The length of mercury column at 30°C is l_0. Suppose the length of the mercury column, if it were at 0°C, is l_0. Then,

$$l_\theta = l_0\,[1 + \frac{1}{3}\gamma\,(30°C)]. \quad \dots \text{(ii)}$$

By (i) and (ii),

$$l_0[1 + \frac{1}{3}\gamma\,(30°C)] = 75\ \text{cm}[1 + \alpha(30°C)]$$

or,
$$l_0 = 75\ \text{cm}\frac{[1 + \alpha(30°C)]}{[1 + \frac{1}{3}\gamma\,(30°C)]}$$

$$= 75\ \text{cm}\ [1 + (\alpha - \frac{\gamma}{3})\,(30°C)]$$

$$= 74{\cdot}89\ \text{cm}.$$

□

QUESTIONS FOR SHORT ANSWER

1. If two bodies are in thermal equilibrium in one frame, will they be in thermal equilibrium in all frames ?
2. Does the temperature of a body depend on the frame from which it is observed ?
3. It is heard sometimes that mercury is used in defining the temperature scale because it expands uniformly with the temperature. If the temperature scale is not yet defined, is it logical to say that a substance expands uniformly with the temperature ?
4. In defining the ideal gas temperature scale, it is assumed that the pressure of the gas at constant volume is proportional to the temperature T. How can we verify whether this is true or not ? Are we using the kinetic theory of gases ? Are we using the experimental result that the pressure is proportional to temperature ?
5. Can the bulb of a thermometer be made of an adiabatic wall ?
6. Why do marine animals live deep inside a lake when the surface of the lake freezes ?
7. The length of a brass rod is found to be smaller on a hot summer day than on a cold winter day as measured by the same aluminium scale. Do we conclude that brass shrinks on heating ?
8. If mercury and glass had equal coefficient of volume expansion, could we make a mercury thermometer in a glass tube ?
9. The density of water at 4°C is supposed to be 1000 kg/m^3. Is it same at the sea level and at a high altitude ?
10. A tightly closed metal lid of a glass bottle can be opened more easily if it is put in hot water for some time. Explain.
11. If an automobile engine is overheated, it is cooled by putting water on it. It is advised that the water should be put slowly with engine running. Explain the reason.
12. Is it possible for two bodies to be in thermal equilibrium if they are not in contact ?
13. A spherical shell is heated. The volume changes according to the equation $V_\theta = V_0(1 + \gamma\theta)$. Does the volume refer to the volume enclosed by the shell or the volume of the material making up the shell ?

OBJECTIVE I

1. A system X is neither in thermal equilibrium with Y nor with Z. The systems Y and Z
   (a) must be in thermal equilibrium
   (b) cannot be in thermal equilibrium
   (c) may be in thermal equilibrium.
2. Which of the curves in figure (23-Q1) represents the relation between Celsius and Fahrenheit temperatures ?

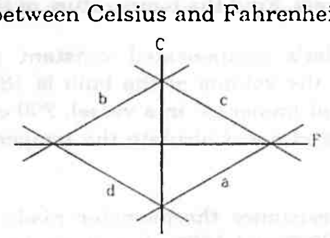


Figure 23-Q1

3. Which of the following pairs may give equal numerical values of the temperature of a body ?
   (a) Fahrenheit and kelvin (b) Celsius and kelvin
   (c) kelvin and platinum.
4. For a constant volume gas thermometer, one should fill the gas at
   (a) low temperature and low pressure
   (b) low temperature and high pressure
   (c) high temperature and low pressure
   (d) high temperature and high pressure.
5. Consider the following statements.
   (A) The coefficient of linear expansion has dimension K^{-1}.
   (B) The coefficient of volume expansion has dimension K^{-1}.
   (a) A and B are both correct.
   (b) A is correct but B is wrong.

(c) B is correct but A is wrong.
(d) A and B are both wrong.

6. A metal sheet with a circular hole is heated. The hole
(a) gets larger (b) gets smaller
(c) remains of the same size (d) gets deformed.

7. Two identical rectangular strips, one of copper and the other of steel, are rivetted together to form a bimetallic strip ($\alpha_{copper} > \alpha_{steel}$). On heating, this strip will
(a) remain straight
(b) bend with copper on convex side
(c) bend with steel on convex side
(d) get twisted.

8. If the temperature of a uniform rod is slightly increased by Δt, its moment of inertia I about a perpendicular bisector increases by
(a) zero (b) $\alpha I \Delta t$ (c) $2\alpha I \Delta t$ (d) $3\alpha I \Delta t$.

9. If the temperature of a uniform rod is slightly increased by Δt, its moment of inertia I about a line parallel to itself will increase by
(a) zero (b) $\alpha I \Delta t$ (c) $2\alpha I \Delta t$ (d) $3\alpha I \Delta t$.

10. The temperature of water at the surface of a deep lake is 2°C. The temperature expected at the bottom is
(a) 0°C (b) 2°C (c) 4°C (d) 6°C.

11. An aluminium sphere is dipped into water at 10°C. If the temperature is increased, the force of buoyancy
(a) will increase (b) will decrease
(c) will remain constant
(d) may increase or decrease depending on the radius of the sphere.

OBJECTIVE II

1. A spinning wheel is brought in contact with an identical wheel spinning at identical speed. The wheels slow down under the action of friction. Which of the following energies of the first wheel decrease ?
(a) kinetic (b) total (c) mechanical (d) internal.

