscillations by this combination. Neglect any induced magnetism.

25. A short magnet makes 40 oscillations per minute when used in an oscillation magnetometer at a place where the earth's horizontal magnetic field is 25 μT. Another short magnet of magnetic moment 1·6 A–m^2 is placed 20 cm east of the oscillating magnet. Find the new frequency of oscillation if the magnet has its north pole (a) towards north and (b) towards south.

□

ANSWERS

OBJECTIVE I

1. (a) 2. (b) 3. (c) 4. (d) 5. (c) 6. (c)
7. (d) 8. (d) 9. (d) 10. (c) 11. (c) 12. (b)
13. (a)

OBJECTIVE II

1. (a), (b) 2. (b), (d) 3. (a), (b)
4. (c), (d) 5. (b), (c), (d)

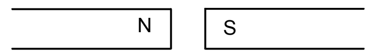
EXERCISES

1. 4×10^{-4} T
2. $2{\cdot}5 \times 10^{-2}$ N
3. decreases by $0{\cdot}10 \times 10^{-3}$ T–m
4. $2{\cdot}0 \times 10^{-4}$ T
5. (a) 1·0 A–m^2 and (b) 2·0 A–m^2
7. 6×10^{-5} A-m
8. at a distance of 20 cm in the plane bisecting the dipole
9. 20 cm south of the dipole
10. 20 cm from the dipole, $\tan^{-1}\sqrt{2}$ south of east
11. 60 μT
12. $6{\cdot}8 \times 10^{-5}$ T
13. 45 μT, 52 μT
14. 30°
15. 39°
16. 570
17. 2×10^{-4} N–m
18. $3{\cdot}75 \times 10^{3} \dfrac{\text{A–m}^2}{\text{T}}$
19. 7·9 cm from the centre
20. 2·0 cm from the needle, north pole pointing towards south
21. 1600 A–m^2
22. 13 : 12
23. 0·076 s
24. $\sqrt{2}$ minutes
25. (a) 18 oscillations/minute (b) 54 oscillations/minute

□

CHAPTER – 36
PERMANENT MAGNETS

1. m = 10 A-m, d = 5 cm = 0.05 m



$B = \frac{\mu_0}{4\pi}\frac{m}{r^2} = \frac{10^{-7} \times 10}{\left(5 \times 10^{-2}\right)^2} = \frac{10^{-2}}{25}$ = 4 × 10^{-4} Tesla

2. $m_1 = m_2$ = 10 A-m
r = 2 cm = 0.02 m
we know

Force exerted by tow magnetic poles on each other = $\frac{\mu_0}{4\pi}\frac{m_1 m_2}{r^2} = \frac{4\pi \times 10^{-7} \times 10^2}{4\pi \times 4 \times 10^{-4}}$ = 2.5 × 10^{-2} N

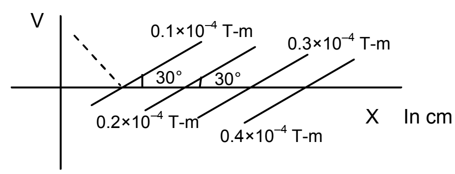
3. $B = -\frac{dv}{d\ell} \Rightarrow dv = -B\,d\ell = -0.2 \times 10^{-3} \times 0.5 = -0.1 \times 10^{-3}$ T-m

Since the sigh is –ve therefore potential decreases.

4. Here dx = 10 sin 30° cm = 5 cm

$\frac{dV}{dx} = B = \frac{0.1 \times 10^{-4}\ \text{T}-\text{m}}{5 \times 10^{-2}\ \text{m}}$

Since B is perpendicular to equipotential surface.
Here it is at angle 120° with (+ve) x-axis and B = 2 × 10^{-4} T



5. B = 2 × 10^{-4} T
d = 10 cm = 0.1 m
(a) if the point at end-on postion.

$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times 2M}{(10^{-1})^3}$

$\Rightarrow \frac{2 \times 10^{-4} \times 10^{-3}}{10^{-7} \times 2} = M \Rightarrow M = 1\ \text{Am}^2$

(b) If the point is at broad-on position

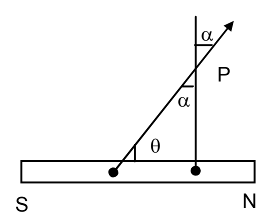
$\frac{\mu_0}{4\pi}\frac{M}{d^3} \Rightarrow 2 \times 10^{-4} = \frac{10^{-7} \times M}{(10^{-1})^3} \Rightarrow M = 2\ \text{Am}^2$

