QUESTIONS FOR SHORT ANSWER

1. Does a gas have just two specific heat capacities or more than two ? Is the number of specific heat capacities of a gas countable ?
2. Can we define specific heat capacity at constant temperature ?
3. Can we define specific heat capacity for an adiabatic process ?
4. Does a solid also have two kinds of molar heat capacities C_p and C_v ? If yes, do we have $C_p > C_v$? $C_p - C_v = R$?
5. In a real gas the internal energy depends on temperature and also on volume. The energy increases when the gas expands isothermally. Looking into the derivation of $C_p - C_v = R$, find whether $C_p - C_v$ will be more than R, less than R or equal to R for a real gas.
6. Can a process on an ideal gas be both adiabatic and isothermal ?
7. Show that the slope of p–V diagram is greater for an adiabatic process as compared to an isothermal process.
8. Is a slow process always isothermal ? Is a quick process always adiabatic ?
9. Can two states of an ideal gas be connected by an isothermal process as well as an adiabatic process ?
10. The ratio C_p/C_v for a gas is 1·29. What is the degree of freedom of the molecules of this gas ?

OBJECTIVE I

1. Work done by a sample of an ideal gas in a process A is double the work done in another process B. The temperature rises through the same amount in the two processes. If C_A and C_B be the molar heat capacities for the two processes,
   (a) $C_A = C_B$ (b) $C_A < C_B$
   (c) $C_A > C_B$ (d) C_A and C_B cannot be defined.
2. For a solid with a small expansion coefficient,
   (a) $C_p - C_v = R$ (b) $C_p = C_v$
   (c) C_p is slightly greater than C_v
   (d) C_p is slightly less than C_v.
3. The value of $C_p - C_v$ is 1·00 R for a gas sample in state A and is 1·08 R in state B. Let p_A, p_B denote the pressures and T_A and T_B denote the temperatures of the states A and B respectively. Most likely
   (a) $p_A < p_B$ and $T_A > T_B$ (b) $p_A > p_B$ and $T_A < T_B$
   (c) $p_A = p_B$ and $T_A < T_B$ (d) $p_A > p_B$ and $T_A = T_B$.
4. Let C_v and C_p denote the molar heat capacities of an ideal gas at constant volume and constant pressure respectively. Which of the following is a universal constant ?
   (a) $\frac{C_p}{C_v}$ (b) $C_p C_v$ (c) $C_p - C_v$ (d) $C_p + C_v$.
5. 70 calories of heat is required to raise the temperature of 2 mole of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is
   (a) 30 calories (b) 50 calories
   (c) 70 calories (d) 90 calories.

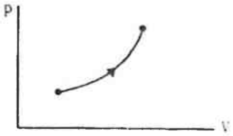


Figure 27-Q1

6. Figure (27-Q1) shows a process on a gas in which pressure and volume both change. The molar heat capacity for this process is C.
   (a) $C = 0$ (b) $C = C_v$ (c) $C > C_v$ (d) $C < C_v$.
7. The molar heat capacity for the process shown in figure (27-Q2) is
   (a) $C = C_p$ (b) $C = C_v$ (c) $C > C_v$ (d) $C = 0$.

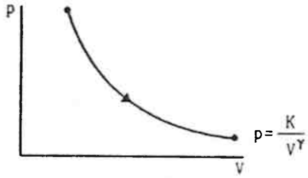


Figure 27-Q2

8. In an isothermal process on an ideal gas, the pressure increases by 0·5%. The volume decreases by about
   (a) 0·25% (b) 0·5% (c) 0·7% (d) 1%.
9. In an adiabatic process on a gas with $\gamma = 1{\cdot}4$, the pressure is increased by 0·5%. The volume decreases by about
   (a) 0·36% (b) 0·5% (c) 0·7% (d) 1%.
10. Two samples A and B are initially kept in the same state. The sample A is expanded through an adiabatic process and the sample B through an isothermal process. The final volumes of the samples are the same. The final pressures in A and B are p_A and p_B respectively.
    (a) $p_A > p_B$. (b) $p_A = p_B$. (c) $p_A < p_B$.
    (d) The relation between p_A and p_B cannot be deduced.
11. Let T_a and T_b be the final temperatures of the samples A and B respectively in the previous question.
    (a) $T_a < T_b$. (b) $T_a = T_b$. (c) $T_a > T_b$.
    (d) The relation between T_a and T_b cannot be deduced.
12. Let ΔW_a and ΔW_b be the work done by the systems A and B respectively in the previous question.
    (a) $\Delta W_a > \Delta W_b$. (b) $\Delta W_a = \Delta W_b$. (c) $\Delta W_a < \Delta W_b$.

(d) The relation between ΔW_a and ΔW_b cannot be deduced.

13. The molar heat capacity of oxygen gas at STP is nearly $2{\cdot}5\,R$. As the temperature is increased, it gradually increases and approaches $3{\cdot}5\,R$. The most appropriate reason for this behaviour is that at high temperatures
(a) oxygen does not behave as an ideal gas
(b) oxygen molecules dissociate in atoms
(c) the molecules collide more frequently
(d) molecular vibrations gradually become effective.

OBJECTIVE II

1. A gas kept in a container of finite conductivity is suddenly compressed. The process
(a) must be very nearly adiabatic
(b) must be very nearly isothermal
(c) may be very nearly adiabatic
(d) may be very nearly isothermal.

2. Let Q and W denote the amount of heat given to an ideal gas and the work done by it in an isothermal process.
(a) $Q = 0$. (b) $W = 0$. (c) $Q \neq W$. (d) $Q = W$.

3. Let Q and W denote the amount of heat given to an ideal gas and the work done by it in an adiabatic process.
(a) $Q = 0$. (b) $W = 0$. (c) $Q = W$. (d) $Q \neq W$.

4. Consider the processes A and B shown in figure (27-Q3). It is possible that

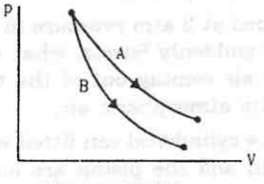


Figure 27-Q3

(a) both the processes are isothermal
(b) both the processes are adiabatic
(c) A is isothermal and B is adiabatic
(d) A is adiabatic and B is isothermal.

5. Three identical adiabatic containers A, B and C contain helium, neon and oxygen respectively at equal pressure. The gases are pushed to half their original volumes.
(a) The final temperatures in the three containers will be the same.
(b) The final pressures in the three containers will be the same.
(c) The pressures of helium and neon will be the same but that of oxygen will be different.
(d) The temperatures of helium and neon will be the same but that of oxygen will be different.

6. A rigid container of negligible heat capacity contains one mole of an ideal gas. The temperature of the gas increases by 1°C if 3·0 cal of heat is added to it. The gas may be
(a) helium (b) argon (c) oxygen (d) carbon dioxide.

