CHAPTER 21

SPEED OF LIGHT

21.1 HISTORICAL INTRODUCTION

The speed of light in vacuum is a fundamental constant in physics. The most interesting fact about this speed is that if an object moves with this speed in one frame, it has the same speed in any other frame. This led to a major revision of our concept of space and time and is the key fact on which the special theory of relativity is based.

In 1983, the speed of light was defined to be exactly 299, 792, 458 m/s. In fact, the length of an object is now defined to be 299, 792, 458 m/s multiplied by the time taken by the light to cross it. Thus, when one sends light from one place to another place and measures the time taken by the light to do so, one is not measuring the speed of light, rather one is measuring the distance between the two places.

Prior to 1983, the length was defined independently and one had a separate metre. The speed of light could then be measured as the length divided by the time taken by the light to cross it.

Perhaps, the great Indian talents in the Vedic age had the knowledge of the speed of light. G.V.Raghavrao in his book quotes a verse from Rigveda (I, 50-4) *Yojananam Sahastra Dwe Dwe Shate Dwe Cha Yojane Aken Nimishardhena Krammana Namostute.* In this verse, the author pays respects to the one (the reference is to the sun light) who moves 2202 *yojans* in half *nimish.* Yojan is a quite common unit in India, it means 4 kose, each kose measuring 8000 British yards and each yard measuring 0·9144 m. The definition of the time unit nimish can be found in Shrimadbhagwat (III, 11-3 to 10) where it is mentioned that 15 nimishas make 1 *kashta,* 15 kashtas make one *laghu,* 30 laghus make 1 *muhurta* and 30 muhurtas make 1 *diva-ratri.* A diva-ratri is, of course, a day-night which is 24 hours in modern language. When you convert 2202 yojans per half nimish into SI units, it turns out to be $3{\cdot}0 \times 10^{8}$ m/s up to two significant digits, a value quite accurate as we know it today.

In the modern era, perhaps the first attempt to measure the speed of light was made by Galileo. The design of the experiment was as follows. Two experimenters A and B, each having a lantern and a shutter, stand on two small hills. The shutter can cover or uncover the lantern. Initially, both the lanterns are covered. One of the persons A uncovers the lantern. The second person B uncovers his lantern when he sees the light from the lantern of A. The first person A covers his lantern when he sees the light from the lantern of B. The time elapsed between the uncovering and covering of the first lantern is measured. During this time, the light travels from the first person to the second person and then back. Knowing the distance and time, the speed of light may be calculated.

The proposed method failed because the speed of light is so large that a human being cannot respond with the required accuracy of timing. If the distance between the hills is as large as 15 km, the time taken by light in going back and forth is only one ten thousandth part of a second. The first recorded speed of light in modern era came through the astronomical observations by the Danish astronomer Olaf Roemer in 1676. The value obtained was about $2{\cdot}1 \times 10^{8}$ m/s, somewhat smaller than the actual. In 1728, English astronomer Bradley measured the speed of light from his observations. The value was quite close to the correct one.

The first measurement of the speed of light from purely terrestrial experiments was reported by the French physicist Fizeau in 1849. The method was improved by another French physicist Foucault. Yet another method was proposed by American physicist Michelson. We now describe these three methods.

21.2 FIZEAU METHOD

Figure (21.1a) shows a schematic diagram of the arrangement used in this method. Light from a source S passes through a convex lens L_1. The transmitted beam is intercepted by a semi-transparent inclined