6. Given :

$\theta = \tan^{-1}\sqrt{2} \Rightarrow \tan\theta = \sqrt{2} \Rightarrow 2 = \tan^2\theta$

$\Rightarrow \tan\theta = 2\cot\theta \Rightarrow \frac{\tan\theta}{2} = \cot\theta$

We know $\frac{\tan\theta}{2} = \tan\alpha$



Comparing we get, $\tan\alpha = \cot\theta$
or, $\tan\alpha = \tan(90 - \theta)$ or $\alpha = 90 - \theta$ or $\theta + \alpha = 90$
Hence magnetic field due to the dipole is ⊥r to the magnetic axis.

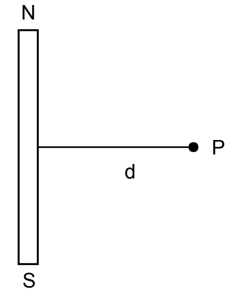
7. Magnetic field at the broad side on position :

$B = \frac{\mu_0}{4\pi}\frac{M}{\left(d^2 + \ell^2\right)^{3/2}}$ 2ℓ = 8 cm d = 3 cm

$\Rightarrow 4 \times 10^{-6} = \frac{10^{-7} \times m \times 8 \times 10^{-2}}{\left(9 \times 10^{-4} + 16 \times 10^{-4}\right)^{3/2}} \Rightarrow 4 \times 10^{-6} = \frac{10^{-9} \times m \times 8}{\left(10^{-4}\right)^{3/2} + (25)^{3/2}}$

$\Rightarrow m = \frac{4 \times 10^{-6} \times 125 \times 10^{-8}}{8 \times 10^{-9}} = 62.5 \times 10^{-5}$ A-m

8. We know for a magnetic dipole with its north pointing the north, the neutral point in the broadside on position.



Again $\vec{B}$ in this case = $\frac{\mu_0 M}{4\pi d^3}$

$\therefore \frac{\mu_0 M}{4\pi d^3} = \overrightarrow{B_H}$ due to earth

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18\ \mu T$

$\Rightarrow \frac{10^{-7} \times 1.44}{d^3} = 18 \times 10^{-6}$

$\Rightarrow d^3 = 8 \times 10^{-3}$

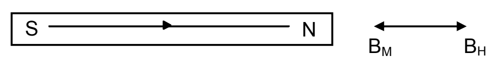
$\Rightarrow d = 2 \times 10^{-1}$ m = 20 cm

In the plane bisecting the dipole.

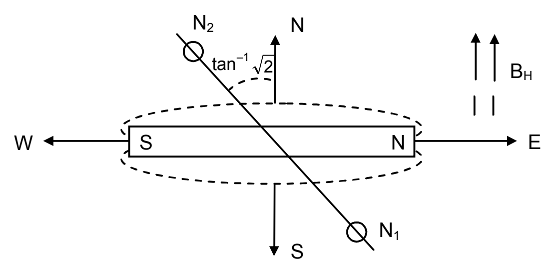
9. When the magnet is such that its North faces the geographic south of earth. The neutral point lies along the axial line of the magnet.

$\frac{\mu_0}{4\pi}\frac{2M}{d^3} = 18 \times 10^{-6} \Rightarrow \frac{10^{-7} \times 2 \times 0.72}{d^3} = 18 \times 10^{-6} \Rightarrow d^3 = \frac{2 \times 0.7 \times 10^{-7}}{18 \times 10^{-6}}$

$\Rightarrow d = \left(\frac{8 \times 10^{-9}}{10^{-6}}\right)^{1/3} = 2 \times 10^{-1}$ m = 20 cm



10. Magnetic moment = $0.72\sqrt{2}$ A-m^2 = M



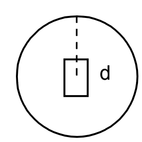
$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ $\quad\quad B_H = 18\ \mu T$

$\Rightarrow \frac{4\pi \times 10^{-7} \times 0.72\sqrt{2}}{4\pi \times d^3} = 18 \times 10^{-6}$

$\Rightarrow d^3 = \frac{0.72 \times 1.414 \times 10^{-7}}{18 \times 10^{-6}} = 0.005656$