7. Four cylinders contain equal number of moles of argon, hydrogen, nitrogen and carbon dioxide at the same temperature. The energy is minimum in
(a) argon (b) hydrogen (c) nitrogen (d) carbon dioxide.

EXERCISES

1. A vessel containing one mole of a monatomic ideal gas (molecular weight = 20 g/mol) is moving on a floor at a speed of 50 m/s. The vessel is stopped suddenly. Assuming that the mechanical energy lost has gone into the internal energy of the gas, find the rise in its temperature.

2. 5 g of a gas is contained in a rigid container and is heated from 15°C to 25°C. Specific heat capacity of the gas at constant volume is 0·172 cal/g–°C and the mechanical equivalent of heat is 4·2 J/cal. Calculate the change in the internal energy of the gas.

3. Figure (27-E1) shows a cylindrical container containing oxygen ($\gamma = 1{\cdot}4$) and closed by a 50 kg frictionless piston. The area of cross-section is 100 cm^2. atmospheric pressure is 100 kPa and g is 10 m/s^2. The cylinder is slowly heated for some time. Find the amount of heat supplied to the gas if the piston moves out through a distance of 20 cm.

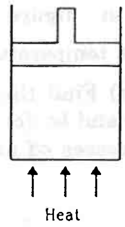


Figure 27-E1

4. The specific heat capacities of hydrogen at constant volume and at constant pressure are 2·4 cal/g–°C and 3·4 cal/g–°C respectively. The molecular weight of hydrogen is 2 g/mol and the gas constant $R = 8{\cdot}3 \times 10^7$ erg/mol–°C. Calculate the value of J.

5. The ratio of the molar heat capacities of an ideal gas is $C_p/C_v = 7/6$. Calculate the change in internal energy of 1·0 mole of the gas when its temperature is raised by

50 K (a) keeping the pressure constant, (b) keeping the volume constant and (c) adiabatically.

6. A sample of air weighing 1·18 g occupies $1\cdot0 \times 10^3\ \text{cm}^3$ when kept at 300 K and $1\cdot0 \times 10^5$ Pa. When 2·0 cal of heat is added to it at constant volume, its temperature increases by 1°C. Calculate the amount of heat needed to increase the temperature of air by 1°C at constant pressure if the mechanical equivalent of heat is $4\cdot2 \times 10^7$ erg/cal. Assume that air behaves as an ideal gas.

7. An ideal gas expands from $100\ \text{cm}^3$ to $200\ \text{cm}^3$ at a constant pressure of $2\cdot0 \times 10^5$ Pa when 50 J of heat is supplied to it. Calculate (a) the change in internal energy of the gas, (b) the number of moles in the gas if the initial temperature is 300 K, (c) the molar heat capacity C_p at constant pressure and (d) the molar heat capacity C_v at constant volume.

8. An amount Q of heat is added to a monatomic ideal gas in a process in which the gas performs a work $Q/2$ on its surrounding. Find the molar heat capacity for the process.

9. An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation $p = kV$. Show that the molar heat capacity of the gas for the process is given by $C = C_v + \frac{R}{2}$.

10. An ideal gas $(C_p/C_v = \gamma)$ is taken through a process in which the pressure and the volume vary as $p = aV^b$. Find the value of b for which the specific heat capacity in the process is zero.

11. Two ideal gases have the same value of $C_p/C_v = \gamma$. What will be the value of this ratio for a mixture of the two gases in the ratio 1 : 2 ?

12. A mixture contains 1 mole of helium $(C_p = 2\cdot5\ R,$ $C_v = 1\cdot5\ R)$ and 1 mole of hydrogen $(C_p = 3\cdot5\ R,$ $C_v = 2\cdot5\ R)$. Calculate the values of C_p, C_v and γ for the mixture.

13. Half mole of an ideal gas $(\gamma = 5/3)$ is taken through the cycle *abcda* as shown in figure (27-E2). Take $R = \frac{25}{3}$ J/mol-K (a) Find the temperature of the gas in the states *a*, *b*, *c* and *d*. (b) Find the amount of heat supplied in the processes *ab* and *bc*. (c) Find the amount of heat liberated in the processes *cd* and *da*.

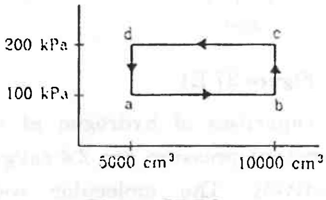


Figure 27-E2

14. An ideal gas $(\gamma = 1\cdot67)$ is taken through the process *abc* shown in figure (27-E3). The temperature at the point *a* is 300 K. Calculate (a) the temperatures at *b* and *c*, (b) the work done in the process, (c) the amount of heat supplied in the path *ab* and in the path *bc* and (d) the change in the internal energy of the gas in the process.

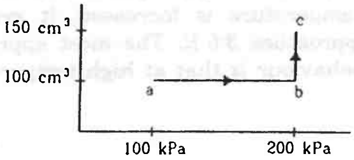


Figure 27-E3

15. In Joly's differential steam calorimeter, 3 g of an ideal gas is contained in a rigid closed sphere at 20°C. The sphere is heated by steam at 100°C and it is found that an extra 0·095 g of steam has condensed into water as the temperature of the gas becomes constant. Calculate the specific heat capacity of the gas in J/g–K. The latent heat of vaporization of water = 540 cal/g.

16. The volume of an ideal gas $(\gamma = 1\cdot5)$ is changed adiabatically from 4·00 litres to 3·00 litres. Find the ratio of (a) the final pressure to the initial pressure and (b) the final temperature to the initial temperature.

17. An ideal gas at pressure $2\cdot5 \times 10^5$ Pa and temperature 300 K occupies 100 cc. It is adiabatically compressed to half its original volume. Calculate (a) the final pressure, (b) the final temperature and (c) the work done by the gas in the process. Take $\gamma = 1\cdot5$.

18. Air $(\gamma = 1\cdot4)$ is pumped at 2 atm pressure in a motor tyre at 20°C. If the tyre suddenly bursts, what would be the temperature of the air coming out of the tyre. Neglect any mixing with the atmospheric air.

19. A gas is enclosed in a cylindrical can fitted with a piston. The walls of the can and the piston are adiabatic. The initial pressure, volume and temperature of the gas are 100 kPa, $400\ \text{cm}^3$ and 300 K respectively. The ratio of the specific heat capacities of the gas is $C_p/C_v = 1\cdot5$. Find the pressure and the temperature of the gas if it is (a) suddenly compressed (b) slowly compressed to $100\ \text{cm}^3$.

20. The initial pressure and volume of a given mass of a gas $(C_p/C_v = \gamma)$ are p_0 and V_0. The gas can exchange heat with the surrounding. (a) It is slowly compressed to a volume $V_0/2$ and then suddenly compressed to $V_0/4$. Find the final pressure. (b) If the gas is suddenly compressed from the volume V_0 to $V_0/2$ and then slowly compressed to $V_0/4$, what will be the final pressure ?

