CHAPTER 20

DISPERSION AND SPECTRA

20.1 DISPERSION

As mentioned earlier, the refractive index of a material depends slightly on the wavelength of light. The relation between the two may be approximately described by the equation

$$\mu = \mu_0 + \frac{A}{\lambda^2},$$

where A is a small positive constant known as Cauchy's constant. The refractive index decreases as the wavelength increases. For visible light, it is maximum for the violet end and minimum for the red end. Figure (20.1) shows the variation of refractive index with wavelength for some transparent materials.

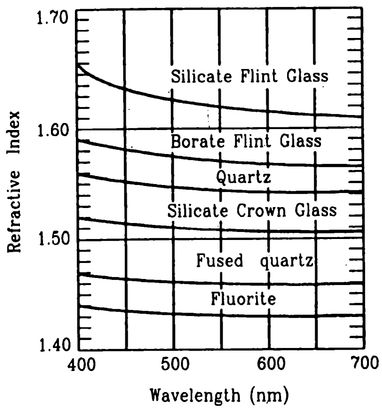


Figure 20.1

Because of the difference in refractive indices, light of different colours bend through different angles on refraction. If white light passes through a glass prism (figure 20.2), the violet rays deviate the most and the red rays deviate the least. Thus, white light is separated into its various component colours. This phenomenon of separation of different constituent colours of light while passing through a transparent medium is known as *dispersion of light.*

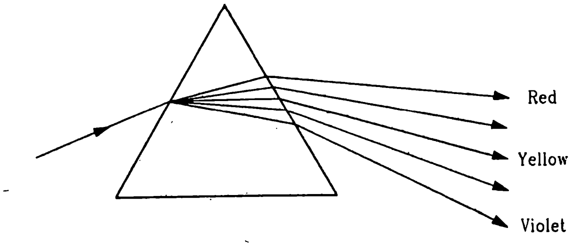


Figure 20.2

20.2 DISPERSIVE POWER

Consider a prism of a transparent material. When a beam of white light is passed through the prism, light of different wavelengths are deviated by different amounts. The overall deviation of the light beam is measured by the deviation of the yellow light as this deviation is roughly the average of all deviations. In figure (20.3), this deviation is shown by the symbol δ_y. It is clear that if δ_r and δ_v are the deviations for red and violet components, the angular divergence of the transmitted beam is $\delta_v - \delta_r$. This divergence is called *angular dispersion.*

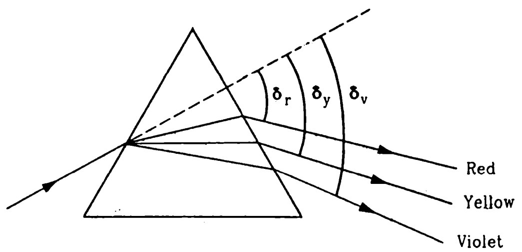


Figure 20.3

The mean deviation depends on the average refractive index μ and the angular dispersion depends on the difference $\mu_v - \mu_r$. It may be seen from figure (20.1) that if the average value of μ is small (fluorite), $\mu_v - \mu_r$ is also small and if the average value of μ is large (silicate flint glass), $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

The *dispersive power* of a material is defined as the ratio of angular dispersion to the average deviation when a light beam is transmitted through a thin prism placed in a position so that the mean ray (ray having the mean wavelength) passes symmetrically through it.

When a light ray passes symmetrically through a prism of refracting angle A, it suffers minimum deviation δ given by

$$\mu = \frac{\sin\frac{A+\delta}{2}}{\sin\frac{A}{2}}.$$

If the refracting angle A is small, the deviation δ is also small. Then,

$$\mu \approx \frac{\frac{A+\delta}{2}}{\frac{A}{2}} = 1 + \frac{\delta}{A}$$

or, $\delta = (\mu - 1)A$.

This equation is also valid if the light ray does not pass symmetrically through the prism, but the angle A and the angle of incidence i are small.

Suppose, a beam of white light goes through such a prism. The deviation of violet, yellow and the red light are

$$\delta_v = (\mu_v - 1)A$$
$$\delta_y = (\mu_y - 1)A$$
and $$\delta_r = (\mu_r - 1)A.$$

The angular dispersion is $\delta_v - \delta_r = (\mu_v - \mu_r)A$. The average deviation is $\delta_y = (\mu_y - 1)A$. Thus, the dispersive power of the medium is

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}. \quad \text{... (20.1)}$$

This equation itself may be taken as the definition of dispersive power.

Refractive index is a continuous function of wavelength. Usually three wavelengths are selected, one from violet, one from yellow and one from red region and dispersive power is defined as (20.1) for these wavelengths.

Example 20.1

Find the dispersive power of flint glass. The refractive indices of flint glass for red, yellow and violet light are 1·613, 1·620 *and* 1·632 *respectively.*

Solution : The dispersive power is $\omega = \frac{\mu_v - \mu_r}{\mu - 1}$

$$= \frac{1{\cdot}632 - 1{\cdot}613}{1{\cdot}620 - 1} = 0{\cdot}0306.$$

Example 20.2

The focal lengths of a thin lens for red and violet light are 90·0 cm *and* 86·4 cm *respectively. Find the dispersive power of the material of the lens. Make appropriate assumptions.*

Solution : We have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\mu - 1 = \frac{1}{f}\cdot\frac{1}{\frac{1}{R_1} - \frac{1}{R_2}} = \frac{K}{f}.$$

Thus, $$\mu_v - 1 = \frac{K}{f_v}$$

and, $$\mu_r - 1 = \frac{K}{f_r}$$

so that $$\mu_v - \mu_r = K\left(\frac{1}{f_v} - \frac{1}{f_r}\right)$$

$$= K\left[\frac{1}{86{\cdot}4\ \text{cm}} - \frac{1}{90\ \text{cm}}\right] = K \times 4{\cdot}6 \times 10^{-4}\ \text{cm}^{-1}.$$

