REST AND MOTION : KINEMATICS

3.1 REST AND MOTION

When do we say that a body is at rest and when do we say that it is in motion ? You may say that if a body does not change its position as time passes it is at rest. If a body changes its position with time, it is said to be moving. But when do we say that it is not changing its position ? A book placed on the table remains on the table and we say that the book is at rest. However, if we station ourselves on the moon (the Appollo missions have made it possible), the whole earth is found to be changing its position and so the room, the table and the book are all continuously changing their positions. The book is at rest if it is viewed from the room, it is moving if it is viewed from the moon.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. Nothing is in absolute rest or in absolute motion. The moon is moving with respect to the book and the book moves with respect to the moon. Take another example. A robber enters a train moving at great speed with respect to the ground, brings out his pistol and says "Don't move, stand still". The passengers stand still. The passengers are at rest with respect to the robber but are moving with respect to the rail track.
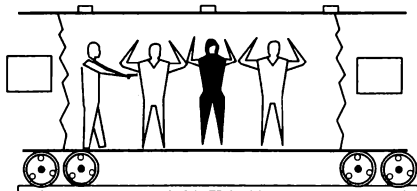

Figure 3.1

To locate the position of a particle we need a *frame of reference*. A convenient way to fix up the frame of reference is to choose three mutually perpendicular axes and name them *X-Y-Z* axes. The coordinates, (x, y, z) of the particle then specify the position of the particle with respect to that frame. Add a clock into the frame of reference to measure the time. If all the three coordinates x, y and z of the particle remain unchanged as time passes, we say that the particle is at rest with respect to this frame. If any one or more coordinates change with time, we say that the body is moving with respect to this frame.

There is no rule or restriction on the choice of a frame. We can choose a frame of reference according to our convenience to describe the situation under study. Thus, when we are in a train it is convenient to choose a frame attached to our compartment. The coordinates of a suitcase placed on the upper berth do not change with time (unless the train gives a jerk) and we say that the suitcase is at rest in the train-frame. The different stations, electric poles, trees etc. change their coordinates and we say that they are moving in the train-frame. Thus, we say that "Bombay is coming" and "Pune has already passed".

In the following sections we shall assume that the frame of reference is already chosen and we are describing the motion of the objects in the chosen frame. Sometimes the choice of the frame is clear from the context and we do not mention it. Thus, when one says the car is travelling and the rickshaw is not, it is clear that all positions are measured from a frame attached to the road.

3.2 DISTANCE AND DISPLACEMENT

Suppose a particle is at A at time t_1 and at B at time t_2 with respect to a given frame (figure 3.2).
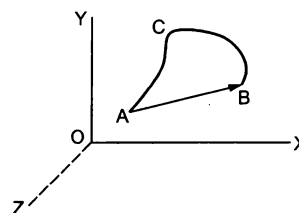

Figure 3.2

During the time interval t_1 to t_2 the particle moves along the path ACB. The length of the path ACB is called the *distance* travelled during the time interval t_1 to t_2. If we connect the initial position A with the final position B by a straight line, we get the *displacement* of the particle. The magnitude of the displacement is the length of the straight line joining the initial and the final position. The direction is from the initial to the final position. The displacement has both the magnitude as well as the direction. Further the displacements add according to the triangle rule of vector addition. Suppose a particle kept on a table is displaced on the table and at the same time the table is also displaced in the room. The net displacement of the particle in the room is obtained by the vector sum of the two displacements. Thus, displacement is a vector quantity. In contrast the distance covered has only a magnitude and is thus, a scalar quantity.

Example 3.1

An old person moves on a semi-circular track of radius 40.0 m during a morning walk. If he starts at one end of the track and reaches at the other end, find the distance covered and the displacement of the person.

Solution : The distance covered by the person equals the length of the track. It is equal to $\pi R = \pi \times 40.0$ m = 126 m.

The displacement is equal to the diameter of the semi-circular track joining the two ends. It is $2R = 2 \times 40.0$ m = 80 m. The direction of this displacement is from the initial point to the final point.

3.3 AVERAGE SPEED AND INSTANTANEOUS SPEED

The average speed of a particle in a time interval is defined as the distance travelled by the particle divided by the time interval. If the particle travels a distance s in time t_1 to t_2, the *average speed* is defined as

$$v_{av} = \frac{s}{t_2 - t_1} . \qquad \ldots (3.1)$$

The average speed gives the overall "rapidity" with which the particle moves in this interval. In a one-day cricket match, the average run rate is quoted as the total runs divided by the total number of overs used to make these runs. Some of the overs may be expensive and some may be economical. Similarly, the average speed gives the total effect in the given interval. The rapidity or slowness may vary from instant to instant. When an athelete starts running, he or she runs slowly and gradually
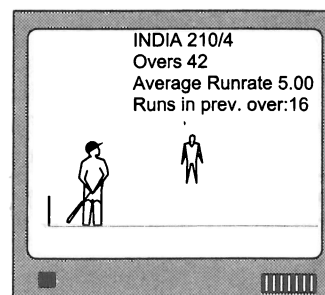

Figure 3.3

increases the rate. We define the *instantaneous speed* at a time t as follows.

Let Δs be the distance travelled in the time interval t to $t + \Delta t$. The average speed in this time interval is

$$v_{av} = \frac{\Delta s}{\Delta t} .$$

Now make Δt vanishingly small and look for the value of $\frac{\Delta s}{\Delta t}$. Remember Δs is the distance travelled in the chosen time interval Δt. As Δt approaches 0, the distance Δs also approaches zero but the ratio $\frac{\Delta s}{\Delta t}$ has a finite limit.

The instantaneous speed at a time t is defined as

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \ldots (3.2)$$

where s is the distance travelled in time t. The average speed is defined for a time interval and the instantaneous speed is defined at a particular instant. Instantaneous speed is also called "speed".

Example 3.2

The distance travelled by a particle in time t is given by $s = (2.5 \text{ m/s}^2) t^2$. Find (a) the average speed of the particle during the time 0 to 5.0 s, and (b) the instantaneous speed at $t = 5.0$ s.

Solution : (a) The distance travelled during time 0 to 5.0 s is

$$s = (2.5 \text{ m/s}^2)(5.0 \text{ s})^2 = 62.5 \text{ m}.$$

The average speed during this time is

$$v_{av} = \frac{62.5 \text{ m}}{5 \text{ s}} = 12.5 \text{ m/s}.$$

(b) $$s = (2.5 \text{ m/s}^2) t^2$$

or, $$\frac{ds}{dt} = (2.5 \text{ m/s}^2)(2t) = (5.0 \text{ m/s}^2) t.$$

At $t = 5.0$ s the speed is

$$v = \frac{ds}{dt} = (5.0 \text{ m/s}^2)(5.0 \text{ s}) = 25 \text{ m/s}.$$

If we plot the distance s as a function of time (figure 3.4), the speed at a time t equals the slope of

the tangent to the curve at the time t. The average speed in a time interval t to $t + \Delta t$ equals the slope of the chord AB where A and B are the points on the
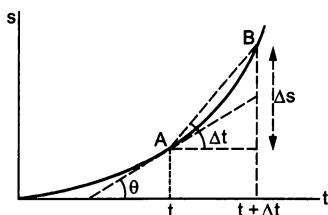

Figure 3.4

curve corresponding to the time t and $t + \Delta t$. As Δt approaches zero, the chord AB becomes the tangent at A and the average speed $\frac{\Delta s}{\Delta t}$ becomes the slope of the tangent which is $\frac{ds}{dt}$.

If the speed of the particle at time t is v, the distance ds travelled by it in the short time interval t to $t + dt$ is $v\, dt$. Thus, $ds = v\, dt$. The total distance travelled by the particle in a finite time interval t_1 to t_2 can be obtained by summing over these small distances ds as time changes from t_1 to t_2. Thus, the distance travelled by a particle in the time interval t_1 to t_2 is
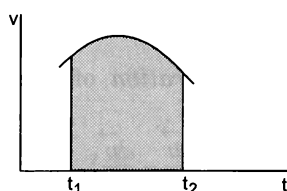

Figure 3.5

t_1 to t_2 is

$$s = \int_{t_1}^{t_2} v\, dt. \qquad \ldots (3.3)$$

If we plot a graph of the speed v versus time t, the distance travelled by the particle can be obtained by finding the area under the curve. Figure (3.5) shows such a speed-time graph. To find the distance travelled in the time interval t_1 to t_2 we draw ordinates from $t = t_1$ and $t = t_2$. The area bounded by the curve $v - t$, the X-axis and the two ordinates at $t = t_1$ and $t = t_2$ (shown shaded in the figure) gives the total distance covered.

The dimension of velocity is LT^{-1} and its SI unit is metre/second abbreviated as m/s.

Example 3.3

Figure (3.6) shows the speed versus time graph for a particle. Find the distance travelled by the particle during the time $t = 0$ to $t = 3$ s.
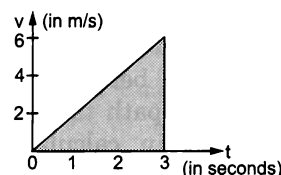

Figure 3.6

Solution : The distance travelled by the particle in the time 0 to 3 s is equal to the area shaded in the figure. This is a right angled triangle with height = 6 m/s and the base = 3 s. The area is $\frac{1}{2}$ (base) (height) $= \frac{1}{2} \times$ (3 s) (6 m/s) = 9 m. Thus, the particle covered a distance of 9 m during the time 0 to 3 s.

3.4 AVERAGE VELOCITY AND INSTANTANEOUS VELOCITY

The *average velocity* of a particle in a time interval t_1 to t_2 is defined as its displacement divided by the time interval. If the particle is at a point A (figure 3.7) at time $t = t_1$ and at B at time $t = t_2$, the displacement in this time interval is the vector $\overrightarrow{AB}$. The average velocity in this time interval is then,

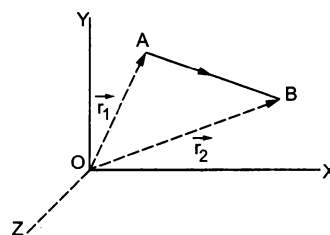$$\overrightarrow{v}_{av} = \frac{\overrightarrow{AB}}{t_2 - t_1}.$$

Figure 3.7

Like displacement, it is a vector quantity.

Position vector : If we join the origin to the position of the particle by a straight line and put an arrow towards the position of the particle, we get the *position vector* of the particle. Thus, the position vector of the particle shown in figure (3.7) at time $t = t_1$ is $\overrightarrow{OA}$ and that at $t = t_2$ is $\overrightarrow{OB}$. The displacement of the particle in the time interval t_1 to t_2 is

$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{r_2} - \overrightarrow{r_1}.$$

The average velocity of a particle in the time interval t_1 to t_2 can be written as

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}. \qquad \ldots (3.4)$$

Note that only the positions of the particle at time $t = t_1$ and $t = t_2$ are used in calculating the average velocity. The positions in between t_1 and t_2 are not needed, hence the actual path taken in going from A to B is not important in calculating the average velocity.

Example 3.4

A table clock has its minute hand 4·0 cm long. Find the average velocity of the tip of the minute hand (a) between 6·00 a.m. to 6·30 a.m. and (b) between 6·00 a.m. to 6·30 p.m.

Solution : At 6·00 a.m. the tip of the minute hand is at 12 mark and at 6·30 a.m. or 6·30 p.m. it is 180° away. Thus, the straight line distance between the initial and final position of the tip is equal to the diameter of the clock.

Displacement = $2\,R$ = 2 × 4·0 cm = 8·0 cm.

The displacement is from the 12 mark to the 6 mark on the clock panel. This is also the direction of the average velocity in both cases.

(a) The time taken from 6·00 a.m. to 6·30 a.m. is 30 minutes = 1800 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8·0 \text{ cm}}{1800 \text{ s}} = 4·4 \times 10^{-3} \text{ cm/s}.$$

(b) The time taken from 6·00 a.m. to 6·30 p.m. is 12 hours and 30 minutes = 45000 s. The average velocity is

$$v_{av} = \frac{\text{Displacement}}{\text{time}} = \frac{8·0 \text{ cm}}{45000 \text{ s}} = 1·8 \times 10^{-4} \text{ cm/s}.$$

The *instantaneous velocity* of a particle at a time t is defined as follows. Let the average velocity of the particle in a short time interval t to $t + \Delta t$ be $\vec{v}_{av}$. This average velocity can be written as

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

where $\Delta \vec{r}$ is the displacement in the time interval Δt. We now make Δt vanishingly small and find the limiting value of $\frac{\Delta \vec{r}}{\Delta t}$. This value is instantaneous velocity $\vec{v}$ of the particle at time t.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}. \qquad \ldots (3.5)$$

For very small intervals the displacement $\Delta \vec{r}$ is along the line of motion of the particle. Thus, the length

Δr equals the distance Δs travelled in that interval. So the magnitude of the velocity is

$$v = \left| \frac{d\vec{r}}{dt} \right| = \frac{|d\vec{r}|}{dt} = \frac{ds}{dt} \qquad \ldots (3.6)$$

which is the instantaneous speed at time t. Instantaneous velocity is also called the "velocity".

3.5 AVERAGE ACCELERATION AND INSTANTANEOUS ACCELERATION

If the velocity of a particle remains constant as time passes, we say that it is moving with uniform velocity. If the velocity changes with time, it is said to be accelerated. The acceleration is the rate of change of velocity. Velocity is a vector quantity hence a change in its magnitude or direction or both will change the velocity.

Suppose the velocity of a particle at time t_1 is $\vec{v}_1$ and at time t_2 it is $\vec{v}_2$. The change produced in time interval t_1 to t_2 is $\vec{v}_2 - \vec{v}_1$. We define the *average acceleration* $\vec{a}_{av}$ as the change in velocity divided by the time interval. Thus,

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}. \qquad \ldots (3.7)$$

Again the average acceleration depends only on the velocities at time t_1 and t_2. How the velocity changed in between t_1 and t_2 is not important in defining the average acceleration.

Instantaneous acceleration of a particle at time t is defined as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad \ldots (3.8)$$

where $\Delta \vec{v}$ is the change in velocity between the time t and $t + \Delta t$. At time t the velocity is $\vec{v}$ and at time $t + \Delta t$ it becomes $\vec{v} + \Delta \vec{v}$. $\frac{\Delta \vec{v}}{\Delta t}$ is the average acceleration of the particle in the interval Δt. As Δt approaches zero, this average acceleration becomes the instantaneous acceleration. Instantaneous acceleration is also called "acceleration".

The dimension of acceleration is LT^{-2} and its SI unit is metre/second^2 abbreviated as m/s^2.

3.6 MOTION IN A STRAIGHT LINE

When a particle is constrained to move on a straight line, the description becomes fairly simple. We choose the line as the X-axis and a suitable time instant as $t = 0$. Generally the origin is taken at the point where the particle is situated at $t = 0$. The position of the particle at time t is given by its coordinate x at that time. The velocity is

$$v = \frac{dx}{dt} \qquad \ldots (3.9)$$

and the acceleration is $a = \frac{dv}{dt}$ $\qquad \ldots (3.10)$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}. \quad \ldots (3.11)$$

If $\frac{dx}{dt}$ is positive, the direction of the velocity is along the positive X-axis and if $\frac{dx}{dt}$ is negative, the direction, is along the negative X-axis. Similarly if $\frac{dv}{dt}$ is positive, the acceleration is along the positive X-axis and if $\frac{dv}{dt}$ is negative, the acceleration is along the negative X-axis. The magnitude of v is speed. If the velocity and the acceleration are both positive, the speed increases. If both of them are negative then also the speed increases but if they have opposite signs, the speed decreases. When the speed decreases, we say that the particle is decelerating. Deceleration is equivalent to negative acceleration. An acceleration of 2.0 m/s^2 towards east is same as a deceleration of 2.0 m/s^2 towards west.

Motion with Constant Acceleration

Suppose the acceleration of a particle is a and remains constant. Let the velocity at time 0 be u and the velocity at time t be v. Thus,

$$\frac{dv}{dt} = a, \quad \text{or}, \quad dv = a\,dt$$

or, $\qquad \int_u^v dv = \int_0^t a\,dt.$

As time changes from 0 to t the velocity changes from u to v. So on the left hand side the summation is made over v from u to v whereas on the right hand side the summation is made on time from 0 to t. Evaluating the integrals we get,

$$[v]_u^v = a[t]_0^t$$

or, $\qquad v - u = at$

or, $\qquad v = u + at.$ $\qquad \ldots (3.12)$

Equation (3.12) may be written as

$$\frac{dx}{dt} = u + at$$

or, $\qquad dx = (u + at)dt$

or, $\qquad \int_0^x dx = \int_0^t (u + at)dt.$

At $t = 0$ the particle is at $x = 0$. As time changes from 0 to t the position changes from 0 to x. So on the left hand side the summation is made on position from

0 to x whereas on the right hand side the summation is made on time from 0 to t. Evaluating the integrals, the above equation becomes

$$[x]_0^x = \int_0^t u\,dt + \int_0^t at\,dt$$

or, $\qquad x = u\int_0^t dt + a\int_0^t t\,dt$

$$= u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t$$

or, $\qquad x = ut + \frac{1}{2}at^2.$ $\qquad \ldots (3.13)$

From equation (3.12),

$$v^2 = (u + at)^2$$

or, $\qquad = u^2 + 2\,uat + a^2t^2$

or, $\qquad = u^2 + 2a\left(ut + \frac{1}{2}at^2\right)$

or, $\qquad = u^2 + 2ax.$ $\qquad \ldots (3.14)$

The three equations (3.12) to (3.14) are collected below in table 3.1. They are very useful in solving the problems of motion in a straight line with constant acceleration.