21. Consider a given sample of an ideal gas $(C_p/C_v = \gamma)$ having initial pressure p_0 and volume V_0. (a) The gas is isothermally taken to a pressure $p_0/2$ and from there adiabatically to a pressure $p_0/4$. Find the final volume. (b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $p_0/2$ and from there isothermally to a pressure $p_0/4$. Find the final volume.

22. A sample of an ideal gas $(\gamma = 1\cdot5)$ is compressed adiabatically from a volume of $150\ \text{cm}^3$ to $50\ \text{cm}^3$. The initial pressure and the initial temperature are 150 kPa and 300 K. Find (a) the number of moles of the gas in the sample, (b) the molar heat capacity at constant volume, (c) the final pressure and temperature, (d) the

work done by the gas in the process and (e) the change in internal energy of the gas.

23. Three samples A, B and C of the same gas (γ = 1·5) have equal volumes and temperatures. The volume of each sample is doubled, the process being isothermal for A, adiabatic for B and isobaric for C. If the final pressures are equal for the three samples, find the ratio of the initial pressures.

24. Two samples A and B of the same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to double its volume and the gas in B is expanded adiabatically to double its volume. If the work done by the gas is the same for the two cases, show that γ satisfies the equation $1 - 2^{1-\gamma} = (\gamma - 1)\ln 2$.

25. 1 litre of an ideal gas (γ = 1·5) at 300 K is suddenly compressed to half its original volume. (a) Find the ratio of the final pressure to the initial pressure. (b) If the original pressure is 100 kPa, find the work done by the gas in the process. (c) What is the change in internal energy? (d) What is the final temperature? (e) The gas is now cooled to 300 K keeping its pressure constant. Calculate the work done during the process. (f) The gas is now expanded isothermally to achieve its original volume of 1 litre. Calculate the work done by the gas. (g) Calculate the total work done in the cycle.

26. Figure (27-E4) shows a cylindrical tube with adiabatic walls and fitted with an adiabatic separator. The separator can be slid into the tube by an external mechanism. An ideal gas (γ = 1·5) is injected in the two sides at equal pressures and temperatures. The separator remains in equilibrium at the middle. It is now slid to a position where it divides the tube in the ratio 1 : 3. Find the ratio of the temperatures in the two parts of the vessel.

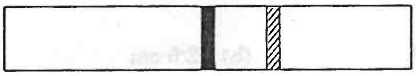


Figure 27-E4

27. Figure (27-E5) shows two rigid vessels A and B, each of volume 200 cm^3 containing an ideal gas (C_v = 12·5 J/mol–K). The vessels are connected to a manometer tube containing mercury. The pressure in both the vessels is 75 cm of mercury and the temperature is 300 K. (a) Find the number of moles of the gas in each vessel. (b) 5·0 J of heat is supplied to the gas in the vessel A and 10 J to the gas in the vessel B. Assuming no appreciable transfer of heat from A to B calculate the difference in the heights of mercury in the two sides of the manometer. Gas constant R = 8·3 J/mol–K.

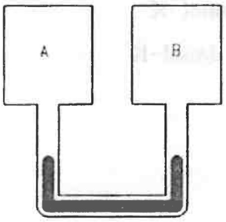


Figure 27-E5

28. Figure (27-E6) shows two vessels with adiabatic walls, one containing 0·1 g of helium (γ = 1·67, M = 4 g/mol) and the other containing some amount of hydrogen (γ = 1·4, M = 2 g/mol). Initially, the temperatures of the two gases are equal. The gases are electrically heated for some time during which equal amounts of heat are given to the two gases. It is found that the temperatures rise through the same amount in the two vessels. Calculate the mass of hydrogen.

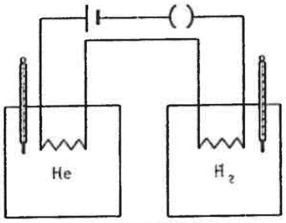


Figure 27-E6

29. Two vessels A and B of equal volume V_0 are connected by a narrow tube which can be closed by a valve. The vessels are fitted with pistons which can be moved to change the volumes. Initially, the valve is open and the vessels contain an ideal gas ($C_p/C_v = \gamma$) at atmospheric pressure p_0 and atmospheric temperature T_0. The walls of the vessel A are diathermic and those of B are adiabatic. The valve is now closed and the pistons are slowly pulled out to increase the volumes of the vessels to double the original value. (a) Find the temperatures and pressures in the two vessels. (b) The valve is now opened for sufficient time so that the gases acquire a common temperature and pressure. Find the new values of the temperature and the pressure.

30. Figure (27-E7) shows an adiabatic cylindrical tube of volume V_0 divided in two parts by a frictionless adiabatic separator. Initially, the separator is kept in the middle, an ideal gas at pressure p_1 and temperature T_1 is injected into the left part and another ideal gas at pressure p_2 and temperature T_2 is injected into the right part. $C_p/C_v = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find (a) the volumes of the two parts, (b) the heat given to the gas in the left part and (c) the final common pressure of the gases.

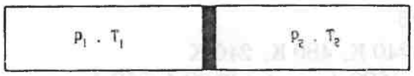


Figure 27-E7

31. An adiabatic cylindrical tube of cross-sectional area 1 cm^2 is closed at one end and fitted with a piston at the other end. The tube contains 0·03 g of an ideal gas. At 1 atm pressure and at the temperature of the surrounding, the length of the gas column is 40 cm. The piston is suddenly pulled out to double the length of the column. The pressure of the gas falls to 0·355 atm. Find the speed of sound in the gas at atmospheric temperature.

32. The speed of sound in hydrogen at 0°C is 1280 m/s. The density of hydrogen at STP is 0·089 kg/m^3. Calculate the molar heat capacities C_p and C_v of hydrogen.

33. 4·0 g of helium occupies 22400 cm^3 at STP. The specific heat capacity of helium at constant pressure is 5·0 cal/mol–K. Calculate the speed of sound in helium at STP.

34. An ideal gas having density $1{\cdot}7 \times 10^{-3}$ g/cm^3 at a pressure $1{\cdot}5 \times 10^5$ Pa is filled in a Kundt's tube. When the gas is resonated at a frequency of 3·0 kHz, nodes are formed at a separation of 6·0 cm. Calculate the molar heat capacities C_p and C_v of the gas.

35. Standing waves of frequency 5·0 kHz are produced in a tube filled with oxygen at 300 K. The separation between the consecutive nodes is 3·3 cm. Calculate the specific heat capacities C_p and C_v of the gas.

