CHAPTER 17

LIGHT WAVES

17.1 WAVES OR PARTICLES

The question whether light is a wave or a particle has a very interesting and a long history. The investigations about the nature of light has unfolded a huge treasure of knowledge and understanding. This question has great contributions to the development of the theory of quantum mechanics which presents an altogether different picture of the world in which there are no particles, no positions, no momenta in an ordinary sense.

Newton, the greatest among the great, believed that light is a collection of particles. He believed that a light source emits tiny corpuscles of light and these corpuscles travel in straight lines when not acted upon by external forces. The fact that light seems to travel in straight lines and cast shadows behind the obstacles was perhaps the strongest evidence of the particle nature of light. Newton could explain the laws of reflection of light on the basis of elastic collisions of the particles of light with the surface it is incident upon. The laws of refraction were explained by assuming that the particles of denser medium, such as glass or water, strongly attract the particles of light causing a bending at the surface. Newton had performed a number of experiments to study the behaviour of light. The book *Optiks* written by him, gives a classic account of these experiments.

The Dutch physicist Christian Huygens (1629-1695), who was a contemporary of Newton, suggested that light may be a wave phenomenon. The apparent rectilinear propagation of light may be due to the fact that the wavelength of light may be much smaller than the dimensions of these openings and obstacles.

Huygens' proposal remained in a dump for almost about a century. The scientific community by and large had great faith in Newton's writings and the particle theory remained in chair for a long time when it was seriously challenged by the double-slit experiment of Thomas Young (1773-1829) in 1801. This experiment clearly established that light coming from two coherent sources interferes and produces maxima and minima depending on the path difference. A series of experiments on diffraction of light conducted by the French physicist Augustin Jean Fresnel (1788-1827), measurement of velocity of light in water by Foucault in 1850, development of theory of electromagnetic waves by Maxwell in 1860 which correctly predicted the speed of light, were parts of a long activity which put the corpuscle theory of light to an end and convincingly established that light is a wave phenomenon.

But the drama was not yet over. The climax came when the wave theory of light failed to explain Hallwachs and Lenard's observation in 1900 that when light falls on a metal surface, electrons are ejected and that the kinetic energy of the emitted electrons does not depend on the intensity of the light used. Hertz possibly had the first observation of this phenomenon in early 1880's. This observation is known as *photoelectric effect* and we shall study it in detail in a later chapter. Photoelectric effect was explained by another giant, Albert Einstein, in 1905 on a particle model of light only. The old question "waves or particles" was reopened and an amicable understanding was reached in accepting that light has dual character. It can behave as particles as well as waves depending on its interaction with the surrounding. Later, it was found that even the well-established particles such as electrons also have a dual character and can show interference and diffraction under suitable situations. We shall study the wave particle duality in a later chapter. In this chapter, we shall study the wave aspect of light.

17.2 THE NATURE OF LIGHT WAVES

In a wave motion, there is some quantity which changes its value with time and space. In the wave on a string, it is the transverse displacement of the particles that changes with time and is different for different particles at the same instant. In the case of sound waves, it is the pressure at a point in the medium that oscillates as time passes and has different values at different points at the same instant. We also know that it is the elastic properties of the medium that is responsible for the propagation of disturbance in a medium. The speed of a wave is determined by the elastic as well as the inertia properties of the medium.

The case with light waves is a bit different. The light waves need no material medium to travel. They can propagate in vacuum. Light is a nonmechanical wave. It was very difficult for the earlier physicists to conceive a wave propagating without a medium. Once the interference and diffraction experiments established the wave character of light, the search began for the medium responsible for the propagation of light waves. Light comes from the sun to the earth crossing millions of kilometers where apparently there is no material medium. What transmits the wave through this region? Physicists assumed that a very dilute and highly elastic material medium is present everywhere in space and they named it "ether". Ether was never discovered and today we understand that light wave can propagate in vacuum.

The quantity that changes with space and time, in terms of which the wave equation should be written, is the electric field existing in space where light travels. We shall define and study electric field in later chapters, here we need only to know that (a) the electric field is a vector quantity and (b) the electric field is transverse to the direction of propagation of light (there are exceptions but we shall not discuss them).

Because light waves are transverse, they can be polarized. If a plane wave of light is travelling along the x-direction, the electric field may be along the y-direction or along the z-direction or in any other direction in the y-z plane. The equation of such a light wave may be written as

$$E = E_0 \sin \omega(t - x/v), \quad \dots \text{(17.1)}$$

where E_0 is the magnitude of the electric field at point x at time t. The speed of light is itself an interesting quantity. The speed of light in vacuum with respect to any observer is always the same and is very nearly equal to 3×10^8 m/s. This speed is a fixed universal constant irrespective of the state of motion of the observer. This needs a basic revision of our concepts about space and time and is the basis of special theory of relativity. The speed of light in vacuum is generally denoted by the letter c. When a light wave travels in a transparent material, the speed is decreased by a factor μ, called the *refractive index* of the material.

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the material}}. \quad \dots \text{(17.2)}$$

For a spherical wave originating from a point source, the equation of the wave is of the form

$$E = \frac{aE_0}{r} \sin \omega(t - r/v)$$

where a is a constant.

The amplitude is proportional to the inverse of the distance and thus the intensity is proportional to the square of the inverse distance.

Example 17.1

The refractive index of glass is 1·5. Find the speed of light in glass.

Solution : We have

$$\mu = \frac{\text{speed of light in vacuum}}{\text{speed of light in the material}}$$

Thus, speed of light in glass

$$= \frac{\text{speed of light in vacuum}}{\mu}$$

$$= \frac{3{\cdot}0 \times 10^8 \text{ m/s}}{1{\cdot}5} = 2{\cdot}0 \times 10^8 \text{ m/s}.$$

The frequency of visible light varies from about 3800×10^{11} Hz to about 7800×10^{11} Hz. The corresponding wavelengths (obtained from $\lambda = c/\nu$) are 380 nm to 780 nm. The colour sensation to a human eye is related to the wavelength of the light. Light of wavelength close to 780 nm appears red, and that close to 380 nm appears violet. Table (17.1) shows a rough relationship between the colour sensed and the wavelength of light.

Table 17.1

Colour	Wavelength (order)
Red	620-780 nm
Orange	590-620 nm
Yellow	570-590 nm
Green	500-570 nm
Blue	450-500 nm
Violet	380-450 nm

Light of single wavelength is called *monochromatic light*. Equation (17.1) represents a monochromatic light wave. Often, the light emitted by a source is a mixture of light corresponding to different wavelengths. Depending on the composition of the mixture, a human eye senses a large number of colours. White light itself is a mixture of light of all wavelengths from about 380 nm to about 780 nm in appropriate proportion.

In fact, a strictly monochromatic light is not possible to obtain. There is always a spread in wavelength. The best monochromatic light are LASERs in which the spread in wavelength is very small but not zero. We shall use the word "monochromatic light" to mean that the light contains a dominant wavelength with only a little spread.

We discussed in the previous chapter that if a wave is obstructed during its propagation by an obstacle or an opening, it gets diffracted. A plane wave going through a small opening becomes more like a spherical wave on the other side. Thus, the wave bends at the edges. Also, if the dimensions of the obstacle or the opening is much larger than the wavelength, the diffraction is negligible and the rays go along straight lines.

In the case of light, the wavelength is around 380-780 nm. The obstacles or openings encountered in normal situations are generally of the order of millimeters or even larger. Thus, the wavelength is several thousands times smaller than the usual obstacles or openings. The diffraction is almost negligible and the light waves propagate in straight lines and cast shadows of the obstacles. The light can then be treated as light rays which are straight lines drawn from the source and which terminate at an opaque surface and which pass through an opening undeflected. This is known as the *Geometrical optics* approximation and majority of the phenomena in normal life may be discussed in this approximation. The three major rules governing geometrical optics are the following.

1. *Rectilinear propagation of light* : Light travels in straight lines unless it is reflected by a polished surface or the medium of propagation is changed.

2. *Reflection of light* : The angle of incidence and the angle of reflection (i.e., the angles made by the incident and the reflected rays with the normal to the surface) are equal. Also, the incident ray, the reflected ray and the normal to the reflecting surface are coplanar.

3. *Refraction of light* : When light travelling in one medium enters another medium, the angle of incidence i and the angle of refraction r (angle made by the refracted ray with the normal) satisfy

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2},$$

where v_1 and v_2 are the speeds of light in the first and the second media respectively. Also, the incident ray, the refracted ray and the normal to the separating surface are coplanar.

The rectilinear propagation of light is explained on the basis of wave theory by observing that the wavelength of light is much smaller than the obstacles or openings usually encountered. The laws of reflection and refraction can also be explained by wave theory. The rigorous derivation involves somewhat complicated mathematics, but things can be fairly well-understood by a geometrical method proposed by Huygens. This method tells us how to construct the shape of a wavefront of light wave from the given shape at an earlier instant. We refer again from the previous chapters that (a) a surface on which the wave disturbance is in same phase at all points is called a wavefront, (b) the direction of propagation of a wave at a point is perpendicular to the wavefront through that point, (c) the wavefronts of a wave originating from a point source are spherical and (d) the wavefronts for a wave going along a fixed direction are planes perpendicular to that direction.

17.3 HUYGENS' PRINCIPLE

The first proposer of the wave theory of light, Huygens, considered light to be a mechanical wave moving in a hypothetical medium which was named as ether. If we consider a surface σ enclosing a light source S, the optical disturbance at any point beyond σ must reach after crossing σ. The particles of the surface σ vibrate as the wave from S reaches there and these vibrations cause the layer beyond to vibrate. We can thus assume that the particles on σ act as new sources of light waves emitting spherical waves and the disturbance at a point A (figure 17.1) beyond σ, is caused by the superposition of all these spherical waves coming from different points of σ. Huygens called the particles spreading the vibration beyond them as *secondary sources* and the spherical wavefronts emitted from these secondary sources as the *secondary wavelets.*

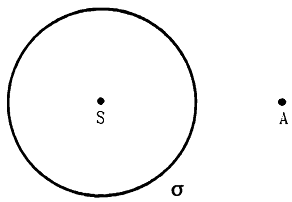


Figure 17.1

Huygens' principle may be stated in its most general form as follows :

Various points of an arbitrary surface, when reached by a wavefront, become secondary sources of light emitting secondary wavelets. The disturbance beyond the surface results from the superposition of these secondary wavelets.

Consider a spherical surface σ with its centre at a point source S emitting a pulse of light (figure 17.2).

The optical disturbance reaches the particles on σ at time $t = 0$ and lasts for a short interval in which the positive and negative disturbances are produced. These particles on σ then send spherical wavelets which spread beyond σ. At time t, each of these wavelets has a radius vt. In figure (17.2), the solid lines represent positive optical disturbance and the dashed lines represent negative optical disturbance. The sphere Σ is the geometrical envelope of all the secondary wavelets which were emitted at time $t = 0$ from the primary wavefront σ.

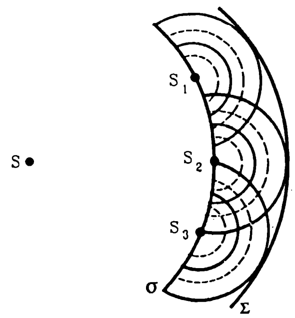


Figure 17.2

It is clear that at the points just inside Σ, only the positive disturbances of various secondary wavelets are meeting. The wavelets, therefore, interfere constructively at these points and produce finite disturbance. For points well inside Σ, some of the wavelets contribute positive disturbance and some others, centred at a nearby point of σ, produce negative disturbance. Thus, the resultant disturbance is zero at these points. The disturbance which was situated at σ at time $t = 0$ is, therefore, confined to a surface Σ at time t. Hence, the secondary wavelets from σ superpose in such a way that they produce a new wavefront at the geometrical envelope of the secondary wavelets.

This allows us to state the method of Huygens' construction as follows :

Huygens' Construction

Various points of an arbitrary surface, as they are reached by a wavefront, become the sources of secondary wavelets. The geometrical envelope of these wavelets at any given later instant represents the new position of the wavefront at that instant.

The method is quite general and although it was developed on the notion of mechanical waves it is valid for light waves. The surface used in the Huygens construction may have any arbitrary shape, not necessarily a wavefront itself. If the medium is homogeneous, (i.e., the optical properties of the medium are same everywhere) light moves forward and does not reflect back. We assume, therefore, that the secondary wavelets are emitted only in the forward direction and the geometrical envelope of the wavelets is to be taken in the direction of advancement of the wave. If there is a change of medium, the wave may be reflected from the discontinuity just as a wave on a string is reflected from a fixed end or a free end. In that case, secondary wavelets on the backward side should also be considered.

Reflection of Light

Let us suppose that a parallel light beam is incident upon a reflecting plane surface σ such as a plane mirror. The wavefronts of the incident wave will be planes perpendicular to the direction of incidence. After reflection, the light returns in the same medium. Consider a particular wavefront AB of the incident light wave at $t = 0$ (figure 17.3). We shall construct the position of this wavefront at time t.

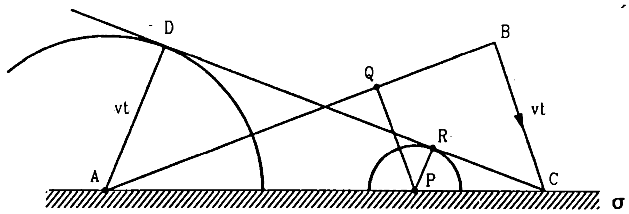


Figure 17.3

To apply Huygens' construction, we use the reflecting surface σ for the sources of secondary wavelets. As the various points of σ are reached by the wavefront AB, they become sources of secondary wavelets. Because of the change of medium, the wavelets are emitted both in forward and backward directions. To study reflection, the wavelets emitted in the backward directions are to be considered.

Suppose, the point A of σ is reached by the wavefront AB at time $t = 0$. This point then emits a secondary wavelet. At time t, this wavelet becomes a hemispherical surface of radius vt centred at A. Here v is the speed of light. Let C be the point which is just reached by the wavefront at time t and hence the wavelet is a point at C itself. Draw the tangent plane CD from C to the hemispherical wavelet originated from A. Consider an arbitrary point P on the surface and let $AP/AC = x$. Let PQ be the perpendicular from P to AB and let PR be the perpendicular from P to CD. By the figure,

$$\frac{PR}{AD} = \frac{PC}{AC} = \frac{AC - AP}{AC} = 1 - x$$

or, $$PR = AD\,(1 - x) = vt(1 - x). \quad \text{... (i)}$$

Also, $$\frac{QP}{BC} = \frac{AP}{AC} = x$$

or, $QP = x\,BC = xvt.$

The time taken by the wavefront to reach the point P is, therefore,

$$t_1 = \frac{QP}{v} = xt.$$

The point P becomes a source of secondary wavelets at time t_1. The radius of the wavelet at time t, originated from P is, therefore,

$$a = v(t - t_1) = v(t - xt) = vt(1 - x). \quad \text{... (ii)}$$

By (i) and (ii), we see that PR is the radius of the secondary wavelet at time t coming from P. As CD is perpendicular to PR, CD touches this wavelet. As P is an arbitrary point on σ, all the wavelets originated from different points of σ, touch CD at time t. Thus, CD is the envelope of all these wavelets at time t. It is, therefore, the new position of the wavefront AB. The reflected rays are perpendicular to this wavefront CD.

In triangles ABC and ADC :

$$AD = BC = vt,$$

AC is common,

and $\angle ADC = \angle ABC = 90°.$

Thus, the triangles are congruent and

$$\angle BAC = \angle DCA. \quad \text{... (iii)}$$

Now, the incident ray is perpendicular to AB and the normal is perpendicular to AC. The angle between the incident ray and the normal is, therefore, equal to the angle between AB and AC. Thus, $\angle BAC$ is equal to the angle of incidence.

Similarly, $\angle DCA$ represents the angle of reflection and we have proved in (iii) that the angle of incidence equals the angle of reflection. From the geometry, it is clear that the incident ray, the reflected ray and the normal to the surface AC lie in the plane of drawing and hence, are coplanar.

Refraction of Light

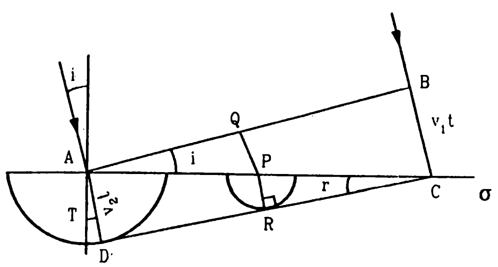


Figure 17.4

Suppose σ represents the surface separating two transparent media, medium 1 and medium 2 in which the speeds of light are v_1 and v_2 respectively. A parallel beam of light moving in medium 1 is incident on the surface and enters medium 2. In figure (17.4), we show the incident wavefront AB in medium 1 at $t = 0$. The incident rays are perpendicular to this wavefront. To find the position of this wavefront after refraction, we apply the method of Huygens' construction to the surface σ. The point A of the surface is reached by the wavefront AB at $t = 0$. This point becomes the source of secondary wavelet which expands in medium 2 at velocity v_2. At time t, this takes the shape of a hemisphere of radius $v_2 t$ centred at A. The point C of the surface is just reached by the wavefront at time t and hence, the wavelet is a point at C itself. Draw the tangent plane CD from C to the wavelet originating from A.

