CHAPTER 14

SOME MECHANICAL PROPERTIES OF MATTER

14.1 MOLECULAR STRUCTURE OF A MATERIAL

Matter is made of molecules and atoms. An atom is made of a nucleus and electrons. The nucleus contains positively charged protons and neutrons, collectively called nucleons. Nuclear forces operating between different nucleons are responsible for the structure of the nucleus. Electromagnetic forces operate between a pair of electrons and between an electron and the nucleus. These forces are responsible for the structure of an atom. The forces between different atoms are responsible for the structure of a molecule and the forces between the molcules are responsible for the structure of the material as seen by us.

Interatomic and Intermolecular Forces

The force between two atoms can be typically represented by the potential energy curve shown in figure (14.1). The horizontal axis represents the separation between the atoms. The zero of potential energy is taken when the atoms are widely separated $(r = \infty)$.

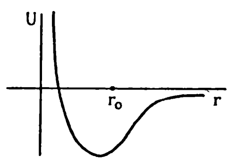


Figure 14.1

As the separation between the atoms is decreased from a large value, the potential energy also decreases, becoming negative. This shows that the force between the atoms is attractive in this range. As the separation is decreased to a particular value r_0, the potential energy is minimum. At this separation, the force is zero and the atoms can stay in equilibrium. If the separation is further decreased, the potential energy increases. This means a repulsive force acts between the atoms at small separations.

A polyatomic molecule is formed when the atoms are arranged in such a fashion that the total potential energy of the system is minimum.

The force between two molecules has the same general nature as shown in figure (14.1). At large separation, the force between two molecules is weak and attractive. The force increases as the separation is decreased to a particular value and then decreases to zero at $r = r_0$. If the separation is further decreased, the force becomes repulsive.

Bonds

The atoms form molecules primarily due to the electrostatic interaction between the electrons and the nuclei. These interactions are described in terms of different kinds of *bonds*. We shall briefly discuss two important bonds that frequently occur in materials.

Ionic Bond

In an *ionic bond* two atoms come close to each other and an electron is completely transferred from one atom to the other. This leaves the first atom positively charged and the other one negatively charged. There is an electrostatic attraction between the ions which keeps them bound. For example, when a sodium atom comes close to a chlorine atom, an electron of the sodium atom is completely transferred to the chlorine atom. The positively charged sodium ion and the negatively charged chlorine ion attract each other to form an ionic bond resulting in sodium chloride molecule.

Covalent Bond

In many of the cases a complete transfer of electron from one atom to another does not take place to form a bond. Rather, electrons from neighbouring atoms are made available for sharing between the atoms. Such bonds are called *covalent bond*. When two hydrogen atoms come close to each other, both the electrons are available to both the nuclei. In other words, each electron moves through the total space occupied by the two atoms. Each electron is pulled by both the nuclei. Chlorine molecule is also formed by this mechanism. Two chlorine atoms share a pair of electrons to form the bond. Another example of covalent bond is hydrogen chloride (HCl) molecule.

Three States of Matter

If two molecules are kept at a separation $r = r_0$, they will stay in equilibrium. If they are slightly pulled apart so that $r > r_0$, an attractive force will operate between them. If they are slightly pushed so that $r < r_0$, a repulsive force will operate. Thus, if a molecule is slightly displaced from its equilibrium position, it will oscillate about its mean position. This is the situation in a solid. The molecules are close to each other, very nearly at the equilibrium separations. The amplitude of vibrations is very small and the molecules remain almost fixed at their positions. This explains why a solid has a fixed shape if no external forces act to deform it.

In liquids, the average separation between the molecules is somewhat larger. The attractive force is weak and the molecules are more free to move inside the whole mass of the liquid. In gases, the separation is much larger and the molecular force is very weak.

Solid State

In solids, the intermolecular forces are so strong that the molecules or ions remain almost fixed at their equilibrium positions. Quite often these equilibrium positions have a very regular three-dimensional arrangement which we call *crystal*. The positions occupied by the molecules or the ions are called *lattice points*. Because of this long range ordering, the molecules or ions combine to form large rigid solids.

The crystalline solids are divided into four categories depending on the nature of the bonding between the basic units.

Molecular Solid

In a molecular solid, the molecules are formed due to covalent bonds between the atoms. The bonding between the molecules depends on whether the molecules are *polar* or *nonpolar* as discussed below. If the centre of negative charge in a molecule coincides with the centre of the positive charge, the molecule is called *nonpolar*. Molecules of hydrogen, oxygen, chlorine etc. are of this type. Otherwise, the molecule is called a *polar* molecule. Water molecule is polar. The bond between polar molecules is called a *dipole-dipole bond*. The bond between nonpolar molecules is called a *Van der Waals bond*. Molecular solids are usually soft and have low melting point. They are poor conductors of electricity.

Ionic Solid

In an ionic solid, the lattice points are occupied by positive and negative ions. The electrostatic attraction between these ions binds the solid. These attraction forces are quite strong so that the material is usually hard and has fairly high melting point. They are poor conductor of electricity.

Covalent Solid

In a covalent solid, atoms are arranged in the crystalline form. The neighbouring atoms are bound by shared electrons. Such covalent bonds extend in space so as to form a large solid structure. Diamond, silicon etc. are examples of covalent solids. Each carbon atom is bonded to four neighbouring carbon atoms in a diamond structure. They are quite hard, have high melting point and are poor conductors of electricity.