Table 3.1

$$v = u + at$$
$$x = ut + \frac{1}{2}at^2$$
$$v^2 = u^2 + 2ax$$

Remember that x represents the position of the particle at time t and not (in general) the distance travelled by it in time 0 to t. For example, if the particle starts from the origin and goes upto $x = 4$ m, then turns and is at $x = 2$ m at time t, the distance travelled is 6 m but the position is still given by $x = 2$ m.

The quantities u, v and a may take positive or negative values depending on whether they are directed along the positive or negative direction. Similarly x may be positive or negative.

Example 3.5 ─────────

A particle starts with an initial velocity 2.5 m/s along the positive x direction and it accelerates uniformly at the rate 0.50 m/s^2. (a) Find the distance travelled by it in the first two seconds. (b) How much time does it take to reach the velocity 7.5 m/s ? (c) How much distance will it cover in reaching the velocity 7.5 m/s ?

Solution : (a) We have,

$$x = ut + \frac{1}{2}at^2$$

$$= (2 \cdot 5 \text{ m/s})\,(2 \text{ s}) + \frac{1}{2}(0 \cdot 50 \text{ m/s}^2)\,(2 \text{ s})^2$$

$$= 5 \cdot 0 \text{ m} + 1 \cdot 0 \text{ m} = 6 \cdot 0 \text{ m}.$$

Since the particle does not turn back it is also the distance travelled.

(b) We have,

$$v = u + at$$

or, $7 \cdot 5 \text{ m/s} = 2 \cdot 5 \text{ m/s} + (0 \cdot 50 \text{ m/s}^2)\,t$

or, $$t = \frac{7 \cdot 5 \text{ m/s} - 2 \cdot 5 \text{ m/s}}{0 \cdot 50 \text{ m/s}^2} = 10 \text{ s}.$$

(c) We have,

$$v^2 = u^2 + 2ax$$

or, $(7 \cdot 5 \text{ m/s})^2 = (2 \cdot 5 \text{ m/s})^2 + 2(0 \cdot 50 \text{ m/s}^2)x$

or, $$x = \frac{(7 \cdot 5 \text{ m/s})^2 - (2 \cdot 5 \text{ m/s})^2}{2 \times 0 \cdot 50 \text{ m/s}^2} = 50 \text{ m}.$$

Example 3.6

A particle having initial velocity u moves with a constant acceleration a for a time t. (a) Find the displacement of the particle in the last 1 second. (b) Evaluate it for $u = 5$ m/s, $a = 2$ m/s^2 and $t = 10$ s.

Solution : (a) The position at time t is

$$s = ut + \frac{1}{2}at^2$$

The position at time $(t - 1 \text{ s})$ is

$$s' = u(t - 1 \text{ s}) + \frac{1}{2}a(t - 1 \text{ s})^2$$

$$= ut - u(1 \text{ s}) + \frac{1}{2}at^2 - at(1 \text{ s}) + \frac{1}{2}a(1 \text{ s})^2$$

Thus, the displacement in the last 1 s is

$$s_t = s - s'$$

$$= u(1 \text{ s}) + at(1 \text{ s}) - \frac{1}{2}a(1 \text{ s})^2$$

or, $s_t = u(1 \text{ s}) + \dfrac{a}{2}(2t - 1 \text{ s})(1 \text{ s}).$ ... (i)

(b) Putting the given values in (i)

$$s_t = \left(5\,\frac{\text{m}}{\text{s}}\right)(1 \text{ s}) + \frac{1}{2}\left(2\,\frac{\text{m}}{\text{s}^2}\right)(2 \times 10 \text{ s} - 1 \text{ s})(1 \text{ s})$$

$$= 5 \text{ m} + \left(1\,\frac{\text{m}}{\text{s}^2}\right)(19 \text{ s})(1 \text{ s})$$

$$= 5 \text{ m} + 19 \text{ m} = 24 \text{ m}.$$

Sometimes, we are not careful in writing the units appearing with the numerical values of physical quantities. If we forget to write the unit of second in equation (i), we get,

$$s_t = u + \frac{a}{2}(2t - 1).$$

This equation is often used to calculate the displacement in the "tth second". However, as you can verify, different terms in this equation have different dimensions and hence the above equation is dimensionally incorrect. Equation (i) is the correct form which was used to solve part (b).

Also note that this equation gives the displacement of the particle in the last 1 second and not necessarily the distance covered in that second.

Freely Falling Bodies

A common example of motion in a straight line with constant acceleration is free fall of a body near the earth's surface. If air resistance is neglected and a body is dropped near the surface of the earth, it falls along a vertical straight line. The acceleration is in the vertically downward direction and its magnitude is almost constant if the height is small as compared with the radius of the earth (6400 km). This magnitude is approximately equal to 9·8 m/s^2 or 32 ft/s^2 and is denoted by the letter g.

If we take vertically upward as the positive Y-axis, acceleration is along the negative Y-axis and we write $a = -g$. The equation (3.12) to (3.14) may be written in this case as

$$v = u - gt$$

$$y = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gy.$$

Here y is the y-coordinate (that is the height above the origin) at time t, u is the velocity in y direction at $t = 0$ and v is the velocity in y direction at time t. The position of the particle at $t = 0$ is $y = 0$.

Sometimes it is convenient to choose vertically downward as the positive Y-axis. Then $a = g$ and the equations (3.12) to (3.14) become

$$v = u + gt$$

$$y = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gy.$$

Example 3.7

A ball is thrown up at a speed of 4·0 m/s. Find the maximum height reached by the ball. Take $g = 10$ m/s^2.

Solution : Let us take vertically upward direction as the positive Y-axis. We have $u = 4 \cdot 0$ m/s and $a = -10$ m/s^2. At the highest point the velocity becomes zero. Using the formula.

$$v^2 = u^2 + 2ay,$$

$$0 = (4 \cdot 0 \text{ m/s})^2 + 2(- 10 \text{ m/s}^2)y$$

or, $$y = \frac{16 \text{ m}^2/\text{s}^2}{20 \text{ m/s}^2} = 0 \cdot 80 \text{ m}.$$

3.7 MOTION IN A PLANE

If a particle is free to move in a plane, its position can be located with two coordinates. We choose the plane of motion as the X-Y plane. We choose a suitable instant as $t = 0$ and choose the origin at the place where the particle is situated at $t = 0$. Any two convenient mutually perpendicular directions in the X-Y plane are chosen as the X and Y-axes.

The position of the particle at a time t is completely specified by its coordinates (x, y). The coordinates at time $t + \Delta t$ are $(x + \Delta x, y + \Delta y)$. Figure (3.8) shows the positions at t and $t + \Delta t$ as A and B respectively. The displacement during the time interval t to $t + \Delta t$ is

$$\vec{\Delta r} = \vec{AB} = \vec{AC} + \vec{CB}$$

$$= \Delta x \, \vec{i} + \Delta y \, \vec{j}$$

or, $$\frac{\vec{\Delta r}}{\Delta t} = \frac{\Delta x}{\Delta t} \vec{i} + \frac{\Delta y}{\Delta t} \vec{j}.$$
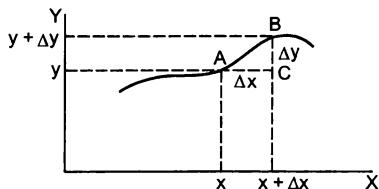
Figure 3.8

Taking limits as $\Delta t \to 0$

$$\vec{v} = \frac{dx}{dt} \vec{i} + \frac{dy}{dt} \vec{j}. \qquad \ldots (3.15)$$

Thus, we see that the x-component of the velocity is

$$v_x = \frac{dx}{dt} \qquad \ldots (3.16)$$

and the y-component is

$$v_y = \frac{dy}{dt}. \qquad \ldots (3.17)$$

Differentiating (3.15) with respect to time,

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \vec{i} + \frac{dv_y}{dt} \vec{j}$$

Thus, the acceleration has components

$$a_x = \frac{dv_x}{dt} \qquad \ldots (3.18)$$

and $$a_y = \frac{dv_y}{dt}. \qquad \ldots (3.19)$$

We see that the x-coordinate, the x-component of velocity v_x and the x-component of acceleration a_x are related by

$$v_x = \frac{dx}{dt} \quad \text{and} \quad a_x = \frac{dv_x}{dt}.$$

These equations are identical to equations (3.9) and (3.10). Thus, if a_x is constant, integrating these equations we get

$$\begin{aligned} v_x &= u_x + a_x t \\ x &= u_x t + \frac{1}{2} a_x t^2 \\ v_x^2 &= u_x^2 + 2a_x x \end{aligned} \qquad \ldots (3.20)$$

where u_x is the x-component of the velocity at $t = 0$. Similarly we have

$$v_y = \frac{dy}{dt} \quad \text{and} \quad a_y = \frac{dv_y}{dt}$$

and if a_y is constant,

$$\begin{aligned} v_y &= u_y + a_y t \\ y &= u_y t + \frac{1}{2} a_y t^2 \\ v_y^2 &= u_y^2 + 2a_y y \end{aligned} \qquad \ldots (3.21)$$

The general scheme for the discussion of motion in a plane is therefore simple. The x-coordinate, the x-component of velocity and the x-component of acceleration are related by equations of straight line motion along the X-axis. Similarly the y-coordinate, the y-component of velocity and the y-component of acceleration are related by the equations of straight line motion along the Y-axis. The problem of motion in a plane is thus, broken up into two independent problems of straight line motion, one along the X-axis and the other along the Y-axis.

Example 3.8

A particle moves in the X-Y plane with a constant acceleration of 1·5 m/s² in the direction making an angle of 37° with the X-axis. At $t = 0$ the particle is at the origin and its velocity is 8·0 m/s along the X-axis. Find the velocity and the position of the particle at $t = 4·0$ s.

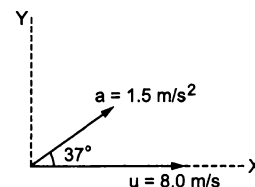Figure 3.9

Solution :

$$a_x = (1 \cdot 5 \text{ m/s}^2) \, (\cos 37°)$$

$$= (1 \cdot 5 \text{ m/s}^2) \times \frac{4}{5} = 1 \cdot 2 \text{ m/s}^2$$

and

$$a_y = (1 \cdot 5 \text{ m/s}^2) \, (\sin 37°)$$

$$= (1 \cdot 5 \text{ m/s}^2) \times \frac{3}{5} = 0 \cdot 90 \text{ m/s}^2.$$

The initial velocity has components

$$u_x = 8 \cdot 0 \text{ m/s}$$

and

$$u_y = 0$$

At $t = 0$, $x = 0$ and $y = 0$.

The x-component of the velocity at time $t = 4 \cdot 0$ s is given by

$$v_x = u_x + a_x t$$

$$= 8 \cdot 0 \text{ m/s} + (1 \cdot 2 \text{ m/s}^2) \, (4 \cdot 0 \text{ s})$$

$$= 8 \cdot 0 \text{ m/s} + 4 \cdot 8 \text{ m/s} = 12 \cdot 8 \text{ m/s}.$$

The y-component of velocity at $t = 4 \cdot 0$ s is given by

$$v_y = u_y + a_y t$$

$$= 0 + (0 \cdot 90 \text{ m/s}^2) \, (4 \cdot 0 \text{ s}) = 3 \cdot 6 \text{ m/s}.$$

The velocity of the particle at $t = 4 \cdot 0$ s is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(12 \cdot 8 \text{ m/s})^2 + (3 \cdot 6 \text{ m/s})^2}$$

$$= 13 \cdot 3 \text{ m/s}.$$

The velocity makes an angle θ with the X-axis where

$$\tan \theta = \frac{v_y}{v_x} = \frac{3 \cdot 6 \text{ m/s}}{12 \cdot 8 \text{ m/s}} = \frac{9}{32}.$$

The x-coordinate at $t = 4 \cdot 0$ s is

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$= (8 \cdot 0 \text{ m/s}) \, (4 \cdot 0 \text{ s}) + \frac{1}{2} (1 \cdot 2 \text{ m/s}^2) \, (4 \cdot 0 \text{ s})^2$$

$$= 32 \text{ m} + 9 \cdot 6 \text{ m} = 41 \cdot 6 \text{ m}.$$

The y-coordinate at $t = 4 \cdot 0$ s is

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$= \frac{1}{2} (0 \cdot 90 \text{ m/s}^2) \, (4 \cdot 0 \text{ s})^2$$

$$= 7 \cdot 2 \text{ m}.$$

Thus, the particle is at $(41 \cdot 6 \text{ m}, 7 \cdot 2 \text{ m})$ at $4 \cdot 0$ s.

3.8 PROJECTILE MOTION

An important example of motion in a plane with constant acceleration is the projectile motion. When a particle is thrown obliquely near the earth's surface, it moves along a curved path. Such a particle is called a *projectile* and its motion is called *projectile motion*. We shall assume that the particle remains close to the surface of the earth and the air resistance is negligible. The acceleration of the particle is then almost constant. It is in the vertically downward direction and its magnitude is g which is about $9 \cdot 8 \text{ m/s}^2$.

Let us first make ourselves familiar with certain terms used in discussing projectile motion. Figure (3.10) shows a particle projected from the point O with an initial velocity u at an angle θ with the horizontal. It goes through the highest point A and falls at B on the horizontal surface through O. The point O is called the *point of projection*, the angle θ is called the *angle of projection* and the distance OB is called the *horizontal range* or simply *range*. The total time taken by the particle in describing the path OAB is called the *time of flight*.

The motion of the projectile can be discussed separately for the horizontal and vertical parts. We take the origin at the point of projection. The instant
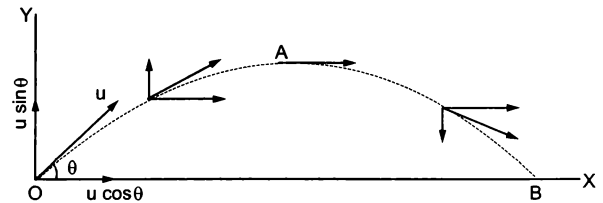

Figure 3.10

when the particle is projected is taken as $t = 0$. The plane of motion is taken as the X-Y plane. The horizontal line OX is taken as the X-axis and the vertical line OY as the Y-axis. Vertically upward direction is taken as the positive direction of the Y-axis.

We have $u_x = u \cos\theta$; $a_x = 0$

$$u_y = u \sin\theta ; \quad a_y = -g.$$

Horizontal Motion

As $a_x = 0$, we have

$$v_x = u_x + a_x t = u_x = u \cos\theta$$

and

$$x = u_x t + \frac{1}{2} a_x t^2 = u_x t = ut \cos\theta.$$

As indicated in figure (3.10), the x-component of the velocity remains constant as the particle moves.

Vertical Motion

The acceleration of the particle is g in the downward direction. Thus, $a_y = -g$. The y-component of the initial velocity is u_y. Thus,

$$v_y = u_y - gt$$

and

$$y = u_y t - \frac{1}{2} g t^2.$$

Also we have,

$$v_y^2 = u_y^2 - 2gy.$$

The vertical motion is identical to the motion of a particle projected vertically upward with speed $u \sin\theta$. The horizontal motion of the particle is identical to a particle moving horizontally with uniform velocity $u \cos\theta$.

Time of Flight

Consider the situation shown in figure (3.10). The particle is projected from the point O and reaches the same horizontal plane at the point B. The total time taken to reach B is the time of flight.

Suppose the particle is at B at a time t. The equation for horizontal motion gives

$$OB = x = ut \cos\theta$$

The y-coordinate at the point B is zero. Thus, from the equation of vertical motion,

$$y = ut \sin\theta - \frac{1}{2} gt^2$$

or, $$0 = ut \sin\theta - \frac{1}{2} gt^2$$

or, $$t(u \sin\theta - \frac{1}{2} gt) = 0.$$

Thus, either $t = 0$ or, $t = \frac{2u \sin\theta}{g}$.

Now $t = 0$ corresponds to the position O of the particle. The time at which it reaches B is thus,

$$T = \frac{2u \sin\theta}{g}. \qquad \ldots (3.22)$$

This is the time of flight.

Range

The distance OB is the horizontal range. It is the distance travelled by the particle in time $T = \frac{2u \sin\theta}{g}$. By the equation of horizontal motion,

$$x = (u\cos\theta)t$$

or, $$OB = (u \cos\theta)\left(\frac{2u \sin\theta}{g}\right)$$

$$= \frac{2u^2 \sin\theta \cos\theta}{g}$$

$$= \frac{u^2 \sin 2\theta}{g}. \qquad \ldots (3.23)$$

Maximum Height Reached

At the maximum height (A in figure 3.10) the velocity of the particle is horizontal. The vertical component of velocity is thus, zero at the highest point. The maximum height is the y-coordinate of the particle when the vertical component of velocity becomes zero.

We have,

$$v_y = u_y - gt$$

$$= u \sin\theta - gt.$$

At the maximum height

$$0 = u \sin\theta - gt$$

or, $$t = \frac{u \sin\theta}{g}. \qquad \ldots (3.24)$$

The maximum height is

$$H = u_y t - \frac{1}{2} gt^2$$

$$= (u \sin\theta)\left(\frac{u \sin\theta}{g}\right) - \frac{1}{2} g\left(\frac{u \sin\theta}{g}\right)^2$$

$$= \frac{u^2 \sin^2\theta}{g} - \frac{1}{2}\frac{u^2 \sin^2\theta}{g}$$

$$= \frac{u^2 \sin^2\theta}{2g}. \qquad \ldots (3.25)$$

Equation (3.24) gives the time taken in reaching the maximum height. Comparison with equation (3.22) shows that it is exactly half the time of the flight. Thus, the time taken in ascending the maximum height equals the time taken in descending back to the same horizontal plane.