2. A spinning wheel A is brought in contact with another wheel B initially at rest. Because of the friction at contact, the second wheel also starts spinning. Which of the following energies of the wheel B increase ?
(a) kinetic (b) total (c) mechanical (d) internal.

3. A body A is placed on a railway platform and an identical body B in a moving train. Which of the following energies of B are greater than those of A as seen from the ground ?
(a) kinetic (b) total (c) mechanical (d) internal.

4. In which of the following pairs of temperature scales, the size of a degree is identical ?
(a) mercury scale and ideal gas scale
(b) Celsius scale and mercury scale
(c) Celsius scale and ideal gas scale
(d) ideal gas scale and absolute scale.

5. A solid object is placed in water contained in an adiabatic container for some time. The temperature of water falls during the period and there is no appreciable change in the shape of the object. The temperature of the solid object
(a) must have increased (b) must have decreased
(c) may have increased
(d) may have remained constant.

6. As the temperature is increased, the time period of a pendulum
(a) increases proportionately with temperature
(b) increases (c) decreases
(d) remains constant.

EXERCISES

1. The steam point and the ice point of a mercury thermometer are marked as 80° and 20°. What will be the temperature in centigrade mercury scale when this thermometer reads 32° ?

2. A constant volume thermometer registers a pressure of 1.500×10^{4} Pa at the triple point of water and a pressure of 2.050×10^{4} Pa at the normal boiling point. What is the temperature at the normal boiling point ?

3. A gas thermometer measures the temperature from the variation of pressure of a sample of gas. If the pressure measured at the melting point of lead is 2.20 times the pressure measured at the triple point of water, find the melting point of lead.

4. The pressure measured by a constant volume gas thermometer is 40 kPa at the triple point of water. What will be the pressure measured at the boiling point of water (100°C) ?

5. The pressure of the gas in a constant volume gas thermometer is 70 kPa at the ice point. Find the pressure at the steam point.

6. The pressures of the gas in a constant volume gas thermometer are 80 cm, 90 cm and 100 cm of mercury at the ice point, the steam point and in a heated wax bath respectively. Find the temperature of the wax bath.

7. In a Callender's compensated constant pressure air thermometer, the volume of the bulb is 1800 cc. When the bulb is kept immersed in a vessel, 200 cc of mercury has to be poured out. Calculate the temperature of the vessel.

8. A platinum resistance thermometer reads 0° when its resistance is 80 Ω and 100° when its resistance is 90 Ω. Find the temperature at the platinum scale at which the resistance is 86 Ω.

9. A resistance thermometer reads $R = 20{\cdot}0\ \Omega$, $27{\cdot}5\ \Omega$, and $50{\cdot}0\ \Omega$ at the ice point (0°C), the steam point (100°C) and the zinc point (420°C) respectively. Assuming that the resistance varies with temperature as $R_\theta = R_0(1 + \alpha\theta + \beta\theta^2)$, find the values of R_0, α and β. Here θ represents the temperature on Celsius scale.

10. A concrete slab has a length of 10 m on a winter night when the temperature is 0°C. Find the length of the slab on a summer day when the temperature is 35°C. The coefficient of linear expansion of concrete is $1{\cdot}0 \times 10^{-5}$/°C.

11. A metre scale made of steel is calibrated at 20°C to give correct reading. Find the distance between 50 cm mark and 51 cm mark if the scale is used at 10°C. Coefficient of linear expansion of steel is $1{\cdot}1 \times 10^{-5}$/°C.

12. A railway track (made of iron) is laid in winter when the average temperature is 18°C. The track consists of sections of 12·0 m placed one after the other. How much gap should be left between two such sections so that there is no compression during summer when the maximum temperature goes to 48°C? Coefficient of linear expansion of iron $= 11 \times 10^{-6}$/°C.

13. A circular hole of diameter 2·00 cm is made in an aluminium plate at 0°C. What will be the diameter at 100°C? α for aluminium $= 2{\cdot}3 \times 10^{-5}$/°C.

14. Two metre scales, one of steel and the other of aluminium, agree at 20°C. Calculate the ratio aluminium-centimetre/steel-centimetre at (a) 0°C, (b) 40°C and (c) 100°C. α for steel $= 1{\cdot}1 \times 10^{-5}$/°C and for aluminium $= 2{\cdot}3 \times 10^{-5}$/°C.

15. A metre scale is made up of steel and measures correct length at 16°C. What will be the percentage error if this scale is used (a) on a summer day when the temperature is 46°C and (b) on a winter day when the temperature is 6°C? Coefficient of linear expansion of steel $= 11 \times 10^{-6}$/°C.

16. A metre scale made of steel reads accurately at 20°C. In a sensitive experiment, distances accurate upto 0·055 mm in 1 m are required. Find the range of temperature in which the experiment can be performed with this metre scale. Coefficient of linear expansion of steel $= 11 \times 10^{-6}$/°C.

17. The density of water at 0°C is 0·998 g/cm^3 and at 4°C is 1·000 g/cm^3. Calculate the average coefficient of volume expansion of water in the temperature range 0 to 4°C.

18. Find the ratio of the lengths of an iron rod and an aluminium rod for which the difference in the lengths is independent of temperature. Coefficients of linear expansion of iron and aluminium are 12×10^{-6}/°C and 23×10^{-6}/°C respectively.