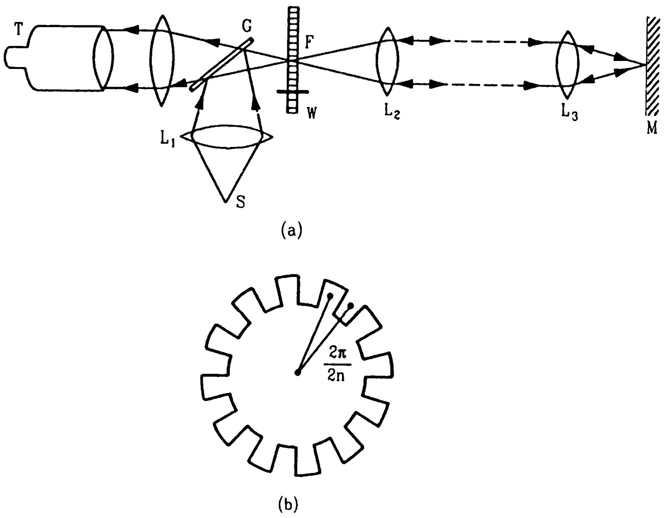

Figure 21.1

glass plate G. A part of the light is reflected and is converged near the rim of a toothed wheel W which can be set into rapid rotation. The light passing through the space between two consecutive teeth is made parallel by a convex lens L_2. This parallel beam travels for several kilometers (in the original Fizeau experiment it was 8·6 km) and is then converged by a convex lens L_3. A plane mirror M is placed in the focal plane of the lens L_3. The reflected light is again made parallel by the lens L_3 and it converges at the rim of the wheel. If it finds a gap, it falls on the glass plate G. The beam is partially transmitted and an observer receives these rays to see the image of S through a telescope.

When the wheel is rotated, it allows light to pass through in separate bursts. Light is passed when a gap comes at F and is stopped when a tooth comes there. The speed of rotation of the wheel is gradually increased while the observer keeps looking for the image. Initially, the image flickers but at a particular angular speed the image cannot be seen at all. This happens when the angular speed is such that by the time light passes through a gap, goes to the mirror M and comes back, the next tooth comes at F. Any light passing through the wheel does not return to the observer and the image cannot be seen. The angular speed of the wheel is carefully measured in this state.

Suppose, D = distance from the wheel W to the mirror M,

ω = angular speed of rotation of the wheel when the image is completely unseen for the first time,

n = number of teeth in the wheel.

The angle rotated by the wheel when a tooth comes in the place of its adjacent gap is $\theta = \frac{2\pi}{2n}$ (figure 21.1b) The time taken by the wheel in doing so is $\theta/\omega = \frac{\pi}{n\omega}$ · In this time interval, the light travels a distance $2D$. The speed of light is, therefore,

$$c = \frac{2D}{\pi/n\omega} = \frac{2Dn\omega}{\pi}.$$

If the number of revolutions of the wheel per unit time is ν, we have $\omega = 2\pi\nu$ and the speed of light is

$$c = 4Dn\nu. \qquad \dots (21.1)$$

One can use a concave mirror in place of the plane mirror. If the radius of curvature of this mirror be equal to its distance from the convex lens L_3 (i.e., equal to the focal length of L_3), a slight error in orientation of lens L_3 does not seriously affect the accuracy of the experiment.

There are two serious difficulties in this method. Since the light has to travel a large distance, the intensity decreases considerably and the final image becomes very dim. Secondly, the experiment cannot be done inside a laboratory. It needs an open space of several kilometers. These difficulties are removed in Foucault method.

21.3 FOUCAULT METHOD

The basic principle of Foucault's method can be understood with the help of figure (21.2). Light from a source S is partly transmitted by a glass plate G and is incident on a convex lens L. The distance of the lens from S is so adjusted that the beam transmitted through the lens is convergent. This beam is intercepted by a plane mirror M_1 which can be rotated about an axis perpendicular to the plane of the figure. The plane mirror reflects the light which converges on a concave mirror M_2. The distance between the two mirrors is equal to the radius of curvature of the concave mirror. The concave mirror reflects the light beam back to the plane mirror. The central ray is always incident on the concave mirror perpendicularly so that it retraces the path. If the plane mirror does not rotate, the rays retrace the path upto the glass plate G. A part of the beam is reflected by the glass plate and forms an image I of the source. Now, suppose the plane mirror M_1 rotates by an angle $\Delta\theta$ by the time light goes from M_1 to M_2 and comes back to it. The light reflected by M_1 then makes an angle $2\Delta\theta$ with the direction of the rays reflected earlier. Because of this deviation, the returning rays (shown dotted in figure 21.2) form an image I' of the source which is slightly shifted from the position I.