Example 3.9

A ball is thrown from a field with a speed of 12·0 m/s at an angle of 45° with the horizontal. At what distance will it hit the field again ? Take g = 10·0 m/s².

Solution : The horizontal range $= \frac{u^2 \sin 2\theta}{g}$

$$= \frac{(12 \text{ m/s})^2 \times \sin(2 \times 45°)}{10 \text{ m/s}^2}$$

$$= \frac{144 \text{ m}^2/\text{s}^2}{10 \cdot 0 \text{ m/s}^2} = 14 \cdot 4 \text{ m}.$$

Thus, the ball hits the field at 14·4 m from the point of projection.

3.9 CHANGE OF FRAME

So far we have discussed the motion of a particle with respect to a given frame of reference. The frame can be chosen according to the convenience of the problem. The position $\vec{r}$, the velocity $\vec{v}$ and the acceleration $\vec{a}$ of a particle depend on the frame chosen. Let us see how can we relate the position, velocity and acceleration of a particle measured in two different frames.

Consider two frames of reference S and S' and suppose a particle P is observed from both the frames. The frames may be moving with respect to each other. Figure (3.11) shows the situation.
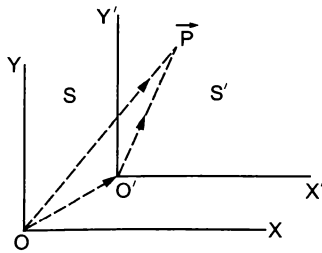
Figure 3.11

The position vector of the particle P with respect to the frame S is $\vec{r}_{P,S} = \overrightarrow{OP}$. The position vector of the particle with respect to the frame S' is $\vec{r}_{P,S'} = \overrightarrow{O'P}$. The position of the frame S' (the origin of frame S' in fact) with respect to the frame S is OO'.

It is clear that

$$\overrightarrow{OP} = \overrightarrow{OO'} + \overrightarrow{O'P} = \overrightarrow{O'P} + \overrightarrow{OO'}$$

or, $\qquad \vec{r}_{P,S} = \vec{r}_{P,S'} + \vec{r}_{S',S}. \qquad \ldots (3.26)$

The position of the particle with respect to S is equal to the position of the particle with respect to S' plus the position of S' with respect to S.

If we differentiate equation (3.26) with respect to time, we get

$$\frac{d}{dt}(\vec{r}_{P,S}) = \frac{d}{dt}(\vec{r}_{P,S'}) + \frac{d}{dt}(\vec{r}_{S',S})$$

or, $\qquad \vec{v}_{P,S} = \vec{v}_{P,S'} + \vec{v}_{S',S} \qquad \ldots (3.27)$

where $\vec{v}_{P,S}$ is the velocity of the particle with respect to S, $\vec{v}_{P,S'}$ is the velocity of the particle with respect to S' and $\vec{v}_{S',S}$ is the velocity of the frame S' with respect to S. The velocity of the particle with respect to S is equal to the velocity of the particle with respect to S' plus the velocity of S' with respect to S.

It is assumed that the meaning of time is same in both the frames. Similarly it is assumed that $\frac{d}{dt}$ has same meaning in both the frames. These assumptions are not correct if the velocity of one frame with respect to the other is so large that it is comparable to 3×10^{8} m/s, or if one frame rotates with respect to the other. If the frames only translate with respect to each other with small velocity, the above assumptions are correct.

Equation (3.27) may be rewritten as

$$\vec{v}_{P,S'} = \vec{v}_{P,S} - \vec{v}_{S',S}. \qquad \ldots (3.28)$$

Thus, if the velocities of two bodies (here the particle and the frame S') are known with respect to a common frame (here S) we can find the velocity of one body with respect to the other body. The velocity of body 1 with respect to the body 2 is obtained by subtracting the velocity of body 2 from the velocity of body 1.

When we say that the muzzle velocity of a bullet is 60 m/s we mean the velocity of the bullet with respect to the gun. If the gun is mounted in a train moving with a speed of 20 m/s with respect to the ground and the bullet is fired in the direction of the train's motion, its velocity with respect to the ground will be 80 m/s. Similarly, when we say that a swimmer can swim at a speed of 5 km/h we mean the velocity of the swimmer with respect to the water. If the water itself is flowing at 3 km/h with respect to the ground and the swimmer swims in the direction of the current, he or she will move at the speed of 8 km/h with respect to the ground.

Example 3.10

A swimmer can swim in still water at a rate 4·0 km/h. If he swims in a river flowing at 3·0 km/h and keeps his direction (with respect to water) perpendicular to the current, find his velocity with respect to the ground.

Solution : The velocity of the swimmer with respect to water is $\vec{v}_{S,R} = 4\cdot0$ km/h in the direction perpendicular to the river. The velocity of river with respect to the ground is $\vec{v}_{R,G} = 3\cdot0$ km/h along the length of the river. The velocity of the swimmer with respect to the ground is $\vec{v}_{S,G}$ where

$$\vec{v}_{S,G} = \vec{v}_{S,R} + \vec{v}_{R,G}.$$
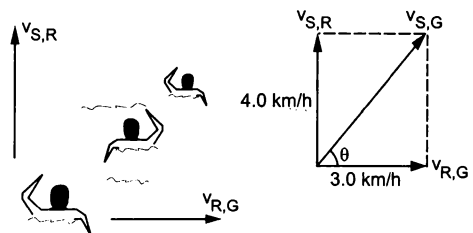
Figure (3.12) shows the velocities. It is clear that,

Figure 3.12

$$v_{S,G} = \sqrt{(4\cdot0 \text{ km/h})^2 + (3\cdot0 \text{ km/h})^2}$$

$$= 5\cdot0 \text{ km/h}$$

The angle θ made with the direction of flow is

$$\tan\theta = \frac{4\cdot0 \text{ km/h}}{3\cdot0 \text{ km/h}} = \frac{4}{3}.$$

Example 3.11

A man is walking on a level road at a speed of 3·0 km/h. Rain drops fall vertically with a speed of 4·0 km/h. Find the velocity of the raindrops with respect to the man.

Solution : We have to find the velocity of raindrops with respect to the man. The velocity of the rain as well as the velocity of the man are given with respect to the street. We have

$$\vec{v}_{rain,\,man} = \vec{v}_{rain,\,street} - \vec{v}_{man,\,street}.$$
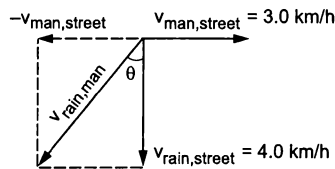
Figure (3.13) shows the velocities.

Figure 3.13

It is clear from the figure that

$$v_{rain,\,man} = \sqrt{(4 \cdot 0\ \text{km/h})^2 + (3 \cdot 0\ \text{km/h})^2}$$
$$= 5 \cdot 0\ \text{km/h}.$$

The angle with the vertical is θ, where

$$\tan\theta = \frac{3 \cdot 0\ \text{km/h}}{4 \cdot 0\ \text{km/h}} = \frac{3}{4}.$$

Thus, the rain appears to fall at an angle $\tan^{-1}(3/4)$ with the speed $5 \cdot 0$ km/h as viewed by the man.

The relation between the accelerations measured from two frames can be obtained by differentiating equation (3.27) with respect to time.

We have,

$$\frac{d}{dt}(\vec{v}_{P,\,S}) = \frac{d}{dt}(\vec{v}_{P,\,S'}) + \frac{d}{dt}(\vec{v}_{S',\,S})$$

or, $$\vec{a}_{P,\,S} = \vec{a}_{P,\,S'} + \vec{a}_{S',\,S}. \qquad \dots (3.29)$$

If S' moves with respect to S at a uniform velocity, $\vec{a}_{S',\,S} = 0$ and so

$$\vec{a}_{P,\,S} = \vec{a}_{P,\,S'}.$$

If two frames are moving with respect to each other with uniform velocity, acceleration of a body is same in both the frames.

Worked Out Examples

1. *A man walks at a speed of 6 km/hr for 1 km and 8 km/hr for the next 1 km. What is his average speed for the walk of 2 km ?*

Solution : Distance travelled is 2 km.

Time taken $= \dfrac{1\ \text{km}}{6\ \text{km/hr}} + \dfrac{1\ \text{km}}{8\ \text{km/hr}}$

$= \left(\dfrac{1}{6} + \dfrac{1}{8}\right)\text{hr} = \dfrac{7}{24}\ \text{hr}.$

Average speed $= \dfrac{2\ \text{km} \times 24}{7\ \text{hr}} = \dfrac{48}{7}\ \text{km/hr}$

$\approx 7\ \text{km/hr}.$

2. *The I.Sc. lecture theatre of a college is 40 ft wide and has a door at a corner. A teacher enters at 12·00 noon through the door and makes 10 rounds along the 40 ft wall back and forth during the period and finally leaves the class-room at 12·50 p.m. through the same door. Compute his average speed and average velocity.*

Solution : Total distance travelled in 50 minutes = 800 ft.

Average speed $= \dfrac{800}{50}\ \text{ft/min} = 16\ \text{ft/min}.$

At 12·00 noon he is at the door and at 12·50 pm he is again at the same door.
The displacement during the 50 min interval is zero.
Average velocity = zero.

3. *The position of a particle moving on X-axis is given by*
$$x = At^3 + Bt^2 + Ct + D.$$
The numerical values of A, B, C, D are 1, 4, –2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4 s, (d) the average velocity during the interval t = 0 to t = 4 s, (e) the average acceleration during the interval t = 0 to t = 4 s.

Solution : (a) Dimensions of x, At^3, Bt^2, Ct and D must be identical and in this case each is length. Thus,

$$[At^3] = \text{L}, \quad \text{or,} \quad [A] = \text{LT}^{-3}$$
$$[Bt^2] = \text{L}, \quad \text{or,} \quad [B] = \text{LT}^{-2}$$
$$[Ct] = \text{L}, \quad \text{or,} \quad [C] = \text{LT}^{-1}$$
and $$[D] = \text{L}.$$

(b) $x = At^3 + Bt^2 + Ct + D$

or, $v = \dfrac{dx}{dt} = 3At^2 + 2Bt + C.$

Thus, at $t = 4$ s, the velocity
$= 3(1\ \text{m/s}^3)(16\ \text{s}^2) + 2(4\ \text{m/s}^2)(4\ \text{s}) + (-2\ \text{m/s})$
$= (48 + 32 - 2)\ \text{m/s} = 78\ \text{m/s}.$

(c) $v = 3At^2 + 2Bt + C$

or, $a = \dfrac{dv}{dt} = 6At + 2B.$

At $t = 4$ s, $a = 6(1\ \text{m/s}^3)(4\ \text{s}) + 2(4\ \text{m/s}^2) = 32\ \text{m/s}^2.$

(d) $x = At^3 + Bt^2 + Ct + D.$
Position at $t = 0$ is $x = D = 5$ m.
Position at $t = 4$ s is
$(1\ \text{m/s}^3)(64\ \text{s}^3) + (4\ \text{m/s}^2)(16\ \text{s}^2) - (2\ \text{m/s})(4\ \text{s}) + 5\ \text{m}$
$= (64 + 64 - 8 + 5)\ \text{m} = 125\ \text{m}.$

Thus, the displacement during 0 to 4 s is
    125 m – 5 m = 120 m.

Average velocity $= \dfrac{120 \text{ m}}{4 \text{ s}} = 30$ m/s.

(e) $v = 3At^2 + 2Bt + C$.

Velocity at $t = 0$ is $C = -2$ m/s.

Velocity at $t = 4$ s is $= 78$ m/s.

Average acceleration $= \dfrac{v_2 - v_1}{t_2 - t_1} = 20$ m/s^2.

4. *From the velocity-time graph of a particle given in figure (3-W1), describe the motion of the particle qualitatively in the interval 0 to 4 s. Find (a) the distance travelled during first two seconds, (b) during the time 2 s to 4 s, (c) during the time 0 to 4 s, (d) displacement during 0 to 4 s, (e) acceleration at $t = 1/2$ s and (f) acceleration at $t = 2$ s.*
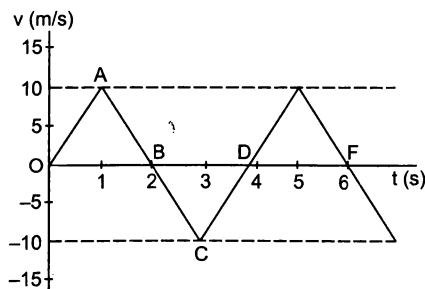
Figure 3-W1

Solution : At $t = 0$, the particle is at rest, say at the origin. After that the velocity is positive, so that the particle moves in the positive x direction. Its speed increases till 1 second when it starts decreasing. The particle continues to move further in positive x direction. At $t = 2$ s, its velocity is reduced to zero, it has moved through a maximum positive x distance. Then it changes its direction, velocity being negative, but increasing in magnitude. At $t = 3$ s velocity is maximum in the negative x direction and then the magnitude starts decreasing. It comes to rest at $t = 4$ s.

(a) Distance during 0 to 2 s = Area of *OAB*

$$= \frac{1}{2} \times 2 \text{ s} \times 10 \text{ m/s} = 10 \text{ m}.$$

(b) Distance during 2 to 4 s = Area of *BCD* = 10 m. The particle has moved in negative x direction during this period.

(c) The distance travelled during 0 to 4s = 10 m + 10 m
                                        = 20 m.

(d) displacement during 0 to 4 s = 10 m + (– 10 m) = 0.

(e) at $t = 1/2$ s acceleration = slope of line *OA* = 10 m/s^2.

(f) at $t = 2$ s acceleration = slope of line *ABC* = – 10 m/s^2.

5. *A particle starts from rest with a constant acceleration. At a time t second, the speed is found to be 100 m/s and one second later the speed becomes 150 m/s. Find (a) the acceleration and (b) the distance travelled during the $(t+1)^{th}$ second.*

Solution : (a) Velocity at time t is

    100 m/s = $a.(t$ second)   ... (1)

and velocity at time $(t + 1)$ second is

    150 m/s = $a. (t + 1)$.   ... (2)

Subtracting (1) from (2), $a = 50$ m/s^2

(b) Consider the interval t second to $(t + 1)$ second,

    time elapsed = 1 s

    initial velocity = 100 m/s

    final velocity = 150 m/s.

Thus, $(150 \text{ m/s})^2 = (100 \text{ m/s})^2 + 2(50 \text{ m/s}^2) x$

or, $x = 125$ m.

6. *A boy stretches a stone against the rubber tape of a catapult or 'gulel' (a device used to detach mangoes from the tree by boys in Indian villages) through a distance of 24 cm before leaving it. The tape returns to its normal position accelerating the stone over the stretched length. The stone leaves the gulel with a speed 2·2 m/s. Assuming that the acceleration is constant while the stone was being pushed by the tape, find its magnitude.*

Solution : Consider the accelerated 24 cm motion of the stone.

    Initial velocity = 0

    Final velocity = 2·2 m/s

Distance travelled = 24 cm = 0·24 m

Using $v^2 = u^2 + 2ax$,

$$a = \frac{4 \cdot 84 \text{ m}^2/\text{s}^2}{2 \times 0 \cdot 24 \text{ m}} = 10 \cdot 1 \text{ m/s}^2.$$

7. *A police inspector in a jeep is chasing a pickpocket on a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pickpocket will be caught if $v \geq \sqrt{2ad}$.*

Solution : Suppose the pickpocket is caught at a time t after the motorcyle starts. The distance travelled by the motorcycle during this interval is
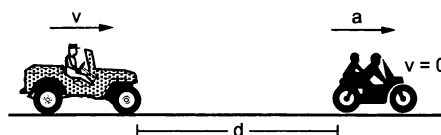

Figure 3-W2

$$s = \frac{1}{2} a t^2. \qquad \ldots \text{(i)}$$

During this interval the jeep travels a distance

$$s + d = vt. \qquad \ldots \text{(ii)}$$

By (i) and (ii),

$$\frac{1}{2} a t^2 - vt + d = 0$$

or, $$t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}.$$

The pickpocket will be caught if t is real and positive. This will be possible if

$$v^2 \geq 2ad \quad \text{or,} \quad v \geq \sqrt{2ad}.$$

8. *A car is moving at a constant speed of* 40 km/h *along a straight road which heads towards a large vertical wall and makes a sharp 90° turn by the side of the wall. A fly flying at a constant speed of* 100 km/h, *starts from the wall towards the car at an instant when the car is* 20 km *away, flies until it reaches the glasspane of the car and returns to the wall at the same speed. It continues to fly between the car and the wall till the car makes the 90° turn. (a) What is the total distance the fly has travelled during this period ? (b) How many trips has it made between the car and the wall ?*

Solution : (a) The time taken by the car to cover 20 km before the turn is $\frac{20 \text{ km}}{40 \text{ km/h}} = \frac{1}{2}$ h. The fly moves at a constant speed of 100 km/h during this time. Hence the total distance coverd by it is $100 \frac{\text{km}}{\text{h}} \times \frac{1}{2}$ h $= 50$ km.