19. A pendulum clock gives correct time at 20°C at a place where $g = 9{\cdot}800$ m/s^2. The pendulum consists of a light steel rod connected to a heavy ball. It is taken to a different place where $g = 9{\cdot}788$ m/s^2. At what temperature will it give correct time? Coefficient of linear expansion of steel $= 12 \times 10^{-6}$/°C.

20. An aluminium plate fixed in a horizontal position has a hole of diameter 2·000 cm. A steel sphere of diameter 2·005 cm rests on this hole. All the lengths refer to a temperature of 10°C. The temperature of the entire system is slowly increased. At what temperature will the ball fall down? Coefficient of linear expansion of aluminium is 23×10^{-6}/°C and that of steel is 11×10^{-6}/°C.

21. A glass window is to be fit in an aluminium frame. The temperature on the working day is 40°C and the glass window measures exactly 20 cm × 30 cm. What should be the size of the aluminium frame so that there is no stress on the glass in winter even if the temperature drops to 0°C? Coefficients of linear expansion for glass and aluminium are $9{\cdot}0 \times 10^{-6}$/°C and 24×10^{-6}/°C respectively.

22. The volume of a glass vessel is 1000 cc at 20°C. What volume of mercury should be poured into it at this temperature so that the volume of the remaining space does not change with temperature? Coefficients of cubical expansion of mercury and glass are $1{\cdot}8 \times 10^{-4}$/°C and $9{\cdot}0 \times 10^{-6}$/°C respectively.

23. An aluminium can of cylindrical shape contains 500 cm^3 of water. The area of the inner cross-section of the can is 125 cm^2. All measurements refer to 10°C. Find the rise in the water level if the temperature increases to 80°C. The coefficient of linear expansion of aluminium $= 23 \times 10^{-6}$/°C and the average coefficient of volume expansion of water $= 3{\cdot}2 \times 10^{-4}$/°C respectively.

24. A glass vessel measures exactly 10 cm × 10 cm × 10 cm at 0°C. It is filled completely with mercury at this temperature. When the temperature is raised to 10°C, 1·6 cm^3 of mercury overflows. Calculate the coefficient of volume expansion of mercury. Coefficient of linear expansion of glass $= 6{\cdot}5 \times 10^{-6}$/°C.

25. The densities of wood and benzene at 0°C are 880 kg/m^3 and 900 kg/m^3 respectively. The coefficients of volume expansion are $1{\cdot}2 \times 10^{-3}$/°C for wood and $1{\cdot}5 \times 10^{-3}$/°C for benzene. At what temperature will a piece of wood just sink in benzene?

26. A steel rod of length 1 m rests on a smooth horizontal base. If it is heated from 0°C to 100°C, what is the longitudinal strain developed?

27. A steel rod is clamped at its two ends and rests on a fixed horizontal base. The rod is unstrained at 20°C. Find the longitudinal strain developed in the rod if the temperature rises to 50°C. Coefficient of linear expansion of steel $= 1{\cdot}2 \times 10^{-5}$/°C.

28. A steel wire of cross-sectional area 0·5 mm^2 is held between two fixed supports. If the wire is just taut at 20°C, determine the tension when the temperature falls to 0°C. Coefficient of linear expansion of steel is $1{\cdot}2 \times 10^{-5}$/°C and its Young's modulus is $2{\cdot}0 \times 10^{11}$ N/m^2.

29. A steel rod is rigidly clamped at its two ends. The rod is under zero tension at 20°C. If the temperature rises to 100°C, what force will the rod exert on one of the

clamps. Area of cross-section of the rod = $2{\cdot}00\ \text{mm}^2$. Coefficient of linear expansion of steel = $12{\cdot}0 \times 10^{-6}$/°C and Young's modulus of steel = $2{\cdot}00 \times 10^{11}\ \text{N/m}^2$.

30. Two steel rods and an aluminium rod of equal length l_0 and equal cross-section are joined rigidly at their ends as shown in the figure below. All the rods are in a state of zero tension at 0°C. Find the length of the system when the temperature is raised to θ. Coefficient of linear expansion of aluminium and steel are α_a and α_s respectively. Young's modulus of aluminium is Y_a and of steel is Y_s.

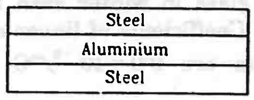


Figure 23-E1

31. A steel ball initially at a pressure of $1{\cdot}0 \times 10^5$ Pa is heated from 20°C to 120°C keeping its volume constant. Find the pressure inside the ball. Coefficient of linear expansion of steel = 12×10^{-6}/°C and bulk modulus of steel = $1{\cdot}6 \times 10^{11}\ \text{N/m}^2$.

32. Show that moment of inertia of a solid body of any shape changes with temperature as $I = I_0(1 + 2\alpha\theta)$, where I_0 is the moment of inertia at 0°C and α is the coefficient of linear expansion of the solid.

33. A torsional pendulum consists of a solid disc connected to a thin wire ($\alpha = 2{\cdot}4 \times 10^{-5}$/°C) at its centre. Find the percentage change in the time period between peak winter (5°C) and peak summer (45°C).

34. A circular disc made of iron is rotated about its axis at a constant velocity ω. Calculate the percentage change in the linear speed of a particle of the rim as the disc is slowly heated from 20°C to 50°C keeping the angular velocity constant. Coefficient of linear expansion of iron = $1{\cdot}2 \times 10^{-5}$/°C.

