Solution : Suppose, the temperature of the water in the smaller vessel is θ at time t. In the next time interval dt, a heat ΔQ is transferred to it where

$$\Delta Q = \frac{KA}{L}(\theta_0 - \theta)\,dt. \qquad \text{... (i)}$$

This heat increases the temperature of the water of mass m to $\theta + d\theta$ where

$$\Delta Q = ms\,d\theta. \qquad \text{... (ii)}$$

From (i) and (ii),

$$\frac{KA}{L}(\theta_0 - \theta)\,dt = ms\,d\theta$$

or, $$dt = \frac{Lms}{KA}\frac{d\theta}{\theta_0 - \theta}$$

or, $$\int_0^T dt = \frac{Lms}{KA}\int_{\theta_1}^{\theta_2}\frac{d\theta}{\theta_0 - \theta}$$

where T is the time required for the temperature of the water to become θ_2.

Thus, $$T = \frac{Lms}{KA}\ln\frac{\theta_0 - \theta_1}{\theta_0 - \theta_2}.$$

13. *One mole of an ideal monatomic gas is kept in a rigid vessel. The vessel is kept inside a steam chamber whose tempeature is 97°C. Initially, the temperature of the gas is 5·0°C. The walls of the vessel have an inner surface of area 800 cm² and thickness 1·0 cm. If the temperature of the gas increases to 9·0°C in 5·0 seconds, find the thermal conductivity of the material of the walls.*

Solution : The initial temperature difference is 97°C − 5°C = 92°C and at 5·0 s the temperature difference becomes 97°C − 9°C = 88°C. As the change in the temperature difference is small, we work with the average temperature difference

$$\frac{92°\text{C} + 88°\text{C}}{2} = 90°\text{C} = 90\ \text{K}.$$

The rise in the temperature of the gas is

$$9{\cdot}0°\text{C} - 5{\cdot}0°\text{C} = 4°\text{C} = 4\ \text{K}.$$

The heat supplied to the gas in 5·0 s is

$$\Delta Q = nC_v\Delta T$$

$$= (1\ \text{mole}) \times \left(\frac{3}{2} \times 8{\cdot}3\ \frac{\text{J}}{\text{mol-K}}\right) \times (4\ \text{K})$$

$$= 49{\cdot}8\ \text{J}.$$

If the thermal conductivity is K,

$$49{\cdot}8\ \text{J} = \frac{K(800 \times 10^{-4}\ \text{m}^2) \times (90\ \text{K})}{1{\cdot}0 \times 10^{-2}\ \text{m}} \times 5{\cdot}0\ \text{s}$$

or, $$K = \frac{49{\cdot}8\ \text{J}}{3600\ \text{m-s-K}} = 0{\cdot}014\ \text{J/m-s-K}.$$

14. *A monatomic ideal gas is contained in a rigid container of volume V with walls of total inner surface area A, thickness x and thermal conductivity K. The gas is at an initial temperature T_0 and pressure p_0. Find the pressure of the gas as a function of time if the temperature of the surrounding air is T_s. All temperatures are in absolute scale.*

Solution : As the volume of the gas is constant, a heat ΔQ given to the gas increases its temperature by $\Delta T = \Delta Q/C_v$. Also, for a monatomic gas, $C_v = \frac{3}{2}R$. If the temperature of the gas at time t is T, the heat current into the gas is

$$\frac{\Delta Q}{\Delta t} = \frac{KA(T_s - T)}{x}$$

or, $$\frac{\Delta T}{\Delta t} = \frac{2\,KA}{3\,xR}(T_s - T)$$

or, $$\int_{T_0}^{T}\frac{dT}{T_s - T} = \int_0^t \frac{2\,KA}{3\,xR}\,dt$$

or, $$\ln\frac{T_s - T_0}{T_s - T} = \frac{2\,KA}{3\,xR}\,t$$

or, $$T_s - T = (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}$$

or, $$T = T_s - (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}.$$

As the volume remains constant,

$$\frac{p}{T} = \frac{p_0}{T_0}$$

or, $$p = \frac{p_0}{T_0}T$$

$$= \frac{p_0}{T_0}\left[T_s - (T_s - T_0)\,e^{-\frac{2KA}{3xR}t}\right].$$

15. *Consider a cubical vessel of edge a having a small hole in one of its walls. The total thermal resistance of the walls is r. At time t = 0, it contains air at atmospheric pressure p_a and temperature T_0. The temperature of the surrounding air is $T_a (> T_0)$. Find the amount of the gas (in moles) in the vessel at time t. Take C_v of air to be 5 R/2.*

Solution : As the gas can leak out of the hole, the pressure inside the vessel will be equal to the atmospheric pressure p_a. Let n be the amount of the gas (moles) in the vessel at time t. Suppose an amount ΔQ of heat is given to the gas in time dt. Its temperature increases by dT where

$$\Delta Q = nC_p dT.$$

If the temperature of the gas is T at time t, we have

$$\frac{\Delta Q}{dt} = \frac{T_a - T}{r}$$

or, $$(C_p r)n\,dT = (T_a - T)dt. \qquad \text{... (i)}$$

We have, $$p_a a^3 = nRT$$

or, $$n\,dT + T\,dn = 0$$

or, $$n\,dT = -T\,dn. \qquad \text{... (ii)}$$

Also, $$T = \frac{p_a a^3}{nR} \quad \text{... (iii)}$$

Using (ii) and (iii) in (i),

$$-\frac{C_p\, p_a a^3}{nR}\, dn = \left(T_a - \frac{p_a a^3}{nR}\right) dt$$

or, $$\frac{dn}{nR\left(T_a - \frac{p_a a^3}{nR}\right)} = -\frac{dt}{C_p\, r\, p_a a^3}$$

or, $$\int_{n_0}^{n} \frac{dn}{nRT_a - p_a a^3} = -\int_0^t \frac{dt}{C_p\, r\, p_a a^3}$$

where $n_0 = \frac{p_a a^3}{RT_0}$ is the initial amount of the gas in the vessel. Thus,

$$\frac{1}{RT_a} \ln \frac{nRT_a - p_a a^3}{n_0 RT_a - p_a a^3} = -\frac{t}{C_p\, r\, p_a a^3}$$

or, $$nRT_a - p_a a^3 = (n_0 RT_a - p_a a^3)\, e^{-\frac{RT_a}{C_p r p_a a^3} t}.$$

Writing $$n_0 = \frac{p_a a^3}{RT_0} \text{ and } C_p = C_v + R = \frac{7R}{2},$$

$$n = \frac{p_a a^3}{RT_a}\left[1 + \left(\frac{T_a}{T_0} - 1\right) e^{-\frac{2T_a}{7 r p_a a^3} t}\right].$$

16 *A blackbody of surface area* 1 cm^2 *is placed inside an enclosure. The enclosure has a constant temperature* 27°C *and the blackbody is maintained at* 327°C *by heating it electrically. What electric power is needed to maintain the temperature?* $\sigma = 6.0 \times 10^{-8}$ W/m^2-K^4.

Solution : The area of the blackbody is $A = 10^{-4}$ m^2, its temperature is $T_1 = 327°C = 600$ K and the temperature of the enclosure is $T_2 = 27°C = 300$ K. The blackbody emits radiation at the rate of $A\sigma T_1^4$. The radiation falls on it (and gets absorbed) at the rate of $A\sigma T_2^4$. The net rate of loss of energy is $A\sigma(T_1^4 - T_2^4)$. The heater must supply this much of power. Thus, the power needed is

$A\sigma(T_1^4 - T_2^4)$

$= (10^{-4}\ \text{m}^2)(6.0 \times 10^{-8}\ \text{W/m}^2\text{-K}^4)\,[(600\ \text{K})^4 - (300\ \text{K})^4]$

$= 0.73$ W.

17. *An electric heater emits* 1000 W *of thermal radiation. The coil has a surface area of* 0.020 m^2. *Assuming that the coil radiates like a blackbody, find its temperature.* $\sigma = 6.00 \times 10^{-8}$ W/m^2-K^4.

Solution : Let the temperature of the coil be T. The coil will emit radiation at a rate $A\sigma T^4$. Thus,

$$1000\ \text{W} = (0.020\ \text{m}^2) \times (6.0 \times 10^{-8}\ \text{W/m}^2\text{-K}^4) \times T^4$$

or, $$T^4 = \frac{1000}{0.020 \times 6.00 \times 10^{-8}}\ \text{K}^4$$

$$= 8.33 \times 10^{11}\ \text{K}^4$$

or, $T = 955$ K.

18. *The earth receives solar radiation at a rate of* 8.2 J/cm^2-minute. *Assuming that the sun radiates like a blackbody, calculate the surface temperature of the sun. The angle subtended by the sun on the earth is* 0.53° *and the Stefan constant* $\sigma = 5.67 \times 10^{-8}$ W/m^2-K^4.

Solution :

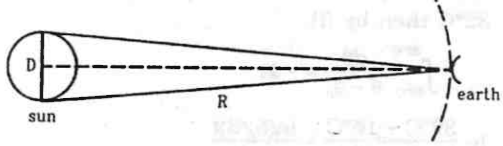


Figure 28-W11

Let the diameter of the sun be D and its distance from the earth be R. From the question,

$$\frac{D}{R} = 0.53 \times \frac{\pi}{180}$$

$$= 9.25 \times 10^{-3}. \quad \text{... (i)}$$

The radiation emitted by the surface of the sun per unit time is

$$4\pi\left(\frac{D}{2}\right)^2 \sigma T^4 = \pi D^2 \sigma T^4.$$

At distance R, this radiation falls on an area $4\pi R^2$ in unit time. The radiation received at the earth's surface per unit time per unit area is, therefore,

$$\frac{\pi D^2 \sigma T^4}{4\pi R^2} = \frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2.$$

Thus, $$\frac{\sigma T^4}{4}\left(\frac{D}{R}\right)^2 = 8.2\ \text{J/cm}^2\text{-minute}$$

or, $$\frac{1}{4} \times \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\text{-K}^4}\right) T^4 \times (9.25 \times 10^{-3})^2$$

$$= \frac{8.2}{10^{-4} \times 60} \frac{\text{W}}{\text{m}^2}$$

or, $T = 5794\ \text{K} \approx 5800$ K.

19. *The temperature of a body falls from* 40°C *to* 36°C *in* 5 *minutes when placed in a surrounding of constant temperature* 16°C. *Find the time taken for the temperature of the body to become* 32°C.

Solution : As the temperature differences are small, we can use Newton's law of cooling.

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$\frac{d\theta}{\theta - \theta_0} = -k\,dt \quad \text{... (i)}$$

where k is a constant, θ is the temperature of the body at time t and $\theta_0 = 16°C$ is the temperature of the surrounding. We have,

$$\int_{40°C}^{36°C} \frac{d\theta}{\theta - \theta_0} = -k(5\text{ min})$$

or, $$\ln \frac{36°C - 16°C}{40°C - 16°C} = -k(5\text{ min})$$

or, $$k = -\frac{\ln(5/6)}{5\text{ min}}.$$

If t be the time required for the temperature to fall from 36°C to 32°C then by (i),

$$\int_{36°C}^{32°C} \frac{d\theta}{\theta - \theta_0} = -kt$$

or, $$\ln \frac{32°C - 16°C}{36°C - 16°C} = \frac{\ln(5/6)t}{5\text{ min}}$$

or, $$t = \frac{\ln(4/5)}{\ln(5/6)} \times 5\text{ min}$$

$$= 6{\cdot}1\text{ min.}$$

Alternative method

The mean temperature of the body as it cools from 40°C to 36°C is $\frac{40°C + 36°C}{2} = 38°C$. The rate of decrease of temperature is $\frac{40°C - 36°C}{5\text{ min}} = 0{\cdot}80°C/\text{min}$.