□

ANSWERS

OBJECTIVE I

1\. (c) 2. (c) 3. (a) 4. (c) 5. (b) 6. (c)
7\. (d) 8. (b) 9. (a) 10. (c) 11. (a) 12. (c)
13\. (d)

OBJECTIVE II

1\. (c), (d) 2. (d) 3. (a, (d)
4\. (c) 5. (c), (d) 6. (a), (b)
7\. (a)

EXERCISES

1\. 2·0 K
2\. 36 J
3\. 1050 J
4\. $4{\cdot}15 \times 10^7$ erg/cal
5\. 2490 J in all cases
6\. 2·08 cal
7\. (a) 30 J (b) 0·008
(c) 20·8 J/mol-K (d) 12·5 J/mol-K
8\. 3 R
10\. $-\gamma$
11\. γ
12\. 3 R, 2 R, 1·5
13\. (a) 120 K, 240 K, 480 K, 240 K
(b) 1250 J. 1500 J (c) 2500 J, 750 J
14\. (a) 600 K, 900 K (b) 10 J
(c) 14·9 J, 24·9 J (d) 29·8 J
15\. 0·90 J/g-K
16\. (a) 1·54 (b) 1·15
17\. (a) $7{\cdot}1 \times 10^5$ Pa (b) 424 K (c) – 21 J
18\. 240 K
19\. 800 kPa, 600 K in both cases
20\. $2^{\gamma+1} p_0$ in both cases
21\. $2^{(\gamma+1)/\gamma} V_0$ in each case
22\. (a) 0·009 (b) 2 R = 16·6 J/mol-K
(c) 780 kPa, 520 K (d) – 33 J (e) 33 J
23\. $2 : 2\sqrt{2} : 1$
25\. (a) $2\sqrt{2}$ (b) – 82 J
(c) 82 J (d) 424 K
(e) – 41·4 J (f) 103 J
(g) – 23·4 J
26\. $\sqrt{3} : 1$
27\. (a) 0·008 (b) 12·5 cm
28\. 0·03 g
29\. (a) T_0, $\frac{p_0}{2}$ in vessel A and $T_0'/2^{\gamma-1}$, $p_0/2^{\gamma}$ in vessel B
(b) T_0, $p_0/2$
30\. (a) $\frac{p_1^{1/\gamma} V_0}{A}$, $\frac{p_2^{1/\gamma} V_0}{A}$ (b) zero
(c) $(A/2)^{\gamma}$ where $A = p_1^{1/\gamma} + p_2^{1/\gamma}$
31\. 447 m/s
32\. 18·0 J/mol-K, 26·3 J/mol-K
33\. 960 m/s
34\. 26 J/mol-K, 17·7 J/mol-K
35\. 29·0 J/mol-K, 20·7 J/mol-K

□

CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

1. N = 1 mole, W = 20 g/mol, V = 50 m/s

   K.E. of the vessel = Internal energy of the gas

   $= (1/2)\,mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$ J

   $25 = n\dfrac{3}{2}r(\Delta T) \Rightarrow 25 = 1 \times \dfrac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \dfrac{50}{3 \times 8.3} \approx 2$ k.

2. m = 5 g, $\Delta t = 25 - 15 = 10°C$

   $C_V = 0.172$ cal/g-°CJ = 4.2 J/Cal.

   dQ = du + dw

   Now, V = 0 (for a rigid body)

   So, dw = 0.

   So dQ = du.

   $Q = msdt = 5 \times 0.172 \times 10 = 8.6$ cal $= 8.6 \times 4.2 = 36.12$ Joule.

3. $\gamma = 1.4$, w or piston = 50 kg., A of piston = 100 cm^2

   Po = 100 kpa, g = 10 m/s^2, x = 20 cm.

   $dw = pdv = \left(\dfrac{mg}{A} + Po\right)Adx = \left(\dfrac{50 \times 10}{100 \times 10^{-4}} + 10^5\right)100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$ J.

   $nRdt = 300 \Rightarrow dT = \dfrac{300}{nR}$

   $dQ = nCpdT = nCp \times \dfrac{300}{nR} = \dfrac{n\gamma R300}{(\gamma - 1)nR} = \dfrac{300 \times 1.4}{0.4} = 1050$ J.

4. $C_VH_2 = 2.4$ Cal/g°C, $C_PH^2 = 3.4$ Cal/g°C

   M = 2 g/ Mol, $R = 8.3 \times 10^7$ erg/mol-°C

   We know, $C_P - C_V = 1$ Cal/g°C

   So, difference of molar specific heats

   $= C_P \times M - C_V \times M = 1 \times 2 = 2$ Cal/g°C

   Now, $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$ erg/mol-°C $\Rightarrow J = 4.15 \times 10^7$ erg/cal.

5. $\dfrac{C_P}{C_V} = 7.6$, n = 1 mole, $\Delta T = 50K$

   (a) Keeping the pressure constant, dQ = du + dw,

   $\Delta T = 50$ K, $\gamma = 7/6$, m = 1 mole,

   $dQ = du + dw \Rightarrow nC_VdT = du + RdT \Rightarrow du = nCpdT - RdT$

   $$= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1}dT - RdT$$

   $= DT - RdT = 7RdT - RdT = 6\,RdT = 6 \times 8.3 \times 50 = 2490$ J.

   (b) Kipping Volume constant, $dv = nC_VdT$

   $$= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$$

   $= 8.3 \times 50 \times 6 = 2490$ J

   (c) Adiabetically dQ = 0, du = – dw

   $$= \left[\frac{n \times R}{\gamma - 1}(T_1 - T_2)\right] = \frac{1 \times 8.3}{\frac{7}{6} - 1}(T_2 - T_1) = 8.3 \times 50 \times 6 = 2490 \text{ J}$$

6. $m = 1.18$ g, $V = 1 \times 10^3$ cm^3 = 1 L $T = 300$ k, $P = 10^5$ Pa

$PV = nRT$ or $n = \frac{PV}{RT} = 10^5 = \text{atm.}$

$N = \frac{PV}{RT} = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2} = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$

Now, $C_v = \frac{1}{n} \times \frac{Q}{dt} = 24.6 \times 2 = 49.2$

$C_p = R + C_v = 1.987 + 49.2 = 51.187$

$Q = nC_p dT = \frac{1}{24.6} \times 51.187 \times 1 = 2.08$ Cal.