□

ANSWERS

OBJECTIVE I

1. (c) 2. (a) 3. (a) 4. (c) 5. (a) 6. (a)
7. (b) 8. (c) 9. (c) 10. (c) 11. (b)

OBJECTIVE II

1. (a), (c) 2. all 3. (a), (b), (c)
4. (c), (d) 5. (a) 6. (b)

EXERCISES

1. 20°C
2. 373·3 K
3. 601 K
4. 55 kPa
5. 96 kPa
6. 200°C
7. 307 K
8. 60°
9. 20·0 Ω, $3{\cdot}8 \times 10^{-3}$/°C, $-5{\cdot}6 \times 10^{-7}/°\text{C}^2$
10. 10·0035 m
11. 1·00011 cm
12. 0·4 cm
13. 2·0046 cm
14. (a) 0·99977 (b) 1·00025 (c) 1·00096
15. (a) 0·033% (b) – 0·011%
16. 15°C to 25°C
17. -5×10^{-4}/°C
18. 23 : 12
19. – 82°C
20. 219°C
21. 20·012 cm × 30·018 cm
22. 50 cc
23. 0·089 cm
24. $1{\cdot}8 \times 10^{-4}$/°C
25. 83°C
26. zero
27. $-3{\cdot}6 \times 10^{-4}$
28. 24 N
29. 384 N
30. $l_0\left[1 + \dfrac{\alpha_a Y_a + 2\alpha_s Y_s}{Y_a + 2Y_s}\theta\right]$
31. $5{\cdot}8 \times 10^8$ Pa
33. $9{\cdot}6 \times 10^{-2}$
34. $3{\cdot}6 \times 10^{-2}$

□

CHAPTER – 23
HEAT AND TEMPERATURE
EXERCISES

1. Ice point = 20° (L_0) L_1 = 32°
   Steam point = 80° (L_{100})
   $$T = \frac{L_1 - L_0}{L_{100} - L_0} \times 100 = \frac{32-20}{80-20} \times 100 = 20°C$$
2. P_{tr} = 1.500 × 10^4 Pa
   P = 2.050 × 10^4 Pa
   We know, For constant volume gas Thermometer
   $$T = \frac{P}{P_{tr}} \times 273.16\,K = \frac{2.050 \times 10^4}{1.500 \times 10^4} \times 273.16 = 373.31$$
3. Pressure Measured at M.P = 2.2 × Pressure at Triple Point
   $$T = \frac{P}{P_{tr}} \times 273.16 = \frac{2.2 \times P_{tr}}{P_{tr}} \times 273.16 = 600.952\,K \approx 601\,K$$
4. P_{tr} = 40 × 10^3 Pa, P = ?
   T = 100°C = 373 K, $T = \frac{P}{P_{tr}} \times 273.16\,K$
   $$\Rightarrow P = \frac{T \times P_{tr}}{273.16} = \frac{373 \times 49 \times 10^3}{273.16} = 54620 \text{ Pa} = 5.42 \times 10^3 \text{ pa} \approx 55 \text{ K Pa}$$
5. P_1 = 70 K Pa, P_2 = ?
   T_1 = 273 K, T_2 = 373K
   $T = \frac{P_1}{P_{tr}} \times 273.16$ $\Rightarrow 273 = \frac{70 \times 10^3}{P_{tr}} \times 273.16$ $\Rightarrow P_{tr}\ \frac{70 \times 273.16 \times 10^3}{273}$
   $T_2 = \frac{P_2}{P_{tr}} \times 273.16$ $\Rightarrow 373 = \frac{P_2 \times 273}{70 \times 273.16 \times 10^3}$ $\Rightarrow P_2 = \frac{373 \times 70 \times 10^3}{273}$ = 95.6 K Pa
6. $P_{\text{ice point}} = P_{0°}$ = 80 cm of Hg
   $P_{\text{steam point}} = P_{100°}$ 90 cm of Hg
   P_0 = 100 cm
   $$t = \frac{P - P_0}{P_{100} - P_0} \times 100° = \frac{80-100}{90-100} \times 100 = 200°C$$
7. $T' = \frac{V}{V - V'} T_0$ T_0 = 273,
   V = 1800 CC, V′ = 200 CC
   $$T' = \frac{1800}{1600} \times 273 = 307.125 \approx 307$$
8. $R_t = 86\Omega$; $R_{0°} = 80\Omega$; $R_{100°} = 90\Omega$
   $$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{86-80}{90-80} \times 100 = 60°C$$
9. R at ice point (R_0) = 20Ω
   R at steam point (R_{100}) = 27.5Ω
   R at Zinc point (R_{420}) = 50Ω
   $R_\theta = R_0 (1 + \alpha\theta + \beta\theta^2)$
   $\Rightarrow R_{100} = R_0 + R_0\,\alpha\theta + R_0\,\beta\theta^2$
   $$\Rightarrow \frac{R_{100} - R_0}{R_0} = \alpha\theta + \beta\theta^2$$

$\Rightarrow \frac{27.5-20}{20} = \alpha \times 100 + \beta \times 10000$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$

$R_{420} = R_0\,(1+\alpha\theta + \beta\theta^2) \Rightarrow \frac{50-R_0}{R_0} = \alpha\theta + \beta\theta^2$