$\Rightarrow d \approx 0.2$ m = 20 cm

11. The geomagnetic pole is at the end on position of the earth.



$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = \frac{10^{-7} \times 2 \times 8 \times 10^{22}}{(6400 \times 10^3)^3} \approx 60 \times 10^{-6}\ T = 60\ \mu T$

12. $\vec{B} = 3.4 \times 10^{-5}$ T

Given $\frac{\mu_0}{4\pi}\frac{M}{R^3} = 3.4 \times 10^{-5}$

$\Rightarrow M = \frac{3.4 \times 10^{-5} \times R^3 \times 4\pi}{4\pi \times 10^{-7}} = 3.4 \times 10^2 R^3$

$\vec{B}$ at Poles = $\frac{\mu_0}{4\pi}\frac{2M}{R^3} = = 6.8 \times 10^{-5}$ T

13. δ(dip) = 60°

$B_H = B \cos 60°$

$\Rightarrow B = 52 \times 10^{-6} = 52\ \mu T$

$B_V = B \sin \delta = 52 \times 10^{-6} \frac{\sqrt{3}}{2} = 44.98\ \mu T \approx 45\ \mu T$

14. If δ_1 and δ_2 be the apparent dips shown by the dip circle in the 2$\perp$r positions, the true dip δ is given by

$\text{Cot}^2 \delta = \text{Cot}^2 \delta_1 + \text{Cot}^2 \delta_2$

$\Rightarrow \text{Cot}^2 \delta = \text{Cot}^2 45° + \text{Cot}^2 53°$

$\Rightarrow \text{Cot}^2 \delta = 1.56 \Rightarrow \delta = 38.6 \approx 39°$

15. We know $B_H = \frac{\mu_0 in}{2r}$

Give : $B_H = 3.6 \times 10^{-5}$ T $\theta = 45°$
$i = 10$ mA $= 10^{-2}$ A $\tan\theta = 1$
$n = ?$ $r = 10$ cm $= 0.1$ m

$n = \frac{B_H \tan\theta \times 2r}{\mu_0 i} = \frac{3.6\times10^{-5}\times2\times1\times10^{-1}}{4\pi\times10^{-7}\times10^{-2}} = 0.5732 \times 10^3 \approx 573$ turns

16. $n = 50$ $A = 2$ cm $\times$ 2 cm $= 2 \times 2 \times 10^{-4}$ m^2
$i = 20 \times 10^{-3}$ A $B = 0.5$ T
$\tau = ni(\vec{A}\times\vec{B}) = niAB \sin 90° = 50 \times 20 \times 10^{-3} \times 4 \times 10^{-4} \times 0.5 = 2 \times 10^{-4}$ N-M

17. Given $\theta = 37°$ $d = 10$ cm $= 0.1$ m
We know

$\frac{M}{B_H} = \frac{4\pi}{\mu_0}\frac{(d^2-\ell^2)^2}{2d}\tan\theta = \frac{4\pi}{\mu_0}\times\frac{d^4}{2d}\tan\theta$ [As the magnet is short]

$= \frac{4\pi}{4\pi\times10^{-7}}\times\frac{(0.1)^3}{2}\times\tan 37° = 0.5 \times 0.75 \times 1 \times 10^{-3} \times 10^7 = 0.375 \times 10^4 = 3.75 \times 10^3$ A-m^2 T^{-1}

18. $\frac{M}{B_H}$ (found in the previous problem) $= 3.75 \times 10^3$ A-m^2 T^{-1}

$\theta = 37°$, $d = ?$

$\frac{M}{B_H} = \frac{4\pi}{\mu_0}(d^2+\ell^2)^{3/2}\tan\theta$

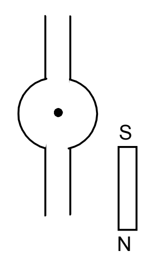
$\ell << d$ neglecting ℓ w.r.t.d

$\Rightarrow \frac{M}{B_H} = \frac{4\pi}{\mu_0}d^3\tan\theta \Rightarrow 3.75 \times 10^3 = \frac{1}{10^{-7}} \times d^3 \times 0.75$

$\Rightarrow d^3 = \frac{3.75\times10^3\times10^{-7}}{0.75} = 5 \times 10^{-4}$

$\Rightarrow d = 0.079$ m $= 7.9$ cm

19. Given $\frac{M}{B_H} = 40$ A-m^2/T



Since the magnet is short 'ℓ' can be neglected

So, $\frac{M}{B_H} = \frac{4\pi}{\mu_0}\times\frac{d^3}{2} = 40$

$\Rightarrow d^3 = \frac{40\times4\pi\times10^{-7}\times2}{4\pi} = 8 \times 10^{-6}$

$\Rightarrow d = 2 \times 10^{-2}$ m $= 2$ cm

with the northpole pointing towards south.