7. $V_1 = 100$ cm^3, $V_2 = 200$ cm^3 $P = 2 \times 10^5$ Pa, $\Delta Q = 50$J

(a) $\Delta Q = du + dw \Rightarrow 50 = du + 20\times 10^5(200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30$ J

(b) $30 = n \times \frac{3}{2} \times 8.3 \times 300$ $[\,U = \frac{3}{2}nRT$ for monoatomic$]$

$\Rightarrow n = \frac{2}{3 \times 83} = \frac{2}{249} = 0.008$

(c) $du = nC_V dT \Rightarrow C_v = \frac{dndTu}{} = \frac{30}{0.008 \times 300} = 12.5$

$C_p = C_v + R = 12.5 + 8.3 = 20.3$

(d) $C_v = 12.5$ (Proved above)

8. Q = Amt of heat given

Work done = $\frac{Q}{2}$, $\Delta Q = W + \Delta U$

for monoatomic gas $\Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$

$V = n\frac{3}{2}RT = \frac{Q}{2} = nT\times \frac{3}{2}R = 3R \times nT$

Again $Q = n\,CpdT$ Where $C_P >$ Molar heat capacity at const. pressure.

$3RnT = ndTC_P \Rightarrow C_P = 3R$

9. $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R\,\Delta T = 2KV\,\Delta U \Rightarrow \frac{R\Delta T}{2KV} = dv$

$dQ = du + dw \Rightarrow mcdT = C_V dT + pdv \Rightarrow msdT = C_V\,dT + \frac{PRdF}{2KV}$

$\Rightarrow ms = C_V + \frac{RKV}{2KV} \Rightarrow C_P + \frac{R}{2}$

10. $\frac{C_P}{C_V} = \gamma$, $C_P - C_V = R$, $C_V = \frac{r}{\gamma - 1}$, $C_P = \frac{\gamma R}{\gamma - 1}$

$Pdv = \frac{1}{b+1}(Rdt)$

$\Rightarrow 0 = C_V dT + \frac{1}{b+1}(Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_V}{R}$

$\Rightarrow b + 1 = \frac{-R}{C_V} = \frac{-(C_P - C_V)}{C_V} = -\gamma + 1 \Rightarrow b = -\gamma$

11. Considering two gases, in Gas(1) we have,

γ, Cp_1 (Sp. Heat at const. ‘P’), Cv_1 (Sp. Heat at const. ‘V’), n_1 (No. of moles)

$\frac{Cp_1}{Cv_1} = \gamma$ & $Cp_1 - Cv_1 = R$

$\Rightarrow \gamma Cv_1 - Cv_1 = R \Rightarrow Cv_1\,(\gamma - 1) = R$

$\Rightarrow Cv_1 = \frac{R}{\gamma - 1}$ & $Cp_1 = \frac{\gamma R}{\gamma - 1}$

In Gas(2) we have, γ, Cp_2 (Sp. Heat at const. 'P'), Cv_2 (Sp. Heat at const. 'V'), n_2 (No. of moles)

$\frac{Cp_2}{Cv_2} = \gamma$ & $Cp_2 - Cv_2 = R \Rightarrow \gamma Cv_2 - Cv_2 = R \Rightarrow Cv_2\,(\gamma - 1) = R \Rightarrow Cv_2 = \frac{R}{\gamma - 1}$ & $Cp_2 = \frac{\gamma R}{\gamma - 1}$

Given $n_1 : n_2 = 1 : 2$

$dU_1 = nCv_1\, dT$ & $dU_2 = 2nCv_2\, dT = 3nCvdT$

$$\Rightarrow C_V = \frac{Cv_1 + 2Cv_2}{3} = \frac{\frac{R}{\gamma-1} + \frac{2R}{\gamma-1}}{3} = \frac{3R}{3(\gamma-1)} = \frac{R}{\gamma-1} \qquad \ldots(1)$$

$$\&Cp = \gamma Cv = \frac{\gamma r}{\gamma - 1} \quad \ldots(2)$$

So, $\frac{Cp}{Cv} = \gamma$ [from (1) & (2)]

12. $Cp' = 2.5\ RCp'' = 3.5\ R$

$Cv' = 1.5\ R \qquad Cv'' = 2.5\ R$

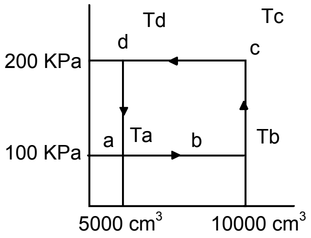
$n_1 = n_2 = 1$ mol $\qquad (n_1 + n_2)C_VdT = n_1\, C_vdT + n_2\, C_vdT$

$$\Rightarrow C_V = \frac{n_1Cv' + n_2Cv''}{n_1 + n_2} = \frac{1.5R + 2.5R}{2}\ 2R$$

$C_P = C_V + R = 2R + R = 3R$

$$\gamma = \frac{C_p}{C_V} = \frac{3R}{2R} = 1.5$$

13. $n = \frac{1}{2}$ mole, $\qquad R = \frac{25}{3}$ J/mol-k, $\qquad \gamma = \frac{5}{3}$



(a) Temp at A = T_a, $P_aV_a = nRT_a$

$$\Rightarrow T_a = \frac{P_aV_a}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^{3}}{\frac{1}{2} \times \frac{25}{3}} = 120\ \text{k}.$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$dQ = nCpdT$ $\qquad$ [since ab is isobaric]

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma - 1}\left(T_b - T_a\right) = \frac{1}{2} \times \frac{\frac{35}{3} \times \frac{5}{3}}{\frac{5}{3} - 1} \times (240 - 120) = \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times 120 = 1250\ \text{J}$$

For bc, $\qquad dQ = du + dw$ $\quad$ [dq = 0, Isochorie process]

$$\Rightarrow dQ = du = nC_vdT = \frac{nR}{\gamma - 1}\left(T_c - T_a\right) = \frac{1}{2} \times \frac{\frac{25}{3}}{\left(\frac{5}{3} - 1\right)}(240) = \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500\ \text{J}$$

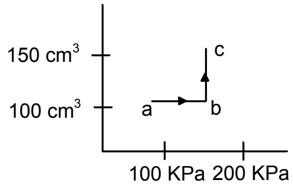
(c) Heat liberated in cd = $-\, nC_pdT$

$$= \frac{-1}{2} \times \frac{nR}{\gamma - 1}\left(T_d - T_c\right) = \frac{-1}{2} \times \frac{125}{3} \times \frac{3}{2} \times 240 = 2500\ \text{J}$$

Heat liberated in da = $-\, nC_vdT$

$$= \frac{-1}{2} \times \frac{R}{\gamma - 1}\left(T_a - T_d\right) = \frac{-1}{2} \times \frac{25}{2} \times (120 - 240) = 750\ \text{J}$$

14. (a) For a, b 'V' is constant



So, $\frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200 \times 300}{100}$ = 600 k

For b,c 'P' is constant

So, $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600 \times 150}{100}$ = 900 k

(b) Work done = Area enclosed under the graph 50 cc × 200 kpa = $50 \times 10^{-6} \times 200 \times 10^3$ J = 10 J

(c) 'Q' Supplied = nC_vdT

Now, $n = \frac{PV}{RT}$ considering at pt. 'b'

$C_v = \frac{R}{\gamma - 1}dT$ = 300 a, b.

