CHAPTER 18

GEOMETRICAL OPTICS

We have learnt that light in many cases behaves as a wave of short wavelength. A ray of light gives the direction of propagation of light. In absence of an obstacle, the rays advance in straight lines without changing directions. When light meets a surface separating two transparent media, reflection and refraction occur and the light rays bend. Light rays also bend round the edge of an obstacle limiting the wavefront. But as the wavelength of light is usually much smaller than the size of the obstacles, this diffraction effect can usually be neglected. We then deal with *geometrical optics.*

18.1 REFLECTION AT SMOOTH SURFACES

A light ray is reflected by a smooth surface in accordance with the two laws of reflection :

(a) the angle of incidence is equal to the angle of reflection

(b) the incident ray, the reflected ray and the normal to the reflecting surface are coplanar.

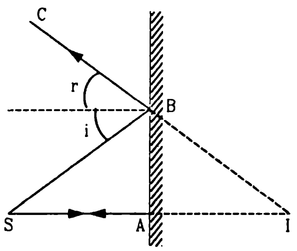


Figure 18.1

Figure (18.1) shows a point source S placed before a plane mirror. Consider a ray SA that falls normally on the mirror. This ray is reflected back along AS. Consider any other ray SB making an angle i with the normal. It is reflected along BC. Suppose AS and BC meet at a point I when produced behind the mirror. It is simple to show that the triangles SAB and ABI are congruent and $SA = AI$. Thus, all the reflected rays meet at I when produced behind the mirror. An eye receiving the reflected rays feels that the rays are diverging from the point I. The point I is called the *image* of the object S.

The basic laws of reflection are same for plane and curved surfaces. A normal can be drawn from any point of the curved surface by first drawing the tangent plane from that point and then drawing the line perpendicular to that plane. Angles of incidence and reflection are defined from this normal (figure 18.2). The angle of incidence is equal to the angle of reflection. The incident ray, the normal and the reflected ray are in the same plane.

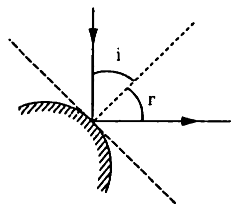


Figure 18.2

18.2 SPHERICAL MIRRORS

A spherical mirror is a part cut from a hollow sphere. Spherical mirrors are generally constructed from glass. One surface of the glass is silvered. The reflection takes place at the other surface. If reflection takes place at the convex surface, it is called a *convex mirror* and if reflection takes place at the concave surface, it is called a *concave mirror.*

Generally, a spherical mirror is constructed with a circular boundary. The centre of the sphere, of which the mirror is a part, is called the *centre of curvature* of the mirror. The radius of this sphere is called the *radius of curvature* of the mirror. The point on the mirror at the middle of the surface is called its *pole.* The line joining the pole and the centre of curvature is called the *principal axis.*

Focus

Suppose a light beam travelling in a direction parallel to the principal axis is incident on a concave mirror. If the aperture of the beam is small so that the light falls "near" the pole only, all the reflected rays cross the principal axis at nearly the same point (figure 18.3a). This point where the reflected rays converge is called the *focus* of the mirror.

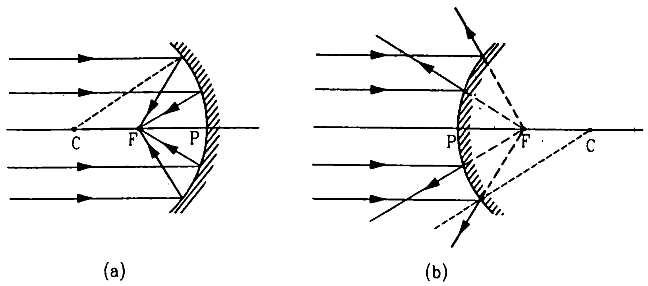


Figure 18.3

In case of a convex mirror, the reflected rays diverge after the reflection. Again, if the incident parallel beam has small aperture, the reflected rays appear to diverge from a point on the principal axis (figure 18.3b). This point is the focus of the convex mirror.

The plane through the focus and perpendicular to the principal axis is called the *focal plane*. The distance of the focus from the pole is called the *focal length* of the mirror.

Paraxial Rays

A ray close to the principal axis is called a paraxial ray. In this chapter, we shall consider only paraxial rays in image formation.

Image Tracing

When a point object is placed before a spherical mirror of small aperture, a point image is formed. To locate the position of the image, we draw two rays from the point object, make them incident on the mirror and trace the reflected rays. The line joining the point of incidence and the centre of curvature is the normal. A reflected ray is traced by applying the laws of reflection. If the reflected rays intersect, the point of intersection is the *real image*. If the rays diverge after reflection, a *virtual image* is formed at the point from where the rays seem to diverge. Figure (18.4) shows some examples.

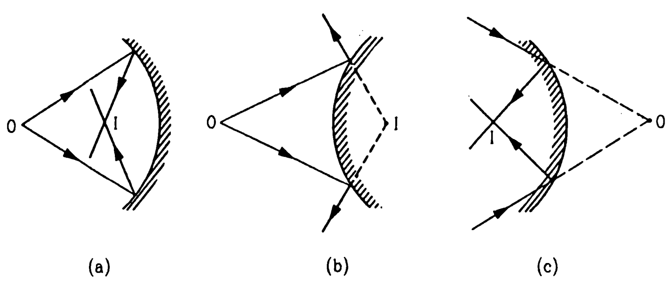


Figure 18.4

If the incident rays diverge from a point object, the object is called a *real object*. Sometimes the rays incident on the mirror do not diverge from a point, rather they converge towards the mirror (figure 18.4c). In this case, the point where these rays would meet if there were no mirror, is treated as the object. Such a point is called a *virtual object*.

Thus, the point of intersection of the incident rays is called the *object* and the point of intersection of the corresponding reflected rays is called its *image*.

Sign Convention

In image tracing, we come across the object distance, the image distance, the focal length and the radius of curvature. A system of signs for these quantities is necessary to derive relations connecting them which are consistent in all types of physical situations. We shall describe coordinate sign convention which is now widely used.

In this method, the pole is taken to be the origin and the principal axis as the X-axis. Usually, the positive of the axis is taken along the incident rays. The quantities u, v, R and f denote the x-coordinates of the object, the image, the centre of curvature and the focus respectively. Any of these quantities is positive if the corresponding point lies on the positive side of the origin and is negative if it is on the negative side. Figure (18.5) shows some typical situations. The signs of various quantities are tabulated below.

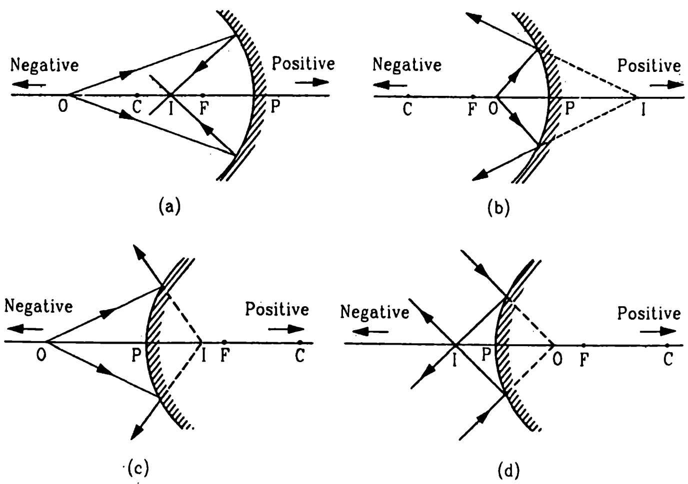


Figure 18.5

Figure	u	v	R	f
18.5a	–	–	–	–
18.5b	–	+	–	–
18.5c	–	+	+	+
18.5d	+	–	+	+

If the lengths perpendicular to the principal axis are needed, we should fix the positive direction of the Y-axis. Generally the upward is taken as positive of the Y-axis and downward as the negative of the Y-axis. Heights along the positive Y-axis are positive and heights along the negative Y-axis are negative. Quite often, we shall use the words "object-distance" and "image-distance" for u and v.

18.3 RELATION BETWEEN u, v AND R FOR SPHERICAL MIRRORS

Consider the situation shown in figure (18.6). A point object is placed at the point O of the principal axis of a concave mirror. A ray OA is incident on the mirror at A. It is reflected in the direction AI. Another ray OP travels along the principal axis. As PO is normal to the mirror at P, the ray is reflected back along PO. The reflected rays PO and AI intersect at I where the image is formed.

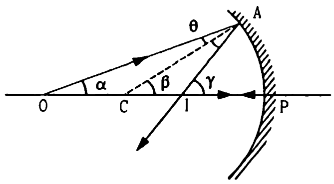


Figure 18.6

Let C be the centre of curvature. The line CA is the normal at A. Thus, by the laws of reflection, $\angle OAC = \angle CAI$. Let α, β, γ and θ denote the angles AOP, ACP, AIP and OAC respectively. As the exterior angle in a triangle equals the sum of the two opposite interior angles, we have,

from triangle OAC $\quad \beta = \alpha + \theta$ ... (i)

and from triangle OAI $\quad \gamma = \alpha + 2\theta$. ... (ii)

Eliminating θ from (i) and (ii),

$$2\beta = \alpha + \gamma. \quad \text{... (iii)}$$

If the point A is close to P, the angles α, β and γ are small and we can write

$$\alpha \approx \frac{AP}{PO}, \quad \beta = \frac{AP}{PC} \quad \text{and} \quad \gamma \approx \frac{AP}{PI}.$$

As C is the centre of curvature, the equation for β is exact whereas the remaining two are approximate. Putting in (iii),

$$2\frac{AP}{PC} = \frac{AP}{PO} + \frac{AP}{PI}$$

or,
$$\frac{1}{PO} + \frac{1}{PI} = \frac{2}{PC}. \quad \text{... (iv)}$$

The pole P is taken as the origin and the principal axis as the X-axis. The rays are incident from left to right. We take the direction from left to right as the positive X-direction. The points O, I and C are situated to the left of the origin P in the figure. The quantities u, v and R are, therefore, negative. As the distances PO, PI and PC are positive, $PO = -u$, $PI = -v$ and $PC = -R$. Putting in (iv),

$$\frac{1}{-u} + \frac{1}{-v} = \frac{2}{-R}$$

or,
$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}. \quad \text{... (18.1)}$$

Although equation (18.1) is derived for a special situation shown in figure (18.6), it is also valid in all other situations with a spherical mirror. This is because we have taken proper care of the signs of u, v and R appearing in figure (18.6),

Example 18.1

A convex mirror has its radius of curvature 20 cm. *Find the position of the image of an object placed at a distance of* 12 cm *from the mirror.*

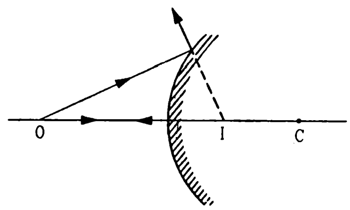


Figure 18.7

Solution : The situation is shown in figure (18.7). Here $u = -12$ cm and $R = +20$ cm. We have,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

or,
$$\frac{1}{v} = \frac{2}{R} - \frac{1}{u}$$

$$= \frac{2}{20\text{ cm}} - \frac{1}{-12\text{ cm}} = \frac{11}{60\text{ cm}}$$

or,
$$v = \frac{60}{11}\text{ cm}.$$

The positive sign of v shows that the image is formed on the right side of the mirror. It is a virtual image.

Relation between the Focal Length and the Radius of Curvature

If the object O in figure (18.6) is taken at a large distance, the rays coming from O and incident on the mirror become almost parallel. The image is then formed close to the focus. Thus, if $u = \infty$, $v = f$. Putting in (18.1),

$$\frac{1}{\infty} + \frac{1}{f} = \frac{2}{R}$$

or,
$$f = R/2. \quad \text{... (18.2)}$$

Equation (18.1) may also be written as

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}. \qquad \dots (18.3)$$

The focus is midway between the pole and the centre of curvature.

18.4 EXTENDED OBJECTS AND MAGNIFICATION

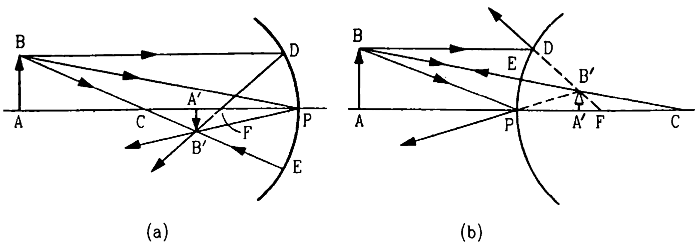


Figure 18.8

Suppose an object AB is placed on the principal axis of a spherical mirror with the length AB perpendicular to the principal axis (figure 18.8). Consider two rays BD and BE, the first parallel to the principal axis and the other directed towards the centre of curvature. The ray BD will go through the focus F after reflection. The ray BE will return along EB as it hits the mirror normally. The image of B is formed at the intersection of these two reflected rays. Thus, the image B' of the point B is traced. If we drop a perpendicular $B'A'$ on the principal axis, it can be shown that A' is the image of A and $A'B'$ is the image of AB. Figures (18.8a) and (18.8b) show the construction in two different situations.

Lateral Magnification

The ratio $\dfrac{\text{height of the image}}{\text{height of the objective}}$ is called *lateral* or *transverse magnification*. The height of the object (placed perpendicular to the principal axis) is taken to be positive. If the image is also on the same side of the principal axis, its height is also positive. The image is then erect. Figure (18.8b) shows an example. If the image is inverted, its height is taken as negative.

Consider the ray BP in figure (18.8) hitting the mirror at the pole P. The reflected ray passes through B' as B' is the image of B. The principal axis is the normal at P to the mirror. By the laws of reflection,

$$\angle BPA = \angle APB'.$$

Thus, the right-angled triangles ABP and $A'B'P$ are similar. Thus,

$$\frac{A'B'}{AB}=\frac{PA'}{PA}. \qquad \dots \text{(i)}$$

In figure (18.8a); $A'B' = -h_2$, $AB = h_1$, $PA' = -v$ and $PA = -u$. Equation (i) gives

$$\frac{-h_2}{h_1}=\frac{-v}{-u}$$

or,
$$m=\frac{h_2}{h_1}=-\frac{v}{u}. \qquad \dots (18.4)$$

Since proper signs are used, this same relation is also valid in all other situations. For example, in figure 18.8(b), $A'B' = +h_2, AB = +h_1, PA' = +v$ and $PA = -u$. Equation (i) gives

$$\frac{h_2}{h_1}=\frac{v}{-u} \quad \text{or,} \quad m=-\frac{v}{u}.$$

Example 18.2

An object of length 2·5 cm is placed at a distance of 1·5 f from a concave mirror where f is the magnitude of the focal length of the mirror. The length of the object is perpendicular to the principal axis. Find the length of the image. Is the image erect or inverted?

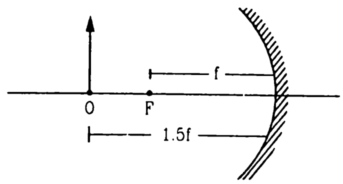


Figure 18.9

Solution : The given situation is shown in figure (18.9). The focal length $F = -f$, and $u = -1{\cdot}5\,f$. We have,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{F} \quad \text{or,} \quad \frac{1}{-1{\cdot}5\,f}+\frac{1}{v}=\frac{1}{-f}$$

or,
$$\frac{1}{v}=\frac{1}{1{\cdot}5\,f}-\frac{1}{f}=\frac{-1}{3\,f}$$

or,
$$v=-3\,f.$$

Now
$$m=-\frac{v}{u}=\frac{3\,f}{-1{\cdot}5\,f}=-2$$

or,
$$\frac{h_2}{h_1}=-2 \quad \text{or,} \quad h_2=-2\,h_1=-5{\cdot}0\text{ cm}.$$

The image is 5·0 cm long. The minus sign shows that it is inverted.

18.5 REFRACTION AT PLANE SURFACES

When a light ray is incident on a surface separating two transparent media, the ray bends at the time of changing the medium. The angle of incidence i and the angle of refraction r follow *Snell's law*

$$\frac{\sin i}{\sin r}=\frac{v_1}{v_2}=\frac{\mu_2}{\mu_1},$$

where v_1 and v_2 are the speeds of light in media 1 and 2 respectively and μ_1 and μ_2 are the refractive indices of media 1 and 2 respectively. For vacuum, the refractive index μ equals 1. For air also, it is very close to 1.

Image due to Refraction at a Plane Surface

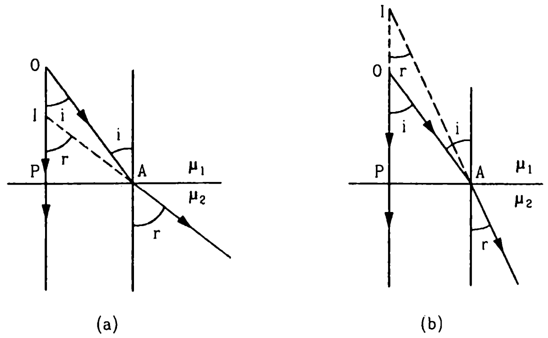


Figure 18.10

Consider the situation shown in figure (18.10). A point object O is placed in a medium of refractive index μ_1. Another medium of refractive index μ_2 has its boundary at PA. Consider two rays OP and OA originating from O. Let OP fall perpendicularly on PA and OA fall at PA at a small angle i with the normal. OP enters the second medium undeviated and OA enters making an angle r with the normal. When produced backward, these rays meet at I which is the virtual image of O. If i and r are small,

$$\sin i \approx \tan i = \frac{PA}{PO}$$

and $$\sin r \approx \tan r = \frac{PA}{PI}.$$

Thus, $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r}$$

$$= \left(\frac{PA}{PO}\right) \cdot \left(\frac{PI}{PA}\right) = \frac{PI}{PO}. \quad \ldots \text{(i)}$$

Suppose medium 2 is air and an observer looks at the image from this medium (figure 18.10a). The real depth of the object inside medium 1 is PO whereas the depth as it appears to the observer is PI. Writing $\mu_2 = 1$ and $\mu_1 = \mu$, equation (i) gives

$$\frac{1}{\mu} = \frac{\text{apparent depth}}{\text{real depth}}$$

or, $$\mu = \frac{\text{real depth}}{\text{apparent depth}}. \quad \ldots (18.5)$$

The image shifts closer to eye by an amount $OI = PO - PI$

$$= \left(\frac{PO - PI}{PO}\right) PO = \left(1 - \frac{PI}{PO}\right) PO$$

or, $$\Delta t = \left(1 - \frac{1}{\mu}\right) t, \quad \ldots (18.6)$$

where t is the thickness of the medium over the object and Δt is the apparent shift in its position towards the observer. Note that Δt is positive in figure (18.10a) and negative in figure (18.10b).

Example 18.3

A printed page is kept pressed by a glass cube (μ = 1·5) of edge 6·0 cm. By what amount will the printed letters appear to be shifted when viewed from the top?

Solution : The thickness of the cube = t = 6·0 cm. The shift in the position of the printed letters is

$$\Delta t = \left(1 - \frac{1}{\mu}\right) t$$

$$= \left(1 - \frac{1}{1{\cdot}5}\right) \times 6{\cdot}0 \text{ cm} = 2{\cdot}0 \text{ cm}.$$

18.6 CRITICAL ANGLE

When a ray passes from an optically denser medium (larger μ) to an optically rarer medium (smaller μ), the angle of refraction r is greater than the corresponding angle of incidence i. We have,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} < 1.$$

If we gradually increase i, the corresponding r will also increase and at a certain stage r will become 90°. Let the angle of incidence for this case be θ_c. If i is increased further, there is no r which can satisfy Snell's law. Thus, the ray will not be refracted. Entire light is then reflected back into the first medium. This is called *total internal reflection*. The angle θ_c is called the *critical angle* for the given pair of media. Generally, critical angle of a medium is quoted for light going from the medium to the air. In this case, $\mu_2 = 1$. Writing $\mu_1 = \mu$, Snell's law gives

$$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{\mu}$$

or, $$\sin\theta_c = (1/\mu)$$

or, $$\theta_c = \sin^{-1}(1/\mu). \quad \ldots (18.7)$$

Example 18.4

The critical angle for water is 48·2°. Find its refractive index.

Solution : $\mu = \dfrac{1}{\sin\theta_c} = \dfrac{1}{\sin 48{\cdot}2°} = 1{\cdot}34.$

18.7 OPTICAL FIBRE

Total internal reflection is the basic principle of a very useful branch of physics known as *fibre optics*. An *optical fibre* is a very thin fibre made of glass or plastic having a radius of the order of a micrometer (10^{-6} m). A bundle of such thin fibres forms a *light pipe*.

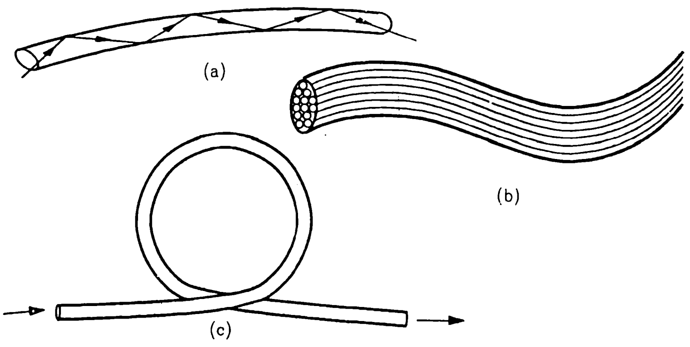


Figure 18.11

Figure (18.11a) shows the principle of light transmission by an optical fibre. Figure (18.11b) sketches a light pipe. Because of the small radius of the fibre, light going into it makes a nearly glancing incidence on the wall. The angle of incidence is greater than the critical angle and hence total internal reflection takes place. The light is thus transmitted along the fibre. Even if a light pipe is put in a complicated shape (figure 18.11c), the light is transmitted without any appreciable loss.

Light pipes using optical fibres may be used to see places which are difficult to reach such as inside of a human body. For example, a patient's stomach can be viewed by inserting one end of a light pipe into the stomach through the mouth. Light is sent down through one set of fibres in the pipe. This illuminates the inside of the stomach. The light from the inside travels back through another set of fibres in the pipe and the viewer gets the image at the outer end.

The other important application of fibre optics is to transmit communication signals through light pipes. For example, about 2000 telephone signals, appropriately mixed with light waves, may be simultaneously transmitted through a typical optical fibre. The clarity of the signals transmitted in this way is much better than other conventional methods.

The fibres in a light pipe must be optically insulated from each other. This is usually done by coating each fibre with a material having refractive index less than that of the fibre.

18.8 PRISM

Figure (18.12) shows the cross-section of a prism. AB and AC represent the refracting surfaces. The angle BAC is the angle of the prism. Consider the prism to be placed in air. A ray PQ, incident on a refracting surface AB, gets refracted along QR. The angle of incidence and the angle of refraction are i and r respectively. The ray QR is incident on the surface AC. Here the light goes from an optically denser medium to an optically rarer medium. If the angle of incidence r' is not greater than the critical angle, the ray is refracted in air along RS. The angle of refraction is i'. The angle i' is also called the angle of emergence. If the prism were not present, the incident ray would have passed undeviated along $PQTU$. Because of the prism, the final ray goes along RS. The angle $UTS = \delta$ is called the *angle of deviation*. From triangle TQR,

$$\angle UTS = \angle TQR + \angle TRQ$$

or,
$$\delta = (\angle TQV - \angle RQV) + (\angle TRV - \angle QRV)$$
$$= (i - r) + (i' - r')$$
$$= (i + i') - (r + r'). \quad \text{... (i)}$$

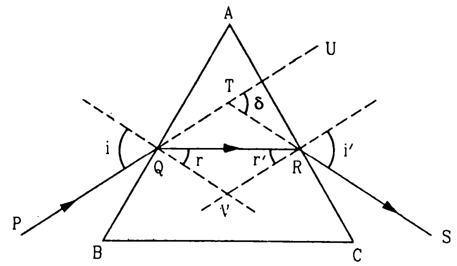


Figure 18.12

Now, the four angles of the quadrangle $AQVR$ add to 360°. The angles AQV and ARV are 90° each. Thus,

$$A + \angle QVR = 180°.$$

Also, from the triangle QRV,

$$r + r' + \angle QVR = 180°.$$

So,
$$r + r' = A. \quad \text{... (18.8)}$$

Substituting in (i),

$$\delta = i + i' - A. \quad \text{... (18.9)}$$

Angle of Minimum Deviation

The angle i' is determined by the angle of incidence i. Thus, the angle of deviation δ is also determined by i. For a particular value of angle of incidence, the angle of deviation is minimum. In this situation, the ray passes symmetrically through the prism, so that $i = i'$.

The above statement can be justified by assuming that there is a unique angle of minimum deviation. Suppose, when deviation is minimum, the angle of incidence is greater than the angle of emergence. Suppose, figure (18.13) shows the situation for minimum deviation. According to our assumption,

$$i > i'.$$

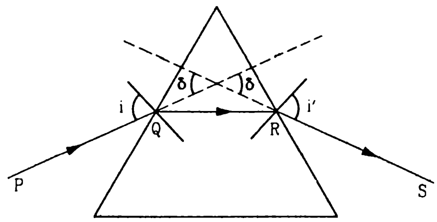


Figure 18.13

Now, if we send a ray along SR, it will retrace the path and will emerge along QP. Thus, the angle of deviation is same as before and hence, is minimum. According to our assumption, the angle of incidence is greater than the angle of emergence. Hence,

$$i' > i.$$

Thus, we get a contradiction. Similarly, if we assume that the angle of incidence is smaller than the angle of emergence for minimum deviation, we again get a contradiction. Hence, for minimum deviation, $i = i'$.

Relation between Refractive Index and the Angle of Minimum Deviation

Let the angle of minimum deviation be δ_m. For minimum deviation, $i = i'$ and $r = r'$. We have,

$$\delta_m = i + i' - A$$

$$= 2\,i - A$$

or, $$i = \frac{A + \delta_m}{2}. \qquad \dots \text{(i)}$$

Also, $$r + r' = A$$

or, $$r = A/2. \qquad \dots \text{(ii)}$$

The refractive index is

$$\mu = \frac{\sin i}{\sin r}.$$

Using (i) and (ii),

$$\mu = \frac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}}. \qquad \dots (18.10)$$

If the angle of prism A is small, δ_m is also small. Equation (18.10) then becomes

$$\mu \approx \frac{\dfrac{A + \delta_m}{2}}{\dfrac{A}{2}}$$

or, $$\delta_m = (\mu - 1)\,A. \qquad \dots (18.11)$$

Example 18.5

The angle of minimum deviation from a prism is 37°. If the angle of prism is 53°, find the refractive index of the material of the prism.

