CHAPTER 19

OPTICAL INSTRUMENTS

19.1 THE EYE

Optical instruments are used primarily to assist the eye in viewing an object. Let us first discuss in brief the construction of a human eye and the mechanism by which we see, the most common but most important experiment we do from the day we open our eyes.

Figure (19.1) shows schematically the basic components of an eye. The eye has a nearly spherical shape of diameter about an inch. The front portion is more sharply curved and is covered by a transparent protective membrane called the *cornea*. It is this portion which is visible from outside. Behind the cornea, we have a space filled with a liquid called the *aqueous humor* and behind that a *crystalline lens*.

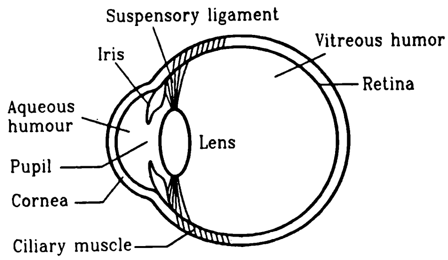


Figure 19.1

Between the aqueous humor and the lens, we have a muscular diaphragm called *iris*, which has a small hole in it called *pupil*. Iris is the coloured part that we see in an eye. The pupil appears black because any light falling on it goes into the eye and there is almost no chance of light coming back to the outside. The amount of light entering the eye, may be controlled by varying the aperture of the pupil with the help of the iris. In low-light condition, the iris expands the pupil to allow more light to go in. In good light conditions, it contracts the pupil.

The lens is hard in the middle and gradually becomes soft towards the outer edge. The curvature of the lens may be altered by the *ciliary muscles* to which it is attached. The light entering the eye forms an image on the *retina* which covers the inside of the rear part of the eyeball. The retina contains about 125 million receptors called *rods* and *cones* which receive the light signal and about one million optic-nerve fibres which transmit the information to the brain. The space between the lens and the retina is filled with another liquid called the *vitreous humor*.

The aqueous humor and the vitreous humor have almost same refractive index 1·336. The refractive index of the material of the lens is different in different portions but on the average it is about 1·396. When light enters the eye from air, most of the bending occurs at the cornea itself because there is a sharp change in the refractive index. Some additional bending is done by the lens which is surrounded by a fluid of somewhat lower refractive index. In normal conditions, the light should be focused on the retina.

The cornea-lens-fluid system is equivalent to a single converging lens whose focal length may be adjusted by the ciliary muscles. Now onwards, we shall use the word eye-lens to mean this equivalent lens.

When the eye is focused on a distant object, the ciliary muscles are relaxed so that the focal length of the eye-lens has its maximum value which is equal to its distance from the retina. The parallel rays coming into the eye are then focused on the retina and we see the object clearly.

When the eye is focused on a closer object, the ciliary muscles are strained and the focal length of the eye-lens decreases. The ciliary muscles adjust the focal length in such a way that the image is again formed on the retina and we see the object clearly. This process of adjusting focal length is called *accommodation*. However, the muscles cannot be strained beyond a limit and hence, if the object is brought too close to the eye, the focal length cannot be adjusted to form the image on the retina. Thus, there is a minimum distance for the clear vision of an object.

The nearest point for which the image can be focused on the retina, is called the *near point* of the eye. The distance of the near point from the eye is called the *least distance for clear vision.* This varies from person to person and with age. At a young age (say below 10 years), the muscles are strong and flexible and can bear more strain. The near point may be as close as 7-8 cm at this age. In old age, the muscles cannot sustain a large strain and the near point shifts to large values, say, 1 to 2 m or even more. We shall discuss about these defects of vision and use of glasses in a later section. The average value of the least distance for clear vision for a normal eye is generally taken to be 25 cm.

19.2 APPARENT SIZE

The size of an object as sensed by us is related to the size of the image formed on the retina. A larger image on the retina activates larger number of rods and cones attached to it and the object looks larger. As is clear from figure (19.2), if an object is taken away from the eye, the size of the image on the retina decreases and hence, the same object looks smaller. It is also clear from figure (19.2) that the size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the *visual angle* and optical instruments are used to increase this angle artificially in order to improve the clarity.

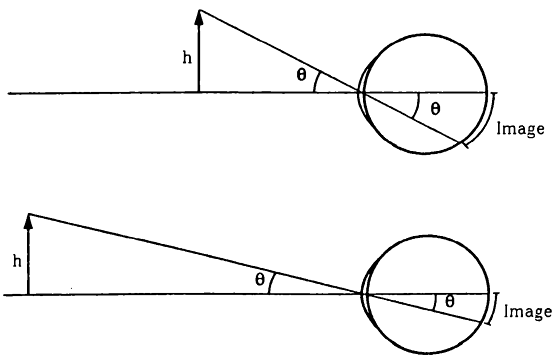


Figure 19.2

Example 19.1

Two boys, one 52 inches *tall and the other* 55 inches *tall, are standing at distances* 4·0 m *and* 5·0 m *respectively from an eye. Which boy will appear taller?*

Solution : The angle subtended by the first boy on the eye is

$$\alpha_1 = \frac{52 \text{ inch}}{4{\cdot}0 \text{ m}} = 13 \text{ inch/m}$$

and the angle subtended by the second boy is

$$\alpha_2 = \frac{55 \text{ inch}}{5{\cdot}0 \text{ m}} = 11 \text{ inch/m}.$$

As $\alpha_1 > \alpha_2$, the first boy will look taller to the eye.

19.3 SIMPLE MICROSCOPE

When we view an object with naked eyes, the object must be placed somewhere between infinity and the near point. The maximum angle is subtended on the eye when the object is placed at the near point. This angle is (figure 19.3a)

$$\theta_o = \frac{h}{D}, \qquad \dots \text{(i)}$$

where h is the size of the object and D is the least distance for clear vision.

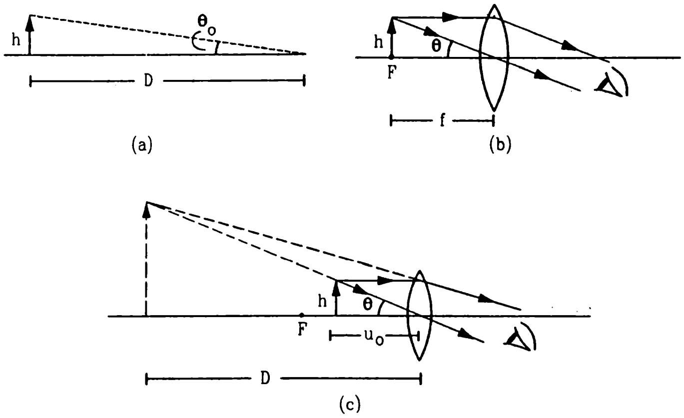


Figure 19.3

This angle can be further increased if a converging lens of short focal length is placed just in front of the eye. When a converging lens is used for this purpose, it is called a *simple microscope* or a *magnifier.*

Suppose, the lens has a focal length f which is less than D and let us move the object to the first focal point F. The eye receives rays which seem to come from infinity (figure 19.3b). The actual size of the image is infinite but the angle subtended on the lens (and hence on the eye) is

$$\theta = \frac{h}{f}. \qquad \dots \text{(ii)}$$

As $f < D$, equations (i) and (ii) show that $\theta > \theta_o$. Hence, the eye perceives a larger image than it could have had without the microscope. As the image is situated at infinity, the ciliary muscles are least strained to focus the final image on the retina. This situation is known as *normal adjustment.* We define *magnifying power* of a microscope as θ/θ_o where θ is the angle subtended by the image on the eye when the microscope is used and θ_o is the angle subtended on the naked eye when the object is placed at the near point. This is also known as the *angular magnification.* Thus, *the magnifying power is the factor by which the* *image on the retina can be enlarged by using the microscope.*

In normal adjustment, the magnifying power of a simple microscope is, by (i) and (ii),

$$m = \frac{\theta}{\theta_o} = \frac{h/f}{h/D}$$

or, $$m = \frac{D}{f}. \qquad \dots (19.1)$$

If $f < D$, the magnifying power is greater than 1.

The magnifying power can be further increased by moving the object still closer to the lens. Suppose, we move the object to a distance u_o from the lens such that the virtual erect image is formed at the near point (figure 19.3c). Though the eye is strained, it can still see the image clearly. The distance u_o can be calculated using the lens formula,

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}.$$

Here $v = -D$ and $u = -u_o$, so that

$$\frac{1}{-u_o} = -\frac{1}{D} - \frac{1}{f}$$

or, $$\frac{D}{u_o} = 1 + \frac{D}{f}. \qquad \dots \text{(iii)}$$

The angle subtended by the image on the lens (and hence on the eye) is

$$\theta' = \frac{h}{u_o}.$$

The angular magnification or magnifying power in this case is

$$m = \frac{\theta'}{\theta_o} = \frac{h/u_o}{h/D}$$

$$= \frac{D}{u_o}$$

$$= 1 + \frac{D}{f}. \qquad \dots (19.2)$$

Equations (19.1) and (19.2) show that the magnification can be made large by choosing the focal length f small. However, due to several other aberrations the image becomes too defective at large magnification with a simple microscope. Roughly speaking, a magnification upto 4 is trouble-free.

The magnifying power is written with a unit X. Thus, if a magnifier produces an angular magnification of 10, it is called a 10 X magnifier.

19.4 COMPOUND MICROSCOPE

Figure (19.4) shows a simplified version of a compound microscope and the ray diagram for image formation. It consists of two converging lenses arranged coaxially. The one facing the object is called the *objective* and the one close to the eye is called the *eyepiece* or *ocular*. The objective has a smaller aperture and smaller focal length than those of the eyepiece. The separation between the objective and the eyepiece can be varied by appropriate screws fixed on the panel of the microscope.