Consider an arbitrary point P on the surface σ and let $AP/AC = x$. Let PQ and PR be the perpendiculars from this arbitrary point P to the planes AB and CD respectively. By the figure,

$$\frac{PR}{AD} = \frac{PC}{AC} = \frac{AC - AP}{AC} = 1 - x$$

or, $$PR = AD(1 - x) = v_2\,t(1 - x). \quad \text{... (i)}$$

Also, $$\frac{QP}{BC} = \frac{AP}{AC} = x.$$

Thus, $$QP = x\,BC = xv_1\,t.$$

The time, at which the wavefront arrives at P, is

$$t_1 = \frac{QP}{v_1} = xt.$$

The radius of the wavelet originated from P and going into the second medium is, therefore,

$$a = v_2(t - t_1) = v_2\,t(1 - x). \quad \text{... (ii)}$$

By (i) and (ii), we see that PR is the radius of the wavelet originating from P. As CD is perpendicular to PR, CD touches this wavelet. As P is an arbitrary point on σ, all the wavelets which originated from different points on σ touch CD at time t. The plane CD is, therefore, the geometrical envelope of all the secondary wavelets at time t. It is, therefore, the position of the wavefront AB at time t. The refracted rays are perpendicular to CD.

The angle BAC is also equal to the angle between the incident ray (which is perpendicular to AB) and the normal to the surface and hence, it is equal to the angle of incidence i. Similarly, $\angle ACD$ is equal to the angle of refraction r.

We have $\sin i = \dfrac{BC}{AC}$

and $\sin r = \dfrac{AD}{AC}$

so that

$$\frac{\sin i}{\sin r} = \frac{BC}{AD} = \frac{v_1\,t}{v_2\,t}$$

or, $$\frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

which is called the *Snell's law*. The ratio v_1/v_2 is called the refractive index of medium 2 with respect to medium 1 and is denoted by μ_{21}. If the medium 1 is vacuum, μ_{21} is simply the refractive index of the medium 2 and is denoted by μ.

Also, $$\mu_{21} = \frac{v_1}{v_2} = \frac{c/v_2}{c/v_1} = \frac{\mu_2}{\mu_1}.$$

From the figure, it is clear that the incident ray, the refracted ray and the normal to the surface σ are all in the plane of the drawing i.e., they are coplanar.

Suppose light from air is incident on water. It bends towards the normal giving $i > r$. From Snell's law proved above, $v_1 > v_2$. Thus, according to the wave theory the speed of light should be greater in air than in water. This is opposite to the prediction of Newton's corpuscle theory. If light bends due to the attraction of the particles of a medium then speed of light should be greater in the medium. Later, experiments on measurement of speed of light confirmed wave theory.

Thus, the basic rules of geometrical optics could be understood in terms of the wave theory of light using Huygens' principle. In the rest of this chapter, we shall study the wave behaviour, such as the interference, diffraction and polarization of light.

17.4 YOUNG'S DOUBLE HOLE EXPERIMENT

Thomas Young in 1801 reported his experiment on the interference of light. He made a pinhole in a cardboard and allowed sunlight to pass through. This light was then allowed to fall upon another cardboard having two pinholes side by side placed symmetrically. The emergent light was received on a plane screen placed at some distance. At a given point on the screen, the waves from the two holes had different phases. These waves interfered to give a pattern of bright and dark areas. The variation of intensity on the screen demonstrated the interference taking place between the light waves reaching the screen from the two pinholes.

The pattern of bright and dark areas is sharply defined only if light of a single wavelength is used. Young's original experiments were performed with white light and he deduced from the experiments that the wavelength of extreme red light was around 1/36000 inch and that of the extreme violet was around 1/60000 inch. These results are quite close to their accurate measurements done with modern instruments.

17.5 YOUNG'S DOUBLE SLIT EXPERIMENT

In the double slit experiment, we use two long parallel slits as the sources of light in place of pin holes. The light coming out of the two slits is intercepted on a screen placed parallel to the plane of the slits. The slits are illuminated by a parallel beam of a monochromatic (of nearly a single wavelength) light. A series of dark and bright strips, called *fringes*, are observed on the screen. The arrangement of the experiment is schematically shown in figure (17.5a). Figure (17.5b) shows a cross-section of the arrangement of the Young's double slit experiment.

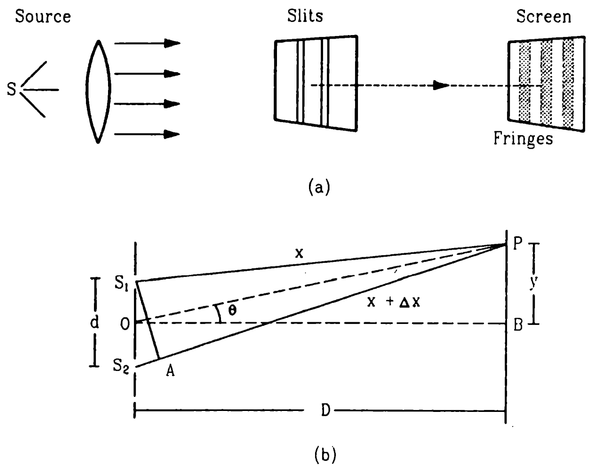


Figure 17.5

The two waves interfering at P have covered different distances $S_1P = x$ and $S_2P = x + \Delta x$. The electric fields at P due to the two waves may be written as

$$E_1 = E_{01} \sin(kx - \omega t)$$

and $$E_2 = E_{02} \sin[k(x + \Delta x) - \omega t]$$

$$= E_{02} \sin[kx - \omega t + \delta]$$

where $$\delta = k\ \Delta x = \frac{2\pi}{\lambda} \Delta x. \quad \text{... (17.3)}$$

The situation is mathematically identical to that discussed in chapter 15, section 15.7. The resultant field at the point P is

$$E = E_0 \sin(kx - \omega t + \varepsilon)$$

where $$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01} E_{02} \cos\delta \quad \text{... (17.4)}$$

and $$\tan\varepsilon = \frac{E_{02} \sin\delta}{E_{01} + E_{02} \cos\delta}.$$

The conditions for constructive (bright fringe) and destructive (dark fringe) interferences are,

$$\left.\begin{array}{lll} & \delta = 2n\pi & \text{for bright fringes} \\ \text{and} & \delta = (2n+1)\pi & \text{for dark fringes} \end{array}\right| \quad \text{... (17.5)}$$

where n is an integer.

Using (17.3), these conditions may also be written as

$$\left.\begin{array}{ll}\Delta x = n\lambda & \text{for bright fringes} \\ \text{and} \quad \Delta x = \left(n + \frac{1}{2}\right)\lambda & \text{for dark fringes.}\end{array}\right| \quad \dots \quad (17.6)$$

At the point B in figure (17.5b), $\Delta x = 0$ as $S_1B = S_2B$. This point is the centre of the bright fringe corresponding to $n = 0$.

Intensity Variation

If the two slits are identical, $E_{01} = E_{02} = E_0'$ and from equation (17.4),

$$E_0^2 = 2E_0'^2(1 + \cos\delta).$$

As the intensity is proportional to the square of the amplitude, we get

$$I = 2I'(1 + \cos\delta) = 4I'\cos^2\frac{\delta}{2} \quad \dots \quad (17.7)$$

where I is the resultant intensity and I' is the intensity due to a single slit.

The equation gives intensity as a function of δ. At the centre of a bright fringe, $\delta = 2n\pi$ and $I = 4I'$. At the centre of a dark fringe, $\delta = (2n + 1)\pi$ and $I = 0$. At other points, the intensity is in between 0 and $4I'$ as given by equation (17.7).

Fringe-width and Determination of Wavelength

The separation on the screen between the centres of two consecutive bright fringes or two consecutive dark fringes is called the *fringe-width*. Suppose S_1A is the perpendicular from S_1 to S_2P (figure 17.5b). Suppose,

$D = OB$ = separation between the slits and the screen,

d = separation between the slits

and $D >> d$.

Under the above approximation ($D >> d$), S_1P and S_2P are nearly parallel and hence S_1A is very nearly perpendicular to S_1P, S_2P and OP. As S_1S_2 is perpendicular to OB and S_1A is perpendicular (nearly) to OP, we have

$$\angle S_2S_1A = \angle POB = \theta.$$

This is a small angle as $D >> d$.

The path difference is

$$\Delta x = PS_2 - PS_1 \approx PS_2 - PA$$
$$= S_2A = d\sin\theta$$
$$\approx d\tan\theta = d\frac{y}{D}.$$

The centres of the bright fringes are obtained at distances y from the point B where

$$\Delta x = d\frac{y}{D} = n\lambda \quad \text{(where } n \text{ is an integer)}$$

or, $$y = \frac{nD\lambda}{d}$$

i.e., at $y = 0, \pm\frac{D\lambda}{d}, \pm\frac{2D\lambda}{d}, \pm\frac{3D\lambda}{d}, \dots$ etc.

The centres of dark fringes will be obtained where

$$\Delta x = d\frac{y}{D} = \left(n + \frac{1}{2}\right)\lambda$$

or, $$y = \left(n + \frac{1}{2}\right)\frac{D\lambda}{d}$$

i.e., at $y = \pm\frac{D\lambda}{2d}, \pm\frac{3D\lambda}{2d}, \pm\frac{5D\lambda}{2d}, \dots$

The fringe-width is, therefore,

$$w = \frac{D\lambda}{d}. \quad \dots \quad (17.8)$$

By measuring D, d and w in an experiment, one can calculate the wavelength of the light used. We see from equation (17.8) that as the separation d between the slits is increased, the fringe-width is decreased. If d becomes much larger than λ, the fringe-width will be very small. The maxima and minima, in this case, will be so closely spaced that it will look like a uniform intensity pattern. This is an example of the general result that the wave effects are difficult to observe, if the wavelength is small compared to the dimensions of the obstructions or openings to the incident wavefront.

Example 17.2

In a Young's double slit experiment, the separation between the slits is 0·10 mm, *the wavelength of light used is* 600 nm *and the interference pattern is observed on a screen* 1·0 m *away. Find the separation between the successive bright fringes.*

Solution : The separation between the successive bright fringes is

$$w = \frac{D\lambda}{d} = \frac{1{\cdot}0\ \text{m} \times 600 \times 10^{-9}\ \text{m}}{0{\cdot}10 \times 10^{-3}\ \text{m}}$$
$$= 6{\cdot}0 \times 10^{-3}\ \text{m} = 6{\cdot}0\ \text{mm}.$$

17.6 OPTICAL PATH

Consider a light wave travelling in a medium of refractive index μ. Its equation may be written as

$$E = E_0\sin\omega(t - x/v) = E_0\sin\omega(t - \mu x/c).$$

If the light wave travels a distance Δx, the phase changes by

$$\delta_1 = \mu\frac{\omega}{c}\Delta x. \quad \dots \quad \text{(i)}$$

Instead, if the light wave travels in vaccum, its equation will be

$$E = E_0 \sin \omega(t - x/c).$$

If the light travels through a distance $\mu\,\Delta x$, the phase changes by

$$\delta_2 = \frac{\omega}{c}(\mu\,\Delta x) = \mu\,\frac{\omega}{c}\,\Delta x. \qquad \ldots \text{(ii)}$$

By (i) and (ii), we see that a wave travelling through a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. In other words, a path length of Δx in a medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vaccum. The quantity $\mu\Delta x$ is called the *optical path* of the light. In dealing with interference of two waves, we need the difference between the optical paths travelled by the waves. The geometrical path and the optical path are equal only when light travels in vacuum or in air where the refractive index is close to 1.

The concept of optical path may also be introduced in terms of the change in wavelength as the wave changes its medium. The frequency of a wave is determined by the frequency of the source and is not changed when the wave enters in a new medium. If the wavelength of light in vacuum is λ_0 and that in the medium is λ_n, then

$$\lambda_0 = \frac{c}{\nu}$$

and $$\lambda_n = \frac{v}{\nu} = \frac{c}{\mu\nu}$$

so that $$\lambda_n = \frac{\lambda_0}{\mu}.$$

At any given instant, the points differing by one wavelength have same phase of vibration. Thus, the points at separation λ_n in the medium have same phase of vibration. On the other hand, in vacuum, points at separation λ_0 will have same phase of vibration. Thus, a path λ_n in a medium is equivalent to a path $\lambda_0 = \mu\lambda_n$ in vacuum. In general, a path Δx, in a medium of refractive index μ, is equivalent to a path $\mu\Delta x$ in vacuum which is called the optical path.

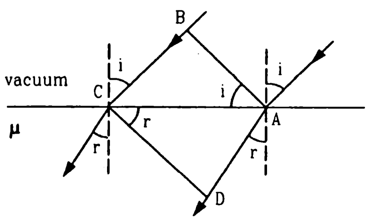


Figure 17.6

We can also understand the idea of optical path with the help of figure (17.6). Suppose a parallel beam of light travelling in vacuum is incident on the surface AC of a medium of refractive index μ. AB is perpendicular to the incident rays and hence represents a wavefront of the incident light. Similaly CD is perpendicular to the refracted rays and represents a wavefront of the refracted light. Now phase of the wave has a constant value at different points of a wavefront. Thus, phase at A = phase at B and phase at C = phase at D.

Thus, the phase difference between A and D = phase difference between B and C. From the figure,

$$\mu = \frac{\sin i}{\sin r} = \frac{BC}{AC} \times \frac{AC}{AD} = \frac{BC}{AD}$$

or, $$BC = \mu AD.$$

The phase of light wave changes by equal amount whether it covers a distance $BC = \mu AD$ in vacuum or AD in the medium. Thus, a path AD in a medium of refractive index μ is equivalent to a path $\mu(AD)$ in vacuum which we call optical path.

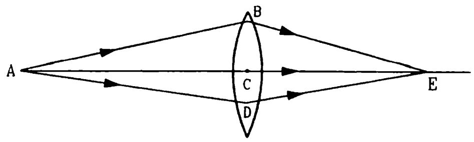


Figure 17.7

Consider the situation in figure (17.7). The geometrical paths ABE, ACE and ADE are different, but the optical paths are equal. This is because each path leads to the same phase difference, phase at E − phase at A. Note that the ray having longer geometrical path covers less distance in the lens as compared to the ray having shorter geometrical path.

Example 17.3

The wavelength of light coming from a sodium source is 589 nm. *What will be its wavelength in water? Refractive index of water* = 1·33.

Solution : The wavelength in water is $\lambda = \lambda_0/\mu$, where λ_0 is the wavelength in vacuum and μ is the refractive index of water. Thus,

$$\lambda = \frac{589}{1{\cdot}33} = 443 \text{ nm}.$$

17.7 INTERFERENCE FROM THIN FILMS

When oil floating on water is viewed in sunlight, beautiful colours appear. These colours appear because of interference between the light waves sent by the film as explained below.

Consider a thin film made of a transparent material with plane parallel faces separated by a distance d. Suppose a parallel beam of light is incident on the film at an angle i as shown in figure (17.8). The wave is divided into two parts at the upper surface, one is reflected and the other is refracted. The refracted part, which enters into the film, again gets divided at the lower surface into two parts; one is transmitted out of the film and the other is reflected back. Multiple reflections and refractions take place and a number of reflected waves as well as transmitted waves are sent by the film. The film may be viewed by the reflected light (more usual case) or by the transmitted light. We shall discuss the transmitted light first.

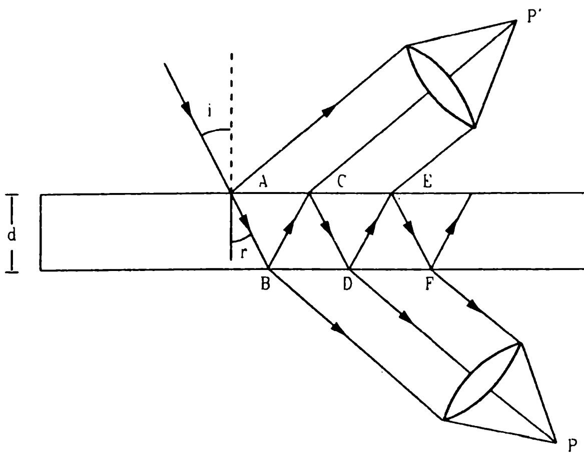


Figure 17.8

In figure (17.8), we have collected the parallel rays transmitted by the film by a converging lens at a point P. The amplitude of the individual transmitted waves is different for different waves; it gradually decreases as more reflections are involved. The wave BP, DP, FP etc. interfere at P to produce a resultant intensity. Let us consider the phase difference between the two waves BP and DP. The two waves moved together and hence, remained in phase up to B where splitting occured and one wave followed the path BP and the other $BCDP$.

Let us discuss the special case for normal incidence when the angle of incidence $i = 0$. Then, the points B and D coincide. The path BP equals DP and the only extra distance travelled by the wave along DP is $BC + CD = 2d$. As this extra path is traversed in a medium of refractive index μ, the optical path difference between the waves BP and DP interfering at P is

$$\Delta x = 2\mu d.$$

The phase difference is

$$\delta = 2\pi \frac{\Delta x}{\lambda} = 2\pi \frac{2\mu d}{\lambda}.$$

This is also the phase difference between the waves DP and FP or in fact, between any consecutively transmitted waves. All these waves are in phase if

$$\delta = 2n\pi$$

or,
$$2\mu d = n\lambda \qquad \dots (17.9)$$

where n is an integer.