Solution : $$\mu = \frac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}} = \frac{\sin \dfrac{53^\circ + 37^\circ}{2}}{\sin \dfrac{53^\circ}{2}} = \frac{\sin 45^\circ}{\sin 26{\cdot}5^\circ}$$

$$= 1{\cdot}58.$$

18.9 REFRACTION AT SPHERICAL SURFACES

When two transparent media are separated by a spherical surface, light incident on the surface gets refracted into the medium on other side. Suppose two transparent media having refractive indices μ_1 and μ_2 are separated by a spherical surface AB (figure 18.14). Let C be the centre of curvature of AB. Consider a point object O in the medium 1. Suppose the line OC cuts the spherical surface at P.

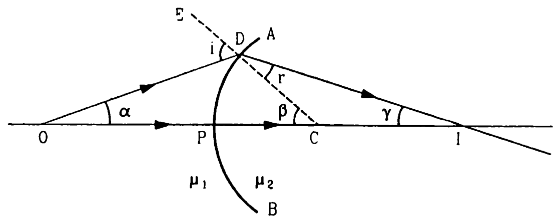


Figure 18.14

Several cases may arise. The surface may be concave towards the higher μ side or it may be convex. The object may be on the convex side or on the concave side. In figure (18.14), it is assumed that $\mu_2 > \mu_1$ and the object O is on the convex side of the surface.

Image Tracing

Consider two rays OD and OP originating from O. The ray OP falls normally on AB. It goes into the medium 2 undeviated. Suppose the ray OD makes a small angle α with the line OPC and falls on the surface AB at a point D. The normal to AB at the point D is DC. The angle $ODE = i$ is the angle of incidence. The ray is refracted along DI. The two refracted rays meet at the point I where the image is formed. The angle $CDI = r$ is the angle of refraction. If the refracted rays actually meet, a real image is formed. If the refracted rays diverge after refraction, a virtual image is formed at the point from where these rays seem to diverge.

Sign Convention

The sign convention for refraction at spherical surface is quite similar to that used for spherical mirrors.

The line joining the object and the centre is taken as the X-axis. The positive direction of the axis is generally chosen along the direction of the incident rays. The point of intersection of the spherical surface with the axis is taken as the origin. The quantities u, v and R denote the x-coordinates of the object, the image and the centre of curvature respectively. Any of these quantities is positive if the corresponding point lies on the positive side of the origin and is negative if it is on the negative side. Similarly for the lengths perpendicular to the X-axis.

Relation between u, v and R

Refer to figure (18.14). Let $\angle DOP = \alpha$, $\angle DCP = \beta$ and $\angle DIC = \gamma$. For paraxial rays, D is close to P and α, i, r, β and γ are all small. From the triangle ODC,

$$\alpha + \beta = i \qquad \text{... (i)}$$

and from DCI,

$$r + \gamma = \beta. \qquad \text{... (ii)}$$

Also, from Snell's law,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}.$$

We can write $\sin i \approx i$ and $\sin r \approx r$ so that the above equation becomes

$$\mu_1 i = \mu_2 r. \qquad \text{... (iii)}$$

Putting i and r from (i) and (ii) into (iii),

$$\mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$$

or, $$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta. \qquad \text{... (iv)}$$

As α, β and γ are small, from figure (18.14),

$$\alpha \approx \frac{DP}{PO}, \; \beta = \frac{DP}{PC} \text{ and } \gamma \approx \frac{DP}{PI}.$$

The expression for β is exact as C is the centre of curvature. Putting in (iv),

$$\mu_1\left(\frac{DP}{PO}\right) + \mu_2\left(\frac{DP}{PI}\right) = (\mu_2 - \mu_1)\frac{DP}{PC}$$

or, $$\frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}. \qquad \text{... (v)}$$

At this stage, proper sign convention must be used so that the formula derived is also valid for situations other than that shown in the figure.

In figure (18.14), the point P is the origin and OPC is the axis. As the incident ray comes from left to right, we choose this direction as the positive direction of the axis. We see that u is negative whereas v and R are positive. As the distances PO, PI and PC are positive, $PO = -u$, $PI = +v$ and $PC = +R$. From (v),

$$\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}. \qquad \text{... (18.12)}$$

Although the formula (18.12) is derived for a particular situation of figure (18.14), it is valid for all other situations of refraction at a single spherical surface. This is because we have used the proper sign convention.

Example 18.6

Locate the image of the point object O in the situation shown in figure (18.15). The point C denotes the centre of curvature of the separating surface.

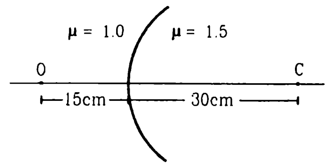


Figure 18.15

Solution : Here $u = -15$ cm, $R = 30$ cm, $\mu_1 = 1$ and $\mu_2 = 1{\cdot}5$. We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{1{\cdot}5}{v} - \frac{1{\cdot}0}{-15 \text{ cm}} = \frac{1{\cdot}5 - 1}{30 \text{ cm}}$$

or, $$\frac{1{\cdot}5}{v} = \frac{0{\cdot}5}{30 \text{ cm}} - \frac{1}{15 \text{ cm}}$$

or, $$v = -30 \text{ cm}.$$

The image is formed 30 cm left to the spherical surface and is virtual.

18.10 EXTENDED OBJECTS : LATERAL MAGNIFICATION

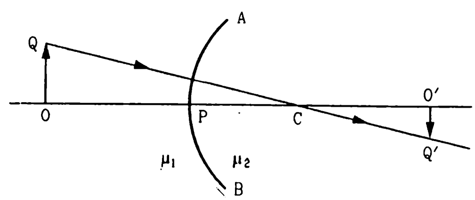


Figure 18.16

Consider the situation shown in figure (18.16). Let OQ be an extended object placed perpendicular to the line OPC. Consider the ray originating from Q and going towards QC. This ray is incident normally on the spherical surface AB. Thus, it goes undeviated in medium 2. The image of Q must be on the line QC.

The image of O will be formed on the line OPC. Let it be formed at O'. The position of O' may be located by using equation (18.12). If we drop a perpendicular from O' on OPC, the intersection Q' of this perpendicular with QC will be the image of Q. Thus, $O'Q'$ will be the image of OQ.

Lateral Magnification

The lateral or transverse magnification is defined as

$$m = \frac{h_2}{h_1},$$

where h_2 = height of the image and h_1 = height of the object. In figure (18.16), $OQ = +h_1$ and $O'Q' = -h_2$.

$$m = \frac{h_2}{h_1} = -\frac{O'Q'}{OQ}.$$

The triangles OCQ and $O'CQ'$ are similar. So,

$$m = -\frac{O'Q'}{OQ} = -\frac{O'C}{OC}$$

$$= -\frac{PO' - PC}{PO + PC}. \quad \ldots \text{(i)}$$

In figure (18.16), $PO = -u$, $PC = +R$ and $PO' = +v$. Equation (i) gives

$$m = -\frac{v - R}{-u + R}$$

$$= \frac{R - v}{R - u}. \quad \ldots \text{(ii)}$$

Also,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{\mu_2 u - \mu_1 v}{uv} = \frac{\mu_2 - \mu_1}{R}$$

or, $$R = \frac{(\mu_2 - \mu_1)uv}{\mu_2 u - \mu_1 v}.$$

This gives

$$R - v = \frac{\mu_1 v(v - u)}{\mu_2 u - \mu_1 v}$$

and $$R - u = \frac{\mu_2 u(v - u)}{\mu_2 u - \mu_1 v}.$$

Thus, by (ii),

$$m = \frac{\mu_1 v}{\mu_2 u}. \quad \ldots (18.13)$$

Example 18.7

Find the size of the image formed in the situation shown in figure (18.17).

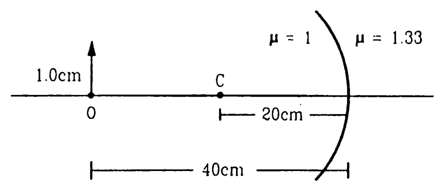


Figure 18.17

Solution: Here $u = -40$ cm, $R = -20$ cm, $\mu_1 = 1$, $\mu_2 = 1{\cdot}33$. We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{1{\cdot}33}{v} - \frac{1}{-40\text{ cm}} = \frac{1{\cdot}33 - 1}{-20\text{ cm}}$$

or, $$\frac{1{\cdot}33}{v} = -\frac{1}{40\text{ cm}} - \frac{0{\cdot}33}{20\text{ cm}}$$

or, $$v = -32\text{ cm}.$$

The magnification is

$$m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}$$

or, $$\frac{h_2}{1{\cdot}0\text{ cm}} = \frac{-32\text{ cm}}{1{\cdot}33 \times (-40\text{ cm})}$$

or, $$h_2 = +0{\cdot}6\text{ cm}.$$

The image is erect.

18.11 REFRACTION THROUGH THIN LENSES

A lens is one of the most familiar optical devices for a human being. We have lenses in our eyes and a good number of us supplement them with another set of lenses in our spectacles. A lens is made of a transparent material bounded by two spherical surfaces. The surfaces may be both convex, both concave or one convex and one concave. When the thickness of the lens is small compared to the other dimensions like object distance etc., we call it a *thin lens*. Figure (18.18) shows several lenses and paths of a number of rays going through them

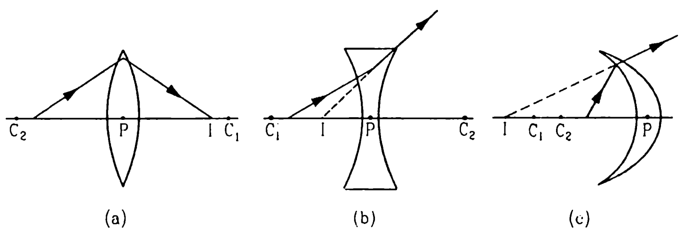


Figure 18.18

As there are two spherical surfaces, there are two centres of curvature C_1 and C_2 and correspondingly two radii of curvature R_1 and R_2. The line joining C_1 and

C_2 is called the *principal axis* of the lens. The centre P of the thin lens which lies on the principal axis, is called the *optical centre*.

Focus

Suppose, a narrow beam of light travelling parallel to the principal axis is incident on the lens near the optical centre (figure 18.19a).

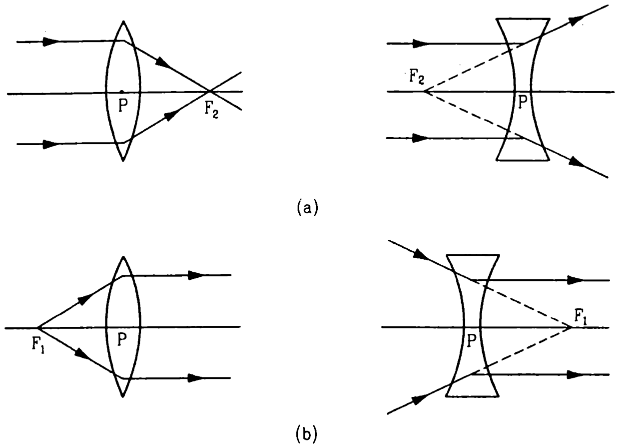


Figure 18.19

The rays are refracted twice and then come out of the lens. The emergent rays may converge at a point F_2 on the principal axis or they may seem to diverge from a point F_2 on the principal axis. In the first case, the lens is called a *convergent lens, converging lens or convex lens*. In the second case, it is called a *divergent lens, diverging lens* or *concave lens*. The point F_2 is called the *second focus* of the lens. The distance PF_2 from the optical centre is called the *second focal length.*

The *first focus* F_1 is defined as a point where an object should be placed to give emergent rays parallel to the principal axis (figure 18.19b). For a convergent lens, such an object is a real object and for a divergent lens, it is a virtual object. The distance PF_1 is the *first focal length.*

If the media on the two sides of a thin lens have same refractive index, the two focal lengths are equal. We shall be largely using the second focus F_2 in our discussions. Thus, when we write just *focus*, we shall mean the second focus and when we write just *focal length*, we shall mean the second focal length.

Sign Conventions

The coordinate sign conventions for a lens are similar to those for mirrors or refraction at spherical surfaces. The optical centre is taken as the origin and the principal axis as the X-axis. The positive direction of the axis is generally taken along the incident rays. The quantities u, v, f, R_1 and R_2 represent the x-coordinates of the object, the image, the focus, first centre of curvature and second centre of curvature respectively. The table below shows the signs of u, v, f, R_1 and R_2 in certain cases shown in figure (18.20).

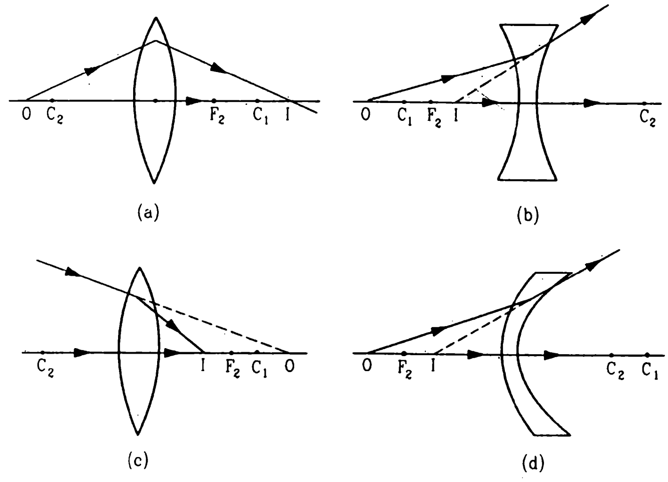


Figure 18.20

Figure	u	v	f	R_1	R_2
18.20a	–	+	+	+	–
18.20b	–	–	–	–	+
18.20c	+	+	+	+	–
18.20d	–	–	–	+	+

Generally, the incident rays and hence the positive direction of the axis is taken from left to right. Heights measured upwards are taken to be positive and the heights measured downward are taken to be negative.

With the usual choice of axes, f of a lens is positive for a converging lens and is negative for a diverging lens.

18.12 LENS MAKER'S FORMULA AND LENS FORMULA

Consider the situation shown in figure (18.21). $ADBE$ is a thin lens. An object O is placed on its principal axis. The two spherical surfaces of the lens have their centres at C_1 and C_2. The optical centre is at P and the principal axis cuts the two spherical surfaces at D and E.

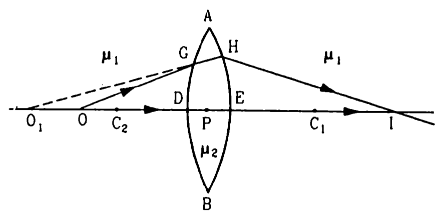


Figure 18.21

Let the refractive index of the material of the lens be μ_2 and suppose it is placed in a medium of refractive index μ_1. To trace the image of O, consider two rays OP and OG originating from O. The ray OP falls on the spherical surfaces perpendicularly and hence, it goes undeviated through the lens. The ray OG is refracted from a medium of refractive index μ_1 to another medium of refractive index μ_2. The centre of curvature of the surface ADB is at C_1. The ray is refracted along GH which meets the principal axis at O_1 (when produced backward in figure 18.21). Thus, due to this single refraction, the image of O is formed at O_1. The ray GH is incident on the spherical surface AEB. It is refracted from medium μ_2 to medium μ_1. The emergent ray HI intersects the principal axis at I where the final image is formed.

The general equation for refraction at a spherical surface is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}. \quad \text{... (i)}$$

To use this equation for the first refraction at ADB, we should take the origin at D, whereas, for the second refraction at AEB, we should take the origin at E. As the lens is thin, the points D, P and E are all close to each other and we may take the origin at P for both these refractions.

For the first refraction, the object is at O, the image is at O_1 and the centre of curvature is at C_1. If u, v, and R, denote their x-coordinates,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R_1}. \quad \text{... (ii)}$$

For the second refraction at AEB, the incident rays GH and DE diverge from O_1. Thus, O_1 is the object for this refraction and its x-coordinate is v_1. The image is formed at I and the centre of curvature is at C_2. Their x-coordinates are v and R respectively. The light goes from the medium μ_2 to medium μ_1. Applying equation (i),

$$\frac{\mu_1}{v} - \frac{\mu_2}{v_1} = \frac{\mu_1 - \mu_2}{R_2}. \quad \text{... (iii)}$$

Adding (ii) and (iii),

$$\mu_1\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_2 - \mu_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\frac{1}{v} - \frac{1}{u} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad \text{... (18.14)}$$

If the object O is taken far away from the lens, the image is formed close to the focus. Thus, for $u = \infty$, $v = f$. Putting in (18.14), we get,

$$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad \text{... (18.15)}$$

If the refractive index of the material of the lens is μ and it is placed in air, $\mu_2 = \mu$ and $\mu_1 = 1$ so that (18.15) becomes

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \quad \text{... (18.16)}$$

This is called *lens maker's formula* because it tells what curvatures will be needed to make a lens of desired focal length. Combining (18.14) and (18.15),

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \text{... (18.17)}$$

which is known as the *lens formula.*

Example 18.8

A biconvex lens has radii of curvature 20 cm *each. If the refractive index of the material of the lens is* 1·5, *what is its focal length?*

Solution : In a biconvex lens, centre of curvature of the first surface is on the positive side of the lens and that of the second surface is on the negative side. Thus, R_1 = 20 cm and R_2 = − 20 cm.

We have,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\frac{1}{f} = (1{\cdot}5 - 1)\left(\frac{1}{20\ \text{cm}} - \frac{1}{-20\ \text{cm}}\right)$$

or, f = 20 cm.

18.13 EXTENDED OBJECTS : LATERAL MAGNIFICATION

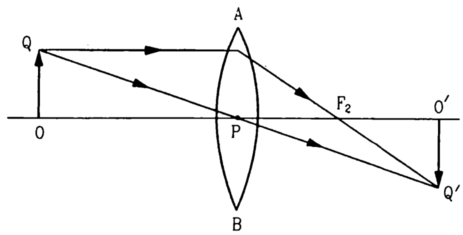


Figure 18.22

Consider the situation shown in figure (18.22). OQ is an extended object placed on the principal axis with its height perpendicular to the principal axis. To locate the image of Q, consider two rays QP and QA, the first one through the optical centre and the other parallel to the principal axis. The parts of the two surfaces at which the ray QP is refracted, are nearly parallel to each other. The lens near the optical centre, therefore, behaves like a rectangular slab. Thus, the ray passing through this region does not bend. Also, the lateral displacement produced is negligible as the thickness of the lens is small. Thus, a ray passing through the optical centre goes undeviated. The ray QP emerges in the same direction PQ'. The ray QA parallel to the principal axis must pass through the focus F_2. Thus, the emergent ray is along AF_2Q'. The image is formed where QPQ' and AF_2Q' intersect. Drop a perpendicular from Q' on the principal axis. This perpendicular $O'Q'$ is the image of OQ. Figure (18.23) shows image formation in some other cases.

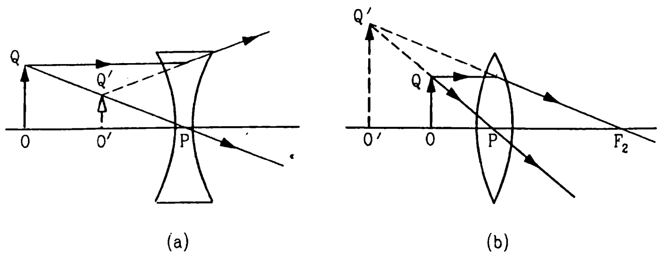


Figure 18.23

The lateral or transverse magnification is defined as

$$m = \frac{h_2}{h_1},$$

where h_2 = height of the image and h_1 = height of the object.

Referring to figure (18.22), the magnification is

$$m = \frac{h_2}{h_1} = \frac{-O'Q'}{OQ}.$$

From the similar triangles OQP and $O'Q'P$,

$$\frac{O'Q'}{OQ} = \frac{PO'}{PO}$$

so that $$m = -\frac{PO'}{PO}. \quad \ldots \text{(i)}$$

But $PO = -u$ and $PO' = +v$

so that by (i), $$m = \frac{v}{u}. \quad \ldots (18.18)$$

As usual, negative m indicates inverted image and positive m indicates erect image.

Since proper sign conventions are used, equation (18.18) is valid in all situations with a single thin lens, although it is derived for the particular situation of figure (18.22).

Example 18.9

An object of length 2·0 cm *is placed perpendicular to the principal axis of a convex lens of focal length* 12 cm. *Find the size of the image if the object is at a distance of* 8·0 cm *from the lens.*

Solution : We have $u = -8{\cdot}0$ cm., and $f = +12$ cm

Using $$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{v} = \frac{1}{12\text{ cm}} + \frac{1}{-8{\cdot}0\text{ cm}}$$

or, $$v = -24\text{ cm}.$$

Thus, $$m = \frac{v}{u} = \frac{-24\text{ cm}}{-8{\cdot}0\text{ cm}} = 3.$$

Thus, $h_2 = 3\,h_1 = 3 \times 2{\cdot}0\text{ cm} = 6{\cdot}0\text{ cm}$. The positive sign shows that the image is erect.

18.14 POWER OF A LENS

The power P of a lens is defined as $P = 1/f$, where f is the focal length. The SI unit of power of a lens is obviously m^{-1}. This is also known as *diopter*. The focal length of a converging lens is positive and that of a diverging lens is negative. Thus, the power of a converging lens is positive and that of a diverging lens is negative.

18.15 THIN LENSES IN CONTACT

Figure (18.24) shows two lenses L_1 and L_2 placed in contact. The focal lengths of the lenses are f_1 and f_2 respectively.

Suppose, a point object is placed at a point O on the common principal axis. The first lens would form its image at O_1. This point O_1 works as the object for the second lens and the final image is formed at I.

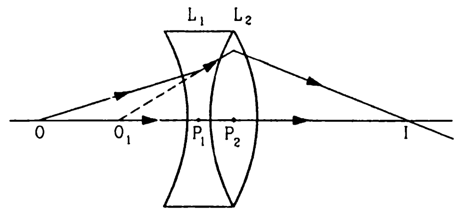


Figure 18.24

Let u = object-distance for the first lens,

v = final image-distance for the second lens,

v_1 = image-distance of the first image O_1 for the first lens. As the lenses are assumed to be thin, v_1 is also the object-distance for the second lens.

Then,

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

and $$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}.$$

Adding these equations, we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}. \quad \ldots \text{(i)}$$

If the combination is replaced by a single lens of focal length F such that it forms the image of O at the same position I,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}. \quad \text{... (ii)}$$

Such a lens is called the equivalent lens for the combination.

Comparing (i) and (ii),

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}. \quad \text{... (18.19)}$$

This F is the focal length of the equivalent lens for the combination. As the power of a lens is $P = 1/F$, equation (18.19) immediately gives

$$P = P_1 + P_2. \quad \text{... (18.20)}$$

Though equation (18.19) is derived for the situation shown in figure (18.24), it is true for any situation involving two thin lenses in contact.

18.16 TWO THIN LENSES SEPARATED BY A DISTANCE

When two thin lenses are separated by a distance, it is *not* equivalent to a single thin lens. In fact, such a combination can only be equivalent to a thick lens which has a more complicated theory.

In a special case when the object is placed at infinity, the combination may be replaced by a single thin lens. We shall now derive the position and focal length of the equivalent lens in this special case. To start with, let us derive an expression for the angle of deviation of a ray when it passes through a lens.

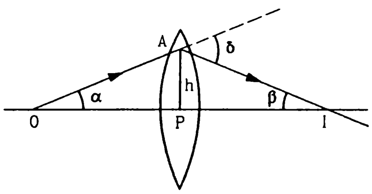


Figure 18.25

Let O be a point object on the principal axis of a lens (figure 18.25). Let OA be a ray incident on the lens at a point A, a height h above the optical centre. It is deviated through an angle δ and comes out along AI. It strikes the principal axis at I where the image is formed.

Let $\angle AOP = \alpha$ and $\angle AIP = \beta$. By triangle OAI,

$$\delta = \alpha + \beta.$$

If the height h is small as compared to PO and PI, the angles α, β are also small. Then,

$$\alpha \approx \tan\alpha = h/OP \text{ and } \beta \approx \tan\beta = h/PI.$$

Thus, $$\delta = \frac{h}{PO} + \frac{h}{PI}. \quad \text{... (i)}$$

Now, $$PO = -u \text{ and } PI = +v$$

so that by (i), $\delta = h\left(\frac{1}{v} - \frac{1}{u}\right)$

or, $$\delta = \frac{h}{f}. \quad \text{... (18.21)}$$

Now, consider the situation shown in figure (18.26). Two thin lenses are placed coaxially at a separation d. The incident ray AB and the emergent ray CD intersect at E. The perpendicular from E to the principal axis falls at P. The equivalent lens should be placed at this position P. A ray ABE going parallel to the principal axis will go through the equivalent lens and emerge along ECD. The angle of deviation is $\delta = \delta_1 + \delta_2$ from triangle BEC. The focal length of the equivalent lens is $F = PD$.

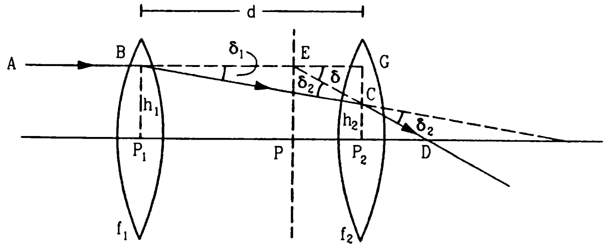


Figure 18.26

Using equation (18.21),

$$\delta_1 = \frac{h_1}{f_1}, \ \delta_2 = \frac{h_2}{f_2} \text{ and } \delta = \frac{h_1}{F}.$$

As $$\delta = \delta_1 + \delta_2,$$

$$\frac{h_1}{F} = \frac{h_1}{f_1} + \frac{h_2}{f_2}. \quad \text{... (ii)}$$

Now,

$$h_1 - h_2 = P_2G - P_2C = CG$$
$$= BG\tan\delta_1 \approx BG\,\delta_1$$

or, $$h_1 - h_2 = d\frac{h_1}{f_1} \quad \text{... (iii)}$$

or, $$h_2 = h_1 - d\frac{h_1}{f_1}. \quad \text{... (iv)}$$

Thus, by (ii),

$$\frac{h_1}{F} = \frac{h_1}{f_1} + \frac{h_1}{f_2} - \frac{d\,(h_1/f_1)}{f_2}.$$

or, $$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}. \quad \text{... (18.22)}$$

Position of the Equivalent Lens

We have, $$PP_2 = EG$$
$$= GC\cot\delta$$
$$= \frac{h_1 - h_2}{\tan\delta} \approx \frac{h_1 - h_2}{\delta}.$$

By (iii), $h_1 - h_2 = \frac{d\,h_1}{f_1} \cdot$ Also, $\delta = \frac{h_1}{F}$ so that

$$PP_2 = \left(\frac{d\,h_1}{f_1}\right)\left(\frac{F}{h_1}\right) = \frac{d\,F}{f_1} \cdot \qquad \dots (18.23)$$

Thus, the equivalent lens is to be placed at a distance $d\,F/f_1$ behind the second lens.