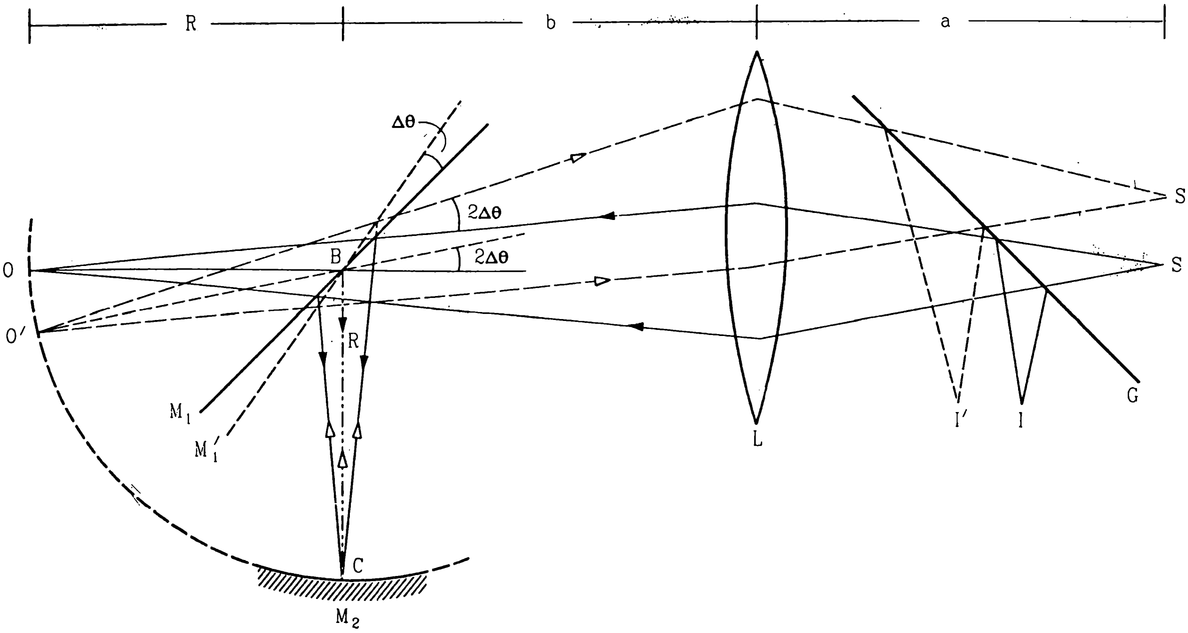

Figure 21.2

Suppose,

R = radius of the concave mirror,

ω = the angular speed of the plane mirror,

s = the shift II',

b = the distance from M_1 to L,

a = the distance from L to S.

When the mirror is in position M_1, the rays reflected by it to the lens seem to come from a point O which is the image of the point C in M_1. When it has rotated by an angle $\Delta\theta$, the rays reflected by it to the lens seem to come from a point O' which is the image of C in the new position M'_1 of the mirror. The distance $BO = BC = R$. It is clear from the figure that

$$OO' = R.(2\Delta\theta). \qquad \dots \text{(i)}$$

Now, the rays reflected by the position M_1 of the mirror retrace the path and would converge at the source S itself. The glass plate partly reflects the beam to converge it at I. Thus, I is the image of S in the plate G acting as a plane mirror. Similarly, the rays reflected by the position M'_1 of the mirror are converged by the lens at a point S'. The glass plate G partly reflects the beam to converge it at I' which is the image of S' in G. It is clear that

$$SS' = II' = s. \qquad \dots \text{(ii)}$$

Thus, the lens L forms an image of O at S and of O' at S'. If we place an object of size OO' at O, its image will have the size SS' at S. Thus,

$$\frac{SS'}{OO'} = \text{magnification produced by } L$$

$$= \frac{\text{image-distance}}{\text{object-distance}}$$

$$= \frac{a}{R+b}.$$

Putting from (i) and (ii),

$$\frac{s}{2R\Delta\theta} = \frac{a}{R+b}. \qquad \dots \text{(iii)}$$

If the speed of light is c, it takes time $\Delta t = 2R/c$ to go from M_1 to M_2 and to come back. As the angular speed of M_1 is ω, the angle rotated by it in time Δt is

$$\Delta\theta = \omega\,\Delta t = \frac{2R\omega}{c}.$$

Putting in (iii),

$$\frac{s}{2R(2R\omega/c)} = \frac{a}{R+b}$$

or,

$$c = \frac{4R^2\omega a}{s(R+b)}. \qquad \dots (21.2)$$

All the quantities on the right side may be measured in the experiment and hence, the speed of light may be calculated. Foucault obtained a value $2{\cdot}98 \times 10^8$ m/s from his measurement.

The space required in this experiment is quite small and hence, it may be performed inside a laboratory. Another advantage with this method is that one can put a tube of a transparent material between the two mirrors. The speed calculated by equation (21.2) then gives the speed of light in that material. It could be experimentally verified that light travels at a slower speed in a medium as compared to its speed in vacuum. This finding was contrary to the predictions of Newton's corpuscular theory.