Metallic Solid

In a metallic solid, positive ions are situated at the lattice points. These ions are formed by detaching one or more electrons from the constituent atoms. These electrons are highly mobile and move throughout the solid just like a gas. They are very good conductors of electricity.

Amorphous or Glassy State

There are several solids which do not exhibit a long range ordering. However, they still show a local ordering so that some molecules (say 4-5) are bonded together to form a structure. Such independent units are randomly arranged to form the extended solid. In this respect the amorphous solid is similar to a liquid which also lacks any long range ordering. However, the intermolecular forces in amorphous solids are much stronger than those in liquids. This prevents the amorphous solid to flow like a fluid. A typical example is glass made of silicon and oxygen together with some other elements like calcium and sodium. The structure contains strong $Si-O-Si$ bonds, but the structure does not extend too far in space.

The amorphous solids do not have a well-defined melting point. Different bonds have different strengths and as the material is heated the weaker bonds break earlier starting the melting process. The stronger bonds break at higher temperatures to complete the melting process.

14.2 ELASTICITY

We have used the concept of a rigid solid body in which the distance between any two particles is always fixed. Real solid bodies do not exactly fulfil this condition. When external forces are applied, the body may get deformed. When deformed, internal forces develop which try to restore the body in its original shape. The extent, to which the shape of a body is restored when the deforming forces are removed, varies from material to material. The property to restore the natural shape or to oppose the deformation is called *elasticity.* If a body completely gains its natural shape after the removal of the deforming forces, it is called a perfectly *elastic* body. If a body remains in the deformed state and does not even partially regain its original shape after the removal of the deforming forces, it is called a perfectly *inelastic* or *plastic* body. Quite often, when the deforming forces are removed, the body partially regains the original shape. Such bodies are partially elastic.

Microscopic Reason of Elasticity

A solid body is composed of a great many molecules or atoms arranged in a particular fashion. Each molecule is acted upon by the forces due to the neighbouring molecules. The solid takes such a shape that each molecule finds itself in a position of stable equilibrium. When the body is deformed, the molecules are displaced from their original positions of stable equilibrium. The intermolecular distances change and restoring forces start acting on the molecules which drive them back to their original positions and the body takes its natural shape.

One can compare this situation to a spring-mass system. Consider a particle connected to several particles through springs. If this particle is displaced a little, the springs exert a resultant force which tries to bring the particle towards its natural position. In fact, the particle will oscillate about this position. In due course, the oscillations will be damped out and the particle will regain its original position.

14.3 STRESS

Longitudinal and Shearing Stress

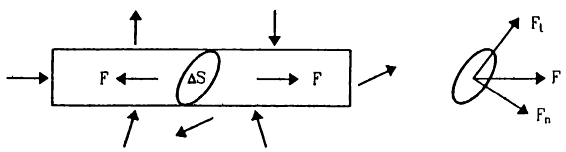


Figure 14.2

Consider a body (figure 14.2) on which several forces are acting. The resultant of these forces is zero so that the centre of mass remains at rest. Due to the forces, the body gets deformed and internal forces appear. Consider any cross-sectional area ΔS of the body. The parts of the body on the two sides of ΔS exert forces $\vec{F}$, $-\vec{F}$ on each other. These internal forces $\vec{F}$, $-\vec{F}$ appear because of the deformation.

The force $\vec{F}$ may be resolved in two components, F_n normal to ΔS and F_t tangential to ΔS. We define the *normal stress* or *longitudinal stress* over the area as

$$\Gamma_n = \frac{F_n}{\Delta S} \quad \text{... (14.1)}$$

and the *tangential stress* or *shearing stress* over the area as

$$\Gamma_t = \frac{F_t}{\Delta S}. \quad \text{... (14.2)}$$

The longitudinal stress can be of two types. The two parts of the body on the two sides of ΔS may pull each other. The longitudinal stress is then called the *tensile stress.* This is the case when a rod or a wire is stretched by equal and opposite forces (figure 14.3). In case of tensile stress in a wire or a rod, the force F_n is just the tension.

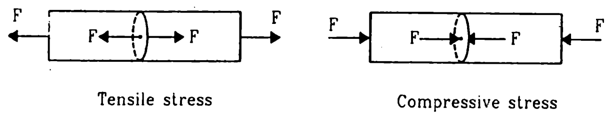


Figure 14.3

If the rod is pushed at the two ends with equal and opposite forces, it will be under compression. Taking any cross-section ΔS of the rod the two parts on the two sides push each other. The longitudinal stress is then called *compressive stress.*

If the area is not specifically mentioned, a cross-section perpendicular to the length is assumed.

Example 14.1

A load of 4·0 kg *is suspended from a ceiling through a steel wire of radius* 2·0 mm. *Find the tensile stress developed in the wire when equilibrium is achieved. Take* $g = 3{\cdot}1\pi \text{ m/s}^2$.