Equation (18.22) and (18.23) are true only for the special case of parallel incident beam. If the object is at a finite distance, one should not use the above equations. The image position should be worked out using the lens equations for the two lenses separately.

18.17 DEFECTS OF IMAGES

The simple theory of image formation developed for mirrors and lenses suffers from various approximations. As a result, the actual images formed contain several defects. These defects can be broadly divided in two categories, (a) *chromatic aberration* and (b) *monochromatic aberration*. The index of refraction of a transparent medium differs for different wavelengths of the light used. The defects arising from such a variation of the refractive index are termed as chromatic aberrations. The other defects, which arise even if light of a single colour is used, are called monochromatic aberrations. We shall first discuss this type of defects.

A. Monochromatic Aberrations

(a) Spherical Aberration

All through the discussion of lenses and mirrors with spherical surfaces, it has been assumed that the aperture of the lens or the mirror is small and the light rays of interest make small angles with the principal axis. It is then possible to have a point image of a point object. However, this is only an approximation even if we neglect diffraction.

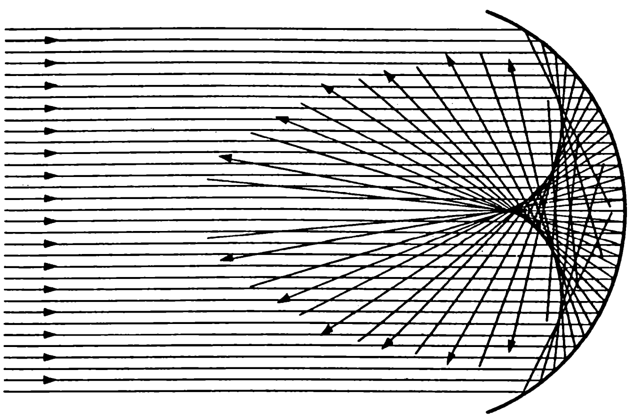


Figure 18.27

The rays reflect or refract from points at different distances from the principal axis. In general, they meet each other at different points. Thus, the image of a point object is a blurred surface. Such a defect is called *spherical aberration.* Figure (18.27) shows *spherical aberration* for a concave mirror for an object at infinity. The rays parallel to the principal axis are incident on the spherical surface of the concave mirror. The rays close to the principal axis (paraxial rays) are focused at the geometrical focus F of the mirror as given by the mirror formula. The rays farthest from the principal axis are called the marginal rays and are focused at a point F' somewhat closer to the mirror. The intermediate rays focus at different points between F and F'. Also, the rays reflected from a small portion away from the pole meet at a point off the axis. Thus, a three-dimensional blurred image is formed. The intersection of this image with the plane of figure is shown blackened in figure (18.27) and is called the *caustic curve.* If a screen is placed perpendicular to the principal axis, a disc image is formed on the screen. As the screen is moved parallel to itself, the disc becomes smallest at one position. This disc is closest to the ideal image and its periphery is called the *circle of least confusion.* The magnitude of spherical aberration may be measured from the distance FF' between the point where the paraxial rays converge and the point where the marginal rays converge.

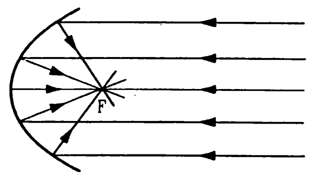


Figure 18.28

The parallel rays may be brought to focus at one point if a parabolic mirror is used. Also, if a point source is placed at the focus of a parabolic mirror, the reflected rays will be very nearly parallel. The reflectors in automobile headlights are made parabolic and the bulb is placed at the focus. The light beam is then nearly parallel and goes up to large distances.

Because of the finite aperture, a lens too produces a blurred disc type image of a point object. Figure (18.29) shows the situation for a convex and a concave

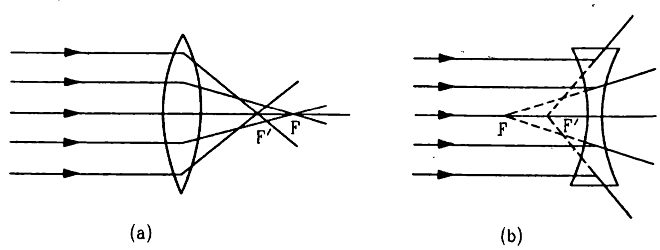


Figure 18.29

lens for the rays coming parallel to the principal axis. We see from the figure that the marginal rays are deviated a bit too strongly and hence, they meet at a point different from that given by geometrical optics formulae. Also, in the situation shown, the spherical aberration is opposite for convex and concave lens. The point F', where the marginal rays meet, is to the left of the focus for convex lens and is to the right of the focus for the concave lens.

The magnitude of spherical aberration for a lens depends on the radii of curvature and the object distance. The spherical aberration for a particular object distance can be reduced by properly choosing the radii of curvature. However, it can not be reduced to zero for a single lens which forms a real image of a real object. A simple method to reduce spherical aberration is to use a stop before and in front of the lens. A stop is an opaque sheet with a small circular opening in it. It only allows a narrow pencil of rays to go through the lens hence reducing the aberration. However, this method reduces the intensity of the image as most of the light is cut off.

The spherical aberration is less if the total deviation of the rays is distributed over the two surfaces of the lens. A striking example is a planoconvex lens forming the image of a distant object. If the plane surface faces the incident rays, the spherical aberration is much larger than that in the case when the curved surface faces the incident rays (figure 18.30). In the former case, the total deviation occurs at a single surface whereas it is distributed at both the surfaces in the latter case.

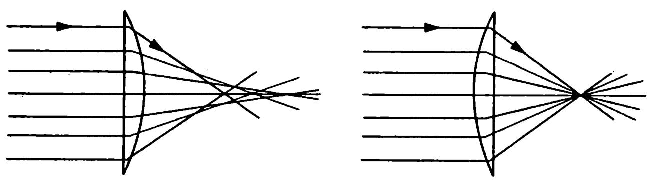


Figure 18.30

The spherical aberration can also be reduced by using a combination of convex and concave lenses. A suitable combination can reduce the spherical aberration by compensation of positive and negative aberrations.

(b) Coma

We have seen that if a point object is placed on the principal axis of a lens and the image is received on a screen perpendicular to the principal axis, the image has a shape of a disc because of spherical aberration. The basic reason of this aberration is that the rays passing through different regions of the lens meet the principal axis at different points. If the point object is placed away from the principal axis and the image is received on a screen perpendicular to the axis, the shape of the image is like a comet. This defect is called *coma*. The basic reason is again the same. The lens fails to converge all the rays passing at different distances from the axis at a single point. Figure (18.31) explains the formation of coma. The paraxial rays form an image of P at P'. The rays passing through the shaded zone forms a circular image on the screen above P'. The rays through outer zones of the lens form bigger circles placed further above P'. The image seen on the screen thus have a comet-like appearance.

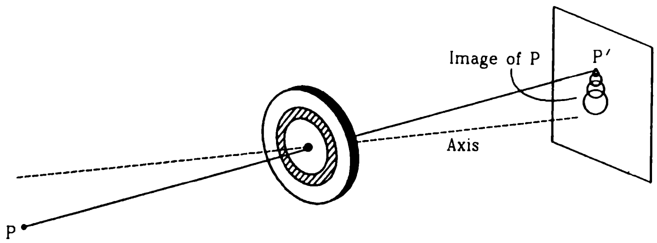


Figure 18.31

Coma can be reduced by properly designing radii of curvature of the lens surfaces. It can also be reduced by appropriate stops placed at appropriate distances from the lens.

(c) Astigmatism

Spherical aberration and coma refer to the spreading of the image of a point object in a plane perpendicular to the principal axis. The image is also spread along the principal axis. Suppose, a point object is placed at a point off the axis of a converging lens. A screen is placed perpendicular to the axis and is moved along the axis. At a certain distance, an approximate line image is focused. If the screen is moved further away, the shape of the image changes but it remains on the screen for quite a distance moved by the screen. The spreading of image along the principal axis is known as *astigmatism* (not to be confused with a defect of vision having the same name).

(d) Curvature

We have so far considered the image formed by a lens on a plane. However, it is not always true that the best image is formed along a plane. For a point object placed off the axis, the image is spread both along and perpendicular to the principal axis. The best image is, in general, obtained not on a plane but on a curved surface. This defect is known as *curvature*. It is intrinsically related to astigmatism. The astigmation or the curvature may be reduced by using proper stops placed at proper locations along the axis.

(e) Distortion

Distortion is the defect arising when extended objects are imaged. Different portions of the object are, in general, at different distances from the axis. The relation between the object distance and the image distance is not linear and hence, the magnification is not the same for all portions of the extended object. As a result, a line object is not imaged into a line but into a curve. Figure (18.32) shows some distorted images.

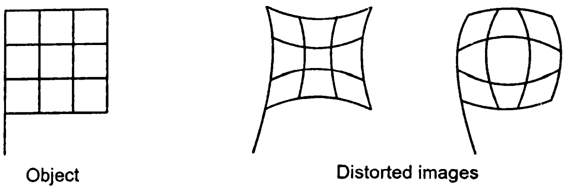


Figure 18.32

B. Chromatic Aberrations

The refractive index of the material of a lens varies slightly with the wavelength and hence, the focal length is also different for different wavelengths. In the visible region, the focal length is maximum for red and minimum for violet. Thus, if white light is used, each colour forms a separate image of the object.

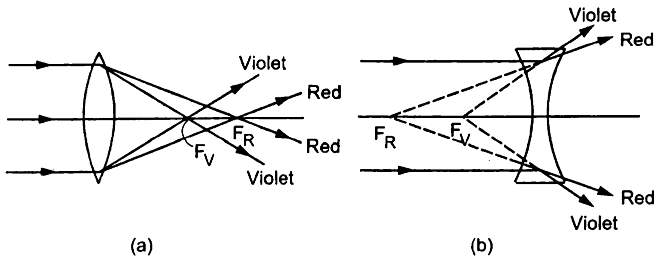


Figure 18.33

The violet rays are deviated more and hence, they form an image closer to the lens as compared to the image formed by the red rays. If light is incident on the lens from left to right, the violet image is to the left of the red image for convex lens and it is to the right of the red image for the concave lens. In the first case, the chromatic aberration is called positive and in the second case, it is negative. Thus, a proper combination of a convex and a concave lens may result in no chromatic aberration. Such a combination is called an *achromatic combination* for the pair of wavelengths. Also, the magnification v/u depends on the focal length and hence, on the wavelength. For an extended object, the images formed by light of different colours are of different sizes. A typical situation is shown in figure (18.33). Monochromatic aberration are assumed to be absent.

The separation between the images formed by extreme wavelengths of the visible range is called the *axial chromatic aberration* or *longitudinal chromatic aberration*. The difference in the size of the images (perpendicular to the principal axis) formed by the extreme wavelengths of the range is called the *lateral chromatic aberration* (figure 18.34).

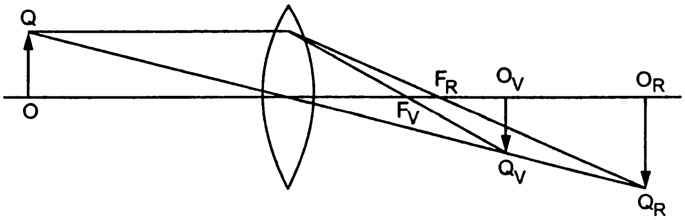


Figure 18.34

Worked Out Examples

1. *An object is placed on the principal axis of a concave mirror of focal length* 10 cm *at a distance of* 8·0 cm *from the pole. Find the position and the nature of the image.*

Solution : Here $u = -8{\cdot}0$ cm and $f = -10$ cm.

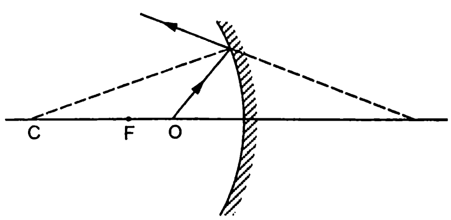


Figure 18-W1

We have, $$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{-10\ \text{cm}} - \frac{1}{-8{\cdot}0\ \text{cm}}$$

$$= \frac{1}{40\ \text{cm}}$$

or, $v = 40$ cm.

The positive sign shows that the image is formed at 40 cm from the pole on the other side of the mirror (figure 18-W1). As the image is formed beyond the mirror, the reflected rays do not intersect, the image is thus virtual.

2. *A rod of length* 10 cm *lies along the principal axis of a concave mirror of focal length* 10 cm *in such a way that the end closer to the pole is* 20 cm *away from it. Find the length of the image.*

Solution :

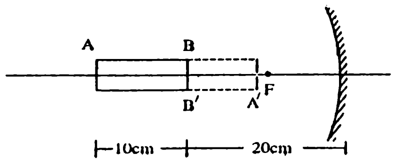


Figure 18-W2

The situation is shown in figure (18-W2). The radius of curvature of the mirror is $r = 2f = 20$ cm. Thus, the nearer end B of the rod AB is at the centre of the curvature and hence, its image will be formed at B itself. We shall now locate the image of A.

Here $u = -30$ cm and $f = -10$ cm. We have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or, $$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$= \frac{1}{-10\text{ cm}} - \frac{1}{-30\text{ cm}}$$

or, $$v = -15\text{ cm}.$$

Thus, the image of A is formed at 15 cm from the pole. The length of the image is, therefore, 5·0 cm.

3. *At what distance from a convex mirror of focal length* 2·5 m *should a boy stand so that his image has a height equal to half the original height? The principal axis is perpendicular to the height.*

Solution : We have,

$$m = -\frac{v}{u} = \frac{1}{2}$$

or, $$v = -\frac{u}{2}.$$

Also, $$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

or, $$\frac{1}{u} + \frac{1}{-u/2} = \frac{1}{2{\cdot}5\text{ m}}$$

or, $$-\frac{1}{u} = \frac{1}{2{\cdot}5\text{ m}}$$

or, $$u = -2{\cdot}5\text{ m}.$$

Thus, he should stand at a distance of 2·5 m from the mirror.

4. *A* 2·0 cm *high object is placed on the principal axis of a concave mirror at a distance of* 12 cm *from the pole. If the image is inverted, real and* 5·0 cm *high, find the location of the image and the focal length of the mirror.*

Solution : The magnification is $m = -\dfrac{v}{u}$

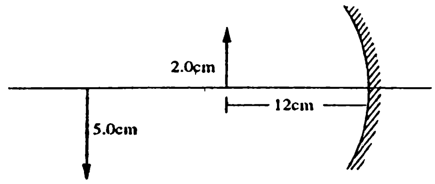


Figure 18-W3

or, $$\frac{-5{\cdot}0\text{ cm}}{2{\cdot}0\text{ cm}} = \frac{-v}{-12\text{ cm}}$$

or, $$v = -30\text{ cm}.$$

The image is formed at 30 cm from the pole on the side of the object. We have,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$= \frac{1}{-30\text{ cm}} + \frac{1}{-12\text{ cm}} = -\frac{7}{60\text{ cm}}$$

or, $$f = -\frac{60\text{ cm}}{7} = -8{\cdot}6\text{ cm}.$$

The focal length of the mirror is 8·6 cm.

5. *Consider the situation shown in figure (18-W4). Find the maximum angle* θ *for which the light suffers total internal reflection at the vertical surface.*

Solution :

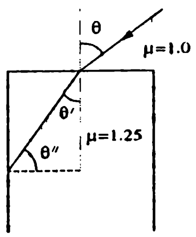


Figure 18-W4

The critical angle for this case is

$$\theta'' = \sin^{-1}\frac{1}{1{\cdot}25} = \sin^{-1}\frac{4}{5}$$

or, $$\sin\theta'' = \frac{4}{5}.$$

Since $\theta'' = \dfrac{\pi}{2} - \theta'$, we have $\sin\theta' = \cos\theta'' = 3/5$. From Snell's law,

$$\frac{\sin\theta}{\sin\theta'} = 1{\cdot}25$$

or, $$\sin\theta = 1{\cdot}25 \times \sin\theta'$$

$$= 1{\cdot}25 \times \frac{3}{5} = \frac{3}{4}$$

or, $\theta = \sin^{-1}\frac{3}{4}$.

If θ'' is greater than the critical angle, θ will be smaller than this value. Thus, the maximum value of θ, for which total reflection takes place at the vertical surface, is $\sin^{-1}(3/4)$.

6. *A right prism is to be made by selecting a proper material and the angles A and B ($B \le A$), as shown in figure (18-W5a). It is desired that a ray of light incident normally on AB emerges parallel to the incident direction after two internal reflections. (a) What should be the minimum refractive index μ for this to be possible? (b) For μ = 5/3, is it possible to achieve this with the angle A equal to* 60 degrees?

Solution :

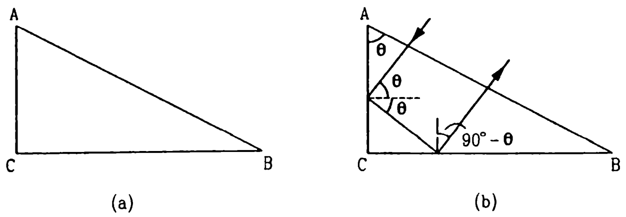


Figure 18-W5

(a) Consider the ray incident normally on *AB* (*figure 18-W5b*). The angle of reflection at the surface *AC* is θ. It is clear from the figure that the angle of incidence at the second surface *CB* is $90° - \theta$. The emergent ray will be parallel to the incident ray after two total internal reflections. The critical angle θ_c should be less than θ as well as $90° - \theta$. Thus, θ_c should be smaller than or equal to the smaller of θ and $90° - \theta$, i.e.,

$$\theta_c \le \min(\theta, 90° - \theta).$$

As $\min(\theta, 90° - \theta) \le 45°$, $\theta_c \le 45°$

or, $\sin\theta_c \le \frac{1}{\sqrt{2}}$ or, $\frac{1}{\mu} \le \frac{1}{\sqrt{2}}$

or, $\mu \ge \sqrt{2}$.

Thus, the refractive index of the material of the prism should be greater than or equal to $\sqrt{2}$. In this case the given ray can undergo two internal reflections for a suitable θ.

(b) For $\mu = 5/3$, the critical angle θ_c is

$$\sin^{-1}(3/5) = 37°.$$

As the figure suggests, we consider the light incident normally on the face *AB*. The angle of incidence θ on the surface *AC* is equal to $\theta = 60°$. As this is larger than the critical angle 37°, total internal reflection takes place here. The angle of incidence at the surface *CB* is $90° - \theta = 30°$. As this is less than the critical angle, total internal reflection does not take place at this surface.

7. *A point object O is placed in front of a transparent slab at a distance x from its closer surface. It is seen from the other side of the slab by light incident nearly normally to the slab. The thickness of the slab is t and its refractive index is μ. Show that the apparent shift in the position of the object is independent of x and find its value.*

Solution :

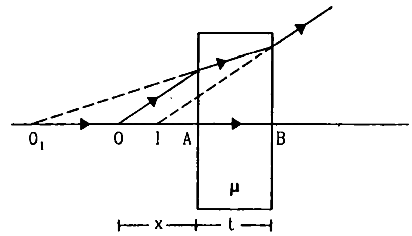


Figure 18-W6

The situation is shown in figure (18-W6). Because of the refraction at the first surface, the image of *O* is formed at O_1. For this refraction, the real depth is $AO = x$ and the apparent depth is AO_1. Also, the first medium is air and the second is the slab. Thus,

$$\frac{x}{AO_1} = \frac{1}{\mu} \quad \text{or,} \quad AO_1 = \mu x.$$

The point O_1 acts as the object for the refraction at the second surface. Due to this refraction, the image of O_1 is formed at *I*. Thus,

$$\frac{BO_1}{BI} = \mu$$

or, $$\frac{AB + AO_1}{BI} = \mu \quad \text{or,} \quad \frac{t + \mu x}{BI} = \mu$$

or, $$BI = x + \frac{t}{\mu}.$$

The net shift is $OI = OB - BI = (x + t) - \left(x + \frac{t}{\mu}\right)$

$$= t\left(1 - \frac{1}{\mu}\right),$$ which is independent of x.

8. *Consider the situation shown in figure (18-W7). A plane mirror is fixed at a height h above the bottom of a beaker containing water (refractive index μ) upto a height d. Find the position of the image of the bottom formed by the mirror.*

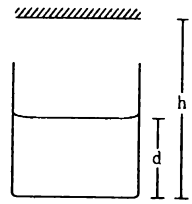


Figure 18-W7

Solution : The bottom of the beaker appears to be shifted up by a distance

$$\Delta t = \left(1 - \frac{1}{\mu}\right) d.$$

Thus, the apparent distance of the bottom from the mirror is $h - \Delta t = h - \left(1 - \frac{1}{\mu}\right) d = h - d + \frac{d}{\mu} \cdot$ The image is formed behind the mirror at a distance $h - d + \frac{d}{\mu} \cdot$

9. *A beaker contains water upto a height h_1 and K.oil above water upto another height h_2. Find the apparent shift in the position of the bottom of the beaker when viewed from above. Refractive index of water is μ_1 and that of K.oil is μ_2.*

Solution : The apparent shift of the bottom due to the water is

$$\Delta t_1 = \left(1 - \frac{1}{\mu_1}\right) h_1$$

and due to the K.oil is

$$\Delta t_2 = \left(1 - \frac{1}{\mu_2}\right) h_2.$$

The total shift $= \Delta t_1 + \Delta t_2 = \left(1 - \frac{1}{\mu_1}\right) h_1 + \left(1 - \frac{1}{\mu_2}\right) h_2.$

10. *Monochromatic light is incident on the plane interface AB between two media of refractive indices μ_1 and μ_2 ($\mu_2 > \mu_1$) at an angle of incidence θ as shown in figure (18-W8). The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now, if a transparent slab DEFG of uniform thickness and of refractive index μ_3 is introduced on the interface (as shown in the figure), show that for any value of μ_3 all light will ultimately be reflected back into medium II.*

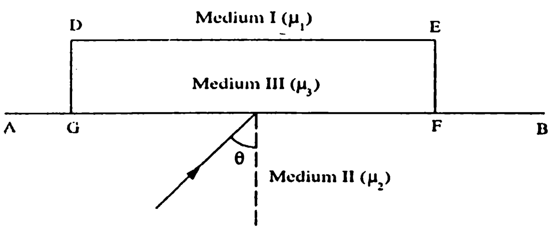


Figure 18-W8

Solution : We shall use the symbol $\bar{>}$ to mean "infinitesimally greater than".

When the slab is not inserted, $\theta \bar{>} \theta_c = \sin^{-1}(\mu_1/\mu_2)$

or, $\sin\theta \bar{>} \mu_1/\mu_2$.

When the slab is inserted, we have two cases $\mu_3 \le \mu_1$ and $\mu_3 > \mu_1$.

Case I : $\mu_3 \le \mu_1$

We have $\sin\theta \bar{>} \mu_1/\mu_2 \ge \mu_3/\mu_2$.

Thus, the light is incident on *AB* at an angle greater than the critical angle $\sin^{-1}(\mu_3/\mu_2)$. It suffers total internal reflection and goes back to medium II.

Case II : $\mu_3 > \mu_1$

$\sin\theta \bar{>} \mu_1/\mu_2 < \mu_3/\mu_2$

Thus, the angle of incidence θ may be smaller than the critical angle $\sin^{-1}(\mu_3/\mu_2)$ and hence it may enter medium III. The angle of refraction θ' is given by (figure 18-W9)

$$\frac{\sin\theta}{\sin\theta'} = \frac{\mu_3}{\mu_2} \qquad \text{... (i)}$$

or, $$\sin\theta' = \frac{\mu_2}{\mu_3}\sin\theta$$

$$\bar{>} \frac{\mu_2}{\mu_3} \cdot \frac{\mu_1}{\mu_2}.$$

Thus, $$\sin\theta' \bar{>} \frac{\mu_1}{\mu_3}$$

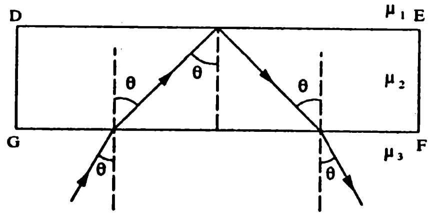


Figure 18-W9

or, $$\theta' \bar{>} \sin^{-1}\left(\frac{\mu_1}{\mu_3}\right). \qquad \text{... (ii)}$$

As the slab has parallel faces, the angle of refraction at the face *FG* is equal to the angle of incidence at the face *DE*. Equation (ii) shows that this angle is infinitesimally greater than the critical angle here. Hence, the light suffers total internal reflection and falls at the surface *FG* at an angle of incidence θ'. At this face, it will refract into medium II and the angle of refraction will be θ as shown by equation (i). Thus, the total light energy is ultimately reflected back into medium II.

11. *A concave mirror of radius* 40 cm *lies on a horizontal table and water is filled in it up to a height of* 5·00 cm *(figure 18-W10). A small dust particle floats on the water surface at a point P vertically above the point of contact of the mirror with the table. Locate the image of the dust particle as seen from a point directly above it. The refractive index of water is* 1·33.

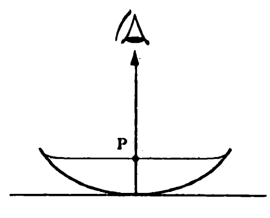


Figure 18-W10

Solution :

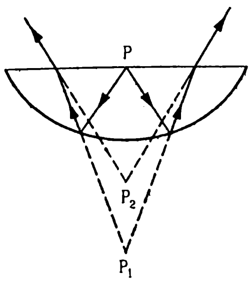


Figure 18-W11

The ray diagram is shown in figure (18-W11). Let us first locate the image formed by the concave mirror. Let us take vertically upward as the negative axis. Then $R = -40$ cm. The object distance is $u = -5$ cm. Using the mirror equation,

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$$

or, $$\frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40 \text{ cm}} - \frac{1}{-5 \text{ cm}} = \frac{6}{40} \text{ cm}$$

or, $v = 6{\cdot}67$ cm.

The positive sign shows that the image P_1 is formed below the mirror and hence, it is virtual. These reflected rays are refracted at the water surface and go to the observer. The depth of the point P_1 from the surface is $6{\cdot}67$ cm + $5{\cdot}00$ cm = $11{\cdot}67$ cm. Due to refraction at the water surface, the image P_1 will be shifted above by a distance

$$(11{\cdot}67 \text{ cm})\left(1 - \frac{1}{1{\cdot}33}\right) = 2{\cdot}92 \text{ cm}.$$

Thus, the final image is formed at a point $(11{\cdot}67 - 2{\cdot}92)$ cm = $8{\cdot}75$ cm below the water surface.

12. *An object is placed* 21 cm *in front of a concave mirror of radius of curvature* 20 cm. *A glass slab of thickness* 3 cm *and refractive index* $1{\cdot}5$ *is placed close to the mirror in the space between the object and the mirror. Find the position of the final image formed. The distance of the nearer surface of the slab from the mirror is* 10 cm.

