OBJECTIVE I

1. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the capacitor is
(a) $\frac{CV}{\varepsilon_0}$ (b) $\frac{2CV}{\varepsilon_0}$ (c) $\frac{CV}{2\varepsilon_0}$ (d) zero.

2. Two capacitors each having capacitance C and breakdown voltage V are joined in series. The capacitance and the breakdown voltage of the combination will be
(a) $2C$ and $2V$ (b) $C/2$ and $V/2$
(c) $2C$ and $V/2$ (d) $C/2$ and $2V$.

3. If the capacitors in the previous question are joined in parallel, the capacitance and the breakdown voltage of the combination will be
(a) $2C$ and $2V$ (b) C and $2V$
(c) $2C$ and V (d) C and V.

4. The equivalent capacitance of the combination shown in figure (31-Q1) is
(a) C (b) $2C$ (c) $C/2$ (d) none of these.

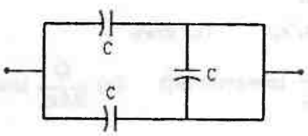


Figure 31-Q1

5. A dielectric slab is inserted between the plates of an isolated capacitor. The force between the plates will
(a) increase (b) decrease
(c) remain unchanged (d) become zero.

6. The energy density in the electric field created by a point charge falls off with the distance from the point charge as
(a) $\frac{1}{r}$ (b) $\frac{1}{r^2}$ (c) $\frac{1}{r^3}$ (d) $\frac{1}{r^4}$.

7. A parallel-plate capacitor has plates of unequal area. The larger plate is connected to the positive terminal of the battery and the smaller plate to its negative terminal. Let Q_+ and Q_- be the charges appearing on the positive and negative plates respectively.
(a) $Q_+ > Q_-$. (b) $Q_+ = Q_-$. (c) $Q_+ < Q_-$.
(d) The information is not sufficient to decide the relation between Q_+ and Q_-.

8. A thin metal plate P is inserted between the plates of a parallel-plate capacitor of capacitance C in such a way that its edges touch the two plates (figure 31-Q2). The capacitance now becomes
(a) $C/2$ (b) $2C$ (c) 0 (d) ∞.

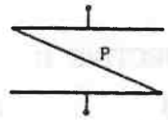


Figure 31-Q2

9. Figure (31-Q3) shows two capacitors connected in series and joined to a battery. The graph shows the variation in potential as one moves from left to right on the branch containing the capacitors.
(a) $C_1 > C_2$. (b) $C_1 = C_2$. (c) $C_1 < C_2$.
(d) The information is not sufficient to decide the relation between C_1 and C_2.

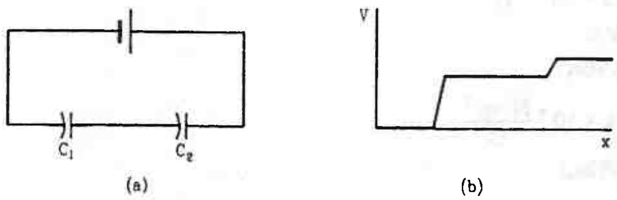


Figure 31-Q3

10. Two metal plates having charges Q, $-Q$ face each other at some separation and are dipped into an oil tank. If the oil is pumped out, the electric field between the plates will
(a) increase (b) decrease
(c) remain the same (d) become zero.

11. Two metal spheres of capacitances C_1 and C_2 carry some charges. They are put in contact and then separated. The final charges Q_1 and Q_2 on them will satisfy
(a) $\frac{Q_1}{Q_2} < \frac{C_1}{C_2}$ (b) $\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$ (c) $\frac{Q_1}{Q_2} > \frac{C_1}{C_2}$ (d) $\frac{Q_1}{Q_2} = \frac{C_2}{C_1}$.

12. Three capacitors of capacitances 6 μF each are available. The minimum and maximum capacitances, which may be obtained are
(a) 6 μF, 18 μF (b) 3 μF, 12 μF
(c) 2 μF, 12 μF (d) 2 μF, 18 μF.

OBJECTIVE II

1. The capacitance of a capacitor does not depend on
(a) the shape of the plates
(b) the size of the plates
(c) the charges on the plates
(d) the separation between the plates.

2. A dielectric slab is inserted between the plates of an isolated charged capacitor. Which of the following quantities will remain the same?
(a) the electric field in the capacitor
(b) the charge on the capacitor

(c) the potential difference between the plates
(d) the stored energy in the capacitor.

3. A dielectric slab is inserted between the plates of a capacitor. The charge on the capacitor is Q and the magnitude of the induced charge on each surface of the dielectric is Q'.
(a) Q' may be larger than Q.
(b) Q' must be larger than Q.
(c) Q' must be equal to Q.
(d) Q' must be smaller than Q.

4. Each plate of a parallel plate capacitor has a charge q on it. The capacitor is now connected to a battery. Now,
(a) the facing surfaces of the capacitor have equal and opposite charges.
(b) the two plates of the capacitor have equal and opposite charges.
(c) the battery supplies equal and opposite charges to the two plates.
(d) the outer surfaces of the plates have equal charges.

5. The separation between the plates of a charged parallel-plate capacitor is increased. Which of the following quantities will change ?
(a) charge on the capacitor
(b) potential difference across the capacitor
(c) energy of the capacitor
(d) energy density between the plates.

6. A parallel-plate capacitor is connected to a battery. A metal sheet of negligible thickness is placed between the plates. The sheet remains parallel to the plates of the capacitor.
(a) The battery will supply more charge.
(b) The capacitance will increase.
(c) The potential difference between the plates will increase.
(d) Equal and opposite charges will appear on the two faces of the metal plate.

7. Following operations can be performed on a capacitor:
X – connect the capacitor to a battery of emf $\mathcal{E}$.
Y – disconnect the battery.
Z – reconnect the battery with polarity reversed.
W – insert a dielectric slab in the capacitor.
(a) In XYZ (perform X, then Y, then Z) the stored electric energy remains unchanged and no thermal energy is developed.
(b) The charge appearing on the capacitor is greater after the action XWY than after the action XYW.
(c) The electric energy stored in the capacitor is greater after the action WXY than after the action XYW.
(d) The electric field in the capacitor after the action XW is the same as that after WX.

EXERCISES

1. When $1{\cdot}0 \times 10^{12}$ electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two-conductor system.

2. The plates of a parallel-plate capacitor are made of circular discs of radii 5·0 cm each. If the separation between the plates is 1·0 mm, what is the capacitance ?

3. Suppose, one wishes to construct a 1·0 farad capacitor using circular discs. If the separation between the discs be kept at 1·0 mm, what would be the radius of the discs ?

4. A parallel-plate capacitor having plate area 25 cm^2 and separation 1·00 mm is connected to a battery of 6·0 V. Calculate the charge flown through the battery. How much work has been done by the battery during the process ?

5. A parallel-plate capacitor has plate area 25·0 cm^2 and a separation of 2·00 mm between the plates. The capacitor is connected to a battery of 12·0 V. (a) Find the charge on the capacitor. (b) The plate separation is decreased to 1·00 mm. Find the extra charge given by the battery to the positive plate.

6. Find the charges on the three capacitors connected to a battery as shown in figure (31-E1). Take $C_1 = 2{\cdot}0\ \mu F$, $C_2 = 4{\cdot}0\ \mu F$, $C_3 = 6{\cdot}0\ \mu F$ and $V = 12$ volt.

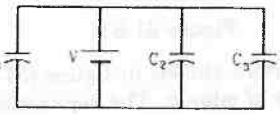


Figure 31-E1

7. Three capacitors having capacitances 20 μF, 30 μF and 40 μF are connected in series with a 12 V battery. Find the charge on each of the capacitors. How much work has been done by the battery in charging the capacitors ?

8. Find the charge appearing on each of the three capacitors shown in figure (31-E2).

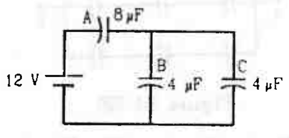


Figure 31-E2

9. Take $C_1 = 4{\cdot}0\ \mu F$ and $C_2 = 6{\cdot}0\ \mu F$ in figure (31-E3). Calculate the equivalent capacitance of the combination between the points indicated.

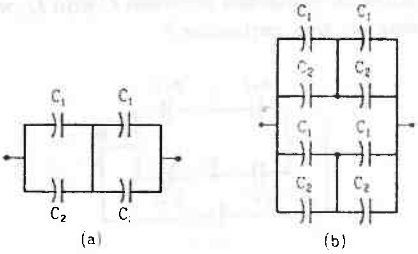


Figure 31-E3

10. Find the charge supplied by the battery in the arrangement shown in figure (31-E4).

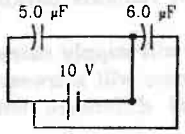


Figure 31-E4

11. The outer cylinders of two cylindrical capacitors of capacitance 2·2 μF each, are kept in contact and the inner cylinders are connected through a wire. A battery of emf 10 V is connected as shown in figure (31-E5). Find the total charge supplied by the battery to the inner cylinders.

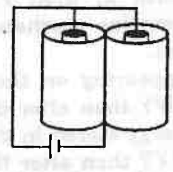


Figure 31-E5

12. Two conducting spheres of radii R_1 and R_2 are kept widely separated from each other. What are their individual capacitances ? If the spheres are connected by a metal wire, what will be the capacitance of the combination ? Think in terms of series–parallel connections.

13. Each of the capacitors shown in figure (31-E6) has a capacitance of 2 μF. Find the equivalent capacitance of the assembly between the points A and B. Suppose, a battery of emf 60 volts is connected between A and B. Find the potential difference appearing on the individual capacitors.

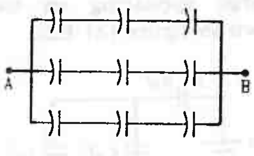


Figure 31-E6

14. It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they can withstand only 50 V. Design a combination which can yield the desired result.

15. Take the potential of the point B in figure (31-E7) to be zero. (a) Find the potentials at the points C and D. (b) If a capacitor is connected between C and D, what charge will appear on this capacitor ?

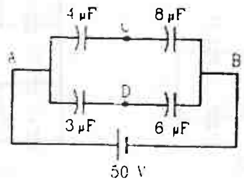


Figure 31-E7

16. Find the equivalent capacitance of the system shown in figure (31-E8) between the points a and b.

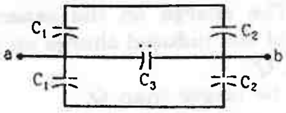


Figure 31-E8

17. A capacitor is made of a flat plate of area A and a second plate having a stair-like structure as shown in figure (31-E9). The width of each stair is a and the height is b. Find the capacitance of the assembly.

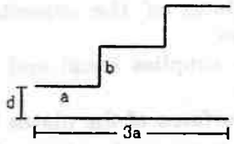


Figure 31-E9

18. A cylindrical capacitor is constructed using two coaxial cylinders of the same length 10 cm and of radii 2 mm and 4 mm. (a) Calculate the capacitance. (b) Another capacitor of the same length is constructed with cylinders of radii 4 mm and 8 mm. Calculate the capacitance.

19. A 100 pF capacitor is charged to a potential difference of 24 V. It is connected to an uncharged capacitor of capacitance 20 pF. What will be the new potential difference across the 100 pF capacitor ?

20. Each capacitor shown in figure (31-E10) has a capacitance of 5·0 μF. The emf of the battery is 50 V. How much charge will flow through AB if the switch S is closed ?

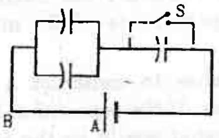


Figure 31-E10

21. The particle P shown in figure (31-E11) has a mass of 10 mg and a charge of – 0·01 μC. Each plate has a surface area 100 cm² on one side. What potential difference V should be applied to the combination to hold the particle P in equilibrium ?

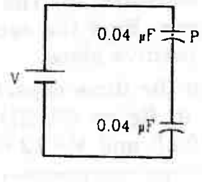


Figure 31-E11

22. Both the capacitors shown in figure (31-E12) are made of square plates of edge a. The separations between the plates of the capacitors are d_1 and d_2 as shown in the

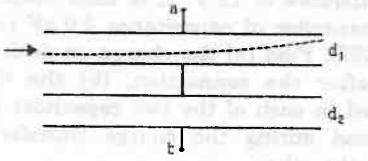


Figure 31-E12

figure. A potential difference V is applied between the points a and b. An electron is projected between the plates of the upper capacitor along the central line. With what minimum speed should the electron be projected so that it does not collide with any plate ? Consider only the electric forces.

23. The plates of a capacitor are 2·00 cm apart. An electron–proton pair is released somewhere in the gap between the plates and it is found that the proton reaches the negative plate at the same time as the electron reaches the positive plate. At what distance from the negative plate was the pair released ?

24. Convince yourself that parts (a), (b) and (c) of figure (31-E13) are identical. Find the capacitance between the points A and B of the assembly.