Solution : The tension in the wire is

$$F = 4{\cdot}0 \times 3{\cdot}1\pi \text{ N}.$$

The area of cross-section is

$$A = \pi r^2 = \pi \times (2{\cdot}0 \times 10^{-3} \text{ m})^2$$
$$= 4{\cdot}0\, \pi \times 10^{-6} \text{ m}^2.$$

Thus, the tensile stress developed

$$= \frac{F}{A} = \frac{4 \cdot 0 \times 3 \cdot 1 \, \pi}{4 \cdot 0 \, \pi \times 10^{-6}} \, \mathrm{N/m^2}$$
$$= 3 \cdot 1 \times 10^{6} \, \mathrm{N/m^2}.$$

Volume Stress

Another type of stress occurs when a body is acted upon by forces acting everywhere on the surface in such a way that (a) the force at any point is normal to the surface and (b) the magnitude of the force on any small surface area is proportional to the area. This is the case when a small solid body is immersed in a fluid. If the pressure at the location of the solid is P, the force on any area ΔS is $P\Delta S$ directed perpendicularly to the area. The force per unit area is then called *volume stress* (figure 14.4). It is

$$\Gamma_v = \frac{F}{A} \qquad \ldots \ (14.3)$$

which is same as the pressure.

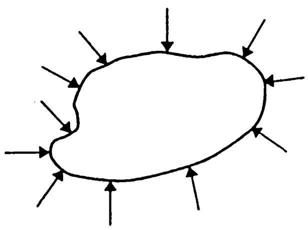

Volume stress

Figure 14.4

14.4 STRAIN

Associated with each type of stress defined above, there is a corresponding type of strain.

Longitudinal Strain

Consider a rod of length l being pulled by equal and opposite forces. The length of the rod increases from its natural value L to $L + \Delta L$. The fractional change $\Delta L/L$ is called the *longitudinal strain.*

$$\text{Longitudinal strain} = \Delta L/L . \qquad \ldots \ (14.4)$$

If the length increases from its natural length, the longitudinal strain is called *tensile strain.* If the length decreases from its natural length, the longitudinal strain is called *compressive strain.*

Shearing Strain

This type of strain is produced when a shearing stress is present over a section. Consider a body with square cross-section and suppose forces parallel to the surfaces are applied as shown in figure (14.5). Note that the resultant of the four forces shown is zero as well as the total torque of the four forces is zero.

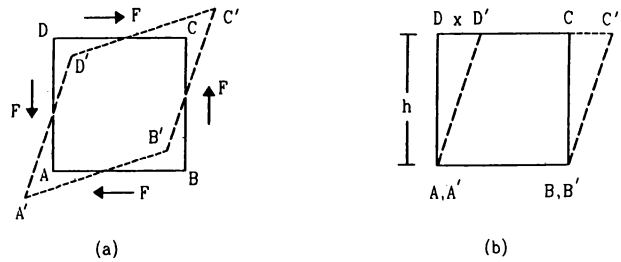


Figure 14.5

This ensures that the body remains in translational and rotational equilibrium after the deformation. Because of the tangential forces parallel to the faces, these faces are displaced. The shape of the cross-section changes from a square to a parallelogram. In figure (14.5a) the dotted area represents the deformed cross-section. To measure the deformation, we redraw the dotted area by rotating it a little so that one edge $A'B'$ coincides with its undeformed position AB. The drawing is presented in part (b) of figure (14.5).

We define the shearing strain as the displacement of a layer divided by its distance from the fixed layer. In the situation of figure (14.5),

$$\text{Shearing strain} = DD'/DA = x/h .$$

Shearing strain is also called *shear.*

Volume Strain

When a body is subjected to a volume stress, its volume changes. The *volume strain* is defined as the fractional change in volume. If V is the volume of unstressed body and $V + \Delta V$ is the volume when the volume stress exists, the volume strain is defined as

$$\text{Volume strain} = \Delta V/V.$$

14.5 HOOKE'S LAW AND THE MODULII OF ELASTICITY

If the deformation is small, the stress in a body is proportional to the corresponding strain.

This fact is known as Hooke's law. Thus, if a rod is stretched by equal and opposite forces F each, a tensile stress F/A is produced in the rod where A is the area of cross-section. The length of the rod increases from its natural value L to $L + \Delta L$. Tensile strain is $\Delta L/L$.

By Hooke's law, for small deformations,

$$\frac{\text{Tensile stress}}{\text{Tensile strain}} = Y \qquad \ldots \ (14.5)$$

is a constant for the given material. This ratio of tensile stress over tensile strain is called *Young's modulus* for the material. In the situation described above, the Young's modulus is

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L} \cdot \qquad \ldots (14.6)$$

If the rod is compressed, compressive stress and compressive strain appear. Their ratio Y is same as that for the tensile case.

Example 14.2

A load of 4·0 kg *is suspended from a ceiling through a steel wire of length* 20 m *and radius* 2·0 mm. *It is found that the length of the wire increases by* 0·031 mm *as equilibrium is achieved. Find Young's modulus of steel. Take* $g = 3{\cdot}1\ \pi\ \mathrm{m/s^2}$.

Solution : The longitudinal stress $= \dfrac{(4{\cdot}0\ \mathrm{kg})\,(3{\cdot}1\ \pi\ \mathrm{m/s^2})}{\pi\,(2{\cdot}0 \times 10^{-3}\ \mathrm{m})^2}$

$= 3{\cdot}1 \times 10^{6}\ \mathrm{N/m^2}$.