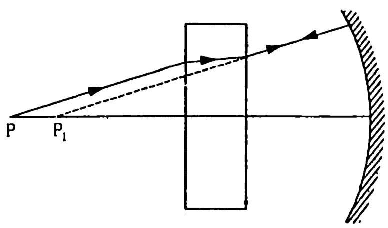


Figure 18-W12

Solution :

The situation is shown in figure (18-W12). Because of the refraction at the two surfaces of the slab, the image of the object P is formed at P_1, shifted towards the mirror by a distance

$$t\left(1 - \frac{1}{\mu}\right) = (3 \text{ cm})\left(1 - \frac{1}{1{\cdot}5}\right) = 1 \text{ cm}.$$

Thus, the rays falling on the concave mirror are diverging from P_1 which is at 21 cm − 1 cm = 20 cm from the mirror. But the radius of curvature is also 20 cm, hence P_1 is at the centre. The rays, therefore, fall normally on the mirror and hence, retrace their path. The final image is formed at P itself.

13. *The refractive indices of silicate flint glass for wavelengths* 400 nm *and* 700 nm *are* $1{\cdot}66$ *and* $1{\cdot}61$ *respectively. Find the minimum angles of deviation of an equilateral prism made of this glass for light of wavelengths* 400 nm *and* 700 nm.

Solution : The minimum angle of deviation δ_m is given by

$$\mu = \frac{\sin \dfrac{A + \delta_m}{2}}{\sin \dfrac{A}{2}} = \frac{\sin\left(30° + \dfrac{\delta_m}{2}\right)}{\sin 30°}$$

$$= 2 \sin\left(30° + \frac{\delta_m}{2}\right).$$

For 400 nm light,

$$1{\cdot}66 = 2 \sin(30° + \delta_m/2)$$

or, $\sin(30° + \delta_m/2) = 0{\cdot}83$

or, $(30° + \delta_m/2) = 56°$

or, $\delta_m = 52°$.

For 700 nm light,

$$1{\cdot}61 = 2 \sin(30° + \delta_m/2).$$

This gives $\delta_m = 48°$.

14. *Consider the situation shown in figure (18-W13). Light from a point source S is made parallel by a convex lens L. The beam travels horizontally and falls on an* 88°-88°-4° *prism as shown in the figure. It passes through the prism symmetrically. The transmitted light falls on a vertical mirror. Through what angle should the mirror be rotated so that the image of S is formed on S itself?*

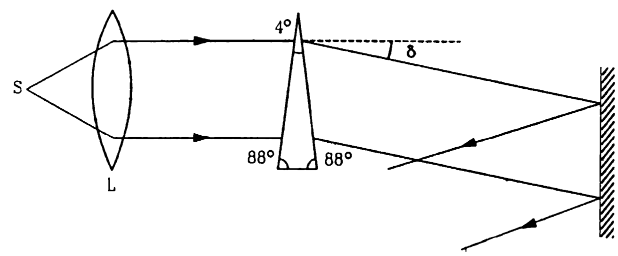


Figure 18-W13

Solution : The parallel beam after going through the prism will be deviated by an angle δ. If the mirror is also rotated by this angle δ, the rays will fall normally on it. The rays will be reflected back along the same path and form the image of S on itself.

As the prism is thin, the angle δ is given by

$$\delta = (\mu - 1) A$$
$$= (1{\cdot}5 - 1) \times 4^\circ = 2^\circ.$$

Thus, the mirror should be rotated by 2°.

15. *Locate the image formed by refraction in the situation shown in figure (18-W14). The point C is the centre of curvature.*

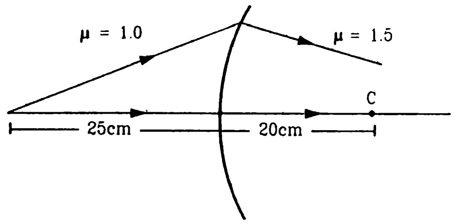


Figure 18-W14

Solution : We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}. \quad \text{... (i)}$$

Here $u = -25$ cm, $R = 20$ cm, $\mu_1 = 1{\cdot}0$ and $\mu_2 = 1{\cdot}5$.

Putting the values in (i),

$$\frac{1{\cdot}5}{v} + \frac{1{\cdot}0}{25\text{ cm}} = \frac{1{\cdot}5 - 1{\cdot}0}{20\text{ cm}}$$

or, $$\frac{1{\cdot}5}{v} = \frac{1}{40\text{ cm}} - \frac{1}{25\text{ cm}}$$

or, $$v = -100\text{ cm}.$$

As v is negative, the image is formed to the left of the separating surface at a distance of 100 cm from it.

16. *One end of a horizontal cylindrical glass rod* (μ = 1·5) *of radius* 5·0 cm *is rounded in the shape of a hemisphere. An object* 0·5 mm *high is placed perpendicular to the axis of the rod at a distance of* 20·0 cm *from the rounded edge. Locate the image of the object and find its height.*

Solution : Taking the origin at the vertex, $u = -20{\cdot}0$ cm and $R = 5{\cdot}0$ cm.

We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{1{\cdot}5}{v} = \frac{1}{-20{\cdot}0\text{ cm}} + \frac{0{\cdot}5}{5{\cdot}0\text{ cm}} = \frac{1}{20\text{ cm}}$$

or, $$v = 30\text{ cm}.$$

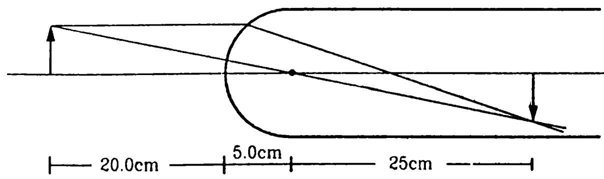


Figure 18-W15

The image is formed inside the rod at a distance of 30 cm from the vertex.

The magnification is $$m = \frac{\mu_1 v}{\mu_2 u}$$
$$= \frac{30\text{ cm}}{-1{\cdot}5 \times 20\text{ cm}} = -1.$$

Thus, the image will be of same height (0·5 mm) as the object but it will be inverted.

17. *There is a small air bubble inside a glass sphere* (μ = 1·5) *of radius* 10 cm. *The bubble is* 4·0 cm *below the surface and is viewed normally from the outside (figure 18-W16). Find the apparent depth of the bubble.*

Solution :

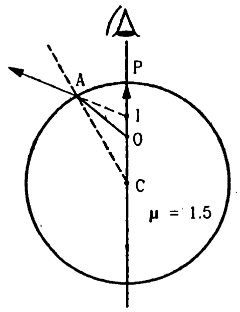


Figure 18-W16

The observer sees the image formed due to refraction at the spherical surface when the light from the bubble goes from the glass to the air.

Here $u = -4{\cdot}0$ cm, $R = -10$ cm, $\mu_1 = 1{\cdot}5$ and $\mu_2 = 1$.

We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or, $$\frac{1}{v} - \frac{1{\cdot}5}{-4{\cdot}0\text{ cm}} = \frac{1 - 1{\cdot}5}{-10\text{ cm}}$$

or, $$\frac{1}{v} = \frac{0{\cdot}5}{10\text{ cm}} - \frac{1{\cdot}5}{4{\cdot}0\text{ cm}}$$

or, $$v = -3{\cdot}0\text{ cm}.$$

Thus, the bubble will apear 3·0 cm below the surface.

18. *A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial, (i) find the position of the image due to*

refraction at the first surface and the position of the final image, and (ii) draw a ray diagram showing the positions of both the images.

Solution :

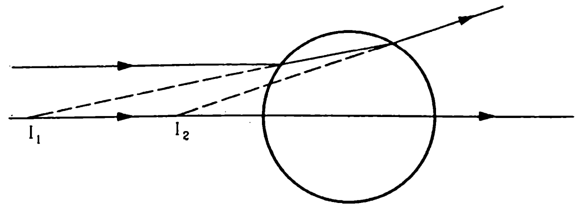


Figure 18-W17

The ray diagram is shown in figure (18-W17). The equation for refraction at a spherical surface is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}. \qquad \text{... (i)}$$

For the first refraction (water to air); $\mu_1 = 1{\cdot}33$, $\mu_2 = 1$, $u = \infty$, $R = +\,2\text{ mm}$.

Thus, $$\frac{1}{v} = \frac{1 - 1{\cdot}33}{2\text{ mm}}$$

or, $$v = -\,6\text{ mm}.$$

The negative sign shows that the image I_1 is virtual and forms at 6 mm from the surface of the bubble on the water side. The refracted rays (which seem to come from I_1) are incident on the farther surface of the bubble. For this refraction,

$$\mu_1 = 1,\ \mu_2 = 1{\cdot}33,\ R = -\,2\text{ mm}.$$

The object distance is $u = -\,(6\text{ mm} + 4\text{ mm}) = -\,10\text{ mm}$. Using equation (i),

$$\frac{1{\cdot}33}{v} + \frac{1}{10\text{ mm}} = \frac{1{\cdot}33 - 1}{-\,2\text{ mm}}$$

or, $$\frac{1{\cdot}33}{v} = -\frac{0{\cdot}33}{2\text{ mm}} - \frac{1}{10\text{ mm}}$$

or, $$v = -\,5\text{ mm}.$$

The minus sign shows that the image is formed on the air side at 5 mm from the refracting surface.

Measuring from the centre of the bubble, the first image is formed at 8·0 mm from the centre and the second image is formed at 3·0 mm from the centre. Both images are formed on the side from which the incident rays are coming.

19. *Calculate the focal length of the thin lens shown in figure (18-W18). The points C_1 and C_2 denote the centres of curvature.*

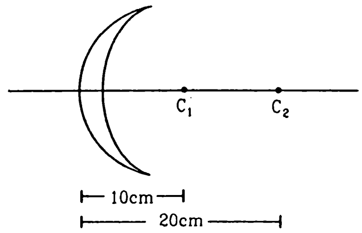


Figure 18-W18

Solution : As is clear from the figure, both the radii of curvature are positive. Thus, $R_1 = +\,10\text{ cm}$ and $R_2 = +\,20\text{ cm}$. The focal length is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1{\cdot}5 - 1)\left(\frac{1}{10\text{ cm}} - \frac{1}{20\text{ cm}}\right)$$

$$= 0{\cdot}5 \times \frac{1}{20\text{ cm}} = \frac{1}{40\text{ cm}}$$

or, $$f = 40\text{ cm}.$$

20. *A point source S is placed at a distance of* 15 cm *from a converging lens of focal length* 10 cm *on its principal axis. Where should a diverging mirror of focal length* 12 cm *be placed so that a real image is formed on the source itself.*

Solution :

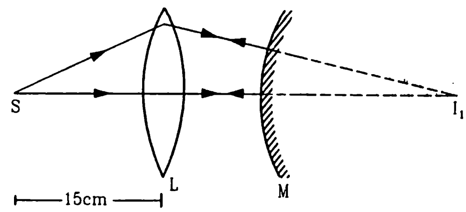


Figure 18-W19

The equation for the lens is

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}. \qquad \text{... (i)}$$

Here $u = -\,15\text{ cm}$ and $f = +\,10\text{ cm}$.

Using equation (i),

$$\frac{1}{v} + \frac{1}{15\text{ cm}} = \frac{1}{10\text{ cm}}$$

$$\frac{1}{v} = \frac{1}{10\text{ cm}} - \frac{1}{15\text{ cm}} = \frac{1}{30\text{ cm}}$$

or, $$v = 30\text{ cm}.$$

The positive sign of v shows that the image I_1 is formed to the right of the lens in the figure. The diverging mirror is to be placed to the right in such a way that the light rays fall on the mirror perpendicularly. Then only the rays will retrace their path and form the final image on the objecct. Thus, the image I_1 formed by the lens should be at the centre of curvature of the mirror.

We have, $LI_1 = 30$ cm,

$$MI_1 = R = 2f = 24 \text{ cm}.$$

Hence, $LM = LI - MI_1 = 6$ cm.

Thus, the mirror should be placed 6 cm to the right of the lens.

21. *A converging lens of focal length* 15 cm *and a converging mirror of focal length* 20 cm *are placed with their principal axes coinciding. A point source S is placed on the principal axis at a distance of* 12 cm *from the lens as shown in figure (18-W20). It is found that the final beam comes out parallel to the principal axis. Find the separation between the mirror and the lens.*

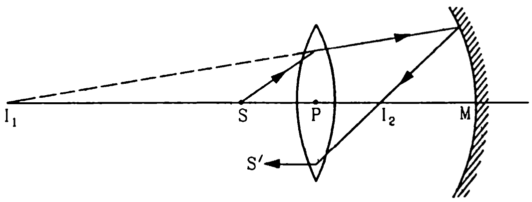


Figure 18-W20

Solution : Let us first locate the image of S formed by the lens. Here $u = -12$ cm and $f = 15$ cm. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or,
$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$= \frac{1}{15 \text{ cm}} - \frac{1}{12 \text{ cm}}$$

or,
$$v = -60 \text{ cm}.$$

The negative sign shows that the image is formed to the left of the lens as suggested in the figure. The image I_1 acts as the source for the mirror. The mirror forms an image I_2 of the source I_1. This image I_2 then acts as the source for the lens and the final beam comes out parallel to the principal axis. Clearly I_2 must be at the focus of the lens. We have,

$$I_1I_2 = I_1P + PI_2 = 60 \text{ cm} + 15 \text{ cm} = 75 \text{ cm}.$$

Suppose the distance of the mirror from I_2 is x cm. For the reflection from the mirror,

$$u = MI_1 = -(75 + x) \text{ cm}, \;\; v = -x \text{ cm} \;\text{ and }\; f = -20 \text{ cm}.$$

Using
$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{x} + \frac{1}{75 + x} = \frac{1}{20}$$

or,
$$\frac{75 + 2x}{(75 + x)\,x} = \frac{1}{20}$$

or,
$$x^2 + 35x - 1500 = 0$$

or,
$$x = \frac{-35 \pm \sqrt{35 \times 35 + 4 \times 1500}}{2}.$$

This gives $x = 25$ or -60.

As the negative sign has no physical meaning, only positive sign should be taken. Taking $x = 25$, the separation between the lens and the mirror is $(15 + 25)$ cm $= 40$ cm.

22. *A biconvex thin lens is prepared from glass* ($\mu = 1{\cdot}5$), *the two bounding surfaces having equal radii* of 25 cm *each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image is formed on the object itself?*

Solution :

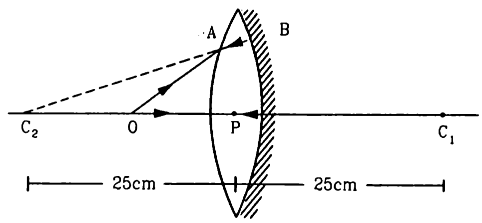


Figure 18-W21

Refer to figure (18-W21). The object is placed at O. A ray OA starting from O gets refracted into the glass at the first surface and hits the silvered surface along AB. To get the image at the object, the rays should retrace their path after reflection from the silvered surface. This will happen only if AB falls normally on the silvered surface. Thus, AB should appear to come from the centre C_2 of the second surface. Thus, due to the refraction at the first surface, a virtual image of O is formed at C_2. For this case,

$$v = -25 \text{ cm}, \;\; R = +25 \text{ cm}, \;\mu_1 = 1, \;\; \mu_2 = 1{\cdot}5.$$

We have,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

or,
$$\frac{1{\cdot}5}{-25 \text{ cm}} - \frac{1}{u} = \frac{1{\cdot}5 - 1}{25 \text{ cm}}$$

or,
$$\frac{1}{u} = -\frac{1{\cdot}5}{25 \text{ cm}} - \frac{0{\cdot}5}{25 \text{ cm}}$$

or,
$$u = -12{\cdot}5 \text{ cm}.$$

Thus, the object should be placed at a distance of 12·5 cm from the lens.

23. *A concavo-convex (figure 18-W22) lens made of glass* ($\mu = 1{\cdot}5$) *has surfaces of radii* 20 cm *and* 60 cm. *(a) Locate the image of an object placed* 80 cm *to the left of the lens along the principal axis. (b) A similar lens is placed coaxially at a distance* of 160 cm *right to it. Locate the position of the image.*

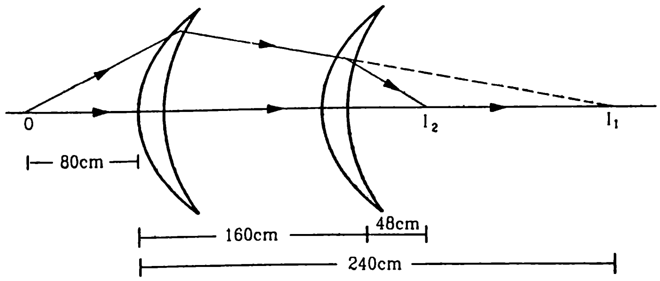


Figure 18-W22

Solution : The focal length of the lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1\cdot5 - 1)\left(\frac{1}{20\text{ cm}} - \frac{1}{60\text{ cm}}\right) = \frac{1}{60\text{ cm}}$$

or, $f = 60$ cm.

(a) For the image formed by the first lens, $u = -80$ cm so that

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{-80\text{ cm}} + \frac{1}{60\text{ cm}} = \frac{1}{240\text{ cm}}$$

or, $v = 240$ cm.

The first image I_1 would form 240 cm to the right of the first lens.

(b) The second lens intercepts the converging beam as suggested by the figure. The image I_1 acts as a virtual source for the second lens. For the image formed by this lens, $u = 240\text{ cm} - 160\text{ cm} = +80$ cm so that

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$$

$$= \frac{1}{80\text{ cm}} + \frac{1}{60\text{ cm}} = \frac{7}{240\text{ cm}}$$

or, $v = 34\cdot3$ cm.

The final image is formed 34·3 cm to the right of the second lens.

24. *A thin lens of focal length* + 12 cm *is immersed in water* ($\mu = 1\cdot33$). *What is its new focal length?*

Solution : We have, $\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

When the lens is placed in air, $f = 12$ cm. Thus,

$$\frac{1}{12\text{ cm}} = (1\cdot5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or, $$\frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{6\text{ cm}}.$$

If the focal length becomes f' when placed in water,

$$\frac{1}{f'} = \left(\frac{1\cdot5}{1\cdot33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{1}{8} \times \frac{1}{6\text{ cm}} = \frac{1}{48\text{ cm}} \quad \text{or, } f' = 48\text{ cm}.$$

25. *A long cylindrical tube containing water is closed by an equiconvex lens of focal length* 10 cm *in air. A point source is placed along the axis of the tube outside it at a distance* of 21 cm *from the lens. Locate the final image of the source. Refractive index of the material of the lens* = 1·5 *and that of water* = 1·33.

Solution :

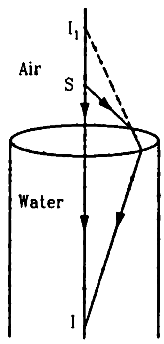


Figure 18-W23

The light from the source S gets refracted at the air-glass interface and then at the glass-water interface. Referring to the figure (18-W23), let us take vertically downward as the positive direction of the axis.

If the image due to the refraction at the first surface is formed at an image-distance v_1 from the surface, we have,

$$\frac{1\cdot5}{v_1} - \frac{1}{u} = \frac{1\cdot5 - 1}{R}, \qquad \ldots \text{(i)}$$

where R is the radius of curvature of the surface. As the lens is equiconvex, the radius of curvature of the second surface is $-R$. Also, the image formed by the first surface acts as the object for the second surface. Thus,

$$\frac{1\cdot33}{v} - \frac{1\cdot5}{v_1} = \frac{1\cdot33 - 1\cdot5}{-R}. \qquad \ldots \text{(ii)}$$

Adding (i) and (ii),

$$\frac{1\cdot33}{v} - \frac{1}{u} = \frac{1}{R}(0\cdot5 + 0\cdot17) = \frac{0\cdot67}{R}$$

or, $$\frac{1\cdot33}{v} - \frac{1}{-21\text{ cm}} = \frac{0\cdot67}{R}$$

or, $$\frac{4}{3v} + \frac{1}{21\text{ cm}} = \frac{2}{3R}$$

or, $$\frac{1}{v} = \frac{1}{2R} - \frac{1}{28\text{ cm}}. \qquad \ldots \text{(iii)}$$

The focal length of the lens in air is 10 cm. Using

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

$$\frac{1}{10\text{ cm}} = (1\cdot5 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

or, $R = 10$ cm.

Thus, by (iii),

$$\frac{1}{v} = \frac{1}{20\text{ cm}} - \frac{1}{28\text{ cm}}$$

or, $v = 70$ cm.

The image is formed 70 cm inside the tube.

26. *A slide projector produces* 500 times *enlarged image of a slide on a screen* 10 m *away. Assume that the projector consists of a single convex lens used for magnification. If the screen is moved* 2·0 m *closer, by what distance should the slide be moved towards or away from the lens so that the image remains focused on the screen? What is the magnification in this case?*

Solution : In the first case, $v = 10$ m and $\frac{v}{u} = -500$.

Thus, $u = -\frac{v}{500} = -\frac{1}{50}\text{ m} = -2{\cdot}0$ cm. The focal length f is given by

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10\text{ m}} + \frac{1}{2{\cdot}0\text{ cm}}.$$

If the screen is moved 2·0 m closer, $v = 8{\cdot}0$ m. The object-distance u' is given by

$$\frac{1}{v} - \frac{1}{u'} = \frac{1}{f}$$

or, $$\frac{1}{u'} = \frac{1}{v} - \frac{1}{f} = \frac{1}{8{\cdot}0\text{ m}} - \frac{1}{10\text{ m}} - \frac{1}{2{\cdot}0\text{ cm}}$$

$$= \frac{1}{40\text{ m}} - \frac{1}{2{\cdot}0\text{ cm}}$$

$$= -\frac{1}{2{\cdot}0\text{ cm}}\left(1 - \frac{1}{2000}\right)$$

or, $$u' = -2{\cdot}0\text{ cm}\left(1 - \frac{1}{2000}\right)^{-1}$$

$$\approx -2{\cdot}0\text{ cm}\left(1 + \frac{1}{2000}\right) = -2{\cdot}0\text{ cm} - \frac{1}{1000}\text{ cm}.$$

Thus, the slide should be taken $\frac{1}{1000}$ cm away from the lens.

27. *A convex lens focuses a distant object on a screen placed* 10 cm *away from it. A glass plate* ($\mu = 1{\cdot}5$) *of thickness* 1·5 cm *is inserted between the lens and the screen. Where should the object be placed so that its image is again focused on the screen.*

Solution :

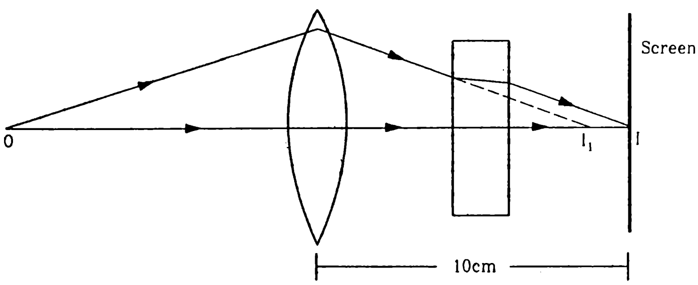


Figure 18-W24

The focal length of the lens is 10 cm. The situation with the glass plate inserted is shown in figure (18-W24). The object is placed at O. The lens would form the image at I_1 but the glass plate intercepts the rays and forms the image at I on the screen.

The shift $I_1I = t\left(1 - \frac{1}{\mu}\right)$

$$= (1{\cdot}5\text{ cm})\left(1 - \frac{1}{1{\cdot}5}\right) = 0{\cdot}5\text{ cm}.$$

Thus, the lens forms the image at a distance of 9·5 cm from itself. Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

$$\frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{9{\cdot}5\text{ cm}} - \frac{1}{10\text{ cm}}$$

or, $u = 190$ cm.

Thus, the object should be placed at a distance of 190 cm from the lens.

28. *Two convex lenses of focal length* 20 cm *each are placed coaxially with a separation of* 60 cm *between them. Find the image of a distant object formed by the combination by (a) using thin lens formula separately for the two lenses and (b) using the equivalent lens. Note that although the combination forms a real image of a distant object on the other side, it is equivalent to a diverging lens as far as the location of the final image is concerned.*

Solution :

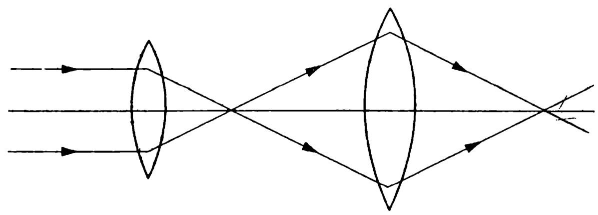


Figure 18-W25

(a) The first image is formed at the focus of the first lens. This is at 20 cm from the first lens and hence at $u = -40$ cm from the second. Using the lens formula for the second lens,

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} = -\frac{1}{40\text{ cm}} + \frac{1}{20\text{ cm}}$$

or, $v = 40$ cm.

The final image is formed 40 cm to the right of the second lens.

(b) The equivalent focal length is

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$= \frac{1}{20 \text{ cm}} + \frac{1}{20 \text{ cm}} - \frac{60 \text{ cm}}{(20 \text{ cm})^2}$$

or, $F = -20$ cm.

It is a divergent lens. It should be kept at a distance

$$D = \frac{dF}{f_1} \text{ behind the second lens.}$$

Here, $D = \frac{(60 \text{ cm})(-20 \text{ cm})}{20 \text{ cm}} = -60$ cm.

Thus, the equivalent divergent lens should be placed at a distance of 60 cm to the right of the second lens. The final image is formed at the focus of this divergent lens i.e., 20 cm to the left of it. It is, therefore, 40 cm to the right of the second lens.

□

QUESTIONS FOR SHORT ANSWER

1. Is the formula "Real depth/Apparent depth = μ" valid if viewed from a position quite away from the normal ?
2. Can you ever have a situation in which a light ray goes undeviated through a prism ?
3. Why does a diamond shine more than a glass piece cut to the same shape ?
4. A narrow beam of light passes through a slab obliquely and is then received by an eye (figure 18-Q1). The index of refraction of the material in the slab fluctuates slowly with time. How will it appear to the eye? The twinkling of stars has a similar explanation.

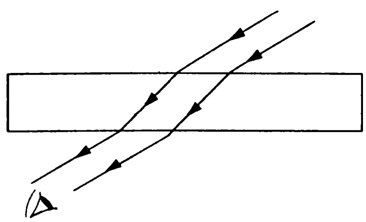


Figure 18-Q1

5. Can a plane mirror ever form a real image ?
6. If a piece of paper is placed at the position of a virtual image of a strong light source, will the paper burn after sufficient time ? What happens if the image is real? What happens if the image is real but the source is virtual ?
7. Can a virtual image be photographed by a camera ?
8. In motor vehicles, a convex mirror is attached near the driver's seat to give him the view of the traffic behind. What is the special function of this convex mirror which a plane mirror can not do ?
9. If an object far away from a convex mirror moves towards the mirror, the image also moves. Does it move faster, slower or at the same speed as compared to the object ?
10. Suppose you are inside the water in a swimming pool near an edge. A friend is standing on the edge. Do you find your friend taller or shorter than his usual height ?
11. The equation of refraction at a spherical surface is

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}.$$

Taking $R = \infty$, show that this equation leads to the equation

$$\frac{\text{Real depth}}{\text{Apparent depth}} = \frac{\mu_1}{\mu_2}$$

for refraction at a plane surface.