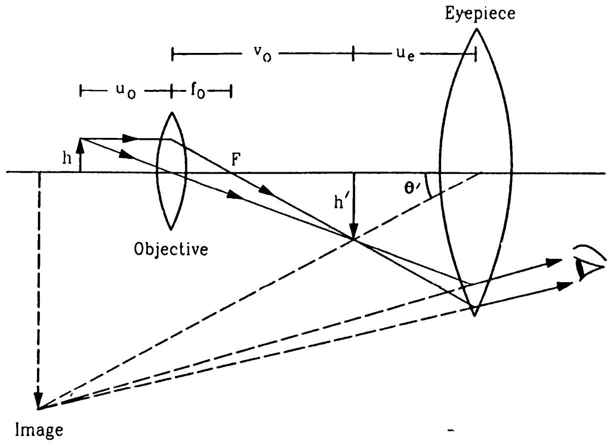


Figure 19.4

The object is placed at a distance u_o from the objective which is slightly greater than its focal length f_o. A real and inverted image is formed at a distance v_o on the other side of the objective. This image works as the object for the eyepiece. For normal adjustment, the position of the eyepiece is so adjusted that the image formed by the objective falls in the focal plane of the eyepiece. The final image is then formed at infinity. It is erect with respect to the first image and hence, inverted with respect to the object. The eye is least strained in this adjustment as it has to focus the parallel rays coming to it. The position of the eyepiece can also be adjusted in such a way that the final virtual image is formed at the near point. The angular magnification is increased in this case. The ray diagram in figure (19.4) refers to this case.

In effect, the eyepiece acts as a simple microscope used to view the first image. Thus, the magnification by a compound microscope is a two step process. In the first step, the objective produces a magnified image of the given object. In the second step, the eyepiece produces an angular magnification. The overall angular magnification is the product of the two.

Magnifying power

Refer to figure (19.4). If an object of height h is seen by the naked eye, the largest image on the retina is formed when it is placed at the near point. The angle formed by the object on the eye in this situation is

$$\theta_o = \frac{h}{D}. \qquad \dots \text{(i)}$$

When a compound microscope is used, the final image subtends an angle θ' on the eyepiece (and hence on the eye) given by

$$\theta' = \frac{h'}{u_e}, \qquad \dots \text{(ii)}$$

where h' is the height of the first image and u_e is its distance from the eyepiece.

The magnifying power of the compound microscope is, therefore,

$$m = \frac{\theta'}{\theta_o} = \frac{h'}{u_e} \times \frac{D}{h} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right). \qquad \dots \text{(iii)}$$

Also from figure (19.4),

$$\frac{h'}{h} = -\frac{v_o}{u_o} = \frac{v}{u}. \qquad \dots \text{(iv)}$$

Now, D/u_e is the magnifying power of the eyepiece treated as a simple microscope. Using (19.1) and (19.2), this is equal to D/f_e in normal adjustment (image at infinity) and $1 + D/f_e$ for the adjustment when the image is formed at the least distance for clear vision i.e., at D. Thus, the magnifying power of the compound microscope is, by (iii),

$$m = \frac{v}{u}\left(\frac{D}{f_e}\right) \qquad \dots \text{(19.3)}$$

for normal adjustment and

$$m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right) \qquad \dots \text{(19.4)}$$

for the adjustment when the final image is formed at the least distance for clear vision.

Using lens equation for the objective,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$1 - \frac{v}{u} = \frac{v}{f_o}$$

or, $$\frac{v}{u} = 1 - \frac{v}{f_o}.$$

In general, the focal length of the objective is very small so that $\frac{v}{f_o} >> 1$. Also, the first image is close to the eyepiece so that $v \approx l$ where l is the tube-length (separation between the objective and the eyepiece).

Thus, $$\frac{v}{u} = 1 - \frac{v}{f_o} \approx -\frac{v}{f_o} \approx -\frac{l}{f_o}.$$

If these conditions are satisfied, the magnifying power of the compound microscope is, by (19.3) and (19.4),

$$m = -\frac{l}{f_o}\frac{D}{f_e}$$

for normal adjustment and

$$m = -\frac{l}{f_o}\left(1 + \frac{D}{f_e}\right)$$

for adjustment for the final image at the least distance for clear vision.

In an actual compound microscope each of the objective and the eyepiece consists of combination of several lenses instead of a single lens as assumed in the simplified version.

Example 19.2

A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2·5 cm. An object has to be placed at a distance of 1·2 cm away from the objective for normal adjustment. (a) Find the angular magnification. (b) Find the length of the microscope tube.

Solution :

(a) If the first image is formed at a distance v from the objective, we have

$$\frac{1}{v} - \frac{1}{(-1{\cdot}2\text{ cm})} = \frac{1}{1\text{ cm}}$$

or, $v = 6$ cm.

The angular magnification in normal adjustment is

$$m = \frac{v}{u}\frac{D}{f_e} = -\frac{6\text{ cm}}{1{\cdot}2\text{ cm}} \cdot \frac{25\text{ cm}}{2{\cdot}5\text{ cm}} = -50.$$

(b) For normal adjustment, the first image must be in the focal plane of the eyepiece.

The length of the tube is, therefore,

$$L = v + f_e = 6\text{ cm} + 2{\cdot}5\text{ cm} = 8{\cdot}5\text{ cm}.$$

19.5 TELESCOPES

A microscope is used to view the objects placed close to it say within few centimeters. To look at distant objects such as a star, a planet or a distant tree etc. we use another instrument called a *telescope.* We shall describe three types of telescopes which are in use.

(A) Astronomical Telescope

Figure (19.5) shows the construction and working of a simplified version of an astronomical telescope.

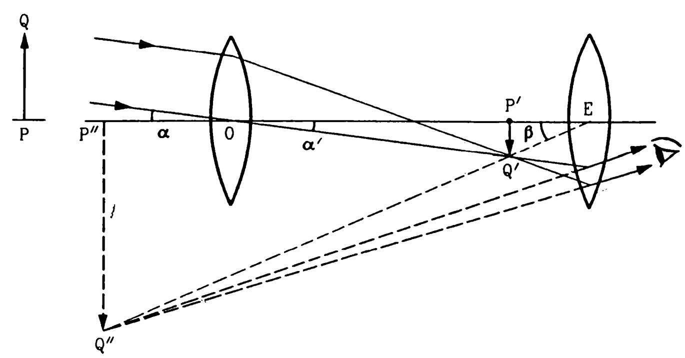


Figure 19.5

It consists of two converging lenses placed coaxially. The one facing the distant object is called the objective and has a large aperture and a large focal length. The other is called the eyepiece, as the eye is placed close to it. It has a smaller aperture and a smaller focal length. The lenses are fixed in tubes. The eyepiece tube can slide within the objective tube so that the separation between the objective and the eyepiece may be changed.

When the telescope is directed towards a distant object PQ, the objective forms a real image of the object in its focal plane. If the point P is on the principal axis, the image point P' is at the second focus of the objective. The rays coming from Q are focused at Q'. The eyepiece forms a magnified virtual image $P''Q''$ of $P'Q'$. This image is finally seen by the eye. In normal adjustment, the position is so adjusted that the final image is formed at infinity. In such a case, the first image $P'Q'$ is formed in the first focal plane of the eyepiece. The eye is least strained to focus this final image. The image can be brought closer by pushing the eyepiece closer to the first image. Maximum angular magnification is produced when the final image is formed at the near point.

Magnifying Power

Suppose, the objective and the eyepiece have focal lengths f_o and f_e respectively and the object is situated at a large distance u_o from the objective. The object PQ in figure (19.5) subtends an angle α on the objective. Since the object is far away, the angle it would subtend on the eye, if there were no telescope, is also essentially α.

As u_o is very large, the first image $P'Q'$ is formed in the focal plane of the objective.

From the figure,

$$|\,\alpha\,| = |\,\alpha'\,| \approx |\,\tan\alpha'\,| = \frac{P'Q'}{OP'} = \frac{P'Q'}{f_o}. \qquad \text{... (i)}$$

The final image $P''Q''$ subtends an angle β on the eyepiece (and hence on the eye). We have from the triangle $P'Q'E$,

$$|\,\beta\,| \approx |\,\tan\beta\,| = \frac{P'Q'}{EP'}$$

or,
$$\left|\frac{\beta}{\alpha}\right| = \frac{f_o}{EP'}. \qquad \text{... (ii)}$$

If the telescope is set for normal adjustment so that the final image is formed at infinity, the first image $P'Q'$ must be in the focal plane of the eyepiece. Then $EP' = f_e$. Thus, equation (ii) becomes

$$\left|\frac{\beta}{\alpha}\right| = \frac{f_o}{f_e}. \qquad \text{... (iii)}$$

The angular magnification or the magnifying power of the telescope is defined as

$$m = \frac{\text{angle subtended by the final image on the eye}}{\text{angle subtended by the object on the unaided eye}}.$$

The angles β and α are formed on the opposite sides of the axis. Hence, their signs are opposite and β/α is negative. Thus,

$$m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right|.$$

Using equation (iii),

$$m = -\frac{f_o}{f_e}. \qquad \text{... (19.5)}$$

If the telescope is adjusted so that the final image is formed at the near point of the eye, the angular magnification is further increased. Let us apply the lens equation to the eyepiece in this case.

Here $u = -EP'$

and $v = -EP'' = -D$.

The lens equation is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or,
$$\frac{1}{-D} - \frac{1}{-EP'} = \frac{1}{f_e}$$

or,
$$\frac{1}{EP'} = \frac{1}{f_e} + \frac{1}{D} = \frac{f_e + D}{f_e D}. \qquad \text{... (iv)}$$

By (ii),

$$\left|\frac{\beta}{\alpha}\right| = \frac{f_o(f_e + D)}{f_e D}.$$

The magnification is

$$m = \frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right| = -\frac{f_o(f_e + D)}{f_e D} = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right). \qquad \text{... (19.6)}$$

Length of the Telescope

From figure (19.5), we see that the length of the telescope is

$$L = OP' + P'E = f_o + P'E.$$

For normal adjustment, $P'E = f_e$ so that $L = f_o + f_e$. For adjustment for near point vision, we have, by (iv) above,

$$P'E = \frac{f_e D}{f_e + D}$$

so that the length is $L = f_o + \dfrac{f_e D}{f_e + D}$.