If this condition is satisfied, constructive interference takes place and the film is seen illuminated. On the other hand, if

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda, \qquad \dots (17.10)$$

$\delta = (2n + 1)\pi$ and the consecutive waves are out of phase. The waves cancel each other although complete cancellation does not take place because the interfering waves do not have equal amplitude. Still, the illumination will be comparatively less.

If white light is used, the film's thickness d will satisfy condition (17.9) for certain wavelengths and these colours will be strongly transmitted due to constructive interference. The colours corresponding to the wavelengths for which (17.10) is satisfied will be poorly transmitted due to destructive interference. This gives coloured appearance of the film.

Next, let us consider the case when the film is viewed by the light reflected by it. The reflected light consists of waves from A, C, E, ... etc. (figure 17.8) which may be brought to a focus at a point P' by a converging lens. The optical path difference between the consecutively reflected waves reaching at P' is again $2\mu d$ in the limit of normal incidence $i = 0$. Experimental arrangement may be a bit difficult for an exact normal incidence and then collection of reflected light along the same direction. However, we can suppose that it is viewed by light falling very nearly normal to it. We may expect that the condition of maximum illumination and minimum illumination will be same as equations (17.9) and (17.10). But a simple argument conflicts the case. If for a given thickness and wavelength, destructive interference takes place both in reflection as well as in transmission; where does the light go then ? What happens to the energy incident on the film ? Similarly, if the intensity is enhanced both in transmission and reflection, where does this extra energy come from ? It seems logical that if the intensity in transmission is increased, it should be at the cost of reflection and vice versa. So the conditions for maximum and minimum illumination in reflection should be opposite to that in transmission. We should have

$2\mu d = n\lambda$ for minimum illumination in reflection ... (17.11) and

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda \text{ for maximum illumination in reflection.} \quad \ldots (17.12)$$

This comes out to be true experimentally. To explain why destructive interference takes place even when the optical path difference is an integral multiple of wavelength, let us recall our discussion of reflection and transmission of waves in chapter 15. If a composite string is prepared by joining a light string to a heavier one and if a wave pulse is sent from the lighter one towards the heavier one, a part is reflected from the junction and other is transmitted into the heavier string (figure 17.9a). The reflected pulse is inverted with respect to the incident pulse.

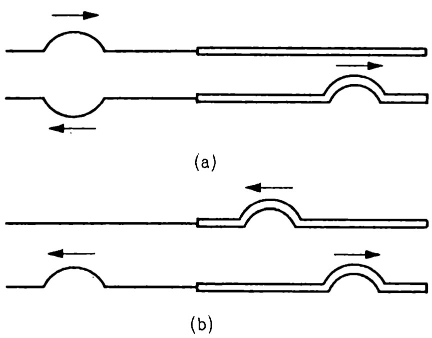


Figure 17.9

There is a sudden phase-change of π when a wave is reflected from a denser string. No such sudden phase change takes place if the wave is reflected from a rarer string (figure 17.9b). Same is true for light waves. The medium with higher refractive index is optically denser. When light is incident from air to a film, the reflected wave suffers a sudden phase-change of π. The next wave, with which it interferes, suffers no such sudden phase-change. If $2\mu d$ is equal to λ or its integral multiple, the second wave is out of phase with the first because the first has suffered a phase change of π. This explains the conditions (17.11) and (17.12).

Example 17.4

Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm. *The refractive index of the material of the film is* 1·25.

Solution : For strong reflection, the least optical path difference introduced by the film should be $\lambda/2$. The optical path difference between the waves reflected from the two surfaces of the film is $2\mu d$. Thus, for strong reflection,

$$2\mu d = \lambda/2$$

or,
$$d = \frac{\lambda}{4\mu} = \frac{589\text{ nm}}{4 \times 1{\cdot}25} = 118\text{ nm}.$$

17.8 FRESNEL'S BIPRISM

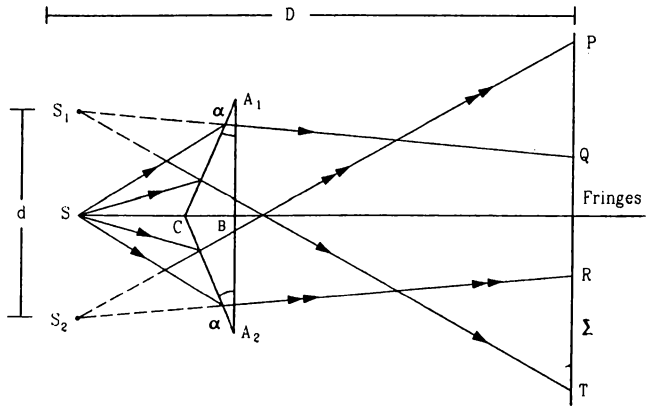


Figure 17.10

Figure (17.10) shows a schematic diagram of Fresnel's biprism and interference of light using it. Two thin prisms A_1BC and A_2BC are joined at the bases to form a biprism. The refracting angles A_1 and A_2 (denoted by α in the figure) are of the order of half a degree each. In fact, it is a simple prism whose base angles are extremely small. A narrow slit S, allowing monochromatic light, is placed parallel to the refracting edge C. The light going through the prism A_1BC appears in a cone S_1QT and the light going through A_2BC appears in a cone S_2PR. Here S_1 and S_2 are the virtual images of S as formed by the prisms A_1BC and A_2BC. A screen Σ is placed to intercept the transmitted light. Interference fringes are formed on the portion QR of the screen where the two cones overlap.

One can treat the points S_1 and S_2 as two coherent sources sending light to the screen. The arrangement is then equivalent to a Young's double slit experiment with S_1 and S_2 acting as the two slits. Suppose the separation between S_1 and S_2 is d and the separation between the plane of S_1S_2 and Σ is D. The fringe-width obtained on the screen is

$$w = \frac{D\lambda}{d}.$$

17.9 COHERENT AND INCOHERENT SOURCES

Two sources of light waves are said to be *coherent* if the initial phase difference δ_0 between the waves emitted by the sources remains constant in time. If δ_0 changes randomly with time, the sources are called *incoherent*. Two waves produce interference pattern only if, they originate from coherent sources. This condition is same as discussed for sound waves in the previous chapter. The process of light emission from ordinary sources such as the sun, a candle, an electric bulb etc. is such that one has to use special techniques to get coherent sources. In an ordinary source, light is emitted by atoms in descrete steps. An atom after emitting a short light pulse becomes inactive for some time. It again gains energy by some interaction, becomes active and emits another pulse of light. Thus, at a particular time a particular group of atoms is active and the rest are inactive. The active time is of the order of 10^{-8} s and during this period, a wavetrain of several meters is emitted.

We can picture the light coming from an ordinary source as a collection of several wavetrains, each several meters long and having no fixed phase relation with each other. Such a source is incoherent in itself. Different wavetrains are emitted by different groups of atoms and these groups act independently of each other, hence the phase varies randomly from train to train. If two lamps are substituted in place of the slits S_1 and S_2 in a Young's interference experiment, no fringe will be seen. This is because each source keeps on changing its phase randomly and hence, the phase difference between the two sources also changes randomly. That is why, a narrow aperture S_0 is used to select a particular wavetrain which is incident on the two slits together. This ensures that the initial phase difference of the wavelets originating from S_1 and S_2 does not change with time. When a new wavetrain is emitted by the lamp, the phase is randomly changed but that change is simultaneously communicated to both S_1 and S_2 and the phase difference remains unchanged. In order to obtain a fairly distinct interference pattern, the path difference between the two waves originating from coherent sources should be kept small. This is so because the wavetrains are finite in length and hence with large difference in path, the waves do not overlap at the same instant in the same region of space. The second wavetrain arrives well after the first train has already passed and hence, no interference takes place. In practice, the path difference should not exceed a few centimeters to observe a good interference pattern.

Because of the incoherent nature of the basic process of light emission in ordinary sources, these sources cannot emit highly monochromatic light. A strictly monochromatic light, having a well-defined single frequency or wavelength, must be a sine wave which has an infinite extension. A wavetrain of finite length may be described by the superposition of a number of sine waves of different wavelengths. Thus, the light emitted by an ordinary source always has a spread in wavelength. An ordinary sodium vapour lamp emits light of wavelength 589·0 nm and 589·6 nm with a spread of about ± 0·01 nm in each line. Shorter the length of the wavetrain, larger is the spread in wavelength.

It has been made possible to produce light sources which emit very long wavetrains, of the order of several hundred metres. The spread in wavelength is accordingly very small. These sources are called *laser* sources. The atoms behave in a cooperative manner in such a source and hence the light is coherent. Two independent laser sources can produce interference fringes and the path difference may be several metres long.

17.10 DIFFRACTION OF LIGHT

When a wave is obstructed by an obstacle, the rays bend round the corner. This phenomenon is known as *diffraction*. We can explain the effect using Huygens' principle. When a wavefront is partially obstructed, only the wavelets from the exposed parts superpose and the resulting wavefront has a different shape. This allows for the bending round the edges. In case of light waves, beautiful fringe patterns comprising maximum and minimum intensity are formed due to diffraction.

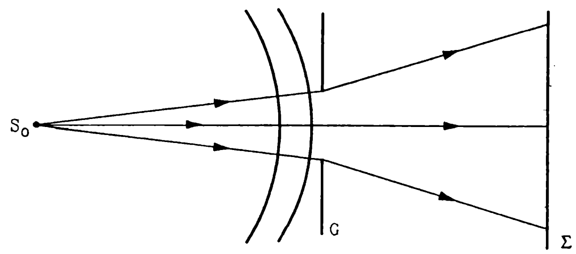


Figure 17.11

Figure (17.11) shows the basic arrangement for observing diffraction effects in light waves. It consists of a narrow source of light S_0, a diffracting element G (an obstacle or an opening) and a screen Σ. The wavefronts emitted by the source S_0 are partially obstructed by the element G. The secondary wavelets originating from different points of the unobstructed part interfere on the screen Σ and produce the diffraction pattern of varying intensity. A special case of diffraction, which is very important in practice and which is simpler to analyse mathematically, arises when the source S_0 and the screen Σ are far away from the diffracting element G. Plane waves are incident on G and the waves interfering at a particular point come parallel to each other. This special class of diffraction is called *Fraunhofer diffraction* after the physicist Joseph von Fraunhofer (1787-1826) who investigated such diffraction cases in great detail. Fraunhofer diffraction can be observed in a laboratory by placing converging lenses before and after G and keeping the source S_0 and the screen Σ in their focal planes respectively (figure 17.12). The source and the screen are effectively at infinite distance from the diffracting element.

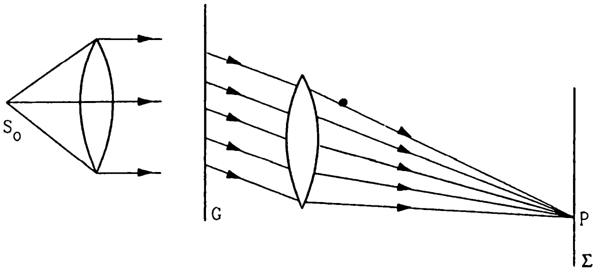


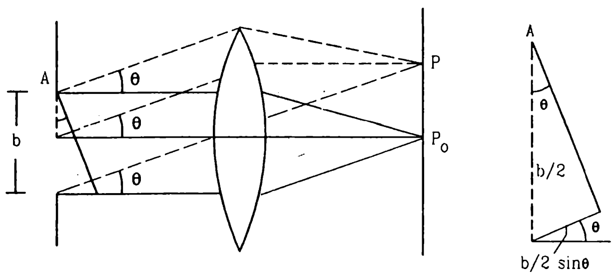
Figure 17.12

If the source or the screen is at a finite distance from the diffracting element G, it is called *Fresnel diffraction* after the physicist Augustin Jean Fresnel (1788-1827).

It is a coincidence that the two great physicists, Fraunhofer and Fresnel, investigating diffraction phenomenon lived a short life of equal number of years.

We shall now discuss some of the important cases of diffraction.

17.11 FRAUNHOFER DIFFRACTION BY A SINGLE SLIT



(a) (b)

Figure 17.13

Suppose a parallel beam of light is incident normally on a slit of width b (figure 17.13). According to Huygens' principle, each and every point of the exposed part of the plane wavefront (i.e., every point of the slit) acts as a source of secondary wavelets spreading in all directions. The light is received by a screen placed at a large distance. In practice, this condition is achieved by placing the screen at the focal plane of a converging lens placed just after the slit. A particular point P on the screen receives waves from all the secondary sources. All these waves start parallel to each other from different points of the slit and interfere at P to give the resultant intensity.

At the point P_0 which is at the bisector plane of the slit, all the waves reach after travelling equal optical path and hence, are in phase. The waves, thus, interfere constructively with each other and maximum intensity is observed. As we move away from P_0, the waves arrive with different phases and the intensity is changed.

Let us consider a point P which collects the waves originating from different points of the slit at an angle θ. Figure (17.13) shows the perpendicular from the point A to the parallel rays. This perpendicular also represents the wavefront of the parallel beam diffracted at an angle θ. The optical paths from any point on this wavefront to the point P are equal. The optical path difference between the waves sent by the upper edge A of the slit and the wave sent by the centre of the slit is $\frac{b}{2}\sin\theta$. This is shown in expanded view in figure (17.13b). Consider the angle for which $\frac{b}{2}\sin\theta = \lambda/2$. The above mentioned two waves will have a phase difference

$$\delta = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi.$$

The two waves will cancel each other. The wave from any point in the upper half of the slit is exactly cancelled by the wave from the point $b/2$ distance below it. The whole slit can be divided into such pairs and hence, the intensity at P will be zero. This is the condition of the first minimum i.e., the first dark fringe.

So, $$\frac{b}{2}\sin\theta = \frac{\lambda}{2}$$

or, $$b\sin\theta = \lambda \text{ (first minimum).}$$

Similar arguments show that other minima (zero intensity) are located at points corresponding to $b\sin\theta = 2\lambda, 3\lambda \ldots$

or, $$b\sin\theta = n\lambda \text{ (dark fringe).} \qquad \ldots (17.13)$$

The points of the maximum intensity lie nearly midway between the successive minima. A detailed mathematical analysis shows that the amplitude E_0' of the electric field at a general point P is,

$$E_0' = E_0\frac{\sin\beta}{\beta} \qquad \ldots (17.14)$$

where, $$\beta = \frac{1}{2}\frac{\omega}{v}b\sin\theta = \frac{\pi}{\lambda}b\sin\theta \qquad \ldots (17.15)$$

and E_0 is the amplitude at the point P_0 which corresponds at $\theta = 0$.

The intensity is proportional to the square of the amplitude. If I_0 represents the intensity at P_0, its value at P is

$$I = I_0 \frac{\sin^2\beta}{\beta^2} \cdot \quad \dots \quad (17.16)$$

We draw, in figure (17.14), variation of the intensity as a function of sinθ.

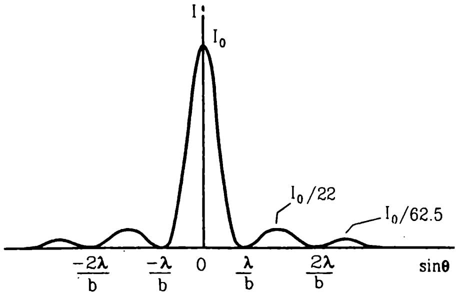


Figure 17.14

Most of the diffracted light is distributed between $\sin\theta = -\frac{\lambda}{b}$ and $+\frac{\lambda}{b} \cdot$ The intensity at the first maximum after the central one is only 1/22 of the intensity of the central maximum. This divergence in θ is inversely proportional to the width b of the slit. If the slit-width b is decreased, the divergence is increased and the light is diffracted in a wider cone. On the other hand, if the slit-width is large compared to the wavelength, $\lambda/b \approx 0$ and the light continues undiffracted in the direction $\theta = 0$. This clearly indicates that diffraction effects are observable only when the obstacle or the opening has dimensions comparable to the wavelength of the wave.

Example 17.5

A parallel beam of monochromatic light of wavelength 450 nm *passes through a long slit of width* 0·2 mm. *Find the angular divergence in which most of the light is diffracted.*

Solution : Most of the light is diffracted between the two first order minima. These minima occur at angles given by $b \sin\theta = \pm \lambda$

or, $\sin\theta = \pm \lambda/b$

$$= \pm \frac{450 \times 10^{-9}\,\text{m}}{0{\cdot}2 \times 10^{-3}\,\text{m}} = \pm 2{\cdot}25 \times 10^{-3}.$$

or, $\theta \approx \pm 2{\cdot}25 \times 10^{-3}$ rad.

The angular divergence $= 4{\cdot}5 \times 10^{-3}$ rad.