(b) Suppose the car is at a distance x away (at A) when the fly is at the wall (at O). The fly hits the glasspane at B, taking a time t. Then

$$AB = (40 \text{ km/h})t,$$

and $$OB = (100 \text{ km/h})t.$$

Thus, $$x = AB + OB$$

$$= (140 \text{ km/h})t$$

or, $$t = \frac{x}{140 \text{ km/h}}, \text{ or } OB = \frac{5}{7} x.$$
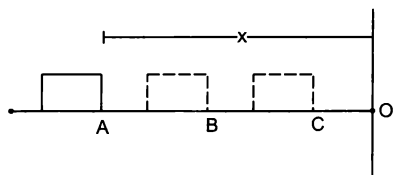
Figure 3-W3

The fly returns to the wall and during this period the car moves the distance BC. The time taken by the fly in this return path is

$$\left(\frac{5 \, x / 7}{100 \text{ km/h}} \right) = \frac{x}{140 \text{ km/h}}.$$

Thus, $$BC = \frac{40 \, x}{140} = \frac{2}{7} x$$

or, $$OC = OB - BC = \frac{3}{7} x.$$

If at the beginning of the round trip (wall to the car and back) the car is at a distance x away, it is $\frac{3}{7} x$ away when the next trip again starts.

Distance of the car at the beginning of the 1st trip = 20 km.

Distance of the car at the beginning of the 2nd trip

$$= \frac{3}{7} \times 20 \text{ km}.$$

Distance of the car at the beginning of the 3rd trip

$$= \left(\frac{3}{7} \right)^2 \times 20 \text{ km}.$$

Distance of the car at the beginning of the 4th trip

$$= \left(\frac{3}{7} \right)^3 \times 20 \text{ km}.$$

Distance of the car at the beginning of the nth trip

$$= \left(\frac{3}{7} \right)^{n-1} \times 20 \text{ km}.$$

Trips will go on till the car reaches the turn that is the distance reduces to zero. This will be the case when n becomes infinity. Hence the fly makes an infinite number of trips before the car takes the turn.

9. *A ball is dropped from a height of* 19·6 m *above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive X-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with the ground.*

Solution : Since the acceleration of the ball during the contact is different from 'g', we have to treat the downward motion and the upward motion separately.

For the downward motion : $a = g = 9·8 \text{ m/s}^2$,

$$x = ut + \frac{1}{2} a t^2 = (4·9 \text{ m/s}^2)t^2.$$

The ball reaches the ground when $x = 19·6$ m. This gives $t = 2$ s. After that it moves up, x decreases and at $t = 4$ s, x becomes zero, the ball reaching the initial point.

We have at $t = 0$, $\quad x = 0$

$$t = 1 \text{ s}, \qquad x = 4·9 \text{ m}$$

$$t = 2 \text{ s}, \qquad x = 19·6 \text{ m}$$

$$t = 3 \text{ s}, \qquad x = 4·9 \text{ m}$$
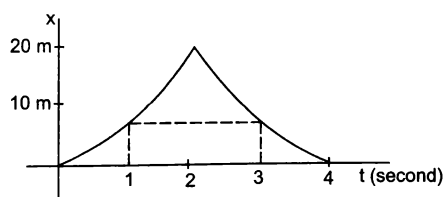
$$t = 4 \text{ s}, \qquad x = 0.$$

Figure 3-W4

Velocity : During the first two seconds,

$$v = u + at = (9.8 \text{ m/s}^2)t$$

at $t = 0$ $v = 0$

at $t = 1$ s, $v = 9.8$ m/s

at $t = 2$ s, $v = 19.6$ m/s.

During the next two seconds the ball goes upward, velocity is negative, magnitude decreasing and at $t = 4$ s, $v = 0$. Thus,

at $t = 2$ s, $v = -19.6$ m/s

at $t = 3$ s, $v = -9.8$ m/s

at $t = 4$ s, $v = 0$.
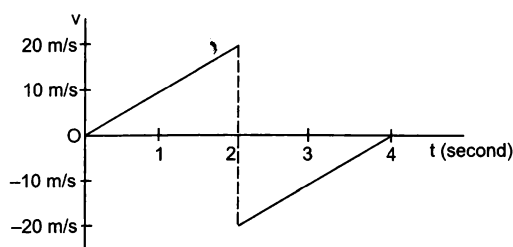
Figure 3-W5

At $t = 2$ s there is an abrupt change in velocity from 19.6 m/s to -19.6 m/s. In fact this change in velocity takes place over a small interval during which the ball remains in contact with the ground.

Acceleration : The acceleration is constant 9.8 m/s^2 throughout the motion (except at $t = 2$ s).
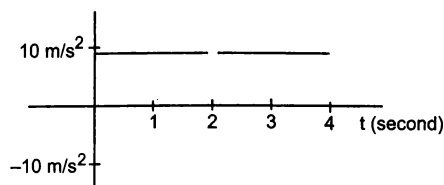
Figure 3-W6

10. *A stone is dropped from a balloon going up with a uniform velocity of 5.0 m/s. If the balloon was 50 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s².*

Solution : At $t = 0$, the stone was going up with a velocity of 5.0 m/s. After that it moved as a freely falling particle with downward acceleration g. Take vertically upward

as the positive X-axis. If it reaches the ground at time t,

$$x = -50 \text{ m}, \quad u = 5 \text{ m/s}, \quad a = -10 \text{ m/s}^2.$$

We have $x = ut + \dfrac{1}{2} at^2$

or, $-50 \text{ m} = (5 \text{ m/s}).t + \dfrac{1}{2} \times (-10 \text{ m/s}^2)t^2$

or, $t = \dfrac{1 \pm \sqrt{41}}{2}$ s.

or, $t = -2.7$ s   or,   3.7 s.

Negative t has no significance in this problem. The stone reaches the ground at $t = 3.7$ s. During this time the balloon has moved uniformly up. The distance covered by it is

$$5 \text{ m/s} \times 3.7 \text{ s} = 18.5 \text{ m}.$$

Hence, the height of the balloon when the stone reaches the ground is 50 m + 18.5 m = 68.5 m.

11. *A football is kicked with a velocity of 20 m/s at an angle of 45° with the horizontal. (a) Find the time taken by the ball to strike the ground. (b) Find the maximum height it reaches. (c) How far away from the kick does it hit the ground ? Take g = 10 m/s².*

Solution : (a) Take the origin at the point where the ball is kicked, vertically upward as the Y-axis and the horizontal in the plane of motion as the X-axis. The initial velocity has the components

$$u_x = (20 \text{ m/s}) \cos 45° = 10 \sqrt{2} \text{ m/s}$$

and $u_y = (20 \text{ m/s}) \sin 45° = 10 \sqrt{2} \text{ m/s}.$

When the ball reaches the ground, $y = 0$.

Using $y = u_y t - \dfrac{1}{2} gt^2$,

$$0 = (10\sqrt{2} \text{ m/s}) \, t - \dfrac{1}{2} \times (10 \text{ m/s}^2) \times t^2$$

or, $t = 2\sqrt{2} \text{ s} = 2.8$ s.

Thus, it takes 2.8 s for the football to fall on the ground.

(b) At the highest point $v_y = 0$. Using the equation

$$v_y^2 = u_y^2 - 2gy,$$

$$0 = (10\sqrt{2} \text{ m/s})^2 - 2 \times (10 \text{ m/s}^2) H$$

or, $H = 10$ m.

Thus, the maximum height reached is 10 m.

(c) The horizontal distance travelled before falling to the ground is $x = u_x t$

$$= (10\sqrt{2} \text{ m/s}) \, (2\sqrt{2} \text{ s}) = 40 \text{ m}.$$

12. *A helicopter on flood relief mission, flying horizontally with a speed u at an altitude H, has to drop a food packet for a victim standing on the ground. At what distance from the victim should the packet be dropped ? The victim stands in the vertical plane of the helicopter's motion.*

Solution : The velocity of the food packet at the time of release is u and is horizontal. The vertical velocity at the time of release is zero.
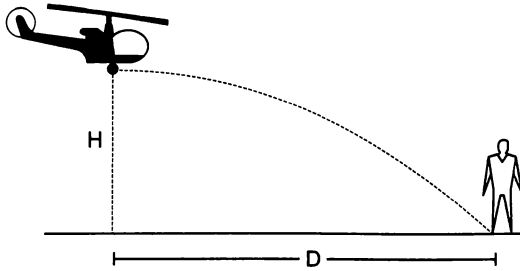

Figure 3-W7

Vertical motion : If t be the time taken by the packet to reach the victim, we have for vertical motion,

$$H = \frac{1}{2} gt^2 \quad \text{or,} \quad t = \sqrt{\frac{2H}{g}}. \qquad \ldots \text{(i)}$$

Horizontal motion : If D be the horizontal distance travelled by the packet, we have $D = ut$. Putting t from (i),

$$D = u\sqrt{\frac{2H}{g}}.$$

The distance between the victim and the packet at the time of release is

$$\sqrt{D^2 + H^2} = \sqrt{\frac{2u^2 H}{g} + H^2}.$$

13. *A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane ?*

Solution : Take X,Y-axes as shown in figure (3-W8). Suppose that the particle strikes the plane at a point P with coordinates (x, y). Consider the motion between A and P.
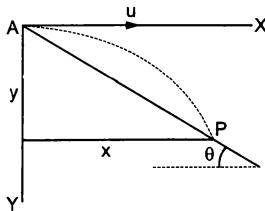
Figure 3-W8

Motion in x-direction :

Initial velocity $= u$

Acceleration $= 0$

$$x = ut. \qquad \ldots \text{(i)}$$

Motion in y-direction :

Initial velocity $= 0$

Acceleration $= g$

$$y = \frac{1}{2} gt^2. \qquad \ldots \text{(ii)}$$

Eliminating t from (i) and (ii)

$$y = \frac{1}{2} g \frac{x^2}{u^2}.$$

Also $\qquad\qquad y = x \tan\theta.$

Thus, $\dfrac{gx^2}{2u^2} = x \tan\theta$ giving $x = 0$, or, $\dfrac{2u^2 \tan\theta}{g}.$

Clearly the point P corresponds to $x = \dfrac{2u^2 \tan\theta}{g}$,

then $y = x \tan\theta = \dfrac{2u^2 \tan^2\theta}{g}.$

The distance $AP = l = \sqrt{x^2 + y^2}$

$$= \frac{2u^2}{g} \tan\theta \sqrt{1 + \tan^2\theta}$$

$$= \frac{2u^2}{g} \tan\theta \sec\theta.$$

Alternatively : Take the axes as shown in figure 3-W9. Consider the motion between A and P.
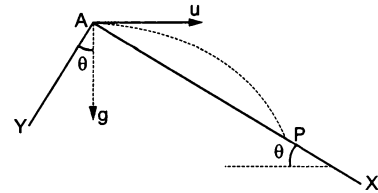
Figure 3-W9

Motion along the X-axis :

Initial velocity $= u \cos\theta$

Acceleration $= g \sin\theta$

Displacement $= AP$.

Thus, $AP = (u \cos\theta)\, t + \dfrac{1}{2} (g \sin\theta)\, t^2.$ $\qquad \ldots \text{(i)}$

Motion along the Y-axis :

Initial velocity $= - u \sin\theta$

Acceleration $= g \cos\theta$

Displacement $= 0$.

Thus, $\qquad 0 = - ut \sin\theta + \dfrac{1}{2} gt^2 \cos\theta$

or, $\qquad\qquad t = 0, \quad \dfrac{2u \sin\theta}{g \cos\theta}.$

Clearly, the point P corresponds to $t = \dfrac{2u \sin\theta}{g \cos\theta}.$

Putting this value of t in (i),

$$AP = (u \cos\theta)\left(\frac{2u \sin\theta}{g \cos\theta}\right) + \frac{g \sin\theta}{2}\left(\frac{2u \sin\theta}{g \cos\theta}\right)^2$$

$$= \frac{2u^2 \sin\theta}{g} + \frac{2u^2 \sin\theta \tan^2\theta}{g}$$

$$= \frac{2u^2}{g} \sin\theta \sec^2\theta = \frac{2u^2}{g} \tan\theta \sec\theta.$$

14. *A projectile is fired with a speed u at an angle θ with the horizontal. Find its speed when its direction of motion makes an angle α with the horizontal.*

Solution : Let the speed be v when it makes an angle α with the horizontal. As the horizontal component of velocity remains constant,

$$v \cos\alpha = u \cos\theta$$

or, $$v = u \cos\theta \sec\alpha.$$

15. *A bullet is fired horizontally aiming at an object which starts falling at the instant the bullet is fired. Show that the bullet will hit the object.*

Solution : The situation is shown in figure (3-W10). The object starts falling from the point B. Draw a vertical line BC through B. Suppose the bullet reaches the line BC at a point D and it takes a time t in doing so.
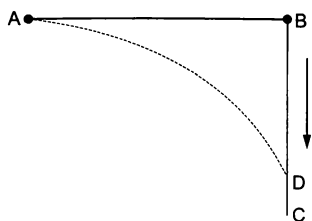
Figure 3-W10

Consider the vertical motion of the bullet. The initial vertical velocity = 0. The distance travelled vertically $= BD = \frac{1}{2} g t^2$. In time t the object also travels a distance $\frac{1}{2} g t^2 = BD$. Hence at time t, the object will also be at the same point D. Thus, the bullet hits the object at point D.

16. *A man can swim in still water at a speed of 3 km/h. He wants to cross a river that flows at 2 km/h and reach the point directly opposite to his starting point. (a) In which direction should he try to swim (that is, find the angle his body makes with the river flow) ? (b) How much time will he take to cross the river if the river is 500 m wide ?*

Solution : (a) The situation is shown in figure (3-W11). The X-axis is chosen along the river flow and the origin at the starting position of the man. The direction of the velocity of man with respect to ground is along the Y-axis (perpendicular to the river). We have to find the direction of velocity of the man with respect to water.

Let $\overrightarrow{v}_{r,g}$ = velocity of the river with respect to the ground

= 2 km/h along the X-axis
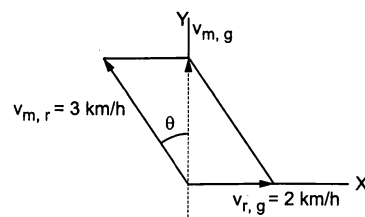
Figure 3-W11

$\overrightarrow{v}_{m,r}$ = velocity of the man with respect to the river

= 3 km/h making an angle θ with the Y-axis

and $\overrightarrow{v}_{m,g}$ = velocity of the man with respect to the ground along the Y-axis.

We have

$$\overrightarrow{v}_{m,g} = \overrightarrow{v}_{m,r} + \overrightarrow{v}_{r,g} \, . \qquad \qquad \ldots \text{(i)}$$

Taking components along the X-axis

$$0 = -(3 \text{ km/h})\sin\theta + 2 \text{ km/h}$$

or, $$\sin\theta = \frac{2}{3} \, .$$

(b) Taking components in equation (i) along the Y-axis,

$$v_{m,g} = (3 \text{ km/h}) \cos\theta + 0$$

or, $$v_{m,g} = \sqrt{5} \text{ km/h}.$$

$$\text{Time} = \frac{\text{Displacement in } y \text{ direction}}{\text{Velocity in } y \text{ direction}}$$

$$= \frac{0.5 \text{ km}}{\sqrt{5} \text{ km/h}} = \frac{\sqrt{5}}{10} \text{ h}.$$

17. *A man can swim at a speed of 3 km/h in still water. He wants to cross a 500 m wide river flowing at 2 km/h. He keeps himself always at an angle of 120° with the river flow while swimming.*
(a) Find the time he takes to cross the river. (b) At what point on the opposite bank will he arrive ?

Solution : The situation is shown in figure (3-W12).
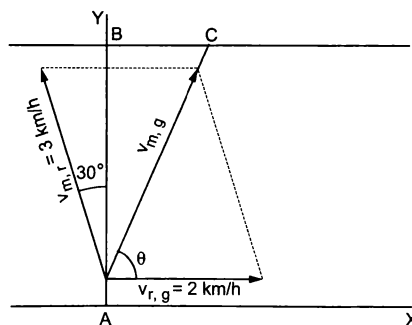
Figure 3-W12

Here $\overrightarrow{v}_{r,g}$ = velocity of the river with respect to the ground

$\overrightarrow{v}_{m,r}$ = velocity of the man with respect to the river

$\overrightarrow{v}_{m,g}$ = velocity of the man with respect to the ground.

(a) We have,

$$\vec{v}_{m,g} = \vec{v}_{m,r} + \vec{v}_{r,g} \qquad \ldots \text{(i)}$$

Hence, the velocity with respect to the ground is along AC. Taking y-components in equation (i),

$$\vec{v}_{m,g} \sin\theta = 3 \text{ km/h } \cos30° + 2 \text{ km/h } \cos90° = \frac{3\sqrt{3}}{2} \text{ km/h.}$$

Time taken to cross the river

$$= \frac{\text{displacement along the } Y\text{-axis}}{\text{velocity along the } Y\text{-axis}}$$

$$= \frac{1/2 \text{ km}}{3\sqrt{3}/2 \text{ km/h}} = \frac{1}{3/3} \text{ h.}$$

(b) Taking x-components in equation (i),

$$\vec{v}_{m,g} \cos\theta = -3 \text{ km/h } \sin30° + 2 \text{ km/h}$$

$$= \frac{1}{2} \text{ km/h.}$$

Displacement along the X-axis as the man crosses the river

$$= (\text{velocity along the } X\text{-axis}) \cdot (\text{time})$$

$$= \left(\frac{1 \text{ km}}{2 \text{ h}}\right) \times \left(\frac{1}{3\sqrt{3}} \text{ h}\right) = \frac{1}{6\sqrt{3}} \text{ km.}$$

18. *A man standing on a road has to hold his umbrella at 30° with the vertical to keep the rain away. He throws the umbrella and starts running at 10 km/h. He finds that raindrops are hitting his head vertically. Find the speed of raindrops with respect to (a) the road, (b) the moving man.*

Solution : When the man is at rest with respect to the ground, the rain comes to him at an angle 30° with the vertical. This is the direction of the velocity of raindrops with respect to the ground. The situation when the man runs is shown in the figure (3-W13b).