20. According to oscillation magnetometer,

$T = 2\pi\sqrt{\frac{I}{MB_H}}$

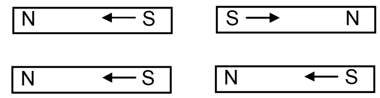
$\Rightarrow \frac{\pi}{10} = 2\pi\sqrt{\frac{1.2\times10^{-4}}{M\times30\times10^{-6}}}$

$\Rightarrow \left(\frac{1}{20}\right)^2 = \frac{1.2\times10^{-4}}{M\times30\times10^{-6}}$

$\Rightarrow M = \frac{1.2\times10^{-4}\times400}{30\times10^{-6}} = 16 \times 10^2$ A-m^2 $= 1600$ A-m^2

21. We know : $\upsilon = \frac{1}{2\pi}\sqrt{\frac{mB_H}{I}}$

For like poles tied together



$M = M_1 - M_2$

For unlike poles $M' = M_1 + M_2$

$\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}} \Rightarrow \left(\frac{10}{2}\right)^2 = \frac{M_1 - M_2}{M_1 + M_2} \Rightarrow 25 = \frac{M_1 - M_2}{M_1 + M_2}$

$\Rightarrow \frac{26}{24} = \frac{2M_1}{2M_2} \Rightarrow \frac{M_1}{M_2} = \frac{13}{12}$

22. $B_H = 24 \times 10^{-6}$ T $\qquad T_1 = 0.1\,'$

$B = B_H - B_{wire} = 2.4 \times 10^{-6} - \frac{\mu_o}{2\pi}\frac{i}{r} = 24 \times 10^{-6} - \frac{2 \times 10^{-7} \times 18}{0.2} = (24 - 10) \times 10^{-6} = 14 \times 10^{-6}$

$T = 2\pi\sqrt{\frac{I}{MB_H}} \qquad \frac{T_1}{T_2} = \sqrt{\frac{B}{B_H}}$

$\Rightarrow \frac{0.1}{T_2} = \sqrt{\frac{14 \times 10^{-6}}{24 \times 10^{-6}}} \Rightarrow \left(\frac{0.1}{T_2}\right)^2 = \frac{14}{24} \Rightarrow T_2^{\,2} = \frac{0.01 \times 14}{24} \Rightarrow T_2 = 0.076$

23. $T = 2\pi\sqrt{\frac{I}{MB_H}} \qquad$ Here $I' = 2I$

$T_1 = \frac{1}{40}$ min $\qquad T_2 = ?$

$\frac{T_1}{T_2} = \sqrt{\frac{I}{I'}}$

$\Rightarrow \frac{1}{40T_2} = \sqrt{\frac{1}{2}} \Rightarrow \frac{1}{1600T_2^{\,2}} = \frac{1}{2} \Rightarrow T_2^{\,2} = \frac{1}{800} \Rightarrow T_2 = 0.03536$ min

For 1 oscillation Time taken = 0.03536 min.

For 40 Oscillation Time = 4 × 0.03536 = 1.414 = $\sqrt{2}$ min

24. γ_1 = 40 oscillations/minute

$B_H = 25\ \mu T$

m of second magnet = 1.6 A-m^2

d = 20 cm = 0.2 m

(a) For north facing north

$\gamma_1 = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \qquad \gamma_2 = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$

$B = \frac{\mu_0}{4\pi}\frac{m}{d^3} = \frac{10^{-7} \times 1.6}{8 \times 10^{-3}} = 20\ \mu T$

$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B}{B_H - B}} \Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{5}} \Rightarrow \gamma_2 = \frac{40}{\sqrt{5}}$ = 17.88 ≈ 18 osci/min

(b) For north pole facing south

$\gamma_1 = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}} \qquad \gamma_2 = \frac{1}{2\pi}\sqrt{\frac{M(B_H - B)}{I}}$

$\frac{\gamma_1}{\gamma_2} = \sqrt{\frac{B}{B_H - B}} \Rightarrow \frac{40}{\gamma_2} = \sqrt{\frac{25}{45}} \Rightarrow \gamma_2 = \frac{40}{\sqrt{\left(\frac{25}{45}\right)}}$ = 53.66 ≈ 54 osci/min

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