Also, we can assume that

$$\mu_y - 1 = \frac{\mu_v + \mu_r}{2} - 1 = \frac{\mu_v - 1}{2} + \frac{\mu_r - 1}{2}$$

$$= \frac{K}{2}\left(\frac{1}{f_v} + \frac{1}{f_r}\right)$$

$$= \frac{K}{2}\left[\frac{1}{86{\cdot}4\ \text{cm}} + \frac{1}{90\ \text{cm}}\right] = K \times 1{\cdot}1 \times 10^{-2}\ \text{cm}^{-1}.$$

Thus, the dispersive power of the material of the lens is

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{4{\cdot}6 \times 10^{-4}}{1{\cdot}1 \times 10^{-2}} = 0{\cdot}042.$$

20.3 DISPERSION WITHOUT AVERAGE DEVIATION AND AVERAGE DEVIATION WITHOUT DISPERSION

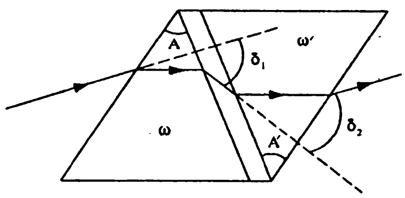


Figure 20.4

Figure (20.4) shows two thin prisms placed in contact in such a way that the two refracting angles are reversed with respect to each other. Suppose, the refracting angles of the two prisms are A and A' and their dispersive powers are ω and ω' respectively.

Consider a ray of light for which the refractive indices of the materials of the two prisms are μ and μ'. Assuming that the ray passes through the prisms in symmetrical situation, the deviations produced by the two prisms are

$$\delta_1 = (\mu - 1) A$$

and $$\delta_2 = (\mu' - 1) A'.$$

As the two deviations are opposite to each other, the net deviation is

$$\delta = \delta_1 - \delta_2$$
$$= (\mu - 1)A - (\mu' - 1)A'. \quad \ldots \text{(i)}$$

If white light passes through the combination, the net deviation of the violet ray is

$$\delta_v = (\mu_v - 1)A - (\mu'_v - 1)A'$$

and that of the red ray is

$$\delta_r = (\mu_r - 1)A - (\mu'_r - 1)A'.$$

The angular dispersion produced by the combination is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'. \quad \ldots \text{(ii)}$$

The dispersive powers are given by

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

and $$\omega' = \frac{\mu'_v - \mu'_r}{\mu'_y - 1}.$$

Thus, by (ii), the net angular dispersion is

$$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A'. \quad \ldots \text{(iii)}$$

The net deviation of the yellow ray i.e., the average deviation, is, by (i),

$$\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A'. \quad \ldots \text{(iv)}$$

Dispersion without Average Deviation

If the combination is not to produce a net average deviation in the beam, δ_y should be 0. By (iv), the required condition is

$$(\mu_y - 1)A = (\mu'_y - 1)A'. \quad \ldots (20.2)$$

Using this in (iii), the net angular dispersion produced is

$$\delta_v - \delta_r = (\mu_y - 1)A\,(\omega - \omega'). \quad \ldots (20.3)$$

By choosing ω and ω' different and the refracting angles to satisfy (20.2), one can get dispersion without average deviation.

Average Deviation without Dispersion

If the combination is not to produce a net dispersion, $\delta_v - \delta_r = 0$. By (iii),

$$(\mu_y - 1)\omega\, A = (\mu'_y - 1)\omega' A'. \quad \ldots (20.4)$$

By (ii), this condition may also be written as

$$(\mu_v - \mu_r)A = (\mu'_v - \mu'_r)A'. \quad \ldots (20.5)$$

The net average deviation produced is, by (i),

$$\delta = (\mu_y - 1)A - (\mu'_y - 1)A'$$
$$= (\mu_y - 1)A\left[1 - \frac{\mu'_y - 1}{\mu_y - 1}\frac{A'}{A}\right].$$

By (20.4),

$$\frac{(\mu'_y - 1)A'}{(\mu_y - 1)A} = \frac{\omega}{\omega'}$$

so that the net average deviation produced by the combination is

$$\delta = (\mu_y - 1)\,A\left(1 - \frac{\omega}{\omega'}\right). \quad \ldots (20.6)$$

20.4 SPECTRUM

When light coming from a source is dispersed by a prism or by any other dispersing element, light of different wavelengths are deviated through different angles and get separated. Such a dispersed light may be received on a screen, on a photographic plate or it may be viewed directly by the eye. A collection of dispersed light giving its wavelength composition is called a *spectrum*. As a very simple demonstration, let white light fall on a prism and collect the transmitted light on a white wall or a white paper. A spectrum consisting of different colours from red to violet is obtained.

Pure and Impure Spectrum

The spectrum of visible light shows different colours. In an ideal situation, light of one wavelength should occupy one particular spatial position in the spectrum. In such a case, no two wavelengths overlap in the dispersed beam. Each colour then gives its sharp impression. Such a spectrum is called a *pure spectrum*. To get a pure spectrum,

(a) the light beam incident on the dispersing element (prism, grating etc.) should be parallel, and

(b) the dispersed light should be focused in such a way that all the rays of a particular wavelength are collected at one place.

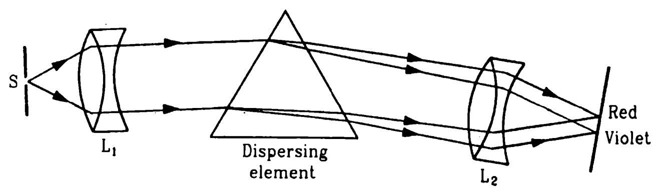


Figure 20.5

These conditions may be achieved to a good approximation using the arrangement shown in figure (20.5). A narrow slit S allows a thin pencil of light to be analyzed. The slit is placed in the focal plane of an achromatic lens combination L_1. The light is dispersed by the dispersing element such as a prism or a grating. The emergent rays for a particular wavelength are all parallel. Another achromatic lens combination L_2 is used to focus the emergent rays in its focal plane. Rays of one wavelength being parallel to each other are finally focused at one place.