(a)   (b)   (c)
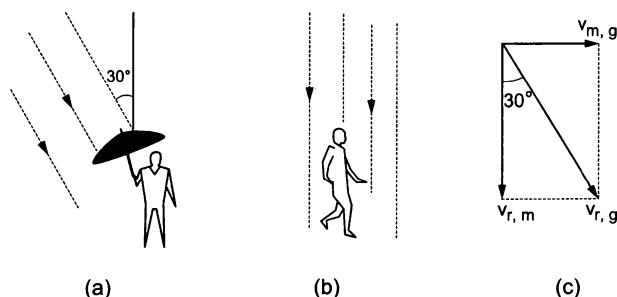
Figure 3-W13

Here $\vec{v}_{r,g}$ = velocity of the rain with respect to the ground
$\vec{v}_{m,g}$ = velocity of the man with respect to the ground
and $\vec{v}_{r,m}$ = velocity of the rain with respect to the man.
We have, $\qquad \vec{v}_{r,g} = \vec{v}_{r,m} + \vec{v}_{m,g}. \qquad \ldots \text{(i)}$

Taking horizontal components, equation (i) gives

$$v_{r,g} \sin30° = v_{m,g} = 10 \text{ km/h}$$

or, $v_{r,g} = \dfrac{10 \text{ km/h}}{\sin30°} = 20 \text{ km/h,}$

Taking vertical components, equation (i) gives

$$v_{r,g} \cos30° = v_{r,m}$$

or, $\qquad\qquad v_{r,m} = (20 \text{ km/h}) \dfrac{\sqrt{3}}{2}$

$$= 10\sqrt{3} \text{ km/h.}$$

19. *A man running on a horizontal road at 8 km/h finds the rain falling vertically. He increases his speed to 12 km/h and finds that the drops make angle 30° with the vertical. Find the speed and direction of the rain with respect to the road.*

Solution :

We have $\qquad \vec{v}_{rain,road} = \vec{v}_{rain,man} + \vec{v}_{man,road} \qquad \ldots \text{(i)}$

The two situations given in the problem may be represented by the following figure.
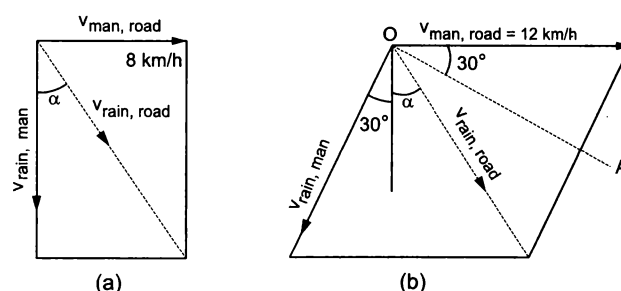
Figure 3-W14

$v_{rain,road}$ is same in magnitude and direction in both the figures.

Taking horizontal components in equation (i) for figure (3-W14a),

$$v_{rain,road} \sin\alpha = 8 \text{ km/h.} \qquad \ldots \text{(ii)}$$

Now consider figure (3-W14b). Draw a line $OA \perp v_{rain,man}$ as shown.

Taking components in equation (i) along the line OA.

$$v_{rain,road} \sin(30° + \alpha) = 12 \text{ km/h } \cos30°. \qquad \ldots \text{(iii)}$$

From (ii) and (iii),

$$\frac{\sin(30° + \alpha)}{\sin\alpha} = \frac{12 \times \sqrt{3}}{8 \times 2}$$

or, $\qquad \dfrac{\sin30° \cos\alpha + \cos30° \sin\alpha}{\sin\alpha} = \dfrac{3\sqrt{3}}{4}$

or, $\qquad \dfrac{1}{2} \cot\alpha + \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{4}$

or, $\qquad\qquad \cot\alpha = \dfrac{\sqrt{3}}{2}$

or, $\qquad\qquad \alpha = \cot^{-1} \dfrac{\sqrt{3}}{2}.$

From (ii), $\qquad v_{rain,road} = \dfrac{8 \text{ km/h}}{\sin\alpha} = 4\sqrt{7} \text{ km/h.}$

20. *Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side d at t = 0. Each*

of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other ?

Solution : The motion of the particles is roughly sketched in figure (3-W15). By symmetry they will meet at the
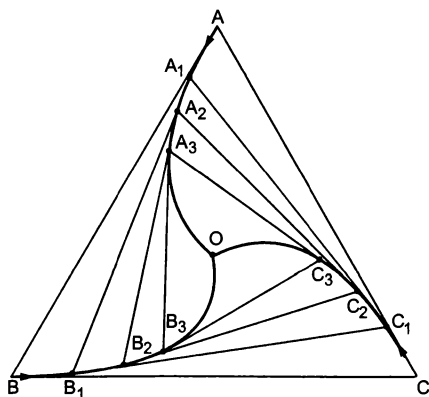

Figure 3-W15

centroid O of the triangle. At any instant the particles will form an equilateral triangle ABC with the same centriod O. Concentrate on the motion of any one

particle, say A. At any instant its velocity makes angle 30° with AO.

The component of this velocity along AO is v cos30°. This component is the rate of decrease of the distance AO. Initially,

$$AO = \frac{2}{3}\sqrt{d^2 - \left(\frac{d}{2}\right)^2} = \frac{d}{\sqrt{3}}.$$

Therefore, the time taken for AO to become zero

$$= \frac{d/\sqrt{3}}{v \cos30°} = \frac{2d}{\sqrt{3}\,v \times \sqrt{3}} = \frac{2d}{3v}.$$

Alternative : Velocity of A is v along AB. The velocity of B is along BC. Its component along BA is v cos 60° $= v/2$. Thus, the separation AB decreases at the rate

$$v + \frac{v}{2} = \frac{3v}{2}.$$

Since this rate is constant, the time taken in reducing the separation AB from d to zero is

$$t = \frac{d}{\frac{3v}{2}} = \frac{2d}{3v}.$$

□

QUESTIONS FOR SHORT ANSWER

1. Galileo was punished by the Church for teaching that the sun is stationary and the earth moves around it. His opponents held the view that the earth is stationary and the sun moves around it. If the absolute motion has no meaning, are the two viewpoints not equally correct or equally wrong ?

2. When a particle moves with constant velocity, its average velocity, its instantaneous velocity and its speed are all equal. Comment on this statement.

3. A car travels at a speed of 60 km/hr due north and the other at a speed of 60 km/hr due east. Are the velocities equal ? If no, which one is greater ? If you find any of the questions irrelevant, explain.

4. A ball is thrown vertically upward with a speed of 20 m/s. Draw a graph showing the velocity of the ball as a function of time as it goes up and then comes back.

5. The velocity of a particle is towards west at an instant. Its acceleration is not towards west, not towards east, not towards north and not towards south. Give an example of this type of motion.

6. At which point on its path a projectile has the smallest speed ?

7. Two particles A and B start from rest and move for equal time on a straight line. The particle A has an acceleration a for the first half of the total time and $2a$ for the second half. The particle B has an acceleration

$2a$ for the first half and a for the second half. Which particle has covered larger distance ?

8. If a particle is accelerating, it is either speeding up or speeding down. Do you agree with this statement ?

9. A food packet is dropped from a plane going at an altitude of 100 m. What is the path of the packet as seen from the plane ? What is the path as seen from the ground ? If someone asks "what is the actual path", what will you answer ?

10. Give examples where (a) the velocity of a particle is zero but its acceleration is not zero, (b) the velocity is opposite in direction to the acceleration, (c) the velocity is perpendicular to the acceleration.

11. Figure (3-Q1) shows the x coordinate of a particle as a function of time. Find the signs of v_x and a_x at $t = t_1$, $t = t_2$ and $t = t_3$.
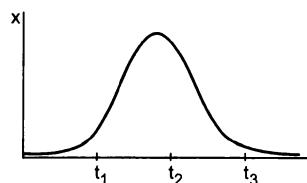
Figure 3-Q1

12. A player hits a baseball at some angle. The ball goes high up in space. The player runs and catches the ball before it hits the ground. Which of the two (the player or the ball) has greater displacement ?

13. The increase in the speed of a car is proportional to the additional petrol put into the engine. Is it possible to accelerate a car without putting more petrol or less petrol into the engine ?

14. Rain is falling vertically. A man running on the road keeps his umbrella tilted but a man standing on the street keeps his umbrella vertical to protect himself from the rain. But both of them keep their umbrella vertical to avoid the vertical sun-rays. Explain.

OBJECTIVE I

1. A motor car is going due north at a speed of 50 km/h. It makes a 90° left turn without changing the speed. The change in the velocity of the car is about
   (a) 50 km/h towards west
   (b) 70 km/h towards south-west
   (c) 70 km/h towards north-west
   (d) zero.

2. Figure (3-Q2) shows the displacement-time graph of a particle moving on the X-axis.
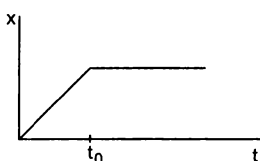
   Figure 3-Q2

   (a) the particle is continuously going in positive x direction
   (b) the particle is at rest
   (c) the velocity increases up to a time t_0, and then becomes constant
   (d) the particle moves at a constant velocity up to a time t_0, and then stops.

3. A particle has a velocity u towards east at $t = 0$. Its acceleration is towards west and is constant. Let x_A and x_B be the magnitude of displacements in the first 10 seconds and the next 10 seconds
   (a) $x_A < x_B$ (b) $x_A = x_B$ (c) $x_A > x_B$
   (d) the information is insufficient to decide the relation of x_A with x_B.

4. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by
   (a) $v = \dfrac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$
   (c) $\dfrac{2}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$ (d) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$.

5. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next equal distance. The average velocity v is given by

   (a) $v = \dfrac{v_1 + v_2}{2}$ (b) $v = \sqrt{v_1 v_2}$
   (c) $\dfrac{2}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$ (d) $\dfrac{1}{v} = \dfrac{1}{v_1} + \dfrac{1}{v_2}$.

6. A stone is released from an elevator going up with an acceleration a. The acceleration of the stone after the release is
   (a) a upward   (b) $(g - a)$ upward
   (c) $(g - a)$ downward   (d) g downward.

7. A person standing near the edge of the top of a building throws two balls A and B. The ball A is thrown vertically upward and B is thrown vertically downward with the same speed. The ball A hits the ground with a speed v_A and the ball B hits the ground with a speed v_B. We have
   (a) $v_A > v_B$, (b) $v_A < v_B$ (c) $v_A = v_B$
   (d) the relation between v_A and v_B depends on height of the building above the ground.

8. In a projectile motion the velocity
   (a) is always perpendicular to the acceleration
   (b) is never perpendicular to the acceleration
   (c) is perpendicular to the acceleration for one instant only
   (d) is perpendicular to the acceleration for two instants.

9. Two bullets are fired simultaneously, horizontally and with different speeds from the same place. Which bullet will hit the ground first ?
   (a) the faster one   (b) the slower one
   (c) both will reach simultaneously
   (d) depends on the masses.

10. The range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be
   (a) 25 m   (b) 37 m   (c) 50 m   (d) 100 m.

11. Two projectiles A and B are projected with angle of projection 15° for the projectile A and 45° for the projectile B. If R_A and R_B be the horizontal range for the two projectiles, then
   (a) $R_A < R_B$ (b) $R_A = R_B$ (c) $R_A > R_B$
   (d) the information is insufficient to decide the relation of R_A with R_B.

12. A river is flowing from west to east at a speed of 5 metres per minute. A man on the south bank of the river, capable of swimming at 10 metres per minute in still water, wants to swim across the river in the shortest time. He should swim in a direction

(a) due north   (b) 30° east of north
(c) 30° north of west   (d) 60° east of north.

13. In the arrangement shown in figure (3-Q3), the ends P and Q of an inextensible string move downwards with uniform speed u. Pulleys A and B are fixed. The mass M moves upwards with a speed
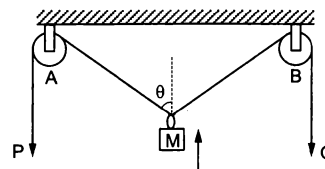(a) $2u\cos\theta$ (b) $u/\cos\theta$ (c) $2u/\cos\theta$ (d) $u\cos\theta$.

Figure 3-Q3

OBJECTIVE II

1. Consider the motion of the tip of the minute hand of a clock. In one hour
(a) the displacement is zero
(b) the distance covered is zero
(c) the average speed is zero
(d) the average velocity is zero

2. A particle moves along the X-axis as
$$x = u(t - 2\text{ s}) + a(t - 2\text{ s})^2.$$
(a) the initial velocity of the particle is u
(b) the acceleration of the particle is a
(c) the acceleration of the particle is $2a$
(d) at $t = 2$ s particle is at the origin.

3. Pick the correct statements :
(a) Average speed of a particle in a given time is never less than the magnitude of the average velocity.
(b) It is possible to have a situation in which $\left|\dfrac{d\vec{v}}{dt}\right| \neq 0$ but $\dfrac{d}{dt}\left|\vec{v}\right| = 0$.
(c) The average velocity of a particle is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval.
(d) The average velocity of a particle moving on a straight line is zero in a time interval. It is possible that the instantaneous velocity is never zero in the interval. (Infinite accelerations are not allowed.)

4. An object may have
(a) varying speed without having varying velocity
(b) varying velocity without having varying speed
(c) nonzero acceleration without having varying velocity
(d) nonzero acceleration without having varying speed.

5. Mark the correct statements for a particle going on a straight line :
(a) If the velocity and acceleration have opposite sign, the object is slowing down.
(b) If the position and velocity have opposite sign, the particle is moving towards the origin.
(c) If the velocity is zero at an instant, the acceleration should also be zero at that instant.
(d) If the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval.

6. The velocity of a particle is zero at $t = 0$.
(a) The acceleration at $t = 0$ must be zero.
(b) The acceleration at $t = 0$ may be zero.
(c) If the acceleration is zero from $t = 0$ to $t = 10$ s, the speed is also zero in this interval.
(d) If the speed is zero from $t = 0$ to $t = 10$ s the acceleration is also zero in this interval.

7. Mark the correct statements :
(a) The magnitude of the velocity of a particle is equal to its speed.
(b) The magnitude of average velocity in an interval is equal to its average speed in that interval.
(c) It is possible to have a situation in which the speed of a particle is always zero but the average speed is not zero.
(d) It is possible to have a situation in which the speed of the particle is never zero but the average speed in an interval is zero.

8. The velocity-time plot for a particle moving on a straight line is shown in the figure (3-Q4).
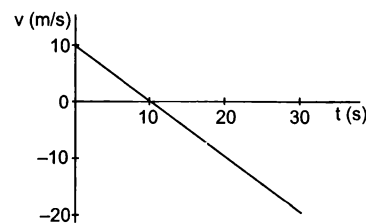
Figure 3-Q4

(a) The particle has a constant acceleration.
(b) The particle has never turned around.
(c) The particle has zero displacement.
(d) The average speed in the interval 0 to 10 s is the same as the average speed in the interval 10 s to 20 s.

9. Figure (3-Q5) shows the position of a particle moving on the X-axis as a function of time.
(a) The particle has come to rest 6 times.
(b) The maximum speed is at $t = 6$ s.
(c) The velocity remains positive for $t = 0$ to $t = 6$ s.
(d) The average velocity for the total period shown is negative.
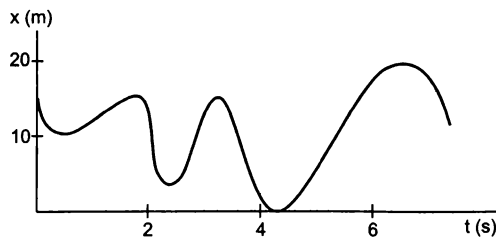
Figure 3-Q5

10. The accelerations of a particle as seen from two frames S_1 and S_2 have equal magnitude 4 m/s^2.
(a) The frames must be at rest with respect to each other.
(b) The frames may be moving with respect to each other but neither should be accelerated with respect to the other.
(c) The acceleration of S_2 with respect to S_1 may either be zero or 8 m/s^2.
(d) The acceleration of S_2 with respect to S_1 may be anything between zero and 8 m/s$_2$.

EXERCISES

1. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field. (a) What distance he has to walk to reach the field ? (b) What is his displacement from his house to the field ?

2. A particle starts from the origin, goes along the X-axis to the point (20 m, 0) and then returns along the same line to the point (–20 m, 0). Find the distance and displacement of the particle during the trip.

3. It is 260 km from Patna to Ranchi by air and 320 km by road. An aeroplane takes 30 minutes to go from Patna to Ranchi whereas a delux bus takes 8 hours. (a) Find the average speed of the plane. (b) Find the average speed of the bus. (c) Find the average velocity of the plane. (d) Find the average velocity of the bus.

4. When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km. (a) What is the average speed of the car during this period ? (b) What is the average velocity ?

5. An athelete takes 2·0 s to reach his maximum speed of 18·0 km/h. What is the magnitude of his average acceleration ?

6. The speed of a car as a function of time is shown in figure (3-E1). Find the distance travelled by the car in 8 seconds and its acceleration.

Figure 3-E1

7. The acceleration of a cart started at $t = 0$, varies with time as shown in figure (3-E2). Find the distance travelled in 30 seconds and draw the position-time graph.