$\Rightarrow \frac{50-20}{20} = 420 \times \alpha + 176400 \times \beta$ $\qquad \Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

$\Rightarrow \frac{7.5}{20} = 100\,\alpha + 10000\,\beta$ $\qquad \Rightarrow \frac{3}{2} = 420\,\alpha + 176400\,\beta$

10. $L_1 = ?$, $L_0 = 10$ m, $\alpha = 1 \times 10^{-5}/°C$, t= 35
    $L_1 = L_0\,(1 + \alpha t) = 10(1 + 10^{-5} \times 35) = 10 + 35 \times 10^{-4} = 10.0035$m
11. $t_1 = 20°C$, $t_2 = 10°C$, $L_1 = 1$cm = 0.01 m, L_2 =?
    $\alpha_{steel} = 1.1 \times 10^{-5}/°C$
    $L_2 = L_1\,(1 + \alpha_{steel}\Delta T) = 0.01(1 + 101 \times 10^{-5} \times 10) = 0.01 + 0.01 \times 1.1 \times 10^{-4}$
    $= 10^4 \times 10^{-6} + 1.1 \times 10^{-6} = 10^{-6}\,(10000 + 1.1) = 10001.1$
    $=1.00011 \times 10^{-2}$ m = 1.00011 cm
12. $L_0 = 12$ cm, $\alpha = 11 \times 10^{-5}\,/°C$
    tw = 18°C ts = 48°C
    $Lw = L_0(1 + \alpha tw) = 12\,(1 + 11 \times 10^{-5} \times 18) = 12.002376$ m
    $Ls = L_0\,(1 + \alpha ts) = 12\,(1 + 11 \times 10^{-5} \times 48) = 12.006336$ m
    $\Delta L = 12.006336 - 12.002376 = 0.00396$ m ≈ 0.4cm
13. $d_1 = 2$ cm $= 2 \times 10^{-2}$
    $t_1 = 0°C$, $t_2 = 100°C$
    $\alpha_{al} = 2.3 \times 10^{-5}\,/°C$
    $d_2 = d_1\,(1 + \alpha\Delta t) = 2 \times 10^{-2}\,(1 + 2.3 \times 10^{-5}\,10^2)$
    = 0.02 + 0.000046 = 0.020046 m = 2.0046 cm
14. $L_{st} = L_{Al}$ at 20°C $\qquad \alpha_{Al} = 2.3 \times 10^{-5}/°C$
    So, $Lo_{st}\,(1 - \alpha_{st} \times 20) = Lo_{Al}\,(1 - \alpha_{Al} \times 20)$ $\qquad \alpha_{st} = 1.1 \times 10^{-5}/°C$

    (a) $\Rightarrow \frac{Lo_{st}}{Lo_{Al}} = \frac{(1-\alpha_{Al} \times 20)}{(1-\alpha_{st} \times 20)} = \frac{1-2.3\times10^{-5}\times20}{1-1.1\times10^{-5}\times20} = \frac{0.99954}{0.99978} = 0.999$

    (b) $\Rightarrow \frac{Lo_{40st}}{Lo_{40Al}} = \frac{(1-\alpha_{Al} \times 40)}{(1-\alpha_{st} \times 40)} = \frac{1-2.3\times10^{-5}\times20}{1-1.1\times10^{-5}\times20} = \frac{0.99954}{0.99978} = 0.999$

    $= \frac{Lo_{Al}}{Lo_{st}} \times \frac{1+2.3\times10^{-5}\times10}{273} = \frac{0.99977\times1.00092}{1.00044} = 1.0002496 \approx 1.00025$

    $\frac{Lo_{100Al}}{Lo_{100St}} = \frac{(1+\alpha_{Al}\times100)}{(1+\alpha_{st}\times100)} = \frac{0.99977\times1.00092}{1.00011} = 1.00096$
15. (a) Length at 16°C = L
    L = ? T_1 =16°C, T_2 = 46°C
    $\alpha = 1.1 \times 10^{-5}/°C$
    $\Delta L = L\alpha\Delta\theta = L \times 1.1 \times 10^{-5} \times 30$
    % of error = $\left(\frac{\Delta L}{L}\times100\right)\% = \left(\frac{L\alpha\Delta\theta}{2}\times100\right)\% = 1.1 \times 10^{-5} \times 30 \times 100\% = 0.033\%$

    (b) T_2 = 6°C
    % of error = $\left(\frac{\Delta L}{L}\times100\right)\% = \left(\frac{L\alpha\Delta\theta}{L}\times100\right)\% = -1.1 \times 10^{-5} \times 10 \times 100 = -0.011\%$

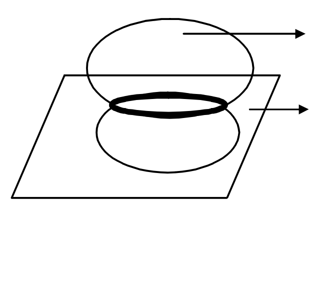
16. $T_1 = 20°C$, $\Delta L = 0.055mm = 0.55 \times 10^{-3}$ m
t_2 = ? $\alpha_{st} = 11 \times 10^{-6}/°C$
We know,
$\Delta L = L_0 \alpha \Delta T$
In our case,
$0.055 \times 10^{-3} = 1 \times 1.1 \text{ I } 10^{-6} \times (T_1 + T_2)$
$0.055 = 11 \times 10^{-3} \times 20 \pm 11 \times 10^{-3} \times T_2$
$T_2 = 20 + 5 = 25°C$ or $20 - 5 = 15°C$
The expt. Can be performed from 15 to 25°C