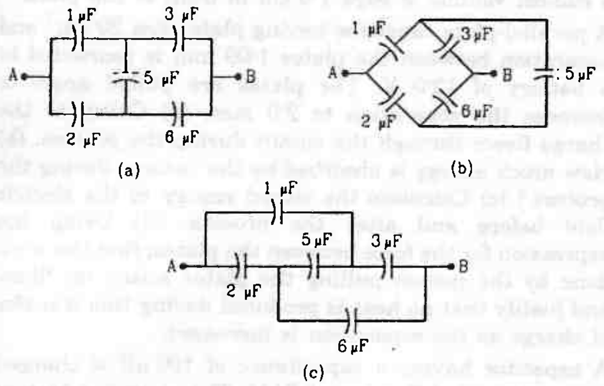


Figure 31-E13

25. Find the potential difference $V_a - V_b$ between the points a and b shown in each part of the figure (31-E14).

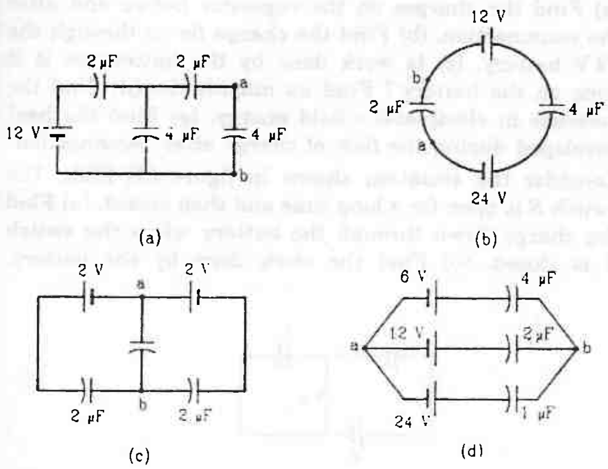


Figure 31-E14

26. Find the equivalent capacitances of the combinations shown in figure (31-E15) between the indicated points.

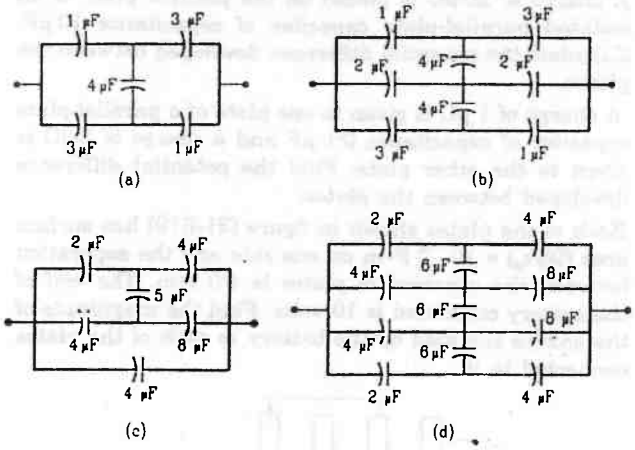


Figure 31-E15

27. Find the capacitance of the combination shown in figure (31-E16) between A and B.

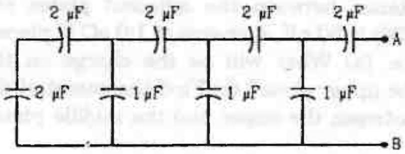


Figure 31-E16

28. Find the equivalent capacitance of the infinite ladder shown in figure (31-E17) between the points A and B.

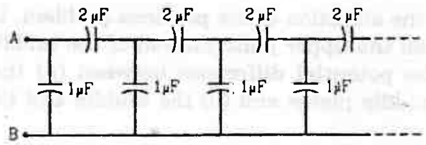


Figure 31-E17

29. A finite ladder is constructed by connecting several sections of 2 μF, 4 μF capacitor combinations as shown in figure (31-E18). It is terminated by a capacitor of capacitance C. What value should be chosen for C, such that the equivalent capacitance of the ladder between the points A and B becomes independent of the number of sections in between ?

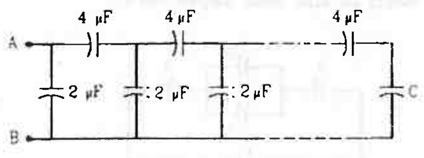


Figure 31-E18

30. A charge of $+2.0 \times 10^{-8}$ C is placed on the positive plate and a charge of -1.0×10^{-8} C on the negative plate of a parallel-plate capacitor of capacitance 1.2×10^{-3} μF.

Calculate the potential difference developed between the plates.

31. A charge of 20 μC is placed on the positive plate of an isolated parallel-plate capacitor of capacitance 10 μF. Calculate the potential difference developed between the plates.

32. A charge of 1 μC is given to one plate of a parallel-plate capacitor of capacitance 0·1 μF and a charge of 2 μC is given to the other plate. Find the potential difference developed between the plates.

33. Each of the plates shown in figure (31-E19) has surface area $(96/\varepsilon_0) \times 10^{-12}$ F–m on one side and the separation between the consecutive plates is 4·0 mm. The emf of the battery connected is 10 volts. Find the magnitude of the charge supplied by the battery to each of the plates connected to it.

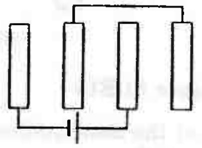


Figure 31-E19

34. The capacitance between the adjacent plates shown in figure (31-E20) is 50 nF. A charge of 1·0 μC is placed on the middle plate. (a) What will be the charge on the outer surface of the upper plate ? (b) Find the potential difference developed between the upper and the middle plates.

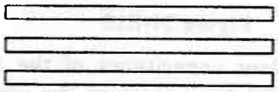


Figure 31-E20

35. Consider the situation of the previous problem. If 1·0 μC is placed on the upper plate instead of the middle, what will be the potential difference between (a) the upper and the middle plates and (b) the middle and the lower plates ?

36. Two capacitors of capacitances 20·0 pF and 50·0 pF are connected in series with a 6·00 V battery. Find (a) the potential difference across each capacitor and (b) the energy stored in each capacitor.

37. Two capacitors of capacitances 4·0 μF and 6·0 μF are connected in series with a battery of 20 V. Find the energy supplied by the battery.

38. Each capacitor in figure (31-E21) has a capacitance of 10 μF. The emf of the battery is 100 V. Find the energy stored in each of the four capacitors.

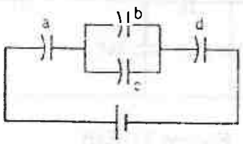


Figure 31-E21

39. A capacitor with stored energy 4·0 J is connected with an identical capacitor with no electric field in between. Find the total energy stored in the two capacitors.

40. A capacitor of capacitance 2·0 μF is charged to a potential difference of 12 V. It is then connected to an uncharged capacitor of capacitance 4·0 μF as shown in figure (31-E22). Find (a) the charge on each of the two capacitors after the connection, (b) the electrostatic energy stored in each of the two capacitors and (c) the heat produced during the charge transfer from one capacitor to the other.

Figure 31-E22

41. A point charge Q is placed at the origin. Find the electrostatic energy stored outside the sphere of radius R centred at the origin.

42. A metal sphere of radius R is charged to a potential V. (a) Find the electrostatic energy stored in the electric field within a concentric sphere of radius $2R$. (b) Show that the electrostatic field energy stored outside the sphere of radius $2R$ equals that stored within it.

43. A large conducting plane has a surface charge density $1{\cdot}0 \times 10^{-4}$ C/m^2. Find the electrostatic energy stored in a cubical volume of edge 1·0 cm in front of the plane.

44. A parallel-plate capacitor having plate area 20 cm^2 and separation between the plates 1·00 mm is connected to a battery of 12·0 V. The plates are pulled apart to increase the separation to 2·0 mm. (a) Calculate the charge flown through the circuit during the process. (b) How much energy is absorbed by the battery during the process ? (c) Calculate the stored energy in the electric field before and after the process. (d) Using the expression for the force between the plates, find the work done by the person pulling the plates apart. (e) Show and justify that no heat is produced during this transfer of charge as the separation is increased.

45. A capacitor having a capacitance of 100 μF is charged to a potential difference of 24 V. The charging battery is disconnected and the capacitor is connected to another battery of emf 12 V with the positive plate of the capacitor joined with the positive terminal of the battery. (a) Find the charges on the capacitor before and after the reconnection. (b) Find the charge flown through the 12 V battery. (c) Is work done by the battery or is it done on the battery ? Find its magnitude. (d) Find the decrease in electrostatic field energy. (e) Find the heat developed during the flow of charge after reconnection.

46. Consider the situation shown in figure (31-E23). The switch S is open for a long time and then closed. (a) Find the charge flown through the battery when the switch S is closed. (b) Find the work done by the battery.

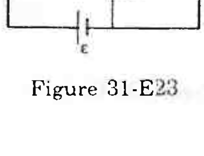


Figure 31-E23

(c) Find the change in energy stored in the capacitors.
(d) Find the heat developed in the system.

47. A capacitor of capacitance 5·00 μF is charged to 24·0 V and another capacitor of capacitance 6·0 μF is charged to 12·0 V. (a) Find the energy stored in each capacitor. (b) The positive plate of the first capacitor is now connected to the negative plate of the second and vice versa. Find the new charges on the capacitors. (c) Find the loss of electrostatic energy during the process. (d) Where does this energy go ?

48. A 5·0 μF capacitor is charged to 12 V. The positive plate of this capacitor is now connected to the negative terminal of a 12 V battery and vice versa. Calculate the heat developed in the connecting wires.

49. The two square faces of a rectangular dielectric slab (dielectric constant 4·0) of dimensions 20 cm × 20 cm × 1·0 mm are metal-coated. Find the capacitance between the coated surfaces.

50. If the above capacitor is connected across a 6·0 V battery, find (a) the charge supplied by the battery, (b) the induced charge on the dielectric and (c) the net charge appearing on one of the coated surfaces.

51. The separation between the plates of a parallel-plate capacitor is 0·500 cm and its plate area is 100 cm^2. A 0·400 cm thick metal plate is inserted into the gap with its faces parallel to the plates. Show that the capacitance of the assembly is independent of the position of the metal plate within the gap and find its value.

52. A capacitor stores 50 μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 100 μC flows through the battery. Find the dielectric constant of the material inserted.

53. A parallel-plate capacitor of capacitance 5 μF is connected to a battery of emf 6 V. The separation between the plates is 2 mm. (a) Find the charge on the positive plate. (b) Find the electric field between the plates. (c) A dielectric slab of thickness 1 mm and dielectric constant 5 is inserted into the gap to occupy the lower half of it. Find the capacitance of the new combination. (d) How much charge has flown through the battery after the slab is inserted ?

54. A parallel-plate capacitor has plate area 100 cm^2 and plate separation 1·0 cm. A glass plate (dielectric constant 6·0) of thickness 6·0 mm and an ebonite plate (dielectric constant 4·0) are inserted one over the other to fill the space between the plates of the capacitor. Find the new capacitance.

55. A parallel-plate capacitor having plate area 400 cm^2 and separation between the plates 1·0 mm is connected to a power supply of 100 V. A dielectric slab of thickness 0·5 mm and dielectric constant 5·0 is inserted into the gap. (a) Find the increase in electrostatic energy. (b) If the power supply is now disconnected and the dielectric slab is taken out, find the further increase in energy. (c) Why does the energy increase in inserting the slab as well as in taking it out ?

56. Find the capacitances of the capacitors shown in figure (31-E24). The plate area is A and the separation between

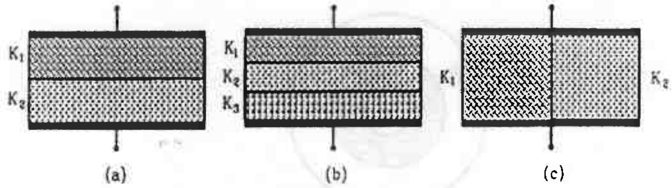


Figure 31-E24

the plates is d. Different dielectric slabs in a particular part of the figure are of the same thickness and the entire gap between the plates is filled with the dielectric slabs.

57. A capacitor is formed by two square metal-plates of edge a, separated by a distance d. Dielectrics of dielectric constants K_1 and K_2 are filled in the gap as shown in figure (31-E25). Find the capacitance.

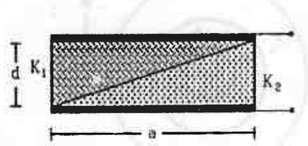


Figure 31-E25

58. Figure (31-E26) shows two identical parallel plate capacitors connected to a battery through a switch S. Initially, the switch is closed so that the capacitors are completely charged. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant 3. Find the ratio of the initial total energy stored in the capacitors to the final total energy stored.

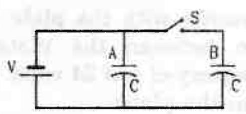


Figure 31-E26

59. A parallel-plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant K is then inserted between the plates of the capacitor so as to fill the space between the plates. Find the work done on the system in the process of inserting the slab.

60. A capacitor having a capacitance of 100 μF is charged to a potential difference of 50 V. (a) What is the magnitude of the charge on each plate ? (b) The charging battery is disconnected and a dielectric of dielectric constant 2·5 is inserted. Calculate the new potential difference between the plates. (c) What charge would have produced this potential difference in absence of the dielectric slab. (d) Find the charge induced at a surface of the dielectric slab.

61. A sphercial capacitor is made of two conducting spherical shells of radii a and b. The space between the shells is filled with a dielectric of dielectric constant K upto a

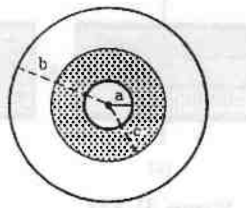


Figure 31-E27

radius c as shown in figure (31-E27). Calculate the capacitance.

62. Consider an assembly of three conducting concentric spherical shells of radii a, b and c as shown in figure (31-E28). Find the capacitance of the assembly between the points A and B.