The longitudinal strain $= \dfrac{0{\cdot}31 \times 10^{-3}\ \mathrm{m}}{2{\cdot}0\ \mathrm{m}}$

$= 0{\cdot}0155 \times 10^{-3}$.

Thus, $Y = \dfrac{3{\cdot}1 \times 10^{6}\ \mathrm{N/m^2}}{0{\cdot}0155 \times 10^{-3}} = 2{\cdot}0 \times 10^{11}\ \mathrm{N/m^2}$.

The ratio of shearing stress over shearing strain is called the *Shear modulus, Modulus of rigidity* or *Torsional modulus*. In the situation of figure (14.5) the shear modulus is

$$\eta = \frac{F/A}{x/h} = \frac{Fh}{Ax} \cdot \qquad \ldots (14.7)$$

The ratio of volume stress over volume strain is called *Bulk modulus*. If P be the volume stress (same as pressure) and ΔV be the increase in volume, the Bulk modulus is defined as

$$B = -\frac{P}{\Delta V/V} \cdot \qquad \ldots (14.8)$$

The minus sign makes B positive as volume actually decreases on applying pressure. Quite often, the change in volume is measured corresponding to a change in pressure. The bulk modulus is then defined as

$$B = -\frac{\Delta P}{\Delta V/V} = -V\frac{dP}{dV} \cdot$$

Compressibility K is defined as the reciprocal of the bulk modulus.

$$K = \frac{1}{B} = -\frac{1}{V}\frac{dV}{dP} \cdot \qquad \ldots (14.9)$$

Yet another kind of modulus of elasticity is associated with the longitudinal stress and strain. When a rod or a wire is subjected to a tensile stress, its length increases in the direction of the tensile force. At the same time the length perpendicular to the tensile force decreases. For a cylindrical rod, the length increases and the diameter decreases when the rod is stretched (Figure 14.6).

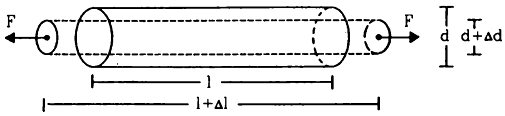


Figure 14.6

The fractional change in the transverse length is proportional to the fractional change in the longitudinal length. The constant of proportionality is called *Poisson's ratio.* Thus, Poisson's ratio is

$$\sigma = -\frac{\Delta d/d}{\Delta L/L} \cdot \qquad \ldots (14.10)$$

The minus sign ensures that σ is positive. Table (14.1) lists the elastic constants of some of the common materials. Table (14.2) lists compressibilities of some liquids.

Table 14.1 : *Elastic constants*

Material	Young's Modulus Y $10^{11}\ \mathrm{N/m^2}$	Shear Modulus η $10^{11}\ \mathrm{N/m^2}$	Bulk Modulus B $10^{11}\ \mathrm{N/m^2}$	Poisson's ratio σ
Aluminium	0·70	0·30	0·70	0·16
Brass	0·91	0·36	0·61	0·26
Copper	1·1	0·42	1·4	0·32
Iron	1·9	0·70	1·0	0·27
Steel	2·0	0·84	1·6	0·19
Tungsten	3·6	1·5	2·0	0·20

Table 14.2 : *Compressibilities of liquids*

Liquid	Compressibility K $10^{-11}\ \mathrm{m^2/N}$
Carbon disulphide	64
Ethyl alcohol	110
Glycerine	21
Mercury	3·7
Water	49

14.6 RELATION BETWEEN LONGITUDINAL STRESS AND STRAIN

For a small deformation, the longitudinal stress is proportional to the longitudinal strain. What happens if the deformation is not small ? The relation of stress and strain is much more complicated in such a case and the nature depends on the material under study. We describe here the behaviour for two representative materials, a metal wire and a rubber piece.

Metal Wire

Suppose a metal wire is stretched by equal forces at the ends so that its length increases from its natural value. Figure (14.7) shows qualitatively the relation between the stress and the strain as the deformation gradually increases.

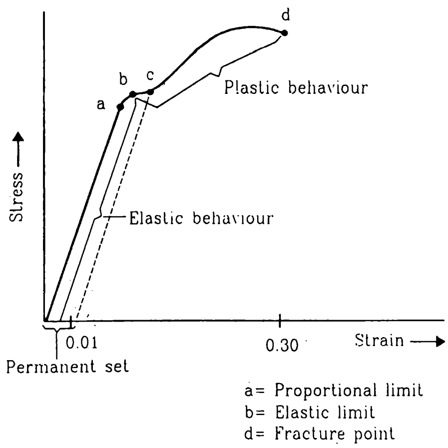


Figure 14.7

When the strain is small (say < 0·01), the stress is proportional to the strain. This is the region where Hooke's law is valid and where Young's modulus is defined. The point *a* on the curve represents the *proportional limit* up to which stress and strain are proportional.