Newton's law of cooling is

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$-0{\cdot}8°C/\text{min} = -k(38°C - 16°C) = -k(22°C)$$

or, $$k = \frac{0{\cdot}8}{22}\text{ min}^{-1}.$$

Let the time taken for the temperature to become 32°C be t.

During this period,

$$\frac{d\theta}{dt} = -\frac{36°C - 32°C}{t} = -\frac{4°C}{t}.$$

The mean temperature is $\frac{36°C + 32°C}{2} = 34°C$.

Now,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

or, $$\frac{-4°C}{t} = -\frac{0{\cdot}8}{22} \times (34°C - 16°C)/\text{min}$$

or, $$t = \frac{22 \times 4}{0{\cdot}8 \times 18}\text{ min} = 6{\cdot}1\text{ min.}$$

□

20. *A hot body placed in air is cooled down according to Newton's law of cooling, the rate of decrease of temperature being k times the temperature difference from the surrounding. Starting from t = 0, find the time in which the body will lose half the maximum heat it can lose.*

Solution : We have,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

where θ_0 is the temperature of the surrounding and θ is the temperature of the body at time t. Suppose $\theta = \theta_1$ at $t = 0$.

Then,

$$\int_{\theta_1}^{\theta} \frac{d\theta}{\theta - \theta_0} = -k\int_0^t dt$$

or, $$\ln \frac{\theta - \theta_0}{\theta_1 - \theta_0} = -kt$$

or, $$\theta - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt}. \qquad \text{... (i)}$$

The body continues to lose heat till its temperature becomes equal to that of the surrounding. The loss of heat in this entire period is

$$\Delta Q_m = ms(\theta_1 - \theta_0).$$

This is the maximum heat the body can lose. If the body loses half this heat, the decrease in its temperature will be,

$$\frac{\Delta Q_m}{2\,ms} = \frac{\theta_1 - \theta_0}{2}.$$

If the body loses this heat in time t_1, the temperature at t_1 will be

$$\theta_1 - \frac{\theta_1 - \theta_0}{2} = \frac{\theta_1 + \theta_0}{2}.$$

Putting these values of time and temperature in (i),

$$\frac{\theta_1 + \theta_0}{2} - \theta_0 = (\theta_1 - \theta_0)\, e^{-kt_1}$$

or, $$e^{-kt_1} = \frac{1}{2}$$

or, $$t_1 = \frac{\ln 2}{k}.$$

QUESTIONS FOR SHORT ANSWER

1. The heat current is written as $\frac{\Delta Q}{\Delta t}$. Why don't we write $\frac{dQ}{dt}$?

2. Does a body at 20°C radiate in a room, where the room temperature is 30°C? If yes, why does its temperature not fall further?

3. Why does blowing over a spoonful of hot tea cools it ? Does evaporation play a role ? Does radiation play a role ?
4. On a hot summer day we want to cool our room by opening the refrigerator door and closing all the windows and doors. Will the process work ?
5. On a cold winter night you are asked to sit on a chair. Would you like to choose a metal chair or a wooden chair ? Both are kept in the same lawn and are at the same temperature.
6. Two identical metal balls one at $T_1 = 300$ K and the other at $T_2 = 600$ K are kept at a distance of 1 m in vacuum. Will the temperatures equalise by radiation ? Will the rate of heat gained by the colder sphere be proportional to $T_2^4 - T_1^4$ as may be expected from the Stefan's law ?
7. An ordinary electric fan does not cool the air, still it gives comfort in summer. Explain.
8. The temperature of the atmosphere at a high altitude is around 500°C. Yet an animal there would freeze to death and not boil. Explain.
9. Standing in the sun is more pleasant on a cold winter day than standing in shade. Is the temperature of air in the sun considerably higher than that of the air in shade ?
10. Cloudy nights are warmer than the nights with clean sky. Explain.
11. Why is a white dress more comfortable than a dark dress in summer ?

OBJECTIVE I

1. The thermal conductivity of a rod depends on
(a) length (b) mass
(c) area of cross-section (d) material of the rod.
2. In a room containing air, heat can go from one place to another
(a) by conduction only (b) by convection only
(c) by radiation only (d) by all the three modes.
3. A solid at temperature T_1 is kept in an evacuated chamber at temperature $T_2 > T_1$. The rate of increase of temperature of the body is proportional to
(a) $T_2 - T_1$ (b) $T_2^2 - T_1^2$
(c) $T_2^3 - T_1^3$ (d) $T_2^4 - T_1^4$
4. The thermal radiation emitted by a body is proportional to T^n where T is its absolute temperature. The value of n is exactly 4 for
(a) a blackbody (b) all bodies
(c) bodies painted black only (d) polished bodies only.
5. Two bodies A and B having equal surface areas are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by A and B are in the ratio
(a) 1 : 1·15 (b) 1 : 2
(c) 1 : 4 (d) 1 : 16.
6. One end of a metal rod is kept in a furnace. In steady state, the temperature of the rod
(a) increases (b) decreases
(c) remains constant (d) is nonuniform.
7. Newton's law of cooling is a special case of
(a) Wien's displacement law (b) Kirchoff's law
(c) Stefan's law (d) Planck's law.
8. A hot liquid is kept in a big room. Its temperature is plotted as a function of time. Which of the following curves may represent the plot ?

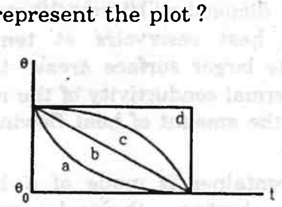


Figure 28-Q1

9. A hot liquid is kept in a big room. The logarithm of the numerical value of the temperature difference between the liquid and the room is plotted against time. The plot will be very nearly
(a) a straight line (b) a circular arc
(c) a parabola (d) an ellipse.
10. A body cools down from 65°C to 60°C in 5 minutes. It will cool down from 60°C to 55°C in
(a) 5 minutes (b) less than 5 minutes
(c) more than 5 minutes
(d) less than or more than 5 minutes depending on whether its mass is more than or less than 1 kg.

OBJECTIVE II

1. One end of a metal rod is dipped in boiling water and the other is dipped in melting ice.
(a) All parts of the rod are in thermal equilibrium with each other.
(b) We can assign a temperature to the rod.
(c) We can assign a temperature to the rod after steady state is reached.
(d) The state of the rod does not change after steady state is reached.
2. A blackbody does not
(a) emit radiation (b) absorb radiation
(c) reflect radiation (d) refract radiation.

3. In summer, a mild wind is often found on the shore of a calm river. This is caused due to
(a) difference in thermal conductivity of water and soil
(b) convection currents
(c) conduction between air and the soil
(d) radiation from the soil.

4. A piece of charcoal and a piece of shining steel of the same area are kept for a long time in an open lawn in bright sun.
(a) The steel will absorb more heat than the charcoal.
(b) The temperature of the steel will be higher than that of the charcoal.
(c) If both are picked up by bare hands, the steel will be felt hotter than the charcoal.
(d) If the two are picked up from the lawn and kept in a cold chamber, the charcoal will lose heat at a faster rate than the steel.

5. A heated body emits radiation which has maximum intensity near the frequency ν_0. The emissivity of the material is 0·5. If the absolute temperature of the body is doubled,
(a) the maximum intensity of radiation will be near the frequency $2\nu_0$
(b) the maximum intensity of radiation will be near the frequency $\nu_0/2$
(c) the total energy emitted will increase by a factor of 16
(d) the total energy emitted will increase by a factor of 8.

6. A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature.
(a) Both will emit equal amount of radiation per unit time in the biginning.
(b) Both will absorb equal amount of radiation from the surrounding in the biginning.
(c) The initial rate of cooling (dT/dt) will be the same for the two spheres.
(d) The two spheres will have equal temperatures at any instant.

EXERCISES

1. A uniform slab of dimension 10 cm × 10 cm × 1 cm is kept between two heat reservoirs at temperatures 10°C and 90°C. The larger surface areas touch the reservoirs. The thermal conductivity of the material is 0·80 W/m–°C. Find the amount of heat flowing through the slab per minute.

2. A liquid-nitrogen container is made of a 1 cm thick thermocoal sheet having thermal conductivity 0·025 J/m–s–°C. Liquid nitrogen at 80 K is kept in it. A total area of 0·80 m^2 is in contact with the liquid nitrogen. The atmospheric temperature is 300 K. Calculate the rate of heat flow from the atmosphere to the liquid nitrogen.

3. The normal body-temperature of a person is 97°F. Calculate the rate at which heat is flowing out of his body through the clothes assuming the following values. Room temperature = 47°F, surface of the body under clothes = 1·6 m^2, conductivity of the cloth = 0·04 J/m–s–°C, thickness of the cloth = 0·5 cm.

4. Water is boiled in a container having a bottom of surface area 25 cm^2, thickness 1·0 mm and thermal conductivity 50 W/m–°C. 100 g of water is converted into steam per minute in the steady state after the boiling starts. Assuming that no heat is lost to the atmosphere, calculate the temperature of the lower surface of the bottom. Latent heat of vaporization of water = 2·26 × 10^6 J/kg

5. One end of a steel rod (K = 46 J/m–s–°C) of length 1·0 m is kept in ice at 0°C and the other end is kept in boiling water at 100°C. The area of cross-section of the rod is 0·04 cm^2. Assuming no heat loss to the atmosphere, find the mass of the ice melting per second. Latent heat of fusion of ice = 3·36 × 10^5 J/kg.

6. An icebox almost completely filled with ice at 0°C is dipped into a large volume of water at 20°C. The box has walls of surface area 2400 cm^2, thickness 2·0 mm and thermal conductivity 0·06 W/m–°C. Calculate the rate at which the ice melts in the box. Latent heat of fusion of ice = 3·4 × 10^5 J/kg.

7. A pitcher with 1 mm thick porous walls contains 10 kg of water. Water comes to its outer surface and evaporates at the rate of 0·1 g/s. The surface area of the pitcher (one side) = 200 cm^2. The room temperature = 42°C, latent heat of vaporization = 2·27 × 10^6 J/kg, and the thermal conductivity of the porous walls = 0·80 J/m–s–°C. Calculate the temperature of water in the pitcher when it attains a constant value.

8. A steel frame (K = 45 W/m–°C) of total length 60 cm and cross-sectional area 0·20 cm^2, forms three sides of a square. The free ends are maintained at 20°C and 40°C. Find the rate of heat flow through a cross-section of the frame.

9. Water at 50°C is filled in a closed cylindrical vessel of height 10 cm and cross-sectional area 10 cm^2. The walls of the vessel are adiabatic but the flat parts are made of 1 mm thick aluminium (K = 200 J/m–s–°C). Assume that the outside temperature is 20°C. The density of water is 1000 kg/m^3, and the specific heat capacity of water = 4200 J/kg–°C. Estimate the time taken for the temperature to fall by 1·0°C. Make any simplifying assumptions you need but specify them.

10. The left end of a copper rod (length = 20 cm, area of cross-section = 0·20 cm^2) is maintained at 20°C and the right end is maintained at 80°C. Neglecting any loss of heat through radiation, find (a) the temperature at a point 11 cm from the left end and (b) the heat current through the rod. Thermal conductivity of copper = 385 W/m–°C.

11. The ends of a metre stick are maintained at 100°C and 0°C. One end of a rod is maintained at 25°C. Where should its other end be touched on the metre stick so that there is no heat current in the rod in steady state ?

12. A cubical box of volume 216 cm^3 is made up of 0·1 cm thick wood. The inside is heated electrically by a 100 W heater. It is found that the temperature difference between the inside and the outside surface is 5°C in steady state. Assuming that the entire electrical energy spent appears as heat, find the thermal conductivity of the material of the box.

13. Figure (28-E1) shows water in a container having 2·0 mm thick walls made of a material of thermal conductivity 0·50 W/m–°C. The container is kept in a melting-ice bath at 0°C. The total surface area in contact with water is 0·05 m^2. A wheel is clamped inside the water and is coupled to a block of mass M as shown in the figure. As the block goes down, the wheel rotates. It is found that after some time a steady state is reached in which the block goes down with a constant speed of 10 cm/s and the temperature of the water remains constant at 1·0°C. Find the mass M of the block. Assume that the heat flows out of the water only through the walls in contact. Take g = 10 m/s^2.