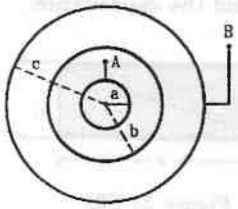


Figure 31-E28

63. Suppose the space between the two inner shells of the previous problem is filled with a dielectric of dielectric constant K. Find the capacitance of the system between A and B.

64. An air-filled parallel-plate capacitor is to be constructed which can store 12 μC of charge when operated at 1200 V. What can be the minimum plate area of the capacitor ? The dielectric strength of air is 3×10^6 V/m.

65. A parallel-plate capacitor with the plate area 100 cm^2 and the separation between the plates 1·0 cm is connected across a battery of emf 24 volts. Find the force of attraction between the plates.

66. Consider the situation shown in figure (31-E29). The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf $\mathcal{E}$. All surfaces are frictionless. Calculate the value of M for which the dielectric slab will stay in equilibrium.

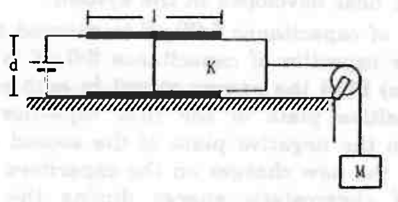


Figure 31-E29

67. Figure (31-E30) shows two parallel plate capacitors with fixed plates and connected to two batteries. The separation between the plates is the same for the two capacitors. The plates are rectangular in shape with width b and lengths l_1 and l_2. The left half of the dielectric slab has a dielectric constant K_1 and the right half K_2. Neglecting any friction, find the ratio of the emf of the left battery to that of the right battery for which the dielectric slab may remain in equilibrium.

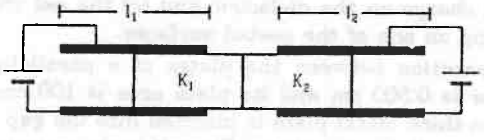


Figure 31-E30

68. Consider the situation shown in figure (31-E31). The plates of the capacitor have plate area A and are clamped in the laboratory. The dielectric slab is released from rest with a length a inside the capacitor. Neglecting any effect of friction or gravity, show that the slab will execute periodic motion and find its time period.

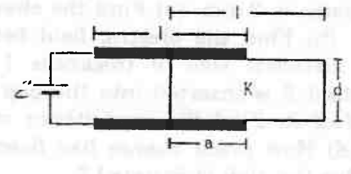


Figure 31-E31

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (c)	4. (b)	5. (c)	6. (d)
7. (b)	8. (d)	9. (c)	10. (a)	11. (b)	12. (d)

OBJECTIVE II

1. (c)	2. (b)	3. (d)	4. (a), (c), (d)	5. (b), (c)
6. (d)	7. (b), (c), (d)			

EXERCISES

1. $1·6 \times 10^{-8}$ F
2. $6·95 \times 10^{-5}$ μF
3. 6 km
4. $1·33 \times 10^{-10}$ C, $8·0 \times 10^{-10}$ J
5. (a) $1·33 \times 10^{-10}$ C (b) $1·33 \times 10^{-10}$ C

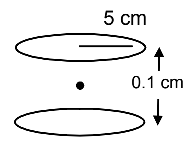
6. 24 μC, 48 μC, 72 μC
7. 110 μC on each, $1\cdot33 \times 10^{-3}$ J
8. 48 μC on the 8 μF capacitor and 24 μC on each of the 4 μF capacitors
9. (a) 5 μF (b) 10 μF
10. 110 μC
11. 44 μC
12. $4\pi\varepsilon_0 R_1, 4\pi\varepsilon_0 R_2, 4\pi\varepsilon_0 (R_1 + R_2)$
13. 2 μF, 20 V
15. (a) 50/3 μV at each point (b) zero
16. $C_3 + \dfrac{2C_1C_2}{C_1 + C_2}$
17. $\dfrac{\varepsilon_0 A(3d^2 + 6bd + 2b^2)}{3d(d+b)(d+2b)}$
18. (a) 8 pF (b) same as in (a)
19. 20 V
20. $3\cdot3 \times 10^{-4}$ C
21. 43 mV
22. $\left(\dfrac{Vea^2}{md_1(d_1 + d_2)}\right)^{1/2}$
23. $1\cdot08 \times 10^{-8}$ cm
24. 2·25 μF
25. (a) $\dfrac{12}{11}$ V (b) − 8 V (c) zero (d) − 10·3 V
26. (a) $\dfrac{11}{6}$ μF (b) $\dfrac{11}{4}$ μF (c) 8 μF (d) 8 μF
27. 1 μF
28. 2 μF
29. 4 μF
30. 12·5 V
31. 1 V
32. 5 V
33. 0·16 μC
34. (a) 0·50 μC (b) 10 V
35. (a) 10 V (b) 10 V
36. (a) 1·71 V, 4·29 V (b) 184 pJ, 73·5 pJ
37. 960 μJ
38. 8 mJ in (a) and (d), 2 mJ in (b) and (c)
39. 2 0 J
40. (a) 8 μC, 16 μC (b) 16 μJ, 32 μJ, (c) 96 μJ
41. $\dfrac{Q^2}{8\pi\varepsilon_0 R}$
42. (a) $\pi\varepsilon_0 RV^2$
43. $5\cdot6 \times 10^{-4}$ J
44. (a) $1\cdot06 \times 10^{-10}$ C (b) $12\cdot7 \times 10^{-10}$ J (c) $12\cdot7 \times 10^{-10}$ J, $6\cdot35 \times 10^{-10}$ J (d) $6\cdot35 \times 10^{-10}$ J
45. (a) 2400 μC, 1200 μC (b) 1200 μC (c) 14·4 mJ (d) 21·6 mJ (e) 7·2 mJ
46. (a) $C\mathcal{E}/2$, (b) $C\mathcal{E}^2/2$ (c) $C\mathcal{E}^2/4$ (d) $C\mathcal{E}^2/4$
47. (a) 1·44 mJ, 0·432 mJ (b) 21·8 μC, 26·2 μC, (c) 1·77 mJ
48. 1·44 mJ
49. 1·42 nF
50. (a) 8·5 nC (b) 6·4 nC (c) 2·1 nC
51. 88 pF
52. 3
53. (a) 30 μC (b) 3×10^3 V/m (c) 8·3 μF (d) 20 μC
54. 44 pF
55. (a) 7·1 μJ (b) 35·4 μJ
56. (a) $\dfrac{2K_1K_2\varepsilon_0 A}{d(K_1 + K_2)}$ (b) $\dfrac{3\varepsilon_0 A K_1K_2K_3}{d(K_1K_2 + K_2K_3 + K_3K_1)}$ (c) $\dfrac{\varepsilon_0 A}{2d}(K_1 + K_2)$
57. $\dfrac{\varepsilon_0 K_1K_2a^2 \ln \dfrac{K_1}{K_2}}{(K_1 - K_2)d}$
58. 3 : 5
59. $\dfrac{\varepsilon_0 AV^2}{2d}\left(\dfrac{1}{K} - 1\right)$
60. (a) 5 mC (b) 20 V (c) 2 mC (d) 3 mC
61. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(b-c) + b(c-a)}$
62. $\dfrac{4\pi\varepsilon_0 ac}{c - a}$
63. $\dfrac{4\pi\varepsilon_0 Kabc}{Ka(c-b) + c(b-a)}$
64. 0·45 m^2
65. $2\cdot5 \times 10^{-7}$ N
66. $\dfrac{\varepsilon_0 b\mathcal{E}^2(K-1)}{2dg}$
67. $\sqrt{\dfrac{K_2 - 1}{K_1 - 1}}$
68. $8\sqrt{\dfrac{(l-a)lmd}{\varepsilon_0 A\mathcal{E}^2(K-1)}}$

□

CHAPTER – 31
CAPACITOR

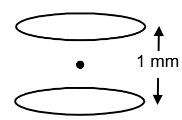
1. Given that
   Number of electron = 1×10^{12}
   Net charge Q = $1 \times 10^{12} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-7}$ C
   $\therefore$ The net potential difference = 10 L.
   $\therefore$ Capacitance – C = $\frac{q}{v} = \frac{1.6 \times 10^{-7}}{10} = 1.6 \times 10^{-8}$ F.

2. $A = \pi r^2 = 25\ \pi cm^2$
   d = 0.1 cm



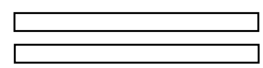
   $c = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 25 \times 3.14}{0.1} = 6.95 \times 10^{-5}\ \mu F$.

3. Let the radius of the disc = R
   $\therefore$Area = πR^2
   C = 1f
   D = 1 mm = 10^{-3} m



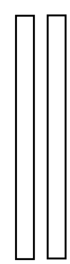
   $\therefore C = \frac{\varepsilon_0 A}{d}$
   $\Rightarrow 1 = \frac{8.85 \times 10^{-12} \times \pi r^2}{10^{-3}} \Rightarrow r^2 = \frac{10^{-3} \times 10^{12}}{8.85 \times \pi} = \frac{10^9}{27.784} = 5998.5$ m = 6 Km

4. A = 25 cm^2 = 2.5×10^{-3} cm^2
   d = 1 mm = 0.01 m
   V = 6V   Q = ?



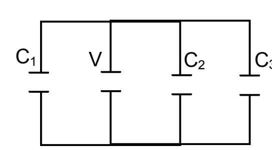
   $C = \frac{\varepsilon_0 A}{d} = \frac{8.854 \times 10^{-12} \times 2.5 \times 10^{-3}}{0.01}$
   $Q = CV = \frac{8.854 \times 10^{-12} \times 2.5 \times 10^{-3}}{0.01} \times 6 = 1.32810 \times 10^{-10}$ C
   $W = Q \times V = 1.32810 \times 10^{-10} \times 6 = 8 \times 10^{-10}$ J.

5. Plate area A = 25 cm^2 = 2.5×10^{-3} m
   Separation d = 2 mm = 2×10^{-3} m
   Potential v = 12 v

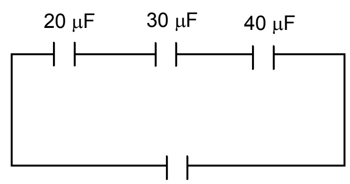


   (a) We know $C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 2.5 \times 10^{-3}}{2 \times 10^{-3}} = 11.06 \times 10^{-12}$ F
   $C = \frac{q}{v} \Rightarrow 11.06 \times 10^{-12} = \frac{q}{12}$
   $\Rightarrow q_1 = 1.32 \times 10^{-10}$ C.
   (b) Then d = decreased to 1 mm
   $\therefore$ d = 1 mm = 1×10^{-3} m
   $C = \frac{\varepsilon_0 A}{d} = \frac{q}{v} = \frac{8.85 \times 10^{-12} \times 2.5 \times 10^{-3}}{1 \times 10^{-3}} = \frac{2}{12}$
   $\Rightarrow q_2 = 8.85 \times 2.5 \times 12 \times 10^{-12} = 2.65 \times 10^{-10}$ C.
   $\therefore$ The extra charge given to plate = $(2.65 - 1.32) \times 10^{-10} = 1.33 \times 10^{-10}$ C.

6. $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$,
   $C_3 = 6\ \mu F$ V = 12 V



   $cq = C_1 + C_2 + C_3 = 2 + 4 + 6 = 12\ \mu F = 12 \times 10^{-6}$ F
   $q_1 = 12 \times 2 = 24\ \mu C$, $q_2 = 12 \times 4 = 48\ \mu C$, $q_3 = 12 \times 6 = 72\ \mu C$

7. 

$\therefore$ The equivalent capacity.

$$C = \frac{C_1C_2C_3}{C_2C_3 + C_1C_3 + C_1C_2} = \frac{20 \times 30 \times 40}{30 \times 40 + 20 \times 40 + 20 \times 30} = \frac{24000}{2600} = 9.23\ \mu F$$

(a) Let Equivalent charge at the capacitor = q

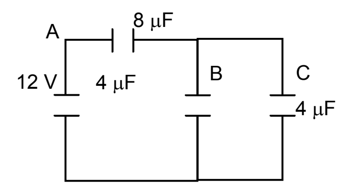
$C = \frac{q}{V} \Rightarrow q = C \times V = 9.23 \times 12 = 110\ \mu C$ on each.

As this is a series combination, the charge on each capacitor is same as the equivalent charge which is 110 μC.

(b) Let the work done by the battery = W

$\therefore V = \frac{W}{q} \Rightarrow W = Vq = 110 \times 12 \times 10^{-6} = 1.33 \times 10^{-3}$ J.