If the slit is wide, different points of the slit produce separate spectra which overlap each other. The colour impression gets diffused due to the overlap. Such a spectrum is called an *impure spectrum.*

20.5 KINDS OF SPECTRA

A. Emission Spectra

Light is emitted by an object when it is suitably excited by heating or by passing an electric discharge, etc. When a light beam emitted by such a source is dispersed to get the spectrum, it is called an *emission spectrum*. An emission spectrum carries information about the source material. An emission spectrum can be of three types :

(a) Continuous Spectrum

Quite often, a source emits light which has continuously varying wavelengths in it. An electric bulb, a candle or a red hot iron piece emits light of this type. When such a light is dispersed, a bright spectrum continuously distributed on a dark background is obtained. The colours gradually change and there are no sharp boundaries in between. Such a spectrum is known as a *continuous emission spectrum.*

(b) Line Spectrum

All objects are made of atoms and molecules. The atoms and molecules can have certain fixed energies. An atom or a molecule having the lowest possible energy is said to be in its ground state, otherwise, in an excited state. An atom or molecule, in an excited state, can emit light to lower its energy. Light emitted in such a process has certain fixed wavelengths. The light emitted by one kind of atoms generally have widely separated wavelength components (figure 20.6a). When such a light is dispersed, we get certain sharp bright lines on a dark background. Such a spectrum is called *line emission spectrum.* It carries information about the atoms of the source. For example, when electric discharge is passed through sodium vapour, the vapour emits light of the wavelengths 589·0 nm and 589·6 nm. When dispersed by a high resolution grating, one obtains two bright yellow lines on a dark background.

(c) Band Spectrum

The molecular energy levels are generally grouped into several bunches, each bunch widely separated from the other, and the levels in a bunch being close to each other. Thus, the wavelengths emitted by such molecules are also grouped, each group being well-separated from the other. The wavelengths in a group are close to one another and appear as continuous. The spectrum looks like separate bands of varying colours. Such a spectrum is called a *band emission spectrum.* Figure (20.6b) shows schematically the production and appearance of band spectra.

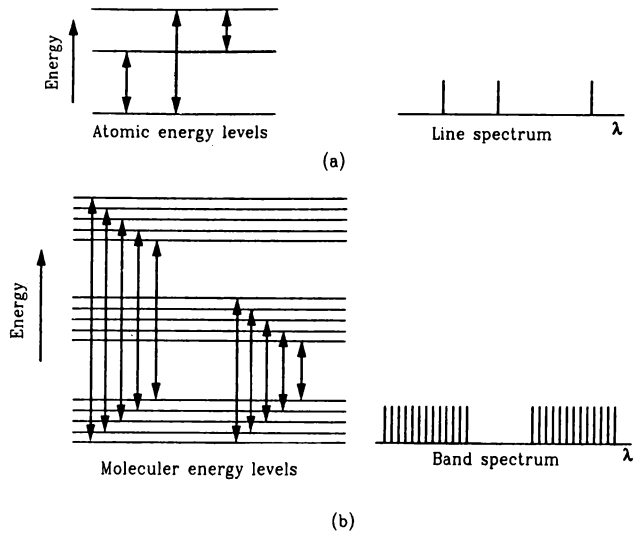


Figure 20.6

B. Absorption Spectrum

When white light having all wavelengths is passed through an absorbing material, the material may absorb certain wavelengths selectively. When the transmitted light is dispersed, we get dark lines or bands at the positions of the missing wavelengths superposed on an otherwise bright continuous coloured background (figure 20.7).

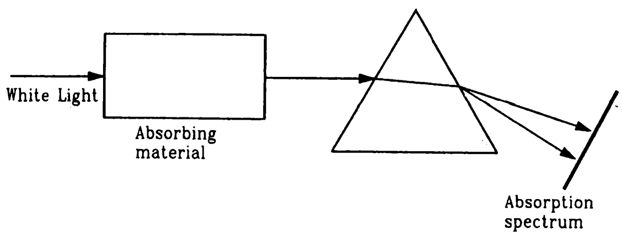


Figure 20.7

The missing wavelengths provide information about the absorbing material. Such a spectrum is called an *absorption spectrum.*

Absorption spectrum may be of two types depending on the absorbing material and the conditions such as temperature etc. of the experiment.

(a) Line Absorption Spectrum

Light may be absorbed by atoms to take them from lower energy states to higher energy states. In this case, the missing wavelengths are widely separated and we get sharp dark lines on a continuous bright background. Such a spectrum is called a *line absorption spectrum*. When light coming from the sun is dispersed, it shows certain sharply defined dark lines. This shows that certain wavelengths are absent. These missing lines are called *Fraunhofer lines.*

(b) Band Absorption Spectra

If light is absorbed by molecules of the absorbing material, exciting them from lower energy to higher energy states, the missing wavelengths are grouped into bunches. Thus, when the transmitted light is dispersed, we get separate dark bands on a continuous bright background. Such a spectrum is called a *band absorption spectrum*. Light passing through hydrogen gas at moderate temperature or through certain solutions of organic and inorganic compounds shows such a spectrum.

20.6 ULTRAVIOLET AND INFRARED SPECTRUM

When an object is suitably excited by heating or in some other way, it emits light. The light, that causes visual sensation to the eye, has a wavelength range from about 380 nm (violet) to about 780 nm (red). The light emitted by an excited object may have wavelengths beyond this visible region. We generally use the word light to mean visible light. That beyond the visible region is called by the general name *radiation*. The radiation with wavelength less than the lower end of the visible region (that is less than about 380 nm) is called *ultraviolet radiation* and the radiation with wavelength greater than the upper end of the visible region is called the *infrared radiation.* The range of ultraviolet radiation is roughly from 15 nm to 380 nm and that of infrared radiation is roughly 780 nm to 40000 nm. Beyond ultraviolet, we have X-rays and gamma-rays and above infrared, we have radiowaves.