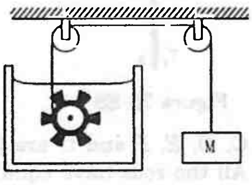


Figure 28-E1

14. On a winter day when the atmospheric temperature drops to −10°C, ice forms on the surface of a lake. (a) Calculate the rate of increase of thickness of the ice when 10 cm of ice is already formed. (b) Calculate the total time taken in forming 10 cm of ice. Assume that the temperature of the entire water reaches 0°C before the ice starts forming. Density of water = 1000 kg/m^3, latent heat of fusion of ice = 3·36 × 10^5 J/kg and thermal conductivity of ice = 1·7 W/m–°C. Neglect the expansion of water on freezing.

15. Consider the situation of the previous problem. Assume that the temperature of the water at the bottom of the lake remains constant at 4°C as the ice forms on the surface (the heat required to maintain the temperature of the bottom layer may come from the bed of the lake). The depth of the lake is 1·0 m. Show that the thickness of the ice formed attains a steady state maximum value. Find this value. The thermal conductivity of water = 0·50 W/m–°C. Take other relevant data from the previous problem.

16. Three rods of lengths 20 cm each and area of cross-section 1 cm^2 are joined to form a triangle ABC. The conductivities of the rods are K_{AB} = 50 J/m-s-°C, K_{BC} = 200 J/m-s-°C and K_{AC} = 400 J/m-s-°C. The junctions A, B and C are maintained at 40°C, 80°C and 80°C respectively. Find the rate of heat flowing through the rods AB, AC and BC.

17. A semicircular rod is joined at its end to a straight rod of the same material and the same cross-sectional area. The straight rod forms a diameter of the other rod. The junctions are maintained at different temperatures. Find the ratio of the heat transferred through a cross-section of the semicircular rod to the heat transferred through a cross-section of the straight rod in a given time.

18. A metal rod of cross-sectional area 1·0 cm^2 is being heated at one end. At one time, the temperature gradient is 5·0°C/cm at cross-section A and is 2·5°C/cm at cross-section B. Calculate the rate at which the temperature is increasing in the part AB of the rod. The heat capacity of the part AB = 0·40 J/°C, thermal conductivity of the material of the rod = 200 W/m-°C. Neglect any loss of heat to the atmosphere.

19. Steam at 120°C is continuously passed through a 50 cm long rubber tube of inner and outer radii 1·0 cm and 1·2 cm. The room temperature is 30°C. Calculate the rate of heat flow through the walls of the tube. Thermal conductivity of rubber = 0·15 J/m–s–°C.

20. A hole of radius r_1 is made centrally in a uniform circular disc of thickness d and radius r_2. The inner surface (a cylinder of length d and radius r_1) is maintained at a temperature θ_1 and the outer surface (a cylinder of length d and radius r_2) is maintained at a temperature $\theta_2(\theta_1 > \theta_2)$. The thermal conductivity of the material of the disc is K. Calculate the heat flowing per unit time through the disc.

21. A hollow tube has a length l, inner radius R_1 and outer radius R_2. The material has a thermal conductivity K. Find the heat flowing through the walls of the tube if (a) the flat ends are maintained at temperatures T_1 and $T_2(T_2 > T_1)$ (b) the inside of the tube is maintained at temperature T_1 and the outside is maintained at T_2.

22. A composite slab is prepared by pasting two plates of thicknesses L_1 and L_2 and thermal conductivities K_1 and K_2. The slabs have equal cross-sectional area. Find the equivalent conductivity of the slab.

23. Figure (28-E2) shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0°C and that of the steel rod is kept at 100°C. Find the temperature at the junction of the rods. Conductivity of copper = 390 W/m-°C and that of steel = 46 W/m-°C.

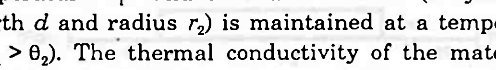


Figure 28-E2

24. An aluminium rod and a copper rod of equal length 1·0 m and cross-sectional area 1 cm^2 are welded together as shown in figure (28-E3). One end is kept at a temperature of 20°C and the other at 60°C. Calculate the amount of heat taken out per second from the hot end. Thermal conductivity of aluminium = 200 W/m–°C and of copper = 390 W/m-°C.

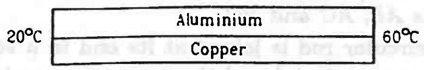


Figure 28-E3

25. Figure (28-E4) shows an aluminium rod joined to a copper rod. Each of the rods has a length of 20 cm and area of cross-section $0{\cdot}20\ \text{cm}^2$. The junction is maintained at a constant temperature 40°C and the two ends are maintained at 80°C. Calculate the amount of heat taken out from the cold junction in one minute after the steady state is reached. The conductivities are $K_{Al} = 200$ W/m-°C and $K_{Cu} = 400$ W/m-°C.

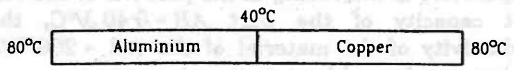


Figure 28-E4

26. Consider the situation shown in figure (28-E5). The frame is made of the same material and has a uniform cross-sectional area everywhere. Calculate the amount of heat flowing per second through a cross-section of the bent part if the total heat taken out per second from the end at 100°C is 130 J.

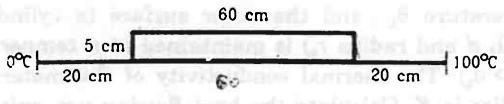


Figure 28-E5

27. Suppose the bent part of the frame of the previous problem has a thermal conductivity of 780 J/m-s-°C whereas it is 390 J/m-s-°C for the straight part. Calculate the ratio of the rate of heat flow through the bent part to the rate of heat flow through the straight part.

28. A room has a window fitted with a single $1{\cdot}0\ \text{m} \times 2{\cdot}0\ \text{m}$ glass of thickness 2 mm. (a) Calculate the rate of heat flow through the closed window when the temperature inside the room is 32°C and that outside is 40°C. (b) The glass is now replaced by two glasspanes, each having a thickness of 1 mm and separated by a distance of 1 mm. Calculate the rate of heat flow under the same conditions of temperature. Thermal conductivity of window glass = $1{\cdot}0$ J/m–s–°C and that of air = $0{\cdot}025$ J/m–s–°C.

29. The two rods shown in figure (28-E6) have identical geometrical dimensions. They are in contact with two heat baths at temperatures 100°C and 0°C. The temperature of the junction is 70°C. Find the temperature of the junction if the rods are interchanged.

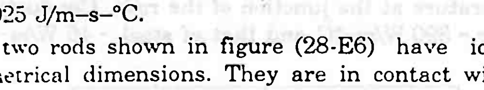


Figure 28-E6

30. The three rods shown in figure (28-E7) have identical geometrical dimensions. Heat flows from the hot end at a rate of 40 W in the arrangement (a). Find the rates of heat flow when the rods are joined as in arrangement (b) and in (c). Thermal conductivities of aluminium and copper are 200 W/m-°C and 400 W/m-°C respectively.

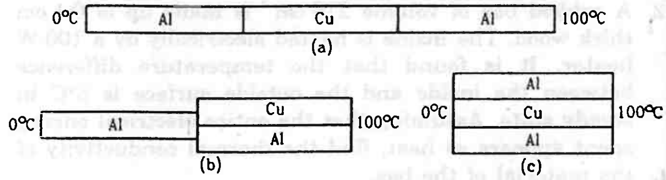


Figure 28-E7

31. Four identical rods AB, CD, CF and DE are joined as shown in figure (28-E8). The length, cross-sectional area and thermal conductivity of each rod are l, A and K respectively. The ends A, E and F are maintained at temperatures T_1, T_2 and T_3 respectively. Assuming no loss of heat to the atmosphere, find the temperature at B.

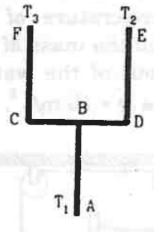


Figure 28-E8

32. Seven rods A, B, C, D, E, F and G are joined as shown in figure (28-E9). All the rods have equal cross-sectional area A and length l. The thermal conductivities of the rods are $K_A = K_C = K_0$, $K_B = K_D = 2K_0$, $K_E = 3K_0$, $K_F = 4K_0$ and $K_G = 5K_0$. The rod E is kept at a constant temperature T_1 and the rod G is kept at a constant temperature $T_2 (T_2 > T_1)$. (a) Show that the rod F has a uniform temperature $T = (T_1 + 2T_2)/3$. (b) Find the rate of heat flow from the source which maintains the temperature T_2.

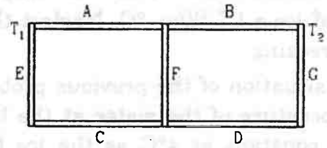


Figure 28-E9

33. Find the rate of heat flow through a cross-section of the rod shown in figure (28-E10) $(\theta_2 > \theta_1)$. Thermal conductivity of the material of the rod is K.

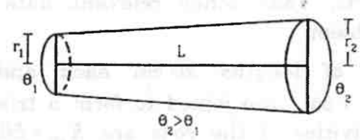


Figure 28-E10

34. A rod of negligible heat capacity has length 20 cm, area of cross-section $1 \cdot 0 \text{ cm}^2$ and thermal conductivity 200 W/m–°C. The temperature of one end is maintained at 0°C and that of the other end is slowly and linearly varied from 0°C to 60°C in 10 minutes. Assuming no loss of heat through the sides, find the total heat transmitted through the rod in these 10 minutes.

35. A hollow metallic sphere of radius 20 cm surrounds a concentric metallic sphere of radius 5 cm. The space between the two spheres is filled with a nonmetallic material. The inner and outer spheres are maintained at 50°C and 10°C respectively and it is found that 100 J of heat passes from the inner sphere to the outer sphere per second. Find the thermal conductivity of the material between the spheres.

36. Figure (28-E11) shows two adiabatic vessels, each containing a mass m of water at different temperatures. The ends of a metal rod of length L, area of cross-section A and thermal conductivity K, are inserted in the water as shown in the figure. Find the time taken for the difference between the temperatures in the vessels to become half of the original value. The specific heat capacity of water is s. Neglect the heat capacity of the rod and the container and any loss of heat to the atmosphere.

Figure 28-E11

37. Two bodies of masses m_1 and m_2 and specific heat capacities s_1 and s_2 are connected by a rod of length l, cross-sectional area A, thermal conductivity K and negligible heat capacity. The whole system is thermally insulated. At time $t = 0$, the temperature of the first body is T_1 and the temperature of the second body is T_2 ($T_2 > T_1$). Find the temperature difference between the two bodies at time t.

38. An amount n (in moles) of a monatomic gas at an initial temperature T_0 is enclosed in a cylindrical vessel fitted with a light piston. The surrounding air has a temperature $T_s (> T_0)$ and the atmospheric pressure is p_a. Heat may be conducted between the surrounding and the gas through the bottom of the cylinder. The bottom has a surface area A, thickness x and thermal conductivity K. Assuming all changes to be slow, find the distance moved by the piston in time t.

39. Assume that the total surface area of a human body is $1 \cdot 6 \text{ m}^2$ and that it radiates like an ideal radiator. Calculate the amount of energy radiated per second by the body if the body temperature is 37°C. Stefan constant σ is $6 \cdot 0 \times 10^{-8} \text{ W/m}^2\text{–K}^4$.

40. Calculate the amount of heat radiated per second by a body of surface area 12 cm^2 kept in thermal equilibrium in a room at temperature 20°C. The emissivity of the surface = 0·80 and $\sigma = 6 \cdot 0 \times 10^{-8} \text{ W/m}^2\text{–K}^4$.