17. $f_{0°C} = 0.098 \text{ g/m}^3$, $f_{4°C} = 1 \text{ g/m}^3$
$$f_{0°C} = \frac{f_{4°C}}{1+\gamma\Delta T} \Rightarrow 0.998 = \frac{1}{1+\gamma \times 4} \Rightarrow 1 + 4\gamma = \frac{1}{0.998}$$
$$\Rightarrow 4 + \gamma = \frac{1}{0.998} - 1 \Rightarrow \gamma = 0.0005 \approx 5 \times 10^{-4}$$
As density decreases $\gamma = -5 \times 10^{-4}$

18. Iron rod Aluminium rod
L_{Fe} L_{Al}
$\alpha_{Fe} = 12 \times 10^{-8}$ /°C $\alpha_{Al} = 23 \times 10^{-8}$ /°C
Since the difference in length is independent of temp. Hence the different always remains constant.
$L'_{Fe} = L_{Fe}(1 + \alpha_{Fe} \times \Delta T)$ …(1)
$L'_{Al} = L_{Al}(1 + \alpha_{Al} \times \Delta T)$ …(2)
$L'_{Fe} - L'_{Al} = L_{Fe} - L_{Al} + L_{Fe} \times \alpha_{Fe} \times \Delta T - L_{Al} \times \alpha_{Al} \times \Delta T$
$$\frac{L_{Fe}}{L_{Al}} = \frac{\alpha_{Al}}{\alpha_{Fe}} = \frac{23}{12} = 23 : 12$$

19. $g_1 = 9.8 \text{ m/s}^2$, $g_2 = 9.788 \text{ m/s}^2$
$$T_1 = 2\pi\frac{\sqrt{l_1}}{g_1} \qquad T_2 = 2\pi\frac{\sqrt{l_2}}{g_2} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g}$$
$\alpha_{Steel} = 12 \times 10^{-6}$ /°C
$T_1 = 20°C$ T_2 = ?
$T_1 = T_2$
$$\Rightarrow 2\pi\frac{\sqrt{l_1}}{g_1} = 2\pi\frac{\sqrt{l_1(1+\Delta T)}}{g_2} \qquad \Rightarrow \frac{l_1}{g_1} = \frac{l_1(1+\Delta T)}{g_2}$$
$$\Rightarrow \frac{1}{9.8} = \frac{1+12\times10^{-6}\times\Delta T}{9.788} \qquad \Rightarrow \frac{9.788}{9.8} = 1 + 12 \times 10^{-6} \times \Delta T$$
$$\Rightarrow \frac{9.788}{9.8} - 1 = 12 \times 10^{-6}\,\Delta T \qquad \Rightarrow \Delta T = \frac{-0.00122}{12\times10^{-6}}$$
$$\Rightarrow T_2 - 20 = -101.6 \qquad \Rightarrow T_2 = -101.6 + 20 = -81.6 \approx -82°C$$

20. Given
$d_{St} = 2.005$ cm, $d_{Al} = 2.000$ cm
$\alpha_S = 11 \times 10^{-6}$ /°C $\alpha_{Al} = 23 \times 10^{-6}$ /°C



$d's = 2.005 (1+ \alpha_s \Delta T)$ (where ΔT is change in temp.)
$\Rightarrow d's = 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T$
$d'_{Al} = 2(1+ \alpha_{Al}\,\Delta T) = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
The two will slip i.e the steel ball with fall when both the diameters become equal.
So,
$\Rightarrow 2.005 + 2.005 \times 11 \times 10^{-6}\,\Delta T = 2 + 2 \times 23 \times 10^{-6}\,\Delta T$
$\Rightarrow (46 - 22.055)10^{-6} \times \Delta T = 0.005$
$$\Rightarrow \Delta T = \frac{0.005\times10^6}{23.945} = 208.81$$

Now $\Delta T = T_2 - T_1 = T_2 - 10°C$ [$\therefore$ $T_1 = 10°C$ given]

$\Rightarrow T_2 = \Delta T + T_1 = 208.81 + 10 = 281.81$

21. The final length of aluminium should be equal to final length of glass.

Let the initial length o faluminium = l

$l(1 - \alpha_{Al}\Delta T) = 20(1 - \alpha_0\Delta\theta)$

$\Rightarrow l(1 - 24 \times 10^{-6} \times 40) = 20\ (1 - 9 \times 10^{-6} \times 40)$

$\Rightarrow l(1 - 0.00096) = 20\ (1 - 0.00036)$

$\Rightarrow l = \dfrac{20 \times 0.99964}{0.99904} = 20.012$ cm

Let initial breadth of aluminium = b

$b(1 - \alpha_{Al}\Delta T) = 30(1 - \alpha_0\Delta\theta)$

$\Rightarrow b = \dfrac{30 \times (1 - 9 \times 10^{-6} \times 40)}{(1 - 24 \times 10^{-6} \times 40)} = \dfrac{30 \times 0.99964}{0.99904} = 30.018$ cm

22. $V_g = 1000$ CC, $T_1 = 20°C$

$V_{Hg} = ?$ $\gamma_{Hg} = 1.8 \times 10^{-4}$ /°C

$\gamma_g = 9 \times 10^{-6}$ /°C

ΔT remains constant

Volume of remaining space = $V'_g - V'_{Hg}$

Now

$V'_g = V_g(1 + \gamma_g\Delta T)$ …(1)