(B) Terrestrial Telescope

In an astronomical telescope, the final image is inverted with respect to the object. This creates some practical difficulty if the telescope is used to see earthly objects.

Imagine, how would you feel if you are viewing a cricket match from the spectator's gallery using an astronomical telescope. You would clearly see the turns and breaks of the ball, but the players would look like hanging from the field and not standing on the field.

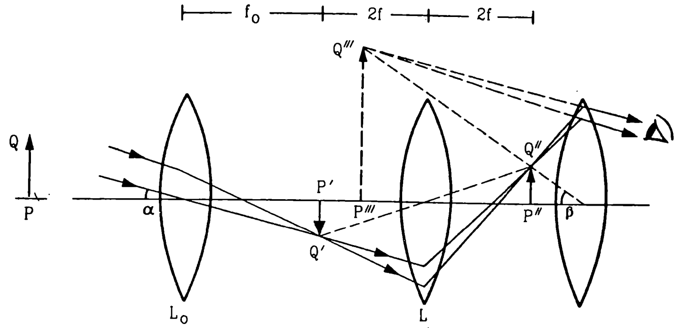


Figure 19.6

To remove this difficulty, a convex lens of focal length f is included between the objective and the eyepiece in such a way that the focal plane of the objective is a distance $2f$ away from this lens (figure 19.6). The objective forms the image $P'Q'$ of a distant object in its focal plane. The lens L forms an image $P''Q''$ which is inverted with respect to $P'Q'$. The eyepiece is adjusted in appropriate position to give the magnified view of $P''Q''$.

The role of the intermediate lens L is only to invert the image. The magnification produced by it is, therefore, -1. The magnifying power of a terrestrial telescope is, therefore, obtained from (19.5) for normal adjustment and from (19.6) for near point vision by multiplying by -1 on the right hand side. Thus, for normal adjustment,

$$m = \frac{f_o}{f_e} \qquad \ldots (19.7)$$

and for final image at near point,

$$m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right). \qquad \ldots (19.8)$$

To have an inverted image of same size, the object should be placed at a distance of $2f$ from a convex lens of focal length f. Thus, $P'P'' = 4f$ in figure (19.6) so that the length of a terrestrial telescope is $f_o + 4f + u_e$. For normal adjustment, u_e equals f_e so that the length is

$$L = f_o + 4f + f_e.$$

If the final image is formed at the near point,

$$u_e = \frac{f_e D}{f_e + D}$$

as derived for astronomical telescope. Thus,

$$L = f_o + 4f + \frac{f_e D}{f_e + D}.$$

(C) Galilean Telescope

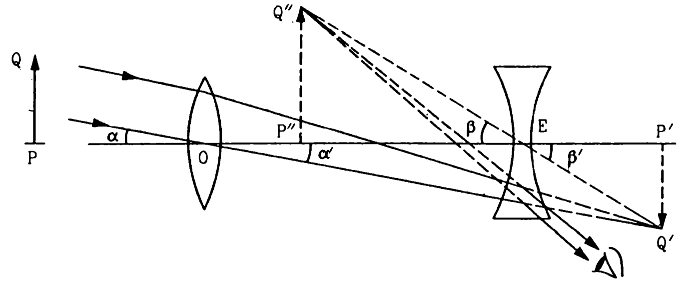


Figure 19.7

Figure (19.7) shows a simple model of Galilean telescope. A convergent lens is used as the objective and a divergent lens as the eyepiece. The objective L would form a real inverted image $P'Q'$ of a distinct object in its focal plane. The eyepiece intercepts the converging rays in between. $P'Q'$ then acts as a virtual object for the eyepiece. The position of the eyepiece is so adjusted that the final image is formed at the desired position. For normal adjustment, the final image is formed at infinity producing least strain on the eyes. If the final image is formed at the least distance of clear vision, the angular magnification is maximum.

Magnifying Power

Suppose the objective and the eyepiece have focal lengths f_o and f_e respectively and the object is situated at a large distance u_o from the objective. The object PQ subtends an angle α on the objective. Since the object is far away, the angle it would subtend on an unaided eye is also essentially α.

As u_o is very large, the first image $P'Q'$ is formed in the focal plane of the objective. Thus, from figure (19.8),

$$|\alpha| = |\alpha'| \approx |\tan\alpha'| = \frac{P'Q'}{OP'} = \frac{P'Q'}{f_o}. \qquad \ldots \text{(i)}$$

The final image $P''Q''$ subtends an angle β on the eyepiece. If the eye is placed close to the eyepiece, this is also the angle formed by the final image on the eye. From the figure,

$$|\beta| = |\beta'| \approx |\tan\beta'| = \frac{P'Q'}{EP'}$$

or, $$\left|\frac{\beta}{\alpha}\right| = \frac{f_o}{EP'}. \qquad \ldots \text{(ii)}$$

As β and α are formed on the same side of the axis, β and α have same sign. Thus,

$$\frac{\beta}{\alpha} = \left|\frac{\beta}{\alpha}\right|.$$

The angular magnification is, therefore,

$$m = \frac{\beta}{\alpha} = \left|\frac{\beta}{\alpha}\right| = \frac{f_o}{P'E}. \qquad \ldots \text{(iii)}$$

If the telescope is set for normal adjustment, the final image $P''Q''$ is formed at infinity. Then $P'E = -f_e$ and the angular magnification is

$$m = -\frac{f_o}{f_e}. \qquad \ldots (19.9)$$

Note that the focal length f_e is negative because the eyepiece is a diverging lens. Thus, m is positive as expected for an erect image. If the final image is formed at the near point, the magnification is increased.

The lens formula is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

For the eyepiece,

$$v = -P''E = -D, \; u = P'E$$

and $f = f_e$ (f_e is itself negative).

Thus,

$$\frac{1}{-D} - \frac{1}{P'E} = \frac{1}{f_e}$$

or, $$\frac{1}{P'E} = -\frac{1}{f_e} - \frac{1}{D}$$

$$= -\frac{1}{f_e}\left(1 + \frac{f_e}{D}\right). \qquad \ldots \text{(iv)}$$

By (iii), the angular magnification is

$$m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right).$$

Length of the Telescope

The length of a Galilean telescope is

$$L = OE = OP' - P'E$$
$$= f_o - P'E.$$

For normal adjustment, $P'E = -f_e$ and hence the length of the tube is

$$L = f_o + f_e = f_o - |f_e|.$$

For the adjustment for near point vision, by (iv),

$$P'E = \frac{-f_e D}{D + f_e} \text{ and } L = f_o + \frac{f_e D}{D + f_e}$$

$$= f_o - \frac{|f_e| D}{D - |f_e|}.$$

19.6 RESOLVING POWER OF A MICROSCOPE AND A TELESCOPE

The resolving power of a microscope is defined as the reciprocal of the distance between two objects which can be just resolved when seen through the microscope. It depends on the wavelength λ of the light, the refractive index μ of the medium between the object and the objective of the microscope, and the angle θ subtended by a radius of the objective on one of the objects. It is given by

$$R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}.$$

To increase the resolving power, the objective and the object are kept immersed in oil. This increases μ and hence R.

The resolving power of a telescope is defined as the reciprocal of the angular separation between two distant objects which are just resolved when viewed through a telescope. It is given by

$$R = \frac{1}{\Delta\theta} = \frac{a}{1{\cdot}22\,\lambda},$$

where a is the diameter of the objective of the telescope. That is why, the telescopes with larger objective aperture (1 m or more) are used in astronomical studies.

19.7 DEFECTS OF VISION

As described earlier, the ciliary muscles control the curvature of the lens in the eye and hence can alter the effective focal length of the system. When the muscles are fully relaxed, the focal length is maximum. When the muscles are strained, the curvature of the lens increases and the focal length decreases. For a clear vision, the image must be formed on the retina. The image-distance is, therefore, fixed for clear vision and it equals the distance of the retina from the eye-lens. It is about 2·5 cm for a grown-up person. If we apply the lens formula to eye, the magnitudes of the object-distance, the image-distance and the effective focal length satisfy

$$\frac{1}{v_o} + \frac{1}{u_o} = \frac{1}{f}$$

or, $$\frac{1}{u_o} = \frac{1}{f} - \frac{1}{v_o}. \qquad \ldots \text{(i)}$$

Here v_o is fixed, hence by changing f, the eye can be focused on objects placed at different values of u_o.

We see from (i) that as f increases, u_o increases and as f decreases, u_o decreases. The maximum distance one can see is given by

$$\frac{1}{u_{\max}} = \frac{1}{f_{\max}} - \frac{1}{v_o}, \qquad \ldots \text{(ii)}$$

where f_{max} is the maximum focal length possible for the eye-lens.

The focal length is maximum when the ciliary muscles are fully relaxed. In a normal eye, this focal length equals the distance v_o from the lens to the retina. Thus,

$v_o = f_{max}$, by (ii), $u_{max} = \infty$.

A person can theoretically have clear vision of objects situated at any large distance from the eye. For the closer objects, u is smaller and hence f should be smaller. The smallest distance at which a person can clearly see, is related to the minimum possible focal length f. The ciliary muscles are most strained in this position. By (ii), the closest distance for clear vision is given by

$$\frac{1}{u_{min}} = \frac{1}{f_{min}} - \frac{1}{v_o} . \quad \ldots \text{(iii)}$$

For an average grown-up person, u_{min} should be around 25 cm or less. This is a convenient distance at which one can hold an object in his/her hand and see. Thus, a normal eye can clearly see objects placed in the range starting from about 25 cm from the eye to a large distance, say, of the order of several kilometers. The nearest point where an eye can clearly see is called the *near point* and the farthest point upto which an eye can clearly see is called the *far point*. For a normal eye, the distance of the near point should be around 25 cm or less and the far point should be at infinity. We now describe some common decfects of vision.