8. $C_1 = 8\ \mu F$, $C_2 = 4\ \mu F$, $C_3 = 4\ \mu F$



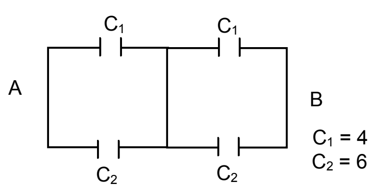
$$Ceq = \frac{(C_2 + C_3) \times C_1}{C_1 + C_2 + C_3}$$

$$= \frac{8 \times 8}{16} = 4\ \mu F$$

Since B & C are parallel & are in series with A

So, $q_1 = 8 \times 6 = 48\ \mu C$ $q_2 = 4 \times 6 = 24\ \mu C$ q3 = 4 × 6 = 24 μC

9. (a)



$\therefore C_1, C_1$ are series & C_2, C_2 are series as the V is same at p & q. So no current pass through p & q.

$$\frac{1}{C} = \frac{1}{C_1} = \frac{1}{C_2} \Rightarrow \frac{1}{C} = \frac{1+1}{C_1C_2}$$

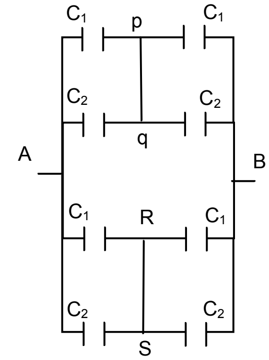
$$C_p = \frac{C_1}{2} = \frac{4}{2} = 2\ \mu F$$

And $C_q = \frac{C_2}{2} = \frac{6}{2} = 3\ \mu F$

$\therefore C = C_p + C_q = 2 + 3 = 5\ \mu F$

(b) $C_1 = 4\ \mu F$, $C_2 = 6\ \mu F$,

In case of p & q, q = 0



$\therefore C_p = \frac{C_1}{2} = \frac{4}{2} = 2\ \mu F$

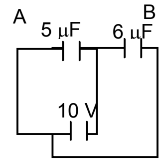
$C_q = \frac{C_2}{2} = \frac{6}{2} = 3\ \mu F$

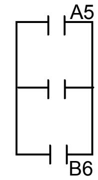
& $C' = 2 + 3 = 5\ \mu F$

C & $C' = 5\ \mu F$

$\therefore$ The equation of capacitor $C = C' + C'' = 5 + 5 = 10\ \mu F$

10. $V = 10$ v



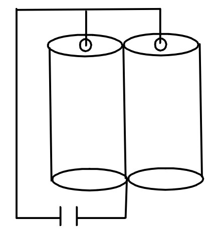


$C_{eq} = C_1 + C_2$ [$\therefore$ They are parallel]

$= 5 + 6 = 11\ \mu F$

$q = CV = 11 \times 10\ 110\ \mu C$

11. The capacitance of the outer sphere = 2.2 μF



$C = 2.2\ \mu F$

Potential, $V = 10$ v

Let the charge given to individual cylinder = q.

$C = \dfrac{q}{V}$

$\Rightarrow q = CV = 2.2 \times 10 = 22\ \mu F$

$\therefore$ The total charge given to the inner cylinder = 22 + 22 = 44 μF

12. $C = \dfrac{q}{V}$, Now $V = \dfrac{Kq}{R}$

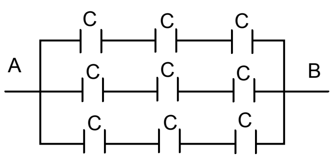
So, $C_1 = \dfrac{q}{(Kq/R_1)} = \dfrac{R_1}{K} = 4\ \pi\varepsilon_0 R_1$

Similarly $c_2 = 4\ \pi\varepsilon_0 R_2$

The combination is necessarily parallel.

Hence $C_{eq} = 4\ \pi\varepsilon_0 R_1 + 4\ \pi\varepsilon_0 R_2 = 4\ \pi\varepsilon_0 (R_1 + R_2)$

13.



$\therefore C = 2\ \mu F$

$\therefore$ In this system the capacitance are arranged in series. Then the capacitance is parallel to each other.

(a) $\therefore$ The equation of capacitance in one row

$C = \dfrac{C}{3}$

(b) and three capacitance of capacity $\dfrac{C}{3}$ are connected in parallel

$\therefore$ The equation of capacitance

$C = \dfrac{C}{3} + \dfrac{C}{3} + \dfrac{C}{3} = C = 2\ \mu F$

As the volt capacitance on each row are same and the individual is

$= \dfrac{\text{Total}}{\text{No. of capacitance}} = \dfrac{60}{3} = 20\ V$

14. Let there are 'x' no of capacitors in series ie in a row

So, $x \times 50 = 200$

$\Rightarrow x = 4$ capacitors.

Effective capacitance in a row = $\dfrac{10}{4}$

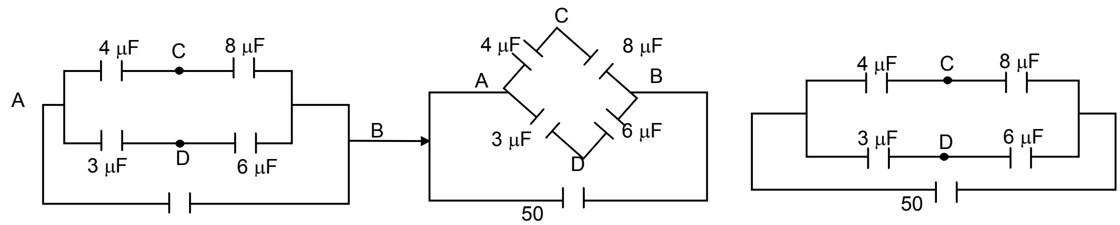
Now, let there are 'y' such rows,

So, $\dfrac{10}{4} \times y = 10$

$\Rightarrow y = 4$ capacitor.

So, the combinations of four rows each of 4 capacitors.

15.



(a) Capacitor = $\frac{4 \times 8}{4 + 8} = \frac{8}{3}$ μ

and $\frac{6 \times 3}{6 + 3}$ = 2 μF

(i) The charge on the capacitance $\frac{8}{3}$ μF

$\therefore$ Q = $\frac{8}{3}$ × 50 = $\frac{400}{3}$

$\therefore$ The potential at 4 μF = $\frac{400}{3 \times 4} = \frac{100}{3}$

at 8 μF = $\frac{400}{3 \times 8} = \frac{100}{6}$

The Potential difference = $\frac{100}{3} - \frac{100}{6} = \frac{50}{3}$ μV

(ii) Hence the effective charge at 2 μF = 50 × 2 = 100 μF

$\therefore$ Potential at 3 μF = $\frac{100}{3}$; Potential at 6 μF = $\frac{100}{6}$

$\therefore$ Difference = $\frac{100}{3} - \frac{100}{6} = \frac{50}{3}$ μV

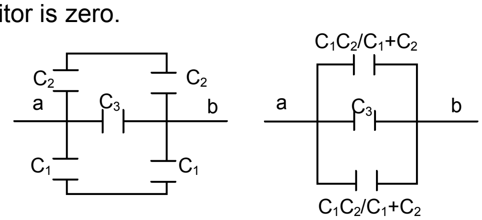
$\therefore$ The potential at C & D is $\frac{50}{3}$ μV

(b) $\therefore$ $\frac{P}{q} = \frac{R}{S} = \frac{1}{2} = \frac{1}{2}$ = It is balanced. So from it is cleared that the wheat star bridge balanced. So the potential at the point C & D are same. So no current flow through the point C & D. So if we connect another capacitor at the point C & D the charge on the capacitor is zero.

16. Ceq between a & b

$= \frac{C_1C_2}{C_1 + C_2} + C_3 + \frac{C_1C_2}{C_1 + C_2}$

$= C_3 + \frac{2C_1C_2}{C_1 + C_2}$ ($\therefore$ The three are parallel)



17. In the figure the three capacitors are arranged in parallel.

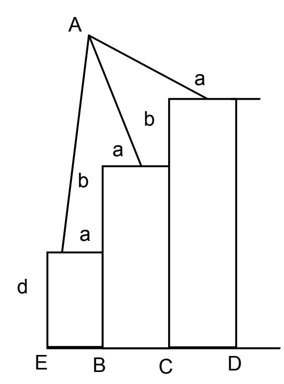
All have same surface area = a = $\frac{A}{3}$

First capacitance $C_1 = \frac{\varepsilon_0 A}{3d}$

2[nd] capacitance $C_2 = \frac{\varepsilon_0 A}{3(b + d)}$

3[rd] capacitance $C_3 = \frac{\varepsilon_0 A}{3(2b + d)}$

Ceq = $C_1 + C_2 + C_3$



$= \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{3(b+d)} + \frac{\varepsilon_0 A}{3(2b+d)} = \frac{\varepsilon_0 A}{3}\left(\frac{1}{d} + \frac{1}{b+d} + \frac{1}{2b+d}\right)$

$= \frac{\varepsilon_0 A}{3}\left(\frac{(b+d)(2b+d)+(2b+d)d+(b+d)d}{d(b+d)(2b+d)}\right)$

$= \frac{\varepsilon_0 A\left(3d^2+6bd+2b^2\right)}{3d(b+d)(2b+d)}$

18. (a) $C = \frac{2\varepsilon_0 L}{\ln(R_2/R_1)} = \frac{e \times 3.14 \times 8.85 \times 10^{-2} \times 10^{-1}}{\ln 2}$ [ln2 = 0.6932]

$= 80.17 \times 10^{-13} \Rightarrow 8$ PF

(b) Same as R_2/R_1 will be same.

19. Given that

$C = 100$ PF $= 100 \times 10^{-12}$ F $C_{cq} = 20$ PF $= 20 \times 10^{-12}$ F

$V = 24$ V $q = 24 \times 100 \times 10^{-12} = 24 \times 10^{-10}$

$q_2 = ?$

Let q_1 = The new charge 100 PF V_1 = The Voltage.

Let the new potential is V_1

After the flow of charge, potential is same in the two capacitor

$V_1 = \frac{q_2}{C_2} = \frac{q_1}{C_1}$

$= \frac{q - q_1}{C_2} = \frac{q_1}{C_1}$

$= \frac{24 \times 10^{-10} - q_1}{24 \times 10^{-12}} = \frac{q_1}{100 \times 10^{-12}}$

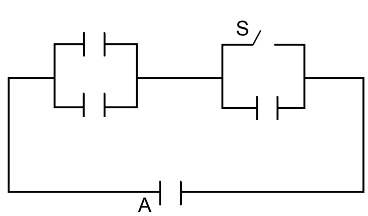
$= 24 \times 10^{-10} - q_1 = \frac{q_1}{5}$

$= 6q_1 = 120 \times 10^{-10}$

$= q_1 = \frac{120}{6} \times 10^{-10} = 20 \times 10^{-10}$

$\therefore V_1 = \frac{q_1}{C_1} = \frac{20 \times 10^{-10}}{100 \times 10^{-12}} = 20$ V

20.



Initially when 's' is not connected,

$C_{eff} = \frac{2C}{3} q = \frac{2C}{3} \times 50 = \frac{5}{2} \times 10^{-4} = 1.66 \times 10^{-4}$ C

After the switch is made on,

Then $C_{eff} = 2C = 10^{-5}$

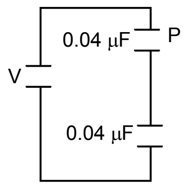
$Q = 10^{-5} \times 50 = 5 \times 10^{-4}$

Now, the initial charge will remain stored in the stored in the short capacitor

Hence net charge flowing

$= 5 \times 10^{-4} - 1.66 \times 10^{-4} = 3.3 \times 10^{-4}$ C.

21.



Given that mass of particle m = 10 mg

Charge 1 = – 0.01 μC

A = 100 cm^2 Let potential = V

The Equation capacitance C = $\frac{0.04}{2}$ = 0.02 μF

The particle may be in equilibrium, so that the wt. of the particle acting down ward, must be balanced by the electric force acting up ward.

$\therefore$ qE = Mg

Electric force = qE = $q\frac{V}{d}$ where V – Potential, d – separation of both the plates.

$= q\frac{VC}{\varepsilon_0 A}$ $C = \frac{\varepsilon_0 A}{q}$ $d = \frac{\varepsilon_0 A}{C}$

qE = mg

$= \frac{QVC}{\varepsilon_0 A}$ = mg

$= \frac{0.01 \times 0.02 \times V}{8.85 \times 10^{-12} \times 100}$ = 0.1 × 980

$\Rightarrow V = \frac{0.1 \times 980 \times 8.85 \times 10^{-10}}{0.0002}$ = 0.00043 = 43 MV

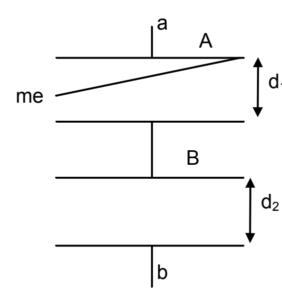
22. Let mass of electron = μ

Charge electron = e

We know, 'q'

For a charged particle to be projected in side to plates of a parallel plate capacitor with electric field E,

$y = \frac{1qE}{2m}\left(\frac{x}{\mu}\right)^2$



where y – Vertical distance covered or

x – Horizontal distance covered

μ – Initial velocity

From the given data,

$y = \frac{d_1}{2}$, $E = \frac{V}{R} = \frac{qd_1}{\varepsilon_0 a^2 \times d_1} = \frac{q}{\varepsilon_0 a^2}$, x = a, μ = ?