Ordinary glass highly absorbs infrared and ultraviolet radiation. A prism made of quartz may be used for studying the spectrum in ultraviolet region. The dispersed radiation may be collected on a photographic plate. To study infrared spectrum, one can use a prism made of rocksalt. Infrared radiation considerably heats the object on which it falls. One way of detecting infrared radiation is from its heating effect. An instrument known as thermopile, which is sensitive to heat, is used to measure the dispersed infrared spectrum.

20.7 SPECTROMETER

Spectrometer is an instrument which is used to produce and study pure spectrum in visible region. It consists of basically three parts.

(a) Collimator

It consists of a long cylindrical tube fitted with an achromatic converging lens at one end. Another tube of slightly smaller diameter can slide into the first tube by a rack-pinion arrangement and has a linear slit at the outer end. The width of the slit may be adjusted by a screw. The distance between the slit and the lens may be changed by sliding the second tube into the first. The incident light is passed through the collimator to make it parallel before falling on the dispersing element.

(b) Prism Table

This is a horizontal platform which can be rotated about its axis and whose height may be adjusted. The dispersing element (prism, grating etc.) is placed on the prism table. When the prism table rotates, a horizontal circular scale (graduated in degrees) rotates with it.

(c) Telescope

This is an astronomical telescope. The objective lens is fitted at one end of a long cylindrical tube. Another cylindrical tube can slide into it and contains the eyepiece. The dispersed light is passed through the telescope before falling to the eye which is placed just behind the eyepiece. A vernier scale is attached to the telescope which rotates on the horizontal circular scale when the telescope is rotated.

Levelling screws are provided under the main base, the collimator tube, the telescope tube and the prism table.

Adjustment and Working

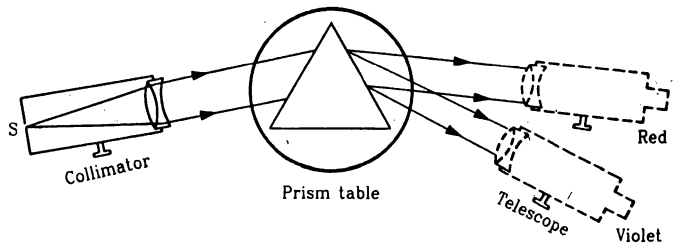


Figure 20.8

The collimator, the prism table and the telescope are fitted in one compact unit (figure 20.8). The prism table and the telescope can be independently rotated about the vertical axis of the prism table. The angle of rotation can be accurately measured by the vernier scale and the horizontal circular scale.

The axis of the collimator tube, of the telescope tube and the surface of the prism table are made horizontal with the help of levelling screws. The light source to be examined is placed behind the slit of the collimator. The distance between the slit and the collimating lens is so adjusted that the slit lies in the first focal plane of the lens and the rays coming from the collimator become parallel. This parallel beam is incident upon the dispersing element (prism, grating etc.) placed on the prism table. The dispersed beam is received by the telescope which is focused for parallel rays, that is, for normal adjustment. The telescope tube is rotated and light rays of different wavelengths are received at different angular positions of the telescope.

Application of Spectrometer

The spectrometer can be used in a wide variety of applications. We mention here a few simple ones. In all these applications, the spectrometer is adjusted as described above.

(a) Measuring the Angle of a Prism

The spectrometer is levelled and adjusted for parallel rays. The prism is placed on the prism table with its refracting edge facing the collimator. The slit is illuminated by a sodium vapour lamp. The parallel beam coming from the collimator is divided into two parts falling on the two surfaces of the prism (figure 20.9).

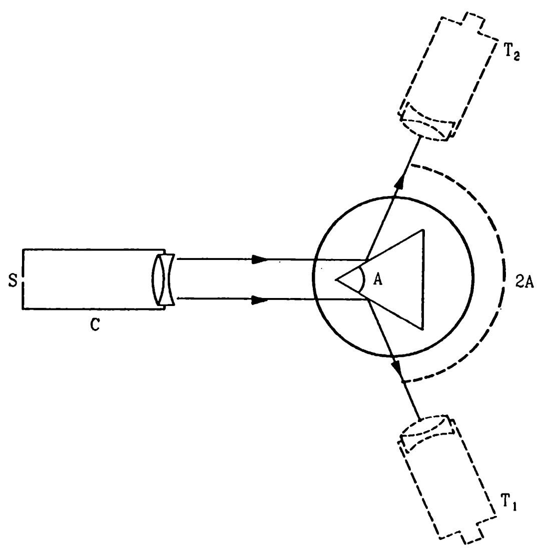


Figure 20.9

The telescope is rotated to a position T_1 where it receives the beam reflected by one surface. The image of the slit should coincide with the vertical crosswire of the telescope. The angular position is read on the base with the help of the vernier scale and the scale fixed in the base. The telescope is now rotated to the position T_2 where it receives the beam reflected by the other surface. The image of the slit should again coincide with the vertical crosswire. The angular position is again read on the base. The difference of these two readings gives the angle rotated by the telescope. This angle is equal to $2A$ where A is the angle of prism. Sodium vapour lamp gives a nearly monochromatic light which makes the image identification easier.

(b) Measuring the Angle of Minimum Deviation for a Prism for a Given Wavelength

The spectrometer is adjusted as described before. The source emitting the light of the given wavelength is placed behind the slit.