(A) Nearsightedness

A person suffering from this defect cannot see distant objects clearly. This is because f_{max} is less than the distance from the lens to the retina and the parallel rays coming from the distant object focus short of the retina. The ciliary muscles are fully relaxed in this case and any strain in it can only further decrease the focal length which is of no help to see distant objects.

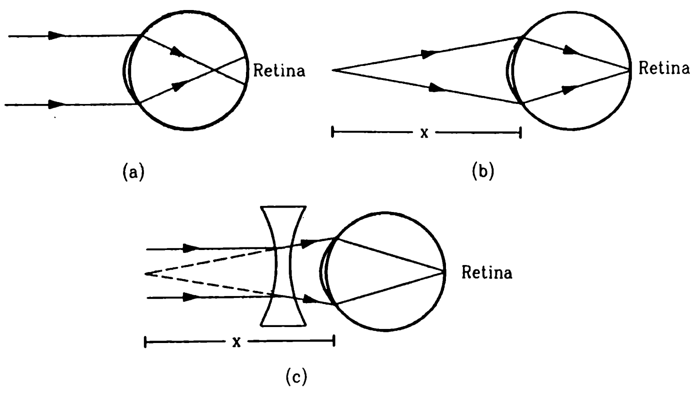


Figure 19.8

Nearsightedness is also called *Myopia*. This may result because the lens is too thick or the diameter of the eyeball is larger than usual. The remedy of myopia is quite simple. The rays should be made a bit divergent before entering the eye so that they may focus a little later. Thus, a divergent lens should be given to a myopic person to enable him/her to see distant objects clearly.

Power of the Lens Needed

Suppose, a person can see an object at a maximum distance x. Thus, with fully relaxed muscles, rays coming from the distance x converge on the retina. Figure (19.8) shows the situation. As is clear from the figure, if the eye is to see a distant object clearly, the diverging lens should form the virtual image of this distant object at a distance x. Thus, the required focal length of the diverging lens is $f = -x$ and the power is

$$P = \frac{1}{f} = \frac{1}{-x} .$$

Example 19.3

A nearsighted man can clearly see objects up to a distance of 1·5 m. *Calculate the power of the lens of the spectacles necessary for the remedy of this defect.*

Solution : The lens should form a virtual image of a distant object at 1·5 m from the lens. Thus, it should be a divergent lens and its focal length should be – 1·5 m. Hence,

$$f = -1{\cdot}5 \text{ m}$$

or, $$P = \frac{1}{f} = -\frac{1}{1{\cdot}5} \text{ m}^{-1} = -0{\cdot}67 \text{ D}.$$

(B) Farsightedness

A person suffering from farsightedness cannot clearly see objects close to the eye. The least distance for clear vision is appreciably larger than 25 cm and the person has to keep the object inconveniently away from the eye. Thus, reading a newspaper or viewing a small thing held in the hands is difficult for such a person.

Farsightedness is also known as *hyperopia*. Generally, it occurs when the eye-lens is too thin at the centre and/or the eyeball is shorter than normal. The ciliary muscles even in their most strained position are not able to reduce the focal length to appropriate value. The defect can also arise if the ciliary muscles become weak and are not able to strain enough to reduce the focal length to appropriate value. When farsightedness develops due to this reason, it is known as *presbyopia*.

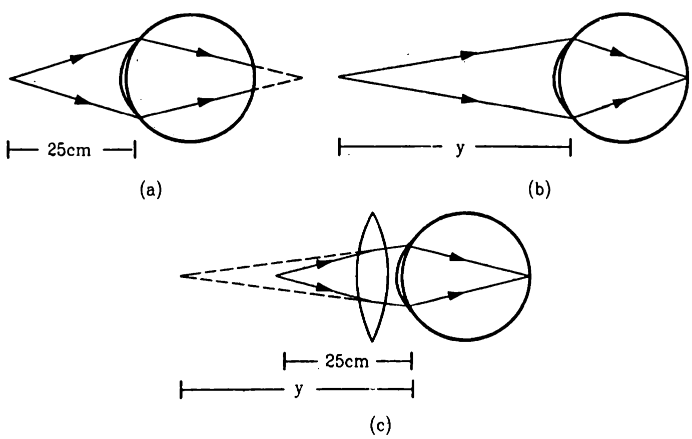


Figure 19.9

Figure (19.9) shows the situation and the remedy for farsightedness. The rays starting from the normal near point 25 cm would focus behind the retina. They should be made a bit less divergent before sending them to the eye so that they may focus on the retina. This can be achieved by putting a converging lens in front of the eye.

Suppose, the eye can clearly see an object at a minimum distance y. If the eye is to see clearly an object at 25 cm, the converging lens should form an image of this object at a distance y (figure 19.9c).

Here $u = -25$ cm and $v = -y$.

Using the lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

we get

$$\frac{1}{-y} - \frac{1}{-25 \text{ cm}} = \frac{1}{f}$$

or, $$P = \frac{1}{f} = \frac{1}{25 \text{ cm}} - \frac{1}{y}.$$

(C) Astigmatism

Another kind of defect arises in the eye when the eye-lens develops different curvatures along different planes. Such a person cannot see all the directions equally well. A particular direction in the plane perpendicular to the line of sight is most visible. The direction perpendicular to this is least visible. Here is a 'do it yourself' test for astigmatism. Figure (19.10) shows four lines passing through a point. The lines are assumed to be drawn with equal intensity (you can draw such lines on a paper with equal intensity and do the test). If you can see all the lines equally distinct and intense, you are not astigmatic. If a particular line say (2)-(2) appears to be most intense and the perpendicular line (4)-(4) appears least intense, you are most likely astigmatic. If it is so, rotate the book through a right angle so that (2)-(2) takes the place of (4)-(4) and vice versa. If you are really astigmatic, you will find that now (4)-(4) appears most intense and (2)-(2) appears least intense.

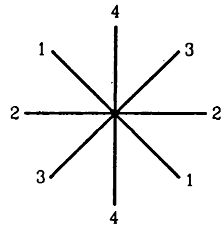


Figure 19.10

The remedy to astigmatism is also painless. Glasses with different curvatures in different planes are used to compensate for the deshaping of the eye-lens. Opticians call them cylindrical glasses.

A person may develop any of the above defects or a combination of more than one. Quite common in old age is the combination of nearsightedness and farsightedness. Such a person may need a converging glass for reading purpose and a diverging glass for seeing at a distance. Such persons either keep two sets of spectacles or a spectacle with upper portion divergent and lower portion convergent (bifocal).

Worked Out Examples

1. *An object is seen through a simple microscope of focal length* 12 cm. *Find the angular magnification produced if the image is formed at the near point of the eye which is* 25 cm *away from it.*

Solution : The angular magnification produced by a simple microscope when the image is formed at the near point of the eye is given by

$$m = 1 + \frac{D}{f}.$$

Here $f = 12$ cm, $D = 25$ cm. Hence,

$$m = 1 + \frac{25}{12} = 3{\cdot}08.$$

2. *A* 10 D *lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification for a normal eye (near point* = 25 cm*)?*

Solution : Maximum angular magnification is achieved when the final image is formed at the near point. Thus, $v = -25$ cm. The focal length is $f = \frac{1}{10}$ m $= 10$ cm.

We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$-\frac{1}{25\text{ cm}} - \frac{1}{u} = \frac{1}{10\text{ cm}}$$

or, $$\frac{1}{u} = -\frac{1}{25\text{ cm}} - \frac{1}{10\text{ cm}}$$

or, $$u = -\frac{50}{7}\text{ cm} = -7{\cdot}1\text{ cm}.$$

3. *A small object is placed at a distance of* 3·6 cm *from a magnifier of focal length* 4·0 cm. *(a) Find the position of the image. (b) Find the linear magnification. (c) Find the angular magnification.*

Solution :

(a) Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = \frac{1}{-3{\cdot}6\text{ cm}} + \frac{1}{4{\cdot}0\text{ cm}}$$

or, $v = -36$ cm.

(b) Linear magnification $= \frac{v}{u}$

$$= \frac{-36\text{ cm}}{-3{\cdot}6\text{ cm}} = 10.$$

(c) If the object is placed at a distance u_o from the lens, the angle subtended by the object on the lens is $\beta = \frac{h}{u_o}$ where h is the height of the object. The maximum angle subtended on the unaided eye is $\alpha = \frac{h}{D}$.

Thus, the angular magnification is

$$m = \frac{\beta}{\alpha} = \frac{D}{u_o} = \frac{25\text{ cm}}{3{\cdot}6\text{ cm}} = 7{\cdot}0.$$

4. *A compound microscope consists of an objective of focal length* 1·0 cm *and an eyepiece of focal length* 5·0 cm *separated by* 12·2 cm. *(a) At what distance from the objective should an object be placed to focus it properly so that the final image is formed at the least distance of clear vision* (25 cm)*? (b) Calculate the angular magnification in this case.*

Solution :

(a) For the eyepiece, $v_e = -25$ cm and $f_e = +5$ cm.