41. A solid aluminium sphere and a solid copper sphere of twice the radius are heated to the same temperature and are allowed to cool under identical surrounding temperatures. Assume that the emissivity of both the spheres is the same. Find the ratio of (a) the rate of heat loss from the aluminium sphere to the rate of heat loss from the copper sphere and (b) the rate of fall of temperature of the aluminium sphere to the rate of fall of temperature of the copper sphere. The specific heat capacity of aluminium = 900 J/kg–°C and that of copper = 390 J/kg–°C. The density of copper = 3·4 times the density of aluminium.

42. A 100 W bulb has tungsten filament of total length 1·0 m and radius 4×10^{-5} m. The emissivity of the filament is 0·8 and $\sigma = 6 \cdot 0 \times 10^{-8} \text{ W/m}^2\text{–K}^4$. Calculate the temperature of the filament when the bulb is operating at correct wattage.

43. A spherical ball of surface area 20 cm^2 absorbs any radiation that falls on it. It is suspended in a closed box maintained at 57°C. (a) Find the amount of radiation falling on the ball per second. (b) Find the net rate of heat flow to or from the ball at an instant when its temperature is 200°C. Stefan constant = $6 \cdot 0 \times 10^{-8}$ W/m^2–K^4.

44. A spherical tungsten piece of radius 1·0 cm is suspended in an evacuated chamber maintained at 300 K. The piece is maintained at 1000 K by heating it electrically. Find the rate at which the electrical energy must be supplied. The emissivity of tungsten is 0·30 and the Stefan constant σ is $6 \cdot 0 \times 10^{-8} \text{ W/m}^2\text{–K}^4$.

45. A cubical block of mass 1·0 kg and edge 5·0 cm is heated to 227°C. It is kept in an evacuated chamber maintained at 27°C. Assuming that the block emits radiation like a blackbody, find the rate at which the temperature of the block will decrease. Specific heat capacity of the material of the block is 400 J/kg–K.

46. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

47. A spherical ball A of surface area 20 cm^2 is kept at the centre of a hollow spherical shell B of area 80 cm^2. The surface of A and the inner surface of B emit as blackbodies. Assume that the thermal conductivity of the material of B is very poor and that of A is very high and that the air between A and B has been pumped out. The heat capacities of A and B are 42 J/°C and 82 J/°C respectively. Initially, the temperature of A is 100°C and that of B is 20°C. Find the rate of change of temperature of A and that of B at this instant. Explain the effects of the assumptions listed in the problem.

48. A cylindrical rod of length 50 cm and cross-sectional area 1 cm^2 is fitted between a large ice chamber at 0°C and an evacuated chamber maintained at 27°C as shown in figure (28-E12). Only small portions of the rod are inside the chambers and the rest is thermally insulated from the surrounding. The cross-section going into the

evacuated chamber is blackened so that it completely absorbs any radiation falling on it. The temperature of the blackened end is 17°C when steady state is reached. Stefan constant $\sigma = 6 \times 10^{-8}$ W/m^2–K^4. Find the thermal conductivity of the material of the rod.

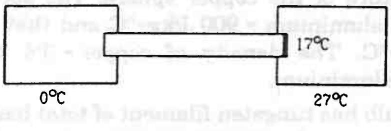


Figure 28-E12

49. One end of a rod of length 20 cm is inserted in a furnace at 800 K. The sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant $\sigma = 6{\cdot}0 \times 10^{-8}$ W/m^2–K^4.

50. A calorimeter of negligible heat capacity contains 100 cc of water at 40°C. The water cools to 35°C in 5 minutes. The water is now replaced by K-oil of equal volume at 40°C. Find the time taken for the temperature to become 35°C under similar conditions. Specific heat capacities of water and K-oil are 4200 J/kg–K and 2100 J/kg–K respectively. Density of K-oil = 800 kg/m^3.

51. A body cools down from 50°C to 45°C in 5 minutes and to 40°C in another 8 minutes. Find the temperature of the surrounding.

52. A calorimeter contains 50 g of water at 50°C. The temperature falls to 45°C in 10 minutes. When the calorimeter contains 100 g of water at 50°C, it takes 18 minutes for the temperature to become 45°C. Find the water equivalent of the calorimeter.

53. A metal ball of mass 1 kg is heated by means of a 20 W heater in a room at 20°C. The temperature of the ball becomes steady at 50°C. (a) Find the rate of loss of heat to the surrounding when the ball is at 50°C. (b) Assuming Newton's law of cooling, calculate the rate of loss of heat to the surrounding when the ball is at 30°C. (c) Assume that the temperature of the ball rises uniformly from 20°C to 30°C in 5 minutes. Find the total loss of heat to the surrounding during this period. (d) Calculate the specific heat capacity of the metal.

54. A metal block of heat capacity 80 J/°C placed in a room at 20°C is heated electrically. The heater is switched off when the temperature reaches 30°C. The temperature of the block rises at the rate of 2 °C/s just after the heater is switched on and falls at the rate of 0·2 °C/s just after the heater is switched off. Assume Newton's law of cooling to hold. (a) Find the power of the heater. (b) Find the power radiated by the block just after the heater is switched off. (c) Find the power radiated by the block when the temperature of the block is 25°C. (d) Assuming that the power radiated at 25°C respresents the average value in the heating process, find the time for which the heater was kept on.

55. A hot body placed in a surrounding of temperature θ_0 obeys Newton's law of cooling $\frac{d\theta}{dt} = -k(\theta - \theta_0)$. Its temperature at $t = 0$ is θ_1. The specific heat capacity of the body is s and its mass is m. Find (a) the maximum heat that the body can lose and (b) the time starting from $t = 0$ in which it will lose 90% of this maximum heat.

□

ANSWERS

OBJECTIVE I

1. (d)	2. (d)	3. (d)	4. (b)	5. (a)	6. (d)
7. (c)	8. (a)	9. (a)	10. (c)		

OBJECTIVE II

1. (d)	2. (c), (d)	3. (b)
4. (c), (d)	5. (a), (c)	6. (a), (b)

EXERCISES

1. 3840 J
2. 440 W
3. 356 J/s
4. 130°C
5. $5{\cdot}5 \times 10^{-3}$ g
6. 1·5 kg/h
7. 28°C
8. 0·03 W
9. 0·035 s
10. (a) 53°C (b) 2·31 J/s
11. 25 cm from the cold end
12. 0·92 W/m–°C
13. 12·5 kg
14. (a) $5{\cdot}0 \times 10^{-7}$ m/s (b) 27·5 hours
15. 89 cm
16. 1 W, 8 W, zero
17. 2 : π

18. 12·5°C/s

19. 233 J/s

20. $\dfrac{2\pi Kd(\theta_1 - \theta_2)}{\ln(r_2/r_1)}$

21. (a) $\dfrac{K\pi(R_2^2 - R_1^2)(T_2 - T_1)}{l}$ (b) $\dfrac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$

22. $\dfrac{K_1K_2(L_1 + L_2)}{L_1K_2 + L_2K_1}$

23. 10·6°C

24. 2·36 J

25. 144 J

26. 60 J

27. 12 : 7

28. (a) 8000 J/s (b) 381 J/s

29. 30°C

30. 75 W, 400 W

31. $\dfrac{3T_1 + 2(T_2 + T_3)}{7}$

32. (b) $\dfrac{4K_0A(T_2 - T_1)}{3l}$

33. $\dfrac{K\pi r_1r_2(\theta_2 - \theta_1)}{L}$

34. 1800 J

35. 3·0 W/m-°C

36. $\dfrac{Lms}{2KA}\ln 2$

37. $(T_2 - T_1)e^{-\lambda t}$ where $\lambda = \dfrac{KA(m_1s_1 + m_2s_2)}{lm_1m_2s_1s_2}$

38. $\dfrac{nR}{P_aA}(T_s - T_0)(1 - e^{[illegible]})$

39. 887 J

40. 0·42 J

41. (a) 1 : 4 (b) 2·9 : 1

42. 1700 K

43. (a) 1·4 J (b) 4·58 W from the ball

44. 22 W

45. 0·12°C/s

46. 0·3

47. 0·05°C/s and 0·01°C/s

48. 1·8 W/m-°C

49. 74 W/m-K

50. 2 min

51. 34°C

52. 12·5 g

53. (a) 20 W (b) $\dfrac{20}{3}$ W (c) 1000 J (d) 500 J/kg-K

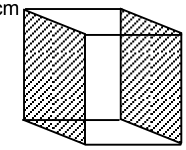
54. (a) 160 W (b) 16 W (c) 8 W (d) 5·2 s

55. (a) $ms(\theta_1 - \theta_0)$ (b) $\dfrac{\ln 10}{k}$

□

CHAPTER 28
HEAT TRANSFER

1. $t_1 = 90°C$, $t_2 = 10°C$
$l = 1\text{ cm} = 1 \times 10^{-3}\text{ m}$
$A = 10\text{ cm} \times 10\text{ cm} = 0.1 \times 0.1\text{ m}^2 = 1 \times 10^{-2}\text{ m}^2$
$K = 0.80\text{ w/m-°C}$
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{8 \times 10^{-1} \times 1 \times 10^{-2} \times 80}{1 \times 10^{-2}} = 64\text{ J/s} = 64 \times 60\ 3840\text{ J.}$$

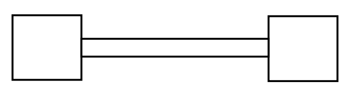


2. $t = 1\text{ cm} = 0.01\text{ m}$, $A = 0.8\text{ m}^2$
$\theta_1 = 300$, $\theta_2 = 80$
$K = 0.025$,
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.025 \times 0.8 \times (30030)}{0.01} = 440\text{ watt.}$$

3. $K = 0.04\text{ J/m-5°C}$, $A = 1.6\text{ m}^2$
$t_1 = 97°F = 36.1°C$ $t_2 = 47°F = 8.33°C$
$l = 0.5\text{ cm} = 0.005\text{ m}$
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{4 \times 10^{-2} \times 1.6 \times 27.78}{5 \times 10^{-3}} = 356\text{ J/s}$$

4. $A = 25\text{ cm}^2 = 25 \times 10^{-4}\text{ m}^2$
$l = 1\text{ mm} = 10^{-3}\text{ m}$
$K = 50\text{ w/m-°C}$
$\frac{Q}{t}$ = Rate of conversion of water into steam
$$= \frac{100 \times 10^{-3} \times 2.26 \times 10^6}{1\ \text{min}} = \frac{10^{-1} \times 2.26 \times 10^6}{60} = 0.376 \times 10^4$$
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \Rightarrow 0.376 \times 10^4 = \frac{50 \times 25 \times 10^{-4} \times (\theta - 100)}{10^{-3}}$$
$$\Rightarrow \theta = \frac{10^{-3} \times 0.376 \times 10^4}{50 \times 25 \times 10^{-4}} = \frac{10^5 \times 0.376}{50 \times 25} = 30.1 \approx 30$$

5. $K = 46\text{ w/m-s°C}$
$l = 1\text{ m}$
$A = 0.04\text{ cm}^2 = 4 \times 10^{-6}\text{ m}^2$
$L_{\text{fussion ice}} = 3.36 \times 10^5\text{ j/Kg}$
$$\frac{Q}{t} = \frac{46 \times 4 \times 10^{-6} \times 100}{1} = 5.4 \times 10^{-8}\text{ kg} \approx 5.4 \times 10^{-5}\text{ g.}$$



6. $A = 2400\text{ cm}^2 = 2400 \times 10^{-4}\text{ m}^2$
$\ell = 2\text{ mm} = 2 \times 10^{-3}\text{ m}$
$K = 0.06\text{ w/m-°C}$
$\theta_1 = 20°C$
$\theta_2 = 0°C$
$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{0.06 \times 2400 \times 10^{-4} \times 20}{2 \times 10^{-3}} = 24 \times 6 \times 10^{-1} \times 10 = 24 \times 6 = 144\text{ J/sec}$$
Rate in which ice melts $= \frac{m}{t} = \frac{Q}{t \times L} = \frac{144}{3.4 \times 10^5}$ Kg/h $= \frac{144 \times 3600}{3.4 \times 10^5}$ Kg/s = 1.52 kg/s.