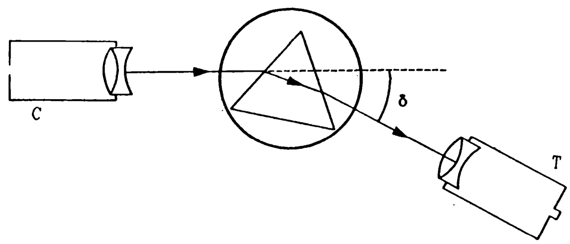


Figure 20.10

The telescope is rotated and placed at a position where the angle between the telescope axis and the collimator axis is large. The prism is placed on the prism table and the table is rotated to such a position that the refracted beam is received by the telescope (figure 20.10). The image of the slit is made to coincide with the crosswire. The angle between the axes of the collimator and the telescope is the angle of deviation δ. Now, the telescope is rotated slightly towards the collimator axis to decrease δ. The prism table is rotated to bring the image of the slit back at the crosswire. The process is repeated till a position comes where if the telescope is further rotated towards the collimator axis, it is not possible to bring the image at the crosswire for any position of the prism table. This position where the image can be last brought to the crosswire is the position of minimum deviation. The angle between the axes of the collimator and the telescope in this position is the angle of minimum deviation.

To measure this angle, the reading of vernier scale attached with the telescope is noted down. The prism is removed and the telescope is brought in line with the collimator so that the image of the slit forms at the crosswire. The reading of the vernier scale is again noted. The difference between the two readings gives the angle of minimum deviation.

(c) Variation of Refractive Index with Wavelength

To study the variation of refractive index μ with the wavelength λ, a source is chosen which emits light of sharply defined discrete wavelengths. Neon discharge tube is one such source.

The spectrometer is adjusted as before and a prism is placed on the prism table. A particular colour is chosen and the angle of minimum deviation for that colour is obtained by the method described above. This is done by always focusing the image of the slit formed by that colour. The refractive index of the material of the prism is calculated by the formula

$$\mu = \frac{\sin \frac{A + \delta_m}{2}}{\sin \frac{A}{2}} .$$

The experiment is repeated for each colour and corresponding values of μ are obtained. Knowing the values of wavelengths, $\mu - \lambda$ variation is studied.

20.8 RAINBOW

When sunlight falls on small water droplets suspended in air during or after a rain, it suffers refraction, total internal reflection and dispersion. If an observer has the sun at the back and the water droplets in front, he or she may see two rainbows, one inside the other. The inner one is called the *primary rainbow* and the outer one is called the *secondary rainbow*. Figure (20.11) shows the path of a typical ray forming the primary bow. It suffers a refraction followed by a total internal reflection and then again a refraction. Dispersion takes place at both the refractions. It turns out that rays of a given colour are strongly returned by the droplet in a direction that corresponds to maximum deviation in its path. For light of red colour this maximum deviation is 137·8° so that the angle θ in figure (20.11) is 180° – 137·8° = 42·2°. For violet this angle is 40·6° and for other colours it is in between 40·6° and 42·2°. Now consider an observer at P (figure 20.11b). Suppose sunrays are incident in a direction parallel to PX. Consider a cone with PX as its axis and semivertical angle 42·2°. All the droplets on the surface of this cone will return the light to P at an angle of 42·2°. This light will be predominantly red. Thus, the red rays coming to the observer will appear to come from a circle which subtends an angle of 42·2° on the eye. Similarly, the violet rays coming to the observer will appear to come from a circle which subtends an angle of 40·6° on the eye. The other colours form their respective circles of intermediate radii. From the ground level, only an arc of the rainbow is usually visible. A complete circular rainbow may be seen from an elevated position such as from an aeroplane.

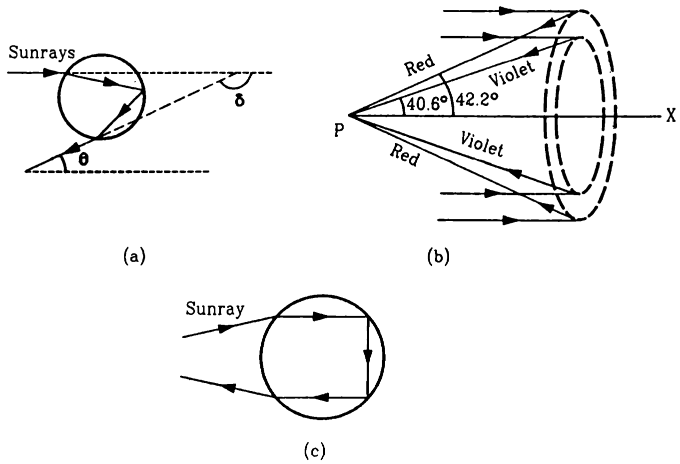


Figure 20.11

The secondary rainbow is formed by rays which suffer two internal reflections before coming out of the water drop (figure 20.11c). In this, the order of colours is reverse. Red appears on the inner circle in the rainbow and violet on the outer. The angle θ is 50·5° for red and 54° for violet.

Worked Out Examples

1. *The refractive indices of flint glass for red and violet light are* 1·613 *and* 1·632 *respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle* 5°.

Solution : Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)A$

$$= (1{\cdot}632 - 1{\cdot}613) \times 5^\circ$$

$$= 0{\cdot}095^\circ.$$

2. *A crown glass prism of angle* 5° *is to be combined with a flint glass prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it.*

Refractive indices for red, yellow and violet light are 1·514, 1·517 and 1·523 respectively for crown glass and 1·613, 1·620 and 1·632 for flint glass.

Solution : The deviation produced by the crown prism is

$$\delta = (\mu - 1)A$$

and by the flint prism is

$$\delta' = (\mu' - 1)A'.$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \quad \ldots \text{(i)}$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A'.$$

or,
$$A' = \frac{(\mu - 1)}{(\mu' - 1)} A = \frac{1{\cdot}517 - 1}{1{\cdot}620 - 1} \times 5°$$
$$= 4{\cdot}2°.$$

(b) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r)A$$

and that by the flint prism is

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A'.$$

The net angular dispersion is,

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$
$$= (1{\cdot}523 - 1{\cdot}514) \times 5° - (1{\cdot}632 - 1{\cdot}613) \times 4{\cdot}2°$$
$$= -\,0{\cdot}0348°.$$

The angular dispersion has magnitude 0·0348°.