For capacitor A –

$V_1 = \frac{q}{C_1} = \frac{qd_1}{\varepsilon_0 a^2}$ as $C_1 = \frac{\varepsilon_0 a^2}{d_1}$

Here q = chare on capacitor.

q = C × V where C = Equivalent capacitance of the total arrangement = $\frac{\varepsilon_0 a^2}{d_1 + d_2}$

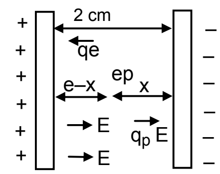
So, $q = \frac{\varepsilon_0 a^2}{d_1 + d_2} \times V$

Hence $E = \frac{q}{\varepsilon_0 a^2} = \frac{\varepsilon_0 a^2 \times V}{(d_1+d_2)\varepsilon_0 a^2} = \frac{V}{(d_1+d_2)}$

Substituting the data in the known equation, we get, $\frac{d_1}{2} = \frac{1}{2} \times \frac{e \times V}{(d_1+d_2)m} \times \frac{a^2}{u^2}$

$\Rightarrow u^2 = \frac{Vea^2}{d_1 m(d_1+d_2)} \Rightarrow u = \left(\frac{Vea^2}{d_1 m(d_1+d_2)}\right)^{1/2}$

23. The acceleration of electron $a_e = \frac{qeme}{Me}$



The acceleration of proton = $\frac{qpe}{Mp}$ = ap

The distance travelled by proton $X = \frac{1}{2}apt^2$ …(1)

The distance travelled by electron …(2)

From (1) and (2) $\Rightarrow 2 - X = \frac{1}{2}a_c t^2 \qquad x = \frac{1}{2}a_c t^2$

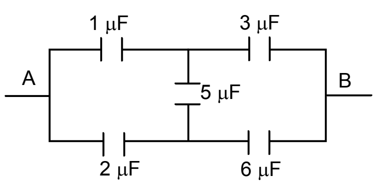
$$\Rightarrow \frac{x}{2-x} = \frac{a_p}{a_c} = \frac{\left(\frac{q_p E}{M_p}\right)}{\left(\frac{q_c F}{M_c}\right)}$$

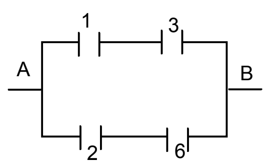
$$= \frac{x}{2-x} = \frac{M_c}{M_p} = \frac{9.1 \times 10^{-31}}{1.67 \times 10^{-27}} = \frac{9.1}{1.67} \times 10^{-4} = 5.449 \times 10^{-4}$$

$\Rightarrow x = 10.898 \times 10^{-4} - 5.449 \times 10^{-4}x$

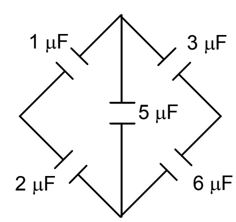
$\Rightarrow x = \frac{10.898 \times 10^{-4}}{1.0005449} = 0.001089226$

24. (a)



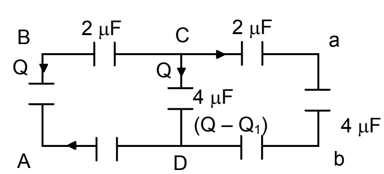


As the bridge in balanced there is no current through the 5 μF capacitor
So, it reduces to
similar in the case of (b) & (c)
as 'b' can also be written as.



$Ceq = \frac{1 \times 3}{1+3} + \frac{2 \times 6}{2+6} = \frac{3}{48} + \frac{12}{8} = \frac{6+12}{8} = 2.25\ \mu F$

25. (a) By loop method application in the closed circuit ABCabDA



$-12 + \frac{2Q}{2\mu F} + \frac{Q_1}{2\mu F} + \frac{Q_1}{4\mu F} = 0$ …(1)

In the close circuit ABCDA

$-12 + \frac{Q}{2\mu F} + \frac{Q+Q_1}{4\mu F} = 0$ …(2)

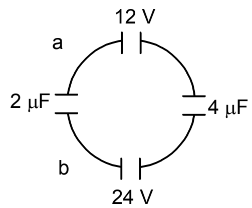
From (1) and (2) $2Q + 3Q_1 = 48$ …(3)

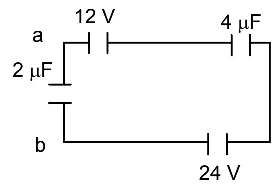
And $3Q - q_1 = 48$ and subtracting $Q = 4Q_1$, and substitution in equation

$2Q + 3Q_1 = 48 \Rightarrow 8\,Q_1 + 3Q_1 = 48 \Rightarrow 11Q_1 = 48,\ q_1 = \dfrac{48}{11}$

$Vab = \dfrac{Q_1}{4\mu F} = \dfrac{48}{11\times 4} = \dfrac{12}{11}$ V

(b)



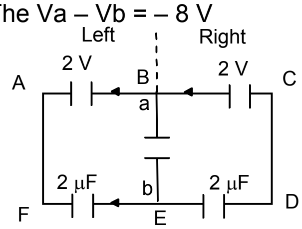


The potential = 24 – 12 = 12

Potential difference V = $\dfrac{(2\times 0 + 12\times 4)}{2+4} = \dfrac{48}{6}$ = 8 V

$\therefore$ The Va – Vb = – 8 V

(c)

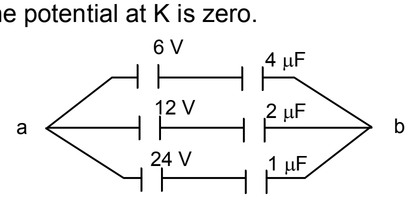


From the figure it is cleared that the left and right branch are symmetry and reversed, so the current go towards BE from BAFEB same as the current from EDCBE.

$\therefore$The net charge Q = 0 $\quad\quad$ $\therefore$ $V = \dfrac{Q}{C} = \dfrac{0}{C} = 0$ $\quad\quad$ $\therefore$Vab = 0

$\therefore$ The potential at K is zero.

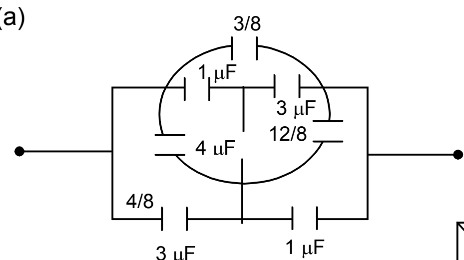
(d)

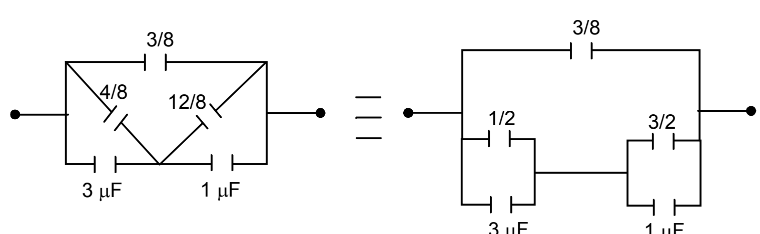


The net potential = $\dfrac{\text{Net charge}}{\text{Net capacitance}} = \dfrac{24+24+24}{7} = \dfrac{72}{7}$ 10.3 V

$\therefore$Va – Vb = – 10.3 V

26. (a)

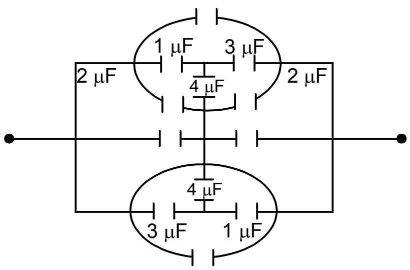




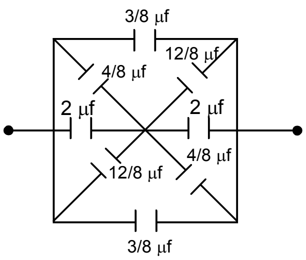
By star Delta conversion

$$C_{eff} = \frac{3}{8} + \left[\frac{\left(3+\frac{1}{2}\right)\times\left(\frac{3}{2}+1\right)}{\left(3+\frac{1}{2}\right)+\left(\frac{3}{2}+1\right)}\right] = \frac{3}{8}+\frac{35}{24} = \frac{9+35}{24} = \frac{11}{6}\ \mu F$$

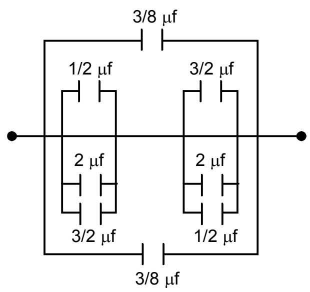
(b)



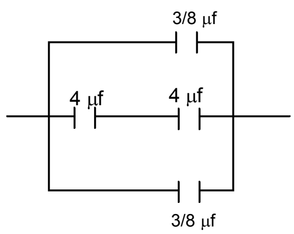
by star Delta convensor



$\equiv$

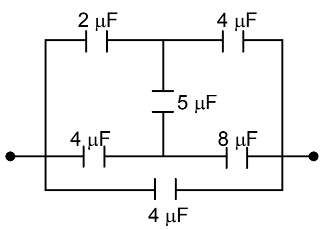


$\equiv$

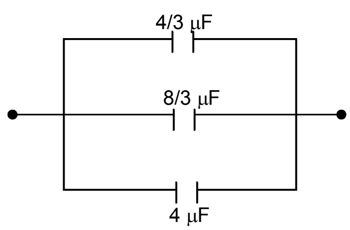


$$= \frac{3}{8}+\frac{16}{8}+\frac{3}{8} = \frac{11}{4}\ \mu f$$

(c)

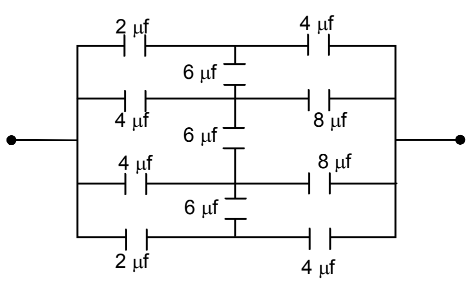


$\equiv$

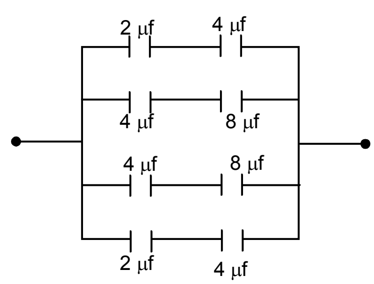


$$\text{Cef} = \frac{4}{3}+\frac{8}{3}+4 = 8\ \mu F$$

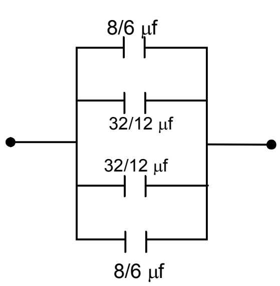
(d)



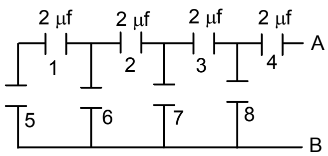
$\equiv$



$\equiv$



$$\text{Cef} = \frac{3}{8}+\frac{32}{12}+\frac{32}{12}+\frac{8}{6} = \frac{16+32}{6} = 8\ \mu f$$

27.


= C_5 and C_1 are in series

$C_{eq} = \frac{2 \times 2}{2+2} = 1$

This is parallel to $C_6 = 1 + 1 = 2$

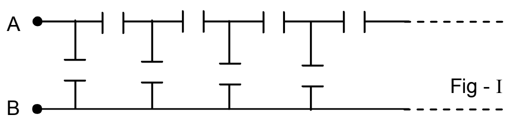
Which is series to $C_2 = \frac{2 \times 2}{2+2} = 1$

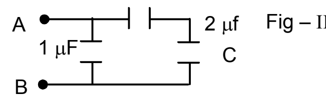
Which is parallel to $C_7 = 1 + 1 = 2$

Which is series to $C_3 = \frac{2 \times 2}{2+2} = 1$

Which is parallel to $C_8 = 1 + 1 = 2$

This is series to $C_4 = \frac{2 \times 2}{2+2} = 1$

28.




Let the equivalent capacitance be C. Since it is an infinite series. So, there will be negligible change if the arrangement is done an in Fig – II

$C_{eq} = \frac{2 \times C}{2+C} + 1 \Rightarrow C = \frac{2C+2+C}{2+C}$

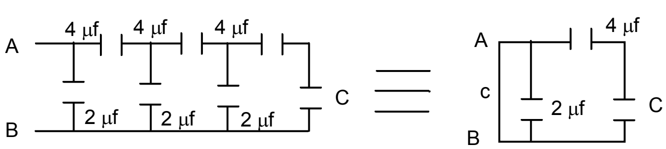
$\Rightarrow (2 + C) \times C = 3C + 2$

$\Rightarrow C^2 - C - 2 = 0$

$\Rightarrow (C - 2)(C + 1) = 0$

$C = -1$ (Impossible)

So, $C = 2\ \mu F$

29.