Using $$\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e},$$

$$\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e}$$

$$= -\frac{1}{25\text{ cm}} - \frac{1}{5\text{ cm}}$$

or, $$u_e = -\frac{25}{6}\text{ cm} = -4{\cdot}17\text{ cm} \approx -4{\cdot}2\text{ cm}.$$

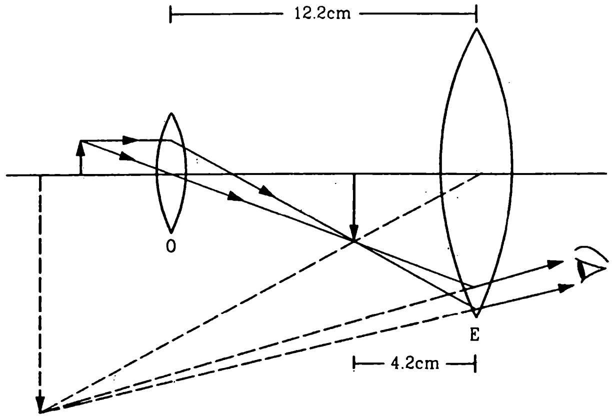


Figure 19-W1

As the objective is 12·2 cm away from the eyepiece, the image formed by the objective is 12·2 cm − 4·2 cm = 8·0 cm away from it. For the objective,

$$v = +8{\cdot}0\text{ cm}, \quad f_o = +1{\cdot}0\text{ cm}.$$

Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o},$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f_o}$$

$$= \frac{1}{8{\cdot}0\text{ cm}} - \frac{1}{1{\cdot}0\text{ cm}}$$

or, $$u = -\frac{8{\cdot}0}{7{\cdot}0}\text{ cm} = -1{\cdot}1\text{ cm}.$$

(b) The angular magnification is

$$m = \frac{v}{u}\left(1 + \frac{D}{f_e}\right)$$

$$= \frac{+8{\cdot}0\text{ cm}}{-1{\cdot}1\text{ cm}}\left(1 + \frac{25\text{ cm}}{5\text{ cm}}\right) \approx -44.$$

5. *The separation L between the objective* ($f = 0{\cdot}5$ cm) *and the eyepiece* ($f = 5$ cm) *of a compound microscope is* 7 cm. *Where should a small object be placed so that the eye is least strained to see the image? Find the angular magnification produced by the microscope.*

Solution : The eye is least strained if the final image is formed at infinity. In such a case, the image formed by the objective should fall at the focus of the eyepiece. As $f_e = 5$ cm and $L = 7$ cm, this first image should be formed at 7 cm − 5 cm = 2 cm from the objective. Thus, $v = +2$ cm. Also, $f_o = 0{\cdot}5$ cm. For the objective, using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_o},$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f_o}$$

$$= \frac{1}{2\text{ cm}} - \frac{1}{0{\cdot}5\text{ cm}}$$

or, $u = -\frac{2}{3}$ cm.

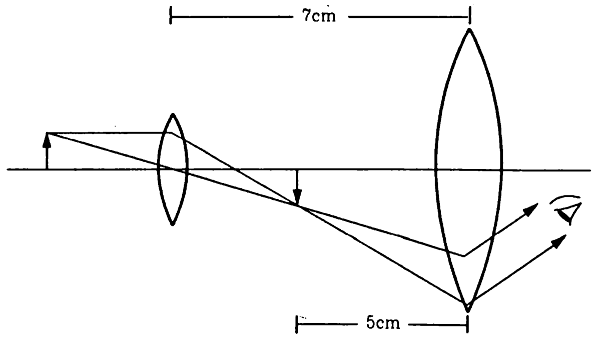


Figure 19-W2

The angular magnification in this case is

$$m = \frac{v}{u}\frac{D}{f_e}$$

$$= \frac{2\text{ cm}}{-(2/3)\text{ cm}}\frac{25\text{ cm}}{5\text{ cm}} = -15.$$

6. *An astronomical telescope has an objective of focal length 200 cm and an eyepiece of focal length 4·0 cm. The telescope is focused to see an object 10 km from the objective. The final image is formed at infinity. Find the length of the tube and the angular magnification produced by the telescope.*

Solution : As the object distance 10 km is much larger than the focal length 200 cm, the first image is formed almost at the focus of the objective. It is thus 200 cm from the objective. This image acts as the object for the eyepiece. To get the final image at infinity, this first image should be at the first focus of the eyepiece. The length of the tube is, therefore, 200 cm + 4 cm = 204 cm. The angular magnification in this case

$$m = -\frac{f_o}{f_e} = -\frac{200}{4} = -50.$$

7. *A Galilean telescope is constructed by an objective of focal length 50 cm and an eyepiece of focal length 5·0 cm. (a) Find the tube length and magnifying power when it is used to see an object at a large distance in normal adjustment. (b) If the telescope is to focus an object 2·0 m away from the objective, what should be the tube length and angular magnification, the image again forming at infinity?*

Solution :

$$f_o = 50\text{ cm}, \quad f_e = -5\text{ cm}.$$

(a) $L = f_o - |f_e| = (50 - 5)\text{ cm} = 45\text{ cm}$

and $m = -\frac{f_o}{f_e} = \frac{50}{5} = 10.$

(b) Using the equation $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ for the objective,

$$\frac{1}{v} = \frac{1}{f_o} + \frac{1}{u}$$

$$= \frac{1}{50\text{ cm}} + \frac{1}{-200\text{ cm}}$$

or, $v = 66·67$ cm.

The tube length $L = v - |f_e| = (66·67 - 5)$ cm

or, $L = 61·67$ cm.

To calculate the angular magnification, we assume that the object remains at large distance from the eye. In this case, the angular magnification

$$m = \frac{v}{f_e} = \frac{66·67}{5} = 13·33.$$

v is the distance of the first image from the objective which is substituted for f_o.

8. *The image of the moon is focused by a converging lens of focal length 50 cm on a plane screen. The image is seen by an unaided eye from a distance of 25 cm. Find the angular magnification achieved due to the converging lens.*

Solution :

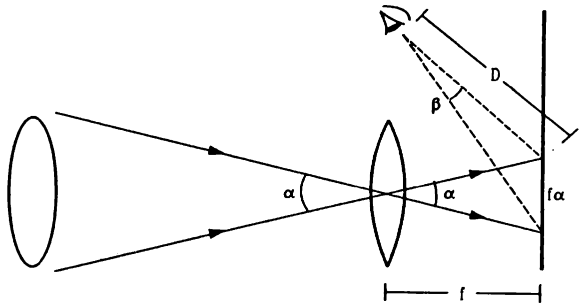


Figure 19-W3

Suppose the moon subtends an angle α on the lens. This will also be the angle subtended by the moon on the eye if the moon is directly viewed. The image is formed in the focal plane. The linear size of the image $\approx f\alpha = (50\text{ cm})\alpha$.

If this image is seen from a distance of 25 cm, the angle formed by the image on the eye

$$|\beta| \approx \frac{(50\text{ cm})|\alpha|}{25\text{ cm}} = 2|\alpha|.$$

The angular magnification is

$$\frac{\beta}{\alpha} = -\left|\frac{\beta}{\alpha}\right| = -2.$$

9. *The near and far points of a person are at 40 cm and 250 cm respectively. Find the power of the lens he/she should use while reading at 25 cm. With this lens on the eye, what maximum distance is clearly visible?*

Solution : If an object is placed at 25 cm from the correcting lens, it should produce the virtual image at 40 cm. Thus, $u = -25$ cm, $v = -40$ cm.

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$= -\frac{1}{40\text{ cm}} + \frac{1}{25\text{ cm}}$$

or, $$f = \frac{200}{3}\text{ cm} = +\frac{2}{3}\text{ m}$$

or, $$P = \frac{1}{f} = +1{\cdot}5\text{ D}.$$

The unaided eye can see a maximum distance of 250 cm. Suppose the maximum distance for clear vision is d when the lens is used. Then the object at a distance d is imaged by the lens at 250 cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

or, $$-\frac{1}{250\text{ cm}} - \frac{1}{d} = \frac{3}{200\text{ cm}}$$

or, $$d = -53\text{ cm}.$$

Thus, the person will be able to see upto a maximum distance of 53 cm.

10. *A young boy can adjust the power of his eye-lens between* 50 D *and* 60 D. *His far point is infinity. (a) What is the distance of his retina from the eye-lens? (b) What is his near point?*

Solution :

(a) When the eye is fully relaxed, its focal length is largest and the power of the eye-lens is minimum. This power is 50 D according to the given data. The focal length is $\frac{1}{50}$ m = 2 cm. As the far point is at infinity, the parallel rays coming from infinity are focused on the retina in the fully relaxed condition. Hence, the distance of the retina from the lens equals the focal length which is 2 cm.

(b) When the eye is focused at the near point, the power is maximum which is 60 D. The focal length in this case is $f = \frac{1}{60}\text{ m} = \frac{5}{3}$ cm. The image is formed on the retina and thus $v = 2$ cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or, $$\frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{2\text{ cm}} - \frac{3}{5\text{ cm}}$$

or, $$u = -10\text{ cm}.$$

The near point is at 10 cm.

□

QUESTIONS FOR SHORT ANSWER

1. Can virtual image be formed on the retina in a seeing process?
2. Can the image formed by a simple microscope be projected on a screen without using any additional lens or mirror?
3. The angular magnification of a system is less than one. Does it mean that the image formed is inverted?
4. A simple microscope using a single lens often shows coloured image of a white source. Why?
5. A magnifying glass is a converging lens placed close to the eye. A farsighted person uses spectacles having converging lenses. Compare the functions of a converging lens used as a magnifying glass and as spectacles.
6. A person is viewing an extended object. If a converging lens is placed in front of his eyes, will he feel that the size has increased?
7. The magnifying power of a converging lens used as a simple microscope is $\left(1 + \frac{D}{f}\right)$. A compound microscope is a combination of two such converging lenses. Why don't we have magnifying power $\left(1 + \frac{D}{f_o}\right)\left(1 + \frac{D}{f_e}\right)$? In other words, why can the objective not be treated as a simple microscope but the eyepiece can?
8. By mistake, an eye surgeon puts a concave lens in place of the lens in the eye after a cataract operation. Will the patient be able to see clearly any object placed at any distance?
9. The magnifying power of a simple microscope is given by $1 + \frac{D}{f}$, where D is the least distance for clear vision. For farsighted persons, D is greater than the usual. Does it mean that the magnifying power of a simple microscope is greater for a farsighted person as compared to a normal person? Does it mean that a farsighted person can see an insect more clearly under a microscope than a normal person?
10. Why are the magnification properties of microscopes and telescopes defined in terms of the ratio of angles and not in terms of the ratio of sizes of objects and images?
11. An object is placed at a distance of 30 cm from a converging lens of focal length 15 cm. A normal eye (near point 25 cm, far point infinity) is placed close to the lens on the other side. (a) Can the eye see the object clearly? (b) What should be the minimum separation between the lens and the eye so that the eye can clearly see the

object ? (c) Can a diverging lens, placed in contact with the converging lens, help in seeing the object clearly when the eye is close to the lens ?