$V'_{Hg} = V_{Hg}(1 + \gamma_{Hg}\Delta T)$ …(2)

Subtracting (2) from (1)

$V'_g - V'_{Hg} = V_g - V_{Hg} + V_g\gamma_g\Delta T - V_{Hg}\gamma_{Hg}\Delta T$

$\Rightarrow \dfrac{V_g}{V_{Hg}} = \dfrac{\gamma_{Hg}}{\gamma_g} \Rightarrow \dfrac{1000}{V_{Hg}} = \dfrac{1.8 \times 10^{-4}}{9 \times 10^{-6}}$

$\Rightarrow V_{HG} = \dfrac{9 \times 10^{-3}}{1.8 \times 10^{-4}} = 500$ CC.

23. Volume of water = $500 cm^3$

Area of cross section of can = $125\ m^2$

Final Volume of water

$= 500(1 + \gamma\Delta\theta) = 500[1 + 3.2 \times 10^{-4} \times (80 - 10)] = 511.2\ cm^3$

The aluminium vessel expands in its length only so area expansion of base cab be neglected.

Increase in volume of water = $11.2\ cm^3$

Considering a cylinder of volume = $11.2\ cm^3$

Height of water increased = $\dfrac{11.2}{125} = 0.089$ cm

24. $V_0 = 10 \times 10 \times 10 = 1000$ CC

$\Delta T = 10°C$, $V'_{HG} - V'_g = 1.6\ cm^3$

$\alpha_g = 6.5 \times 10^{-6}$/°C, $\gamma_{Hg} = ?$, $\gamma_g = 3 \times 6.5 \times 10^{-6}$/°C

$V'_{Hg} = v_{HG}(1 + \gamma_{Hg}\Delta T)$ …(1)

$V'_g = v_g(1 + \gamma_g\Delta T)$ …(2)

$V'_{Hg} - V'_g = V_{Hg} - V_g + V_{Hg}\gamma_{Hg}\,\Delta T - V_g\gamma_g\,\Delta T$

$\Rightarrow 1.6 = 1000 \times \gamma_{Hg} \times 10 - 1000 \times 6.5 \times 3 \times 10^{-6} \times 10$

$\Rightarrow \gamma_{Hg} = \dfrac{1.6 + 6.3 \times 3 \times 10^{-2}}{10000} = 1.789 \times 10^{-4} \approx 1.8 \times 10^{-4}$/°C

25. $f_\omega = 880$ Kg/m^3, $f_b = 900$ Kg/m^3

$T_1 = 0°C$, $\gamma_\omega = 1.2 \times 10^{-3}$ /°C,

$\gamma_b = 1.5 \times 10^{-3}$ /°C

The sphere begins t sink when,

$(mg)_{sphere}$ = displaced water

$\Rightarrow V f'_\omega\, g = V f'_b\, g$

$\Rightarrow \dfrac{f_\omega}{1+\gamma_\omega\Delta\theta} = \dfrac{f_b}{1+\gamma_b\Delta\theta}$

$\Rightarrow \dfrac{880}{1+1.2\times10^{-3}\Delta\theta} = \dfrac{900}{1+1.5\times10^{-3}\Delta\theta}$

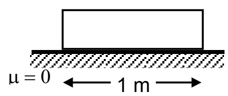
$\Rightarrow 880 + 880 \times 1.5 \times 10^{-3}\,(\Delta\theta) = 900 + 900 \times 1.2 \times 10^{-3}\,(\Delta\theta)$

$\Rightarrow (880 \times 1.5 \times 10^{-3} - 900 \times 1.2 \times 10^{-3})\,(\Delta\theta) = 20$

$\Rightarrow (1320 - 1080) \times 10^{-3}\,(\Delta\theta) = 20$

$\Rightarrow \Delta\theta = 83.3°C \approx 83°C$

26. $\Delta L = 100°C$



A longitudinal strain develops if and only if, there is an opposition to the expansion.
Since there is no opposition in this case, hence the longitudinal stain here = Zero.

27. $\theta_1 = 20°C$, $\theta_2 = 50°C$

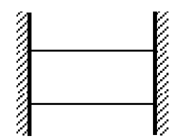
$\alpha_{steel} = 1.2 \times 10^{-5}$ /°C

Longitudinal stain = ?

Stain = $\dfrac{\Delta L}{L} = \dfrac{L\alpha\Delta\theta}{L} = \alpha\Delta\theta$

$= 1.2 \times 10^{-5} \times (50 - 20) = 3.6 \times 10^{-4}$

28. A = 0.5mm^2 = 0.5×10^{-6} m^2



$T_1 = 20°C$, $T_2 = 0°C$

$\alpha_s = 1.2 \times 10^{-5}$ /°C, $\qquad Y = 2 \times 2 \times 10^{11}$ N/m^2

Decrease in length due to compression = $L\alpha\Delta\theta$ …(1)

$Y = \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{F}{A} \times \dfrac{L}{\Delta L} \Rightarrow \Delta L = \dfrac{FL}{AY}$ …(2)

Tension is developed due to (1) & (2)

Equating them,

$L\alpha\Delta\theta = \dfrac{FL}{AY} \Rightarrow F = \alpha\Delta\theta AY$

$= 1.2 \times 10^{-5} \times (20 - 0) \times 0.5 \times 10^{-5}\ 2 \times 10^{11} = 24$ N

29. $\theta_1 = 20°C$, $\qquad \theta_2 = 100°C$

A = 2mm^2 = 2×10^{-6} m^2

$\alpha_{steel} = 12 \times 10^{-6}$ /°C, $\qquad Y_{steel} = 2 \times 10^{11}$ N/m^2

Force exerted on the clamps = ?