12. A thin converging lens is formed with one surface convex and the other plane. Does the position of image depend on whether the convex surface or the plane surface faces the object ?
13. A single lens is mounted in a tube. A parallel beam enters the tube and emerges out of the tube as a divergent beam. Can you say with certainty that there is a diverging lens in the tube ?
14. An air bubble is formed inside water. Does it act as a converging lens or a diverging lens ?
15. Two converging lenses of unequal focal lengths can be used to reduce the aperture of a parallel beam of light without losing the energy of the light. This increases the intensity. Describe how the converging lenses should be placed to do this.
16. If a spherical mirror is dipped in water, does its focal length change ?
17. If a thin lens is dipped in water, does its focal length change ?
18. Can mirrors give rise to chromatic aberration ?
19. A laser light is focused by a converging lens. Will there be a significant chromatic aberration ?

OBJECTIVE I

1. A point source of light is placed in front of a plane mirror.

(a) All the reflected rays meet at a point when produced backward.

(b) Only the reflected rays close to the normal meet at a point when produced backward.

(c) Only the reflected rays making a small angle with

the mirror, meet at a point when produced backward.
(d) Light of different colours make different images.

2. Total internal reflection can take place only if
(a) light goes from optically rarer medium (smaller refractive index) to optically denser medium
(b) light goes from optically denser medium to rarer medium
(c) the refractive indices of the two media are close to each other
(d) the refractive indices of the two media are widely different.

3. In image formation from spherical mirrors, only paraxial rays are considered because they
(a) are easy to handle geometrically
(b) contain most of the intensity of the incident light
(c) form nearly a point image of a point source
(d) show minimum dispersion effect.

4. A point object is placed at a distance of 30 cm from a convex mirror of focal length 30 cm. The image will form at
(a) infinity (b) pole
(c) focus (d) 15 cm behind the mirror.

5. Figure (18-Q2) shows two rays A and B being reflected by a mirror and going as A' and B'. The mirror
(a) is plane (b) is convex
(c) is concave (d) may be any spherical mirror.

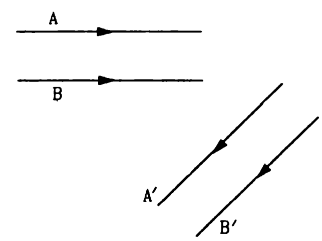


Figure 18-Q2

6. The image formed by a concave mirror
(a) is always real (b) is always virtual
(c) is certainly real if the object is virtual
(d) is certainly virtual if the object is real.

7. Figure (18-Q3) shows three transparent media of refractive indices μ_1, μ_2 and μ_3. A point object O is placed in the medium μ_2. If the entire medium on the right of the spherical surface has refractive index μ_1, the image forms at O'. If this entire medium has refractive index μ_3, the image forms at O''. In the situation shown,
(a) the image forms between O' and O''
(b) the image forms to the left of O'
(c) the image forms to the right of O''
(d) two images form, one at O' and the other at O''.

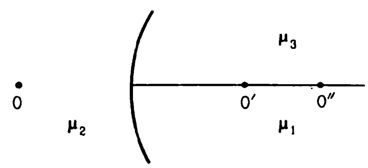


Figure 18-Q3

8. Four modifications are suggested in the lens formula to include the effect of the thickness t of the lens. Which one is likely to be correct?
(a) $\frac{1}{v}-\frac{1}{u}=\frac{t}{uf}$ (b) $\frac{t}{v^2}-\frac{1}{u}=\frac{1}{f}$
(c) $\frac{1}{v-t}-\frac{1}{u+t}=\frac{1}{f}$ (d) $\frac{1}{v}-\frac{1}{u}+\frac{t}{uv}=\frac{t}{f}$.

9. A double convex lens has two surfaces of equal radii R and refractive index $\mu = 1{\cdot}5$. We have,
(a) $f = R/2$ (b) $f = R$ (c) $f = -R$ (d) $f = 2R$.

10. A point source of light is placed at a distance of $2f$ from a converging lens of focal length f. The intensity on the other side of the lens is maximum at a distance
(a) f (b) between f and $2f$ (c) $2f$ (d) more than $2f$.

11. A parallel beam of light is incident on a converging lens parallel to its principal axis. As one moves away from the lens on the other side on its principal axis, the intensity of light
(a) remains constant (b) continuously increases
(c) continuously decreases
(d) first increases then decreases.

12. A symmetric double convex lens is cut in two equal parts by a plane perpendicular to the principal axis. If the power of the original lens was 4 D, the power of a cut-lens will be
(a) 2 D (b) 3 D (c) 4 D (d) 5 D.

13. A symmetric double convex lens is cut in two equal parts by a plane containing the principal axis. If the power of the original lens was 4 D, the power of a divided lens will be
(a) 2 D (b) 3 D (c) 4 D (d) 5 D.

14. Two concave lenses L_1 and L_2 are kept in contact with each other. If the space between the two lenses is filled with a material of refractive index $\mu \approx 1$, the magnitude of the focal length of the combination
(a) becomes undefined (b) remains unchanged
(c) increases (d) decreases.

15. A thin lens is made with a material having refractive index $\mu = 1{\cdot}5$. Both the sides are convex. It is dipped in water ($\mu = 1{\cdot}33$). It will behave like
(a) a convergent lens (b) a divergent lens
(c) a rectangular slab (d) a prism.

16. A convex lens is made of a material having refractive index 1·2. Both the surfaces of the lens are convex. If it is dipped into water ($\mu = 1{\cdot}33$), it will behave like
(a) a convergent lens (b) a divergent lens
(c) a rectangular slab (d) a prism.

17. A point object O is placed on the principal axis of a convex lens of focal length $f = 20$ cm at a distance of 40 cm to the left of it. The diameter of the lens is 10 cm. An eye is placed 60 cm to right of the lens and a distance h below the principal axis. The maximum value of h to see the image is
(a) 0 (b) 2·5 cm (c) 5 cm (d) 10 cm.

18. The rays of different colours fail to converge at a point after going through a converging lens. This defect is called
(a) spherical aberration (b) distortion
(c) coma (d) chromatic aberration.

OBJECTIVE II

1. If the light moving in a straight line bends by a small but fixed angle, it may be a case of
(a) reflection (b) refraction
(c) diffraction (d) dispersion.

2. Mark the correct options.
(a) If the incident rays are converging, we have a real object.
(b) If the final rays are converging, we have a real image.
(c) The image of a virtual object is called a virtual image.
(d) If the image is virtual, the corresponding object is called a virtual object.

3. Which of the following (referred to a spherical mirror) do (does) not depend on whether the rays are paraxial or not ?
(a) Pole. (b) Focus.
(c) Radius of curvature. (d) Principal axis.

4. The image of an extended object, placed perpendicular to the principal axis of a mirror, will be erect if
(a) the object and the image are both real
(b) the object and the image are both virtual
(c) the object is real but the image is virtual
(d) the object is virtual but the image is real.

5. A convex lens forms a real image of a point object placed on its principal axis. If the upper half of the lens is painted black,
(a) the image will be shifted downward
(b) the image will be shifted upward
(c) the image will not be shifted
(d) the intensity of the image will decrease.

6. Consider three converging lenses L_1, L_2 and L_3 having identical geometrical construction. The index of refraction of L_1 and L_2 are μ_1 and μ_2 respectively. The upper half of the lens L_3 has a refractive index μ_1 and the lower half has μ_2 (figure 18-Q4). A point object O is imaged at O_1 by the lens L_1 and at O_2 by the lens L_2 placed in same position. If L_3 is placed at the same place,
(a) there will be an image at O_1
(b) there will be an image at O_2.
(c) the only image will form somewhere between O_1 and O_2
(d) the only image will form away from O_2.

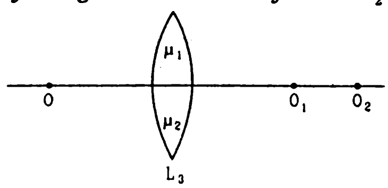


Figure 18-Q4

7. A screen is placed a distance 40 cm away from an illuminated object. A converging lens is placed between the source and the screen and it is attempted to form the image of the source on the screen. If no position could be found, the focal length of the lens
(a) must be less than 10 cm
(b) must be greater than 20 cm
(c) must not be greater than 20 cm
(d) must not be less than 10 cm.

EXERCISES

1. A concave mirror having a radius of curvature 40 cm is placed in front of an illuminated point source at a distance of 30 cm from it. Find the location of the image.

2. A concave mirror forms an image of 20 cm high object on a screen placed 5·0 m away from the mirror. The height of the image is 50 cm. Find the focal length of the mirror and the distance between the mirror and the object.

3. A concave mirror has a focal length of 20 cm. Find the position or positions of an object for which the image-size is double of the object-size.

4. A 1 cm object is placed perpendicular to the principal axis of a convex mirror of focal length 7·5 cm. Find its distance from the mirror if the image formed is 0·6 cm in size.

5. A candle flame 1·6 cm high is imaged in a ball bearing of diameter 0·4 cm. If the ball bearing is 20 cm away from the flame, find the location and the height of the image.

6. A 3 cm tall object is placed at a distance of 7·5 cm from a convex mirror of focal length 6 cm. Find the location, size and nature of the image.

7. A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure (18-E1). Find the total length of the image.

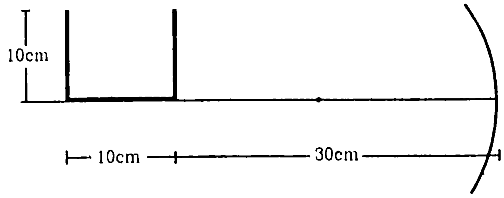


Figure 18-E1

8. A man uses a concave mirror for shaving. He keeps his face at a distance of 25 cm from the mirror and gets an image which is 1·4 times enlarged. Find the focal length of the mirror.

9. Find the diameter of the image of the moon formed by a spherical concave mirror of focal length 7·6 m. The diameter of the moon is 3450 km and the distance between the earth and the moon is $3{\cdot}8 \times 10^5$ km.

10. A particle goes in a circle of radius 2·0 cm. A concave mirror of focal length 20 cm is placed with its principal axis passing through the centre of the circle and perpendicular to its plane. The distance between the pole of the mirror and the centre of the circle is 30 cm. Calculate the radius of the circle formed by the image.

11. A concave mirror of radius R is kept on a horizontal table (figure 18-E2). Water (refractive index = μ) is poured into it upto a height h. Where should an object be placed so that its image is formed on itself ?

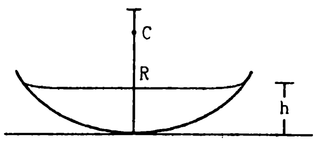


Figure 18-E2

12. A point source S is placed midway between two converging mirrors having equal focal length f as shown in figure (18-E3). Find the values of d for which only one image is formed.

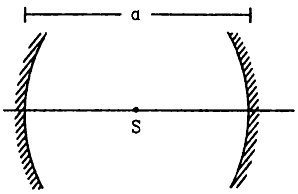


Figure 18-E3

13. A converging mirror M_1, a point source S and a diverging mirror M_2 are arranged as shown in figure (18-E4). The source is placed at a distance of 30 cm from M_1. The focal length of each of the mirrors is 20 cm. Consider only the images formed by a maximum of two reflections. It is found that one image is formed on the source itself. (a) Find the distance between the two mirrors. (b) Find the location of the image formed by the single reflection from M_2.

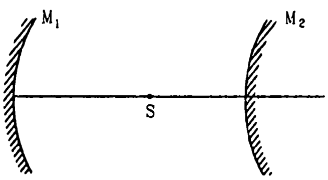


Figure 18-E4

14. A light ray falling at an angle of 45° with the surface of a clean slab of ice of thickness 1·00 m is refracted into it at an angle of 30°. Calculate the time taken by the light rays to cross the slab. Speed of light in vacuum $= 3 \times 10^8$ m/s.

15. A pole of length 1·00 m stands half dipped in a swimming pool with water level 50·0 cm higher than the bed. The refractive index of water is 1·33 and sunlight is coming at an angle of 45° with the vertical. Find the length of the shadow of the pole on the bed.

16. A small piece of wood is floating on the surface of a 2·5 m deep lake. Where does the shadow form on the bottom when the sun is just setting? Refractive index of water = 4/3.

17. An object P is focused by a microscope M. A glass slab of thickness 2·1 cm is introduced between P and M. If the refractive index of the slab is 1·5, by what distance should the microscope be shifted to focus the object again ?

18. A vessel contains water upto a height of 20 cm and above it an oil upto another 20 cm. The refractive indices of the water and the oil are 1·33 and 1·30 respectively. Find the apparent depth of the vessel when viewed from above.

19. Locate the image of the point P as seen by the eye in the figure (18-E5).

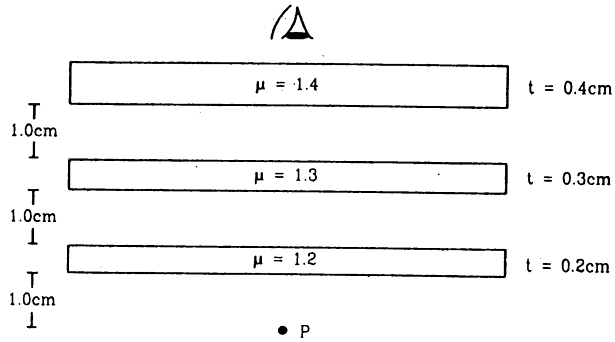


Figure 18-E5

20. k transparent slabs are arranged one over another. The refractive indices of the slabs are $\mu_1, \mu_2, \mu_3, \ldots \mu_k$ and the thicknesses are $t_1, t_2, t_3, \ldots t_k$. An object is seen through this combination with nearly perpendicular light. Find the equivalent refractive index of the system which will allow the image to be formed at the same place.

21. A cylindrical vessel of diameter 12 cm contains 800π cm^3 of water. A cylindrical glass piece of diameter 8·0 cm and height 8·0 cm is placed in the vessel. If the bottom of the vessel under the glass piece is seen by the paraxial rays (see figure 18-E6), locate its image. The index of refraction of glass is 1·50 and that of water is 1·33.

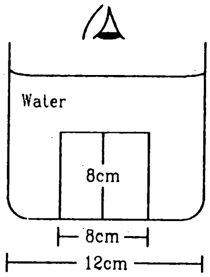


Figure 18-E6

22. Consider the situation in figure (18-E7). The bottom of the pot is a reflecting plane mirror, S is a small fish and T is a human eye. Refractive index of water is μ. (a) At what distance(s) from itself will the fish see the image(s) of the eye ? (b) At what distance(s) from itself will the eye see the image(s) of the fish.

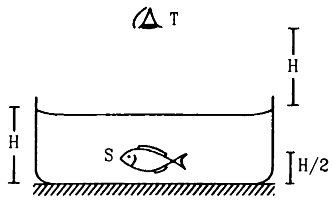


Figure 18-E7

23. A small object is placed at the centre of the bottom of a cylindrical vessel of radius 3 cm and height 4 cm filled completely with water. Consider the ray leaving the vessel through a corner. Suppose this ray and the ray along the axis of the vessel are used to trace the image. Find the apparent depth of the image and the ratio of real depth to the apparent depth under the assumptions taken. Refractive index of water = 1·33.

24. A cylindrical vessel, whose diameter and height both are equal to 30 cm, is placed on a horizontal surface and a small particle P is placed in it at a distance of 5·0 cm from the centre. An eye is placed at a position such that the edge of the bottom is just visible (see figure 18-E8). The particle P is in the plane of drawing. Up to what minimum height should water be poured in the vessel to make the particle P visible ?

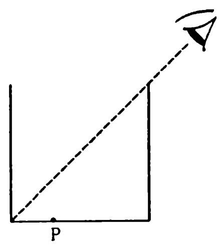


Figure 18-E8

25. A light ray is incident at an angle of 45° with the normal to a $\sqrt{2}$ cm thick plate (μ = 2·0). Find the shift in the path of the light as it emerges out from the plate.

26. An optical fibre (μ = 1·72) is surrounded by a glass coating (μ = 1·50). Find the critical angle for total internal reflection at the fibre-glass interface.

27. A light ray is incident normally on the face AB of a right-angled prism ABC (μ = 1·50) as shown in figure (18-E9). What is the largest angle ϕ for which the light ray is totally reflected at the surface AC ?

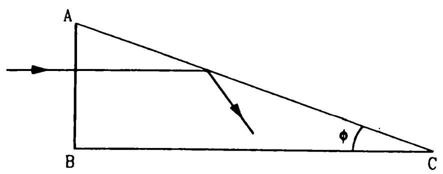


Figure 18-E9

28. Find the maximum angle of refraction when a light ray is refracted from glass (μ = 1·50) to air.

29. Light is incident from glass (μ = 1·5) to air. Sketch the variation of the angle of deviation δ with the angle of incident i for $0 < i < 90°$.

30. Light is incident from glass (μ = 1·50) to water (μ = 1·33). Find the range of the angle of deviation for which there are two angles of incidence.

31. Light falls from glass (μ = 1·5) to air. Find the angle of incidence for which the angle of deviation is 90°.

32. A point source is placed at a depth h below the surface of water (refractive index = μ). (a) Show that light escapes through a circular area on the water surface with its centre directly above the point source. (b) Find the angle subtended by a radius of the area on the source.

33. A container contains water upto a height of 20 cm and there is a point source at the centre of the bottom of the container. A rubber ring of radius r floats centrally on the water. The ceiling of the room is 2·0 m above the water surface. (a) Find the radius of the shadow of the ring formed on the ceiling if r = 15 cm. (b) Find the maximum value of r for which the shadow of the ring is formed on the ceiling. Refractive index of water = 4/3.

34. Find the angle of minimum deviation for an equilateral prism made of a material of refractive index 1·732. What is the angle of incidence for this deviation ?

35. Find the angle of deviation suffered by the light ray shown in figure (18-E10). The refractive index μ = 1·5 for the prism material.

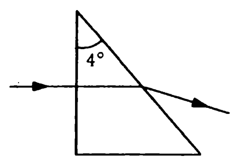


Figure 18-E10

36. A light ray, going through a prism with the angle of prism 60°, is found to deviate by 30°. What limit on the refractive index can be put from these data ?

37. Locate the image formed by refraction in the situation shown in figure (18-E11).

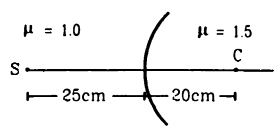


Figure 18-E11

38. A spherical surface of radius 30 cm separates two transparent media A and B with refractive indices 1·33 and 1·48 respectively. The medium A is on the convex side of the surface. Where should a point object be placed in medium A so that the paraxial rays become parallel after refraction at the surface ?

39. Figure (18-E12) shows a transparent hemisphere of radius 3·0 cm made of a material of refractive index 2·0. (a) A narrow beam of parallel rays is incident on the hemisphere as shown in the figure. Are the rays totally reflected at the plane surface? (b) Find the image formed by the refraction at the first surface. (c) Find the image formed by the reflection or by the refraction at the plane surface. (d) Trace qualitatively the final rays as they come out of the hemisphere.

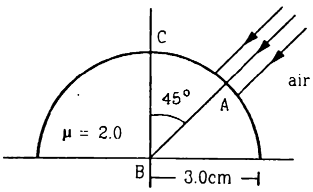


Figure 18-E12

40. A small object is embedded in a glass sphere (μ = 1·5) of radius 5·0 cm at a distance 1·5 cm left to the centre. Locate the image of the object as seen by an observer standing (a) to the left of the sphere and (b) to the right of the sphere.

41. A biconvex thick lens is constructed with glass (μ = 1·50). Each of the surfaces has a radius of 10 cm and the thickness at the middle is 5 cm. Locate the image of an object placed far away from the lens.

42. A narrow pencil of parallel light is incident normally on a solid transparent sphere of radius r. What should be the refractive index if the pencil is to be focused (a) at the surface of the sphere, (b) at the centre of the sphere.

43. One end of a cylindrical glass rod (μ = 1·5) of radius 1·0 cm is rounded in the shape of a hemisphere. The rod is immersed in water (μ = 4/3) and an object is placed in the water along the axis of the rod at a distance of 8·0 cm from the rounded edge. Locate the image of the object.

44. A paperweight in the form of a hemisphere of radius 3·0 cm is used to hold down a printed page. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the centre appear to the observer?

45. Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

46. A hemispherical portion of the surface of a solid glass sphere (μ = 1·5) of radius r is silvered to make the inner side reflecting. An object is placed on the axis of the hemisphere at a distance $3r$ from the centre of the sphere. The light from the object is refracted at the unsilvered part, then reflected from the silvered part and again refracted at the unsilvered part. Locate the final image formed.

47. The convex surface of a thin concavo-convex lens of glass of refractive index 1·5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface as shown in figure (18-E13). (a) Where should a pin be placed on the axis so that its image is formed at the same place? (b) If the concave part is filled with water (μ = 4/3), find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.

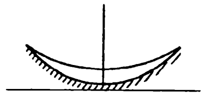


Figure 18-E13

48. A double convex lens has focal length 25 cm. The radius of curvature of one of the surfaces is double of the other. Find the radii, if the refractive index of the material of the lens is 1·5.

49. The radii of curvature of a lens are + 20 cm and + 30 cm. The material of the lens has a refracting index 1·6. Find the focal length of the lens (a) if it is placed in air, and (b) if it is placed in water (μ = 1·33).

50. Lenses are constructed by a material of refractive index 1·50. The magnitude of the radii of curvature are 20 cm and 30 cm. Find the focal lengths of the possible lenses with the above specifications.

51. A thin lens made of a material of refractive index μ_2 has a medium of refractive index μ_1 on one side and a medium of refractive index μ_3 on the other side. The lens is biconvex and the two radii of curvature have equal magnitude R. A beam of light travelling parallel to the principal axis is incident on the lens. Where will the image be formed if the beam is incident from (a) the medium μ_1 and (b) from the medium μ_3?

52. A convex lens has a focal length of 10 cm. Find the location and nature of the image if a point object is placed on the principal axis at a distance of (a) 9·8 cm, (b) 10·2 cm from the lens.

53. A slide projector has to project a 35 mm slide (35 mm × 23 mm) on a 2 m × 2 m screen at a distance of 10 m from the lens. What should be the focal length of the lens in the projector?

54. A particle executes a simple harmonic motion of amplitude 1·0 cm along the principal axis of a convex lens of focal length 12 cm. The mean position of oscillation is at 20 cm from the lens. Find the amplitude of oscillation of the image of the particle.

55. An extended object is placed at a distance of 5·0 cm from a convex lens of focal length 8·0 cm. (a) Draw the ray diagram (to the scale) to locate the image and from this, measure the distance of the image from the lens. (b) Find the position of the image from the lens formula and see how close the drawing is to the correct result.

56. A pin of length 2·00 cm is placed perpendicular to the principal axis of a converging lens. An inverted image of size 1·00 cm is formed at a distance of 40·0 cm from the pin. Find the focal length of the lens and its distance from the pin.

57. A convex lens produces a double size real image when an object is placed at a distance of 18 cm from it. Where should the object be placed to produce a triple size real image ?

58. A pin of length 2·0 cm lies along the principal axis of a converging lens, the centre being at a distance of 11 cm from the lens. The focal length of the lens is 6 cm. Find the size of the image.

59. The diameter of the sun is $1{\cdot}4 \times 10^{9}$ m and its distance from the earth is $1{\cdot}5 \times 10^{11}$ m. Find the radius of the image of the sun formed by a lens of focal length 20 cm.

60. A 5·0 diopter lens forms a virtual image which is 4 times the object placed perpendicularly on the principal axis of the lens. Find the distance of the object from the lens.

61. A diverging lens of focal length 20 cm and a converging mirror of focal length 10 cm are placed coaxially at a separation of 5 cm. Where should an object be placed so that a real image is formed at the object itself?

62. A converging lens of focal length 12 cm and a diverging mirror of focal length 7·5 cm are placed 5·0 cm apart with their principal axes coinciding. Where should an object be placed so that its image falls on itself?

63. A converging lens and a diverging mirror are placed at a separation of 15 cm. The focal length of the lens is 25 cm and that of the mirror is 40 cm. Where should a point source be placed between the lens and the mirror so that the light, after getting reflected by the mirror and then getting transmitted by the lens, comes out parallel to the principal axis ?

64. A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart with common principal axis. A point source is placed in between the lens and the mirror at a distance of 40 cm from the lens. Find the locations of the two images formed.

65. Consider the situation described in the previous problem. Where should a point source be placed on the principal axis so that the two images form at the same place ?

66. A converging lens of focal length 15 cm and a converging mirror of focal length 10 cm are placed 50 cm apart. If a pin of length 2·0 cm is placed 30 cm from the lens farther away from the mirror, where will the final image form and what will be the size of the final image ?

67. A point object is placed on the principal axis of a convex lens (f = 15 cm) at a distance of 30 cm from it. A glass plate (μ = 1·50) of thickness 1 cm is placed on the other side of the lens perpendicular to the axis. Locate the image of the point object.

68. A convex lens of focal length 20 cm and a concave lens of focal length 10 cm are placed 10 cm apart with their principal axes coinciding. A beam of light travelling parallel to the principal axis and having a beam diameter 5·0 mm, is incident on the combination. Show that the emergent beam is parallel to the incident one. Find the beam diameter of the emergent beam.

69. A diverging lens of focal length 20 cm and a converging lens of focal length 30 cm are placed 15 cm apart with their principal axes coinciding. Where should an object be placed on the principal axis so that its image is formed at infinity ?

70. A 5 mm high pin is placed at a distance of 15 cm from a convex lens of focal length 10 cm. A second lens of focal length 5 cm is placed 40 cm from the first lens and 55 cm from the pin. Find (a) the position of the final image, (b) its nature and (c) its size.

71. A point object is placed at a distance of 15 cm from a convex lens. The image is formed on the other side at a distance of 30 cm from the lens. When a concave lens is placed in contact with the convex lens, the image shifts away further by 30 cm. Calculate the focal lengths of the two lenses.

72. Two convex lenses, each of focal length 10 cm, are placed at a separation of 15 cm with their principal axes coinciding, (a) Show that a light beam coming parallel to the principal axis diverges as it comes out of the lens system. (b) Find the location of the virtual image formed by the lens system of an object placed far away. (c) Find the focal length of the equivalent lens. (Note that the sign of the focal length is positive although the lens system actually diverges a parallel beam incident on it).