17.12 FRAUNHOFER DIFFRACTION BY A CIRCULAR APERTURE

When a parallel beam of light is passed through an opaque board with a circular hole in it, the light is diffracted by the hole. If received on a screen at a large distance, the pattern is a bright disc surrounded by alternate dark and bright rings of decreasing intensity as shown in figure (17.15). The wavefront is obstructed by the opaque board and only the points of the wavefront, that are exposed by the hole, send the secondary wavelets. The bright and dark rings are formed by the superposition of these wavelets. The mathematical analysis shows that the first dark ring is formed by the light diffracted from the hole at an angle θ with the axis where,

$$\sin\theta \approx 1{\cdot}22 \frac{\lambda}{b} \cdot \quad \dots \quad (17.17)$$

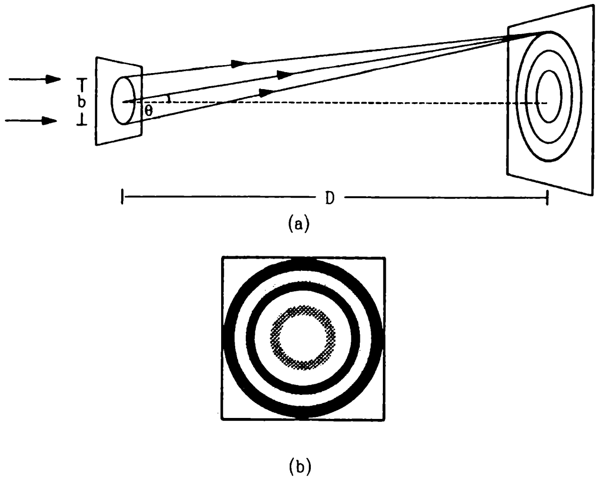


Figure 17.15

Here λ is the wavelength of the light used and b is the diameter of the hole. If the screen is at a distance $D (D >> b)$ from the hole, the radius of the first dark ring is

$$R \approx 1{\cdot}22 \frac{\lambda D}{b} \cdot \quad \dots \quad (17.18)$$

If the light transmitted by the hole is converged by a converging lens at the screen placed at the focal plane of this lens, the radius of the first dark ring is

$$R = 1{\cdot}22 \frac{\lambda f}{b} \cdot \quad \dots \quad (17.19)$$

As most of the light coming from the hole is concentrated within the first dark ring, this radius is also called the *radius of the diffraction disc.*

Diffraction by a circular aperture is of great practical importance. In many of the optical instruments, lenses are used. When light passes through a lens, the wavefront is limited by its rim which is usually circular. If a parallel beam of light is incident on a converging lens, only the part intercepted by the lens gets transmitted into the converging beam. Thus, the light is diffracted by the lens. This lens itself works to converge the diffracted light in its focal plane and hence, we observe a bright disc, surrounded by alternate dark and bright rings, as the image. The radius of the diffraction disc is given by equation (17.19) where, b now stands for the diameter of the aperature of the lens.

The above discussion shows that a converging lens can never form a point image of a distant point source. In the best conditions, it produces a bright disc surrounded by dark and bright rings. If we assume that most of the light is concentrated within the central bright disc, we can say that the lens produces a disc image for a distant point source. This is not only true for a distant point source but also for any point source. The radius of the image disc is

$$R = 1{\cdot}22 \frac{\lambda}{b} D$$

where, D is the distance from the lens at which the light is focused.

Example 17.6

A beam of light of wavelength 590 nm *is focussed by a converging lens of diameter* 10·0 cm *at a distance of* 20 cm *from it. Find the diameter of the disc image formed.*

Solution : The angular radius of the central bright disc in a diffraction pattern from circular aperture is given by

$$\sin\theta \approx \frac{1{\cdot}22\,\lambda}{b}$$

$$= \frac{1{\cdot}22 \times 590 \times 10^{-9}\ \text{m}}{10{\cdot}0 \times 10^{-2}\ \text{m}} = 0{\cdot}7 \times 10^{-5}\ \text{rad.}$$

The radius of the bright disc is

$$0{\cdot}7 \times 10^{-5} \times 20\ \text{cm} = 1{\cdot}4 \times 10^{-4}\ \text{cm.}$$

The diameter of the disc image $= 2{\cdot}8 \times 10^{-4}$ cm.

17.13 FRESNEL DIFFRACTION AT A STRAIGHT EDGE

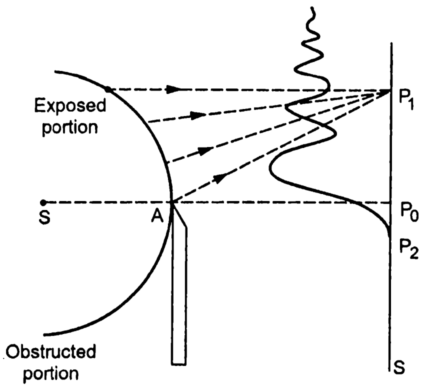


Figure 17.16

Consider the situation shown in figure (17.16). Let S be a narrow slit sending monochromatic light. The light is obstructed by an opaque obstacle having a sharp edge A. The light is collected on a screen Σ. The portion of the screen below P_0 in the figure is the region of the geometrical shadow.

Cylindrical wavefronts emitted from the slit are obstructed by the obstacle. The points on the exposed portion of the wavefront emit secondary wavelets which interfere to produce varying intensity on the screen Σ.

The curve in the figure shows the variation of intensity of light on the screen. We see that the intensity gradually decreases as we go farther inside the region of geometrical shadow i.e., below P_0. As we go above P_0, the intensity alternately increases and decreases. The difference of the maximum intensity and minimum intensity goes on decreasing as we go farther away from P_0 and finally we get uniform illumination.

17.14 LIMIT OF RESOLUTION

The fact that a lens forms a disc image of a point source, puts a limit on resolving two neighbouring points imaged by a lens.

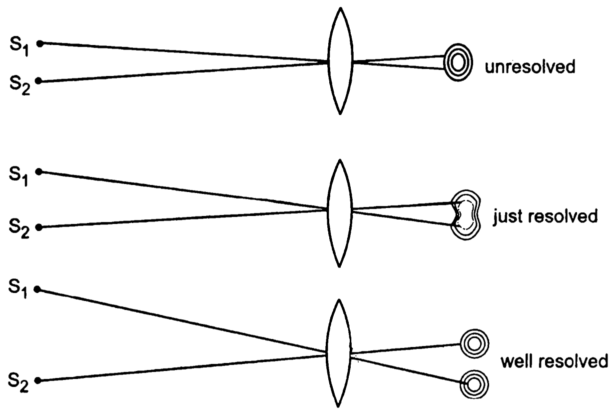


Figure 17.17

Suppose S_1 and S_2 are two point sources placed before a converging lens (figure 17.17). If the separation between the centres of the image-discs is small in comparison to the radii of the discs, the discs will largely overlap on one another and it will appear like a single disc. The two points are then not resolved. If S_1 and S_2 are moved apart, the centres of their image-discs also move apart. For a sufficient separation, one can distinguish the presence of two discs in the pattern. In this case, we say that the points are just resolved.

The angular radius θ of the diffraction disc is given by $\sin\theta = \frac{1{\cdot}22\,\lambda}{b}$, where b is the radius of the lens. Thus, increasing the radius of the lens improves the resolution. This is the reason why objective lenses of powerful microscopes and telescopes are kept large in size.

The human eye is itself a converging lens which forms the image of the points (we see) on the retina. The above discussion then shows that two points very close to each other cannot be seen as two distinct points by the human eye.

Rayleigh Criterion

Whether two disc images of nearby points are resolved or not may depend on the person viewing the images. Rayleigh suggested a quantitative criterion for resolution. Two images are called just resolved in this criterion if the centre of one bright disc falls on the periphery of the second. This means, the radius of each bright disc should be equal to the separation between them. In this case, the resultant intensity has a minimum between the centres of the images. Figure (17.18) shows the variation of intensity when the two images are just resolved.

Figure 17.18

17.15 SCATTERING OF LIGHT

When a parallel beam of light passes through a gas, a part of it appears in directions other than the incident direction. This phenomenon is called *scattering of light*. The basic process in scattering is absorption of light by the molecules followed by its re-radiation in different directions. The strength of scattering can be measured by the loss of energy in the light beam as it passes through the gas. It should be distinguished from the absorption of light as it passes through a medium. In absorption, the light energy is converted into internal energy of the medium whereas in scattering, the light energy is radiated in other directions. The strength of scattering depends on the wavelength of the light beside the size of the particles which cause scattering. If these particles are smaller than the wavelength, the scattering is proportional to $1/\lambda^4$. This is known as *Rayleigh's law of scattering*. Thus, red light is scattered the least and violet is scattered the most. This is why, red signals are used to indicate dangers. Such a signal goes to large distances without an appreciable loss due to scattering.

The blue appearance of sky is due to scattering of sunlight from the atmosphere. When you look at the sky, it is the scattered light that enters the eyes. Among the shorter wavelengths, the colour blue is present in larger proportion in sunlight. Light of short wavelengths, are strongly scattered by the air molecules and reach the observer. This explains the blue colour of sky. Another natural phenomenon related to the scattering of light is the red appearance of sun at the sunset and at the sunrise. At these times, the sunlight has to travel a large distance through the atmosphere. The blue and neighbouring colours are scattered away in the path and the light reaching the observer is predominantly red.

If the earth had no atmosphere, the sky would appear black and stars could be seen during day hours. In fact if you go about 20 km up, where the atmosphere becomes quite thin, the sky does appear black and stars are visible during day hours as astronouts have found.

Besides air molecules, water particles, dust etc. also scatter light. The appearance of sky is affected by the presence of these scattering centres. On a humid day before rains, the sky appears light blue whereas, on a clear day it appears deep blue. The change in the quality of colour of sky results from the fact that the water droplets and the dust particles may have size greater than the wavelength of light. Rayleigh's law of scattering does not operate in this case and colours other than blue may be scattered in larger proportion. The appearance of sky in large industrial cities is also different from villages. An automobile engine typically ejects about 10^{11} particles per second. Similarly for other machines. Such particles remain suspended in air for quite long time unless rain or wind clears them. Often the sky looks hazy with a greyish tinge in such areas.

17.16 POLARIZATION OF LIGHT

In writing equation (17.1) for light wave, we assumed that the direction of electric field is fixed and the magnitude varies sinusoidally with space and time. The electric field in a light wave propagating in free space is perpendicular to the direction of propagation. However, there are infinite number of directions perpendicular to the direction of propagation and the eletric field may be along any of these directions. For example, if the light propagates along the X-axis, the eletric field may be along the Y-axis, or along the Z-axis or along any direction in the Y-Z plane. If the electric field at a point always remains parallel to a fixed direction as the time passes, the light is called *linearly polarized* along that direction. For example, if the electric field at a point is always parallel to the

Y-axis, we say that the light is linearly polarized along the Y-axis. The same is also called *plane polarized* light. The plane containing the electric field and the direction of propagation is called the *plane of polarization.*

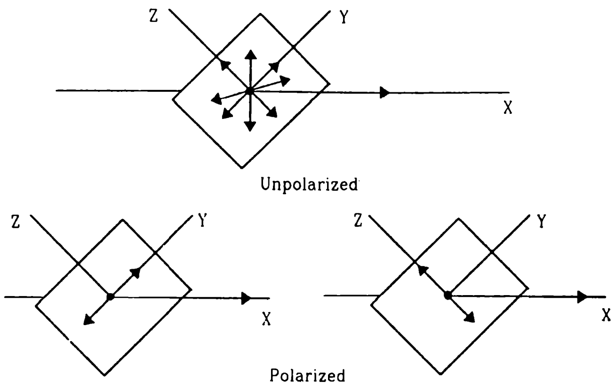


Figure 17.19

As we have mentioned earlier, light is emitted by atoms. The light pulse emitted by one atom in a single event has a fixed direction of electric field. However, the light pulses emitted by different atoms, in general, have electric fields in different directions. Hence, the resultant electric field at a point keeps on changing its direction randomly and rapidly. Such a light is called *unpolarized.* The light emitted by an ordinary source such as an electric lamp, a mercury tube, a candle, the sun etc. are unpolarized.

Suppose an unpolarized light wave travels along the X-axis. The electric field at any instant is in the Y-Z plane, we can break the field into its components E_y and E_z along the Y-axis and the Z-axis respectively. The fact that the resultant electric field changes its direction randomly may be mathematically expressed by saying that E_y and E_z have a phase difference δ that changes randomly with time. Thus,

$$E_y = E_1 \sin(\omega t - kx + \delta)$$

$$E_z = E_2 \sin(\omega t - kx).$$

The resultant electric field makes an angle θ with the Y-axis where

$$\tan\theta = \frac{E_z}{E_y} = \frac{E_2 \sin(\omega t - kx)}{E_1 \sin(\omega t - kx + \delta)}.$$

Since δ changes randomly with time, so does θ and the light is unpolarized.

If δ is zero, $\tan\theta = E_2/E_1 =$ constant and the electric field is always parallel to a fixed direction. The light is linearly polarized.

If $\delta = \pi$, $\tan\theta = -E_2/E_1$ and again the electric field is parallel to a fixed direction and the light is linearly polarized.

If $\delta = \pi/2$ and $E_1 = E_2$, then

$$\tan\theta = \frac{E_z}{E_y} = \frac{E_2 \sin(\omega t - kx)}{E_1 \sin(\omega t - kx + \pi/2)}$$

$$= \tan(\omega t - kx)$$

or, $\theta = \omega t - kx.$

At any point x, the angle θ increases at a uniform rate ω. The electric field, therefore, rotates at a uniform angular speed ω. Also,

$$E^2 = E_y^2 + E_z^2 = E_1^2 \cos^2(\omega t - kx) + E_1^2 \sin^2(\omega t - kx) = E_1^2$$

i.e., the magnitude of the field remains constant. The tip of the electric field, thus, goes in a circle at a uniform angular speed. Such a light is called a *circularly polarized light.*

If $\delta = \pi/2$ but $E_1 \neq E_2$, the tip of the electric field traces out an ellipse. Such a light wave is called an *elliptically polarized light.*

Polaroids

There are several methods to produce polarized light from the unpolarized light. An instrument used to produce polarized light from unpolarized light is called a *polarizer.* Plane sheets in the shape of circular discs called *polaroids* are commercially available which transmit light with E-vector parallel to a special direction in the sheet. These polaroids have long chains of hydrocarbons which become conducting at optical frequencies. When light falls perpendicularly on the sheet, the electric field parallel to the chains is absorbed in setting up electric currents in the chains but the field perpendicular to the chains gets transmitted. The direction perpendicular to the chains is called the *transmission axis* of the polaroid. When light passes through the polaroid, the transmitted light becomes linearly polarized with E-vector parallel to the transmission axis.

If linearly polarized light is incident on a polaroid with the E-vector parallel to the transmission axis, the light is completely transmitted by the polaroid. If the E-vector is perpendicular to the transmission axis, the light is completely stopped by the polaroid. If the E-vector is at an angle θ with the transmission axis, light is partially transmitted. The intensity of the transmitted light is

$$I = I_0 \cos^2\theta \qquad \ldots (17.20)$$

where I_0 is the intensity when the incident E-vector is parallel to the transmission axis. This is known as the *law of Malus.*

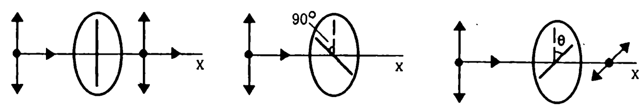


Figure 17.20

Polarization by Reflection and Refraction

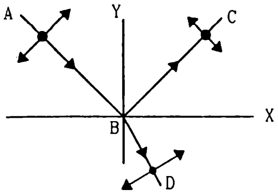


Figure 17.21

Consider a light beam from air incident on the surface of a transparent medium of refractive index μ. The incident ray, the reflected ray and the refracted ray are all in one plane. The plane is called the plane of incidence. In figure (17.21) we have shown this plane as the X-Y plane. Consider the incident light going along AB. The electric field $\vec{E}$ must be perpendicular to AB. If the incident light is unpolarized, the electric field will randomly change its direction, remaining at all times in a plane perpendicular to AB. We can resolve the field in two components, one in the X-Y plane and the other along the Z-direction. In figure (17.21), the component in the X-Y plane is shown by the double-arrow perpendicular to AB and the component along the Z-direction by the solid dot.

Light with electric field along the Z-direction is more strongly reflected as compared to that in the X-Y plane. This is shown in the figure by reduced size of the double arrow. Similarly, the refracted light has a larger component of electric field in the X-Y plane shown in the figure by the reduced size of the solid dot.

If the light is incident on the surface with an angle of incidence i given by

$$\tan i = \mu, \qquad \dots (17.21)$$

the reflected light is completely polarized with the electric field along the Z-direction as suggested by figure (17.22). The refracted ray is never completely polarized. The angle i given by equation (17.21) is called the *Brewster's angle* and equation (17.21) itself is known as the *Brewster's law.*

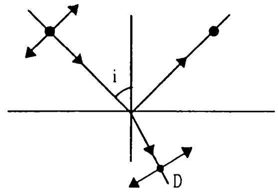


Figure 17.22

The fact that reflected light is polarized is used in preparing 'polarizing sunglasses' which reduce the glare from roads, snow, water surfaces etc. The glasses are, in fact, polaroids with their transmission axis perpendicular to the electric field of the polarized reflected light. The reflected light, which is responsible for the glare, is thus largely absorbed. The direct light coming to the glasses is unpolarized and is less absorbed. In this respect, the polarizing sunglasses are different from the ordinary dark-coloured sunglasses which absorb any light passing through them reducing the intensity to a large extent.

Polarization by Scattering

When unpolarized light is scattered by small particles, the scattered light is partially polarized. The blue light received from the sky is accordingly partially polarized. Though human eye does not distinguish between an unpolarized light and a polarized light, the eyes of a bee can detect the difference. Austrian Novel Laureate Karl Von Frisch performed experiments for several years on bees and concluded that the bees can not only distinguish unpolarized light from polarized light but can also determine the direction of polarization.