= C and 4 μf are in series

So, $C_1 = \frac{4 \times C}{4+C}$

Then C_1 and 2 μf are parallel

$C = C_1 + 2\ \mu f$

$\Rightarrow \frac{4 \times C}{4+C} + 2 \Rightarrow \frac{4C+8+2C}{4+C} = C$

$\Rightarrow 4C + 8 + 2C = 4C + C^2 = C^2 - 2C - 8 = 0$

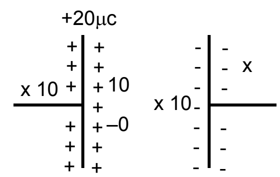
$C = \frac{2 \pm \sqrt{4+4 \times 1 \times 8}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$

$C = \frac{2+6}{2} = 4\ \mu f$

$\therefore$ The value of C is 4 μf

30. $q_1 = +2.0 \times 10^{-8}$ c $\qquad$ $q_2 = -1.0 \times 10^{-8}$ c
$C = 1.2 \times 10^{-3}\ \mu F = 1.2 \times 10^{-9}$ F
$$\text{net } q = \frac{q_1 - q_2}{2} = \frac{3.0 \times 10^{-8}}{2}$$
$$V = \frac{q}{c} = \frac{3 \times 10^{-8}}{2} \times \frac{1}{1.2 \times 10^{-9}} = 12.5 \text{ V}$$

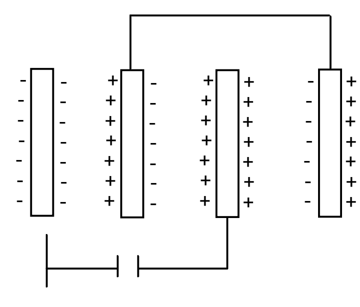
31. $\therefore$ Given that
Capacitance = 10 μF
Charge = 20 μc



$$\therefore \text{The effective charge} = \frac{20 - 0}{2} = 10\ \mu F$$
$$\therefore C = \frac{q}{V} \Rightarrow V = \frac{q}{C} = \frac{10}{10} = 1 \text{ V}$$

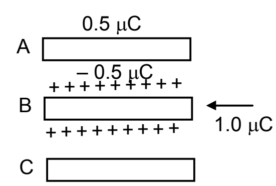
32. $q_1 = 1\ \mu C = 1 \times 10^{-6}$ C $\qquad$ $C = 0.1\ \mu F = 1 \times 10^{-7}$F
$q_2 = 2\ \mu C = 2 \times 10^{-6}$ C
$$\text{net } q = \frac{q_1 - q_2}{2} = \frac{(1-2) \times 10^{-6}}{2} = -0.5 \times 10^{-6} \text{ C}$$
$$\text{Potential 'V'} = \frac{q}{c} = \frac{1 \times 10^{-7}}{-5 \times 10^{-7}} = -5 \text{ V}$$
But potential can never be (–)ve. So, V = 5 V

33. Here three capacitors are formed
And each of



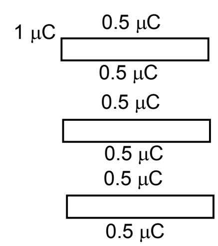
$$A = \frac{96}{\varepsilon_0} \times 10^{-12} \text{ f.m.}$$
$d = 4$ mm $= 4 \times 10^{-3}$ m
$\therefore$Capacitance of a capacitor
$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \dfrac{96 \times 10^{-12}}{\varepsilon_0}}{4 \times 10^{-3}} = 24 \times 10^{-9} \text{ F.}$$
$\therefore$ As three capacitor are arranged is series
$$\text{So, Ceq} = \frac{C}{q} = \frac{24 \times 10^{-9}}{3} = 8 \times 10^{-9}$$
$\therefore$ The total charge to a capacitor $= 8 \times 10^{-9} \times 10 = 8 \times 10^{-8}$ c
$\therefore$ The charge of a single Plate $= 2 \times 8 \times 10^{-8} = 16 \times 10^{-8} = 0.16 \times 10^{-6} = 0.16\ \mu$c.

34. (a) When charge of 1 μc is introduced to the B plate, we also get 0.5 μc charge on the upper surface of Plate 'A'.



(b) Given $C = 50\ \mu F = 50 \times 10^{-9}$ F $= 5 \times 10^{-8}$ F
Now charge $= 0.5 \times 10^{-6}$ C
$$V = \frac{q}{C} = \frac{5 \times 10^{-7} \text{C}}{5 \times 10^{-8} \text{F}} = 10 \text{ V}$$

35. Here given,
Capacitance of each capacitor, C = 50 μf = 0.05 μf



Charge Q = 1 μF which is given to upper plate = 0.5 μc charge appear on outer and inner side of upper plate and 0.5 μc of charge also see on the middle.
(a) Charge of each plate = 0.5 μc
Capacitance = 0.5 μf

$\therefore$ C = $\frac{q}{V}$ $\therefore$ V = $\frac{q}{C}$ = $\frac{0.5}{0.05}$ = 10 v

(b) The charge on lower plate also = 0.5 μc

Capacitance = 0.5 μF

$\therefore$ C = $\frac{q}{V}$ $\Rightarrow$ V = $\frac{q}{C}$ = $\frac{0.5}{0.05}$ = 10 V

$\therefore$ The potential in 10 V

36. C_1 = 20 PF = 20 × 10^{-12} F, C_2 = 50 PF = 50 × 10^{-12} F

Effective C = $\frac{C_1C_2}{C_1+C_2}$ = $\frac{2\times10^{-11}\times5\times10^{-11}}{2\times10^{-11}+5\times10^{-11}}$ = 1.428 × 10^{-11} F

Charge 'q' = 1.428 × 10^{-11} × 6 = 8.568 × 10^{-11} C

$V_1 = \frac{q}{C_1} = \frac{8.568\times10^{-11}}{2\times10^{-11}}$ = 4.284 V

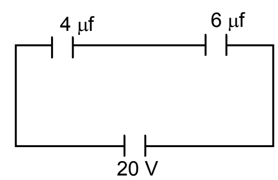
$V_2 = \frac{q}{C_2} = \frac{8.568\times10^{-11}}{5\times10^{-11}}$ = 1.71 V

Energy stored in each capacitor

E_1 = (1/2) $C_1V_1^2$ = (1/2) × 2 × 10^{-11} × $(4.284)^2$ = 18.35 × 10^{-11} ≈ 184 PJ

E_2 = (1/2) $C_2V_2^2$ = (1/2) × 5 × 10^{-11} × $(1.71)^2$ = 7.35 × 10^{-11} ≈ 73.5 PJ

37. $\therefore C_1$ = 4 μF, C_2 = 6 μF, V = 20 V



Eq. capacitor C_{eq} = $\frac{C_1C_2}{C_1+C_2}$ = $\frac{4\times6}{4+6}$ = 2.4

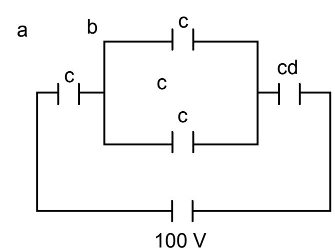
$\therefore$The Eq Capacitance C_{eq} = 2.5 μF

$\therefore$ The energy supplied by the battery to each plate

E = (1/2) CV^2 = (1/2) × 2.4 × 20^2 = 480 μJ

$\therefore$ The energy supplies by the battery to capacitor = 2 × 480 = 960 μJ

38. C = 10 μF = 10 × 10^{-6} F



For a & d

q = 4 × 10^{-4} C

c = 10^{-5} F

E = $\frac{1}{2}\frac{q^2}{c}$ = $\frac{1}{2}\frac{(4\times10^{-4})^2}{10^{-5}}$ = 8 × 10^{-3} J = 8 mJ

For b & c

q = 4 × 10^{-4} c

C_{eq} = 2c = 2 × 10^{-5} F

V = $\frac{4\times10^{-4}}{2\times10^{-5}}$ = 20 V

E = (1/2) cv^2 = (1/2) × 10^{-5} × $(20)^2$ = 2 × 10^{-3} J = 2 mJ

39. Stored energy of capacitor C_1 = 4.0 J

= $\frac{1}{2}\frac{q^2}{c^2}$ = 4.0 J

When then connected, the charge shared

$\frac{1}{2}\frac{q_1^2}{c^2} = \frac{1}{2}\frac{q_2^2}{c^2}$ $\Rightarrow$ $q_1 = q_2$

So that the energy should divided.

$\therefore$ The total energy stored in the two capacitors each is 2 J.

40. Initial charge stored = C × V = 12 × 2 × 10^{-6} = 24 × 10^{-6} c
Let the charges on 2 & 4 capacitors be q_1 & q_2 respectively

There, $V = \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{2} = \frac{q_2}{4} \Rightarrow q_2 = 2q_1$.

or $q_1 + q_2 = 24 \times 10^{-6}$ C

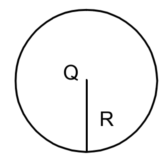
$\Rightarrow q_1 = 8 \times 10^{-6}\ \mu c$

$q_2 = 2q_1 = 2 \times 8 \times 10^{-6} = 16 \times 10^{-6}\ \mu c$

$E_1 = (1/2) \times C_1 \times V_1^2 = (1/2) \times 2 \times \left(\frac{8}{2}\right)^2 = 16\ \mu J$

$E_2 = (1/2) \times C_2 \times V_2^2 = (1/2) \times 4 \times \left(\frac{8}{4}\right)^2 = 8\ \mu J$

41. Charge = Q
Radius of sphere = R
$\therefore$ Capacitance of the sphere = C = $4\pi\varepsilon_0 R$



Energy = $\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}$

42. Q = CV = $4\pi\varepsilon_0 R \times V$

$E = \frac{1}{2}\frac{q^2}{C}$ [$\therefore$ 'C' in a spherical shell = $4\ \pi\varepsilon_0 R$]

$E = \frac{1}{2}\frac{16\pi^2\varepsilon_0{}^2 \times R^2 \times V^2}{4\pi\varepsilon_0 \times 2R} = 2\ \pi\varepsilon_0 RV^2$ ['C' of bigger shell = $4\ \pi\varepsilon_0 R$]

43. $\sigma = 1 \times 10^{-4}$ c/m^2
a = 1 cm = 1 × 10^{-2} m $a^3 = 10^{-6}$ m

The energy stored in the plane = $\frac{1}{2}\frac{\sigma^2}{\varepsilon_0} = \frac{1}{2}\frac{(1\times10^{-4})^2}{8.85\times10^{-12}} = \frac{10^4}{17.7} = 564.97$

The necessary electro static energy stored in a cubical volume of edge 1 cm infront of the plane

$= \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}a^3 = 265 \times 10^{-6} = 5.65 \times 10^{-4}$ J

44. area = a = 20 cm^2 = 2 × 10^{-2} m^2
d = separation = 1 mm = 10^{-3} m

$Ci = \frac{\varepsilon_0 \times 2 \times 10^{-3}}{10^{-3}} = 2\varepsilon_0$ $Cf = \frac{\varepsilon_0 \times 2 \times 10^{-3}}{2 \times 10^{-3}} = \varepsilon_0$

$q_i = 24\ \varepsilon_0$ $qf = 12\ \varepsilon_0$ So, q flown out 12 ε_0. ie, $q_i - q_f$.

(a) So, q = 12 × 8.85 × 10^{-12} = 106.2 × 10^{-12} C = 1.06 × 10^{-10} C

(b) Energy absorbed by battery during the process
= q × v = 1.06 × 10^{-10} C × 12 = 12.7 × 10^{-10} J

(c) Before the process
$E_i = (1/2) \times Ci \times v^2 = (1/2) \times 2 \times 8.85 \times 10^{-12} \times 144 = 12.7 \times 10^{-10}$ J
After the force
$E_i = (1/2) \times Cf \times v^2 = (1/2) \times 8.85 \times 10^{-12} \times 144 = 6.35 \times 10^{-10}$ J

(d) Workdone = Force × Distance

$= \frac{1}{2}\frac{q^2}{\varepsilon_0 A} = 1 \times 10^3$ $= \frac{1}{2} \times \frac{12 \times 12 \times \varepsilon_0 \times \varepsilon_0 \times 10^{-3}}{\varepsilon_0 \times 2 \times 10^{-3}}$

(e) From (c) and (d) we have calculated, the energy loss by the separation of plates is equal to the work done by the man on plate. Hence no heat is produced in transformer.

45. (a) Before reconnection

$C = 100\ \mu f$ $\qquad V = 24$ V

$q = CV = 2400\ \mu c$ (Before reconnection)

After connection

When $C = 100\ \mu f$ $\qquad V = 12$ V

$q = CV = 1200\ \mu c$ (After connection)

(b) $C = 100$, $\qquad V = 12$ V

$\therefore q = CV = 1200$ v

(c) We know $V = \frac{W}{q}$

$W = vq = 12 \times 1200 = 14400$ J = 14.4 mJ

The work done on the battery.

(d) Initial electrostatic field energy $Ui = (1/2)\ CV_1^2$

Final Electrostatic field energy $U_f = (1/2)\ CV_2^2$

$\therefore$Decrease in Electrostatic

Field energy = $(1/2)\ CV_1^2 - (1/2)\ CV_2^2$

$= (1/2)\ C(V_1^2 - V_2^2) = (1/2) \times 100(576 - 144) = 21600$J

$\therefore$ Energy = 21600 j = 21.6 mJ

(e)After reconnection

$C = 100\ \mu c$, $\qquad V = 12$ v

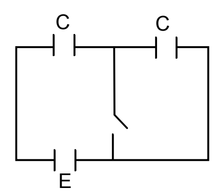
$\therefore$ The energy appeared = $(1/2)\ CV^2 = (1/2) \times 100 \times 144 = 7200$ J = 7.2 mJ

This amount of energy is developed as heat when the charge flow through the capacitor.

46. (a) Since the switch was open for a long time, hence the charge flown must be due to the both, when the switch is closed.

Cef = C/2

So $q = \frac{E \times C}{2}$



(b) Workdone = $q \times v = \frac{EC}{2} \times E = \frac{E^2C}{2}$

(c) $E_i = \frac{1}{2} \times \frac{C}{2} \times E^2 = \frac{E^2C}{4}$

$E_f = (1/2) \times C \times E^2 = \frac{E^2C}{2}$

$E_i - E_f = \frac{E^2C}{4}$

(d) The net charge in the energy is wasted as heat.