7. $\ell = 1\text{ mm} = 10^{-3}\text{ m}$ $m = 10\text{ kg}$
$A = 200\text{ cm}^2 = 2 \times 10^{-2}\text{ m}^2$
$L_{vap} = 2.27 \times 10^6\text{ J/kg}$
$K = 0.80\text{ J/m-s-°C}$

$dQ = 2.27 \times 10^6 \times 10$,

$$\frac{dQ}{dt} = \frac{2.27 \times 10^7}{10^5} = 2.27 \times 10^2 \text{ J/s}$$

Again we know

$$\frac{dQ}{dt} = \frac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{1 \times 10^{-3}}$$

So, $\dfrac{8 \times 2 \times 10^{-3}(42 - T)}{10^{-3}} = 2.27 \times 10^2$

$\Rightarrow 16 \times 42 - 16T = 227 \Rightarrow T = 27.8 \approx 28°C$

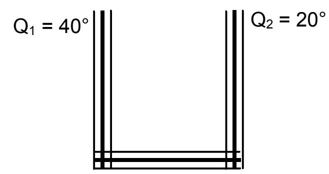
8. $K = 45$ w/m-°C

$\ell = 60 \text{ cm} = 60 \times 10^{-2}$ m

$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

Rate of heat flow,

$$= \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{60 \times 10^{-2}} = 30 \times 10^{-3}\ 0.03 \text{ w}$$



9. $A = 10 \text{ cm}^2$, $h = 10$ cm

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{200 \times 10^{-3} \times 30}{1 \times 10^{-3}} = 6000$$

Since heat goes out from both surfaces. Hence net heat coming out.

$= \dfrac{\Delta Q}{\Delta t} = 6000 \times 2 = 12000$, $\dfrac{\Delta Q}{\Delta t} = MS\dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow 6000 \times 2 = 10^{-3} \times 10^{-1} \times 1000 \times 4200 \times \dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow \dfrac{\Delta\theta}{\Delta t} = \dfrac{72000}{420} = 28.57$

So, in 1 Sec. 28.57°C is dropped

Hence for drop of 1°C $\dfrac{1}{28.57}$ sec. = 0.035 sec. is required

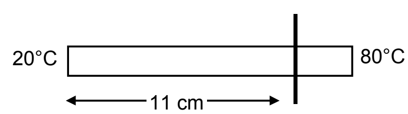
10. $\ell = 20 \text{ cm} = 20 \times 10^{-2}$ m

$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$

$\theta_1 = 80°C$, $\theta_2 = 20°C$, $K = 385$

(a) $\dfrac{Q}{t} = \dfrac{KA(\theta_1 - \theta_2)}{\ell} = \dfrac{385 \times 0.2 \times 10^{-4}(80 - 20)}{20 \times 10^{-2}} = 385 \times 6 \times 10^{-4} \times 10 = 2310 \times 10^{-3} = 2.31$

(b) Let the temp of the 11 cm point be θ



$\dfrac{\Delta\theta}{\Delta l} = \dfrac{Q}{tKA}$

$\Rightarrow \dfrac{\Delta\theta}{\Delta l} = \dfrac{2.31}{385 \times 0.2 \times 10^{-4}}$

$\Rightarrow \dfrac{\theta - 20}{11 \times 10^{-2}} = \dfrac{2.31}{385 \times 0.2 \times 10^{-4}}$

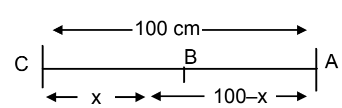
$\Rightarrow \theta - 20 = \dfrac{2.31 \times 10^4}{385 \times 0.2} \times 11 \times 10^{-2} = 33$

$\Rightarrow \theta = 33 + 20 = 53$

11. Let the point to be touched be 'B'

No heat will flow when, the temp at that point is also 25°C

i.e. $Q_{AB} = Q_{BC}$



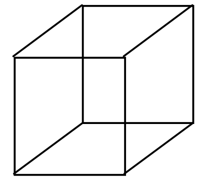
So, $\dfrac{KA(100 - 25)}{100 - x} = \dfrac{KA(25 - 0)}{x}$

$\Rightarrow 75x = 2500 - 25x \Rightarrow 100x = 2500 \Rightarrow x = 25$ cm from the end with 0°C

12. $V = 216\ cm^3$

$a = 6\ cm$, Surface area = $6\ a^2 = 6 \times 36\ m^2$

$t = 0.1\ cm$ $\frac{Q}{t} = 100\ W$,



$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell}$$

$$\Rightarrow 100 = \frac{K \times 6 \times 36 \times 10^{-4} \times 5}{0.1 \times 10^{-2}}$$

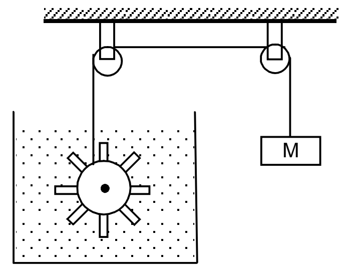
$$\Rightarrow K = \frac{100}{6 \times 36 \times 5 \times 10^{-1}} = 0.9259\ W/m°C \approx 0.92\ W/m°C$$

13. Given $\theta_1 = 1°C$, $\theta_2 = 0°C$

$K = 0.50$ w/m-°C, $d = 2\ mm = 2 \times 10^{-3}\ m$

$A = 5 \times 10^{-2}\ m^2$, $v = 10\ cm/s = 0.1\ m/s$

Power = Force × Velocity = Mg × v



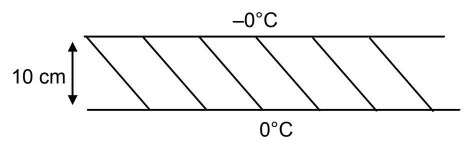
$$\text{Again Power} = \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\text{So, } Mgv = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\Rightarrow M = \frac{KA(\theta_1 - \theta_2)}{dvg} = \frac{5 \times 10^{-1} \times 5 \times^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10} = 12.5\ kg.$$

14. $K = 1.7$ W/m-°C $f_w = 1000\ Kg/m^3$

$L_{ice} = 3.36 \times 10^5$ J/kg $T = 10\ cm = 10 \times 10^{-2}\ m$



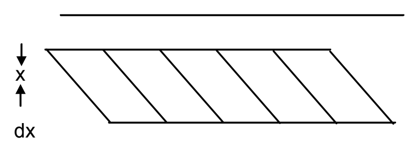
(a) $$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} \Rightarrow \frac{\ell}{t} = \frac{KA(\theta_1 - \theta_2)}{Q} = \frac{KA(\theta_1 - \theta_2)}{mL}$$

$$= \frac{KA(\theta_1 - \theta_2)}{Atf_w L} = \frac{1.7 \times [0 - (-10)]}{10 \times 10^{-2} \times 1000 \times 3.36 \times 10^5}$$

$$= \frac{17}{3.36} \times 10^{-7} = 5.059 \times 10^{-7} \approx 5 \times 10^{-7}\ m/sec$$

(b) let us assume that x length of ice has become formed to form a small strip of ice of length dx, dt time is required.

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{Adx f\omega L}{dt} = \frac{KA(\Delta\theta)}{x}$$



$$\Rightarrow \frac{dx f\omega L}{dt} = \frac{K(\Delta\theta)}{x} \Rightarrow dt = \frac{x dx f\omega L}{K(\Delta\theta)}$$

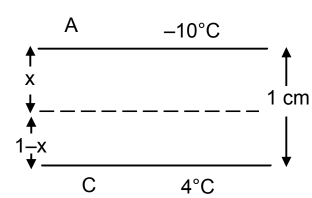
$$\Rightarrow \int_0^t dt = \frac{f\omega L}{K(\Delta\theta)} \int_0^t x dx \qquad \Rightarrow t = \frac{f\omega L}{K(\Delta\theta)} \left[\frac{x^2}{2}\right]_o^l = \frac{f\omega L}{K\Delta\theta} \frac{l^2}{2}$$

Putting values

$$\Rightarrow t = \frac{1000 \times 3.36 \times 10^5 \times (10 \times 10^{-2})^2}{1.7 \times 10 \times 2} = \frac{3.36}{2 \times 17} \times 10^6 \text{ sec.} = \frac{3.36 \times 10^6}{2 \times 17 \times 3600} \text{ hrs} = 27.45 \text{ hrs} \approx 27.5 \text{ hrs.}$$

15. let 'B' be the maximum level upto which ice is formed. Hence the heat conducted at that point from both the levels is the same.

Let AB = x

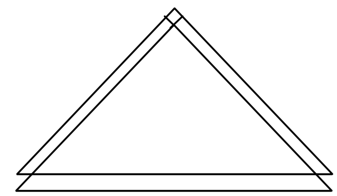


$$\text{i.e. } \frac{Q}{t}\text{ice} = \frac{Q}{t}\text{water} \qquad \Rightarrow \frac{K_{ice} \times A \times 10}{x} = \frac{K_{water} \times A \times 4}{(1-x)}$$

$$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1-x} \Rightarrow \frac{17}{x} = \frac{2}{1-x}$$

$$\Rightarrow 17 - 17x = 2x \Rightarrow 19x = 17 \Rightarrow x = \frac{17}{19} = 0.894 \approx 89\ cm$$

16. K_{AB} = 50 j/m-s-°c θ_A = 40°C
K_{BC} = 200 j/m-s-°c θ_B = 80°C
K_{AC} = 400 j/m-s-°c θ_C = 80°C
Length = 20 cm = 20 × 10^{-2} m
A = 1 cm^2 = 1 × 10^{-4} m^2



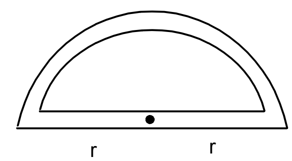
(a) $\frac{Q_{AB}}{t} = \frac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}}$ = 1 W.

(b) $\frac{Q_{AC}}{t} = \frac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}}$ = 800 × 10^{-2} = 8

(c) $\frac{Q_{BC}}{t} = \frac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}}$ = 0

17. We know Q = $\frac{KA(\theta_1 - \theta_2)}{d}$

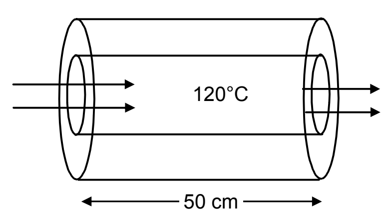
$Q_1 = \frac{KA(\theta_1 - \theta_2)}{d_1}$, $Q_2 = \frac{KA(\theta_1 - \theta_2)}{d_2}$



$$\frac{Q_1}{Q_2} = \frac{\frac{KA(\theta_1 - \theta_1)}{\pi r}}{\frac{KA(\theta_1 - \theta_1)}{2r}} = \frac{2r}{\pi r} = \frac{2}{\pi}$$ [$d_1 = \pi r$, $d_2 = 2r$]

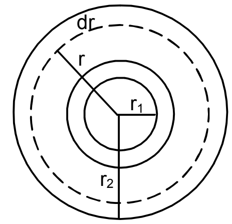
18. The rate of heat flow per sec.
$= \frac{dQ_A}{dt} = KA\frac{d\theta}{dt}$
The rate of heat flow per sec.
$= \frac{dQ_B}{dt} = KA\frac{d\theta_B}{dt}$
This part of heat is absorbed by the red.
$\frac{Q}{t} = \frac{ms\Delta\theta}{dt}$ where $\frac{d\theta}{dt}$ = Rate of net temp. variation
$\Rightarrow \frac{msd\theta}{dt} = KA\frac{d\theta_A}{dt} - KA\frac{d\theta_B}{dt}$ $\Rightarrow ms\frac{d\theta}{dt} = KA\left(\frac{d\theta_A}{dt} - \frac{d\theta_B}{dt}\right)$
$\Rightarrow 0.4 \times \frac{d\theta}{dt}$ = 200 × 1 × 10^{-4} (5 – 2.5) °C/cm
$\Rightarrow 0.4 \times \frac{d\theta}{dt}$ = 200 × 10^{-4} × 2.5
$\Rightarrow \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}}$ °C/m = 1250 × 10^{-2} = 12.5 °C/m

19. Given
K_{rubber} = 0.15 J/m-s-°C $T_2 - T_1$ = 90°C
We know for radial conduction in a Cylinder



$\frac{Q}{t} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$
$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)}$ = 232.5 ≈ 233 j/s.