12. A compound microscope forms an inverted image of an object. In which of the following cases it is likely to create difficulties ? (a) Looking at small germs. (b) Looking at circular spots. (c) Looking at a vertical tube containing some water.

OBJECTIVE I

1. The size of an object as perceived by an eye depends primarily on
   (a) actual size of the object
   (b) distance of the object from the eye
   (c) aperture of the pupil
   (d) size of the image formed on the retina.
2. The muscles of a normal eye are least strained when the eye is focused on an object
   (a) far away from the eye
   (b) very close to the eye
   (c) at about 25 cm from the eye
   (d) at about 1 m from the eye.
3. A normal eye is not able to see objects closer than 25 cm because
   (a) the focal length of the eye is 25 cm
   (b) the distance of the retina from the eye-lens is 25 cm
   (c) the eye is not able to decrease the distance between the eye-lens and the retina beyond a limit
   (d) the eye is not able to decrease the focal length beyond a limit.
4. When objects at different distances are seen by the eye, which of the following remain constant ?
   (a) The focal length of the eye-lens.
   (b) The object-distance from the eye-lens.
   (c) The radii of curvature of the eye-lens.
   (d) The image-distance from the eye-lens.
5. A person A can clearly see objects between 25 cm and 200 cm. Which of the following may represent the range of clear vision for a person B having muscles stronger than A, but all other parameters of eye identical to that of A ?
   (a) 25 cm to 200 cm (b) 18 cm to 200 cm
   (c) 25 cm to 300 cm (d) 18 cm to 300 cm.
6. The focal length of a normal eye-lens is about
   (a) 1 mm (b) 2 cm (c) 25 cm (d) 1 m.
7. The distance of the eye-lens from the retina is x. For a normal eye, the maximum focal length of the eye-lens
   (a) $= x$ (b) $< x$ (c) $> x$ (d) $= 2x$.
8. A man wearing glasses of focal length +1 m cannot clearly see beyond 1 m
   (a) if he is farsighted (b) if he is nearsighted
   (c) if his vision is normal (d) in each of these cases.
9. An object is placed at a distance u from a simple microscope of focal length f. The angular magnification obtained depends
   (a) on f but not on u (b) on u but not on f
   (c) on f as well as u (d) neither on f nor on u.
10. To increase the angular magnification of a simple microscope, one should increase
    (a) the focal length of the lens
    (b) the power of the lens
    (c) the aperture of the lens
    (d) the object size.
11. A man is looking at a small object placed at his near point. Without altering the position of his eye or the object, he puts a simple microscope of magnifying power 5 X before his eyes. The angular magnification achieved is
    (a) 5 (b) 2·5 (c) 1 (d) 0·2.

OBJECTIVE II

1. When we see an object, the image formed on the retina is
   (a) real (b) virtual (c) erect (d) inverted.
2. In which of the following the final image is erect ?
   (a) Simple microscope. (b) Compound microscope.
   (c) Astronomical telescope. (d) Galilean telescope.
3. The maximum focal length of the eye-lens of a person is greater than its distance from the retina. The eye is
   (a) always strained in looking at an object
   (b) strained for objects at large distances only
   (c) strained for objects at short distances only
   (d) unstrained for all distances.
4. Mark the correct options.
   (a) If the far point goes ahead, the power of the divergent lens should be reduced.
   (b) If the near point goes ahead, the power of the convergent lens should be reduced.
   (c) If the far point is 1 m away from the eye, divergent lens should be used.
   (d) If the near point is 1 m away from the eye, divergent lens should be used.
5. The focal length of the objective of a compound microscope is f_o and its distance from the eyepiece is L. The object is placed at a distance u from the objective. For proper working of the instrument,
   (a) $L < u$ (b) $L > u$ (c) $f_o < L < 2f_o$ (d) $L > 2f_o$.

EXERCISES

1. A person looks at different trees in an open space with the following details. Arrange the trees in decreasing order of their apparent sizes.

Tree	Height(m)	Distance from the eye(m)
A	2·0	50
B	2·5	80
C	1·8	70
D	2·8	100

2. An object is to be seen through a simple microscope of focal length 12 cm. Where should the object be placed so as to produce maximum angular magnification ? The least distance for clear vision is 25 cm.
3. A simple microscope has a magnifying power of 3·0 when the image is formed at the near point (25 cm) of a normal eye. (a) What is its focal length ? (b) What will be its magnifying power if the image is formed at infinity ?
4. A child has near point at 10 cm. What is the maximum angular magnification the child can have with a convex lens of focal length 10 cm ?
5. A simple microscope is rated 5 X for a normal relaxed eye. What will be its magnifying power for a relaxed farsighted eye whose near point is 40 cm ?
6. Find the maximum magnifying power of a compound microscope having a 25 diopter lens as the objective, a 5 diopter lens as the eyepiece and the separation 30 cm between the two lenses. The least distance for clear vision is 25 cm.
7. The separation between the objective and the eyepiece of a compound microscope can be adjusted between 9·8 cm to 11·8 cm. If the focal lengths of the objective and the eyepiece are 1·0 cm and 6 cm respectively, find the range of the magnifying power if the image is always needed at 24 cm from the eye.
8. An eye can distinguish between two points of an object if they are separated by more than 0·22 mm when the object is placed at 25 cm from the eye. The object is now seen by a compound microscope having a 20 D objective and 10 D eyepiece separated by a distance of 20 cm. The final image is formed at 25 cm from the eye. What is the minimum separation between two points of the object which can now be distinguished ?
9. A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0·5 cm and the tube length is 6·5 cm. Find the focal length of the eyepiece.
10. A compound microscope consists of an objective of focal length 1 cm and an eyepiece of focal length 5 cm. An object is placed at a distance of 0·5 cm from the objective. What should be the separation between the lenses so that the microscope projects an inverted real image of the object on a screen 30 cm behind the eyepiece ?
11. An optical instrument used for angular magnification has a 25 D objective and a 20 D eyepiece. The tube length is 25 cm when the eye is least strained. (a) Whether it is a microscope or a telescope ? (b) What is the angular magnification produced ?
12. An astronomical telescope is to be designed to have a magnifying power of 50 in normal adjustment. If the length of the tube is 102 cm, find the powers of the objective and the eyepiece.
13. The eyepiece of an astronomical telescope has a focal length of 10 cm. The telescope is focused for normal vision of distant objects when the tube length is 1·0 m. Find the focal length of the objective and the magnifying power of the telescope.
14. A Galilean telescope is 27 cm long when focused to form an image at infinity. If the objective has a focal length of 30 cm, what is the focal length of the eyepiece ?
15. A farsighted person cannot see objects placed closer to 50 cm. Find the power of the lens needed to see the objects at 20 cm.
16. A nearsighted person cannot clearly see beyond 200 cm. Find the power of the lens needed to see objects at large distances.
17. A person wears glasses of power – 2·5 D. Is the person farsighted or nearsighted ? What is the far point of the person without the glasses ?
18. A professor reads a greeting card received on his 50th birthday with + 2·5 D glasses keeping the card 25 cm away. Ten years later, he reads his farewell letter with the same glasses but he has to keep the letter 50 cm away. What power of lens should he now use ?
19. A normal eye has retina 2 cm behind the eye-lens. What is the power of the eye-lens when the eye is (a) fully relaxed, (b) most strained ?
20. The near point and the far point of a child are at 10 cm and 100 cm. If the retina is 2·0 cm behind the eye-lens, what is the range of the power of the eye-lens ?
21. A nearsighted person cannot see beyond 25 cm. Assuming that the separation of the glass from the eye is 1 cm, find the power of lens needed to see distant objects.
22. A person has near point at 100 cm. What power of lens is needed to read at 20 cm if he/she uses (a) contact lens, (b) spectacles having glasses 2·0 cm separated from the eyes?
23. A lady uses + 1·5 D glasses to have normal vision from 25 cm onwards. She uses a 20 D lens as a simple microscope to see an object. Find the maximum magnifying power if she uses the microscope (a) together with her glass (b) without the glass. Do the answers suggest that an object can be more clearly seen through a microscope without using the correcting glasses ?
24. A lady cannot see objects closer than 40 cm from the left eye and closer than 100 cm from the right eye. While on a mountaineering trip, she is lost from her team. She tries to make an astronomical telescope from her reading glasses to look for her teammates. (a) Which glass should she use as the eyepiece ? (b) What magnification can she get with relaxed eye ?

ANSWERS

OBJECTIVE I

1. (d) 2. (a) 3. (d) 4. (d) 5. (b) 6. (b)
7. (a) 8. (d) 9. (c) 10. (b) 11. (c)

OBJECTIVE II

1. (a), (d) 2. (a), (d) 3. (a)
4. (a), (c) 5. (b), (d)

EXERCISES

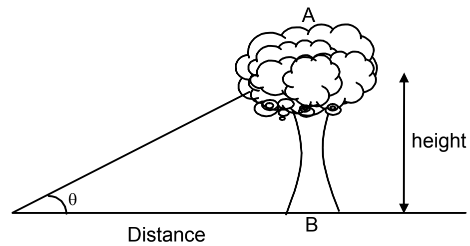
1. *A, B, D, C*
2. 8·1 cm from the lens
3. (a) 12·5 cm (b) 2·0
4. 2
5. 8 X
6. 8·4
7. 20 to 30
8. 0·04 mm
9. 2 cm
10. 5 cm
11. microscope, 20
12. 1 D, 50 D
13. 90 cm, 9
14. 3 cm
15. 3 D
16. – 0·5 D
17. nearsighted, 40 cm
18. + 4·5 D
19. 50 D, 54 D
20. + 60 D to + 51 D
21. – 4·2 D
22. + 4 D, + 4·53 D
23. 6, 9
24. right lens, 2

□

SOLUTIONS TO CONCEPTS
CHAPTER 19

1. The visual angles made by the tree with the eyes can be calculated be below.

$$\theta = \frac{\text{Height of the tree}}{\text{Distance from the eye}} = \frac{AB}{OB} \Rightarrow \theta_A = \frac{2}{50} = 0.04$$



similarly, $\theta_B = 2.5 / 80 = 0.03125$

$\theta_C = 1.8 / 70 = 0.02571$

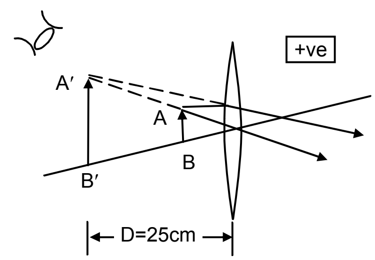
$\theta_D = 2.8 / 100 = 0.028$

Since, $\theta_A > \theta_B > \theta_D > \theta_C$, the arrangement in decreasing order is given by A, B, D and C.