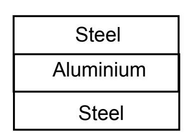
$\dfrac{\left(\frac{F}{A}\right)}{\text{Strain}} = Y \Rightarrow F = \dfrac{Y \times \Delta L}{L} \times L = \dfrac{YL\alpha\Delta\theta A}{L} = YA\alpha\Delta\theta$

$= 2 \times 10^{11} \times 2 \times 10^{-6} \times 12 \times 10^{-6} \times 80 = 384$ N

30. Let the final length of the system at system of temp. 0°C = ℓ_θ

Initial length of the system = ℓ_0

When temp. changes by θ.



Strain of the system = $\ell_1 - \dfrac{\ell_0}{\ell_\theta}$

But the total strain of the system = $\dfrac{\text{total stress of system}}{\text{total young's mod ulusof of system}}$

Now, total stress = Stress due to two steel rod + Stress due to Aluminium

$= \gamma_s\alpha_s\theta + \gamma_s\, ds\, \theta + \gamma_{al}\, at\, \theta = 2\%\ \alpha_s\, \theta + \gamma 2\, A\ell\, \theta$

Now young'' modulus of system = $\gamma_s + \gamma_s + \gamma_{al} = 2\gamma_s + \gamma_{al}$

$\therefore$ Strain of system = $\dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \dfrac{\ell_\theta - \ell_0}{\ell_0} = \dfrac{2\gamma_s\alpha_s\theta + \gamma_s\alpha_{al}\theta}{2\gamma_s + \gamma_{al}}$

$\Rightarrow \ell_\theta = \ell_0 \left[\dfrac{1 + \alpha_{al}\gamma_{al} + 2\alpha_s\gamma_s\theta}{\gamma_{al} + 2\gamma_s}\right]$

31. The ball tries to expand its volume. But it is kept in the same volume. So it is kept at a constant volume. So the stress arises

$\dfrac{P}{\left(\dfrac{\Delta V}{v}\right)} = B \Rightarrow P = B\dfrac{\Delta V}{V} = B \times \gamma\Delta\theta$

$= B \times 3\alpha\Delta\theta = 1.6 \times 10^{11} \times 10^{-6} \times 3 \times 12 \times 10^{-6} \times (120 - 20) = 57.6 \times 19^{7} \approx 5.8 \times 10^{8}$ pa.

32. Given

I_0 = Moment of Inertia at 0°C

α = Coefficient of linear expansion

To prove, $I = I_0 = (1 + 2\alpha\theta)$

Let the temp. change to θ from 0°C

$\Delta T = \theta$

Let 'R' be the radius of Gyration,

Now, $R' = R(1 + \alpha\theta)$, $\quad I_0 = MR^2$ $\quad$ where M is the mass.

Now, $I' = MR'^2 = MR^2 (1 + \alpha\theta)^2 \approx = MR^2 (1 + 2\alpha\theta)$

[By binomial expansion or neglecting $\alpha^2\theta^2$ which given a very small value.]

So, $I = I_0 (1 + 2\alpha\theta)$ (proved)

33. Let the initial m.I. at 0°C be I_0

$T = 2\pi\sqrt{\dfrac{I}{K}}$

$I = I_0 (1 + 2\alpha\Delta\theta)$ (from above question)

At 5°C, $\quad T_1 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha\Delta\theta)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 5)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+10\alpha)}{K}}$

At 45°C, $\quad T_2 = 2\pi\sqrt{\dfrac{I_0(1+2\alpha 45)}{K}} = 2\pi\sqrt{\dfrac{I_0(1+90\alpha)}{K}}$

$\dfrac{T_2}{T_1} = \sqrt{\dfrac{1+90\alpha}{1+10\alpha}} = \sqrt{\dfrac{1+90\times 2.4\times 10^{-5}}{1+10\times 2.4\times 10^{-5}}} \quad \sqrt{\dfrac{1.00216}{1.00024}}$

% change = $\left(\dfrac{T_2}{T_1} - 1\right)\times 100 = 0.0959\% = 9.6 \times 10^{-2}\%$

34. $T_1 = 20°C$, $\quad T_2 = 50°C$, $\quad \Delta T = 30°C$

$\alpha = 1.2 \times 10^{5}$ /°C

ω remains constant

(I) $\omega = \dfrac{V}{R}$ $\quad$ (II) $\omega = \dfrac{V'}{R'}$

Now, $R' = R(1 + \alpha\Delta\theta) = R + R \times 1.2 \times 10^{-5} \times 30 = 1.00036R$

From (I) and (II)

$\dfrac{V}{R} = \dfrac{V'}{R'} = \dfrac{V'}{1.00036R}$

$\Rightarrow V' = 1.00036\ V$

% change = $\dfrac{(1.00036V - V)}{V} \times 100 = 0.00036 \times 100 = 3.6 \times 10^{-2}$

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