3. *The dispersive powers of crown and flint glasses are 0·03 and 0·05 respectively. The refractive indices for yellow light for these glasses are 1·517 and 1·621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Find the refracting angles of the two prisms needed.*

Solution : Suppose, the angle of the crown prism needed is A and that of the flint prism is A'. We have

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1}$$

or,
$$\mu_v - \mu_r = (\mu - 1)\omega.$$

The angular dispersion produced by the crown prism is

$$(\mu_v - \mu_r)A = (\mu - 1)\omega A.$$

Similarly, the angular dispersion produced by the flint prism is

$$(\mu' - 1)\omega' A'.$$

For achromatic combination, the net dispersion should be zero. Thus,

$$(\mu - 1)\omega A = (\mu' - 1)\omega' A'$$

or,
$$\frac{A'}{A} = \frac{(\mu - 1)\omega}{(\mu' - 1)\omega'} = \frac{0{\cdot}517 \times 0{\cdot}03}{0{\cdot}621 \times 0{\cdot}05} = 0{\cdot}50. \quad \ldots \text{(i)}$$

The deviation in the yellow ray produced by the crown prism is $\delta = (\mu - 1)A$ and by the the flint prism is $\delta' = (\mu' - 1)A'$. The net deviation produced by the combination is

$$\delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$$

or,
$$1° = 0{\cdot}517\,A - 0{\cdot}621\,A'. \quad \ldots \text{(ii)}$$

Solving (i) and (ii), $A = 4{\cdot}8°$ and $A' = 2{\cdot}4°$. Thus, the crown prism should have its refracting angle 4·8° and that of the flint prism should be 2·4°.

□

QUESTIONS FOR SHORT ANSWER

1. The equation $\omega = \frac{\mu_v - \mu_r}{\mu - 1}$ was derived for a prism having small refracting angle. Is it also valid for a prism of large refracting angle ? Is it also valid for a glass slab or a glass sphere ?
2. Can the dispersive power $\omega = \frac{\mu_v - \mu_r}{\mu - 1}$ be negative ? What is the sign of ω if a hollow prism is immersed into water ?
3. If three identical prisms are combined, is it possible to pass a beam that emerges undeviated ? Undispersed ?
4. "Monochromatic light should be used to produce pure spectrum". Comment on this statement.
5. Does focal length of a lens depend on the colour of the light used ? Does focal length of a mirror depend on the colour ?
6. Suggest a method to produce a rainbow in your house.

OBJECTIVE I

1. The angular dispersion produced by a prism
   (a) increases if the average refractive index increases
   (b) increases if the average refractive index decreases
   (c) remains constant whether the average refractive index increases or decreases.
   (d) has no relation with average refractive index.

2. If a glass prism is dipped in water, its dispersive power
(a) increases (b) decreases (c) does not change
(d) may increase or decrease depending on whether the angle of the prism is less than or greater than 60°.

3. A prism can produce a minimum deviation δ in a light beam. If three such prisms are combined, the minimum deviation that can be produced in this beam is
(a) 0 (b) δ (c) 2δ (d) 3δ.

4. Consider the following two statements :
(A) Line spectra contain information about atoms.
(B) Band spectra contain information about molecules.
(a) both A and B are wrong
(b) A is correct but B is wrong
(c) B is correct but A is wrong
(d) both A and B are correct.

5. The focal length of a converging lens are f_v and f_r for violet and red light respectively.
(a) $f_v > f_r$. (b) $f_v = f_r$. (c) $f_v < f_r$.
(d) Any of the three is possible depending on the value of the average refractive index μ.

OBJECTIVE II

1. A narrow beam of white light goes through a slab having parallel faces.
(a) The light never splits in different colours.
(b) The emergent beam is white.
(c) The light inside the slab is split into different colours.
(d) The light inside the slab is white.

2. By properly combining two prisms made of different materials, it is possible to
(a) have dispersion without average deviation
(b) have deviation without dispersion
(c) have both dispersion and average deviation
(d) have neither dispersion nor average deviation.

3. In producing a pure spectrum, the incident light is passed through a narrow slit placed in the focal plane of an achromatic lens because a narrow slit
(a) produces less diffraction
(b) increases intensity
(c) allows only one colour at a time
(d) allows a more parallel beam when it passes through the lens.

4. Which of the following quantities related to a lens depend on the wavelength or wavelengths of the incident light ?
(a) Power. (b) Focal length.
(c) Chromatic aberration. (d) Radii of curvature.

5. Which of the following quantities increase when wavelength is increased ? Consider only the magnitudes.
(a) The power of a converging lens.
(b) The focal length of a converging lens.
(c) The power of a diverging lens.
(d) The focal length of a diverging lens.

EXERCISES

1. A flint glass prism and a crown glass prism are to be combined in such a way that the deviation of the mean ray is zero. The refractive index of flint and crown glasses for the mean ray are 1·620 and 1·518 respectively. If the refracting angle of the flint prism is 6·0°, what would be the refracting angle of the crown prism ?

2. A certain material has refractive indices 1·56, 1·60 and 1·68 for red, yellow and violet light respectively. (a) Calculate the dispersive power. (b) Find the angular dispersion produced by a thin prism of angle 6° made of this material.

3. The focal lengths of a convex lens for red, yellow and violet rays are 100 cm, 98 cm and 96 cm respectively. Find the dispersive power of the material of the lens.

4. The refractive index of a material changes by 0·014 as the colour of the light changes from red to violet. A rectangular slab of height 2·00 cm made of this material is placed on a newspaper. When viewed normally in yellow light, the letters appear 1·32 cm below the top surface of the slab. Calculate the dispersive power of the material.