Worked Out Examples

1. *White light is a mixture of light of wavelengths between* 400 nm *and* 700 nm. *If this light goes through water* (μ = 1·33), *what are the limits of the wavelength there ?*

Solution : When a light having wavelength λ_0 in vacuum goes through a medium of refractive index μ, the wavelength in the medium becomes $\lambda = \lambda_0/\mu$.

For $\lambda_0 = 400\text{ nm},\ \lambda = \dfrac{400\text{ nm}}{1{\cdot}33} = 300\text{ nm}$

and for $\lambda_0 = 700\text{ nm},\ \lambda = \dfrac{700\text{ nm}}{1{\cdot}33} = 525\text{ nm}.$

Thus, the limits are 300 nm and 525 nm.

2. *The optical path of a monochromatic light is the same if it goes through* 2·00 cm *of glass or* 2·25 cm *of water. If the refractive index of water is* 1·33, *what is the refractive index of glass ?*

Solution : When light travels through a distance x in a medium of refractive index μ, its optical path is μx. Thus, if μ is the refractive index of glass,

$$\mu(2{\cdot}00\text{ cm}) = 1{\cdot}33 \times (2{\cdot}25\text{ cm})$$

or,
$$\mu = 1{\cdot}33 \times \frac{2{\cdot}25}{2{\cdot}00} = 1{\cdot}50.$$

3. *White light is passed through a double slit and interference pattern is observed on a screen 2·5 m away. The separation between the slits is 0·5 mm. The first violet and red fringes are formed 2·0 mm and 3·5 mm away from the central white fringe. Calculate the wavelengths of the violet and the red light.*

Solution : For the first bright fringe, the distance from the centre is

$$y = \frac{D\lambda}{d}.$$

For violet light, $y = 2{\cdot}0$ mm. Thus,

$$2{\cdot}0\ \text{mm} = \frac{(2{\cdot}5\ \text{m})\lambda}{0{\cdot}5\ \text{mm}}$$

or,
$$\lambda = \frac{(0{\cdot}5\ \text{mm})\,(2{\cdot}0\ \text{mm})}{2{\cdot}5\ \text{m}} = 400\ \text{nm}.$$

Similarly, for red light, $y = 3{\cdot}5$ mm. Thus,

$$3{\cdot}5\ \text{mm} = \frac{(2{\cdot}5\ \text{m})\lambda}{0{\cdot}5\ \text{mm}}$$

or,
$$\lambda = 700\ \text{nm}.$$

4. *A double slit experiment is performed with sodium (yellow) light of wavelength* 589·3 nm *and the interference pattern is observed on a screen* 100 cm *away. The tenth bright fringe has its centre at a distance of* 12 mm *from the central maximum. Find the separation between the slits.*

Solution : For the nth maximum fringe, the distance above the central line is

$$x = \frac{n\lambda D}{d}.$$

According to the data given,

$$x = 12\ \text{mm},\ n = 10,\ \lambda = 589{\cdot}3\ \text{nm},\ D = 100\ \text{cm}.$$

Thus, the separation between the slits is

$$d = \frac{n\lambda D}{x} = \frac{10 \times 589{\cdot}3 \times 10^{-9}\ \text{m} \times 100 \times 10^{-2}\ \text{m}}{12 \times 10^{-3}\ \text{m}}$$

$$= 4{\cdot}9 \times 10^{-4}\ \text{m} = 0{\cdot}49\ \text{mm}.$$

5. *The intensity of the light coming from one of the slits in a Young's double slit experiment is double the intensity from the other slit. Find the ratio of the maximum intensity to the minimum intensity in the interference fringe pattern observed.*

Solution : The intensity of the light originating from the first slit is double the intensity from the second slit. The amplitudes of the two interfering waves are in the ratio $\sqrt{2} : 1$, say $\sqrt{2}\,A$ and A.

At the point of constructive interference, the resultant amplitude becomes $(\sqrt{2} + 1)A$. At the points of destructive interference, this amplitude is $(\sqrt{2} - 1)A$. The ratio of the resultant intensities at the maxima to that at the minima is

$$\frac{(\sqrt{2} + 1)^2 A^2}{(\sqrt{2} - 1)^2 A^2} = 34.$$

6. *The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit-width, find the ratio of the maximum to the minimum intensity in the interference pattern.*

Solution : Suppose the amplitude of the light wave coming from the narrower slit is A and that coming from the wider slit is $2A$. The maximum intensity occurs at a place where constructive interference takes place. Then the resultant amplitude is the sum of the individual amplitudes. Thus,

$$A_{max} = 2A + A = 3A.$$

The minimum intensity occurs at a place where destructive interference takes place. The resultant amplitude is then difference of the individual amplitudes. Thus,

$$A_{min} = 2A - A = A.$$

As the intensity is proportional to the square of the amplitude,

$$\frac{I_{max}}{I_{min}} = \frac{(A_{max})^2}{(A_{min})^2} = \frac{(3A)^2}{A^2} = 9.$$

7. *Two sources S_1 and S_2 emitting light of wavelength* 600 nm *are placed a distance* $1{\cdot}0 \times 10^{-2}$ cm *apart. A detector can be moved on the line S_1P which is perpendicular to S_1S_1. (a) What would be the minimum and maximum path difference at the detector as it is moved along the line S_1P? (b) Locate the position of the farthest minimum detected.*

Solution :

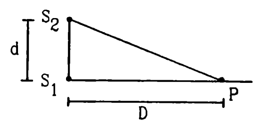


Figure 17-W1

(a) The situation is shown in figure (17-W1). The path difference is maximum when the detector is just at the position of S_1 and its value is equal to $d = 1{\cdot}0 \times 10^{-2}$ cm. The path difference is minimum when the detector is at a large distance from S_1. The path difference is then close to zero.

(b) The farthest minimum occurs at a point P where the path difference is $\lambda/2$. If $S_1P = D$,

$$S_2P - S_1P = \frac{\lambda}{2}$$

or,
$$\sqrt{D^2 + d^2} - D = \frac{\lambda}{2}$$

or, $$D^2 + d^2 = \left(D + \frac{\lambda}{2}\right)^2$$

or, $$d^2 = D\lambda + \frac{\lambda^2}{4}$$

or, $$D = \frac{d^2}{\lambda} - \frac{\lambda}{4}$$

$$= \frac{(1{\cdot}0 \times 10^{-4}\,\text{m})^2}{600 \times 10^{-9}\,\text{m}} - 150 \times 10^{-9}\,\text{m} = 1{\cdot}7\ \text{cm}.$$

8. *A beam of light consisting of two wavelengths,* 6500 Å *and* 5200 Å *is used to obtain interference fringes in a Young's double slit experiment* (1 Å = 10^{-10} m). *The distance between the slits is* 2·0 mm *and the distance between the plane of the slits and the screen is* 120 cm. *(a) Find the distance of the third bright fringe on the screen from the central maximum for the wavelength* 6500 Å. *(b) What is the least distance from the central maximum where the bright fringes due to both the wavelengths coincide?*

Solution : (a) The centre of the nth bright fringe is at a distance $y = \frac{n\lambda D}{d}$ from the central maximum. For the 3rd bright fringe of 6500 Å,

$$y = \frac{3 \times 6500 \times 10^{-10}\,\text{m} \times 1{\cdot}2\,\text{m}}{2 \times 10^{-3}\,\text{m}}$$

$$= 0{\cdot}117\ \text{cm} \approx 0{\cdot}12\ \text{cm}.$$

(b) Suppose the mth bright fringe of 6500 Å coincides with the nth bright fringe of 5200 Å.

Then, $$\frac{m \times 6500\,\text{Å} \times D}{d} = \frac{n \times 5200\,\text{Å} \times D}{d}$$

or, $$\frac{m}{n} = \frac{5200}{6500} = \frac{4}{5}.$$

The minimum values of m and n that satisfy this equation are 4 and 5 respectively. The distance of the 4th bright fringe of 6500 Å or the 5th bright fringe of 5200 Å from the central maximum is

$$y = \frac{4 \times 6500 \times 10^{-10}\,\text{m} \times 1{\cdot}2\,\text{m}}{2 \times 10^{-3}\,\text{m}}$$

$$= 0{\cdot}156\ \text{cm} \approx 0{\cdot}16\ \text{cm}.$$

9. *Monochromatic light of wavelength* 600 nm *is used in a Young's double slit experiment. One of the slits is covered by a transparent sheet of thickness* $1{\cdot}8 \times 10^{-5}$ m *made of a material of refractive index* 1·6. *How many fringes will shift due to the introduction of the sheet?*

Solution : When the light travels through a sheet of thickness t, the optical path travelled is μt where μ is the refractive index. When one of the slits is covered by the sheet, air is replaced by the sheet and hence, the optical path changes by $(\mu - 1)t$. One fringe shifts when the optical path changes by one wavelength. Thus, the number of fringes shifted due to the introduction of the sheet is

$$\frac{(\mu - 1)t}{\lambda} = \frac{(1{\cdot}6 - 1) \times 1{\cdot}8 \times 10^{-5}\,\text{m}}{600 \times 10^{-9}\,\text{m}} = 18.$$

10. *White light is incident normally on a glass plate of thickness* $0{\cdot}50 \times 10^{-6}$ *and index of refraction* 1·50. *Which wavelengths in the visible region* (400 nm-700 nm) *are strongly reflected by the plate?*

Solution : The light of wavelength λ is strongly reflected if

$$2\mu d = \left(n + \frac{1}{2}\right)\lambda, \qquad \text{... (i)}$$

where n is a nonnegative integer.

Here, $$2\mu d = 2 \times 1{\cdot}50 \times 0{\cdot}5 \times 10^{-6}\,\text{m}$$

$$= 1{\cdot}5 \times 10^{-6}\,\text{m}. \qquad \text{... (ii)}$$

Putting $\lambda = 400$ nm in (i) and using (ii),

$$1{\cdot}5 \times 10^{-6}\,\text{m} = \left(n + \frac{1}{2}\right)(400 \times 10^{-9}\,\text{m})$$

or, $$n = 3{\cdot}25.$$

Putting $\lambda = 700$ nm in (ii),

$$1{\cdot}5 \times 10^{-6}\,\text{m} = \left(n + \frac{1}{2}\right)(700 \times 10^{-9}\,\text{m})$$

or, $$n = 1{\cdot}66.$$

Thus, within 400 nm to 700 nm the integer n can take the values 2 and 3. Putting these values of n in (i), the wavelengths become

$$\lambda = \frac{4\mu d}{2n + 1} = 600\ \text{nm and } 429\ \text{nm}.$$

Thus, light of wavelengths 429 nm and 600 nm are strongly reflected.

11. *A parallel beam of green light of wavelength* 546 nm *passes through a slit of width* 0·40 mm. *The transmitted light is collected on a screen* 40 cm *away. Find the distance between the two first order minima.*

Solution : The minima occur at an angular deviation θ given by $b \sin\theta = n\lambda$, where n is an integer. For the first order minima, $n = \pm 1$ so that $\sin\theta = \pm \frac{\lambda}{b}\cdot$ As the fringes are observed at a distance much larger than the width of the slit, the linear distances from the central maximum are given by

$$x = D\tan\theta$$

$$\approx D\sin\theta = \pm \frac{\lambda D}{b}.$$

Thus, the minima are formed at a distance $\frac{\lambda D}{b}$ from the central maximum on its two sides. The separation between the minima is

$$\frac{2\lambda D}{b} = \frac{2 \times 546 \times 10^{-9}\,\text{m} \times 40 \times 10^{-2}\,\text{m}}{0{\cdot}40 \times 10^{-3}\,\text{m}} = 1{\cdot}1\ \text{mm}.$$

QUESTIONS FOR SHORT ANSWER

1. Is the colour of 620 nm light and 780 nm light same? Is the colour of 620 nm light and 621 nm light same? How many colours are there in white light?
2. The wavelength of light in a medium is $\lambda = \lambda_0/\mu$, where λ is the wavelength in vacuum. A beam of red light ($\lambda_0 = 720$ nm) enters into water. The wavelength in water is $\lambda = \lambda_0/\mu = 540$ nm. To a person under water does this light appear green?
3. Whether the diffraction effects from a slit will be more clearly visible or less clearly, if the slit-width is increased ?
4. If we put a cardboard (say 20 cm × 20 cm) between a light source and our eyes, we can't see the light. But when we put the same cardboard between a sound source and our ear, we hear the sound almost clearly. Explain.
5. TV signals broadcast by Delhi studio cannot be directly received at Patna which is about 1000 km away. But the same signal goes some 36000 km away to a satellite, gets reflected and is then received at Patna. Explain.
6. Can we perform Young's double slit experiment with sound waves ? To get a reasonable "fringe pattern", what should be the order of separation between the slits ? How can the bright fringes and the dark fringes be detected in this case ?
7. Is it necessary to have two waves of equal intensity to study interference pattern ? Will there be an effect on clarity if the waves have unequal intenstity ?
8. Can we conclude from the interference phenomenon whether light is a transverse wave or a longitudinal wave ?
9. Why don't we have interference when two candles are placed close to each other and the intensity is seen at a distant screen ? What happens if the candles are replaced by laser sources ?
10. If the separation between the slits in a Young's double slit experiment is increased, what happens to the fringe-width ? If the separation is increased too much, will the fringe pattern remain detectable ?
11. Suppose white light falls on a double slit but one slit is covered by a violet filter (allowing $\lambda = 400$ nm). Describe the nature of the fringe pattern observed.

OBJECTIVE I

1. Light is
(a) wave phenomenon (b) particle phenomenon
(c) both particle and wave phenomenon.
2. The speed of light depends
(a) on elasticity of the medium only
(b) on inertia of the medium only
(c) on elasticity as well as inertia
(d) neither on elasticity nor on inertia.
3. The equation of a light wave is written as $y = A \sin(kx - \omega t)$. Here, y represents
(a) displacement of ether particles
(b) pressure in the medium
(c) density of the medium
(d) electric field.
4. Which of the following properties show that light is a transverse wave ?
(a) Reflection. (b) Interference.
(c) Diffraction. (d) Polarization.
5. When light is refracted into a medium,
(a) its wavelength and frequency both increase
(b) its wavelength increases but frequency remains unchanged
(c) its wavelength decreases but frequency remains unchanged
(d) its wavelength and frequency both decrease.
6. When light is refracted, which of the following does not change ?
(a) Wavelength. (b) Frequency.
(c) Velocity. (d) Amplitude.
7. The amplitude modulated (AM) radio wave bends appreciably round the corners of a 1 m × 1 m board but the frequency modulated (FM) wave only negligibly bends. If the average wavelengths of AM and FM waves are λ_a and λ_f,
(a) $\lambda_a > \lambda_f$ (b) $\lambda_a = \lambda_f$ (c) $\lambda_a < \lambda_f$
(d) we don't have sufficient information to decide about the relation of λ_a and λ_f.
8. Which of the following sources gives best monochromatic light ?
(a) A candle. (b) A bulb. (c) A mercury tube. (d) A laser.
9. The wavefronts of a light wave travelling in vacuum are given by $x + y + z = c$. The angle made by the direction of propagation of light with the X-axis is
(a) 0° (b) 45° (c) 90° (d) $\cos^{-1}(1/\sqrt{3})$.
10. The wavefronts of light coming from a distant source of unknown shape are nearly
(a) plane (b) elliptical (c) cylindrical (d) spherical.
11. The inverse square law of intensity (i.e., the intensity $\propto \frac{1}{r^2}$) is valid for a
(a) point source (b) line source
(c) plane source (d) cylindrical source.
12. Two sources are called coherent if they produce waves
(a) of equal wavelength (b) of equal velocity
(c) having same shape of wavefront
(d) having a constant phase difference.
13. When a drop of oil is spread on a water surface, it displays beautiful colours in daylight because of

(a) disperson of light (b) reflection of light
(c) polarization of light (d) interference of light.

14. Two coherent sources of different intensities send waves which interfere. The ratio of maximum intensity to the minimum intensity is 25. The intensities of the sources are in the ratio
(a) 25 : 1 (b) 5 : 1 (c) 9 : 4 (d) 625 : 1.

15. The slits in a Young's double slit experiment have equal width and the source is placed symmetrically with respect to the slits. The intensity at the central fringe is I_0. If one of the slits is closed, the intensity at this point will be
(a) I_0 (b) $I_0/4$ (c) $I_0/2$ (d) $4I_0$.

16. A thin transparent sheet is placed in front of a Young's double slit. The fringe-width will
(a) increase (b) decrease
(c) remain same (d) become nonuniform.

17. If Young's double slit experiment is performed in water,
(a) the fringe width will decrease
(b) the fringe width will increase
(c) the fringe width will remain unchanged
(d) there will be no fringe.

OBJECTIVE II

1. A light wave can travel
(a) in vacuum (b) in vacuum only
(c) in a material medium (d) in a material medium only.

2. Which of the following properties of light conclusively support wave theory of light ?
(a) Light obeys laws of reflection.
(b) Speed of light in water is smaller than the speed in vacuum.
(c) Light shows interference.
(d) Light shows photoelectric effect.

3. When light propagates in vacuum there is an electric field and a magnetic field. These fields
(a) are constant in time
(b) have zero average value
(c) are perpendicular to the direction of propagation of light.
(d) are mutually perpendicular.