2. For the given simple microscope,

f = 12 cm and D = 25 cm

For maximum angular magnification, the image should be produced at least distance of clear vision.



(Simple Microscope)

So, v = – D = –25 cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{-25} - \frac{1}{12} = -\frac{37}{300}$

$\Rightarrow$ u = –8.1 cm

So, the object should be placed 8.1 cm away from the lens.

3. The simple microscope has, m = 3, when image is formed at D = 25 cm

a) $m = 1 + \frac{D}{f} \Rightarrow 3 = 1 + \frac{25}{f}$

$\Rightarrow$ f = 25/2 = 12.5 cm

b) When the image is formed at infinity (normal adjustment)

Magnifying power = $\frac{D}{f} = \frac{25}{12.5}$ = 2.0

4. The child has D = 10 cm and f = 10 cm

The maximum angular magnification is obtained when the image is formed at near point.

$m = 1 + \frac{D}{f} = 1 + \frac{10}{10}$ = 1 + 1 = 2

5. The simple microscope has magnification of 5 for normal relaxed eye (D = 25 cm).

Because, the eye is relaxed the image is formed at infinity (normal adjustment)

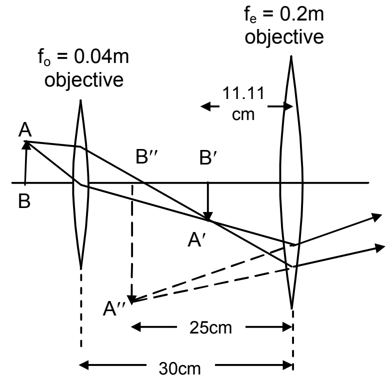
So, m = 5 = $\frac{D}{f} = \frac{25}{f} \Rightarrow$ f = 5 cm

For the relaxed farsighted eye, D = 40 cm

So, m = $\frac{D}{f} = \frac{40}{5}$ = 8

So, its magnifying power is 8X.

6. For the given compound microscope



$f_0 = \dfrac{1}{25 \text{ diopter}}$ = 0.04 m = 4 cm, $f_e = \dfrac{1}{5 \text{ diopter}}$ = 0.2 m = 20 cm

D = 25 cm, separation between objective and eyepiece = 30 cm

The magnifying power is maximum when the image is formed by the eye piece at least distance of clear vision i.e. D = 25 cm

for the eye piece, v_e = –25 cm, f_e = 20 cm

For lens formula, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow \dfrac{1}{-25} - \dfrac{1}{20}$ $\quad \Rightarrow u_e$ = 11.11 cm

So, for the objective lens, the image distance should be

v_0 = 30 – (11.11) = 18.89 cm

Now, for the objective lens,

v_0 = +18.89 cm (because real image is produced)

f_0 = 4 cm

So, $\dfrac{1}{u_o} = \dfrac{1}{v_o} - \dfrac{1}{f_o} \Rightarrow \dfrac{1}{18.89} - \dfrac{1}{4}$ = 0.053 – 0.25 = –0.197

$\Rightarrow u_o$ = –5.07 cm

So, the maximum magnificent power is given by

$m = -\dfrac{v_o}{u_o}\left[1+\dfrac{D}{f_e}\right] = -\dfrac{18.89}{-5.07}\left[1+\dfrac{25}{20}\right]$

= 3.7225 × 2.25 = 8.376

7. For the given compound microscope

f_o = 1 cm, f_e = 6 cm, D = 24 cm

For the eye piece, v_e = –24 cm, f_e = 6 cm

Now, $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e}$

$\Rightarrow \dfrac{1}{u_e} = \dfrac{1}{v_e} - \dfrac{1}{f_e} \Rightarrow -\left[\dfrac{1}{24} + \dfrac{1}{6}\right] = -\dfrac{5}{24}$

$\Rightarrow u_e$ = –4.8 cm

a) When the separation between objective and eye piece is 9.8 cm, the image distance for the objective lens must be (9.8) – (4.8) = 5.0 cm

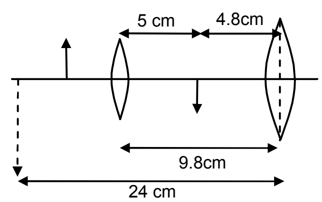


Fig-A

Now, $\dfrac{1}{v_0} - \dfrac{1}{u_0} = \dfrac{1}{f_0}$

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{5} - \dfrac{1}{1} = -\dfrac{4}{5}$

$\Rightarrow u_0 = -\dfrac{5}{4}$ = – 1.25 cm

So, the magnifying power is given by,

$m = \dfrac{v_0}{u_0}\left[1+\dfrac{D}{f}\right] = \dfrac{-5}{-1.25}\left[1+\dfrac{24}{6}\right]$ = 4 × 5 = 20

(b) When the separation is 11.8 cm,

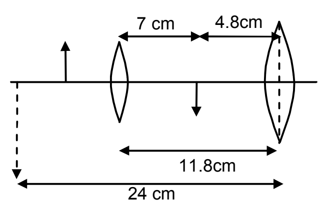


Fig-B

v_0 = 11.8 – 4.8 = 7.0 cm, $\qquad f_0$ = 1 cm

$\Rightarrow \dfrac{1}{u_0} = \dfrac{1}{v_0} - \dfrac{1}{f_0} = \dfrac{1}{7} - \dfrac{1}{1} = -\dfrac{6}{7}$

So, $m = -\frac{v_0}{u_o}\left[1+\frac{D}{f}\right] = \frac{-7}{-\left(\frac{7}{6}\right)}\left[1+\frac{24}{6}\right] = 6 \times 5 = 30$

So, the range of magnifying power will be 20 to 30.

8. For the given compound microscope.

$f_0 = \frac{1}{20D} = 0.05$ m = 5 cm, $\qquad f_e = \frac{1}{10D} = 0.1$ m = 10 cm.

D = 25 cm, separation between objective & eyepiece= 20 cm

For the minimum separation between two points which can be distinguished by eye using the microscope, the magnifying power should be maximum.

For the eyepiece, $v_0 = -25$ cm, $f_e = 10$ cm

So, $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10} = -\left[\frac{2+5}{50}\right] \Rightarrow u_e = -\frac{50}{7}$ cm

So, the image distance for the objective lens should be,

$V_0 = 20 - \frac{50}{7} = \frac{90}{7}$ cm

Now, for the objective lens,

$\frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{7}{90} - \frac{1}{5} = -\frac{11}{90}$

$\Rightarrow u_0 = -\frac{90}{11}$ cm

So, the maximum magnifying power is given by,

$m = \frac{-v_0}{u_0}\left[1+\frac{D}{f_e}\right]$

$= \frac{\left(\frac{90}{7}\right)}{\left(-\frac{90}{11}\right)}\left[1+\frac{25}{10}\right]$

$= \frac{11}{7} \times 3.5 = 5.5$

Thus, minimum separation eye can distinguish = $\frac{0.22}{5.5}$ mm = 0.04 mm

9. For the give compound microscope,

$f_0 = 0.5$cm, tube length = 6.5cm

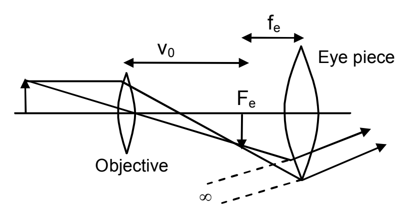
magnifying power = 100 (normal adjustment)

Since, the image is formed at infinity, the real image produced by the objective lens should lie on the focus of the eye piece.

So, $v_0 + f_e = 6.5$ cm $\qquad$ …(1)

Again, magnifying power= $\frac{v_0}{u_0} \times \frac{D}{f_e}$ [for normal adjustment]

$\Rightarrow m = -\left[1-\frac{v_0}{f_0}\right]\frac{D}{f_e} \qquad \left[\because \frac{v_0}{u_0} = 1 - \frac{v_0}{f_0}\right]$



$\Rightarrow 100 = -\left[1-\frac{v_0}{0.5}\right] \times \frac{25}{f_e}$ [Taking D = 25 cm]

$\Rightarrow 100\,f_e = -(1 - 2v_0) \times 25$

$\Rightarrow 2v_0 - 4f_e = 1 \qquad$ …(2)

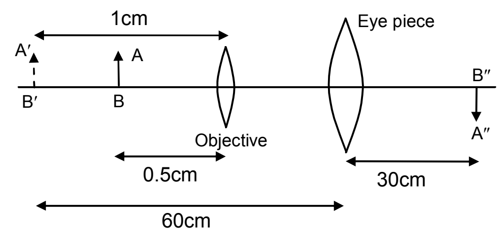
Solving equation (1) and (2) we can get,

V_0 = 4.5 cm and f_e = 2 cm

So, the focal length of the eye piece is 2cm.

10. Given that,

f_o = = 1 cm, f_e = 5 cm, u_0 = 0.5 cm, v_e = 30 cm



For the objective lens, u_0 = – 0.5 cm, f_0 = 1 cm.