73. A ball is kept at a height h above the surface of a heavy transparent sphere made of a material of refractive index μ. The radius of the sphere is R. At $t = 0$, the ball is dropped to fall normally on the sphere. Find the speed of the image formed as a function of time for $t < \sqrt{\dfrac{2h}{g}}$ · Consider only the image by a single refraction.

74. A particle is moving at a constant speed V from a large distance towards a concave mirror of radius R along its principal axis. Find the speed of the image formed by the mirror as a function of the distance x of the particle from the mirror.

75. A small block of mass m and a concave mirror of radius R fitted with a stand lie on a smooth horizontal table with a separation d between them. The mirror together with its stand has a mass m. The block is pushed at $t = 0$ towards the mirror so that it starts moving towards the mirror at a constant speed V and collides with it. The collision is perfectly elastic. Find the velocity of the image (a) at a time $t < d/V$, (b) at a time $t > d/V$.

76. A gun of mass M fires a bullet of mass m with a horizontal speed V. The gun is fitted with a concave mirror of focal length f facing towards the receding bullet. Find the speed of separation of the bullet and the image just after the gun was fired.

77. A mass m = 50 g is dropped on a vertical spring of spring constant 500 N/m from a height h = 10 cm as shown in figure (18-E14). The mass sticks to the spring and

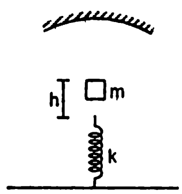


Figure 18-E14

executes simple harmonic oscillations after that. A concave mirror of focal length 12 cm facing the mass is fixed with its principal axis coinciding with the line of motion of the mass, its pole being at a distance of 30 cm from the free end of the spring. Find the length in which the image of the mass oscillates.

78. Two concave mirrors of equal radii of curvature R are fixed on a stand facing opposite directions. The whole system has a mass m and is kept on a frictionless horizontal table (figure 18-E15).

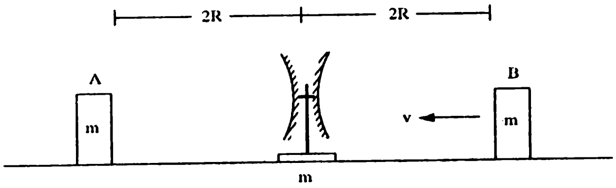


Figure 18-E15

Two blocks A and B, each of mass m, are placed on the two sides of the stand. At $t = 0$, the separation between A and the mirrors is $2R$ and also the separation between B and the mirrors is $2R$. The block B moves towards the mirror at a speed v. All collisions which take place are elastic. Taking the original position of the mirrors-stand system to be $x = 0$ and X-axis along AB, find the position of the images of A and B at $t =$

(a) $\frac{R}{v}$ (b) $\frac{3R}{v}$ (c) $\frac{5R}{v}$.

79. Consider the situation shown in figure (18-E16). The elevator is going up with an acceleration of 2·00 m/s^2 and the focal length of the mirror is 12·0 cm. All the surfaces are smooth and the pulley is light. The mass-pulley system is released from rest (with respect to the elevator) at $t = 0$ when the distance of B from the mirror is 42·0 cm. Find the distance between the image of the block B and the mirror at t = 0·200 s. Take g = 10 m/s^2.

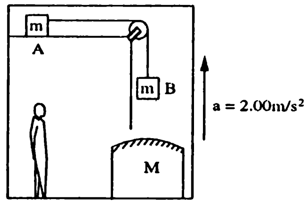


Figure 18-E16

ANSWERS

OBJECTIVE I

1. (a)	2. (b)	3. (c)	4. (d)	5. (a)	6. (c)
7. (d)	8. (c)	9. (b)	10. (c)	11. (d)	12. (a)
13. (c)	14. (c)	15. (a)	16. (b)	17. (b)	18. (d)

OBJECTIVE II

1. (a), (b)	2. (b)	3. (a), (c), (d)
4. (c), (d)	5. (c), (d)	6. (a), (b)
7. (b)		

EXERCISES

1. 60 cm from the mirror on the side of the object
2. 1·43 m, 2·0 m
3. 10 cm or 30 cm from the mirror
4. 5 cm
5. 1·0 mm inside the ball bearing, 0·08 mm
6. $\frac{10}{3}$ cm from the mirror on the side opposite to the object, 1·33 cm, virtual and erect
7. 10 cm
8. 87·5 cm
9. 6·9 cm
10. 4·0 cm
11. $\frac{(R-h)}{\mu}$ above the water surface
12. $2f$, $4f$
13. (a) 50 cm (b) 10 cm from the diverging mirror farther from the converging mirror
14. 5·44 ns
15. 81·5 cm
16. 2·83 m shifted from the position directly below the piece of the wood.
17. 0·70 cm
18. 30·4 cm
19. 0·2 cm above P
20. $\dfrac{\sum_{i=1}^{k} t_i}{\sum_{i=1}^{k} (t_i/\mu_i)}$
21. 7·1 cm above the bottom
22. (a) $H\left(\mu + \frac{1}{2}\right)$ above itself, $H\left(\mu + \frac{3}{2}\right)$ below itself

(b) $H\left(1+\frac{1}{2\mu}\right)$ below itself and $H\left(1+\frac{3}{2\mu}\right)$ below itself

23. 2·25 cm, 1·78
24. 26·7 cm
25. 0·62 cm
26. $\sin^{-1}\frac{75}{86}$
27. $\cos^{-1}(2/3)$
28. 90°
30. 0 to $\cos^{-1}(8/9)$
31. 45°
32. (b) $\sin^{-1}(1/\mu)$
33. (a) 2·8 m (b) 22·6 cm
34. 60°, 60°
35. 2°
36. $\mu \le \sqrt{2}$
37. 100 cm from the surface on the side of S
38. 266·0 cm away from the separating surface
39. (a) They are reflected

    (b) If the sphere is completed, the image forms at the point diametrically opposite to A

    (c) At the mirror image of A in BC
40. (a) 2 cm left to the centre

    (b) 2·65 cm left to the centre
41. 9·1 cm from the farther surface on the other side of the lens
42. (a) 2,

    (b) not possible, it will focus close to the centre if the refractive index is large.
43. At infinity
44. No shift is observed
45. 1 cm
46. At the reflecting surface of the sphere
47. (a) 15 cm from the lens on the axis

    (b) 1·14 cm towards the lens
48. 18·75 cm, 37·5 cm
49. (a) 100 cm (b) 300 cm
50. ± 24 cm, ± 120 cm
51. (a) $\frac{\mu_3 R}{2\mu_2-\mu_1-\mu_3}$ (b) $\frac{\mu_1 R}{2\mu_2-\mu_1-\mu_3}$
52. (a) 490 cm on the side of the object, virtual

    (b) 510 cm on the other side, real
53. 17·2 cm
54. 2·3 cm
56. 8·89 cm, 26·7 cm
57. 16 cm
58. 3 cm
59. 0·93 mm
60. 15 cm
61. 60 cm from the lens further away from the mirror
62. 30 cm from the lens further away from the mirror
63. 1·67 cm from the lens
64. One at 15 cm and the other at 24 cm from the lens away from the mirror
65. 30 cm from the lens towards the mirror
66. At the object itself, of the same size
67. 30·33 cm from the lens
68. 1·0 cm if the light is incident from the side of concave lens and 2·5 mm if it is incident from the side of the convex lens
69. 60 cm from the diverging lens or 210 cm from the converging lens
70. (a) 10 cm from the second lens further away,

    (b) erect and real, (c) 10 mm
71. 10 cm for convex lens and 60 cm for concave lens
72. (b) 5 cm from the first lens towards the second lens

    (c) 20 cm
73. $\frac{\mu R^2 gt}{\left[(\mu-1)\left(h-\frac{1}{2}gt^2\right)-R\right]^2}$
74. $\frac{R^2 V}{(2x-R)^2}$
75. (a) $-\frac{R^2 V}{[2(d-Vt)-R]^2}$

    (b) $V\left[1-\frac{R^2}{[2(Vt-d)-R]^2}\right]$
76. $2(1+m/M)V$
77. 1·2 cm
78. (a) $x=-\frac{2R}{3}$, R (b) $x=-2R, 0$ (c) $x=-3R, -\frac{4R}{3}$
79. 8·57 cm

SOLUTIONS TO CONCEPTS
CHAPTER – 18

SIGN CONVENTION :

1) The direction of incident ray (from object to the mirror or lens) is taken as positive direction.
2) All measurements are taken from pole (mirror) or optical centre (lens) as the case may be.

1. u = –30 cm, R = – 40 cm

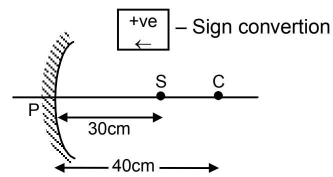
From the mirror equation,

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$$

$$\Rightarrow \frac{1}{v} = \frac{2}{R} - \frac{1}{u} = \frac{2}{-40} - \frac{1}{-30} = -\frac{1}{60}$$

or, v = –60 cm

So, the image will be formed at a distance of 60 cm in front of the mirror.



2. Given that,

H_1 = 20 cm, v = –5 m = –500 cm, h_2 = 50 cm

Since, $\frac{-v}{u} = \frac{h_2}{h_1}$

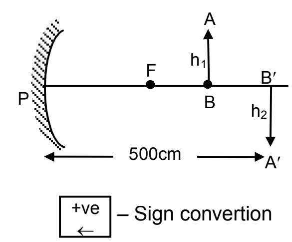
or $\frac{500}{u} = -\frac{50}{20}$ (because the image in inverted)

or u = $-\frac{500 \times 2}{5}$ = –200 cm = – 2 m

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{-5} + \frac{1}{-2} = \frac{1}{f}$

or f = $\frac{-10}{7}$ = –1.44 m

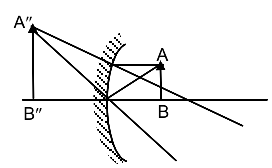
So, the focal length is 1.44 m.



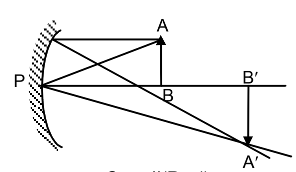
3. For the concave mirror, f = –20 cm, M = –v/u = 2

$\Rightarrow$ v = –2u

<u>1st case</u>	<u>2nd case</u>
$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$	$\frac{-1}{2u} - \frac{1}{u} = -\frac{1}{f}$
$\Rightarrow \frac{1}{2u} - \frac{1}{u} = -\frac{1}{f}$	$\Rightarrow \frac{3}{2u} = \frac{1}{f}$
$\Rightarrow$ u = f/2 = 10 cm	$\Rightarrow$ u = 3f/2 = 30 cm



Case I (Virtual)



Case II(Real)

$\therefore$ The positions are 10 cm or 30 cm from the concave mirror.

4. m = –v/u = 0.6 and f = 7.5 cm = 15/2 cm

From mirror equation,

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{0.6u} - \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow$ u = 5 cm

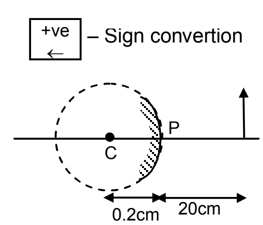
5. Height of the object AB = 1.6 cm

Diameter of the ball bearing = d = 0.4 cm

$\Rightarrow$ R = 0.2 cm

Given, u = 20 cm

We know, $\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$



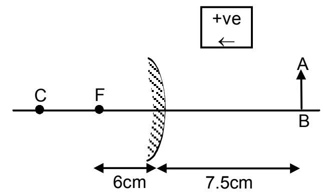
Putting the values according to sign conventions $\frac{1}{-20}+\frac{1}{v}=\frac{2}{0.2}$

$\Rightarrow\ \frac{1}{v}=\frac{1}{20}+10=\frac{201}{20}$ $\Rightarrow$ v = 0.1 cm = 1 mm inside the ball bearing.

Magnification = m = $\frac{A'B'}{AB}=-\frac{v}{u}=-\frac{0.1}{-20}=\frac{1}{200}$

$\Rightarrow$ A′B′ = $\frac{AB}{200}=\frac{16}{200}$ = +0.008 cm = +0.8 mm.

6. Given AB = 3 cm, u = –7.5 cm, f = 6 cm.



Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

Putting values according to sign conventions,

$\frac{1}{v}=\frac{1}{6}-\frac{1}{-7.5}=\frac{3}{10}$

$\Rightarrow$ v = 10/3 cm

$\therefore$ magnification = m = $-\frac{v}{u}=\frac{10}{7.5\times 3}$

$\Rightarrow\ \frac{A'B'}{AB}=\frac{10}{7.5\times 3}\Rightarrow A'B'=\frac{100}{72}=\frac{4}{3}=1.33$ cm.

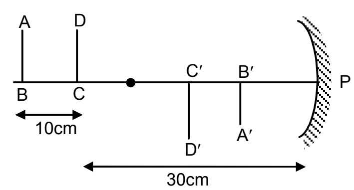
$\therefore$ Image will form at a distance of 10/3 cm. From the pole and image is 1.33 cm (virtual and erect).

7. R = 20 cm, f = R/2 = –10 cm

For part AB, PB = 30 + 10 = 40 cm

So, u = –40 cm $\Rightarrow\ \frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(\frac{1}{-40}\right)=-\frac{3}{40}$

$\Rightarrow$ v = $-\frac{40}{3}$ = –13.3 cm.



So, PB′ = 13.3 cm

m = $\frac{A'B'}{AB}=-\left(\frac{v}{u}\right)=-\left(\frac{-13.3}{-40}\right)=-\frac{1}{3}$

$\Rightarrow$ A′B′ = –10/3 = –3.33 cm

For part CD, PC = 30, So, u = –30 cm

$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=-\frac{1}{10}-\left(-\frac{1}{30}\right)=-\frac{1}{15}$ $\Rightarrow$ v = –15 cm = PC′

So, m = $\frac{C'D'}{CD}=-\frac{v}{u}=-\left(\frac{-15}{-30}\right)=-\frac{1}{2}$

$\Rightarrow$ C′D′ = 5 cm

B′C′ = PC′ – PB′ = 15 – 13.3 = 17 cm

So, total length A′B′ + B′C′ + C′D′ = 3.3 + 1.7 + 5 = 10 cm.

8. u = –25 cm

m = $\frac{A'B'}{AB}=-\frac{v}{u}\Rightarrow 1.4=-\left(\frac{v}{-25}\right)\Rightarrow\frac{14}{10}=\frac{v}{25}$

$\Rightarrow$ v = $\frac{25\times 14}{10}$ = 35 cm.

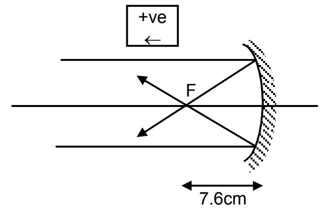
Now, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow\ \frac{1}{f}=\frac{1}{35}-\left(\frac{1}{-25}\right)=\frac{5-7}{175}=-\frac{2}{175}$ $\Rightarrow$ f = –87.5 cm.

So, focal length of the concave mirror is 87.5 cm.

9. $u = -3.8 \times 10^5$ km
diameter of moon = 3450 km ; f = –7.6 m

$\therefore \ \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\left(-\frac{1}{3.8\times10^5}\right)=\left(-\frac{1}{7.6}\right)$



Since, distance of moon from earth is very large as compared to focal length it can be taken as ∞.

$\Rightarrow$ Image will be formed at focus, which is inverted.

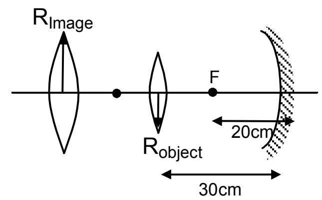
$\Rightarrow \ \frac{1}{v}=-\left(\frac{1}{7.6}\right) \Rightarrow v=-7.6 \text{ m}.$

m = $-\frac{v}{u}=\frac{d_{image}}{d_{object}} \Rightarrow \frac{-(-7.6)}{(-3.8\times10^8)}=\frac{d_{image}}{3450\times10^3}$

$d_{image} = \frac{3450\times7.6\times10^3}{3.8\times10^8}$ = 0.069 m = 6.9 cm.

10. u = –30 cm, f = –20 cm

We know, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$



$\Rightarrow \ \frac{1}{v}+\left(-\frac{1}{30}\right)=\left(-\frac{1}{20}\right) \Rightarrow$ v = –60 cm.

Image of the circle is formed at a distance 60 cm in front of the mirror.

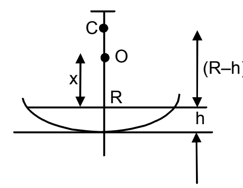
$\therefore$ m = $-\frac{v}{u}=\frac{R_{image}}{R_{object}} \Rightarrow -\frac{-60}{-30}=\frac{R_{image}}{2}$

$\Rightarrow R_{image}$ = 4 cm

Radius of image of the circle is 4 cm.

11. Let the object be placed at a height x above the surface of the water.

The apparent position of the object with respect to mirror should be at the centre of curvature so that the image is formed at the same position.

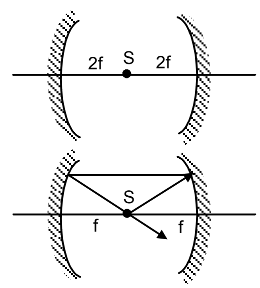


Since, $\frac{\text{Real depth}}{\text{Apparent depth}}=\frac{1}{\mu}$ (with respect to mirror)

Now, $\frac{x}{R-h}=\frac{1}{\mu} \Rightarrow x=\frac{R-h}{\mu}$.

12. Both the mirrors have equal focal length f.

They will produce one image under two conditions.



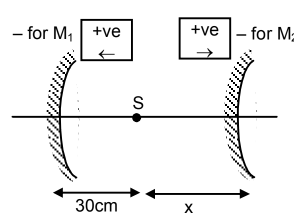
Case I : When the source is at distance '2f' from each mirror i.e. the source is at centre of curvature of the mirrors, the image will be produced at the same point S. So, d = 2f + 2f = 4f.

Case II : When the source S is at distance 'f' from each mirror, the rays from the source after reflecting from one mirror will become parallel and so these parallel rays after the reflection from the other mirror the object itself. So, only sine image is formed.

Here, d = f + f = 2f.

13. As shown in figure, for 1st reflection in M_1, u = –30 cm, f = –20 cm

$\Rightarrow \ \frac{1}{v}+\frac{1}{-30}=-\frac{1}{20} \Rightarrow$ v = –60 cm.



So, for 2nd reflection in M_2

u = 60 – (30 + x) = 30 – x

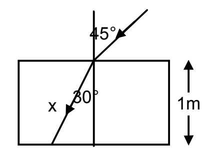
v = –x ; f = 20 cm

$\Rightarrow \ \frac{1}{30-x}-\frac{1}{x}=\frac{1}{20} \Rightarrow x^2+10x-600=0$

$\Rightarrow$ x = $\frac{10 \pm 50}{2} = \frac{40}{2}$ = 20 cm or –30 cm

$\therefore$ Total distance between the two lines is 20 + 30 = 50 cm.

14. We know, $\frac{\sin i}{\sin r} = \frac{3 \times 10^8}{v} = \frac{\sin 45^\circ}{\sin 30^\circ} = \sqrt{2}$



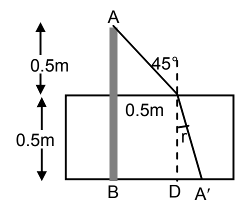
$\Rightarrow$ v = $\frac{3 \times 10^8}{\sqrt{2}}$ m/sec.

Distance travelled by light in the slab is,

x = $\frac{1\,m}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$ m

So, time taken = $\frac{2 \times \sqrt{2}}{\sqrt{3} \times 3 \times 10^8}$ = 0.54 × 10^{-8} = 5.4 × 10^{-9} sec.

15. Shadow length = BA′ = BD + A′D = 0.5 + 0.5 tan r



Now, 1.33 = $\frac{\sin 45^\circ}{\sin r}$ $\Rightarrow$ sin r = 0.53.

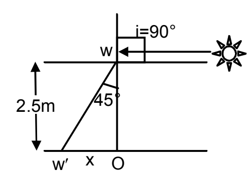
$\Rightarrow$ cos r = $\sqrt{1 - \sin^2 r} = \sqrt{1 - (0.53)^2}$ = 0.85

So, tan r = 0.6235

So, shadow length = (0.5) (1 + 0.6235) = 81.2 cm.

16. Height of the lake = 2.5 m

When the sun is just setting, θ is approximately = 90°



$\therefore$ $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{1}{\sin r} = \frac{4/3}{1} \Rightarrow \sin r = \frac{3}{4} \Rightarrow$ r = 49°

As shown in the figure, x/2.5 = tan r = 1.15

$\Rightarrow$ x = 2.5 × 1.15 = 2.8 m.

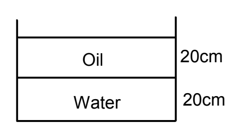
17. The thickness of the glass is d = 2.1 cm and μ =1.5

Shift due to the glass slab

ΔT = $\left(1 - \frac{1}{\mu}\right) d = \left(1 - \frac{1}{1.5}\right) 2.1$ = 0.7 CM

So, the microscope should be shifted 0.70 cm to focus the object again.

18. Shift due to water Δt_w = $\left(1 - \frac{1}{\mu}\right) d = \left(1 - \frac{1}{1.33}\right) 20$ = 5 cm



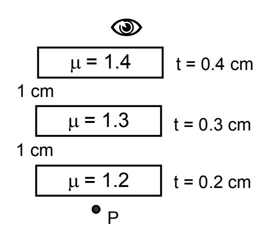
Shift due to oil, Δt_o = $\left(1 - \frac{1}{1.3}\right) 20$ = 4.6 cm

Total shift Δt = 5 + 4.6 = 9.6 cm

Apparent depth = 40 – (9.6) = 30.4 cm below the surface.

19. The presence of air medium in between the sheets does not affect the shift.

The shift will be due to 3 sheets of different refractive index other than air.



= $\left(1 - \frac{1}{1.2}\right)(0.2) + \left(1 - \frac{1}{13}\right)(0.3) + \left(1 - \frac{1}{14}\right)(0.4)$

= 0.2 cm above point P.

20. Total no. of slabs = k, thickness = t_1, t_2, t_3 … t_k

Refractive index = μ_1, μ_2, μ_3, μ_4,… μ_k

$\therefore$ The shift Δt = $\left(1 - \frac{1}{\mu_1}\right) t_1 + \left(1 - \frac{1}{\mu_2}\right) t_2 + \ldots\ldots + \left(1 - \frac{1}{\mu_k}\right) t_k$ …(1)

If, $\mu \rightarrow$ refractive index of combination of slabs and image is formed at same place,

Δt = $\left(1 - \frac{1}{\mu}\right)(t_1 + t_2 + \ldots + t_k)$ …(2)

Equation (1) and (2), we get,

$$\left(1-\frac{1}{\mu}\right)(t_1+t_2+\ldots+t_k) = \left(1-\frac{1}{\mu_1}\right)t_1+\left(1-\frac{1}{\mu_2}\right)t_2+\ldots\ldots+\left(1-\frac{1}{\mu_k}\right)t_k$$

$$= (t_1+t_2+\ldots+t_k)-\left(\frac{t_1}{\mu_1}+\frac{t_2}{\mu_2}+\ldots+\frac{t_k}{\mu_k}\right)$$

$$= -\frac{1}{\mu}\sum_{i=1}^{k}t_1 = -\sum_{i=1}^{k}\left(\frac{t_1}{\mu_1}\right) \Rightarrow \mu = \frac{\sum_{i=1}^{k}t_i}{\sum_{i=1}^{k}(t_1/\mu_1)}.$$

21. Given r = 6 cm, r_1 = 4 cm, h_1 = 8 cm

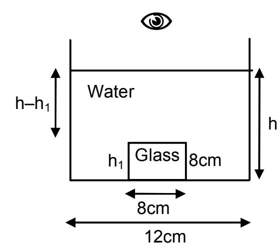
Let, h = final height of water column.

The volume of the cylindrical water column after the glass piece is put will be,

$\pi r^2 h = 800\,\pi + \pi r_1^2 h_1$

or $r^2 h = 800 + r_1^2 h_1$

or $6^2\,h = 800 + 4^2 \times 8 = 25.7$ cm



There are two shifts due to glass block as well as water.

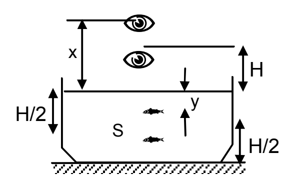
So, $\Delta t_1 = \left(1-\frac{1}{\mu_0}\right)t_0 = \left(1-\frac{1}{3/2}\right)8 = 2.26$ cm

And, $\Delta t_2 = \left(1-\frac{1}{\mu_w}\right)t_w = \left(1-\frac{1}{4/3}\right)(25.7-8) = 4.44$ cm.

Total shift = (2.66 + 4.44) cm = 7.1 cm above the bottom.

22. a) Let x = distance of the image of the eye formed above the surface as seen by the fish

So, $\frac{H}{x} = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{1}{\mu}$ or $x = \mu H$



So, distance of the direct image = $\frac{H}{2}+\mu H = H(\mu+\frac{1}{2})$

Similarly, image through mirror = $\frac{H}{2}+(H+x) = \frac{3H}{2}+\mu H = H(\mu+\frac{3}{2})$

b) Here, $\frac{H/2}{y} = \mu$, so, $y = \frac{H}{2\mu}$

Where, y = distance of the image of fish below the surface as seen by eye.

So, Direct image = H + y = $H+\frac{H}{2\mu} = H\left(1+\frac{1}{2\mu}\right)$

Again another image of fish will be formed H/2 below the mirror.

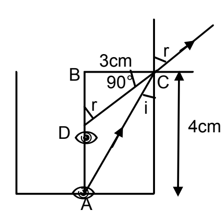
So, the real depth for that image of fish becomes H + H/2 = 3H/2

So, Apparent depth from the surface of water = $3H/2\mu$

So, distance of the image from the eye = $H+\frac{3H}{2\mu} = H(1+\frac{3}{2\mu})$.

23. According to the figure, x/3 = cot r …(1)

Again, $\frac{\sin i}{\sin r} = \frac{1}{1.33} = \frac{3}{4}$



$\Rightarrow$ sin r = $\frac{4}{3}\sin i = \frac{4}{3}\times\frac{3}{5} = \frac{4}{5}$ (because sin i = $\frac{BC}{AC} = \frac{3}{5}$)

$\Rightarrow$ cot r = 3/4 …(2)

From (1) and (2) $\Rightarrow$ x/3 = ¾

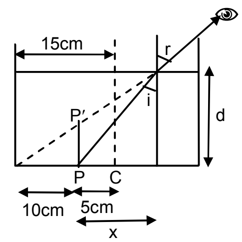
$\Rightarrow$ x = 9/4 = 2.25 cm.