4. Huygens' principle of secondary wavelets may be used to
(a) find the velocity of light in vacuum
(b) explain the particle behaviour of light
(c) find the new position of a wavefront
(d) explain Snell's law.

5. Three observers A, B and C measure the speed of light coming from a source to be v_A, v_B and v_C. The observer A moves towards the source and C moves away from the source at the same speed. The observer B stays stationary. The surrounding space is vacuum everywhere.
(a) $v_A > v_B > v_C$. (b) $v_A < v_B < v_C$.
(c) $v_A = v_B = v_C$. (d) $v_B = \frac{1}{2}(v_A + v_C)$.

6. Suppose the medium in the previous question is water. Select the correct option(s) from the list given in that question.

7. Light waves travel in vacuum along the X-axis. Which of the following may represent the wavefronts ?
(a) $x = c$. (b) $y = c$. (c) $z = c$. (d) $x + y + z = c$.

8. If the source of light used in a Young's double slit experiment is changed from red to violet,
(a) the fringes will become brighter
(b) consecutive fringes will come closer
(c) the intensity of minima will increase
(d) the central bright fringe will become a dark fringe.

9. A Young's double slit experiment is performed with white light.
(a) The central fringe will be white.
(b) There will not be a completely dark fringe.
(c) The fringe next to the central will be red.
(d) The fringe next to the central will be violet.

10. Four light waves are represented by
(i) $y = a_1 \sin \omega t$. (ii) $y = a_2 \sin(\omega t + \varepsilon)$.
(iii) $y = a_1 \sin 2\omega t$. (iv) $y = a_2 \sin 2(\omega t + \varepsilon)$.
Interference fringes may be observed due to superposition of
(a) (i) and (ii) (b) (i) and (iii)
(c) (ii) and (iv) (d) (iii) and (iv).

EXERCISES

1. Find the range of frequency of light that is visible to an average human being $(400 \text{ nm} < \lambda < 700 \text{ nm})$

2. The wavelength of sodium light in air is 589 nm. (a) Find its frequency in air. (b) Find its wavelength in water (refractive index = 1·33). (c) Find its frequency in water. (d) Find its speed in water.

3. The index of refraction of fused quartz is 1·472 for light of wavelength 400 nm and is 1·452 for light of wavelength 760 nm. Find the speeds of light of these wavelengths in fused quartz.

4. The speed of the yellow light in a certain liquid is $2{\cdot}4 \times 10^8$ m/s. Find the refractive index of the liquid.

5. Two narrow slits emitting light in phase are separated by a distance of 1·0 cm. The wavelength of the light is $5{\cdot}0 \times 10^{-7}$ m. The interference pattern is observed on a screen placed at a distance of 1·0 m. (a) Find the separation between the consecutive maxima. Can you

expect to distinguish between these maxima? (b) Find the separation between the sources which will give a separation of 1·0 mm between the consecutive maxima.

6. The separation between the consecutive dark fringes in a Young's double slit experiment is 1·0 mm. The screen is placed at a distance of 2·5 m from the slits and the separation between the slits is 1·0 mm. Calculate the wavelength of light used for the experiment.

7. In a double slit interference experiment, the separation between the slits is 1·0 mm, the wavelength of light used is $5{\cdot}0 \times 10^{-7}$ m and the distance of the screen from the slits is 1·0 m. (a) Find the distance of the centre of the first minimum from the centre of the central maximum. (b) How many bright fringes are formed in one centimeter width on the screen?

8. In a Young's double slit experiment, two narrow vertical slits placed 0·800 mm apart are illuminated by the same source of yellow light of wavelength 589 nm. How far are the adjacent bright bands in the interference pattern observed on a screen 2·00 m away?

9. Find the angular separation between the consecutive bright fringes in a Young's double slit experiment with blue-green light of wavelength 500 nm. The separation between the slits is $2{\cdot}0 \times 10^{-3}$ m.

10. A source emitting light of wavelengths 480 nm and 600 nm is used in a double slit interference experiment. The separation between the slits is 0·25 mm and the interference is observed on a screen placed at 150 cm from the slits. Find the linear separation between the first maximum (next to the central maximum) corresponding to the two wavelengths.

11. White light is used in a Young's double slit experiment. Find the minimum order of the violet fringe ($\lambda = 400$ nm) which overlaps with a red fringe ($\lambda = 700$ nm).

12. Find the thickness of a plate which will produce a change in optical path equal to half the wavelength λ of the light passing through it normally. The refractive index of the plate is μ.

13. A plate of thickness t made of a material of refractive index μ is placed in front of one of the slits in a double slit experiment. (a) Find the change in the optical path due to introduction of the plate. (b) What should be the minimum thickness t which will make the intensity at the centre of the fringe pattern zero? Wavelength of the light used is λ. Neglect any absorption of light in the plate.

14. A transparent paper (refractive index = 1·45) of thickness 0·02 mm is pasted on one of the slits of a Young's double slit experiment which uses monochromatic light of wavelength 620 nm. How many fringes will cross through the centre if the paper is removed?

15. In a Young's double slit experiment using monochromatic light, the fringe pattern shifts by a certain distance on the screen when a mica sheet of refractive index 1·6 and thickness 1·964 micron (1 micron $= 10^{-6}$ m) is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the screen and the slits is doubled. It is found that the distance between the successive maxima now is the same as the observed fringe-shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in the experiment.

16. A mica strip and a polysterene strip are fitted on the two slits of a double slit apparatus. The thickness of the strips is 0·50 mm and the separation between the slits is 0·12 cm. The refractive index of mica and polysterene are 1·58 and 1·55 respectively for the light of wavelength 590 nm which is used in the experiment. The interference is observed on a screen a distance one meter away. (a) What would be the fringe-width? (b) At what distance from the centre will the first maximum be located?

17. Two transparent slabs having equal thickness but different refractive indices μ_1 and μ_2 are pasted side by side to form a composite slab. This slab is placed just after the double slit in a Young's experiment so that the light from one slit goes through one material and the light from the other slit goes through the other material. What should be the minimum thickness of the slab so that there is a minimum at the point P_0 which is equidistant from the slits?

18. A thin paper of thickness 0·02 mm having a refractive index 1·45 is pasted across one of the slits in a Young's double slit experiment. The paper transmits 4/9 of the light energy falling on it. (a) Find the ratio of the maximum intensity to the minimum intensity in the fringe pattern. (b) How many fringes will cross through the centre if an identical paper piece is pasted on the other slit also? The wavelength of the light used is 600 nm.

19. A Young's double slit apparatus has slits separated by 0·28 mm and a screen 48 cm away from the slits. The whole apparatus is immersed in water and the slits are illuminated by the red light ($\lambda = 700$ nm in vacuum). Find the fringe-width of the pattern formed on the screen.

20. A parallel beam of monochromatic light is used in a Young's double slit experiment. The slits are separated by a distance d and the screen is placed parallel to the plane of the slits. Show that if the incident beam makes an angle $\theta = \sin^{-1}\left(\frac{\lambda}{2d}\right)$ with the normal to the plane of the slits, there will be a dark fringe at the centre P_0 of the pattern.

21. A narrow slit S transmitting light of wavelength λ is placed a distance d above a large plane mirror as shown in figure (17-E1). The light coming directly from the slit

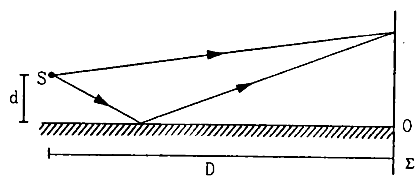


Figure 17-E1

and that coming after the reflection interfere at a screen Σ placed at a distance D from the slit. (a) What will be the intensity at a point just above the mirror, i.e., just above O ? (b) At what distance from O does the first maximum occur ?

22. A long narrow horizontal slit is placed 1 mm above a horizontal plane mirror. The interference between the light coming directly from the slit and that after reflection is seen on a screen 1·0 m away from the slit. Find the fringe-width if the light used has a wavelength of 700 nm.

23. Consider the situation of the previous problem. If the mirror reflects only 64% of the light energy falling on it, what will be the ratio of the maximum to the minimum intensity in the interference pattern observed on the screen ?

24. A double slit $S_1 - S_2$ is illuminated by a coherent light of wavelength λ. The slits are separated by a distance d. A plane mirror is placed in front of the double slit at a distance D_1 from it and a screen Σ is placed behind the double slit at a distance D_2 from it (figure 17-E2). The screen Σ receives only the light reflected by the mirror. Find the fringe-width of the interference pattern on the screen.

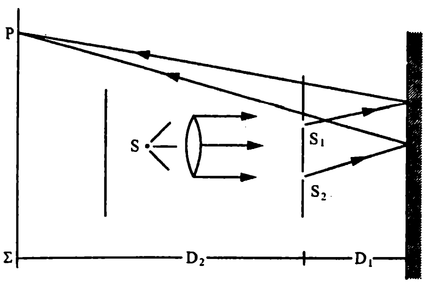


Figure 17-E2

25. White coherent light (400 nm-700 nm) is sent through the slits of a Young's double slit experiment (figure 17-E3). The separation between the slits is 0·5 mm and the screen is 50 cm away from the slits. There is a hole in the screen at a point 1·0 mm away (along the width of the fringes) from the central line. (a) Which wavelength(s) will be absent in the light coming from the hole ? (b) which wavelength(s) will have a strong intensity ?

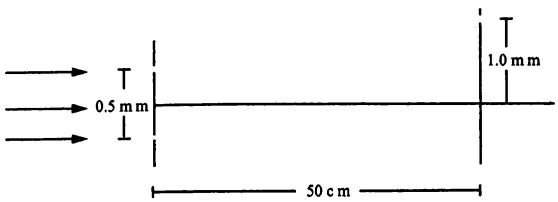


Figure 17-E3

26. Consider the arrangement shown in figure (17-E4). The distance D is large compared to the separation d between the slits. (a) Find the minimum value of d so that there is a dark fringe at O. (b) Suppose d has this value. Find the distance x at which the next bright fringe is formed. (c) Find the fringe-width.

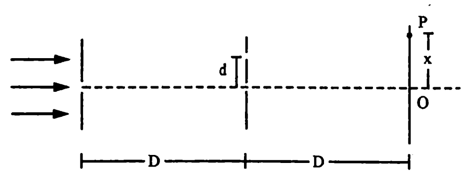


Figure 17-E4

27. Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. Consider a line passing through S_2 and perpendicular to the line $S_1 S_2$. What is the smallest distance from S_2 where a minimum of intensity occurs ?

28. Figure (17-E5) shows three equidistant slits being illuminated by a monochromatic parallel beam of light. Let $BP_0 - AP_0 = \lambda/3$ and $D >> \lambda$. (a) Show that in this case $d = \sqrt{2\lambda D/3}$. (b) Show that the intensity at P_0 is three times the intensity due to any of the three slits individually.

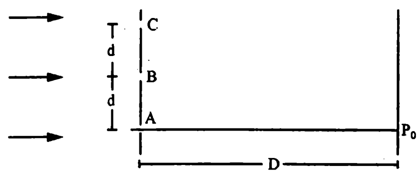


Figure 17-E5

29. In a Young's double slit experiment, the separation between the slits = 2·0 mm, the wavelength of the light = 600 nm and the distance of the screen from the slits = 2·0 m. If the intensity at the centre of the central maximum is 0·20 W/m^2, what will be the intensity at a point 0·5 cm away from this centre along the width of the fringes ?

30. In a Young's double slit interference experiment the fringe pattern is observed on a screen placed at a distance D from the slits. The slits are separated by a distance d and are illuminated by monochromatic light of wavelength λ. Find the distance from the central point where the intensity falls to (a) half the maximum, (b) one fourth of the maximum.

31. In a Young's double slit experiment $\lambda = 500$ nm, d = 1·0 mm and D = 1·0 m. Find the minimum distance from the central maximum for which the intensity is half of the maximum intensity.

32. The linewidth of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum. Find the linewidth of a bright fringe in a Young's double slit experiment in terms of λ, d and D where the symbols have their usual meanings.

33. Consider the situation shown in figure (17-E6). The two slits S_1 and S_2 placed symmetrically around the central line are illuminated by a monochromatic light of wavelength λ. The separation between the slits is d. The light transmitted by the slits falls on a screen Σ_1 placed at a distance D from the slits. The slit S_3 is at the central line and the slit S_4 is at a distance z from S_3. Another screen Σ_2 is placed a further distance D away from Σ_1. Find the ratio of the maximum to minimum intensity observed on Σ_2 if z is equal to

(a) $z = \frac{\lambda D}{2d}$, (b) $\frac{\lambda D}{d}$, (c) $\frac{\lambda D}{4d}$.

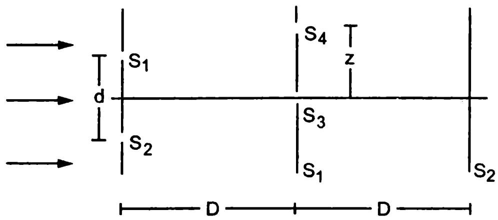


Figure 17-E6

34. Consider the arrangement shown in figure (17-E7). By some mechanism, the separation between the slits S_3 and S_4 can be changed. The intensity is measured at the point P which is at the common perpendicular bisector of S_1S_2 and S_3S_4. When $z = \frac{D\lambda}{2d}$, the intensity measured at P is I. Find this intensity when z is equal to

(a) $\frac{D\lambda}{d}$, (b) $\frac{3D\lambda}{2d}$ and (c) $\frac{2D\lambda}{d}$.

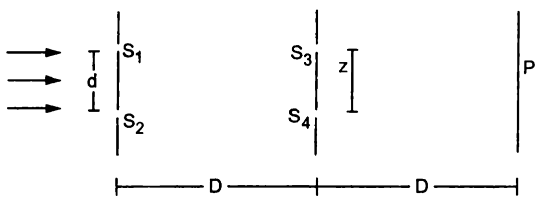


Figure 17-E7

35. A soap film of thickness 0·0011 mm appears dark when seen by the reflected light of wavelength 580 nm. What is the index of refraction of the soap solution, if it is known to be between 1·2 and 1·5?

36. A parallel beam of light of wavelength 560 nm falls on a thin film of oil (refractive index = 1·4). What should be the minimum thickness of the film so that it strongly reflects the light?

37. A parallel beam of white light is incident normally on a water film $1·0 \times 10^{-4}$ cm thick. Find the wavelength in the visible range (400 nm-700 nm) which are strongly transmitted by the film. Refractive index of water = 1·33.

38. A glass surface is coated by an oil film of uniform thickness $1·00 \times 10^{-4}$ cm. The index of refraction of the oil is 1·25 and that of the glass is 1·50. Find the wavelengths of light in the visible region (400 nm-750 nm) which are completely transmitted by the oil film under normal incidence.

39. Plane microwaves are incident on a long slit having a width of 5·0 cm. Calculate the wavelength of the microwaves if the first diffraction minimum is formed at $\theta = 30°$.

40. Light of wavelength 560 nm goes through a pinhole of diameter 0·20 mm and falls on a wall at a distance of 2·00 m. What will be the radius of the central bright spot formed on the wall?

41. A convex lens of diameter 8·0 cm is used to focus a parallel beam of light of wavelength 620 nm. If the light be focused at a distance of 20 cm from the lens, what would be the radius of the central bright spot formed?

□

ANSWERS

OBJECTIVE I

1. (c)	2. (d)	3. (d)	4. (d)	5. (c)	6. (b)
7. (a)	8. (d)	9. (d)	10. (a)	11. (a)	12. (d)
13. (d)	14. (c)	15. (b)	16. (c)	17. (a)	

OBJECTIVE II

1. (a), (c)	2. (b), (c)	3. (b), (c), (d)
4. (c), (d)	5. (c), (d)	6. (a), (d)
7. (a)	8. (b)	9. (a), (b), (d)
10. (a), (d)		

EXERCISES

1. $4·3 \times 10^{14}$ Hz-$7·5 \times 10^{14}$ Hz
2. (a) $5·09 \times 10^{14}$ Hz (b) 443 nm
   (c) $5·09 \times 10^{14}$ Hz (d) $2·25 \times 10^{8}$ m/s
3. $2·04 \times 10^{8}$ m/s, $2·07 \times 10^{8}$ m/s
4. 1·25
5. (a) 0·05 mm (b) 0·50 mm
6. 400 nm
7. (a) 0·25 mm (b) 20
8. 1·47 mm

9. 0·014 degree
10. 0·72 mm
11. 7
12. $\frac{\lambda}{2(\mu - 1)}$
13. (a) $(\mu - 1)t$, (b) $\frac{\lambda}{2(\mu - 1)}$
14. 14·5
15. 590 nm
16. (a) $4\cdot9 \times 10^{-4}$ m
    (b) 0·021 cm on one side and 0·028 cm on the other side
17. $\frac{\lambda}{2|\mu_1 - \mu_2|}$
18. (a) 25 (b) 15
19. 0·90 mm
21. (a) zero (b) $\frac{D\lambda}{4d}$
22. 0·35 mm
23. 81 : 1
24. $\lambda(2D_1 + D_2)/d$
25. (a) 400 nm, 667 nm, (b) 500 nm
26. (a) $\sqrt{\frac{\lambda D}{2}}$ (b) d (c) $2d$
27. $7\lambda/12$
29. 0·05 W/m ^{2}
30. (a) $\frac{D\lambda}{4d}$ (b) $\frac{D\lambda}{3d}$
31. $1\cdot25 \times 10^{-4}$ m
32. $\frac{D\lambda}{2d}$
33. (a) 1 (b) ∞ (c) 34
34. (a) zero (b) I (c) $2I$
35. 1·32
36. 100 nm
37. 443 nm, 532 nm and 666 nm
38. 455 nm, 556 nm, 714 nm
39. 2·5 cm
40. 1·37 cm
41. $3\cdot8 \times 10^{-6}$ m

□

SOLUTIONS TO CONCEPTS
CHAPTER 17

1. Given that, 400 m < λ < 700 nm.

$$\frac{1}{700\text{nm}} < \frac{1}{\lambda} < \frac{1}{400\text{nm}}$$

$$\Rightarrow \frac{1}{7\times10^{-7}} < \frac{1}{\lambda} < \frac{1}{4\times10^{-7}} \Rightarrow \frac{3\times10^{8}}{7\times10^{-7}} < \frac{c}{\lambda} < \frac{3\times10^{8}}{4\times10^{-7}}$$ (Where, c = speed of light = 3×10^8 m/s)

$\Rightarrow 4.3 \times 10^{14} < c/\lambda < 7.5 \times 10^{14}$

$\Rightarrow 4.3 \times 10^{14}$ Hz < f < 7.5×10^{14} Hz.