8. Figure (3-E3) shows the graph of velocity versus time for a particle going along the X-axis. Find (a) the
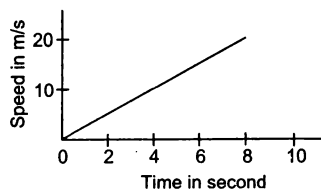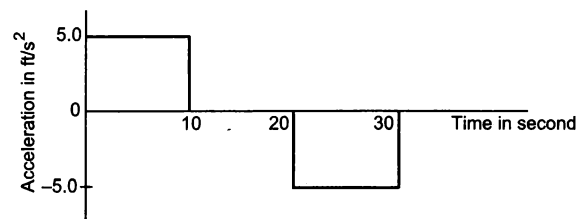
Figure 3-E2

acceleration, (b) the distance travelled in 0 to 10 s and (c) the displacement in 0 to 10 s.
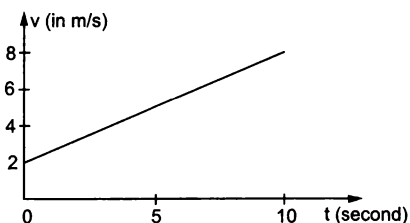
Figure 3-E3

9. Figure (3-E4) shows the graph of the x-coordinate of a particle going along the X-axis as a function of time. Find (a) the average velocity during 0 to 10 s, (b) instantaneous velocity at 2, 5, 8 and 12s.

Figure 3-E4

10. From the velocity–time plot shown in figure (3-E5), find the distance travelled by the particle during the first 40

seconds. Also find the average velocity during this period.
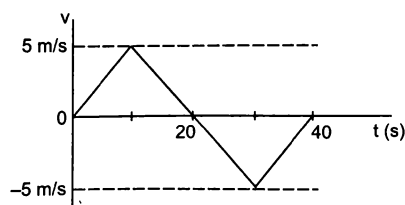
Figure 3-E5

11. Figure (3-E6) shows x-t graph of a particle. Find the time t such that the average velocity of the particle during the period 0 to t is zero.
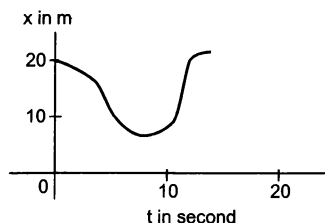
Figure 3-E6

12. A particle starts from a point A and travels along the solid curve shown in figure (3-E7). Find approximately the position B of the particle such that the average velocity between the positions A and B has the same direction as the instantaneous velocity at B.
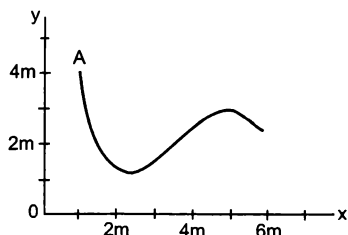
Figure 3-E7

13. An object having a velocity 4·0 m/s is accelerated at the rate of 1·2 m/s² for 5·0 s. Find the distance travelled during the period of acceleration.

14. A person travelling at 43·2 km/h applies the brake giving a deceleration of 6·0 m/s² to his scooter. How far will it travel before stopping ?

15. A train starts from rest and moves with a constant acceleration of 2·0 m/s² for half a minute. The brakes are then applied and the train comes to rest in one minute. Find (a) the total distance moved by the train, (b) the maximum speed attained by the train and (c) the position(s) of the train at half the maximum speed.

16. A bullet travelling with a velocity of 16 m/s penetrates a tree trunk and comes to rest in 0·4 m. Find the time taken during the retardation.

17. A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of 5·0 cm before coming to rest. Find the deceleration.

18. A particle starting from rest moves with constant acceleration. If it takes 5·0 s to reach the speed 18·0 km/h find (a) the average velocity during this period, and (b) the distance travelled by the particle during this period.

19. A driver takes 0·20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6·0 m/s², find the distance travelled by the car after he sees the need to put the brakes on.

20. Complete the following table :

Car Model	Driver X Reaction time 0·20 s	Driver Y Reaction time 0·30 s
A (deceleration on hard braking = 6·0 m/s²)	Speed = 54 km/h Braking distance a = ... Total stopping distance b = ...	Speed = 72 km/h Braking distance c = ... Total stopping distance d = ...
B (deceleration on hard braking = 7·5 m/s²)	Speed = 54 km/h Braking distance e = ... Total stopping distance f = ...	Speed = 72km/h Braking distance g = ... Total stopping distance h = ...

21. A police jeep is chasing a culprit going on a motorbike. The motorbike crosses a turning at a speed of 72 km/h. The jeep follows it at a speed of 90 km/h, crossing the turning ten seconds later than the bike. Assuming that they travel at constant speeds, how far from the turning will the jeep catch up with the bike ?

22. A car travelling at 60 km/h overtakes another car travelling at 42 km/h. Assuming each car to be 5·0 m long, find the time taken during the overtake and the total road distance used for the overtake.

23. A ball is projected vertically upward with a speed of 50 m/s. Find (a) the maximum height, (b) the time to reach the maximum height, (c) the speed at half the maximum height. Take g = 10 m/s².

24. A ball is dropped from a balloon going up at a speed of 7 m/s. If the balloon was at a height 60 m at the time of dropping the ball, how long will the ball take in reaching the ground ?

25. A stone is thrown vertically upward with a speed of 28 m/s. (a) Find the maximum height reached by the stone. (b) Find its velocity one second before it reaches the maximum height. (c) Does the answer of part (b) change if the initial speed is more than 28 m/s such as 40 m/s or 80 m/s ?

26. A person sitting on the top of a tall building is dropping balls at regular intervals of one second. Find the positions of the 3rd, 4th and 5th ball when the 6th ball is being dropped.

27. A healthy youngman standing at a distance of 7 m from a 11·8 m high building sees a kid slipping from the top floor. With what speed (assumed uniform) should he run to catch the kid at the arms height (1·8 m) ?

28. An NCC parade is going at a uniform speed of 6 km/h through a place under a berry tree on which a bird is sitting at a height of 12·1 m. At a particular instant the bird drops a berry. Which cadet (give the distance from the tree at the instant) will receive the berry on his uniform ?

29. A ball is dropped from a height. If it takes 0·200 s to cross the last 6·00 m before hitting the ground, find the height from which it was dropped. Take $g = 10 \text{ m/s}^2$.

30. A ball is dropped from a height of 5 m onto a sandy floor and penetrates the sand up to 10 cm before coming to rest. Find the retardation of the ball in sand assuming it to be uniform.

31. An elevator is descending with uniform acceleration. To measure the acceleration, a person in the elevator drops a coin at the moment the elevator starts. The coin is 6 ft above the floor of the elevator at the time it is dropped. The person observes that the coin strikes the floor in 1 second. Calculate from these data the acceleration of the elevator.

32. A ball is thrown horizontally from a point 100 m above the ground with a speed of 20 m/s. Find (a) the time it takes to reach the ground, (b) the horizontal distance it travels before reaching the ground, (c) the velocity (direction and magnitude) with which it strikes the ground.

33. A ball is thrown at a speed of 40 m/s at an angle of 60° with the horizontal. Find (a) the maximum height reached and (b) the range of the ball. Take $g = 10 \text{ m/s}^2$.

34. In a soccer practice session the football is kept at the centre of the field 40 yards from the 10 ft high goalposts. A goal is attempted by kicking the football at a speed of 64 ft/s at an angle of 45° to the horizontal. Will the ball reach the goal post ?

35. A popular game in Indian villages is *goli* which is played with small glass balls called golis. The goli of one player is situated at a distance of 2·0 m from the goli of the second player. This second player has to project his goli by keeping the thumb of the left hand at the place of his goli, holding the goli between his two middle fingers and making the throw. If the projected goli hits the goli of the first player, the second player wins. If the height from which the goli is projected is 19·6 cm from the ground and the goli is to be projected horizontally, with what speed should it be projected so that it directly hits the stationary goli without falling on the ground earlier ?

36. Figure (3-E8) shows a 11·7 ft wide ditch with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motorbike be moving on the road so that it safely crosses the ditch ?

11.7 ft

15°   15°
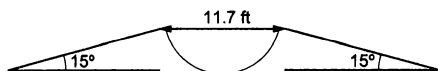
Figure 3-E8

Assume that the length of the bike is 5 ft, and it leaves the road when the front part runs out of the approach road.

37. A person standing on the top of a cliff 171 ft high has to throw a packet to his friend standing on the ground 228 ft horizontally away. If he throws the packet directly aiming at the friend with a speed of 15·0 ft/s, how short will the packet fall ?

38. A ball is projected from a point on the floor with a speed of 15 m/s at an angle of 60° with the horizontal. Will it hit a vertical wall 5 m away from the point of projection and perpendicular to the plane of projection without hitting the floor ? Will the answer differ if the wall is 22 m away ?

39. Find the average velocity of a projectile between the instants it crosses half the maximum height. It is projected with a speed u at an angle θ with the horizontal.

40. A bomb is dropped from a plane flying horizontally with uniform speed. Show that the bomb will explode vertically below the plane. Is the statement true if the plane flies with uniform speed but not horizontally ?

41. A boy standing on a long railroad car throws a ball straight upwards. The car is moving on the horizontal road with an acceleration of 1 m/s^2 and the projection velocity in the vertical direction is 9·8 m/s. How far behind the boy will the ball fall on the car ?

42. A staircase contains three steps each 10 cm high and 20 cm wide (figure 3-E9). What should be the minimum horizontal velocity of a ball rolling off the uppermost plane so as to hit directly the lowest plane ?
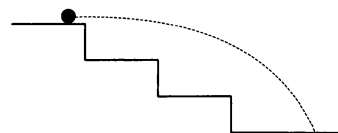

Figure 3-E9

43. A person is standing on a truck moving with a constant velocity of 14·7 m/s on a horizontal road. The man throws a ball in such a way that it returns to the truck after the truck has moved 58·8 m. Find the speed and the angle of projection (a) as seen from the truck, (b) as seen from the road.

44. The benches of a gallery in a cricket stadium are 1 m wide and 1 m high. A batsman strikes the ball at a level one metre above the ground and hits a mammoth sixer. The ball starts at 35 m/s at an angle of 53° with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit ?

45. A man is sitting on the shore of a river. He is in the line of a 1·0 m long boat and is 5·5 m away from the centre of the boat. He wishes to throw an apple into the boat. If he can throw the apple only with a speed of 10 m/s, find the minimum and maximum angles of projection for successful shot. Assume that the point of

projection and the edge of the boat are in the same horizontal level.

46. A river 400 m wide is flowing at a rate of 2·0 m/s. A boat is sailing at a velocity of 10 m/s with respect to the water, in a direction perpendicular to the river. (a) Find the time taken by the boat to reach the opposite bank. (b) How far from the point directly opposite to the starting point does the boat reach the opposite bank ?

47. A swimmer wishes to cross a 500 m wide river flowing at 5 km/h. His speed with respect to water is 3 km/h. (a) If he heads in a direction making an angle θ with the flow, find the time he takes to cross the river. (b) Find the shortest possible time to cross the river.

48. Consider the situation of the previous problem. The man has to reach the other shore at the point directly opposite to his starting point. If he reaches the other shore somewhere else, he has to walk down to this point. Find the minimum distance that he has to walk.

49. An aeroplane has to go from a point A to another point B, 500 km away due 30° east of north. A wind is blowing due north at a speed of 20 m/s. The air-speed of the plane is 150 m/s. (a) Find the direction in which the

pilot should head the plane to reach the point B. (b) Find the time taken by the plane to go from A to B.

50. Two friends A and B are standing a distance x apart in an open field and wind is blowing from A to B. A beats a drum and B hears the sound t_1 time after he sees the event. A and B interchange their positions and the experiment is repeated. This time B hears the drum t_2 time after he sees the event. Calculate the velocity of sound in still air v and the velocity of wind u. Neglect the time light takes in travelling between the friends.

51. Suppose A and B in the previous problem change their positions in such a way that the line joining them becomes perpendicular to the direction of wind while maintaining the separation x. What will be the time lag B finds between seeing and hearing the drum beating by A ?

52. Six particles situated at the corners of a regular hexagon of side a move at a constant speed v. Each particle maintains a direction towards the particle at the next corner. Calculate the time the particles will take to meet each other.

□

ANSWERS

OBJECTIVE I

1. (b)	2. (d)	3. (d)	4. (a)	5. (c)	6. (d)
7. (c)	8. (c)	9. (c)	10. (d)	11. (d)	12. (a)
13. (b)					

OBJECTIVE II

1. (a), (d)	2. (c), (d)	3. (a), (b), (c)
4. (b), (d)	5. (a), (b), (d)	6. (b), (c), (d)
7. (a)	8. (a), (d)	9. (a)
10. (d)		

EXERCISES

1. (a) 110 m   (b) 50 m, $\tan^{-1} 3/4$ north to east
2. 60 m, 20 m in the negative direction
3. (a) 520 km/h   (b) 40 km/h
   (c) 520 km/h Patna to Ranchi
   (d) 32·5 km/h Patna to Ranchi
4. 32 km/h   (b) zero
5. 2·5 m/s^2
6. 80 m, 2·5 m/s^2
7. 1000 ft
8. (a) 0·6 m/s^2 (b) 50 m   (c) 50 m
9. (a) 10 m/s   (b) 20 m/s, zero, 20 m/s, − 20 m/s
10. 100 m, zero
11. 12 s
12. $x = 5$ m, $y = 3$ m
13. 35 m
14. 12 m
15. (a) 2·7 km   (b) 60 m/s   (c) 225 m and 2·25 km
16. 0·05 s
17. $12 \cdot 2 \times 10^5$ m/s^2
18. (a) 2·5 m/s   (b) 12·5 m
19. 22 m
20. (a) 19 m   (b) 22 m   (c) 33 m   (d) 39 m
    (e) 15 m   (f) 18 m   (g) 27 m   (h) 33 m
21. 1·0 km
22. 2 s, 38 m
23. (a) 125 m   (b) 5 s   (c) 35 m/s
24. 4·3 s
25. (a) 40 m   (b) 9·8 m/s   (c) No
26. 44·1 m, 19·6 m and 4·9 m below the top
27. 4·9 m/s
28. 2·62 m
29. 48 m
30. 490 m/s^2
31. 20 ft/s^2
32. (a) 4·5 s   (b) 90 m   (c) 49 m/s, $\theta = 66°$ with horizontal
33. (a) 60 m   (b) 80√3 m
34. Yes
35. 10 m/s
36. 32 ft/s

37. 192 ft

38. Yes, Yes

39. $u \cos\theta$, horizontal in the plane of projection

41. 2 m

42. 2 m/s

43. (a) 19·6 m/s upward
    (b) 24·5 m/s at 53° with horizontal

44. Sixth

45. Minimum angle 15°, maximum angle 75° but there is an interval of 53° between 15° and 75°, which is not allowed for successful shot

46. (a) 40 s   (b) 80 m

47. (a) $\dfrac{10 \text{ minutes}}{\sin\theta}$ (b) 10 minutes

48. 2/3 km

49. (a) $\sin^{-1}(1/15)$ east of the line AB (b) 50 min

50. $\dfrac{x}{2}\left(\dfrac{1}{t_1}+\dfrac{1}{t_2}\right), \dfrac{x}{2}\left(\dfrac{1}{t_1}-\dfrac{1}{t_2}\right)$

51. $\dfrac{x}{\sqrt{v^2-u^2}}$

52. $2\,a/v.$

□

SOLUTIONS TO CONCEPTS
CHAPTER – 3

1. a) Distance travelled = 50 + 40 + 20 = 110 m

    b) AF = AB – BF = AB – DC = 50 – 20 = 30 M

    His displacement is AD

    AD = $\sqrt{AF^2 - DF^2} = \sqrt{30^2 + 40^2} = 50m$

    In $\triangle AED$ tan θ = DE/AE = 30/40 = 3/4

    $\Rightarrow \theta = \tan^{-1}(3/4)$

    His displacement from his house to the field is 50 m,

    $\tan^{-1}(3/4)$ north to east.
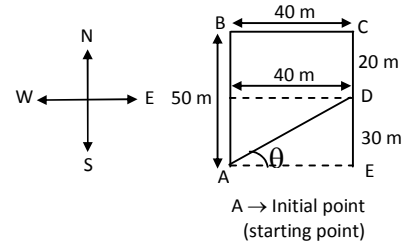
2. O → Starting point origin.

    i) Distance travelled = 20 + 20 + 20 = 60 m

    ii) Displacement is only OB = 20 m in the negative direction.

    Displacement → Distance between final and initial position.
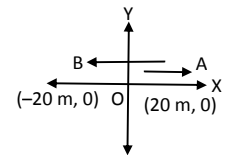
3. a) V_{ave} of plane (Distance/Time) = 260/0.5 = 520 km/hr.

    b) V_{ave} of bus = 320/8 = 40 km/hr.

    c) plane goes in straight path

    velocity = $\vec{V}_{ave}$ = 260/0.5 = 520 km/hr.

    d) Straight path distance between plane to Ranchi is equal to the displacement of bus.

    ∴ Velocity = $\vec{V}_{ave}$ = 260/8 = 32.5 km/hr.

4. a) Total distance covered 12416 – 12352 = 64 km in 2 hours.

    Speed = 64/2 = 32 km/h

    b) As he returns to his house, the displacement is zero.

    Velocity = (displacement/time) = 0 (zero).

5. Initial velocity u = 0 (∴ starts from rest)

    Final velocity v = 18 km/hr = 5 sec

    (i.e. max velocity)

    Time interval t = 2 sec.

    ∴ Acceleration = $a_{ave} = \dfrac{v - u}{t} = \dfrac{5}{2}$ = 2.5 m/s².

6. In the interval 8 sec the velocity changes from 0 to 20 m/s.

    Average acceleration = 20/8 = 2.5 m/s² $\left(\dfrac{\text{change in velocity}}{\text{time}} \right)$

    Distance travelled S = ut + 1/2 at²

    $\Rightarrow 0 + 1/2(2.5)8^2$ = 80 m.

7. In 1st 10 sec S_1 = ut + 1/2 at² $\Rightarrow 0 + (1/2 \times 5 \times 10^2)$ = 250 ft.

    At 10 sec v = u + at = 0 + 5 × 10 = 50 ft/sec.