$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma - 1}dT = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925$ $(\therefore \gamma = 1.67)$

Q supplied to be nC_pdT $[\therefore C_p = \frac{\gamma R}{\gamma - 1}]$

$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1}dT = \frac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \frac{1.67 \times 8.3}{0.67} \times 300 = 24.925$

(d) $Q = \Delta U + w$

Now, $\Delta U = Q - w$ = Heat supplied – Work done = (24.925 + 14.925) – 1 = 29.850

15. In Joly's differential steam calorimeter

$C_v = \frac{m_2 L}{m_1(\theta_2 - \theta_1)}$

m_2 = Mass of steam condensed = 0.095 g, L = 540 Cal/g = 540 × 4.2 J/g

m_1 = Mass of gas present = 3 g, $\theta_1 = 20°C$, $\theta_2 = 100°C$

$\Rightarrow C_v = \frac{0.095 \times 540 \times 4.2}{3(100 - 20)} = 0.89 \approx 0.9$ J/g-K

16. $\gamma = 1.5$

Since it is an adiabatic process, So PV^γ = const.

(a) $P_1V_1^\gamma = P_2V_2^\gamma$ Given $V_1 = 4$ L, $V_2 = 3$ L, $\frac{P_2}{P_1} = ?$

$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{4}{3}\right)^{1.5} = 1.5396 \approx 1.54$

(b) $TV^{\gamma-1}$ = Const.

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5} = 1.154$

17. $P_1 = 2.5 \times 10^5$ Pa, $V_1 = 100$ cc, $T_1 = 300$ k

(a) $P_1V_1^\gamma = P_2V_2^\gamma$

$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$

$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$

(b) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5 - 1} = T_2 \times (50)^{1.5 - 1}$

$\Rightarrow T_2 = \frac{3000}{7.07} = 424.32\text{ k} \approx 424\text{ k}$

(c) Work done by the gas in the process

$$W = \frac{mR}{\gamma - 1}\left[T_2 - T_1\right] = \frac{P_1V_1}{T(\gamma - 1)}\left[T_2 - T_1\right]$$

$$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1,5 - 1)}[424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21 \text{ J}$$

18. $\gamma = 1.4$, $T_1 = 20°C = 293$ k, $P_1 = 2$ atm, $p_2 = 1$ atm

We know for adiabatic process,

$P_1^{1-\gamma} \times T_1^{\gamma} = P_2^{1-\gamma} \times T_2^{\gamma}$ or $(2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$

$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3$ K

19. $P_1 = 100$ KPa $= 10^5$ Pa, $V_1 = 400 \text{ cm}^3 = 400 \times 10^{-6} \text{ m}^3$, $T_1 = 300$ k,

$\gamma = \frac{C_P}{C_V} = 1.5$

(a) Suddenly compressed to $V_2 = 100 \text{ cm}^3$

$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow 10^5 (400)^{1.5} = P_2 \times (100)^{1.5}$

$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800$ KPa

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5\text{-}1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600$ K

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain, $P_2 = 800$ KPa, $T_2 = 600$ K.

20. Given $\frac{C_P}{C_V} = \gamma$ P_0 (Initial Pressure), V_0 (Initial Volume)

(a) (i) Isothermal compression, $P_1V_1 = P_2V_2$ or, $P_0V_0 = \frac{P_2V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $2P_0\left(\frac{V_0}{2}\right)^{\gamma} = P1\left(\frac{V_0}{4}\right)^{\gamma}$

$\Rightarrow P' = \frac{V_0^{\ \gamma}}{2^{\gamma}} \times 2P_0 \times \frac{4^{\gamma}}{V_0^{\ \gamma}} = 2^{\gamma} \times 2\,P_0 \Rightarrow P_0 2^{\gamma+1}$

(b) (i) Adiabatic compression $P_1V_1^{\gamma} = P_2V_2^{\gamma}$ or $P_0V_0^{\gamma} = P'\left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P' = P_0 2^{\gamma}$

(ii) Isothermal compression $P_1V_1 = P_2V_2$ or $2^{\gamma}\,P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure = P_0

Initial Volume = V_0

$\gamma = \frac{C_P}{C_V}$

(a) Isothermally to pressure $\frac{P_0}{2}$

$P_0V_0 = \frac{P_0}{2}V_1 \Rightarrow V_1 = 2\,V_0$

Adiabetically to pressure = $\frac{P_0}{4}$

$\frac{P_0}{2}(V_1)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma} \Rightarrow \frac{P_0}{2}(2V_0)^{\gamma} = \frac{P_0}{4}(V_2)^{\gamma}$

$\Rightarrow 2^{\gamma+1}\,V_0^{\gamma} = V_2^{\gamma} \Rightarrow V_2 = 2^{(\gamma+1)/\gamma}\,V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma}\,V_0$

(b) Adiabetically to pressure $\frac{P_0}{2}$ to P_0

$P_0 \times (2^{\gamma+1} V_0^{\gamma}) = \frac{P_0}{2} \times (V')^{\gamma}$

Isothermal to pressure $\frac{P_0}{4}$

$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$

$\therefore$ Final Volume = $2^{(\gamma+1)/\gamma} V_0$

22. PV = nRT

Given P = 150 KPa = 150 × 10^3 Pa, V = 150 cm^3 = 150 × 10^{-6} m^3, T = 300 k

(a) $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300}$ = 9.036 × 10^{-3} = 0.009 moles.

(b) $\frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma-1)C_V} = \gamma$ $\left[\therefore C_P = \frac{\gamma R}{\gamma - 1}\right]$

$\Rightarrow C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.5 - 1} = \frac{8.3}{0.5}$ = 2R = 16.6 J/mole

(c) Given P_1 = 150 KPa = 150 × 10^3 Pa, P_2 =?

V_1 = 150 cm^3 = 150× 10^{-6} m^3, γ = 1.5

V_2 = 50 cm^3 = 50 × 10^{-6} m^3, T_1 = 300 k, T_2 = ?

Since the process is adiabatic Hence – $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow 150\times 10^3 (150 \times 10^{-6})^{\gamma} = P_2 \times (50 \times 10^{-6})^{\gamma}$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\frac{150 \times 10^{-6}}{50 \times 10^{-6}}\right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3$ Pa ≈ 780 KPa

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$ [$\therefore \Delta U = 0$, in adiabatic]

$= -nC_VdT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8$ J ≈ – 33 J

(e) $\Delta U = nC_VdT = 0.009 \times 16.6 \times 220 \approx 33$ J

23. $V_A = V_B = V_C$

For A, the process is isothermal

$P_AV_A = P_A'V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$

For B, the process is adiabatic,

$P_A(V_B)^{\gamma} = P_A'(V_B)^{\gamma} = P_B' = P_B\left(\frac{V_B}{V_B'}\right)^{\gamma} = P_B \times \left(\frac{1}{2}\right)^{1.5} = \frac{P_B}{2^{1.5}}$

For, C, the process is isobaric

$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$

Final pressures are equal.