5. A thin prism is made of a material having refractive indices 1·61 and 1·65 for red and violet light. The dispersive power of the material is 0·07. It is found that a beam of yellow light passing through the prism suffers a minimum deviation of 4·0° in favourable conditions. Calculate the angle of the prism.

6. The minimum deviations suffered by red, yellow and violet beams passing through an equilateral transparent prism are 38·4°, 38·7° and 39·2° respectively. Calculate the dispersive power of the medium.

7. Two prisms of identical geometrical shape are combined with their refracting angles oppositely directed. The materials of the prisms have refractive indices 1·52 and 1·62 for violet light. A violet ray is deviated by 1·0° when passes symmetrically through this combination. What is the angle of the prisms ?

8. Three thin prisms are combined as shown in figure (20-E1). The refractive indices of the crown glass for red, yellow and violet rays are μ_r, μ_y and μ_v respectively and

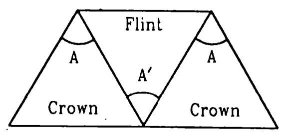


Figure 20-E1

those for the flint glass are μ'_r, μ'_y and μ'_v respectively. Find the ratio A'/A for which (a) there is no net angular dispersion, and (b) there is no net deviation in the yellow ray.

9. A thin prism of crown glass ($\mu_r = 1{\cdot}515$, $\mu_v = 1{\cdot}525$) and a thin prism of flint glass ($\mu_r = 1{\cdot}612$, $\mu_v = 1{\cdot}632$) are placed in contact with each other. Their refracting angles are 5·0° each and are similarly directed. Calculate the angular dispersion produced by the combination.

10. A thin prism of angle 6·0°, $\omega = 0{\cdot}07$ and $\mu_y = 1{\cdot}50$ is combined with another thin prism having $\omega = 0{\cdot}08$ and $\mu_y = 1{\cdot}60$. The combination produces no deviation in the mean ray. (a) Find the angle of the second prism. (b) Find the net angular dispersion produced by the combination when a beam of white light passes through it. (c) If the prisms are similarly directed, what will be the deviation in the mean ray? (d) Find the angular dispersion in the situation described in (c).

11. The refractive index of a material *M*1 changes by 0·014 and that of another material *M*2 changes by 0·024 as the colour of the light is changed from red to violet. Two thin prisms one made of $M1 (A = 5{\cdot}3°)$ and other made of $M2 (A = 3{\cdot}7°)$ are combined with their refracting angles oppositely directed. (a) Find the angular dispersion produced by the combination. (b) The prisms are now combined with their refracting angles similarly directed. Find the angular dispersion produced by the combination.

□

ANSWERS

OBJECTIVE I

1. (a) 2. (b) 3. (b) 4. (d) 5. (c)

OBJECTIVE II

1. (b), (c) 2. (a), (b), (c) 3. (d) 4. (a), (b), (c)
5. (b), (d)

EXERCISES

1. 7·2°
2. (a) 0·2 (b) 0·72°
3. 0·041
4. 0·026
5. 7°
6. 0·0206
7. 10°
8. (a) $\dfrac{2(\mu_y - 1)}{\mu'_y - 1}$ (b) $\dfrac{2(\mu_v - \mu_r)}{\mu'_v - \mu'_r}$
9. 0·15°
10. (a) 5° (b) 0·03° (c) 6° (d) 0·45°
11. (a) 0·0146° (b) 0·163°

□

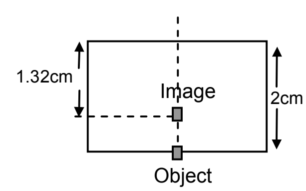
SOLUTIONS TO CONCEPTS
CHAPTER – 20

1. Given that,
   Refractive index of flint glass = μ_f = 1.620
   Refractive index of crown glass = μ_c = 1.518
   Refracting angle of flint prism = A_f = 6.0°
   For zero net deviation of mean ray
   $(\mu_f - 1)A_f = (\mu_c - 1)A_c$
   $\Rightarrow A_c = \frac{\mu_f - 1}{\mu_c - 1}A_f = \frac{1.620 - 1}{1.518 - 1}(6.0)^\circ = 7.2^\circ$

2. Given that
   $\mu_r = 1.56$, $\mu_y = 1.60$, and $\mu_v = 1.68$
   (a) Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{1.68 - 1.56}{1.60 - 1} = 0.2$
   (b) Angular dispersion = $(\mu_v - \mu_r)A = 0.12 \times 6^\circ = 7.2^\circ$

3. The focal length of a lens is given by
   $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
   $\Rightarrow (\mu - 1) = \frac{1}{f} \times \frac{1}{\left(\frac{1}{R_1} - \frac{1}{R_2}\right)} = \frac{K}{f}$ …(1)
   So, $\mu_r - 1 = \frac{K}{100}$ …(2)
   $\mu_y - 1 = \frac{K}{98}$ …(3)
   And $\mu_v - 1 = \frac{K}{96}$ (4)
   So, Dispersive power = $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \frac{\frac{K}{96} - \frac{K}{100}}{\frac{K}{98}} = \frac{98 \times 4}{9600} = 0.0408$

4. Given that, $\mu_v - \mu_r = 0.014$
   Again, $\mu_y = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{2.00}{1.30} = 1.515$
   So, dispersive power = $\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{0.014}{1.515 - 1} = 0.027$



5. Given that, $\mu_r = 1.61$, $\mu_v = 1.65$, $\omega = 0.07$ and $\delta_y = 4^\circ$
   Now, $\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$
   $\Rightarrow 0.07 = \frac{1.65 - 1.61}{\mu_y - 1}$
   $\Rightarrow \mu_y - 1 = \frac{0.04}{0.07} = \frac{4}{7}$
   Again, $\delta = (\mu - 1)A$
   $\Rightarrow A = \frac{\delta_y}{\mu_y - 1} = \frac{4}{(4/7)} = 7^\circ$