    ∴ From 10 to 20 sec (Δt = 20 – 10 = 10 sec) it moves with uniform
    velocity 50 ft/sec,

3.1
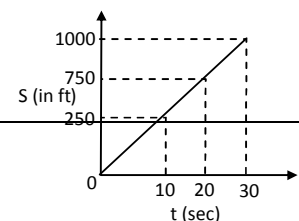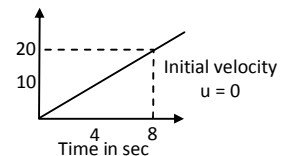
Distance $S_2 = 50 \times 10 = 500$ ft

Between 20 sec to 30 sec acceleration is constant i.e. -5 ft/s^2. At 20 sec velocity is 50 ft/sec.

t = 30 − 20 = 10 s

$S_3 = ut + 1/2\ at^2$

$= 50 \times 10 + (1/2)(-5)(10)^2 = 250$ m

Total distance travelled is 30 sec = $S_1 + S_2 + S_3 = 250 + 500 + 250 = 1000$ ft.

8. a) Initial velocity u = 2 m/s.

    final velocity v = 8 m/s

    time = 10 sec,

    acceleration = $\dfrac{v-u}{ta} = \dfrac{8-2}{10} = 0.6$ m/s^2

    b) $v^2 - u^2 = 2aS$

    $\Rightarrow$ Distance S = $\dfrac{v^2-u^2}{2a} = \dfrac{8^2-2^2}{2\times0.6} = 50$ m.

    c) Displacement is same as distance travelled.

    Displacement = 50 m.

9. a) Displacement in 0 to 10 sec is 1000 m.

    time = 10 sec.

    $V_{ave} = s/t = 100/10 = 10$ m/s.

    b) At 2 sec it is moving with uniform velocity 50/2.5 = 20 m/s.

    at 2 sec. $V_{inst} = 20$ m/s.

    At 5 sec it is at rest.

        V_{inst} = zero.

    At 8 sec it is moving with uniform velocity 20 m/s

        $V_{inst} = 20$ m/s

    At 12 sec velocity is negative as it move towards initial position. $V_{inst} = -20$ m/s.

10. Distance in first 40 sec is, Δ OAB + ΔBCD

$= \dfrac{1}{2} \times 5 \times 20 + \dfrac{1}{2} \times 5 \times 20 = 100$ m.

Average velocity is 0 as the displacement is zero.

11. Consider the point B, at t = 12 sec

At t = 0 ; s = 20 m

and t = 12 sec s = 20 m

So for time interval 0 to 12 sec

Change in displacement is zero.

So, average velocity = displacement/ time = 0

$\therefore$ The time is 12 sec.

12. At position B instantaneous velocity has direction along $\overrightarrow{BC}$. For average velocity between A and B.

V_{ave} = displacement / time = $(\overrightarrow{AB}/t)$ t = time

3.2
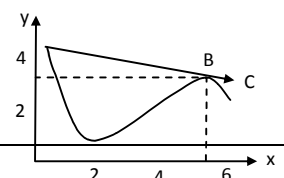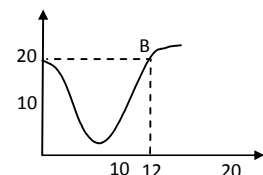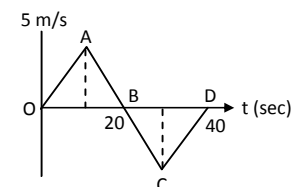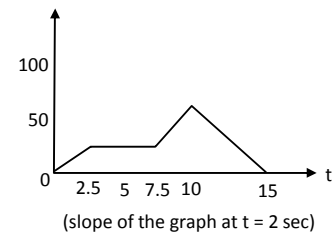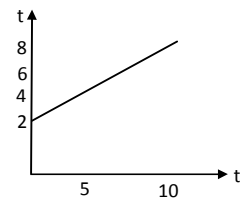
We can see that $\overrightarrow{AB}$ is along $\overrightarrow{BC}$ i.e. they are in same direction.

The point is B (5m, 3m).

13. u = 4 m/s, a = 1.2 m/s^2, t = 5 sec

Distance = s = $ut + \dfrac{1}{2}at^2$

= 4(5) + 1/2 (1.2)5^2 = 35 m.

14. Initial velocity u = 43.2 km/hr = 12 m/s

u = 12 m/s, v = 0

a = −6 m/s^2 (deceleration)

Distance S = $\dfrac{v^2 - u^2}{2(-6)}$ = 12 m

15. Initial velocity u = 0

    Acceleration a = 2 m/s². Let final velocity be v (before applying breaks)

    t = 30 sec

    v = u + at $\Rightarrow$ 0 + 2 × 30 = 60 m/s

    a) $S_1 = ut + \frac{1}{2}at^2$ = 900 m

       when breaks are applied u' = 60 m/s

       v' = 0, t = 60 sec (1 min)

       Declaration a' = (v – u)/t = = (0 – 60)/60 = –1 m/s².

       $S_2 = \frac{v'^2 - u'^2}{2a'}$ = 1800 m

       Total S = S_1 + S_2 = 1800 + 900 = 2700 m = 2.7 km.

    b) The maximum speed attained by train v = 60 m/s

    c) Half the maximum speed = 60/2= 30 m/s

       Distance S = $\frac{v^2 - u^2}{2a}$ = $\frac{30^2 - 0^2}{2 \times 2}$ = 225 m from starting point

       When it accelerates the distance travelled is 900 m. Then again declarates and attain 30 m/s.

       $\therefore$ u = 60 m/s, v = 30 m/s, a = –1 m/s²

       Distance = $\frac{v^2 - u^2}{2a}$ = $\frac{30^2 - 60^2}{2(-1)}$ = 1350 m

       Position is 900 + 1350 = 2250 = 2.25 km from starting point.

16. u = 16 m/s (initial), v = 0, s = 0.4 m.

    Deceleration a = $\frac{v^2 - u^2}{2s}$ = –320 m/s².

    Time = t = $\frac{v - u}{a}$ = $\frac{0 - 16}{-320}$ = 0.05 sec.

17. u = 350 m/s, s = 5 cm = 0.05 m, v = 0

    Deceleration = a = $\frac{v^2 - u^2}{2s}$ = $\frac{0 - (350)^2}{2 \times 0.05}$ = –12.2 × 10⁵ m/s².

    Deceleration is 12.2 × 10⁵ m/s².

18. u = 0, v = 18 km/hr = 5 m/s, t = 5 sec

    a = $\frac{v - u}{t}$ = $\frac{5 - 0}{5}$ = 1 m/s².

    s = $ut + \frac{1}{2}at^2$ = 12.5 m

    a) Average velocity V_{ave} = (12.5)/5 = 2.5 m/s.

    b) Distance travelled is 12.5 m.

19. In reaction time the body moves with the speed 54 km/hr = 15 m/sec (constant speed)

    Distance travelled in this time is S_1 = 15 × 0.2 = 3 m.

    When brakes are applied,

       u = 15 m/s, v = 0, a = –6 m/s² (deceleration)

$$S_2 = \frac{v^2 - u^2}{2a} = \frac{0 - 15^2}{2(-6)} = 18.75 \text{ m}$$

Total distance $s = s_1 + s_2 = 3 + 18.75 = 21.75 = 22$ m.

20.

	Driver X Reaction time 0.25	Driver Y Reaction time 0.35
A (deceleration on hard braking = 6 m/s^2)	Speed = 54 km/h Braking distance a= 19 m Total stopping distance b = 22 m	Speed = 72 km/h Braking distance c = 33 m Total stopping distance d = 39 m.
B (deceleration on hard braking = 7.5 m/s^2)	Speed = 54 km/h Braking distance e = 15 m Total stopping distance f = 18 m	Speed = 72 km/h Braking distance g = 27 m Total stopping distance h = 33 m.

   $a = \dfrac{0^2 - 15^2}{2(-6)} = 19$ m

So, b = 0.2 × 15 + 19 = 33 m

Similarly other can be calculated.

Braking distance : Distance travelled when brakes are applied.

Total stopping distance = Braking distance + distance travelled in reaction time.

21. V_P = 90 km/h = 25 m/s.

V_C = 72 km/h = 20 m/s.

In 10 sec culprit reaches at point B from A.

Distance converted by culprit S = vt = 20 × 10 = 200 m.

At time t = 10 sec the police jeep is 200 m behind the culprit.

Time = s/v = 200 / 5 = 40 s. (Relative velocity is considered).

In 40 s the police jeep will move from A to a distance S, where

S = vt = 25 × 40 = 1000 m = 1.0 km away.

∴ The jeep will catch up with the bike, 1 km far from the turning.
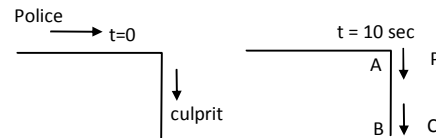
22. v_1 = 60 km/hr = 16.6 m/s.

v_2 = 42 km/h = 11.6 m/s.

Relative velocity between the cars = (16.6 – 11.6) = 5 m/s.

Distance to be travelled by first car is 5 + t = 10 m.

Time = t = s/v = 0/5 = 2 sec to cross the 2nd car.

In 2 sec the 1st car moved = 16.6 × 2 = 33.2 m

H also covered its own length 5 m.

∴ Total road distance used for the overtake = 33.2 + 5 = 38 m.
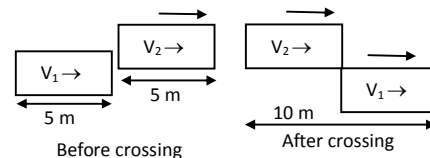
23. u = 50 m/s, g = –10 m/s^2 when moving upward, v = 0 (at highest point).

a) $S = \dfrac{v^2 - u^2}{2a} = \dfrac{0 - 50^2}{2(-10)} = 125$ m

   maximum height reached = 125 m

b) t = (v – u)/a = (0 – 50)/–10 = 5 sec

c) s' = 125/2 = 62.5 m, u = 50 m/s, a = –10 m/s^2,

$v^2 - u^2 = 2as$

$\Rightarrow v = \sqrt{(u^2 + 2as)} = \sqrt{50^2 + 2(-10)(62.5)} = 35$ m/s.

24. Initially the ball is going upward

   u = –7 m/s, s = 60 m, a = g = 10 m/s^2

   $s = ut + \dfrac{1}{2}at^2 \Rightarrow 60 = -7t + 1/2 \ 10t^2$

   $\Rightarrow 5t^2 - 7t - 60 = 0$

   $t = \dfrac{7 \pm \sqrt{49 - 4.5(-60)}}{2 \times 5} = \dfrac{7 \pm 35.34}{10}$

   taking positive sign t = $\dfrac{7 + 35.34}{10}$ = 4.2 sec ($\because$ t $\neq$ –ve)

   Therefore, the ball will take 4.2 sec to reach the ground.

25. u = 28 m/s, v = 0, a = –g = –9.8 m/s^2

   a) S = $\dfrac{v^2 - u^2}{2a} = \dfrac{0^2 - 28^2}{2(9.8)}$ = 40 m

   b) time t = $\dfrac{v - u}{a} = \dfrac{0 - 28}{-9.8}$ = 2.85

      t' = 2.85 – 1 = 1.85

      v' = u + at' = 28 – (9.8) (1.85) = 9.87 m/s.

      $\therefore$ The velocity is 9.87 m/s.

   c) No it will not change. As after one second velocity becomes zero for any initial velocity and deceleration is g = 9.8 m/s^2 remains same. Fro initial velocity more than 28 m/s max height increases.

26. For every ball, u = 0, a = g = 9.8 m/s^2

   $\therefore$ 4th ball move for 2 sec, 5th ball 1 sec and 3rd ball 3 sec when 6th ball is being dropped.

   For 3rd ball t = 3 sec

   $S_3 = ut + \dfrac{1}{2}at^2 = 0 + 1/2 \ (9.8)3^2$ = 4.9 m below the top.

   For 4th ball, t = 2 sec

   $S_2 = 0 + 1/2 \ gt^2 = 1/2 \ (9.8)2^2$ = 19.6 m below the top (u = 0)

   For 5th ball, t = 1 sec

   $S_3 = ut + 1/2 \ at^2 = 0 + 1/2 \ (9.8)t^2$ = 4.98 m below the top.

27. At point B (i.e. over 1.8 m from ground) the kid should be caught.

   For kid initial velocity u = 0

   Acceleration = 9.8 m/s^2

   Distance S = 11.8 – 1.8 = 10 m

   $S = ut + \dfrac{1}{2}at^2 \Rightarrow 10 = 0 + 1/2 \ (9.8)t^2$

   $\Rightarrow t^2 = 2.04 \Rightarrow t = 1.42$.

   In this time the man has to reach at the bottom of the building.

   Velocity s/t = 7/1.42 = 4.9 m/s.

28. Let the true of fall be 't' initial velocity u = 0
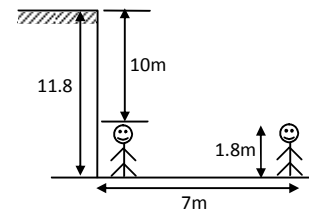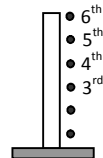
Acceleration a = 9.8 m/s^2

Distance S = 12/1 m

$\therefore$ S = $ut + \dfrac{1}{2}at^2$

$\Rightarrow$ 12.1 = 0 + 1/2 (9.8) × t^2

$\Rightarrow t^2 = \dfrac{12.1}{4.9}$ = 2.46 $\Rightarrow$ t = 1.57 sec

For cadet velocity = 6 km/hr = 1.66 m/sec

Distance = vt = 1.57 × 1.66 = 2.6 m.

The cadet, 2.6 m away from tree will receive the berry on his uniform.
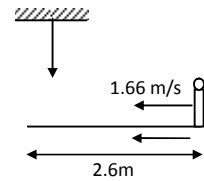
29. For last 6 m distance travelled s = 6 m, u = ?

t = 0.2 sec, a = g = 9.8 m/s^2

S = $ut + \dfrac{1}{2}at^2 \Rightarrow$ 6 = u(0.2) + 4.9 × 0.04

$\Rightarrow$ u = 5.8/0.2 = 29 m/s.

For distance x, u = 0, v = 29 m/s, a = g = 9.8 m/s^2

S = $\dfrac{v^2 - u^2}{2a} = \dfrac{29^2 - 0^2}{2 \times 9.8}$ = 42.05 m

Total distance = 42.05 + 6 = 48.05 = 48 m.

30. Consider the motion of ball form A to B.

B → just above the sand (just to penetrate)

u = 0, a = 9.8 m/s^2, s = 5 m

S = $ut + \dfrac{1}{2}at^2$

$\Rightarrow$ 5 = 0 + 1/2 (9.8)t^2

$\Rightarrow t^2$ = 5/4.9 = 1.02 $\Rightarrow$ t = 1.01.

$\therefore$ velocity at B, v = u + at = 9.8 × 1.01 (u = 0) =9.89 m/s.

From motion of ball in sand

u$_1$ = 9.89 m/s, v$_1$ = 0, a = ?, s = 10 cm = 0.1 m.

a = $\dfrac{v_1^2 - u_1^2}{2s} = \dfrac{0 - (9.89)^2}{2 \times 0.1}$ = − 490 m/s^2

The retardation in sand is 490 m/s^2.

31. For elevator and coin u = 0

As the elevator descends downward with acceleration a' (say)

The coin has to move more distance than 1.8 m to strike the floor. Time taken t = 1 sec.

S$_c$ = $ut + \dfrac{1}{2}a't^2$ = 0 + 1/2 g(1)^2 = 1/2 g

S$_e$ = $ut + \dfrac{1}{2}at^2$ = u + 1/2 a(1)^2 = 1/2 a

Total distance covered by coin is given by = 1.8 + 1/2 a = 1/2 g

$\Rightarrow$ 1.8 +a/2 = 9.8/2 = 4.9

$\Rightarrow$ a = 6.2 m/s^2 = 6.2 × 3.28 = 20.34 ft/s^2.
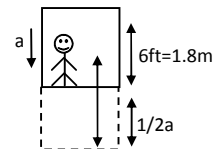
32. It is a case of projectile fired horizontally from a height.
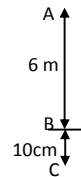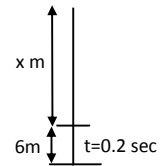
h = 100 m, g = 9.8 m/s^2

a) Time taken to reach the ground t = $\sqrt{(2h/g)}$

$= \sqrt{\dfrac{2 \times 100}{9.8}}$ = 4.51 sec.

b) Horizontal range x = ut = 20 × 4.5 = 90 m.

c) Horizontal velocity remains constant through out the motion.

   At A, V = 20 m/s

   A V_y = u + at = 0 + 9.8 × 4.5 = 44.1 m/s.

   Resultant velocity V_r = $\sqrt{(44.1)^2 + 20^2}$ = 48.42 m/s.

   Tan β = $\dfrac{V_y}{V_x} = \dfrac{44.1}{20}$ = 2.205

   $\Rightarrow$ β = tan^{-1} (2.205) = 60°.

   The ball strikes the ground with a velocity 48.42 m/s at an angle 66° with horizontal.
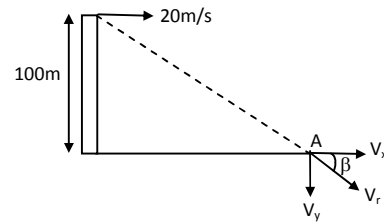
33. u = 40 m/s, a = g= 9.8 m/s^2, θ = 60° Angle of projection.

   a) Maximum height h = $\dfrac{u^2 \sin^2 \theta}{2g} = \dfrac{40^2 (\sin 60°)^2}{2 \times 10}$ = 60 m

   b) Horizontal range X = (u^2 sin 2θ) / g = (40^2 sin 2(60°)) / 10 = $80\sqrt{3}$ m.