If the strain is increased a little bit, the stress is not proportional to the strain. However, the wire still remains elastic. This means, if the stretching force is removed, the wire acquires its natural length. This behaviour is shown up to a point *b* on the curve known as the *elastic limit* or the *yield point.* If the wire is stretched beyond the elastic limit, the strain increases much more rapidly. If the stretching force is removed, the wire does not come back to its natural length. Some permanent increase in length takes place. In figure (14.7), we have shown this behaviour by the dashed line from *c*. The behaviour of the wire is now plastic. If the deformation is increased further, the wire breaks at a point *d* known as *fracture point.* The stress corresponding to this point is called *breaking stress.*

If large deformation takes place between the elastic limit and the fracture point, the material is called *ductile.* If it breaks soon after the elastic limit is crossed, it is called *brittle.*

Rubber

A distinctly different stress-strain relation exists for vulcanized rubber, the behaviour is qualitatively shown in figure (14.8). The material remains elastic even when it is stretched to over several times its original length. In the case shown in figure (14.8), the length is increased to 8 times its natural length, even then if the stretching forces are removed, it will come back to its original length.

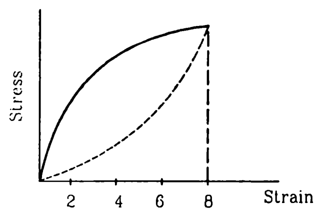


Figure 14.8

In this respect rubber is more elastic than a ductile metal like steel. However, the magnitude of stress for a given strain is much larger in steel than in rubber. This means large internal forces appear if the steel wire is deformed. In this sense, steel is more elastic than rubber. There are two important phenomena to note from figure (14.8). Firstly, in no part of this large deformation stress is proportional to strain. There is almost no region of proportionality. Secondly, when the deforming force is removed the original curve is not retraced although the sample finally acquires its natural length. The work done by the material in returning to its original shape is less than the work done by the deforming force when it was deformed. A particular amount of energy is, thus, absorbed by the material in the cycle which appears as heat. This phenomenon is called *elastic hysteresis.*

Elastic hysteresis has an important application in shock absorbers. If a padding of vulcanized rubber is given between a vibrating system and say a flat board, the rubber is compressed and released in every cycle of vibration. As energy is absorbed in the rubber in each cycle, only a part of the energy of vibrations is transmitted to the board.

14.7 ELASTIC POTENTIAL ENERGY OF A STRAINED BODY

When a body is in its natural shape, its potential energy corresponding to the molecular forces is minimum. We may take the potential energy in this state to be zero. When deformed, internal forces appear and work has to be done against these forces. Thus, the potential energy of the body is increased. This is called the *elastic potential energy.* We shall derive an expression for the increase in elastic potential energy when a wire is stretched from its natural length.

Suppose a wire having natural length L and cross-sectional area A is fixed at one end and is stretched

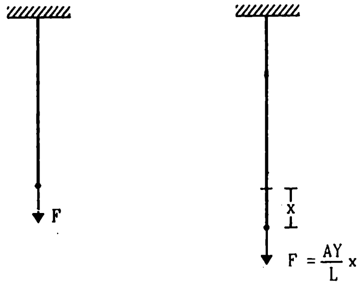


Figure 14.9

by an external force applied at the other end (figure 14.9). The force is so adjusted that the wire is only slowly stretched. This ensures that at any time during the extension the external force equals the tension in the wire. When the extension is x, the wire is under a longitudinal stress F/A, where F is the tension at this time. The strain is x/L.

If Young's modulus is Y,

$$\frac{F/A}{x/L} = Y$$

or, $$F = \frac{AY}{L} x. \quad \dots \text{(i)}$$

The work done by the external force in a further extension dx is

$$dW = F\,dx.$$

Using (i),

$$dW = \frac{AY}{L} x\,dx.$$

The total work by the external force in an extension 0 to l is

$$W = \int_0^l \frac{AY}{L} x\,dx$$

$$= \frac{AY}{2L} l^2.$$

This work is stored into the wire as its elastic potential energy.

Thus, the elastic potential energy of the stretched wire is,

$$U = \frac{AY}{2L} l^2. \quad \dots (14.11)$$

This may be written as

$$U = \frac{1}{2}\left(AY\frac{l}{L}\right) l$$

$$= \frac{1}{2} \text{(maximum stretching force) (extension)}.$$

Equation (14.11) may also be written as

$$U = \frac{1}{2}\left(Y\frac{l}{L}\right)\frac{l}{L}(AL)$$

or, *Potential energy* $= \frac{1}{2}$ × *stress × strain × volume.*

... (14.12)

Example 14.3

A steel wire of length 2·0 m is stretched through 2·0 mm. The cross-sectional area of the wire is 4·0 mm^2. Calculate the elastic potential energy stored in the wire in the stretched condition. Young's modulus of steel = 2·0 × 10^{11} N/m^2.

Solution : The strain in the wire $\frac{\Delta l}{l} = \frac{2\cdot0\text{ mm}}{2\cdot0\text{ m}} = 10^{-3}$.

The stress in the wire = Y × strain

$= 2\cdot0 \times 10^{11}\text{ N/m}^2 \times 10^{-3} = 2\cdot0 \times 10^{8}\text{ N/m}^2$.

The volume of the wire $= (4 \times 10^{-6}\text{ m}^2) \times (2\cdot0\text{ m})$

$= 8\cdot0 \times 10^{-6}\text{ m}^3$.

The elastic potential energy stored

$= \frac{1}{2} \times$ stress × strain × volume

$= \frac{1}{2} \times 2\cdot0 \times 10^{8}\text{ N/m}^2 \times 10^{-3} \times 8\cdot0 \times 10^{-6}\text{ m}^3$

= 0·8 J.