21.4 MICHELSON METHOD

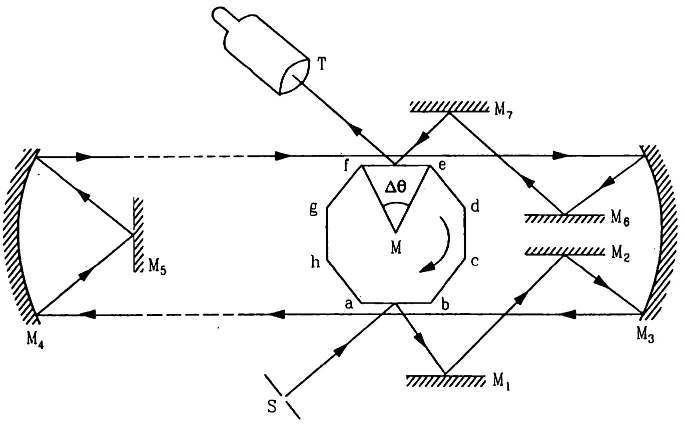

Figure 21.3

The scheme of Michelson method to measure the speed of light is shown in figure (21.3). Light from an intense source S is incident upon one face of a polygon-shaped mirror M. The light reflected from this surface is sent to the lower portion of a concave mirror M_3 after reflections from two plane mirrors M_1 and M_2. The geometry is set so that the light reflected from the concave mirror becomes parallel. This parallel beam of light is allowed to travel through a long distance (several kilometers) and falls on the lower portion of another concave mirror M_4. The parallel beam is converged at the focus of M_4 where a plane mirror M_5 is placed. M_5 reflects the beam back to the concave mirror M_4, this time at the upper portion. As M_5 is at the focus, the beam reflected by M_4 becomes parallel and travels back to the concave mirror M_3. After proper reflections from M_3 and the plane mirrors, it is sent to the polygonal mirror. A telescope is adjusted to receive the rays reflected by the polygonal mirror and hence, to form an image of the source.

Suppose the polygonal mirror M is stationary. Light from the source falls on the face ab of the mirror M and after reflections from all the mirrors, finally falls on the face ef of the mirror M. The image of S is seen in the telescope. If the polygonal mirror rotates, the face ef also turns a little while light travels between the two reflections from the polygonal mirror. The light thus fails to enter into the telescope and the image is not seen. If the rotational speed of the mirror is gradually increased, a stage comes when the adjacent face fg takes the place of ef by the time light comes there. Then, the light is again sent into the telescope.

In the experiment, one looks through the telescope and gradually increases the angular speed of the polygonal mirror. The image flickers initially and becomes steady at a particular angular speed of the mirror. This angular speed is measured.

Suppose,

N = the number of faces in the polygonal mirror,

ω = the angular speed of rotation of the mirror when the image becomes steady,

D = the distance travelled by the light between the reflections from the polygonal mirror.

If the speed of light is c, the time taken by the light to travel the distance D is $\Delta t = D/c$. The angle rotated by the mirror during this time is $\Delta\theta = 2\pi/N$.

The angular speed of the mirror is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi/N}{D/c} = \frac{2\pi c}{DN}$$

or, $$c = \frac{D\omega N}{2\pi}.$$

If ν be the frequency of rotation, $\omega/2\pi = \nu$ and

$$c = D\nu N. \qquad \text{... (21.3)}$$

Michelson and his co-workers made a series of similar experiments. The first determination was made in 1879 with an octagonal rotating mirror. The latest in the series was underway at the time of the death of Michelson and was completed in 1935 by Pease and Pearson. This experiment used a rotating mirror with 32 faces.

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QUESTIONS FOR SHORT ANSWER

1. The speed of sound in air is 332 m/s. Is it advisable to define the length 1 m as the distance travelled by sound in 1/332 s?
2. Consider Galileo's method of measuring the speed of light using two lanterns. To get an accuracy of about 10%, the time taken by the experimenter in closing or opening the shutter should be about one tenth of the time taken by the light in going from one experimenter to the other. Assume that it takes 1/100 second for an experimenter to close or open the shutter. How far should the two experimenters be to get a 10% accuracy? What are the difficulties in having this separation?
3. In Fizeau method of measuring the speed of light, the toothed wheel is placed in the focal plane of a converging

lens. How would the experiment be affected if the wheel is slightly away from the focal plane?