47. $C_1 = 5\ \mu f$ $\qquad V_1 = 24$ V

$q_1 = C_1V_1 = 5 \times 24 = 120\ \mu c$

and $C_2 = 6\ \mu f$ $\qquad V_2 = R$

$q_2 = C_2V_2 = 6 \times 12 = 72$

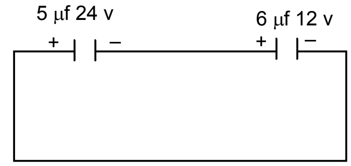
$\therefore$ Energy stored on first capacitor

$E_i = \frac{1}{2}\frac{q_1^2}{C_1} = \frac{1}{2} \times \frac{(120)^2}{2} = 1440$ J = 1.44 mJ

Energy stored on 2nd capacitor

$E_2 = \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2} \times \frac{(72)^2}{6} = 432$ J = 4.32 mJ

(b) C_1V_1 C_2V_2



Let the effective potential = V

$$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2} = \frac{120 - 72}{5 + 6} = 4.36$$

The new charge $C_1V = 5 \times 4.36 = 21.8\ \mu c$

and $C_2V = 6 \times 4.36 = 26.2\ \mu c$

(c) $U_1 = (1/2)\ C_1V^2$

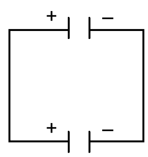
$U_2 = (1/2)\ C_2V^2$

$U_f = (1/2)\ V^2\ (C_1 + C_2) = (1/2)\ (4.36)^2\ (5 + 6) = 104.5 \times 10^{-6}\ J = 0.1045\ mJ$

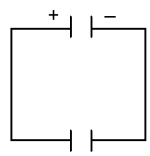
But $U_i = 1.44 + 0.433 = 1.873$

$\therefore$ The loss in KE = 1.873 – 0.1045 = 1.7687 = 1.77 mJ

48. (i)



(ii)



When the capacitor is connected to the battery, a charge Q = CE appears on one plate and –Q on the other. When the polarity is reversed, a charge –Q appears on the first plate and +Q on the second. A charge 2Q, therefore passes through the battery from the negative to the positive terminal.

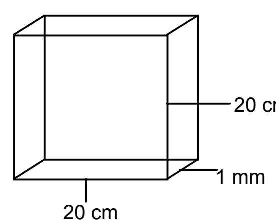
The battery does a work.

$W = Q \times E = 2QE = 2CE^2$

In this process. The energy stored in the capacitor is the same in the two cases. Thus the workdone by battery appears as heat in the connecting wires. The heat produced is therefore,

$2CE^2 = 2 \times 5 \times 10^{-6} \times 144 = 144 \times 10^{-5}\ J = 1.44\ mJ$ [have C = 5 μf V = E = 12V]

49. $A = 20\ cm \times 20\ cm = 4 \times 10^{-2}\ m$



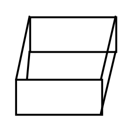
$d = 1\ m = 1 \times 10^{-3}\ m$

k = 4 t = d

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - d + \frac{d}{k}} = \frac{\varepsilon_0 Ak}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 4 \times 10^{-2} \times 4}{1 \times 10^{-3}} = 141.6 \times 10^{-9}\ F = 1.42\ nf$$

50. Dielectric const. = 4



F = 1.42 nf, V = 6 V

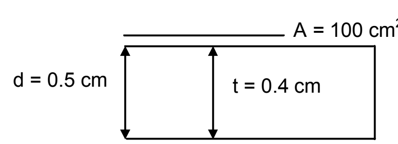
Charge supplied = $q = CV = 1.42 \times 10^{-9} \times 6 = 8.52 \times 10^{-9}\ C$

Charge Induced = $q(1 - 1/k) = 8.52 \times 10^{-9} \times (1 - 0.25) = 6.39 \times 10^{-9} = 6.4\ nc$

Net charge appearing on one coated surface = $\frac{8.52 \mu c}{4} = 2.13\ nc$

51. Here

Plate area = $100\ cm^2 = 10^{-2}\ m^2$



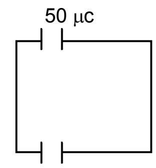
Separation d = $.5\ cm = 5 \times 10^{-3}\ m$

Thickness of metal t = $.4\ cm = 4 \times 10^{-3}\ m$

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - t} = \frac{8.585 \times 10^{-12} \times 10^{-2}}{(5 - 4) \times 10^{-3}} = 88\ pF$$

Here the capacitance is independent of the position of metal. At any position the net separation is d – t. As d is the separation and t is the thickness.

52. Initial charge stored = 50 μc



Let the dielectric constant of the material induced be 'k'.
Now, when the extra charge flown through battery is 100.
So, net charge stored in capacitor = 150 μc

Now $C_1 = \dfrac{\varepsilon_0 A}{d}$ or $\dfrac{q_1}{V} = \dfrac{\varepsilon_0 A}{d}$ …(1)

$C_2 = \dfrac{\varepsilon_0 Ak}{d}$ or, $\dfrac{q_2}{V} = \dfrac{\varepsilon_0 Ak}{d}$ …(2)

Deviding (1) and (2) we get $\dfrac{q_1}{q_2} = \dfrac{1}{k}$

$\Rightarrow \dfrac{50}{150} = \dfrac{1}{k} \Rightarrow k = 3$

53. $C = 5\ \mu f$ $V = 6\ V$ $d = 2\ mm = 2 \times 10^{-3}\ m$.

(a) the charge on the +ve plate

$q = CV = 5\ \mu f \times 6\ V = 30\ \mu c$

(b) $E = \dfrac{V}{d} = \dfrac{6V}{2\times10^{-3}m} = 3 \times 10^3\ V/M$

(c) $d = 2 \times 10^{-3}$ m

$t = 1 \times 10^{-3}$ m

$k = 5$ or $C = \dfrac{\varepsilon_0 A}{d}$ $\Rightarrow 5 \times 10^{-6} = \dfrac{8.85\times A\times10^{-12}}{2\times10^{-3}}\times10^{-9} \Rightarrow A = \dfrac{10^4}{8.85}$

When the dielectric placed on it

$$C_1 = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{8.85\times10^{-12}\times\frac{10^4}{8.85}}{10^{-3}+\frac{10^{-3}}{5}} = \frac{10^{-12}\times10^4\times5}{6\times10^{-3}} = \frac{5}{6}\times10^{-5} = 0.00000833 = 8.33\ \mu F.$$

(d) $C = 5 \times 10^{-6}$ f. $V = 6\ V$

$\therefore Q = CV = 3 \times 10^{-5}\ f = 30\ \mu f$

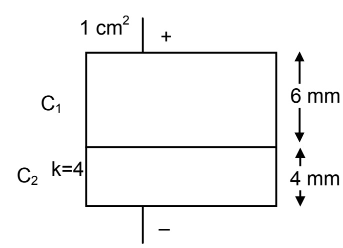
$C' = 8.3 \times 10^{-6}$ f

$V = 6\ V$

$\therefore Q' = C'V = 8.3 \times 10^{-6} \times 6 \approx 50\ \mu F$

$\therefore$ charge flown $= Q' - Q = 20\ \mu F$

54. Let the capacitances be C_1 & C_2 net capacitance 'C' $= \dfrac{C_1C_2}{C_1 + C_2}$



Now $C_1 = \dfrac{\varepsilon_0 Ak_1}{d_1}$ $C_2 = \dfrac{\varepsilon_0 Ak_2}{d_2}$

$$C = \frac{\frac{\varepsilon_0 Ak_1}{d_1}\times\frac{\varepsilon_0 Ak_2}{d_2}}{\frac{\varepsilon_0 Ak_1}{d_1}+\frac{\varepsilon_0 Ak_2}{d_2}} = \frac{\varepsilon_0 A\left(\frac{k_1k_2}{d_1d_2}\right)}{\varepsilon_0 A\left(\frac{k_1d_2+k_2d_1}{d_1d_2}\right)} = \frac{8.85\times10^{-12}\times10^{-2}\times24}{6\times4\times10^{-3}+4\times6\times10^{-3}}$$

$= 4.425 \times 10^{-11}$ C = 44.25 pc.

55. $A = 400\ cm^2 = 4 \times 10^{-2}\ m^2$

$d = 1\ cm = 1\times 10^{-3}$ m

$V = 160\ V$

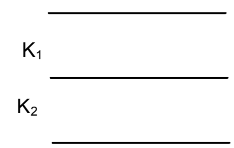
$t = 0.5 = 5 \times 10^{-4}$ m

$k = 5$

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{8.85\times10^{-12}\times4\times10^{-2}}{10^{-3} - 5\times10^{-4} + \frac{5\times10^{-4}}{5}} = \frac{35.4\times10^{-4}}{10^{-3} - 0.5}$$

56. (a) Area = A

Separation = d



$C_1 = \frac{\varepsilon_0 A k_1}{d/2}$ $\quad$ $C_2 = \frac{\varepsilon_0 A k_2}{d/2}$

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{\frac{2\varepsilon_0 Ak_1}{d}\times\frac{2\varepsilon_0 Ak_2}{d}}{\frac{2\varepsilon_0 Ak_1}{d}+\frac{2\varepsilon_0 Ak_2}{d}} = \frac{\frac{(2\varepsilon_0 A)^2k_1k_2}{d^2}}{(2\varepsilon_0 A)\frac{k_1d+k_2d}{d^2}} = \frac{2k_1k_2\varepsilon_0 A}{d(k_1+k_2)}$$

(b) similarly

$$\frac{1}{C} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} = \frac{1}{\frac{3\varepsilon_0 Ak_1}{d}}+\frac{1}{\frac{3\varepsilon_0 Ak_2}{d}}+\frac{1}{\frac{3\varepsilon_0 Ak_3}{d}}$$

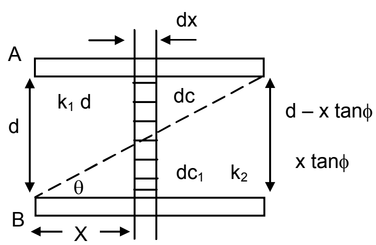
$$= \frac{d}{3\varepsilon_0 A}\left[\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right] = \frac{d}{3\varepsilon_0 A}\left[\frac{k_2k_3+k_1k_3+k_1k_2}{k_1k_2k_3}\right]$$

$$\therefore C = \frac{3\varepsilon_0 Ak_1k_2k_3}{d(k_1k_2+k_2k_3+k_1k_3)}$$

(c) $C = C_1 + C_2$

$$= \frac{\varepsilon_0\frac{A}{2}k_1}{d}+\frac{\varepsilon_0\frac{A}{2}k_2}{d} = \frac{\varepsilon_0 A}{2d}(k_1+k_2)$$

57.



Consider an elemental capacitor of with dx our at a distance 'x' from one end. It is constituted of two capacitor elements of dielectric constants k_1 and k_2 with plate separation x$\tan\phi$ and d –x$\tan\phi$ respectively in series

$$\frac{1}{dcR} = \frac{1}{dc_1}+\frac{1}{dc_2} = \frac{x\tan\phi}{\varepsilon_0 k_2(bdx)}+\frac{d - x\tan\phi}{\varepsilon_0 k_1(bdx)}$$

$$dcR = \frac{\varepsilon_0 bdx}{\frac{x\tan\phi}{k_2}+\frac{(d-x\tan\phi)}{k_1}}$$

or $C_R = \varepsilon_0 bk_1k_2 \int \frac{dx}{k_2d+(k_1-k_2)x\tan\phi}$

$$= \frac{\varepsilon_0 bk_1k_2}{\tan\phi(k_1-k_2)}[\log_e k_2d + (k_1 - k_2)\, x\tan\phi]a$$

$$= \frac{\varepsilon_0 bk_1k_2}{\tan\phi(k_1-k_2)}[\log_e k_2d + (k_1 - k_2)\, a\tan\phi - \log_e k_2d]$$

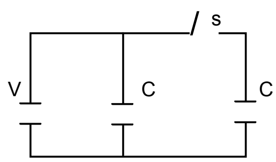
$\therefore \tan\phi = \frac{d}{a}$ and A = a × a

$C_R = \dfrac{\varepsilon_0 a k_1 k_2}{\frac{d}{a}(k_1 - k_2)}$ $\left[\log_e\left(\dfrac{k_1}{k_2}\right)\right]$

$C_R = \dfrac{\varepsilon_0 a^2 k_1 k_2}{d(k_1 - k_2)}$ $\left[\log_e\left(\dfrac{k_1}{k_2}\right)\right]$

$C_R = \dfrac{\varepsilon_0 a^2 k_1 k_2}{d(k_1 - k_2)}$ $\ln \dfrac{k_1}{k_2}$

58.