6. Given that, $\delta_r = 38.4°$, $\delta_y = 38.7°$ and $\delta_v = 39.2°$

Dispersive power = $\dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{(\mu_v - 1) - (\mu_r - 1)}{(\mu_y - 1)} = \dfrac{\left(\dfrac{\delta_v}{A}\right) - \left(\dfrac{\delta_r}{A}\right)}{\left(\dfrac{\delta_v}{A}\right)}$ $\quad [\because\ \delta = (\mu - 1)\,A]$

$= \dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{39.2 - 38.4}{38.7} = 0.0204$

7. Two prisms of identical geometrical shape are combined.

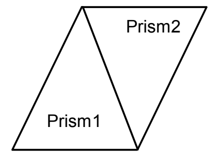
Let A = Angle of the prisms

$\mu'_v = 1.52$ and $\mu_v = 1.62$, $\delta_v = 1°$

$\delta_v = (\mu_v - 1)A - (\mu'_v - 1)\,A$ [since $A = A'$]

$\Rightarrow \delta_v = (\mu_v - \mu'_v)A$

$\Rightarrow A = \dfrac{\delta_v}{\mu_v - \mu'_v} = \dfrac{1}{1.62 - 1.52} = 10°$



8. Total deviation for yellow ray produced by the prism combination is

$\delta_y = \delta_{cy} - \delta_{fy} + \delta_{cy} = 2\,\delta_{cy} - \delta_{fy} = 2(\mu_{cy} - 1)A - (\mu_{cy} - 1)A'$

Similarly the angular dispersion produced by the combination is

$\delta_v - \delta_r = [(\mu_{vc} - 1)A - (\mu_{vf} - 1)A' + (\mu_{vc} - 1)A] - [(\mu_{rc} - 1)\,A - (\mu_{rf} - 1)A' + (\mu_r - 1)\,A)]$

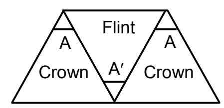
$= 2(\mu_{vc} - 1)A - (\mu_{vf} - 1)A'$

(a) For net angular dispersion to be zero,

$\delta_v - \delta_r = 0$

$\Rightarrow\ 2(\mu_{vc} - 1)A = (\mu_{vf} - 1)A'$

$\Rightarrow\ \dfrac{A'}{A} = \dfrac{2(\mu_{cv} - \mu_{rc})}{(\mu_{vf} - \mu_{rf})} = \dfrac{2(\mu_v - \mu_r)}{(\mu'_v - \mu'_r)}$



(b) For net deviation in the yellow ray to be zero,

$\delta_y = 0$

$\Rightarrow 2(\mu_{cy} - 1)A = (\mu_{fy} - 1)A'$

$\Rightarrow\ \dfrac{A'}{A} = \dfrac{2(\mu_{cy} - 1)}{(\mu_{fy} - 1)} = \dfrac{2(\mu_y - 1)}{(\mu'_y - 1)}$

9. Given that, $\mu_{cr} = 1.515$, $\mu_{cv} = 1.525$ and $\mu_{fr} = 1.612$, $\mu_{fv} = 1.632$ and $A = 5°$

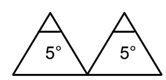
Since, they are similarly directed, the total deviation produced is given by,

$\delta = \delta_c + \delta_r = (\mu_c - 1)A + (\mu_r - 1)\,A = (\mu_c + \mu_r - 2)A$

So, angular dispersion of the combination is given by,

$\delta_v - \delta_y = (\mu_{cv} + \mu_{fv} - 2)A - (\mu_{cr} + \mu_{fr} - 2)A$

$= (\mu_{cv} + \mu_{fv} - \mu_{cr} - \mu_{fr})A = (1.525 + 1.632 - 1.515 - 1.612)\ 5 = 0.15°$



10. Given that, $A' = 6°$, $\omega' = 0.07$, $\mu'_y = 1.50$

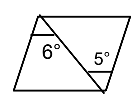
$A = ?$ $\omega = 0.08$, $\mu_y = 1.60$

The combination produces no deviation in the mean ray.

(a) $\delta_y = (\mu_y - 1)A - (\mu'_y - 1)A' = 0$ [Prism must be oppositely directed]

$\Rightarrow (1.60 - 1)A = ((1.50 - 1)A'$

$\Rightarrow A = \dfrac{0.50 \times 6°}{0.60} = 5°$



(b) When a beam of white light passes through it,

Net angular dispersion = $(\mu_y - 1)\omega A - (\mu'_y - 1)\omega' A'$

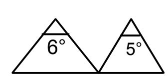
$\Rightarrow (1.60 - 1)(0.08)(5°) - (1.50 - 1)(0.07)(6°)$

$\Rightarrow 0.24° - 0.21° = 0.03°$

(c) If the prisms are similarly directed,

$\delta_y = (\mu_y - 1)A + (\mu'_y - 1)A$

$= (1.60 - 1)5° + (1.50 - 1)6° = 3° + 3° = 6°$



(d) Similarly, if the prisms are similarly directed, the net angular dispersion is given by,

$\delta_v - \delta_r = (\mu_y - 1)\omega A - (\mu'_y - 1)\ \omega' A' = 0.24° + 0.21° = 0.45°$

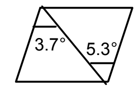
11. Given that, $\mu'_v - \mu'_r = 0.014$ and $\mu_v - \mu_r = 0.024$

$A' = 5.3°$ and $A = 3.7°$

(a) When the prisms are oppositely directed,

angular dispersion = $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$

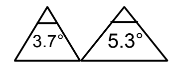
$= 0.024 \times 3.7° - 0.014 \times 5.3° = 0.0146°$



(b) When they are similarly directed,

angular dispersion = $(\mu_v - \mu_r)A + (\mu'_v - \mu'_r)A'$

$= 0.024 \times 3.7° + 0.014 \times 5.3° = 0.163°$



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