20. $\frac{dQ}{dt}$ = Rate of flow of heat
Let us consider a strip at a distance r from the center of thickness dr.
$\frac{dQ}{dt} = \frac{K \times 2\pi rd \times d\theta}{dr}$ [$d\theta$ = Temperature diff across the thickness dr]



$\Rightarrow C = \dfrac{K \times 2\pi rd \times d\theta}{dr}$ $\left[c = \dfrac{d\theta}{dr}\right]$

$\Rightarrow C\dfrac{dr}{r} = K2\pi d\ d\theta$

Integrating

$\Rightarrow C\int_{r_1}^{r_2}\dfrac{dr}{r} = K2\pi d \int_{\theta_1}^{\theta_2} d\theta$ $\Rightarrow C[\log r]_{r_1}^{r_2} = K2\pi d\ (\theta_2 - \theta_1)$

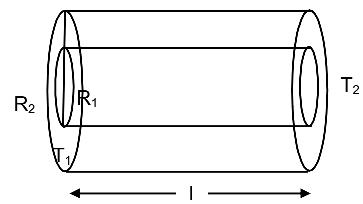
$\Rightarrow C(\log r_2 - \log r_1) = K2\pi d\ (\theta_2 - \theta_1) \Rightarrow C \log\left(\dfrac{r_2}{r_1}\right) = K2\pi d\ (\theta_2 - \theta_1)$

$\Rightarrow C = \dfrac{K2\pi d(\theta_2 - \theta_1)}{\log(r_2 / r_1)}$

21. $T_1 > T_2$

$A = \pi(R_2^2 - R_1^2)$

So, $Q = \dfrac{KA(T_2 - T_1)}{l} = \dfrac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$



Considering a concentric cylindrical shell of radius 'r' and thickness 'dr'. The radial heat flow through the shell

$H = \dfrac{dQ}{dt} = -KA\dfrac{d\theta}{dt}$ [(-)ve because as r – increases θ decreases]

$A = 2\pi rl$ $H = -2\pi rl\ K\ \dfrac{d\theta}{dt}$

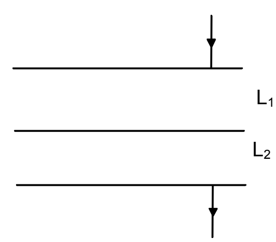
or $\displaystyle\int_{R_1}^{R_2}\frac{dr}{r} = -\frac{2\pi LK}{H}\int_{T_1}^{T_2} d\theta$

Integrating and simplifying we get

$H = \dfrac{dQ}{dt} = \dfrac{2\pi KL(T_2 - T_1)}{Loge(R_2 / R_1)} = \dfrac{2\pi KL(T_2 - T_1)}{ln(R_2 / R_1)}$

22. Here the thermal conductivities are in series,

$\therefore \dfrac{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} \times \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}}{\dfrac{K_1A(\theta_1 - \theta_2)}{l_1} + \dfrac{K_2A(\theta_1 - \theta_2)}{l_2}} = \dfrac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$



$\Rightarrow \dfrac{\dfrac{K_1}{l_1} \times \dfrac{K_2}{l_2}}{\dfrac{K_1}{l_1} + \dfrac{K_2}{l_2}} = \dfrac{K}{l_1 + l_2}$

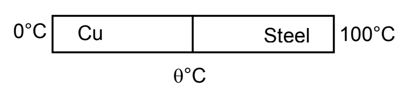
$\Rightarrow \dfrac{K_1K_2}{K_1l_2 + K_2l_1} = \dfrac{K}{l_1 + l_2} \Rightarrow K = \dfrac{(K_1K_2)(l_1 + l_2)}{K_1l_2 + K_2l_1}$

23. $K_{Cu} = 390$ w/m-°C $K_{St} = 46$ w/m-°C

Now, Since they are in series connection,

So, the heat passed through the crossections in the same.

So, $Q_1 = Q_2$

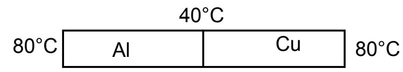


Or $\dfrac{K_{Cu} \times A \times (\theta - 0)}{l} = \dfrac{K_{St} \times A \times (100 - \theta)}{l}$

$\Rightarrow 390(\theta - 0) = 46 \times 100 - 46\ \theta \Rightarrow 436\ \theta = 4600$

$\Rightarrow \theta = \dfrac{4600}{436} = 10.55 \approx 10.6°C$

24. As the Aluminum rod and Copper rod joined are in parallel



$$\frac{Q}{t} = \left(\frac{Q}{t_1}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$

$$\Rightarrow \frac{KA(\theta_1-\theta_2)}{l} = \frac{K_1A(\theta_1-\theta_2)}{l} + \frac{K_2A(\theta_1-\theta_2)}{l}$$

$\Rightarrow K = K_1 + K_2 = (390 + 200) = 590$

$$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{l} = \frac{590\times1\times10^{-4}\times(60-20)}{1} = 590 \times 10^{-4} \times 40 = 2.36 \text{ Watt}$$

25. $K_{Al} = 200$ w/m-°C $\qquad K_{Cu} = 400$ w/m-°C

$A = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$

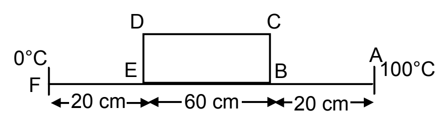
$l = 20 \text{ cm} = 2 \times 10^{-1} \text{ m}$

Heat drawn per second

$$= Q_{Al} + Q_{Cu} = \frac{K_{Al}\times A(80-40)}{l} + \frac{K_{Cu}\times A(80-40)}{l} = \frac{2\times10^{-5}\times40}{2\times10^{-1}}[200+400] = 2.4 \text{ J}$$

Heat drawn per min = 2.4 × 60 = 144 J

26. $(Q/t)_{AB} = (Q/t)_{BE\ bent} + (Q/t)_{BE}$



$$(Q/t)_{BE\ bent} = \frac{KA(\theta_1-\theta_2)}{70} \qquad (Q/t)_{BE} = \frac{KA(\theta_1-\theta_2)}{60}$$

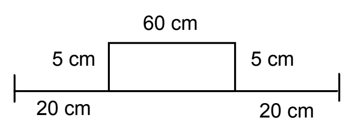
$$\frac{(Q/t)_{BE\,bent}}{(Q/t)_{BE}} = \frac{60}{70} = \frac{6}{7}$$

$(Q/t)_{BE\ bent} + (Q/t)_{BE} = 130$

$\Rightarrow (Q/t)_{BE\ bent} + (Q/t)_{BE}\ 7/6 = 130$

$$\Rightarrow \left(\frac{7}{6}+1\right)(Q/t)_{BE\ bent} = 130 \qquad \Rightarrow (Q/t)_{BE\ bent} = \frac{130\times6}{13} = 60$$

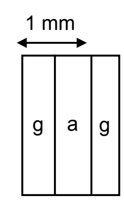
27. $$\frac{Q}{t}\text{bent} = \frac{780\times A\times100}{70}$$



$$\frac{Q}{t}\text{str} = \frac{390\times A\times100}{60}$$

$$\frac{(Q/t)\text{bent}}{(Q/t)\text{ str}} = \frac{780\times A\times100}{70}\times\frac{60}{390\times A\times100} = \frac{12}{7}$$

28. (a) $$\frac{Q}{t} = \frac{KA(\theta_1-\theta_2)}{\ell} = \frac{1\times2\times1(40-32)}{2\times10^{-3}} = 8000 \text{ J/sec.}$$



(b) Resistance of glass = $\dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_g}$

Resistance of air = $\dfrac{\ell}{ak_a}$

Net resistance = $\dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_g} + \dfrac{\ell}{ak_a}$

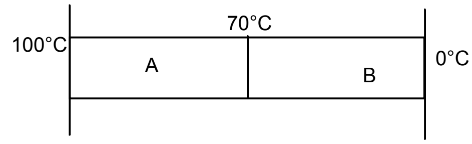
$$= \frac{\ell}{a}\left(\frac{2}{k_g}+\frac{1}{k_a}\right) = \frac{\ell}{a}\left(\frac{2k_a+k_g}{K_gk_a}\right)$$

$$= \frac{1\times10^{-3}}{2}\left(\frac{2\times0.025+1}{0.025}\right)$$

$$= \frac{1\times10^{-3}\times1.05}{0.05}$$

$$\frac{Q}{t} = \frac{\theta_1-\theta_2}{R} = \frac{8\times0.05}{1\times10^{-3}\times1.05} = 380.9 \approx 381 \text{ W}$$

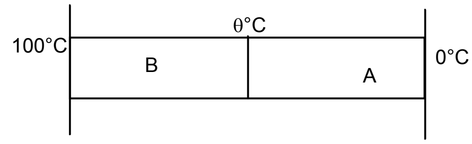
29. Now; Q/t remains same in both cases



In Case I : $\dfrac{K_A \times A \times (100-70)}{\ell} = \dfrac{K_B \times A \times (70-0)}{\ell}$

$\Rightarrow$ 30 K_A = 70 K_B

In Case II : $\dfrac{K_B \times A \times (100-\theta)}{\ell} = \dfrac{K_A \times A \times (\theta-0)}{\ell}$

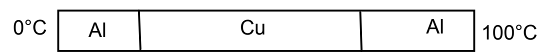


$\Rightarrow 100K_B - K_B\,\theta = K_A\,\theta$

$\Rightarrow 100K_B - K_B\,\theta = \dfrac{70}{30} K_B\,\theta$

$\Rightarrow 100 = \dfrac{7}{3}\theta + \theta$ $\qquad\qquad$ $\Rightarrow \theta = \dfrac{300}{10} = 30°C$

30. $\theta_1 - \theta_2 = 100$



$\dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R}$

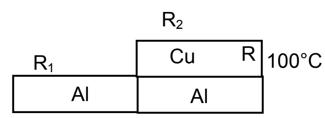
$R = R_1 + R_2 + R_3 = \dfrac{\ell}{aK_{Al}} + \dfrac{\ell}{aK_{Cu}} + \dfrac{\ell}{aK_{Al}} = \dfrac{\ell}{a}\left(\dfrac{2}{200} + \dfrac{1}{400}\right) = \dfrac{\ell}{a}\left(\dfrac{4+1}{400}\right) = \dfrac{\ell}{a}\dfrac{1}{80}$

$\dfrac{Q}{t} = \dfrac{100}{(\ell/a)(1/80)} \Rightarrow 40 = 80 \times 100 \times \dfrac{a}{\ell}$

$\Rightarrow \dfrac{a}{\ell} = \dfrac{1}{200}$

For (b)

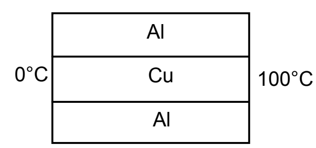
$R = R_1 + R_2 = R_1 + \dfrac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = R_{Al} + \dfrac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = \dfrac{\dfrac{l}{AK_{Al}} + \dfrac{l}{AK_{Cu}} + \dfrac{l}{AK_{Al}}}{\dfrac{l}{A_{Cu}} + \dfrac{l}{A_{Al}}}$