34. $g = 9.8$ m/s^2, 32.2 ft/s^2 ; 40 yd = 120 ft

    horizontal range x = 120 ft, u = 64 ft/s, $\theta = 45°$

    We know that horizontal range X = u cos θt

    $$\Rightarrow t = \frac{x}{u\cos\theta} = \frac{120}{64\cos 45°} = 2.65 \text{ sec.}$$

    y = u sin θ(t) $-$ 1/2 gt^2 = $64\dfrac{1}{\sqrt{2}(2.65)} - \dfrac{1}{2}(32.2)(2.65)^2$

    = 7.08 ft which is less than the height of goal post.

    In time 2.65, the ball travels horizontal distance 120 ft (40 yd) and vertical height 7.08 ft which is less than 10 ft. The ball will reach the goal post.
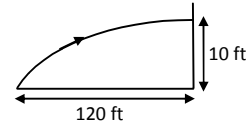
35. The goli move like a projectile.

    Here h = 0.196 m

    Horizontal distance X = 2 m

    Acceleration g = 9.8 m/s^2.

    Time to reach the ground i.e.

    $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 0.196}{9.8}} = 0.2 \text{ sec}$$

    Horizontal velocity with which it is projected be u.

    $\therefore$ x = ut

    $$\Rightarrow u = \frac{x}{t} = \frac{2}{0.2} = 10 \text{ m/s.}$$
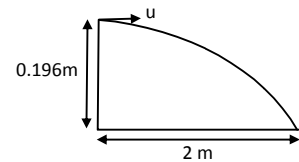
36. Horizontal range X = 11.7 + 5 = 16.7 ft covered by te bike.

    g = 9.8 m/s^2 = 32.2 ft/s^2.

    $$y = x \tan\theta - \frac{gx^2\sec^2\theta}{2u^2}$$

    To find, minimum speed for just crossing, the ditch

    y = 0 ($\therefore$ A is on the x axis)

    $$\Rightarrow x\tan\theta = \frac{gx^2\sec^2\theta}{2u^2} \Rightarrow u^2 = \frac{gx^2\sec^2\theta}{2x\tan\theta} = \frac{gx}{2\sin\theta\cos\theta} = \frac{gx}{\sin 2\theta}$$

    $$\Rightarrow u = \sqrt{\frac{(32.2)(16.7)}{1/2}} \text{ (because sin 30° = 1/2)}$$

    $\Rightarrow$ u = 32.79 ft/s = 32 ft/s.
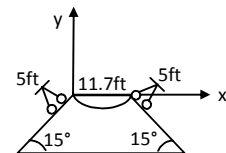
37. tan θ = 171/228 $\Rightarrow \theta = \tan^{-1}(171/228)$

    The motion of projectile (i.e. the packed) is from A. Taken reference axis at A.

    $\therefore \theta = -37°$ as u is below x-axis.

    u = 15 ft/s, g = 32.2 ft/s^2, y = $-$171 ft

    $$y = x\tan\theta - \frac{x^2 g\sec^2\theta}{2u^2}$$

    $$\therefore -171 = -x\,(0.7536) - \frac{x^2 g(1.568)}{2(225)}$$

    $\Rightarrow 0.1125x^2 + 0.7536\,x - 171 = 0$

    x = 35.78 ft (can be calculated)

3.10
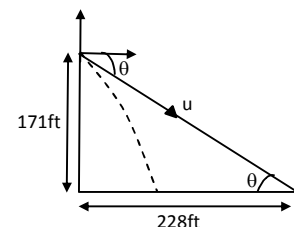
Horizontal range covered by the packet is 35.78 ft.

So, the packet will fall 228 − 35.78 = 192 ft short of his friend.

38. Here u = 15 m/s, θ = 60°, g = 9.8 m/s^2

    Horizontal range X = $\dfrac{u^2 \sin 2\theta}{g} = \dfrac{(15)^2 \sin(2 \times 60°)}{9.8}$ = 19.88 m

    In first case the wall is 5 m away from projection point, so it is in the horizontal range of projectile. So the ball will hit the wall. In second case (22 m away) wall is not within the horizontal range. So the ball would not hit the wall.

39. Total of flight T = $\dfrac{2u \sin\theta}{g}$

    Average velocity = $\dfrac{\text{change in displacement}}{\text{time}}$
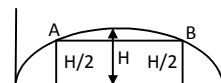
    From the figure, it can be said AB is horizontal. So there is no effect of vertical component of the velocity during this displacement.

    So because the body moves at a constant speed of 'u cos θ' in horizontal direction.

    The average velocity during this displacement will be u cos θ in the horizontal direction.

40. During the motion of bomb its horizontal velocity u remains constant and is same

    as that of aeroplane at every point of its path. Suppose the bomb explode i.e. reach the ground in time t. Distance travelled in horizontal direction by bomb = ut = the distance travelled by aeroplane. So bomb explode vertically below the aeroplane.
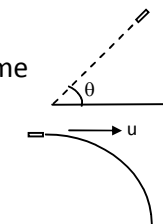
    Suppose the aeroplane move making angle θ with horizontal. For both bomb and aeroplane, horizontal distance is u cos θ t. t is time for bomb to reach the ground.

    So in this case also, the bomb will explode vertically below aeroplane.

41. Let the velocity of car be u when the ball is thrown. Initial velocity of car is = Horizontal velocity of ball.

    Distance travelled by ball B S_b = ut (in horizontal direction)

    And by car S_c = ut + 1/2 at^2 where t → time of flight of ball in air.

    ∴ Car has travelled extra distance $S_c - S_b$ = 1/2 at^2.

    Ball can be considered as a projectile having θ = 90°.

    ∴ t = $\dfrac{2u \sin\theta}{g} = \dfrac{2 \times 9.8}{9.8}$ = 2 sec.

    ∴ $S_c - S_b$ = 1/2 at^2 = 2 m

    ∴ The ball will drop 2m behind the boy.
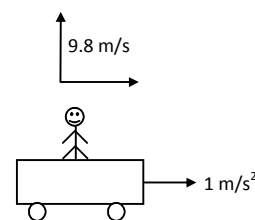
42. At minimum velocity it will move just touching point E reaching the ground.

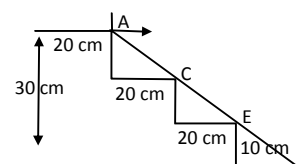   A is origin of reference coordinate.

    If u is the minimum speed.

    X = 40, Y = −20, θ = 0°

    ∴ Y = $x \tan\theta - g\dfrac{x^2 \sec^2\theta}{2u^2}$ (because g = 10 m/s^2 = 1000 cm/s^2)

    $\Rightarrow -20 = x \tan\theta - \dfrac{1000 \times 40^2 \times 1}{2u^2}$

3.12

$\Rightarrow$ u = 200 cm/s = 2 m/s.

$\therefore$ The minimum horizontal velocity is 2 m/s.

43. a) As seen from the truck the ball moves vertically upward comes back. Time taken = time taken by truck to cover 58.8 m.

$\therefore$ time = $\dfrac{s}{v} = \dfrac{58.8}{14.7}$ = 4 sec. (V = 14.7 m/s of truck)

u = ?, v = 0, g = –9.8 m/s^2 (going upward), t = 4/2 = 2 sec.

v = u + at $\Rightarrow$ 0 = u – 9.8 × 2 $\Rightarrow$ u = 19.6 m/s. (vertical upward velocity).

b) From road it seems to be projectile motion.

Total time of flight = 4 sec

In this time horizontal range covered 58.8 m = x

$\therefore$ X = u cos θ t

$\Rightarrow$ u cos θ = 14.7   ...(1)

Taking vertical component of velocity into consideration.

y = $\dfrac{0^2 - (19.6)^2}{2 \times (-9.8)}$ = 19.6 m [from (a)]

$\therefore$ y = u sin θ t – 1/2 gt^2

$\Rightarrow$ 19.6 = u sin θ (2) – 1/2 (9.8)2^2 $\Rightarrow$ 2u sin θ = 19.6 × 2

$\Rightarrow$ u sin θ = 19.6   ...(ii)

$\dfrac{u \sin \theta}{u \cos \theta}$ = tan θ $\Rightarrow$ $\dfrac{19.6}{14.7}$ = 1.333

$\Rightarrow$ θ = tan^{-1} (1.333) = 53°

Again u cos θ = 14.7

$\Rightarrow$ u = $\dfrac{14.7}{u \cos 53°}$ = 24.42 m/s.

The speed of ball is 42.42 m/s at an angle 53° with horizontal as seen from the road.

44. θ = 53°, so cos 53° = 3/5

Sec^2 θ = 25/9 and tan θ = 4/3

Suppose the ball lands on nth bench

So, y = (n – 1)1   ...(1)   [ball starting point 1 m above ground]

Again y = x tan θ – $\dfrac{gx^2 \sec^2 \theta}{2u^2}$ [x = 110 + n – 1 = 110 + y]

$\Rightarrow$ y = (110 + y)(4/3) – $\dfrac{10(110 + y)^2 (25/9)}{2 \times 35^2}$

$\Rightarrow$ $\dfrac{440}{3} + \dfrac{4}{3} y - \dfrac{250(110 + y)^2}{18 \times 35^2}$
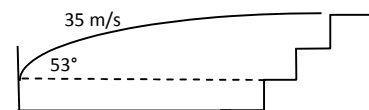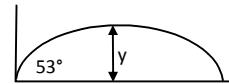
From the equation, y can be calculated.

$\therefore$ y = 5

$\Rightarrow$ n – 1 = 5 $\Rightarrow$ n = 6.

The ball will drop in sixth bench.

45. When the apple just touches the end B of the boat.

x = 5 m, u = 10 m/s, g = 10 m/s^2, θ = ?

$$x = \frac{u^2 \sin 2\theta}{g}$$

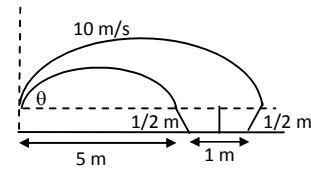$$\Rightarrow 5 = \frac{10^2 \sin 2\theta}{10} \Rightarrow 5 = 10 \sin 2\theta$$

$\Rightarrow \sin 2\theta = 1/2 \Rightarrow \sin 30°$ or $\sin 150°$

$\Rightarrow \theta = 15°$ or $75°$

Similarly for end C, x = 6 m

Then $2\theta_1 = \sin^{-1}(gx/u^2) = \sin^{-1}(0.6) = 182°$ or $71°$.

So, for a successful shot, θ may very from $15°$ to $18°$ or $71°$ to $75°$.

46. a) Here the boat moves with the resultant velocity R. But the vertical component 10 m/s takes him to the opposite shore.

Tan θ = 2/10 = 1/5

Velocity = 10 m/s

distance = 400 m

Time = 400/10 = 40 sec.

b) The boat will reach at point C.

In $\triangle ABC$, tan θ = $\dfrac{BC}{AB} = \dfrac{BC}{400} = \dfrac{1}{5}$

$\Rightarrow$ BC = 400/5 = 80 m.
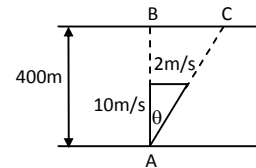
47. a) The vertical component 3 sin θ takes him to opposite side.

Distance = 0.5 km, velocity = 3 sin θ km/h

Time = $\dfrac{\text{Distance}}{\text{Velocity}} = \dfrac{0.5}{3\sin\theta}$ hr

= 10/sinθ min.
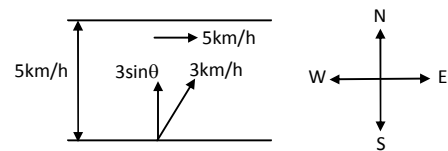
b) Here vertical component of velocity i.e. 3 km/hr takes him to opposite side.

Time = $\dfrac{\text{Distance}}{\text{Velocity}} = \dfrac{0.5}{3} = 0.16$ hr

$\therefore$ 0.16 hr = 60 × 0.16 = 9.6 = 10 minute.
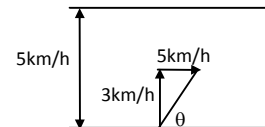
48. Velocity of man $\bar{V}_m$ = 3 km/hr

BD horizontal distance for resultant velocity R.

X-component of resultant R_x = 5 + 3 cos θ

t = 0.5 / 3sinθ
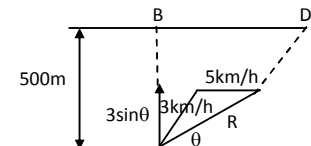
which is same for horizontal component of velocity.

H = BD = (5 + 3 cos θ) (0.5 / 3 sin θ) = $\dfrac{5 + 3\cos\theta}{6\sin\theta}$

For H to be min (dH/dθ) = 0

$\Rightarrow \dfrac{d}{d\theta}\left(\dfrac{5 + 3\cos\theta}{6\sin\theta}\right) = 0$

$\Rightarrow -18(\sin^2\theta + \cos^2\theta) - 30\cos\theta = 0$

$\Rightarrow -30\cos\theta = 18 \Rightarrow \cos\theta = -18/30 = -3/5$

3.14

Sin θ = $\sqrt{1 - \cos^2 \theta}$ = 4/5

∴ H = $\dfrac{5 + 3\cos\theta}{6\sin\theta} = \dfrac{5 + 3(-3/5)}{6 \times (4/5)} = \dfrac{2}{3}$ km.

49. In resultant direction $\vec{R}$ the plane reach the point B.

Velocity of wind $\vec{V}_w$ = 20 m/s

Velocity of aeroplane $\vec{V}_a$ = 150 m/s

In ∆ACD according to sine formula

∴ $\dfrac{20}{\sin A} = \dfrac{150}{\sin 30°} \Rightarrow \sin A = \dfrac{20}{150}\sin 30° = \dfrac{20}{150} \times \dfrac{1}{2} = \dfrac{1}{15}$

⇒ A = $\sin^{-1}$ (1/15)

a) The direction is $\sin^{-1}$ (1/15) east of the line AB.

b) $\sin^{-1}$ (1/15) = 3°48′

⇒ 30° + 3°48′ = 33°48′

R = $\sqrt{150^2 + 20^2 + 2(150)20\cos 33°48′}$ = 167 m/s.

Time = $\dfrac{s}{v} = \dfrac{500000}{167}$ = 2994 sec = 49 = 50 min.
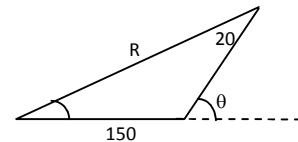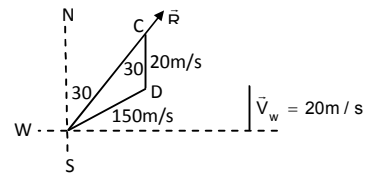
50. Velocity of sound v, Velocity of air u, Distance between A and B be x.

In the first case, resultant velocity of sound = v + u

⇒ (v + u) t_1 = x

⇒ v + u = x/t_1 …(1)

In the second case, resultant velocity of sound = v − u

∴ (v − u) t_2 = x

⇒ v − u = x/t_2 …(2)
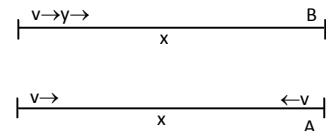
From (1) and (2) 2v = $\dfrac{x}{t_1} + \dfrac{x}{t_2} = x\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

⇒ v = $\dfrac{x}{2}\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

From (i) u = $\dfrac{x}{t_1} - v = \dfrac{x}{t_1} - \left(\dfrac{x}{2t_1} + \dfrac{x}{2t_2}\right) = \dfrac{x}{2}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$

∴ Velocity of air V = $\dfrac{x}{2}\left(\dfrac{1}{t_1} + \dfrac{1}{t_2}\right)$

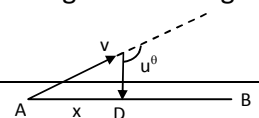And velocity of wind u = $\dfrac{x}{2}\left(\dfrac{1}{t_1} - \dfrac{1}{t_2}\right)$

51. Velocity of sound v, velocity of air u

Velocity of sound be in direction AC so it can reach B with resultant velocity AD.

Angle between v and u is θ > π/2.

Resultant $\overrightarrow{AD} = \sqrt{(v^2 - u^2)}$

Here time taken by light to reach B is neglected. So time lag between seeing and hearing = time to here the drum sound.

$$t = \frac{\text{Displacement}}{\text{velocity}} = \frac{x}{\sqrt{v^2 - u^2}}$$

$$\Rightarrow \frac{x}{\sqrt{(v+u)(v-u)}} = \frac{x}{\sqrt{(x/t_1)(x/t_2)}} \quad \text{[from question no. 50]}$$
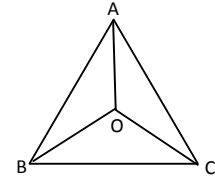
$$= \sqrt{t_1 t_2} \ .$$

52. The particles meet at the centroid O of the triangle. At any instant the particles will form an equilateral ΔABC with the same centroid.

    Consider the motion of particle A. At any instant its velocity makes angle 30°. This component is the rate of decrease of the distance AO.

    Initially AO = $\frac{2}{3}\sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{a}{\sqrt{3}}$

    Therefore, the time taken for AO to become zero.

    $$= \frac{a/\sqrt{3}}{v\cos 30°} = \frac{2a}{\sqrt{3}v \times \sqrt{3}} = \frac{2a}{3v} \ .$$

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