$\therefore$ Ratio of real and apparent depth = 4 : (2.25) = 1.78.

24. For the given cylindrical vessel, dimetre = 30 cm

$\Rightarrow$ r = 15 cm and h = 30 cm

Now, $\frac{\sin i}{\sin r} = \frac{3}{4}\left[\mu_w = 1.33 = \frac{4}{3}\right]$



$\Rightarrow$ sin i = $3/4\sqrt{2}$ [because r = 45°]

The point P will be visible when the refracted ray makes angle 45° at point of refraction.

Let x = distance of point P from X.

Now, tan 45° = $\frac{x+10}{d}$

$\Rightarrow$ d = x + 10 …(1)

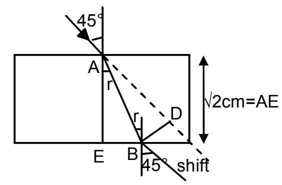
Again, tan i = x/d

$\Rightarrow \frac{3}{\sqrt{23}} = \frac{d-10}{d}$ $\left[\text{since, } \sin i = \frac{3}{4\sqrt{2}} \Rightarrow \tan i = \frac{3}{\sqrt{23}}\right]$

$\Rightarrow \frac{3}{\sqrt{23}} - 1 = -\frac{10}{d} \Rightarrow d = \frac{\sqrt{23}\times 10}{\sqrt{23}-3}$ = 26.7 cm.

25. As shown in the figure,

$\frac{\sin 45°}{\sin r} = \frac{2}{1} \Rightarrow \sin r = \frac{\sin 45°}{2} = \frac{1}{2\sqrt{2}} \Rightarrow r = 21°$



Therefore, θ = (45° – 21°) = 24°

Here, BD = shift in path = AB sin 24°

= $0.406 \times AB = \frac{AE}{\cos 21°} \times 0.406$ = 0.62 cm.

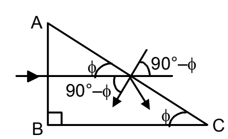
26. For calculation of critical angle,

$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin C}{\sin 90} = \frac{15}{1.72} = \frac{75}{86}$

$\Rightarrow$ C = $\sin^{-1}\left(\frac{75}{26}\right)$.

27. Let θ_c be the critical angle for the glass

$\frac{\sin\theta_c}{\sin 90°} = \frac{1}{x} \Rightarrow \sin\theta_c = \frac{1}{1.5} = \frac{2}{3} \Rightarrow \theta_c = \sin^{-1}\left(\frac{2}{3}\right)$



From figure, for total internal reflection, 90° – ϕ > θ_c

$\Rightarrow \phi < 90° - \theta_c \Rightarrow \phi < \cos^{-1}(2/3)$

So, the largest angle for which light is totally reflected at the surface is $\cos^{-1}(2/3)$.

28. From the definition of critical angle, if refracted angle is more than 90°, then reflection occurs, which is known as total internal reflection.

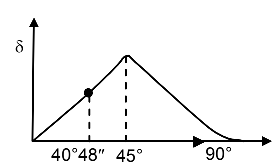
So, maximum angle of refraction is 90°.

29. Refractive index of glass μ_g = 1.5

Given, 0° < i < 90°

Let, C $\rightarrow$ Critical angle.

$\frac{\sin C}{\sin r} = \frac{\mu_a}{\mu_g} \Rightarrow \frac{\sin C}{\sin 90°} = \frac{1}{15}$ = 0.66

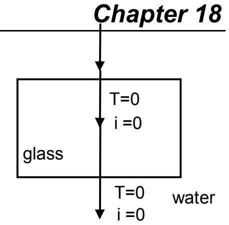


$\Rightarrow$ C = 40°48″

The angle of deviation due to refraction from glass to air increases as the angle of incidence increases from 0° to 40°48″. The angle of deviation due to total internal reflection further increases for 40°48″ to 45° and then it decreases.

30. μ_g = 1.5 = 3/2 ; μ_w = 1.33 = 4/3

For two angles of incidence,



1) When light passes straight through normal,

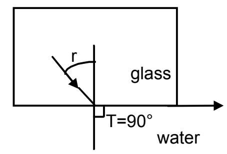
   $\Rightarrow$ Angle of incidence = 0°, angle of refraction = 0°, angle of deviation = 0

2) When light is incident at critical angle,

   $$\frac{\sin C}{\sin r} = \frac{\mu_w}{\mu_g} \quad \text{(since light passing from glass to water)}$$

   $\Rightarrow$ sin C = 8/9 $\Rightarrow$ C = $\sin^{-1}(8/9)$ = 62.73°.

   $\therefore$ Angle of deviation = 90° – C = 90° – $\sin^{-1}(8/9)$ = $\cos^{-1}(8/9)$ = 37.27°



Here, if the angle of incidence is increased beyond critical angle, total internal reflection occurs and deviation decreases. So, the range of deviation is 0 to $\cos^{-1}(8/9)$.

31. Since, μ = 1.5, Critial angle = $\sin^{-1}(1/\mu)$ = $\sin^{-1}(1/1.5)$ = 41.8°

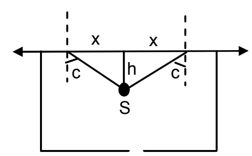
    We know, the maximum attainable deviation in refraction is (90° – 41.8°) = 47.2°

    So, in this case, total internal reflection must have taken place.

    In reflection,

    Deviation = 180° – 2i = 90° $\Rightarrow$ 2i = 90° $\Rightarrow$ i = 45°.

32. a) Let, x = radius of the circular area



$$\frac{x}{h} = \tan C \quad \text{(where C is the critical angle)}$$

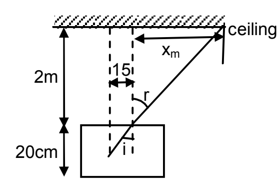
$$\Rightarrow \frac{x}{h} = \frac{\sin C}{\sqrt{1-\sin^2 C}} = \frac{1/\mu}{\sqrt{1-\frac{1}{\mu^2}}} \qquad \text{(because } \sin C = 1/\mu\text{)}$$

$$\Rightarrow \frac{x}{h} = \frac{1}{\sqrt{\mu^2-1}} \text{ or } x = \frac{h}{\sqrt{\mu^2-1}}$$

So, light escapes through a circular area on the water surface directly above the point source.

b) Angle subtained by a radius of the area on the source, C = $\sin^{-1}(1/\mu)$.

33. a) As shown in the figure, sin i = 15/25



$$\text{So, } \frac{\sin i}{\sin r} = \frac{1}{\mu} = \frac{3}{4}$$

$\Rightarrow$ sin r = 4/5

Again, x/2 = tan r (from figure)

$$\text{So, } \sin r = \frac{\tan r}{\sqrt{1+\tan^2 r}} = \frac{x/2}{\sqrt{1-x^2/4}}$$

$$\Rightarrow \frac{x}{\sqrt{4+x^2}} = \frac{4}{5}$$

$\Rightarrow$ $25x^2 = 16(4 + x^2) \Rightarrow 9x^2 = 64 \Rightarrow x = 8/3$ m

$\therefore$ Total radius of shadow = 8/3 + 0.15 = 2.81 m

b) For maximum size of the ring, i = critical angle = C

   Let, R = maximum radius

   $$\Rightarrow \sin C = \frac{\sin C}{\sin r} = \frac{R}{\sqrt{20^2+R^2}} = \frac{3}{4} \quad \text{(since, sin r = 1)}$$

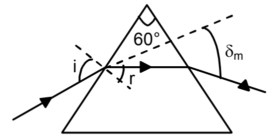
   $\Rightarrow 16R^2 = 9R^2 + 9 \times 400$

   $\Rightarrow 7R^2 = 9 \times 400$

   $\Rightarrow$ R = 22.67 cm.

34. Given, A = 60°, μ = 1.732

Since, angle of minimum deviation is given by,



$$\mu = \frac{\sin\left(\frac{A+\delta m}{2}\right)}{\sin A/2} \Rightarrow 1.732 \times ½ = \sin(30 + \delta m/2)$$

$\Rightarrow \sin^{-1}(0.866) = 30 + \delta m/2 \Rightarrow 60° = 30\ \delta m/2 \Rightarrow \delta m = 60°$

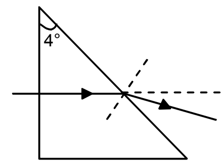
Now, $\delta m = i + i' - A$

$\Rightarrow$ 60° = i + i′ – 60° (δ = 60° minimum deviation)

$\Rightarrow$ i = 60°. So, the angle of incidence must be 60°.

35. Given μ = 1.5

And angle of prism = 4°



$$\therefore\ \mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{(A+\delta_m)/2}{(A/2)} \quad \text{(for small angle } \sin\theta = \theta)$$

$$\Rightarrow\ \mu = \frac{A+\delta_m}{2} \Rightarrow 1.5 = \frac{4°+\delta_m}{4°} \Rightarrow \delta_m = 4° \times (1.5) - 4° = 2°.$$

36. Given A = 60° and δ = 30°

We know that,

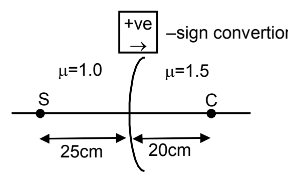
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin A/2} = \frac{\sin\frac{60°+\delta_m}{2}}{\sin 30°} = 2\sin\frac{60°+\delta_m}{2}$$

Since, one ray has been found out which has deviated by 30°, the angle of minimum deviation should be either equal or less than 30°. (It can not be more than 30°).

So, $\mu \le 2\sin\frac{60°+\delta_m}{2}$ (because μ will be more if δ_m will be more)

or, $\mu \le 2 \times 1/\sqrt{2}$ or, $\mu \le \sqrt{2}$.

37. $\mu_1 = 1$, $\mu_2 = 1.5$, R = 20 cm (Radius of curvature), u = –25 cm



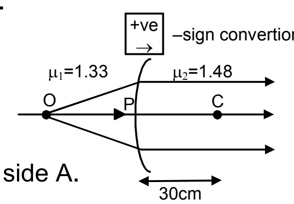
$$\therefore\ \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{25} = \frac{1}{40} - \frac{1}{25} = \frac{-3}{200}$$

$\Rightarrow$ v = –200 × 0.5 = –100 cm.

So, the image is 100 cm from (P) the surface on the side of S.

38. Since, paraxial rays become parallel after refraction i.e. image is formed at ∞.

$v = \infty$, $\mu_1 = 1.33$, u = ?, $\mu_2 = 1.48$, R = 30 cm



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{1.48}{\infty} - \frac{1.33}{u} = \frac{1.48-1.33}{30} \Rightarrow -\frac{1.33}{u} - \frac{0.15}{30}$$

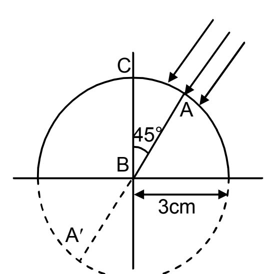
$\Rightarrow$ u = –266.0 cm

$\therefore$ Object should be placed at a distance of 266 cm from surface (convex) on side A.

39. Given, $\mu_2 = 2.0$

So, critical angle = $\sin^{-1}\left(\frac{1}{\mu_2}\right) = \sin^{-1}\left(\frac{1}{2}\right)$ = 30°

a) As angle of incidence is greater than the critical angle, the rays are totally reflected internally.



b) Here, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$

$\Rightarrow \frac{2}{v} - \left(-\frac{1}{\infty}\right) = \frac{2-1}{3}$ [For parallel rays, u = ∞]

$\Rightarrow \frac{2}{v} = \frac{1}{3} \Rightarrow$ v = 6 cm

$\Rightarrow$ If the sphere is completed, image is formed diametrically opposite of A.

c) Image is formed at the mirror in front of A by internal reflection.

40. a) Image seen from left :

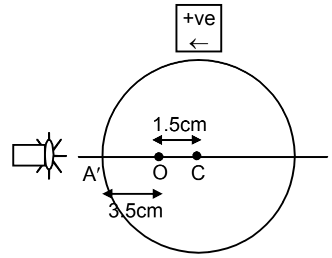
$u = (5 - 15) = -3.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{3.5} = -\frac{1-1.5}{5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{7} \Rightarrow v = \frac{-70}{23} = -3$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2 cm left to centre.



b) Image seen from right :

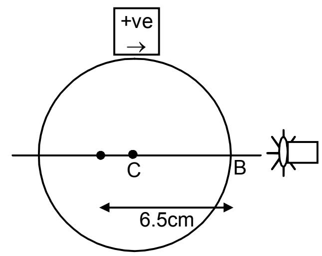
$u = -(5 + 1.5) = -6.5$ cm

$R = -5$ cm

$\therefore \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \frac{1}{v} + \frac{1.5}{6.5} = \frac{1-1.5}{-5}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{3}{13} \Rightarrow v = -\frac{130}{17} = -7.65$ cm (inside the sphere).

$\Rightarrow$ Image will be formed, 2.65 cm left to centre.



41. $R_1 = R_2 = 10$ cm, $t = 5$ cm, $u = -\infty$

For the first refraction, (at A)

$\frac{\mu_g}{v} - \frac{\mu_a}{u} = \frac{\mu_g - \mu_a}{R_1}$ or $\frac{1.5}{v} - 0 = \frac{1.5}{10}$

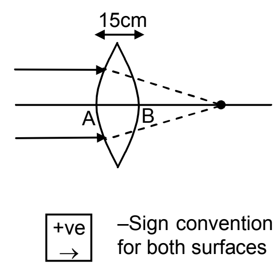
$\Rightarrow v = 30$ cm.

Again, for 2nd surface, $u = (30 - 5) = 25$ cm (virtual object)

$R_2 = -10$ cm

So, $\frac{1}{v} - \frac{15}{25} = \frac{-0.5}{-10} \Rightarrow v = 9.1$ cm.

So, the image is formed 9.1 cm further from the 2nd surface of the lens.



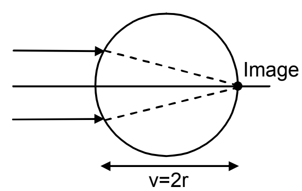
42. For the refraction at convex surface A.

$\mu = -\infty$, $\mu_1 = 1$, $\mu_2 = ?$

a) When focused on the surface, $v = 2r$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{2r} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = 2\mu_2 - 2 \Rightarrow \mu_2 = 2$



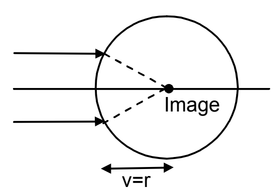
b) When focused at centre, $u = r_1$, $R = r$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{\mu_2}{R} = \frac{\mu_2 - 1}{r} \Rightarrow \mu_2 = \mu_2 - 1$.

This is not possible.

So, it cannot focus at the centre.



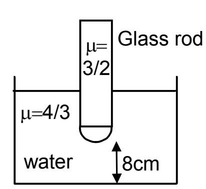
43. Radius of the cylindrical glass tube = 1 cm

We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

Here, $u = -8$ cm, $\mu_2 = 3/2$, $\mu_1 = 4/3$, $R = +1$ cm

So, $\frac{3}{2v} + \frac{4}{3 \times 8} \Rightarrow \frac{3}{2v} + \frac{1}{6} = \frac{1}{6}$ $v = \infty$

$\therefore$ The image will be formed at infinity.

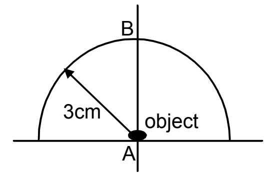


44. In the first refraction at A.

$\mu_2 = 3/2$, $\mu_1 = 1$, $u = 0$, $R = \infty$

So, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow$ v = 0 since ($R \Rightarrow \infty$ and u = 0)



$\therefore$ The image will be formed at the point, Now for the second refraction at B,

u = –3 cm, R = –3 cm, $\mu_1 = 3/2$, $\mu_2 = 1$

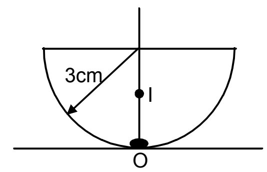
So, $\frac{1}{v} + \frac{3}{2 \times 3} = \frac{1-1.5}{-3} = \frac{1}{6}$

$\Rightarrow \frac{1}{v} = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$

$\Rightarrow$ v = –3 cm, $\therefore$ There will be no shift in the final image.

45. Thickness of glass = 3 cm, $\mu_g = 1.5$

Image shift = $3\left(1 - \frac{1}{1.5}\right)$



[Treating it as a simple refraction problem because the upper surface is flat and the spherical surface is in contact with the object]

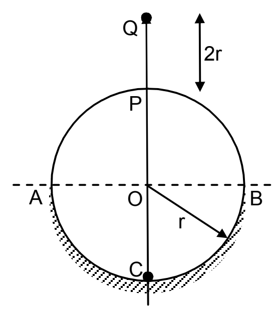
$= 3 \times \frac{0.5}{1.5}$ = 1 cm.

The image will appear 1 cm above the point P.

46. As shown in the figure, OQ = 3r, OP = r

So, PQ = 2r

For refraction at APB



We know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{1.5}{v} - \frac{1}{-2r} = \frac{0.5}{r} = \frac{1}{2r}$ [because u = –2r]

$\Rightarrow v = \infty$

For the reflection in concave mirror

$u = \infty$

So, v = focal length of mirror = r/2

For the refraction of APB of the reflected image.

Here, u = –3r/2

$\frac{1}{v} - \frac{1.5}{-3r/2} = \frac{-0.5}{-r}$ [Here, $\mu_1 = 1.5$ and $\mu_2 = 1$ and R = –r]

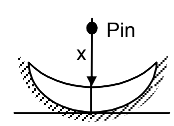
$\Rightarrow$ v = –2r

As, negative sign indicates images are formed inside APB. So, image should be at C.

So, the final image is formed on the reflecting surface of the sphere.

47. a) Let the pin is at a distance of x from the lens.

Then for 1st refraction, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$



Here $\mu_2 = 1.5$, $\mu_1 = 1$, u = –x, R = –60 cm

$\therefore \frac{1.5}{v} - \frac{1}{-x} = \frac{0.5}{-60}$

$\Rightarrow$ 120(1.5x + v) = –vx …(1)

$\Rightarrow$ v(120 + x) = –180x

$\Rightarrow v = \frac{-180x}{120 + x}$

This image distance is again object distance for the concave mirror.

$u = \dfrac{-180x}{120+x}$, f = –10 cm ($\therefore$ f = R/2)

$\therefore \ \dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f} \Rightarrow \dfrac{1}{v_1}=\dfrac{1}{-10}-\dfrac{-(120+x)}{180x}$

$\Rightarrow \ \dfrac{1}{v_1}=\dfrac{120+x-18x}{180x} \Rightarrow v_1 = \dfrac{180x}{120-17x}$

Again the image formed is refracted through the lens so that the image is formed on the object taken in the 1st refraction. So, for 2nd refraction.

According to sign conversion v = –x, μ_2 = 1, μ_1 = 1.5, R = –60

Now, $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$ $\qquad$ [$u = \dfrac{180x}{120-17x}$]

$\Rightarrow \ \dfrac{1}{-x}-\dfrac{1.5}{180x}(120-17x)=\dfrac{-0.5}{-60}$

$\Rightarrow \ \dfrac{1}{x}+\dfrac{120-17x}{120x}=\dfrac{-1}{120}$

Multiplying both sides with 120 m, we get

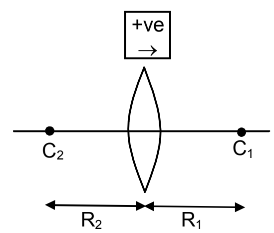
120 + 120 – 17x = –x

$\Rightarrow$ 16x = 240 $\Rightarrow$ x = 15 cm

$\therefore$ Object should be placed at 15 cm from the lens on the axis.

48. For the double convex lens

f = 25 cm, R_1 = R and R_2 = –2R (sign convention)



$\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$

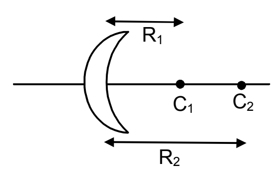
$\Rightarrow \ \dfrac{1}{25}=(15-1)\left(\dfrac{1}{R}-\dfrac{1}{-2R}\right) = 0.5\left(\dfrac{3R}{2}\right)$

$\Rightarrow \ \dfrac{1}{25}=\dfrac{3}{4}\dfrac{1}{R} \Rightarrow$ R = 18.75 cm

R_1 = 18.75 cm, R_2 = 2R = 37.5 cm.

49. R_1 = +20 cm ; R_2 = +30 cm ; μ = 1.6

a) If placed in air :



$\dfrac{1}{f}=(\mu_g-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) = \left(\dfrac{1.6}{1}-1\right)\left(\dfrac{1}{20}-\dfrac{1}{30}\right)$

$\Rightarrow$ f = 60/6 = 100 cm

b) If placed in water :

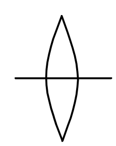
$\dfrac{1}{f}=(\mu_w-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) = \left(\dfrac{1.6}{1.33}-1\right)\left(\dfrac{1}{20}-\dfrac{1}{30}\right)$

$\Rightarrow$ f = 300 cm

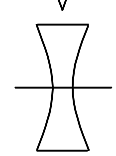
50. Given μ = 1.5

Magnitude of radii of curvatures = 20 cm and 30 cm

The 4types of possible lens are as below.



$\dfrac{1}{f}=(\mu-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$



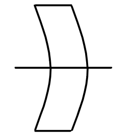
Case (1) : (Double convex) [R_1 = +ve, R_2 = –ve]

$\dfrac{1}{f}=(15-1)\left(\dfrac{1}{20}-\dfrac{1}{-30}\right) \Rightarrow$ f = 24 cm

Case (2) : (Double concave) [R_1 = –ve, R_2 = +ve]

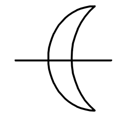
$\dfrac{1}{f}=(15-1)\left(\dfrac{-1}{20}-\dfrac{1}{30}\right) \Rightarrow$ f = –24 cm

Case (3) : (Concave concave) [R_1 = –ve, R_2 = –ve]



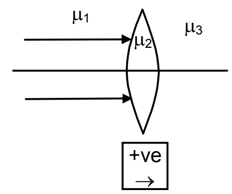
$$\frac{1}{f} = (15-1)\left(\frac{1}{-20} - \frac{1}{-30}\right) \Rightarrow f = -120 \text{ cm}$$

Case (4) : (Concave convex) [R_1 = +ve, R_2 = +ve]



$$\frac{1}{f} = (15-1)\left(\frac{1}{20} - \frac{1}{30}\right) \Rightarrow f = +120 \text{ cm}$$

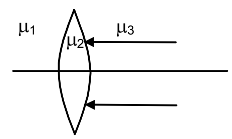
51. a) When the beam is incident on the lens from medium μ_1.



Then $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ or $\frac{\mu_2}{v} - \frac{\mu_1}{(-\infty)} = \frac{\mu_2 - \mu_1}{R}$

or $\frac{1}{v} = \frac{\mu_2 - \mu_1}{\mu_2 R}$ or v = $\frac{\mu_2 R}{\mu_2 - \mu_1}$

Again, for 2nd refraction, $\frac{\mu_3}{v} - \frac{\mu_2}{u} = \frac{\mu_3 - \mu_2}{R}$



or, $\frac{\mu_3}{v} = -\left[\frac{\mu_3 - \mu_2}{R} - \frac{\mu_2}{\mu_2 R}(\mu_2 - \mu_1)\right] \Rightarrow -\left[\frac{\mu_3 - \mu_2 - \mu_2 + \mu_1}{R}\right]$

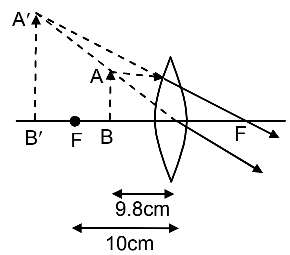
or, v = $-\left[\frac{\mu_3 R}{\mu_3 - 2\mu_2 + \mu_1}\right]$

So, the image will be formed at = $\frac{\mu_3 R}{2\mu_2 - \mu_1 - \mu_3}$

b) Similarly for the beam from μ_3 medium the image is formed at $\frac{\mu_1 R}{2\mu_2 - \mu_1 - \mu_3}$.

52. Given that, f = 10 cm

a) When u = –9.5 cm



(Virtual image)

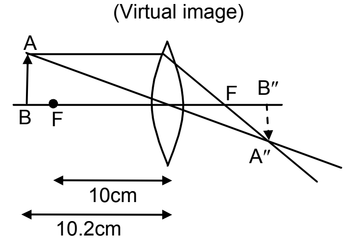
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{9.8} = \frac{-0.2}{98}$$

⇒ v = – 490 cm

So, ⇒ m = $\frac{v}{u} = \frac{-490}{-9.8}$ = 50 cm

So, the image is erect and virtual.

b) When u = –10.2 cm



(Real image)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{-10.2} = \frac{102}{0.2}$$

⇒ v = 510 cm

So, m = $\frac{v}{u} = \frac{510}{-9.8}$

The image is real and inverted.

53. For the projector the magnification required is given by

m = $\frac{v}{u} = \frac{200}{3.5}$ ⇒ u = 17.5 cm

[35 mm > 23 mm, so the magnification is calculated taking object size 35 mm]

Now, from lens formula,

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{1000} + \frac{1}{17.5} = \frac{1}{f}$$

⇒ f = 17.19 cm.

54. When the object is at 19 cm from the lens, let the image will be at, v_1.

$\Rightarrow \frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f} \Rightarrow \frac{1}{v_1} - \frac{1}{-19} = \frac{1}{12}$

$\Rightarrow v_1 = 32.57$ cm

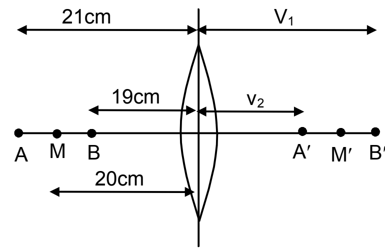
Again, when the object is at 21 cm from the lens, let the image will be at, v_2

$\Rightarrow \frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f} \Rightarrow \frac{1}{v_2} + \frac{1}{21} = \frac{1}{12}$

$\Rightarrow v_2 = 28$ cm

$\therefore$ Amplitude of vibration of the image is $A = \frac{A'B'}{2} = \frac{v_1 - v_2}{2}$

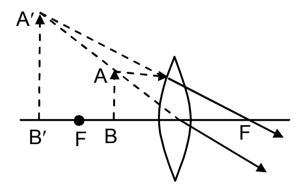
$\Rightarrow A = \frac{32.57 - 28}{2} = 2.285$ cm.



55. Given, u = –5 cm, f = 8 cm

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{8} - \frac{1}{5} = \frac{-3}{40}$

$\Rightarrow v = -13.3$ cm (virtual image).