$= \dfrac{l}{AK_{Al}} + \dfrac{l}{A} + \dfrac{l}{K_{Al} + K_{Cu}} = \dfrac{l}{A}\left(\dfrac{1}{200} + \dfrac{1}{200+400}\right) = \dfrac{l}{A} \times \dfrac{4}{600}$

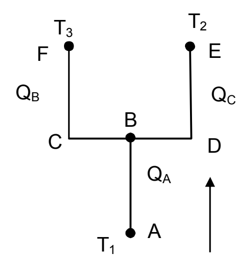
$\dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R} = \dfrac{100}{(l/A)(4/600)} = \dfrac{100 \times 600}{4}\dfrac{A}{l} = \dfrac{100 \times 600}{4} \times \dfrac{1}{200} = 75$

For (c)



$\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{\dfrac{l}{aK_{Al}}} + \dfrac{1}{\dfrac{l}{aK_{Cu}}} + \dfrac{1}{\dfrac{l}{aK_{Al}}}$

$= \dfrac{a}{l}(K_{Al} + K_{Cu} + K_{Al}) = \dfrac{a}{l}(2 \times 200 + 400) = \dfrac{a}{l}(800)$

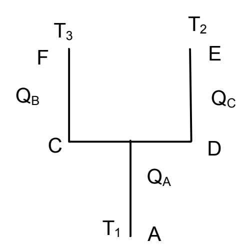


$\Rightarrow R = \dfrac{l}{a} \times \dfrac{1}{800}$

$\Rightarrow \dfrac{Q}{t} = \dfrac{\theta_1 - \theta_2}{R} = \dfrac{100 \times 800 \times a}{l}$

$= \dfrac{100 \times 800}{200} = 400$ W

31. Let the temp. at B be T



$\dfrac{Q_A}{t} = \dfrac{Q_B}{t} + \dfrac{Q_C}{t}$ $\qquad\qquad$ $\Rightarrow \dfrac{KA(T_1 - T)}{l} = \dfrac{KA(T - T_3)}{l + (l/2)} + \dfrac{KA(T - T_2)}{l + (l/2)}$

$\Rightarrow \dfrac{T_1 - T}{l} = \dfrac{T - T_3}{3l/2} + \dfrac{T - T_2}{3l/2}$ $\qquad\qquad$ $\Rightarrow 3T_1 - 3T = 4T - 2(T_2 + T_3)$

$\Rightarrow -7T = -3T_1 - 2(T_2 + T_3)$ $\qquad\qquad$ $\Rightarrow T = \dfrac{3T_1 + 2(T_2 + T_3)}{7}$

32. The temp at the both ends of bar F is same

Rate of Heat flow to right = Rate of heat flow through left

$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$

$\Rightarrow \dfrac{K_A(T_1 - T)A}{l} + \dfrac{K_C(T_1 - T)A}{l} = \dfrac{K_B(T - T_2)A}{l} + \dfrac{K_D(T - T_2)A}{l}$

$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$

$\Rightarrow T_1 - T = 2T - 2T_2$

$\Rightarrow T = \dfrac{T_1 + 2T_2}{3}$

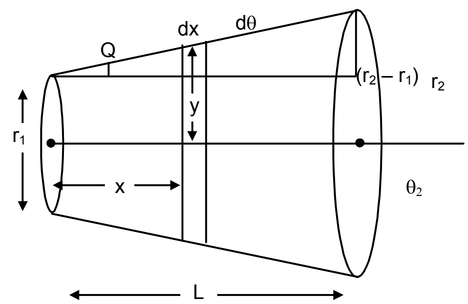
33. $\text{Tan } \phi = \dfrac{r_2 - r_1}{L} = \dfrac{(y - r_1)}{x}$

$\Rightarrow xr_2 - xr_1 = yL - r_1L$

Differentiating wr to 'x'

$\Rightarrow r_2 - r_1 = \dfrac{Ldy}{dx} - 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{r_2 - r_1}{L} \Rightarrow dx = \dfrac{dyL}{(r_2 - r_1)}$ …(1)



Now $\dfrac{Q}{T} = \dfrac{K\pi y^2 d\theta}{dx} \Rightarrow \dfrac{\theta dx}{T} = k\pi y^2 d\theta$

$\Rightarrow \dfrac{\theta L dy}{r_2 r_1} = K\pi y^2 d\theta$ from(1)

$\Rightarrow d\theta \; \dfrac{QLdy}{(r_2 - r_1)K\pi y^2}$

Integrating both side

$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \dfrac{QL}{(r_2 - r_1)k\pi} \int_{r_1}^{r_2} \dfrac{dy}{y}$

$\Rightarrow (\theta_2 - \theta_1) = \dfrac{QL}{(r_2 - r_1)K\pi} \times \left[\dfrac{-1}{y}\right]_{r_1}^{r_2}$

$\Rightarrow (\theta_2 - \theta_1) = \dfrac{QL}{(r_2 - r_1)K\pi} \times \left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$

$\Rightarrow (\theta_2 - \theta_1) = \dfrac{QL}{(r_2 - r_1)K\pi} \times \left[\dfrac{r_2 - r_1}{r_1 + r_2}\right]$

$\Rightarrow Q = \dfrac{K\pi r_1 r_2 (\theta_2 - \theta_1)}{L}$

34. $\dfrac{d\theta}{dt} = \dfrac{60}{10 \times 60} = 0.1°\text{C/sec}$

$\dfrac{dQ}{dt} = \dfrac{KA}{d}(\theta_1 - \theta_2)$

$= \dfrac{KA \times 0.1}{d} + \dfrac{KA \times 0.2}{d} + \ldots\ldots + \dfrac{KA \times 60}{d}$

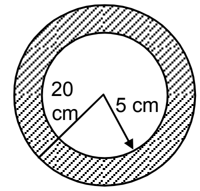
$= \dfrac{KA}{d}(0.1 + 0.2 + \ldots\ldots + 60) = \dfrac{KA}{d} \times \dfrac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$

[$\therefore$ a + 2a +……….+ na = n/2{2a + (n – 1)a}]

$= \dfrac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \dfrac{200 \times 10^{-2} \times 300 \times 60.1}{20}$

= 3 × 10 × 60.1 = 1803 w ≈ 1800 w

35. $a = r_1 = 5$ cm $= 0.05$ m
$b = r_2 = 20$ cm $= 0.2$ m
$\theta_1 = T_1 = 50°C$ $\qquad\qquad$ $\theta_2 = T_2 = 10°C$

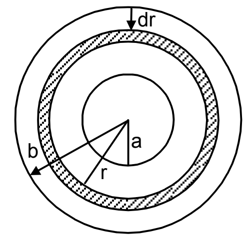


Now, considering a small strip of thickness 'dr' at a distance 'r'.
$A = 4\pi r^2$

$H = -4\pi r^2 K\dfrac{d\theta}{dr}$ [(–)ve because with increase of r, θ decreases]

$= \displaystyle\int_a^b \frac{dr}{r^2} = \frac{-4\pi K}{H}\int_{\theta_1}^{\theta_2} d\theta$ $\qquad$ On integration,

$H = \dfrac{dQ}{dt} = K\dfrac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$



Putting the values we get

$$\frac{K \times 4 \times 3.14 \times 5 \times 20 \times 40 \times 10^{-3}}{15 \times 10^{-2}} = 100$$

$$\Rightarrow K = \frac{15}{4 \times 3.14 \times 4 \times 10^{-1}} = 2.985 \approx 3 \text{ w/m-°C}$$

36. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ $\qquad$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lms}$

Fall in Temp in $T_1 = \dfrac{KA(T_1 - T_2)}{Lms}$ $\qquad$ Final Temp. $T_1 \Rightarrow T_1 - \dfrac{KA(T_1 - T_2)}{Lms}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lms}$

Final $\dfrac{\Delta T}{dt} = T_1 - \dfrac{KA(T_1 - T_2)}{Lms} - T_2 - \dfrac{KA(T_1 - T_2)}{Lms}$

$= (T_1 - T_2) - \dfrac{2KA(T_1 - T_2)}{Lms}$ $= \dfrac{dT}{dt} = -\dfrac{2KA(T_1 - T_2)}{Lms}$ $\Rightarrow \displaystyle\int_{(T_1-T_2)}^{(T_1-T_2)} \frac{dt}{(T_1 - T_2)} = \frac{-2KA}{Lms}dt$

$\Rightarrow Ln\dfrac{(T_1 - T_2)/2}{(T_1 - T_2)} = \dfrac{-2KAt}{Lms}$ $\qquad$ $\Rightarrow \ln(1/2) = \dfrac{-2KAt}{Lms}$ $\qquad$ $\Rightarrow \ln_2 = \dfrac{2KAt}{Lms}$ $\qquad$ $\Rightarrow t = \ln_2\dfrac{Lms}{2KA}$

37. $\dfrac{Q}{t} = \dfrac{KA(T_1 - T_2)}{L}$ $\qquad$ Rise in Temp. in $T_2 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Fall in Temp in $T_1 \Rightarrow \dfrac{KA(T_1 - T_2)}{Lm_2s_2}$ $\qquad$ Final Temp. $T_1 = T_1 - \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

Final Temp. $T_2 = T_2 + \dfrac{KA(T_1 - T_2)}{Lm_1s_1}$

$$\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1} - T_2 - \frac{KA(T_1 - T_2)}{Lm_2s_2} = (T_1 - T_2) - \left[\frac{KA(T_1 - T_2)}{Lm_1s_1} + \frac{KA(T_1 - T_2)}{Lm_2s_2}\right]$$

$$\Rightarrow \frac{dT}{dt} = -\frac{KA(T_1 - T_2)}{L}\left(\frac{1}{m_1s_1} + \frac{1}{m_2s_2}\right) \qquad \Rightarrow \frac{dT}{(T_1 - T_2)} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)dt$$

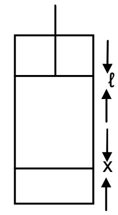
$$\Rightarrow \ln\Delta t = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t + C$$

At time $t = 0$, $T = T_0$, $\qquad$ $\Delta T = \Delta T_0$ $\qquad$ $\Rightarrow C = \ln\Delta T_0$

$$\Rightarrow \ln\frac{\Delta T}{\Delta T_0} = -\frac{KA}{L}\left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2}\right)t \Rightarrow \frac{\Delta T}{\Delta T_0} = e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$$

$$\Rightarrow \Delta T = \Delta T_0\, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t} = (T_2 - T_1)\, e^{-\frac{KA}{L}\left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2}\right)t}$$

38. $\frac{Q}{t} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_P dT}{dt} = \frac{KA(T_s - T_0)}{x}$



$\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_S - T_0)$

$\Rightarrow \frac{dT}{(T_S - T_0)} = -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_S - T_0)_{T_0}^{T} = -\frac{2KAdt}{5nRx}$

$\Rightarrow \ln\frac{T_S - T}{T_S - T_0} = -\frac{2KAdt}{5nRx} \Rightarrow T_S - T = (T_S - T_0)e^{-\frac{2KAt}{5nRx}}$

$\Rightarrow T = T_S - (T_S - T_0)e^{-\frac{2KAt}{5nRx}} = T_S + (T_S + T_0)e^{+\frac{2KAt}{5nRx}}$

$\Rightarrow \Delta T = T - T_0 = (T_S - T_0) + (T_S - T_0)e^{+\frac{2KAt}{5nRx}} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$

$\Rightarrow \frac{P_a AL}{nR} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$ [$p_a dv = nRdt$ $P_a Al = nRdt$ $dT = \frac{P_a AL}{nR}$]

$\Rightarrow L = \frac{nR}{P_a A}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$

39. $A = 1.6\ m^2$, $T = 37°C = 310\ K$, $\sigma = 6.0 \times 10^{-8}\ w/m^2\text{-}K^4$

Energy radiated per second

$= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887\ J$

40. $A = 12\ cm^2 = 12 \times 10^{-4}\ m^2$ $T = 20°C = 293\ K$

$e = 0.8$ $\sigma = 6 \times 10^{-8}\ w/m^2\text{-}k^4$

$\frac{Q}{t} = Ae\ \sigma T^4 = 12 \times 10^{-4}\ 0.8 \times 6 \times 10^{-8}\ (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42$

41. $E \rightarrow$ Energy radiated per unit area per unit time

Rate of heat flow $\rightarrow$ Energy radiated

(a) Per time = E × A

So, $E_{Al} = \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4}$ $\therefore$ 1 : 4

(b) Emissivity of both are same

$= \frac{m_1 S_1 dT_1}{m_2 S_2 dT_2} = 1$

$\Rightarrow \frac{dT_1}{dT_2} = \frac{m_2 S_2}{m_1 S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1 : 2 : 9$

42. $\frac{Q}{t} = Ae\ \sigma T^4$

$\Rightarrow T^4 = \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}}$

$\Rightarrow T = 1697.0 \approx 1700\ K$

43. (a) $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$, $T = 57°C = 330\ K$

$E = A\ \sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42\ J$

(b) $\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$, $A = 20\ cm^2 = 20 \times 10^{-4}\ m^2$

$\sigma = 6 \times 10^{-8}$ $T_1 = 473\ K$, $T_2 = 330\ K$

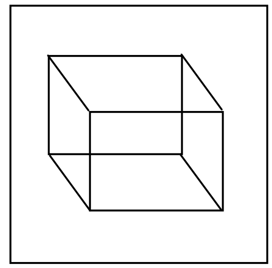
$= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4]$

$= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}]$

$= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58\ w$ from the ball.