4. In the original Fizeau method, the light travelled 8·6 km and then returned. What could be the difficulty if this distance is taken as 8·6 m?

5. What is the advantage of using a polygonal mirror with larger number of faces in Michelson method of measuring the speed of light?

OBJECTIVE I

1. Light passes through a closed cylindrical tube containing a gas. If the gas is gradually pumped out, the speed of light inside the tube will
(a) increase (b) decrease (c) remain constant
(d) first increase and then decrease.

2. The speeds of red light and yellow light are exactly same
(a) in vacuum but not in air
(b) in air but not in vacuum
(c) in vacuum as well as in air
(d) neither in vacuum nor in air.

3. An illuminated object is placed on the principal axis of a converging lens so that a real image is formed on the other side of the lens. If the object is shifted a little,
(a) the image will be shifted simultaneously with the object
(b) the image will be shifted a little later than the object
(c) the image will be shifted a little earlier than the object
(d) the image will not shift.

OBJECTIVE II

1. The speed of light is 299,792,458 m/s
(a) with respect to the earth
(b) with respect to the sun
(c) with respect to a train moving on the earth
(d) with respect to a spaceship going in outer space.

2. Which of the following methods can be used to measure the speed of light in laboratory?
(a) Roemer method. (b) Fizeau method.
(c) Focault method. (d) Michelson method.

3. Which of the following methods can be used to measure the speed of light in water?
(a) Roemer method. (b) Fizeau method.
(c) Focault method. (d) Michelson method.

EXERCISES

1. In an experiment to measure the speed of light by Fizeau's apparatus, following data are used:
Distance between the mirrors = 12·0 km,
Number of teeth in the wheel = 180.
Find the minimum angular speed of the wheel for which the image is not seen.

2. In an experiment with Foucault's apparatus, the various distances used are as follows:
Distance between the rotating and the fixed mirror = 16 m
Distance between the lens and the rotating mirror = 6 m,
Distance between the source and the lens = 2 m.
When the mirror is rotated at a speed of 356 revolutions per second, the image shifts by 0·7 mm. Calculate the speed of light from these data.

3. In a Michelson experiment for measuring speed of light, the distance travelled by light between two reflections from the rotating mirror is 4·8 km. The rotating mirror has a shape of a regular octagon. At what minimum angular speed of the mirror (other than zero) the image is formed at the position where a non-rotating mirror forms it?

□

ANSWERS

OBJECTIVE I

1. (a) 2. (a) 3. (b)

OBJECTIVE II

1. (a), (b), (c), (d) 2. (c) 3. (c)

EXERCISES

1. 1·25 × 10^4 deg/s
2. 2·984 × 10^8 m/s
3. 7·8 × 10^3 rev/s

□

SOLUTIONS TO CONCEPTS

CHAPTER 21

1. In the given Fizeau'' apparatus,

   D = 12 km = 12 × 10^3 m

   n = 180

   c = 3 × 10^8 m/sec

   We know, $c = \frac{2Dn\omega}{\pi}$

   $\Rightarrow \omega = \frac{\pi c}{2Dn}$ rad/sec = $\frac{\pi c}{2Dn} \times \frac{180}{\pi}$ deg/sec

   $\Rightarrow \omega = \frac{180 \times 3 \times 10^8}{24 \times 10^3 \times 180}$ = 1.25 × 10^4 deg/sec

2. In the given Focault experiment,

   R = Distance between fixed and rotating mirror = 16m

   ω = Angular speed = 356 rev/′ = 356 × 2π rad/sec

   b = Distance between lens and rotating mirror = 6m

   a = Distance between source and lens = 2m

   s = shift in image = 0.7 cm = 0.7 × 10^{-3} m

   So, speed of light is given by,

   $C = \frac{4R^2\omega a}{s(R+b)} = \frac{4 \times 16^2 \times 356 \times 2\pi \times 2}{0.7 \times 10^{-3}(16+6)}$ = 2.975 × 10^8 m/s

3. In the given Michelson experiment,

   D = 4.8 km = 4.8 × 10^3 m

   N = 8

   We know, $c = \frac{D\omega N}{2\pi}$

   $\Rightarrow \omega = \frac{2\pi c}{DN}$ rad/sec = $\frac{c}{DN}$ rev/sec = $\frac{3 \times 10^8}{4.8 \times 10^3 \times 8}$ = 7.8 × 10^3 rev/sec

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