14.8 DETERMINATION OF YOUNG'S MODULUS IN LABORATORY

Figure (14.10) shows the experimental set up of a simple method to determine Young's modulus in a laboratory. A long wire A (say 2-3 m) is suspended from a fixed support. It carries a fixed graduated scale and below it a heavy fixed load. This load keeps the wire straight and free from kinks. The wire itself serves as a reference. The experimental wire B of almost equal length is also suspended from the same support close to the reference wire. A vernier scale is attached at the free end of the experimental wire. This vernier scale can slide against the main scale attached to the reference wire.

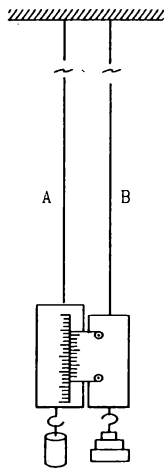


Figure 14.10

A hanger is attached at the lower end of the vernier scale. A number of slotted half kilogram or one kilogram weights may be slipped into the hanger.

First of all, the radius of the experimental wire is measured at several places with a screw gauge. From the average radius r, the breaking weight is determined using the standard value of the breaking stress for the material. Half of this breaking weight is the permissible weight.

Some initial load, say 1 kg or 2 kg, is kept on the hanger (this should be much smaller than the permissible weight). This keeps the experimental wire straight and kink-free. The reading of the main scale and vernier coincidence are noted. A known weight say 1/2 kg or 1 kg is slipped into the hanger. The set up is left for about a minute so that the elongation takes place fully. The readings on the scale are noted. The difference of the scale readings gives the extension due to the extra weight put. The weight is gradually increased upto the permissible weight and every time the extension is noted.

The experiment is repeated in reverse order decreasing the weight gradually in the same steps and everytime noting the extension.

From the data, extension versus load curve is plotted. This curve should be a straight line passing through the origin (figure 14.11). The slope of this line gives

$$\tan\theta = \frac{l}{Mg}.$$

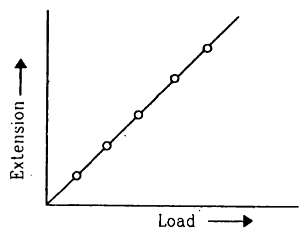


Figure 14.11

Now the stress due to the weight Mg at the end is

$$\text{stress} = \frac{Mg}{\pi r^2}$$

and $$\text{strain} = \frac{l}{L}.$$

Thus, $$Y = \frac{MgL}{\pi r^2 l} = \frac{L}{\pi r^2 \tan\theta}.$$

All the quantities on the right hand side are known and hence Young's modulus Y may be calculated.

14.9 SURFACE TENSION

The properties of a surface are quite often markedly different from the properties of the bulk material. A molecule well inside a body is surrounded by similar particles from all sides. But a molecule on the surface has particles of one type on one side and of a different type on the other side. Figure (14.12) shows an example. A molecule of water well inside the bulk experiences forces from water molecules from all sides but a molecule at the surface interacts with air molecules from above and water molecules from below. This asymmetric force distribution is responsible for surface tension.

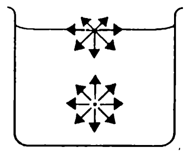


Figure 14.12

By a surface we shall mean a layer approximately 10-15 molecular diameters. The force between two molecules decreases as the separation between them increases. The force becomes negligible if the separation exceeds 10-15 molecular diameters. Thus, if we go 10-15 molecular diameters deep, a molecule finds equal forces from all directions.

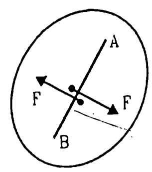


Figure 14.13

Imagine a line AB drawn on the surface of a liquid (figure 14.13). The line divides the surface in two parts, surface on one side and the surface on the other side of the line. Let us call them surface to the left of the line and surface to the right of the line. It is found that the two parts of the surface pull each other with a force proportional to the length of the line AB. These forces of pull are perpendicular to the line separating the two parts and are tangential to the surface. In this respect the surface of the liquid behaves like a stretched rubber sheet. The rubber sheet which is stretched from all sides is in the state of tension. Any part of the sheet pulls the adjacent part towards itself.

Let F be the common magnitude of the forces exerted on each other by the two parts of the surface across a line of length l. We define the surface tension S of the liquid as

$$S = F/l \qquad \text{... (14.13)}$$

The SI unit of surface tension is N/m.

Example 14.4

Water is kept in a beaker of radius 5·0 cm. *Consider a diameter of the beaker on the surface of the water. Find the force by which the surface on one side of the diameter pulls the surface on the other side. Surface tension of water* = 0·075 N/m.

Solution : The length of the diameter is

$$l = 2r = 10 \text{ cm}$$
$$= 0{\cdot}1 \text{ m}.$$

The surface tension is $S = F/l$. Thus,

$$F = Sl$$
$$= (0{\cdot}075 \text{ N/m}) \times (0{\cdot}1 \text{ m}) = 7{\cdot}5 \times 10^{-3} \text{ N}.$$

The fact that a liquid surface has the property of surface tension can be demonstrated by a number of simple experiments.

(a) Take a ring of wire and dip it in soap solution. When the ring is taken out, a soap film bounded by the ring is formed. Now take a loop of thread, wet it and place it gently on the soap film. The loop stays on the film in an irregular fashion as it is placed. Now prick a hole in the film inside the loop with a needle. The thread is radially pulled by the film surface outside and it takes a circular shape (figure 14.14).