$= \frac{p_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. P_1 = Initial Pressure V_1 = Initial Volume P_2 = Final Pressure V_2 = Final Volume

Given, $V_2 = 2V_1$, Isothermal workdone = $nRT_1 \, Ln\left(\frac{V_2}{V_1}\right)$

Adiabatic workdone = $\frac{P_1V_1 - P_2V_2}{\gamma - 1}$

Given that workdone in both cases is same.

Hence $nRT_1 \text{ Ln}\left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln \left(\frac{V_2}{V_1}\right) = \frac{P_1V_1 - P_2V_2}{nRT_1}$

$\Rightarrow (\gamma - 1)\ln \left(\frac{V_2}{V_1}\right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \frac{T_1 - T_1}{T_1}$ …(i) $[\therefore V_2 = 2V_1]$

We know $TV^{\gamma-1}$ = const. in adiabatic Process.

$T_1V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$, or $T_1 (V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$

Or, $T_1 = 2^{\gamma-1} \times T_2$ or $T_2 = T_1{}^{1-\gamma}$ …(ii)

From (i) & (ii)

$(\gamma - 1) \ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{\,1-\gamma}$

25. $\gamma = 1.5$, $T = 300$ k, $V = 1Lv = \frac{1}{2} l$

(a) The process is adiabatic as it is sudden,

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \Rightarrow P_1 (V_0)^{\gamma} = P_2 \left(\frac{V_0}{2}\right)^{\gamma} \Rightarrow P_2 = P_1 \left(\frac{1}{1/2}\right)^{1.5} = P_1 (2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$

(b) $P_1 = 100$ KPa $= 10^5$ Pa $W = \frac{nR}{\gamma - 1}[T_1 - T_2]$

$T_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2 (0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$

$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2}$ K

$P_1 V_1 = nRT_1 \Rightarrow n = \frac{P_1V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R}$ (V in m^3)

$w = \frac{nR}{\gamma - 1}[T_1 - T_2] = \frac{1R}{3R(1.5 - 1)}\left[300 - 300\sqrt{2}\right] = \frac{300}{3 \times 0.5}\left(1 - \sqrt{2}\right) = -82.8 \text{ J} \approx -82 \text{ J.}$

(c) Internal Energy,

$dQ = 0$, $\Rightarrow du = -dw = -(-82.8)\text{J} = 82.8 \text{ J} \approx 82 \text{ J.}$

(d) Final Temp $= 300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2$ k ≈ 424 k.

(e) The pressure is kept constant. ∴ The process is isobaric.

Work done $= nRdT = \frac{1}{3R} \times R \times (300 - 300\sqrt{2})$ Final Temp = 300 K

$= -\frac{1}{3} \times 300 (0.414) = -41.4$ J. Initial Temp $= 300\sqrt{2}$

(f) Initial volume $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}}$ L.

Final volume = 1L

Work done in isothermal $= nRT\ln\frac{V_2}{V_1}$

$= \frac{1}{3R} \times R \times 300 \ln\left(\frac{1}{1/2\sqrt{2}}\right) = 100 \times \ln\left(2\sqrt{2}\right) = 100 \times 1.039 \approx 103$

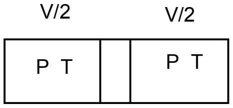
(g) Net work done $= W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4$ J.

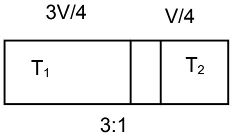
26. Given $\gamma = 1.5$

We know fro adiabatic process $TV^{\gamma-1}$ = Const.

So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ …(eq)

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.



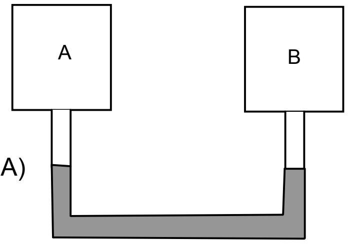


$$\text{So, } T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}} \qquad \text{So, } T_1 : T_2 = 1 : \sqrt{3}$$

27. $V = 200\ \text{cm}^3$, C = 12.5 J/mol-k, T = 300 k, P = 75 cm

(a) No. of moles of gas in each vessel,



$$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$$

(b) Heat is supplied to the gas but dv = 0

$$dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5} \text{ for (A)}$$

$$\text{For (B) } dT = \frac{10}{0.008 \times 12.5} \qquad \because \frac{P}{T} = \frac{P_A}{T_A} \text{ [For container A]}$$

$$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5 \text{ cm of Hg.}$$

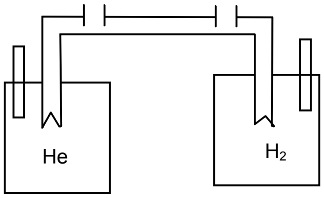
$$\because \frac{P}{T} = \frac{P_B}{T_B} \text{ [For Container B]} \Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2\,P_A = 25 \text{ cm of Hg.}$$

Mercury moves by a distance $P_B - P_A = 25 - 12.5 = 12.5$ Cm.

28. mHe = 0.1 g, $\gamma = 1.67$, $\mu = 4$ g/mol, mH_2 =?

$\mu = 28$/mol $\gamma_2 = 1.4$

Since it is an adiabatic surrounding



$$\text{He } dQ = nC_V dT = \frac{0.1}{4} \times \frac{R}{\gamma - 1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67 - 1)} \times dT \qquad \text{…(i)}$$

$$H_2 = nC_V dT = \frac{m}{2} \times \frac{R}{\gamma - 1} \times dT = \frac{m}{2} \times \frac{R}{1.4 - 1} \times dT$$

[Where m is the rqd. Mass of H_2]

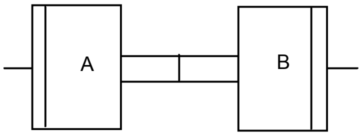
Since equal amount of heat is given to both and ΔT is same in both.

Equating (i) & (ii) we get

$$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03 \text{ g}$$

29. Initial pressure = P_0, Initial Temperature = T_0

Initial Volume = V_0



$$\frac{C_P}{C_V} = \gamma$$

(a) For the diathermic vessel the temperature inside remains constant

$$P_1 V_1 - P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}, \qquad \text{Temperature} = T_o$$

For adiabatic vessel the temperature does not remains constant. The process is adiabatic

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0\left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$$

$P_1\,V_1^{\gamma} = P_2\,V_2^{\gamma} \Rightarrow P_0\,V_0^{\gamma} = p_1\,(2V_0)^{\gamma} \Rightarrow\ P_1\ = P_0\left(\dfrac{V_0}{2V_0}\right)^{\gamma}\ = \dfrac{P_0}{2^{\gamma}}$

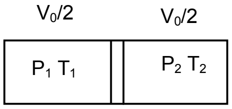
(b) When the values are opened, the temperature remains T_0 through out

$P_1 = \dfrac{n_1RT_0}{4V_0}$, $P_2 = \dfrac{n_2RT_0}{4V_0}$ [Total value after the expt = $2V_0 + 2V_0 = 4V_0$]

$P = P_1 + P_2 = \dfrac{(n_1+n_2)RT_0}{4V_0} = \dfrac{2nRT_0}{4V_0} = \dfrac{nRT_0}{2V} = \dfrac{P_0}{2}$

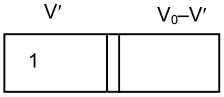
30. For an adiabatic process, Pv^{γ} = Const.