2. Given that, for sodium light, λ = 589 nm = 589×10^{-9} m

a) $f_a = \frac{3\times10^8}{589\times10^{-9}} = 5.09 \times 10^{14}\ \text{sec}^{-1} \left[\because f = \frac{c}{\lambda}\right]$

b) $\frac{\mu_a}{\mu_w} = \frac{\lambda_w}{\lambda_a} \Rightarrow \frac{1}{1.33} = \frac{\lambda_w}{589\times10^{-9}} \Rightarrow \lambda_w$ = 443 nm

c) $f_w = f_a = 5.09 \times 10^{14}$ sec^{-1} [Frequency does not change]

d) $\frac{\mu_a}{\mu_w} = \frac{v_w}{v_a} \Rightarrow v_w = \frac{\mu_a v_a}{\mu_w} = \frac{3\times10^8}{1.33} = 2.25 \times 10^8$ m/sec.

3. We know that, $\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$

So, $\frac{1472}{1} = \frac{3\times10^8}{v_{400}} \Rightarrow v_{400} = 2.04\times10^8 \text{m/sec}.$

[because, for air, μ = 1 and v = 3×10^8 m/s]

Again, $\frac{1452}{1} = \frac{3\times10^8}{v_{760}} \Rightarrow v_{760} = 2.07\times10^8 \text{m/sec}.$

4. $\mu_t = \frac{1\times3\times10^8}{(2.4)\times10^8} = 1.25$ $\left[\text{since, } \mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in the given medium}}\right]$

5. Given that, d = 1 cm = 10^{-2} m, $\lambda = 5 \times 10^{-7}$ m and D = 1 m

a) Separation between two consecutive maxima is equal to fringe width.

So, $\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times1}{10^{-2}}$ m = 5×10^{-5} m = 0.05 mm.

b) When, β = 1 mm = 10^{-3} m

$10^{-3}\text{m} = \frac{5\times10^{-7}\times1}{D} \Rightarrow D = 5 \times 10^{-4}$ m = 0.50 mm.

6. Given that, β = 1 mm = 10^{-3} m, D = 2.t m and d = 1 mm = 10^{-3} m

So, $10^{-3}\text{m} = \frac{25\times\lambda}{10^{-3}} \Rightarrow \lambda = 4 \times 10^{-7}$ m = 400 nm.

7. Given that, d = 1 mm = 10^{-3} m, D = 1 m.

So, fringe with = $\frac{D\lambda}{d}$ = 0.5 mm.

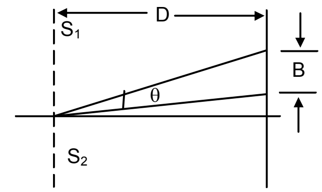
a) So, distance of centre of first minimum from centre of central maximum = 0.5/2 mm = 0.25 mm

b) No. of fringes = 10 / 0.5 = 20.

8. Given that, d = 0.8 mm = 0.8×10^{-3} m, λ = 589 nm = 589×10^{-9} m and D = 2 m.

So, $\beta = \frac{D\lambda}{d} = \frac{589\times10^{-9}\times2}{0.8\times10^{-3}} = 1.47 \times 10^{-3}$ m = 147 mm.

9. Given that, λ = 500 nm = 500×10^{-9} m and d = 2×10^{-3} m



As shown in the figure, angular separation $\theta = \frac{\beta}{D} = \frac{\lambda D}{dD} = \frac{\lambda}{d}$

So, $\theta = \frac{\beta}{D} = \frac{\lambda}{d} = \frac{500 \times 10^{-9}}{2 \times 10^{-3}}$ = 250×10^{-6}

= 25×10^{-5} radian = 0.014 degree.

10. We know that, the first maximum (next to central maximum) occurs at $y = \frac{\lambda D}{d}$

Given that, λ_1 = 480 nm, λ_2 = 600 nm, D = 150 cm = 1.5 m and d = 0.25 mm = 0.25×10^{-3} m

So, $y_1 = \frac{D\lambda_1}{d} = \frac{1.5 \times 480 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 2.88 mm

$y_2 = \frac{1.5 \times 600 \times 10^{-9}}{0.25 \times 10^{-3}}$ = 3.6 mm.

So, the separation between these two bright fringes is given by,

$\therefore$ separation = $y_2 - y_1$ = 3.60 – 2.88 = 0.72 mm.

11. Let m^{th} bright fringe of violet light overlaps with n^{th} bright fringe of red light.

$\therefore \quad \frac{m \times 400\text{nm} \times D}{d} = \frac{n \times 700\text{nm} \times D}{d} \Rightarrow \frac{m}{n} = \frac{7}{4}$

$\Rightarrow$ 7^{th} bright fringe of violet light overlaps with 4^{th} bright fringe of red light (minimum). Also, it can be seen that 14^{th} violet fringe will overlap 8^{th} red fringe.

Because, m/n = 7/4 = 14/8.

12. Let, t = thickness of the plate

Given, optical path difference = $(\mu - 1)t = \lambda/2$

$\Rightarrow t = \frac{\lambda}{2(\mu - 1)}$

13. a) Change in the optical path = $\mu t - t = (\mu - 1)t$

b) To have a dark fringe at the centre the pattern should shift by one half of a fringe.

$\Rightarrow (\mu - 1)t = \frac{\lambda}{2} \Rightarrow t = \frac{\lambda}{2(\mu - 1)}$.

14. Given that, μ = 1.45, t = 0.02 mm = 0.02×10^{-3} m and λ = 620 nm = 620×10^{-9} m

We know, when the transparent paper is pasted in one of the slits, the optical path changes by $(\mu - 1)t$.

Again, for shift of one fringe, the optical path should be changed by λ.

So, no. of fringes crossing through the centre is given by,

$n = \frac{(\mu - 1)t}{\lambda} = \frac{0.45 \times 0.02 \times 10^{-3}}{620 \times 10^{-9}}$ = 14.5

15. In the given Young's double slit experiment,

μ = 1.6, t = 1.964 micron = 1.964×10^{-6} m

We know, number of fringes shifted = $\frac{(\mu - 1)t}{\lambda}$

So, the corresponding shift = No.of fringes shifted $\times$ fringe width

$= \frac{(\mu - 1)t}{\lambda} \times \frac{\lambda D}{d} = \frac{(\mu - 1)tD}{d}$ … (1)

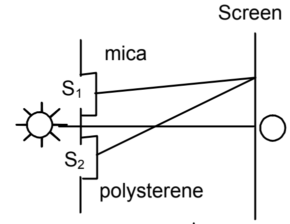
Again, when the distance between the screen and the slits is doubled,

Fringe width = $\frac{\lambda(2D)}{d}$ …(2)

From (1) and (2), $\frac{(\mu - 1)tD}{d} = \frac{\lambda(2D)}{d}$

$\Rightarrow \lambda = \frac{(\mu - 1)t}{\lambda} = \frac{(1.6 - 1) \times (1.964) \times 10^{-6}}{2}$ = 589.2×10^{-9} = 589.2 nm.

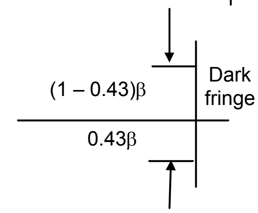
16. Given that, $t_1 = t_2 = 0.5$ mm $= 0.5 \times 10^{-3}$ m, $\mu_m = 1.58$ and $\mu_p = 1.55$,
$\lambda = 590$ nm $= 590 \times 10^{-9}$ m, d = 0.12 cm $= 12 \times 10^{-4}$ m, D = 1 m



a) Fringe width $= \dfrac{D\lambda}{d} = \dfrac{1 \times 590 \times 10^{-9}}{12 \times 10^{-4}}$ $= 4.91 \times 10^{-4}$ m.

b) When both the strips are fitted, the optical path changes by
$\Delta x = (\mu_m - 1)t_1 - (\mu_p - 1)t_2 = (\mu_m - \mu_p)t$
$= (1.58 - 1.55) \times (0.5)(10^{-3}) = 0.015 \times 10^{-13}$ m.

So, No. of fringes shifted $= \dfrac{0.015 \times 10^{-3}}{590 \times 10^{-3}}$ = 25.43.



$\Rightarrow$ There are 25 fringes and 0.43 th of a fringe.
$\Rightarrow$ There are 13 bright fringes and 12 dark fringes and 0.43 th of a dark fringe.
So, position of first maximum on both sides will be given by
$\therefore$ $x = 0.43 \times 4.91 \times 10^{-4} = 0.021$ cm
$x' = (1 - 0.43) \times 4.91 \times 10^{-4} = 0.028$ cm (since, fringe width $= 4.91 \times 10^{-4}$ m)

17. The change in path difference due to the two slabs is $(\mu_1 - \mu_2)t$ (as in problem no. 16).
For having a minimum at P_0, the path difference should change by $\lambda/2$.

So, $\Rightarrow \lambda/2 = (\mu_1 - \mu_2)t \Rightarrow t = \dfrac{\lambda}{2(\mu_1 - \mu_2)}$.

18. Given that, t = 0.02 mm $= 0.02 \times 10^{-3}$ m, $\mu_1 = 1.45$, $\lambda = 600$ nm $= 600 \times 10^{-9}$ m
a) Let, I_1 = Intensity of source without paper = I
b) Then I_2 = Intensity of source with paper = (4/9)I

$\Rightarrow \dfrac{I_1}{I_2} = \dfrac{9}{4} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{3}{2}$ [because $I \propto r^2$]

where, r_1 and r_2 are corresponding amplitudes.

So, $\dfrac{I_{max}}{I_{min}} = \dfrac{(r_1 + r_2)^2}{(r_1 - r_2)^2}$ = 25 : 1

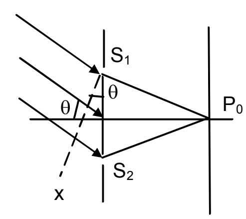
b) No. of fringes that will cross the origin is given by,

n = $\dfrac{(\mu - 1)t}{\lambda}$ = $\dfrac{(1.45 - 1) \times 0.02 \times 10^{-3}}{600 \times 10^{-9}}$ = 15.

19. Given that, d = 0.28 mm $= 0.28 \times 10^{-3}$ m, D = 48 cm = 0.48 m, $\lambda_a = 700$ nm in vacuum
Let, λ_w = wavelength of red light in water
Since, the fringe width of the pattern is given by,

$\beta = \dfrac{\lambda_w D}{d} = \dfrac{525 \times 10^{-9} \times 0.48}{0.28 \times 10^{-3}}$ $= 9 \times 10^{-4}$ m = 0.90 mm.

20. It can be seen from the figure that the wavefronts reaching O from S_1 and S_2 will have a path difference of S_2X.

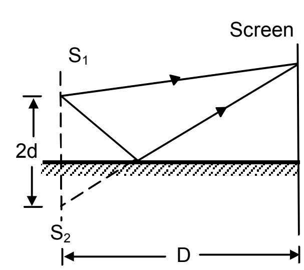


In the Δ S_1S_2X,

$\sin\ \theta = \dfrac{S_2X}{S_1S_2}$

So, path difference $= S_2X = S_1S_2 \sin\theta = d \sin\theta = d \times \lambda/2d = \lambda/2$
As the path difference is an odd multiple of $\lambda/2$, there will be a dark fringe at point P_0.

21. a) Since, there is a phase difference of π between direct light and reflecting light, the intensity just above the mirror will be zero.



b) Here, 2d = equivalent slit separation
D = Distance between slit and screen.

We know for bright fringe, $\Delta x = \dfrac{y \times 2d}{D} = n\lambda$

But as there is a phase reversal of $\lambda/2$.

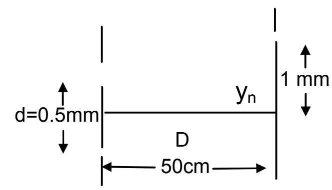
$\Rightarrow \dfrac{y \times 2d}{D} + \dfrac{\lambda}{2} = n\lambda$ $\qquad \Rightarrow \dfrac{y \times 2d}{D} = n\lambda - \dfrac{\lambda}{2} \Rightarrow y = \dfrac{\lambda D}{4d}$

22. Given that, D = 1 m, λ = 700 nm = 700×10^{-9} m
Since, a = 2 mm, d = 2a = 2mm = 2×10^{-3} m (L loyd's mirror experiment)
Fringe width = $\frac{\lambda D}{d} = \frac{700 \times 10^{-9} m \times 1m}{2 \times 10^{-3} m}$ = 0.35 mm.

23. Given that, the mirror reflects 64% of energy (intensity) of the light.
So, $\frac{I_1}{I_2} = 0.64 = \frac{16}{25} \Rightarrow \frac{r_1}{r_2} = \frac{4}{5}$
So, $\frac{I_{max}}{I_{min}} = \frac{(r_1 + r_2)^2}{(r_1 - r_2)^2}$ = 81 : 1.

24. It can be seen from the figure that, the apparent distance of the screen from the slits is,
$D = 2D_1 + D_2$
So, Fringe width = $\frac{D\lambda}{d} = \frac{(2D_1 + D_2)\lambda}{d}$

25. Given that, λ = (400 nm to 700 nm), d = 0.5 mm = 0.5×10^{-3} m,
D = 50 cm = 0.5 m and on the screen y_n = 1 mm = 1×10^{-3} m



a) We know that for zero intensity (dark fringe)
$y_n = \left(\frac{2n+1}{2}\right)\frac{\lambda_n D}{d}$ where n = 0, 1, 2, …….

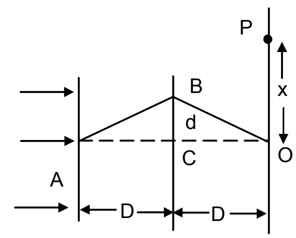
$\Rightarrow \lambda_n = \frac{2}{(2n+1)}\frac{\lambda_n d}{D} = \frac{2}{2n+1} \times \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} \Rightarrow \frac{2}{(2n+1)} \times 10^{-6} m = \frac{2}{(2n+1)} \times 10^{3} nm$

If n = 1, λ_1 = (2/3) × 1000 = 667 nm
If n = 1, λ_2 = (2/5) × 1000 = 400 nm

So, the light waves of wavelengths 400 nm and 667 nm will be absent from the out coming light.

b) For strong intensity (bright fringes) at the hole
$y_n = \frac{n\lambda_n D}{d} \Rightarrow \lambda_n = \frac{y_n d}{nD}$
When, n = 1, $\lambda_1 = \frac{y_n d}{D} = \frac{10^{-3} \times 0.5 \times 10^{-3}}{0.5} = 10^{-6} m = 1000 nm$.
1000 nm is not present in the range 400 nm – 700 nm
Again, where n = 2, $\lambda_2 = \frac{y_n d}{2D}$ = 500 nm
So, the only wavelength which will have strong intensity is 500 nm.

26. From the diagram, it can be seen that at point O.
Path difference = (AB + BO) – (AC + CO)
= 2(AB – AC) [Since, AB = BO and AC = CO] = $2(\sqrt{d^2 + D^2} - D)$
For dark fringe, path difference should be odd multiple of $\lambda/2$.

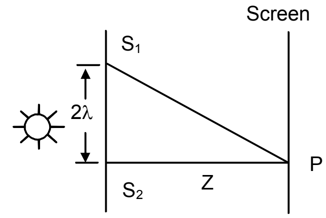


So, $2(\sqrt{d^2 + D^2} - D)$ = (2n + 1)(λ/2)
$\Rightarrow \sqrt{d^2 + D^2}$ = D + (2n + 1) λ/4
$\Rightarrow D^2 + d^2 = D^2 + (2n+1)^2 \lambda^2/16 + (2n + 1) \lambda D/2$
Neglecting, $(2n+1)^2 \lambda^2/16$, as it is very small
We get, d = $\sqrt{(2n+1)\frac{\lambda D}{2}}$
For minimum 'd', putting n = 0 $\Rightarrow d_{min} = \sqrt{\frac{\lambda D}{2}}$.