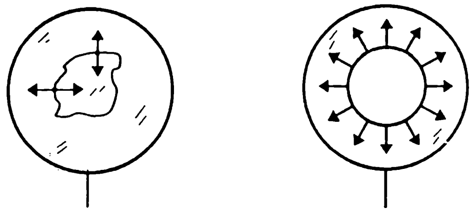


Figure 14.14

Before the pricking, there were surfaces both inside and outside the thread loop. Taking any small part of the thread, surfaces on both sides pulled it and the net force was zero. The thread could remain in any shape. Once the surface inside was punctured, the outside surface pulled the thread to take the circular shape.

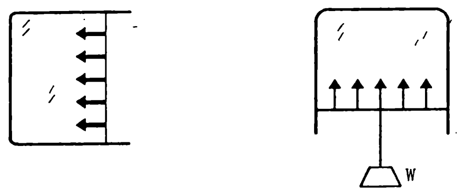


Figure 14.15

(b) Take a U-shaped frame of wire on which a light wire can slide (figure 14.15). Dip the frame in a soap solution and take it out. A soap film is formed between the frame and the sliding wire. If the frame is kept in a horizontal position and the friction is negligible, the sliding wire quickly slides towards the closing arm of the frame. This shows that the soap surface in contact with the wire pulls it parallel to the surface. If the frame is kept vertical with the sliding wire at the lower position, one can hang some weight from it to keep it in equilibrium. The force due to surface tension by the surface in contact with the sliding wire balances the weight.

Tendency to Decrease the Surface Area

The property of surface tension may also be described in terms of the tendency of a liquid to decrease its surface area. Because of the existence of forces across any line in the surface, the surface tends to shrink whenever it gets a chance to do so. The two demonstrations described above may help us in understanding the relation between the force of surface tension and the tendency to shrink the surface.

In the first example, the soap film is pricked in the middle. The remaining surface readjusts its shape so that a circular part bounded by the thread loop is excluded. The loop has a fixed length and the largest area that can be formed with a fixed periphery is a circle. This ensures that the surface of the soap solution takes the minimum possible area.

In the second example, the wire can slide on the frame. When kept in horizontal position, the wire slides to the closing arm of the U-shaped frame so that the surface shrinks.

There are numerous examples which illustrate that the surface of a liquid tries to make its area minimum. When a painting brush is inside a liquid, the bristles of the brush wave freely. When the brush is taken out of the liquid, surfaces are formed between the bristles. To minimise the area of these surfaces, they stick together.

A small drop of liquid takes a nearly spherical shape. This is because, for a given volume, the sphere assume the smallest surface area. Because of gravity there is some deviation from the spherical shape but for small drops this may be neglected.

Table(14.3)gives the values of surface tension of some liquids.

Table 14.3 : *Surface tension*

Liquid	Surface tension N/m	Liquid	Surface Tension N/m
Mercury	0·465	Glycerine	0·063
Water	0·075	Carbon tetra chloride	0·027
Soap solution	0·030	Ethyl alcohol	0·022

14.10 SURFACE ENERGY

We have seen that a molecule well within the volume of a liquid is surrounded by the similar liquid molecules from all sides and hence there is no resultant force on it (figure 14.12). On the other hand, a molecule in the surface is surrounded by similar liquid molecules only on one side of the surface while on the other side it may be surrounded by air molecules or the molecules of the vapour of the liquid etc. These vapours having much less density exert only a small force. Thus, there is a resultant inward force on a molecule in the surface. This force tries to pull the molecule into the liquid. Thus, the surface layer remains in microscopic turbulence. Molecules are pulled back from the surface layer to the bulk and new molecules from the bulk go to the surface to fill the empty space.

When a molecule is taken from the inside to the surface layer, work is done against the inward resultant force while moving up in the layer. The potential energy is increased due to this work. A molecule in the surface has greater potential energy than a molecule well inside the liquid. The extra energy that a surface layer has is called the *surface energy*. The surface energy is related to the surface tension as discussed below.

Consider a U-shaped frame with a sliding wire on its arm. Suppose it is dipped in a soap solution, taken out and placed in a horizontal position (figure 14.16).

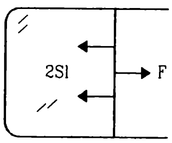


Figure 14.16

The soap film that is formed may look quite thin, but on the molecular scale its thickness is not small. It may have several hundred thousands molecular layers. So it has two surfaces enclosing a bulk of soap solution. Both the surfaces are in contact with the sliding wire and hence exert forces of surface tension on it. If S be the surface tension of the solution and l be the length of the sliding wire, each surface will pull the wire parallel to itself with a force Sl. The net force of pull F on the wire due to both the surfaces is

$$F = 2Sl.$$

One has to apply an external force equal and opposite to F so as to keep the wire in equilibrium.

Now suppose the wire is slowly pulled out by the external force through a distance x so that the area of the frame is increased by lx. As there are two surfaces of the solution, a new surface area $2lx$ is created. The liquid from the inside is brought to create the new surface.

The work done by the external force in the displacement is

$$W = Fx = 2Sl\,x = S(2lx).$$

As there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the new surface.