There will be a common pressure 'P' when the equilibrium is reached



Hence $P_1\left(\dfrac{V_0}{2}\right)^{\gamma} = P(V')^{\gamma}$

For left $P = P_1\left(\dfrac{V_0}{2}\right)^{\gamma}(V')^{\gamma}$ …(1)



For Right $P = P_2\left(\dfrac{V_0}{2}\right)^{\gamma}(V_0-V')^{\gamma}$ …(2)

Equating 'P' for both left & right

$= \dfrac{P_1}{(V')^{\gamma}} = \dfrac{P_2}{(V_0-V')^{\gamma}}$ or $\dfrac{V_0-V'}{V'} = \left(\dfrac{P_2}{P_1}\right)^{1/\gamma}$

$\Rightarrow \dfrac{V_0}{V'} - 1 = \dfrac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \dfrac{V_0}{V'} = \dfrac{P_2^{1/\gamma}+P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \dfrac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma}+P_2^{1/\gamma}}$ For left …….(3)

Similarly $V_0 - V' = \dfrac{V_0P_2^{1/\gamma}}{P_1^{1/\gamma}+P_2^{1/\gamma}}$ For right ……(4)

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

(c) From (1) Final pressure $P = \dfrac{P_1\left(\dfrac{V_0}{2}\right)^{y}}{(V')^{\gamma}}$

Again from (3) $V' = \dfrac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma}+P_2^{1/\gamma}}$ or $P = \dfrac{P_1\dfrac{(V_0)^{\gamma}}{2^{\gamma}}}{\left(\dfrac{V_0P_1^{1/\gamma}}{P_1^{1/\gamma}+P_2^{1/\gamma}}\right)^{\gamma}} = \dfrac{P_1(V_0)^{\gamma}}{2^{\gamma}} \times \dfrac{\left(P_1^{1/\gamma}+P_2^{1/\gamma}\right)^{\gamma}}{(V_0)^{\gamma}P_1} = \left(\dfrac{P_1^{1/\gamma}+P_2^{1/\gamma}}{2}\right)^{\gamma}$

31. A = 1 cm^2 = 1 × 10^{-4} m^2, M = 0.03 g = 0.03 × 10^{-3} kg,

P = 1 atm = 10^5 pascal, L= 40 cm = 0.4 m.

L_1 = 80 cm = 0.8 m, P = 0.355 atm

The process is adiabatic

$P(V)^{\gamma} = P(V')^{\gamma} = \Rightarrow 1 \times (AL)^{\gamma} = 0.355 \times (A2L)^{\gamma} \Rightarrow 1\ \ 1 = 0.355\ \ 2^{\gamma} \Rightarrow \dfrac{1}{0.355} = 2^{\gamma}$

$= \gamma \log 2 = \log\left(\dfrac{1}{0.355}\right) = 1.4941$

$V = \sqrt{\dfrac{\gamma P}{f}} = \sqrt{\dfrac{1.4941\times10^5}{m/v}} = \sqrt{\dfrac{1.4941\times10^5}{\left(\dfrac{0.03\times10^{-3}}{10^{-4}\times1\times0.4}\right)}} = \sqrt{\dfrac{1.441\times10^5\times4\times10^{-5}}{3\times10^{-5}}} = 446.33 \approx 447$ m/s

32. V = 1280 m/s, T = 0°C, $f\mathrm{o}H_2 = 0.089\ \mathrm{kg/m^3}$, rR = 8.3 J/mol-k,
At STP, P = 10^5 Pa, We know

$$V_{\mathrm{sound}} = \sqrt{\frac{\gamma P}{f\mathrm{o}}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1\ \text{J/mol-k}$$

Again, $\frac{C_P}{C_V} = \gamma$ or $C_P = \gamma C_V = 1.458 \times 18.1 = 26.3$ J/mol-k

33. $\mu = 4\mathrm{g} = 4 \times 10^{-3}$ kg, V = 22400 cm^3 = 22400 × 10^{-6} m^3
C_P = 5 cal/mol-ki = 5 × 4.2 J/mol-k = 21 J/mol-k

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma\,(8.3) \Rightarrow 21\,\gamma - 21 = 8.3\,\gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP, P = 1 atm = 10^5 pa

$$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{\frac{4 \times 10^{-3}}{22400 \times 10^{-6}}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28\ \text{m/s}$$

34. Given $f\mathrm{o} = 1.7 \times 10^{-3}\ \mathrm{g/cm^3} = 1.7\ \mathrm{kg/m^3}$, P = 1.5 × 10^5 Pa, R = 8.3 J/mol-k,
f = 3.0 KHz.

Node separation in a Kundt″ tube = $\frac{\lambda}{2}$ = 6 cm, $\Rightarrow \lambda$ = 12 cm = 12 × 10^{-3} m

So, V = $f\lambda$ = 3 × 10^3 × 12 × 10^{-2} = 360 m/s

We know, Speed of sound = $\sqrt{\frac{\gamma P}{f\mathrm{o}}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.488 - 1} = 17.72$ J/mol-k

Again $\frac{C_P}{C_V} = \gamma$ So, $C_P = \gamma C_V = 17.72 \times 1.468 = 26.01 \approx 26$ J/mol-k

35. f = 5 × 10^3 Hz, T = 300 Hz, $\frac{\lambda}{2}$ = 3.3 cm $\Rightarrow \lambda$ = 6.6 × 10^{-2} m

V = $f\lambda$ = 5 × 10^3 × 6.6 × 10^{-2} = (66 × 5) m/s

$$V = \frac{\lambda P}{f} \quad \left[Pv = nRT \Rightarrow P = \frac{m}{mV} \times Rt \Rightarrow PM = f\mathrm{o}RT \Rightarrow \frac{P}{f\mathrm{o}} = \frac{RT}{m}\right]$$

$$= \sqrt{\frac{\gamma RT}{m}}(66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300} = 1.3995$$

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995} = 20.7\ \text{J/mol-k},$$

$C_P = C_V + R$ = 20.77 + 8.3 = 29.07 J/mol-k.

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