I. Initially when switch 's' is closed

Total Initial Energy = (1/2) CV^2 + (1/2) CV^2 = CV^2 …(1)

II. When switch is open the capacitance in each of capacitors varies, hence the energy also varies.

i.e. in case of 'B', the charge remains

Same i.e. cv

$C_{eff} = 3C$

$E = \dfrac{1}{2} \times \dfrac{q^2}{c} = \dfrac{1}{2} \times \dfrac{c^2v^2}{3c} = \dfrac{cv^2}{6}$

In case of 'A'

$C_{eff} = 3c$

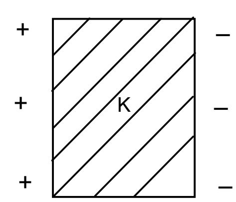
$E = \dfrac{1}{2} \times C_{eff} v^2 = \dfrac{1}{2} \times 3c \times v^2 = \dfrac{3}{2} cv^2$

Total final energy = $\dfrac{cv^2}{6} + \dfrac{3cv^2}{2} = \dfrac{10cv^2}{6}$

Now, $\dfrac{\text{Initial Energy}}{\text{Final Energy}} = \dfrac{cv^2}{\frac{10cv^2}{6}} = 3$

59. Before inserting

$C = \dfrac{\varepsilon_0 A}{d} C$ $Q = \dfrac{\varepsilon_0 AV}{d} C$



After inserting

$C = \dfrac{\varepsilon_0 A}{\frac{d}{k}} = \dfrac{\varepsilon_0 Ak}{d}$ $Q_1 = \dfrac{\varepsilon_0 Ak}{d} V$

The charge flown through the power supply

$Q = Q_1 - Q$

$= \dfrac{\varepsilon_0 AkV}{d} - \dfrac{\varepsilon_0 AV}{d} = \dfrac{\varepsilon_0 AV}{d}(k-1)$

Workdone = Charge in emf

$= \dfrac{1}{2}\dfrac{q^2}{C} = \dfrac{1}{2}\dfrac{\frac{\varepsilon_0{}^2 A^2 V^2}{d^2}(k-1)^2}{\frac{\varepsilon_0 A}{d}(k-1)} = \dfrac{\varepsilon_0 AV^2}{2d}(k-1)$

60. Capacitance = 100 μF = 10^{-4} F

P.d = 30 V

(a) q = CV = $10^{-4} \times 50 = 5 \times 10^{-3}$ c = 5 mc

Dielectric constant = 2.5

(b) New C = C′ = 2.5 × C = 2.5 × 10^{-4} F

New p.d = $\dfrac{q}{c^1}$ [$\therefore$'q' remains same after disconnection of battery]

$= \dfrac{5 \times 10^{-3}}{2.5 \times 10^{-4}}$ = 20 V.

(c) In the absence of the dielectric slab, the charge that must have produced

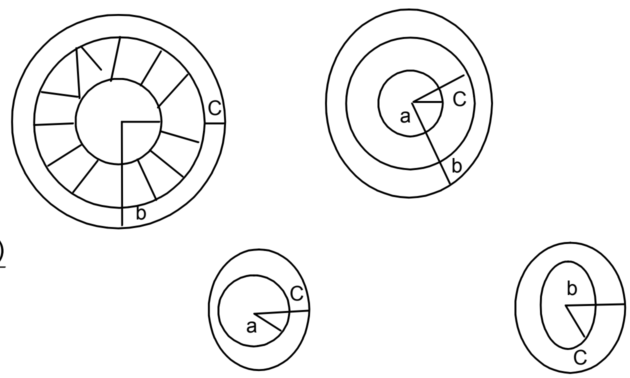
C × V = $10^{-4} \times 20 = 2 \times 10^{-3}$ c = 2 mc

(d) Charge induced at a surface of the dielectric slab

= q (1 –1/k) (where k = dielectric constant, q = charge of plate)

$= 5 \times 10^{-3}\left(1 - \dfrac{1}{2.5}\right) = 5 \times 10^{-3} \times \dfrac{3}{5} = 3 \times 10^{-3}$ = 3 mc.

61. Here we should consider a capacitor cac and cabc in series



$\text{Cac} = \dfrac{4\pi\varepsilon_0 ack}{k(c-a)}$

$\text{Cbc} = \dfrac{4\pi\varepsilon_0 bc}{(b-c)}$

$\dfrac{1}{C} = \dfrac{1}{\text{Cac}} + \dfrac{1}{\text{Cbc}}$

$= \dfrac{(c-a)}{4\pi\varepsilon_0 ack} + \dfrac{(b-c)}{4\pi\varepsilon_0 bc} = \dfrac{b(c-a)+ka(b-c)}{k4\pi\varepsilon_0 abc}$

$C = \dfrac{4\pi\varepsilon_0 kabc}{ka(b-c)+b(c-a)}$

62. These three metallic hollow spheres form two spherical capacitors, which are connected in series. Solving them individually, for (1) and (2)

$C_1 = \dfrac{4\pi\varepsilon_0 ab}{b-a}$ ($\therefore$ for a spherical capacitor formed by two spheres of radii $R_2 > R_1$)

$C = \dfrac{4\pi\varepsilon_0 R_2 R_1}{R_2 - R_2}$

Similarly for (2) and (3)

$C_2 = \dfrac{4\pi\varepsilon_0 bc}{c-b}$

$$C_{eff} = \frac{C_1C_2}{C_1+C_2} \; \frac{\dfrac{(4\pi\varepsilon_0)^2 ab^2c}{(b-a)(c-a)}}{4\pi\varepsilon_0\left[\dfrac{ab(c-b)+bc(b-a)}{(b-a)(c-b)}\right]}$$

$$= \frac{4\pi\varepsilon_0 ab^2c}{abc - ab^2 + b^2c - abc} = \frac{4\pi\varepsilon_0 ab^2c}{b^2(c-a)} = \frac{4\pi\varepsilon_0 ac}{c-a}$$

63. Here we should consider two spherical capacitor of capacitance cab and cbc in series

$\text{Cab} = \dfrac{4\pi\varepsilon_0 abk}{(b-a)}$ $\qquad$ $\text{Cbc} = \dfrac{4\pi\varepsilon_0 bc}{(c-b)}$

$$\frac{1}{C} = \frac{1}{Cab} + \frac{1}{Cbc} = \frac{(b-a)}{4\pi\varepsilon_0 abk} + \frac{(c-b)}{4\pi\varepsilon_0 bc} = \frac{c(b-a)+ka(c-b)}{k4\pi\varepsilon_0 abc}$$

$$C = \frac{4\pi\varepsilon_0 kabc}{c(b-a)+ka(c-b)}$$

64. $Q = 12\ \mu c$

$V = 1200\ V$

$\frac{v}{d} = 3 \times {}^{10-6}\ \frac{v}{m}$

$d = \frac{V}{(v/d)} = \frac{1200}{3\times10^{-6}} = 4 \times 10^{-4}\ m$

$c = \frac{Q}{v} = \frac{12\times10^{-6}}{1200} = 10^{-8}\ f$

$\therefore C = \frac{\varepsilon_0 A}{d} = 10^{-8}\ f$

$\Rightarrow A = \frac{10^{-8}\times d}{\varepsilon_0} = \frac{10^{-8}\times4\times10^{-4}}{8.854\times10^{-4}}\ 0.45\ m^2$

65. $A = 100\ cm^2 = 10^{-2}\ m^2$

$d = 1\ cm = 10^{-2}\ m$

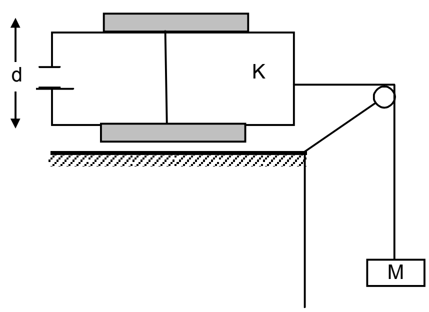
$V = 24\ V_0$

$\therefore$ The capacitance $C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times10^{-2}}{10^{-2}} = 8.85 \times 10^{-12}$

$\therefore$ The energy stored $C_1 = (1/2)\ CV^2 = (1/2) \times 10^{-12} \times (24)^2 = 2548.8 \times 10^{-12}$

$\therefore$ The forced attraction between the plates $= \frac{C_1}{d} = \frac{2548.8\times10^{-12}}{10^{-2}} = 2.54 \times 10^{-7}$ N.

66.



We knows

In this particular case the electricfield attracts the dielectric into the capacitor with a force $\frac{\varepsilon_0 bV^2(k-1)}{2d}$

Where b – Width of plates

k – Dielectric constant

d – Separation between plates

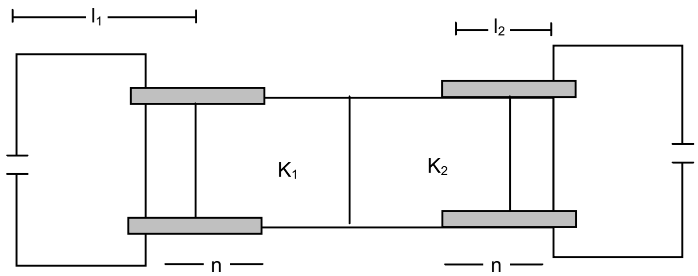
V = E = Potential difference.

Hence in this case the surfaces are frictionless, this force is counteracted by the weight.

So, $\frac{\varepsilon_0 bE^2(k-1)}{2d} = Mg$

$\Rightarrow M = \frac{\varepsilon_0 bE^2(k-1)}{2dg}$

67.



(a) Consider the left side

The plate area of the part with the dielectric is by its capacitance

$C_1 = \frac{k_1 \varepsilon_0 bx}{d}$ and with out dielectric $C_2 = \frac{\varepsilon_0 b(L_1 - x)}{d}$

These are connected in parallel

$C = C_1 + C_2 = \frac{\varepsilon_0 b}{d}[L_1 + x(k_1 - 1)]$

Let the potential V_1

$U = (1/2)\,CV_1^2 = \frac{\varepsilon_0 b v_1^2}{2d}\left[L_1 + x(k - 1)\right]$ …(1)

Suppose dielectric slab is attracted by electric field and an external force F consider the part dx which makes inside further, As the potential difference remains constant at V.

The charge supply, dq = (dc) v to the capacitor

The work done by the battery is $dw_b = v.dq = (dc)\,v^2$

The external force F does a work $dw_e = (-f.dx)$

during a small displacement

The total work done in the capacitor is $dw_b + dw_e = (dc)\,v^2 - fdx$

This should be equal to the increase dv in the stored energy.

Thus $(1/2)\,(dk)v^2 = (dc)\,v^2 - fdx$

$f = \frac{1}{2} v^2 \frac{dc}{dx}$

from equation (1)

$F = \frac{\varepsilon_0 b v^2}{2d}(k_1 - 1)$

$\Rightarrow V_1^2 = \frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)} \Rightarrow V_1 = \sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)}}$

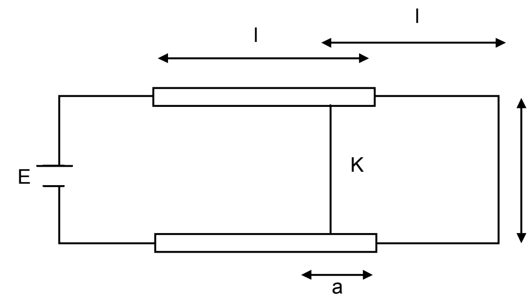
For the right side, $V_2 = \sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_2 - 1)}}$

$$\frac{V_1}{V_2} = \frac{\sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_1 - 1)}}}{\sqrt{\frac{F \times 2d}{\varepsilon_0 b(k_2 - 1)}}}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{\sqrt{k_2 - 1}}{\sqrt{k_1 - 1}}$$

$\therefore$ The ratio of the emf of the left battery to the right battery $= \frac{\sqrt{k_2 - 1}}{\sqrt{k_1 - 1}}$

68. Capacitance of the portion with dielectrics,



$C_1 = \dfrac{k\varepsilon_0 A}{\ell d}$

Capacitance of the portion without dielectrics,

$C_2 = \dfrac{\varepsilon_0(\ell - a)A}{\ell d}$

$\therefore$ Net capacitance $C = C_1 + C_2 = \dfrac{\varepsilon_0 A}{\ell d}[ka + (\ell - a)]$

$C = \dfrac{\varepsilon_0 A}{\ell d}[\ell + a(k-1)]$

Consider the motion of dielectric in the capacitor.

Let it further move a distance dx, which causes an increase of capacitance by dc

$\therefore dQ = (dc)\,E$

The work done by the battery dw = Vdg = E (dc) E = E^2 dc

Let force acting on it be f

$\therefore$ Work done by the force during the displacement, dx = fdx

$\therefore$ Increase in energy stored in the capacitor

$\Rightarrow (1/2)\,(dc)\,E^2 = (dc)\,E^2 - fdx$

$\Rightarrow fdx = (1/2)\,(dc)\,E^2 \Rightarrow f = \dfrac{1}{2}\dfrac{E^2 dc}{dx}$

$C = \dfrac{\varepsilon_0 A}{\ell d}[\ell + a(k-1)]$ (here x = a)

$\Rightarrow \dfrac{dc}{da} = \dfrac{-d}{da}\left[\dfrac{\varepsilon_0 A}{\ell d}\{\ell + a(k-1)\}\right]$

$\Rightarrow \dfrac{\varepsilon_0 A}{\ell d}(k-1) = \dfrac{dc}{dx}$

$\Rightarrow f = \dfrac{1}{2}E^2\dfrac{dc}{dx} = \dfrac{1}{2}E^2\left\{\dfrac{\varepsilon_0 A}{\ell d}(k-1)\right\}$

$\therefore a_d = \dfrac{f}{m} = \dfrac{E^2\varepsilon_0 A(k-1)}{2\ell dm}$ $\therefore (\ell - a) = \dfrac{1}{2}a_d t^2$

$\Rightarrow t = \sqrt{\dfrac{2(\ell - a)}{a_d}} = \sqrt{\dfrac{2(\ell - a)2\ell dm}{E^2\varepsilon_0 A(k-1)}} = \sqrt{\dfrac{4m\ell d(\ell - a)}{\varepsilon_0 AE^2(k-1)}}$

$\therefore$ Time period = $2t = \sqrt{\dfrac{8m\ell d(\ell - a)}{\varepsilon_0 AE^2(k-1)}}$

♣ ♣ ♣ ♣ ♣