27. For minimum intensity

$\therefore$ $S_1P - S_2P = x = (2n + 1)\ \lambda/2$

From the figure, we get



$$\Rightarrow \sqrt{Z^2 + (2\lambda)^2} - Z = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow Z^2 + 4\lambda^2 = Z^2 + (2n+1)^2\frac{\lambda^2}{4} + Z(2n+1)\lambda$$

$$\Rightarrow Z = \frac{4\lambda^2 - (2n+1)^2(\lambda^2/4)}{(2n+1)\lambda} = \frac{16\lambda^2 - (2n+1)^2\lambda^2}{4(2n+1)\lambda} \quad \ldots(1)$$

Putting, $n = 0 \Rightarrow Z = 15\lambda/4$ $\quad$ $n = -1 \Rightarrow Z = -15\lambda/4$

$n = 1 \Rightarrow Z = 7\lambda/12$ $\quad$ $n = 2 \Rightarrow Z = -9\lambda/20$

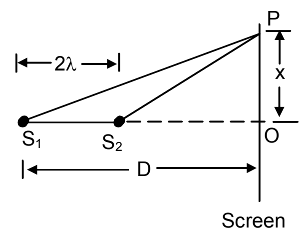
$\therefore$ $Z = 7\lambda/12$ is the smallest distance for which there will be minimum intensity.

28. Since S_1, S_2 are in same phase, at O there will be maximum intensity.

Given that, there will be a maximum intensity at P.

$\Rightarrow$ path difference = $\Delta x = n\lambda$

From the figure,



$(S_1P)^2 - (S_2P)^2 = (\sqrt{D^2 + X^2})^2 - (\sqrt{(D-2\lambda)^2 + X^2})^2$

$= 4\lambda D - 4\lambda^2 = 4\ \lambda D$ (λ^2 is so small and can be neglected)

$$\Rightarrow S_1P - S_2P = \frac{4\lambda D}{2\sqrt{x^2 + D^2}} = n\lambda$$

$$\Rightarrow \frac{2D}{\sqrt{x^2 + D^2}} = \nu$$

$$\Rightarrow n^2\ (X^2 + D^2) = 4D^2 = \Delta X = \frac{D}{n}\sqrt{4 - n^2}$$

when $n = 1$, $x = \sqrt{3}\ D$ $\quad$ (1^{st} order)

$n = 2$, $x = 0$ $\quad$ (2^{nd} order)

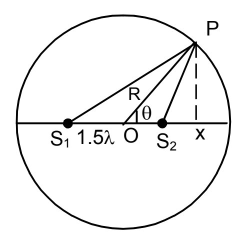
$\therefore$ When $X = \sqrt{3}\ D$, at P there will be maximum intensity.

29. As shown in the figure,

$(S_1P)^2 = (PX)^2 + (S_1X)^2$ $\quad \ldots(1)$

$(S_2P)^2 = (PX)^2 + (S_2X)^2$ $\quad \ldots(2)$

From (1) and (2),



$(S_1P)^2 - (S_2P)^2 = (S_1X)^2 - (S_2X)^2$

$= (1.5\ \lambda + R\cos\theta)^2 - (R\cos\theta - 15\ \lambda)^2$

$= 6\lambda\ R\cos\theta$

$$\Rightarrow (S_1P - S_2P) = \frac{6\lambda R\cos\theta}{2R} = 3\lambda\cos\theta.$$

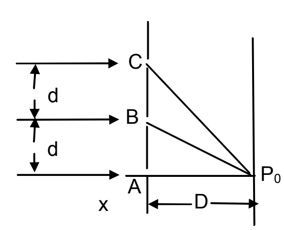
For constructive interference,

$(S_1P - S_2P)^2 = x = 3\lambda\cos\theta = n\lambda$

$\Rightarrow$ $\cos\theta = n/3 \Rightarrow \theta = \cos^{-1}(n/3)$, where $n = 0, 1, 2, \ldots.$

$\Rightarrow$ $\theta = 0°, 48.2°, 70.5°, 90°$ and similar points in other quadrants.

30. a) As shown in the figure, $BP_0 - AP_0 = \lambda/3$



$$\Rightarrow \sqrt{(D^2 + d^2)} - D = \lambda/3$$

$\Rightarrow$ $D^2 + d^2 = D^2 + (\lambda^2/9) + (2\lambda D)/3$

$\Rightarrow$ $d = \sqrt{(2\lambda D)/3}$ $\quad$ (neglecting the term $\lambda^2/9$ as it is very small)

b) To find the intensity at P_0, we have to consider the interference of light waves coming from all the three slits.

Here, $CP_0 - AP_0 = \sqrt{D^2 + 4d^2} - D$

$= \sqrt{D^2 + \frac{8\lambda D}{3}} - D = D\left\{1 + \frac{8\lambda}{3D}\right\}^{1/2} - D$

$= D\left\{1 + \frac{8\lambda}{3D \times 2} + ...\right\} - D = \frac{4\lambda}{3}$ [using binomial expansion]

So, the corresponding phase difference between waves from C and A is,

$\phi_c = \frac{2\pi x}{\lambda} = \frac{2\pi \times 4\lambda}{3\lambda} = \frac{8\pi}{3} = \left(2\pi + \frac{2\pi}{3}\right) = \frac{2\pi}{3}$ …(1)

Again, $\phi_B = \frac{2\pi x}{3\lambda} = \frac{2\pi}{3}$ …(2)

So, it can be said that light from B and C are in same phase as they have some phase difference with respect to A.

So, R $= \sqrt{(2r)^2 + r^2 + 2 \times 2r \times r\cos(2\pi/3)}$ (using vector method)

$= \sqrt{4r^2 + r^2 - 2r^2} = \sqrt{3}r$

$\therefore\ I_{P_0} - K(\sqrt{3}r)^2 = 3Kr^2 = 3I$

As, the resulting amplitude is $\sqrt{3}$ times, the intensity will be three times the intensity due to individual slits.

31. Given that, d = 2 mm = 2×10^{-3} m, λ = 600 nm = 6×10^{-7} m, I_{max} = 0.20 W/m^2, D = 2m

For the point, y = 0.5 cm

We know, path difference = x = $\frac{yd}{D} = \frac{0.5 \times 10^{-2} \times 2 \times 10^{-3}}{2}$ = 5×10^{-6} m

So, the corresponding phase difference is,

$\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 5 \times 10^{-6}}{6 \times 10^{-7}} \Rightarrow \frac{50\pi}{3} = 16\pi + \frac{2\pi}{3} \Rightarrow \phi = \frac{2\pi}{3}$

So, the amplitude of the resulting wave at the point y = 0.5 cm is,

$A = \sqrt{r^2 + r^2 + 2r^2\cos(2\pi/3)} = \sqrt{r^2 + r^2 - r^2} = r$

Since, $\frac{I}{I_{max}} = \frac{A^2}{(2r)^2}$ [since, maximum amplitude = 2r]

$\Rightarrow \frac{I}{0.2} = \frac{A^2}{4r^2} = \frac{r^2}{4r^2}$

$\Rightarrow I = \frac{0.2}{4} = 0.05$ W/m^2.

32. i) When intensity is half the maximum $\frac{I}{I_{max}} = \frac{1}{2}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{2}$

$\Rightarrow \cos^2(\phi/2) = 1/2 \Rightarrow \cos(\phi/2) = 1/\sqrt{2}$

$\Rightarrow \phi/2 = \pi/4 \Rightarrow \phi = \pi/2$

$\Rightarrow$ Path difference, x = $\lambda/4$

$\Rightarrow$ y = xD/d = λD/4d

ii) When intensity is 1/4th of the maximum $\frac{I}{I_{max}} = \frac{1}{4}$

$\Rightarrow \frac{4a^2\cos^2(\phi/2)}{4a^2} = \frac{1}{4}$

$\Rightarrow \cos^2(\phi/2) = 1/4 \Rightarrow \cos(\phi/2) = 1/2$

$\Rightarrow \phi/2 = \pi/3 \Rightarrow \phi = 2\pi/3$

$\Rightarrow$ Path difference, x = $\lambda/3$

$\Rightarrow$ y = xD/d = λD/3d

33. Given that, D = 1 m, d = 1 mm = 10^{-3} m, λ = 500 nm = 5×10^{-7} m
For intensity to be half the maximum intensity.

$$y = \frac{\lambda D}{4d} \qquad \text{(As in problem no. 32)}$$

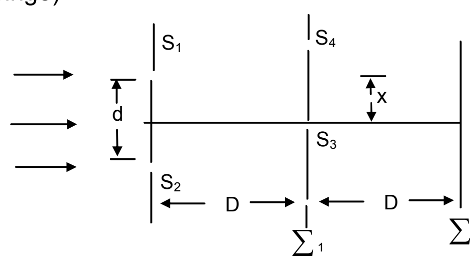
$$\Rightarrow y = \frac{5\times10^{-7}\times1}{4\times10^{-3}} \Rightarrow y = 1.25 \times 10^{-4} \text{ m.}$$

34. The line width of a bright fringe is sometimes defined as the separation between the points on the two sides of the central line where the intensity falls to half the maximum.
We know that, for intensity to be half the maximum

$$y = \pm\frac{\lambda D}{4d}$$

$$\therefore \text{ Line width} = \frac{\lambda D}{4d} + \frac{\lambda D}{4d} = \frac{\lambda D}{2d}.$$

35. i) When, z = $\lambda D/2d$, at S_4, minimum intensity occurs (dark fringe)



$\Rightarrow$ Amplitude = 0,
At S_3, path difference = 0
$\Rightarrow$ Maximum intensity occurs.
$\Rightarrow$ Amplitude = 2r.
So, on $\Sigma 2$ screen,

$$\frac{I_{max}}{I_{min}} = \frac{(2r+0)^2}{(2r-0)^2} = 1$$

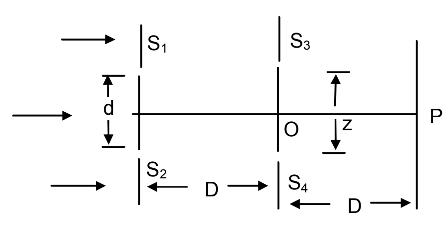
ii) When, z = $\lambda D/2d$, At S_4, minimum intensity occurs. (dark fringe)
$\Rightarrow$ Amplitude = 0.
At S_3, path difference = 0
$\Rightarrow$ Maximum intensity occurs.
$\Rightarrow$ Amplitude = 2r.
So, on $\Sigma 2$ screen,

$$\frac{I_{max}}{I_{min}} = \frac{(2r+2r)^2}{(2r-0)^2} = \infty$$

iii) When, z = $\lambda D/4d$, At S_4, intensity = I_{max} / 2
$\Rightarrow$ Amplitude = $\sqrt{2}r$.
$\therefore$ At S_3, intensity is maximum.
$\Rightarrow$ Amplitude = 2r

$$\therefore \frac{I_{max}}{I_{min}} = \frac{(2r+\sqrt{2}r)^2}{(2r-\sqrt{2}r)^2} = 34.$$

36. a) When, z = $D\lambda/d$
So, $OS_3 = OS_4 = D\lambda/2d \Rightarrow$ Dark fringe at S_3 and S_4.



$\Rightarrow$ At S_3, intensity at $S_3 = 0 \Rightarrow I_1 = 0$
At S_4, intensity at $S_4 = 0 \Rightarrow I_2 = 0$
At P, path difference = 0 $\Rightarrow$ Phase difference = 0.
$\Rightarrow I = I_1 + I_2 + \sqrt{I_1 I_2} \cos 0° = 0 + 0 + 0 = 0 \Rightarrow$ Intensity at P = 0.

b) Given that, when z = $D\lambda/2d$, intensity at P = I
Here, $OS_3 = OS_4 = y = D\lambda/4d$

$$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda}\times\frac{yd}{D} = \frac{2\pi}{\lambda}\times\frac{D\lambda}{4d}\times\frac{d}{D} = \frac{\pi}{2}. \qquad \text{[Since, x = path difference = yd/D]}$$

Let, intensity at S_3 and S_4 = I′
$\therefore$ At P, phase difference = 0
So, I′ + I′ + 2I′ cos 0° = I.
$\Rightarrow$ 4I′ = I $\Rightarrow$ I′ = 1/4.

When, $z = \frac{3D\lambda}{2d}$, $\Rightarrow y = \frac{3D\lambda}{4d}$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{3D\lambda}{4d} \times \frac{d}{D} = \frac{3\pi}{2}$

Let, I'' be the intensity at S_3 and S_4 when, $\phi = 3\pi/2$

Now comparing,

$\frac{I''}{I} = \frac{a^2 + a^2 + 2a^2\cos(3\pi/2)}{a^2 + a^2 + 2a^2\cos\pi/2} = \frac{2a^2}{2a^2} = 1$ $\Rightarrow I'' = I' = I/4.$

$\therefore$ Intensity at P = I/4 + I/4 + 2 × (I/4) cos 0° = I/2 + I/2 = I.

c) When $z = 2D\lambda/d$

$\Rightarrow y = OS_3 = OS_4 = D\lambda/d$

$\therefore \phi = \frac{2\pi x}{\lambda} = \frac{2\pi}{\lambda} \times \frac{yd}{D} = \frac{2\pi}{\lambda} \times \frac{D\lambda}{d} \times \frac{d}{D} = 2\pi$.

Let, I''' = intensity at S_3 and S_4 when, $\phi = 2\pi$.

$\frac{I'''}{I'} = \frac{a^2 + a^2 + 2a^2\cos 2\pi}{a^2 + a^2 + 2a^2\cos\pi/2} = \frac{4a^2}{2a^2} = 2$

$\Rightarrow I''' = 2I' = 2(I/4) = I/2$

At P, $I_{resultant}$ = I/2 + I/2 + 2(I/2) cos 0° = I + I = 2I.

So, the resultant intensity at P will be 2I.

37. Given $d = 0.0011 \times 10^{-3}$ m

For minimum reflection of light, $2\mu d = n\lambda$

$\Rightarrow \mu = \frac{n\lambda}{2d} = \frac{2n\lambda}{4d} = \frac{580 \times 10^{-9} \times 2n}{4 \times 11 \times 10^{-7}} = \frac{5.8}{44}(2n) = 0.132\ (2n)$

Given that, μ has a value in between 1.2 and 1.5.

$\Rightarrow$ When, n = 5, $\mu = 0.132 \times 10 = 1.32$.

38. Given that, $\lambda = 560 \times 10^{-9}$ m, $\mu = 1.4$.

For strong reflection, $2\mu d = (2n + 1)\lambda/2 \Rightarrow d = \frac{(2n+1)\lambda}{4d}$

For minimum thickness, putting n = 0.

$\Rightarrow d = \frac{\lambda}{4d} \Rightarrow d = \frac{560 \times 10^{-9}}{14} = 10^{-7}$ m = 100 nm.

39. For strong transmission, $2\mu d = n\lambda \Rightarrow \lambda = \frac{2\mu d}{n}$

Given that, $\mu = 1.33$, $d = 1 \times 10^{-4}$ cm $= 1 \times 10^{-6}$ m.

$\Rightarrow \lambda = \frac{2 \times 1.33 \times 1 \times 10^{-6}}{n} = \frac{2660 \times 10^{-9}}{n}$ m

when, n = 4, λ_1 = 665 nm

n = 5, λ_2 = 532 nm

n = 6, λ_3 = 443 nm

40. For the thin oil film,

$d = 1 \times 10^{-4}$ cm $= 10^{-6}$ m, $\mu_{oil} = 1.25$ and $\mu_x = 1.50$

$\lambda = \frac{2\mu d}{(n+1/2)} \frac{2 \times 10^{-6} \times 1.25 \times 2}{2n+1} = \frac{5 \times 10^{-6} m}{2n+1}$

$\Rightarrow \lambda = \frac{5000\ nm}{2n+1}$

For the wavelengths in the region (400 nm – 750 nm)

When, n = 3, $\lambda = \frac{5000}{2 \times 3 + 1} = \frac{5000}{7}$ = 714.3 nm

When, n = 4, $\lambda = \frac{5000}{2 \times 4 + 1} = \frac{5000}{9}$ = 555.6 nm

When, n = 5, $\lambda = \frac{5000}{2 \times 5 + 1} = \frac{5000}{11}$ = 454.5 nm

41. For first minimum diffraction, b sin $\theta = \lambda$
Here, $\theta = 30°$, b = 5 cm
$\therefore$ $\lambda = 5 \times \sin 30° = 5/2 = 2.5$ cm.

42. λ = 560 nm = 560×10^{-9} m, b = 0.20 mm = 2×10^{-4} m, D = 2 m
Since, $R = 1.22\frac{\lambda D}{b} = 1.22 \times \frac{560 \times 10^{-9} \times 2}{2 \times 10^{-4}}$ = 6.832×10^{-3} M = 0.683 cm.
So, Diameter = 2R = 1.37 cm.

43. λ = 620 nm = 620×10^{-9} m,
D = 20 cm = 20×10^{-2} m, b = 8 cm = 8×10^{-2} m
$\therefore$ $R = 1.22 \times \frac{620 \times 10^{-4} \times 20 \times 10^{-2}}{8 \times 10^{-2}}$ = 1891×10^{-9} = 1.9×10^{-6} m
So, diameter = 2R = 3.8×10^{-6} m

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