From lens formula,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{v_0} = \frac{1}{u_0} + \frac{1}{f_0} = \frac{1}{-0.5} + \frac{1}{1} = -1$$

$\Rightarrow v_0 = -1$ cm

So, a virtual image is formed by the objective on the same side as that of the object at a distance of 1 cm from the objective lens. This image acts as a virtual object for the eyepiece.

For the eyepiece,

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{30} - \frac{1}{5} = \frac{-5}{30} = \frac{-1}{6} \quad \Rightarrow u_0 = -6 \text{ cm}$$

So, as shown in figure,

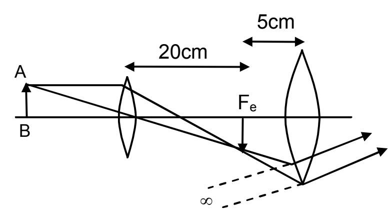
Separation between the lenses = $u_0 - v_0$ = 6 – 1 = 5 cm

11. The optical instrument has

$$f_0 = \frac{1}{25D} = 0.04 \text{ m} = 4 \text{ cm}$$

$$f_e = \frac{1}{20D} = 0.05 \text{ m} = 5 \text{ cm}$$

tube length = 25 cm (normal adjustment)



(a) The instrument must be a microscope as $f_0 < f_e$

(b) Since the final image is formed at infinity, the image produced by the objective should lie on the focal plane of the eye piece.

So, image distance for objective = v_0 = 25 – 5 = 20 cm

Now, using lens formula.

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \quad \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{20} - \frac{1}{4} = \frac{-4}{20} = \frac{-1}{5} \quad \Rightarrow u_0 = -5 \text{ cm}$$

So, angular magnification = $m = -\frac{v_0}{u_0} \times \frac{D}{f_e}$ [Taking D = 25 cm]

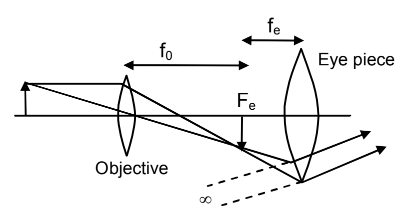
$$= -\frac{20}{-5} \times \frac{25}{5} = 20$$

12. For the astronomical telescope in normal adjustment.

Magnifying power = m = 50, length of the tube = L = 102 cm

Let f_0 and f_e be the focal length of objective and eye piece respectively.

$m = \frac{f_0}{f_e} = 50 \Rightarrow f_0 = 50 f_e$ …(1)



and, L = $f_0 + f_e$ = 102 cm …(2)

Putting the value of f_0 from equation (1) in (2), we get,

$f_0 + f_e = 102 \Rightarrow 51f_e = 102 \Rightarrow f_e$ = 2 cm = 0.02 m

So, f_0 = 100 cm = 1 m

$\therefore$ Power of the objective lens = $\frac{1}{f_0}$ = 1D

And Power of the eye piece lens = $\frac{1}{f_e} = \frac{1}{0.02}$ = 50D

13. For the given astronomical telescope in normal adjustment,
    F_e = 10 cm, L = 1 m = 100cm
    S0, $f_0 = L - f_e = 100 - 10 = 90$ cm
    and, magnifying power = $\dfrac{f_0}{f_e} = \dfrac{90}{10} = 9$
14. For the given Galilean telescope, (When the image is formed at infinity)
    f_0 = 30 cm, L = 27 cm
    Since $L = f_0 - |f_e|$
    [Since, concave eyepiece lens is used in Galilean Telescope]
    $\Rightarrow f_e = f_0 - L = 30 - 27 = 3$ cm
15. For the far sighted person,
    u = – 20 cm, v = – 50 cm
    from lens formula $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$
    $\dfrac{1}{f} = \dfrac{1}{-50} - \dfrac{1}{-20} = \dfrac{1}{20} - \dfrac{1}{50} = \dfrac{3}{100}$ $\Rightarrow f = \dfrac{100}{3}$ cm = $\dfrac{1}{3}$ m
    So, power of the lens = $\dfrac{1}{f}$ = 3 Diopter
16. For the near sighted person,
    u = ∞ and v = – 200 cm = – 2m
    So, $\dfrac{1}{f} = \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{-2} - \dfrac{1}{\infty} = -\dfrac{1}{2} = -0.5$
    So, power of the lens is –0.5D
17. The person wears glasses of power –2.5D
    So, the person must be near sighted.
    u = ∞, v = far point, f= $\dfrac{1}{-2.5}$ = – 0.4m = – 40 cm
    Now, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$
    $\Rightarrow \dfrac{1}{v} = \dfrac{1}{u} + \dfrac{1}{f} = 0 + \dfrac{1}{-40}$ $\Rightarrow$ v = – 40 cm
    So, the far point of the person is 40 cm
18. On the 50th birthday, he reads the card at a distance 25cm using a glass of +2.5D.
    Ten years later, his near point must have changed.
    So after ten years,
    u = – 50 cm, f = $\dfrac{1}{2.5D}$ = 0.4m = 40 cm v = near point
    Now, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$ $\Rightarrow \dfrac{1}{v} = \dfrac{1}{u} + \dfrac{1}{f} = \dfrac{1}{-50} + \dfrac{1}{40} = \dfrac{1}{200}$
    So, near point = v = 200cm
    To read the farewell letter at a distance of 25 cm,
    U = – 25 cm
    For lens formula,
    $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{f} = \dfrac{1}{200} - \dfrac{-}{-25} = \dfrac{1}{200} + \dfrac{1}{25} = \dfrac{9}{200}$ $\Rightarrow f = \dfrac{200}{9}$ cm = $\dfrac{2}{9}$ m
    $\Rightarrow$ Power of the lens = $\dfrac{1}{f} = \dfrac{9}{2}$ = 4.5D
    $\therefore$He has to use a lens of power +4.5D.

19. Since, the retina is 2 cm behind the eye-lens

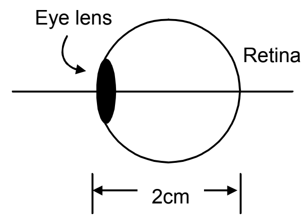
v = 2cm

(a) When the eye-lens is fully relaxed

$u = \infty$, v = 2cm = 0.02 m

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{\infty} = 50D$

So, in this condition power of the eye-lens is 50D



(b) When the eye-lens is most strained,

u = – 25 cm = – 0.25 m, v = +2 cm = +0.02 m

$\Rightarrow \frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.25} = 50 + 4 = 54D$

In this condition power of the eye lens is 54D.

20. The child has near point and far point 10 cm and 100 cm respectively.

Since, the retina is 2 cm behind the eye-lens, v = 2cm

For near point u = – 10 cm = – 0.1 m, v = 2 cm = 0.02 m

So, $\frac{1}{f_{near}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-0.1} = 50 + 10 = 60D$

For far point, u = – 100 cm = – 1 m, v = 2 cm = 0.02 m

So, $\frac{1}{f_{far}} = \frac{1}{v} - \frac{1}{u} = \frac{1}{0.02} - \frac{1}{-1} = 50 + 1 = 51D$

So, the rage of power of the eye-lens is +60D to +51D

21. For the near sighted person,

v = distance of image from glass

= distance of image from eye – separation between glass and eye

= 25 cm – 1cm = 24 cm = 0.24m

So, for the glass, $u = \infty$ and v = – 24 cm = –0.24m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-0.24} - \frac{1}{\infty} = -4.2\ D$

22. The person has near point 100 cm. It is needed to read at a distance of 20cm.

(a) When contact lens is used,

u = – 20 cm = – 0.2m, v = – 100 cm = –1 m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.2} = -1 + 5 = +4D$

(b) When spectacles are used,

u = – (20 – 2) = – 18 cm = – 0.18m, v = – 100 cm = –1 m

So, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{-1} - \frac{1}{-0.18} = -1 + 5.55 = +4.5D$

23. The lady uses +1.5D glasses to have normal vision at 25 cm.

So, with the glasses, her least distance of clear vision = D = 25 cm

Focal length of the glasses = $\frac{1}{1.5}$ m = $\frac{100}{1.5}$ cm

So, without the glasses her least distance of distinct vision should be more

If, u = – 25cm, $f = \frac{100}{1.5}$ cm

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \frac{1.5}{100} - \frac{1}{25} = \frac{1.5-4}{100} = \frac{-2.5}{100}$ $\Rightarrow$ v = – 40cm = near point without glasses.

Focal length of magnifying glass = $\frac{1}{20}$ m = 0.05m = 5 cm = f

(a) The maximum magnifying power with glasses

$$m = 1+\frac{D}{f} = 1+\frac{25}{5} = 6 \qquad [\because D = 25cm]$$

(b) Without the glasses, D = 40cm

$$\text{So, } m = 1+\frac{D}{f} = 1+\frac{40}{5} = 9$$

24. The lady can not see objects closer than 40 cm from the left eye and 100 cm from the right eye.

For the left glass lens,

$v = -40$ cm, $u = -25$ cm

$$\therefore \frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{-40}-\frac{1}{-25} = \frac{1}{25}-\frac{1}{40} = \frac{3}{200} \qquad \Rightarrow f = \frac{200}{3}\text{ cm}$$

For the right glass lens,

$v = -100$ cm, $u = -25$ cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} = \frac{1}{-100}-\frac{1}{-25} = \frac{1}{25}-\frac{1}{100} = \frac{3}{100} \qquad \Rightarrow f = \frac{100}{3}\text{ cm}$$

(a) For an astronomical telescope, the eye piece lens should have smaller focal length. So, she should use the right lens ($f = \frac{100}{3}$ cm) as the eye piece lens.

(b) With relaxed eye, (normal adjustment)

$$f_0 = \frac{200}{3}\text{ cm}, \quad f_e = \frac{100}{3}\text{ cm}$$

$$\text{magnification} = m = \frac{f_0}{f_e} = \frac{(200/3)}{(100/3)} = 2$$

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