56. Given that,

(–u) + v = 40 cm = distance between object and image

$h_o = 2$ cm, $h_i = 1$ cm

Since $\frac{h_i}{h_o} = \frac{v}{-u}$ = magnification

$\Rightarrow \frac{1}{2} = \frac{v}{-u} \Rightarrow u = -2v$ …(1)

Now, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{2v} = \frac{1}{f}$

$\Rightarrow \frac{3}{2v} = \frac{1}{f} \Rightarrow f = \frac{2v}{3}$ …(2)

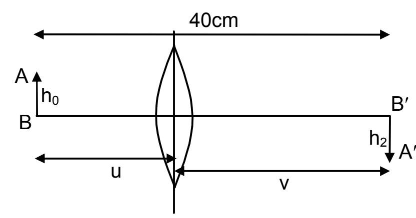
Again, (–u) + v = 40

$\Rightarrow 3v = 40 \Rightarrow v = 40/3$ cm

$\therefore f = \frac{2 \times 40}{3 \times 3} = 8.89$ cm = focal length

From eqn. (1) and (2)

u = –2v = –3f = –3(8.89) = 26.7 cm = object distance.



57. A real image is formed. So, magnification m = –2 (inverted image)

$\therefore \frac{v}{u} = -2 \Rightarrow v = -2u = (-2)(-18) = 36$

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{36} - \frac{1}{-18} = \frac{1}{f}$

$\Rightarrow f = 12$ cm

Now, for triple sized image m = –3 = (v/u)

$\therefore \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-3u} - \frac{1}{u} = \frac{1}{12}$

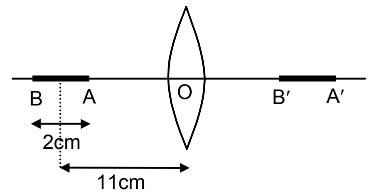
$\Rightarrow 3u = -48 \Rightarrow u = -16$ cm

So, object should be placed 16 cm from lens.

58. Now we have to calculate the image of A and B. Let the images be A′, B′. So, length of A′ B′ = size of image.

For A, u = –10 cm, f = 6 cm

Since, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}-\frac{1}{-10}=\frac{1}{6}$



$\Rightarrow$ v = 15 cm = OA′

For B, u = –12 cm, f = 6 cm

Again, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{6}-\frac{1}{12}$

$\Rightarrow$ v = 12 cm = OB′

$\therefore$A′B′ = OA′ – OB′ = 15 – 12 = 3 cm.

So, size of image = 3 cm.

59. u = -1.5×10^{11} m ; f = $+20 \times 10^{-2}$ m

Since, f is very small compared to u, distance is taken as ∞. So, image will be formed at focus.

$\Rightarrow$ v = $+20 \times 10^{-2}$ m

$\therefore$ We know, m = $\frac{v}{u}=\frac{h_{image}}{h_{object}}$

$\Rightarrow \frac{20\times 10^{-2}}{1.5\times 10^{11}}=\frac{D_{image}}{1.4\times 10^{9}}$

$\Rightarrow$ D_{image} = 1.86 mm

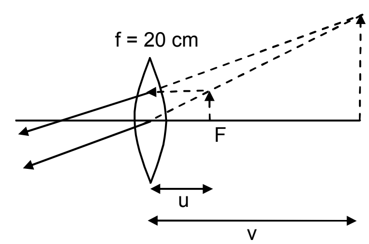
So, radius = $\frac{D_{image}}{2}$ = 0.93 mm.

60. Given, P = 5 diopter (convex lens)

$\Rightarrow$ f = 1/5 m = 20 cm

Since, a virtual image is formed, u and v both are negative.

Given, v/u = 4



$\Rightarrow$ v = 4u …(1)

From lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

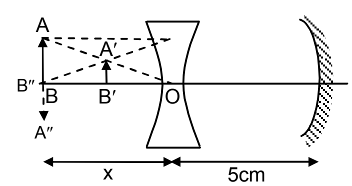
$\Rightarrow \frac{1}{f}=\frac{1}{4u}-\frac{1}{u} \Rightarrow \frac{1}{20}=\frac{1-4}{4u}=-\frac{3}{4u}$

$\Rightarrow$ u = –15 cm

$\therefore$ Object is placed 15 cm away from the lens.

61. Let the object to placed at a distance x from the lens further away from the mirror.

For the concave lens (1^{st} refraction)



u = –x, f = –20 cm

From lens formula,

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{-20}+\frac{1}{-x}$

$\Rightarrow$ v = $-\left(\frac{20x}{x+20}\right)$

So, the virtual image due to fist refraction lies on the same side as that of object. (A′B′)

This image becomes the object for the concave mirror.

For the mirror,

u = $-\left(5+\frac{20x}{x+20}\right)=-\left(\frac{25x+100}{x+20}\right)$

f = –10 cm

From mirror equation,

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=\frac{1}{-10}+\frac{x+20}{25x+100}$

$\Rightarrow v = \dfrac{50(x+4)}{3x-20}$

So, this image is formed towards left of the mirror.

Again for second refraction in concave lens,

$u = -\left[5 - \dfrac{50(x+4)}{3x-20}\right]$ (assuming that image of mirror is formed between the lens and mirro)

v = +x (Since, the final image is produced on the object)

Using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} + \frac{1}{5 - \dfrac{50(x+4}{3x-20}} = \frac{1}{-20}$$

$\Rightarrow$ x = 60 cm

The object should be placed at a distance 60 cm from the lens further away from the mirror.

So that the final image is formed on itself.

62. It can be solved in a similar manner like question no.61, by using the sign conversions properly. Left as an exercise for the student.

63. If the image in the mirror will form at the focus of the converging lens, then after transmission through the lens the rays of light will go parallel.

Let the object is at a distance x cm from the mirror

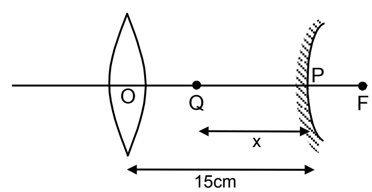
$\therefore$ u = –x cm ; v = 25 – 15 = 10 cm (because focal length of lens = 25 cm)

f = 40 cm

$$\Rightarrow \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{x} = \frac{1}{10} - \frac{1}{40}$$

$\Rightarrow$ x = 400/30 = 40/3

$\therefore$ The object is at distance $\left(15 - \dfrac{40}{3}\right) = \dfrac{5}{3}$ = 1.67 cm from the lens.



64. The object is placed in the focus of the converging mirror.

There will be two images.

a) One due to direct transmission of light through lens.

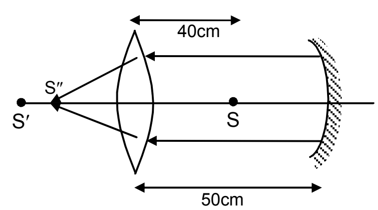
b) One due to reflection and then transmission of the rays through lens.

Case I : (S′) For the image by direct transmission,

u = –40 cm, f = 15 cm

$$\Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{15} + \frac{1}{-40}$$

$\Rightarrow$ v = 24 cm (left of lens)



Case II : (S″) Since, the object is placed on the focus of mirror, after reflection the rays become parallel for the lens.

So, u = ∞

$\Rightarrow$ f = 15 cm

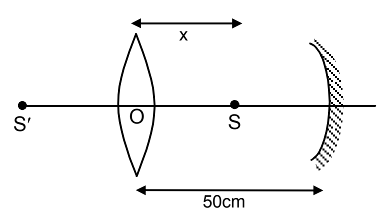
$\Rightarrow \dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow$ v = 15 cm (left of lens)

65. Let the source be placed at a distance 'x' from the lens as shown, so that images formed by both coincide.

For the lens, $\dfrac{1}{v_\ell} - \dfrac{1}{-x} = \dfrac{1}{15} \Rightarrow v_\ell = \dfrac{15x}{x-15}$ …(1)

Fro the mirror, u = –(50 – x), f = –10 cm

So, $\dfrac{1}{v_m} + \dfrac{1}{-(50-x)} = -\dfrac{1}{10}$



$\Rightarrow \frac{1}{v_m} = \frac{1}{-(50-x)} - \frac{1}{10}$

So, $v_m = \frac{10(50-x)}{x-40}$ …(2)

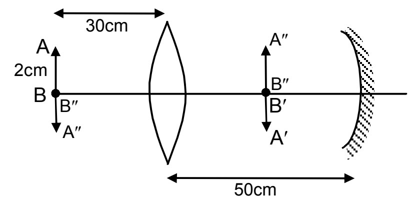
Since the lens and mirror are 50 cm apart,

$v_\ell - v_m = 50 \Rightarrow \frac{15x}{x-15} - \frac{10(50-x)}{(x-40)} = 50$

$\Rightarrow$ x = 30 cm.

So, the source should be placed 30 cm from the lens.

66. Given that, f_1 = 15 cm, F_m = 10 cm, h_o = 2 cm



The object is placed 30 cm from lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.

$\Rightarrow v = \frac{uf}{u+f}$

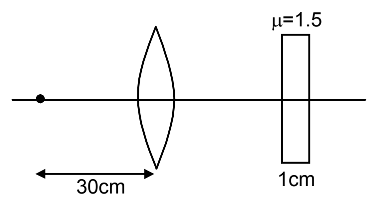
Since, u = –30 cm and f = 15 cm

So, v = 30 cm

So, real and inverted image (A′B′) will be formed at 30 cm from the lens and it will be of same size as the object. Now, this real image is at a distance 20 cm from the concave mirror. Since, f_m = 10 cm, this real image is at the centre of curvature of the mirror. So, the mirror will form an inverted image A″B″ at the same place of same size.

Again, due to refraction in the lens the final image will be formed at AB and will be of same size as that of object. (A‴B‴)

67. For the lens, f = 15 cm, u = –30 cm



From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} = \frac{1}{15} - \frac{1}{30} = \frac{1}{30} \Rightarrow$ v = 30 cm

The image is formed at 30 cm of right side due to lens only.

Again, shift due to glass slab is,

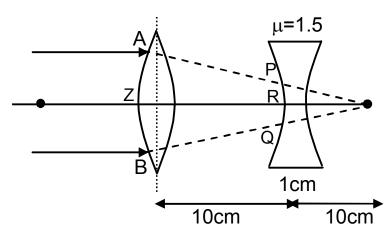
$= \Delta t = \left(1 - \frac{1}{15}\right)1$ [since, μ_g = 1.5 and t = 1 cm]

= 1 – (2/3) = 0.33 cm

$\therefore$ The image will be formed at 30 + 0.33 = 30.33 cm from the lens on right side.

68. Let, the parallel beam is first incident on convex lens.

d = diameter of the beam = 5 mm



Now, the image due to the convex lens should be formed on its focus (point B)

So, for the concave lens,

u = +10 cm (since, the virtual object is on the right of concave lens)

f = –10 cm

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{1}{10} = 0 \Rightarrow v = \infty$

So, the emergent beam becomes parallel after refraction in concave lens.

As shown from the triangles XYB and PQB,

$\frac{PQ}{XY} = \frac{RB}{ZB} = \frac{10}{20} = \frac{1}{2}$

So, PQ = ½ × 5 = 25 mm

So, the beam diameter becomes 2.5 mm.

Similarly, it can be proved that if the light is incident of the concave side, the beam diameter will be 1cm.

69. Given that, f_1 = focal length of converging lens = 30 cm

f_2 = focal length of diverging lens = –20 cm

and d = distance between them = 15 cm

Let, F = equivalent focal length

So, $\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{30} + \left(-\frac{1}{20}\right) - \left(\frac{15}{30(-200}\right) = \frac{1}{120}$

$\Rightarrow$ F = 120 cm

$\Rightarrow$ The equivalent lens is a converging one.

Distance from diverging lens so that emergent beam is parallel (image at infinity),

$d_1 = \frac{dF}{f_1} = \frac{15 \times 120}{30}$ = 60 cm

It should be placed 60 cm left to diverging lens

$\Rightarrow$ Object should be placed (120 – 60) = 60 cm from diverging lens.

Similarly, $d_2 = \frac{dF}{f_2} = \frac{15 \times 120}{20}$ = 90 cm

So, it should be placed 90 cm right to converging lens.

$\Rightarrow$ Object should be placed (120 + 90) = 210 cm right to converging lens.

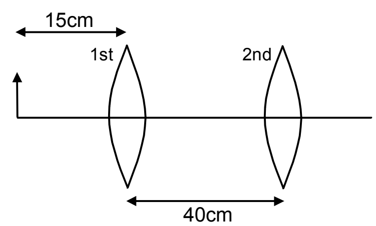
70. **a) First lens** :

u = –15 cm, f = 10 cm

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{15}\right) = -\frac{1}{10}$

$\Rightarrow$ v = 30 cm

So, the final image is formed 10 cm right of second lens.



b) m for 1st lens :

$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{30}{-15}\right) = \frac{h_{image}}{5mm}$

$\Rightarrow h_{image}$ = –10 mm (inverted)

Second lens :

u = –(40 – 30) = –10 cm ; f = 5 cm

[since, the image of 1st lens becomes the object for the second lens].

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \left(-\frac{1}{10}\right) = \frac{1}{5}$

$\Rightarrow$ v = 10 cm

m for 2nd lens :

$\frac{v}{u} = \frac{h_{image}}{h_{object}} \Rightarrow \left(\frac{10}{10}\right) = \frac{h_{image}}{-10}$

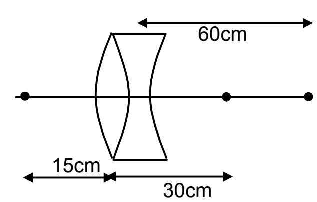
$\Rightarrow h_{image}$ = 10 mm (erect, real).

c) So, size of final image = 10 mm

71. Let u = object distance from convex lens = –15 cm

v_1 = image distance from convex lens when alone = 30 cm

f_1 = focal length of convex lens



Now, $\therefore \frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$

or, $\frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1}{30} + \frac{1}{15}$

or f_1 = 10 cm

Again, Let v = image (final) distance from concave lens = +(30 + 30) = 60 cm

v_1 = object distance from concave lens = +30 m

f_2 = focal length of concave lens

Now, $\therefore \frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_1}$

or, $\frac{1}{f_1} = \frac{1}{60} - \frac{1}{30} \Rightarrow f_2 = -60$ cm.

So, the focal length of convex lens is 10 cm and that of concave lens is 60 cm.

72. a) The beam will diverge after coming out of the two convex lens system because, the image formed by the first lens lies within the focal length of the second lens.

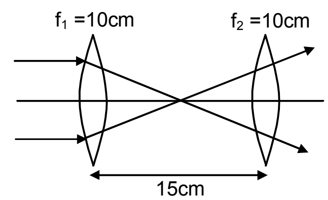
b) For 1st convex lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{10}$ (since, u = –∞)

or, v = 10 cm

for 2nd convex lens, $\frac{1}{v'} = \frac{1}{f} + \frac{1}{u}$

or, $\frac{1}{v'} = \frac{1}{10} + \frac{1}{-(15-10)} = \frac{-1}{10}$

or, v′ = –10 cm



So, the virtual image will be at 5 cm from 1st convex lens.

c) If, F be the focal length of equivalent lens,

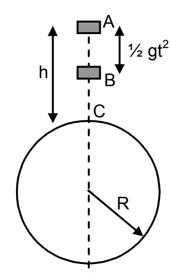
Then, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow \frac{1}{10} + \frac{1}{10} - \frac{15}{100} = \frac{1}{20}$

$\Rightarrow$ F = 20 cm.

73. Let us assume that it has taken time 't' from A to B.

$\therefore$ AB = $\frac{1}{2}gt^2$

$\therefore$ BC = h – $\frac{1}{2}gt^2$



This is the distance of the object from the lens at any time 't'.

Here, u = – (h – $\frac{1}{2}gt^2$)

$\mu_2 = \mu$(given) and μ_1 = i (air)

So, $\Rightarrow \frac{\mu}{v} - \frac{1}{-(h - \frac{1}{2}gf^2)} = \frac{\mu - 1}{R}$

$$\Rightarrow \frac{\mu}{v} = \frac{\mu-1}{R} - \frac{1}{(h - \frac{1}{2}gt^2)} = \frac{(\mu-1)(h - \frac{1}{2}gt^2) - R}{R(h - \frac{1}{2}gt^2)}$$

So, v = image distance at any time 't' = $\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$

So, velocity of the image = V = $\frac{dv}{dt} = \frac{d}{dt}\left[\frac{\mu R(h - \frac{1}{2}gt^2)}{(\mu-1)(h - \frac{1}{2}gt^2) - R}\right] = \frac{\mu R^2 gt}{(\mu-1)(h - \frac{1}{2}gt^2) - R}$ (can be found out).

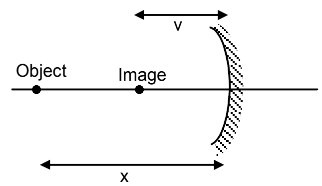
74. Given that, u = distance of the object = –x

f = focal length = –R/2

and, V = velocity of object = dx/dt

From mirror equation, $\frac{1}{-x}+\frac{1}{v}=-\frac{2}{R}$

$\frac{1}{v}=-\frac{2}{R}+\frac{1}{x}=\frac{R-2x}{Rx} \Rightarrow v = \frac{Rx}{R-2x}$ = Image distance



So, velocity of the image is given by,

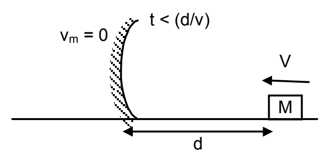
$$V_1 = \frac{dv}{dt}=\frac{[\frac{d}{dt}(xR)(R-2x)]-[\frac{d}{dt}(R-2x)][xR]}{(R-2x)^2}$$

$$= \frac{R[\frac{dx}{dt}(R-2x)]-[-2\frac{dx}{dt}x]}{(R-2x)^2}=\frac{R[v(R-2x)+2vx0}{(R-2x)^2}$$

$$= \frac{VR^2}{(2x-R)^2}=\frac{R[VR-2xV+2xV}{(R-2x)^2}.$$

75. a) When t < d/V, the object is approaching the mirror

As derived in the previous question,



$$V_{image} = \frac{\text{Velocity of object} \times R^2}{[2\times \text{distance between them } -R]^2}$$

$$\Rightarrow V_{image} = \frac{VR^2}{[2(d-Vt)-R]^2}$$ [At any time, x = d – Vt]

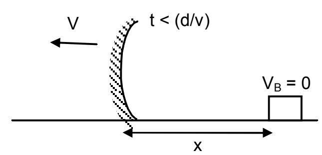
b) After a time t > d/V, there will be a collision between the mirror and the mass.

As the collision is perfectly elastic, the object (mass) will come to rest and the mirror starts to move away with same velocity V.

At any time t > d/V, the distance of the mirror from the mass will be

$$x = V\left(t-\frac{d}{V}\right)=Vt-d$$

Here, u = –(Vt – d) = d – Vt ; f = –R/2



So, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}=-\frac{1}{d-Vt}+\frac{1}{(-R/2)}=-\left[\frac{R+2(d-Vt)}{R(d-Vt)}\right]$

$\Rightarrow v = -\left[\frac{R(d-Vt)}{R-2(d-Vt)}\right]$ = Image distance

So, Velocity of the image will be,

$$V_{image} = \frac{d}{dt}(\text{Image distance}) = \frac{d}{dt}\left[\frac{R(d-Vt)}{R+2(d-Vt)}\right]$$

Let, y = (d – Vt)

$$\Rightarrow \frac{dy}{dt}=-V$$

So, $V_{image} = \frac{d}{dt}\left[\frac{Ry}{R+2y}\right]=\frac{(R+2y)R(-V)-Ry(+2)(-V)}{(R+2y)^2}$

$$= -Vr\left[\frac{R+2y-2y}{(R+2y)^2}\right]=\frac{-VR^2}{(R+2y)^2}$$

Since, the mirror itself moving with velocity V,

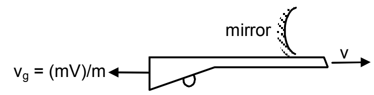
Absolute velocity of image = $V\left[1-\frac{R^2}{(R+2y)^2}\right]$ (since, V = $V_{mirror} + V_{image}$)

$$= V\left[1-\frac{R^2}{[2(Vt-d)-R^2}\right].$$

76. Recoil velocity of gun = $V_g = \frac{mV}{M}$.

At any time 't', position of the bullet w.r.t. mirror = $Vt + \frac{mV}{M}t = \left(1+\frac{m}{M}\right)Vt$

For the mirror, u = $-\left(1+\frac{m}{M}\right)Vt = kVt$



v = position of the image

From lens formula,

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} \Rightarrow \frac{1}{v} = \frac{1}{-f} + \frac{1}{kVt} = \frac{1}{kVt} - \frac{1}{f} = \frac{f - kVt}{kVtf}$$

Let $\left(1+\frac{m}{M} = k\right)$,

So, v = $\frac{kVft}{-kVt + f} = \left(\frac{kVtf}{f - kVt}\right)$

So, velocity of the image with respect to mirror will be,

$$v_1 = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{kVtf}{f - kVt}\right] = \frac{(f - kVt)kVf - kVtf(-kV)}{(f - kVt)^2} = \frac{kVt^2}{(f - kVt)^2}$$

Since, the mirror itself is moving at a speed of mV/M and the object is moving at 'V', the velocity of separation between the image and object at any time 't' will be,

$$v_s = V + \frac{mV}{M} + \frac{kVf^2}{(f - kVt)^2}$$

When, t = 0 (just after the gun is fired),

$$v_s = V + \frac{mV}{M} + kV = V + \frac{m}{M}V + \left(1+\frac{m}{M}\right)V = 2\left(1+\frac{m}{M}\right)V$$

77. Due to weight of the body suppose the spring is compressed by which is the mean position of oscillation.

$m = 50 \times 10^{-3}$ kg, g = 10 ms^{-2}, k = 500 Nm^{-2}, h = 10 cm = 0.1 m

For equilibrium, mg = kx $\Rightarrow$ x = mg/k = 10^{-3}m = 0.1 cm

So, the mean position is at 30 + 0.1 = 30.1 cm from P (mirror).

Suppose, maximum compression in spring is δ.

Since, E.K.E. – I.K.E. = Work done

$\Rightarrow 0 - 0 = mg(h + \delta) - \frac{1}{2}k\delta^2$ (work energy principle)

$\Rightarrow mg(h + \delta) = \frac{1}{2}k\delta^2 \Rightarrow 50 \times 10^{-3} \times 10(0.1 + \delta) = \frac{1}{2}\, 500\, \delta^2$

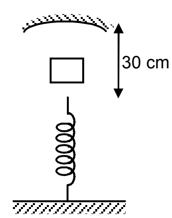


Fig-A

So, $\delta = \frac{0.5 \pm \sqrt{0.25 + 50}}{2 \times 250}$ = 0.015 m = 1.5 cm.

From figure B,

Position of B is 30 + 1.5 = 31.5 cm from pole.

Amplitude of the vibration = 31.5 – 30.1 – 1.4.

Position A is 30.1 – 1.4 = 28.7 cm from pole.

For A u = –31.5, f = –12 cm

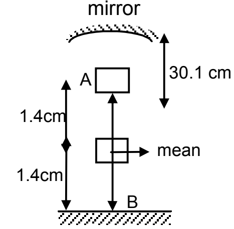


Fig-B

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{31.5}$$

$\Rightarrow v_A = -19.38$ cm

For B f = –12 cm, u = –28.7 cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{12} + \frac{1}{28.7}$$

$\Rightarrow v_B = -20.62$ cm

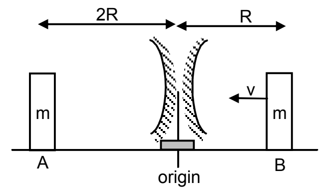
The image vibrates in length (20.62 – 19.38) = 1.24 cm.

78. a) In time, t = R/V the mass B must have moved (v × R/v) = R closer to the mirror stand

So, For the block B :

u = –R, f = –R/2

$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{2}{R} + \frac{1}{R} = -\frac{1}{R}$



$\Rightarrow$ v = –R at the same place.

For the block A : u = –2R, f = –R/2

$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{-2}{R} + \frac{1}{2R} = \frac{-3}{2R}$

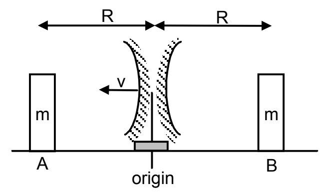
$\Rightarrow$ v = $\frac{-2R}{3}$ image of A at $\frac{2R}{3}$ from PQ in the x-direction.

So, with respect to the given coordinate system,

$\therefore$ Position of A and B are $\frac{-2R}{3}$, R respectively from origin.

b) When t = 3R/v, the block B after colliding with mirror stand must have come to rest (elastic collision) and the mirror have travelled a distance R towards left form its initial position.

So, at this point of time,



For block A :

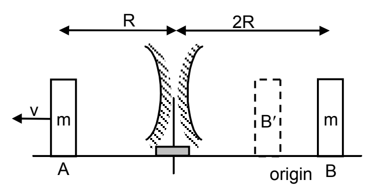
u = –R, f = –R/2

Using lens formula, v = –R (from the mirror),

So, position x_A = –2R (from origin of coordinate system)

For block B :

Image is at the same place as it is R distance from mirror. Hence, position of image is '0'.



Distance from PQ (coordinate system)

$\therefore$ positions of images of A and B are = –2R, 0 from origin.

c) Similarly, it can be proved that at time t = 5R/v,

the position of the blocks will be –3R and –4R/3 respectively.

79. Let a = acceleration of the masses A and B (w.r.t. elevator). From the freebody diagrams,

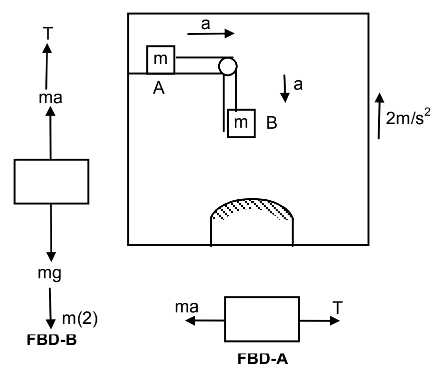
T – mg + ma – 2m = 0 …(1)

Similarly, T – ma = 0 …(2)

From (1) and (2), 2ma – mg – 2m = 0

$\Rightarrow$ 2ma = m(g + 2)

$\Rightarrow$ a = $\frac{10+2}{2} = \frac{12}{2}$ = 6 ms^{-2}



so, distance travelled by B in t = 0.2 sec is,

s = $\frac{1}{2}at^2 = \frac{1}{2} \times 6 \times (0.2)^2$ = 0.12 m = 12 cm.

So, Distance from mirror, u = –(42 – 12) = –30 cm ; f = +12 cm

From mirror equation, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \left(-\frac{1}{30}\right) = \frac{1}{12}$

$\Rightarrow$ v = 8.57 cm

Distance between image of block B and mirror = 8.57 cm.

♠ ♠ ♠