44. $r = 1\text{ cm} = 1 \times 10^{-3}\text{ m}$
$A = 4\pi(10^{-2})^2 = 4\pi \times 10^{-4}\text{ m}^2$
$E = 0.3,\qquad \sigma = 6 \times 10^{-8}$
$\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$
$= 0.3 \times 6 \times 10^{-8} \times 4\pi \times 10^{-4} \times [(100)^4 - (300)^4]$
$= 0.3 \times 6 \times 4\pi \times 10^{-12} \times [1 - 0.0081] \times 10^{12}$
$= 0.3 \times 6 \times 4 \times 3.14 \times 9919 \times 10^{-4}$
$= 4 \times 18 \times 3.14 \times 9919 \times 10^{-5} = 22.4 \approx 22\text{ W}$

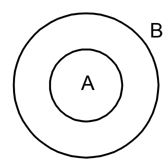
45. Since the Cube can be assumed as black body
$e = \ell$
$\sigma = 6 \times 10^{-8}\text{ w/m}^2\text{-k}^4$
$A = 6 \times 25 \times 10^{-4}\text{ m}^2$
$m = 1\text{ kg}$
$s = 400\text{ J/kg-°K}$
$T_1 = 227°C = 500\text{ K}$
$T_2 = 27°C = 300\text{ K}$



$\Rightarrow ms\frac{d\theta}{dt} = e\sigma A(T_1^4 - T_2^4)$
$\Rightarrow \frac{d\theta}{dt} = \frac{e\sigma A\left(T_1^4 - T_2^4\right)}{ms}$
$= \frac{1 \times 6 \times 10^{-8} \times 6 \times 25 \times 10^{-4} \times [(500)^4 - (300)^4]}{1 \times 400}$
$= \frac{36 \times 25 \times 544}{400} \times 10^{-4} = 1224 \times 10^{-4} = 0.1224°C/s \approx 0.12°C/s.$

46. $Q = e\sigma A(T_2^4 - T_1^4)$
For any body, $210 = eA\sigma[(500)^4 - (300)^4]$
For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$
Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

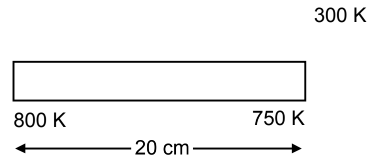
47. $A_A = 20\text{ cm}^2,\qquad A_B = 80\text{ cm}^2$
$(mS)_A = 42\text{ J/°C},\qquad (mS)_B = 82\text{ J/°C},$
$T_A = 100°C,\qquad T_B = 20°C$
K_B is low thus it is a poor conducter and K_A is high.
Thus A will absorb no heat and conduct all



$\left(\frac{E}{t}\right)_A = \sigma A_A[(373)^4 - (293)^4] \qquad \Rightarrow (mS)_A\left(\frac{d\theta}{dt}\right)_A = \sigma A_A[(373)^4 - (293)^4]$
$\Rightarrow \left(\frac{d\theta}{dt}\right)_A = \frac{\sigma A_a\left[(373)^4 - (293)^4\right]}{(mS)_A} = \frac{6 \times 10^{-8}\left[(373)^4 - (293)^4\right]}{42} = 0.03\text{ °C/S}$
Similarly $\left(\frac{d\theta}{dt}\right)_B = 0.043\text{ °C/S}$

48. $\frac{Q}{t} = eAe(T_2^4 - T_1^4)$
$\Rightarrow \frac{Q}{At} = 1 \times 6 \times 10^{-8}[(300)^4 - (290)^4] \qquad = 6 \times 10^{-8}(81 \times 10^8 - 70.7 \times 10^8) = 6 \times 10.3$
$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$
$\Rightarrow \frac{Q}{tA} = \frac{K(\theta_1 - \theta_2)}{l} = \frac{K \times 17}{0.5} = 6 \times 10.3 = \frac{K \times 17}{0.5} \Rightarrow K = \frac{6 \times 10.3 \times 0.5}{17} = 1.8$

49. $\sigma = 6 \times 10^{-8}$ w/m^2-k^4



L = 20 cm = 0.2 m, K = ?

$\Rightarrow E = \dfrac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$

$\Rightarrow K = \dfrac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \dfrac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$

$\Rightarrow K = 73.993 \approx 74.$

50. v = 100 cc

$\Delta\theta = 5°C$

t = 5 min

For water

$\dfrac{mS\Delta\theta}{dt} = \dfrac{KA}{l}\Delta\theta$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 1000 \times 4200}{5} = \dfrac{KA}{l}$

For Kerosene

$\dfrac{ms}{at} = \dfrac{KA}{l}$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 800 \times 2100}{t} = \dfrac{KA}{l}$

$\Rightarrow \dfrac{100 \times 10^{-3} \times 800 \times 2100}{t} = \dfrac{100 \times 10^{-3} \times 1000 \times 4200}{5}$

$\Rightarrow T = \dfrac{5 \times 800 \times 2100}{1000 \times 4200} = 2$ min

51. 50°C 45°C 40°C

Let the surrounding temperature be 'T'°C

Avg. t = $\dfrac{50 + 45}{2} = 47.5$

Avg. temp. diff. from surrounding

T = 47.5 – T

Rate of fall of temp = $\dfrac{50 - 45}{5} = 1$ °C/mm

From Newton's Law

1°C/mm = bA × t

$\Rightarrow bA = \dfrac{1}{t} = \dfrac{1}{47.5 - T}$ …(1)

In second case,

Avg, temp = $\dfrac{40 + 45}{2} = 42.5$

Avg. temp. diff. from surrounding

t′ = 42.5 – t

Rate of fall of temp = $\dfrac{45 - 40}{8} = \dfrac{5}{8}$ °C/mm

From Newton's Law

$\dfrac{5}{B} = bAt'$

$\Rightarrow \dfrac{5}{8} = \dfrac{1}{(47.5 - T)} \times (42.5 - T)$

By C & D [Componendo & Dividendo method]

We find, T = 34.1°C

52. Let the water eq. of calorimeter = m

$$\frac{(m+50\times10^{-3})\times4200\times5}{10} = \text{Rate of heat flow}$$

$$\frac{(m+100\times10^{-3})\times4200\times5}{18} = \text{Rate of flow}$$

$$\Rightarrow \frac{(m+50\times10^{-3})\times4200\times5}{10} = \frac{(m+100\times10^{-3})\times4200\times5}{18}$$

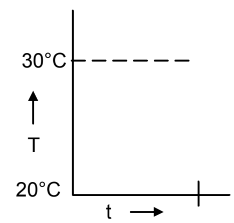
$\Rightarrow (m + 50 \times 10^{-3})18 = 10m + 1000 \times 10^{-3}$

$\Rightarrow 18m + 18 \times 50 \times 10^{-3} = 10m + 1000 \times 10^{-3}$

$\Rightarrow 8m = 100 \times 10^{-3}$ kg

$\Rightarrow m = 12.5 \times 10^{-3}$ kg = 12.5 g

53. In steady state condition as no heat is absorbed, the rate of loss of heat by conduction is equal to that of the supplied.



i.e. H = P

m = 1Kg, Power of Heater = 20 W, Room Temp. = 20°C

(a) $H = \frac{d\theta}{dt} = P = 20$ watt

(b) by Newton's law of cooling

$$\frac{-d\theta}{dt} = K(\theta - \theta_0)$$

$-20 = K(50 - 20) \Rightarrow K = 2/3$

Again, $\frac{-d\theta}{dt} = K(\theta - \theta_0) = \frac{2}{3}\times(30-20) = \frac{20}{3}$ w

(c) $\left(\frac{dQ}{dt}\right)_{20} = 0$, $\quad \left(\frac{dQ}{dt}\right)_{30} = \frac{20}{3}$ $\quad \left(\frac{dQ}{dt}\right)_{avg} = \frac{10}{3}$

T = 5 min = 300 ′

Heat liberated = $\frac{10}{3}\times300 = 1000$ J

Net Heat absorbed = Heat supplied – Heat Radiated = 6000 – 1000 = 5000 J

Now, $m\Delta\theta' = 5000$

$\Rightarrow S = \frac{5000}{m\Delta\theta} = \frac{5000}{1\times10} = 500$ J $Kg^{-1}°C^{-1}$

54. Given:

Heat capacity = m × s = 80 J/°C

$$\left(\frac{d\theta}{dt}\right)_{increase} = 2\ °C/s$$

$$\left(\frac{d\theta}{dt}\right)_{decrease} = 0.2\ °C/s$$

(a) Power of heater = $mS\left(\frac{d\theta}{dt}\right)_{increasing} = 80 \times 2 = 160$ W

(b) Power radiated = $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = 80 \times 0.2 = 16$ W

(c) Now $mS\left(\frac{d\theta}{dt}\right)_{decreasing} = K(T - T_0)$

$\Rightarrow 16 = K(30 - 20) \qquad \Rightarrow K = \frac{16}{10} = 1.6$

Now, $\frac{d\theta}{dt} = K(T - T_0) = 1.6 \times (30 - 25) = 1.6 \times 5 = 8$ W

(d) $P.t = H \Rightarrow 8 \times t$

55. $\dfrac{d\theta}{dt} = -K(T - T_0)$

Temp. at t = 0 is θ_1

(a) Max. Heat that the body can loose = $\Delta Q_m = ms(\theta_1 - \theta_0)$

($\therefore$ as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0)\times 9}{10}$$

If it takes time t_1, for this process, the temp. at t_1

$$= \theta_1 - (\theta_1 - \theta_0)\frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10}\times 1$$

Now, $\dfrac{d\theta}{dt} = -K(\theta - \theta_1)$

Let $\theta = \theta_1$ at t = 0; & θ be temp. at time t

$$\int_{\theta}^{\theta}\frac{d\theta}{\theta - \theta_o} = -K\int_0^t dt$$

or, $\ln\dfrac{\theta - \theta_0}{\theta_1 - \theta_0} = -Kt$

or, $\theta - \theta_0 = (\theta_1 - \theta_0)\,e^{-kt}$ …(2)

Putting value in the Eq (1) and Eq (2)

$$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0)\,e^{-kt}$$

$\Rightarrow t_1 = \dfrac{\ln 10}{k}$

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