The increase in surface energy is

$$U = W = S(2lx).$$

Thus, $$\frac{U}{(2lx)} = S$$

or, $$\frac{U}{A} = S. \quad \ldots (14.14)$$

We see that the surface tension of a liquid is equal to the surface energy per unit surface area.

In this interpretation, the SI unit of surface tension may be written as J/m^2. It may be verified that N/m is equivalent to J/m^2.

Example 14.5

A water drop of radius 10^{-2} m is broken into 1000 equal droplets. Calculate the gain in surface energy. Surface tension of water is 0·075 N/m.

Solution : The volume of the original drop is

$$V = \frac{4}{3}\pi R^3 \text{ where } R = 10^{-2}\text{ m}.$$

If r is the radius of each broken droplet, the volume is also

$$V = 1000 \times \frac{4}{3}\pi r^3.$$

Thus, $$1000\, r^3 = R^3$$

or, $$r = R/10.$$

The surface area of the original drop is $A_1 = 4\pi R^2$ and the surface area of the 1000 droplets is

$$A_2 = 1000 \times 4\pi r^2 = 40\pi R^2.$$

The increase in area is

$$\Delta A = A_2 - A_1 = 40\pi R^2 - 4\pi R^2 = 36\pi R^2.$$

The gain in surface energy is

$$\Delta U = (\Delta A)S = 36\pi R^2 S$$
$$= 36 \times 3{\cdot}14 \times (10^{-4}\text{ m}^2) \times (0{\cdot}075\text{ N/m})$$
$$= 8{\cdot}5 \times 10^{-4}\text{ J}.$$

14.11 EXCESS PRESSURE INSIDE A DROP

Let us consider a spherical drop of liquid of radius R (figure 14.17). If the drop is small, the effect of gravity may be neglected and the shape may be assumed to be spherical.

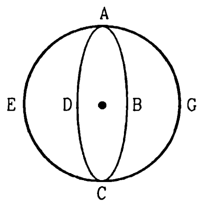


Figure 14.17

Imagine a diametric cross-section $ABCD$ of the drop which divides the drop in two hemispheres. The surfaces of the two hemispheres touch each other along the periphery $ABCDA$. Each hemispherical surface pulls the other hemispherical surface due to the surface tension.

Consider the equilibrium of the hemispherical surface $ABCDE$. Forces acting on this surface are

(i) F_1, due to the surface tension of the surface $ABCDG$ in contact,

(ii) F_2, due to the air outside the surface $ABCDE$ and

(iii) F_3, due to the liquid inside the surface $ABCDE$.

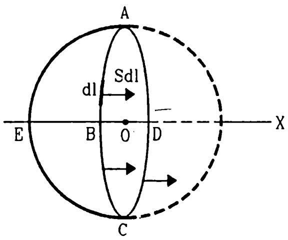


Figure 14.18

The force due to surface tension acts on the points of the periphery $ABCDA$. The force on any small part dl of this periphery is $S\,dl$ (figure 14.18) and acts parallel to the symmetry axis OX. The resultant of all these forces due to surface tension is

$$F_1 = 2\pi RS$$

along OX.

Now consider the forces due to the air outside the surface $ABCDE$. Consider a small part ΔS of the surface as shown in figure (14.19).

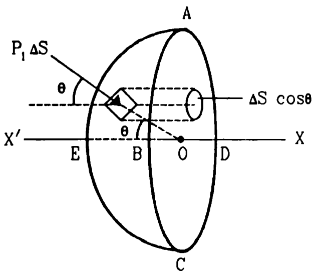


Figure 14.19

If the pressure just outside the surface is P_1, the force on this surface ΔS is $P_1 \Delta S$ along the radial direction. By symmetry, the resultant of all such forces acting on different parts of the hemispherical surface must be along OX.

If the radius through ΔS makes an angle θ with OX, the component of $P_1 \Delta S$ along OX will be $P_1 \Delta S \cos\theta$. If we project the area ΔS on the diametric plane $ABCD$, the area of projection will be $\Delta S \cos\theta$. Thus, we can write,

component of $P_1 \Delta S$ along OX

$$= P_1 \text{ (projection of } \Delta S \text{ on the plane } ABCD).$$

When components of all the forces $P_1 \Delta S$ on different ΔS are added, we get the resultant force due to the air outside the hemispherical surface. This resultant is then

$$F_2 = P_1 \text{ (Projection of the hemispherical surface } ABCDE \text{ on the plane } ABCD).$$

The projection of the hemispherical surface on the plane $ABCD$ is the circular disc $ABCD$ itself, having an area πR^2. Thus,

$$F_2 = P_1 \pi R^2.$$

Similarly, the resultant force on this surface due to the liquid inside is $F_3 = P_2 \cdot \pi R^2$, where P_2 is the pressure just inside the surface. This force will be in the direction OX'.

For equilibrium of the hemispherical surface $ABCDE$ we should have,

$$F_1 + F_2 = F_3$$

or, $$2\pi RS + P_1 \pi R^2 = P_2 \pi R^2$$

or, $$P_2 - P_1 = 2S/R. \quad \text{... (14.15)}$$

The pressure inside the surface is greater than the pressure outside the surface by an amount 2S/R.

In the case of a drop, there is liquid on the concave side of the surface and air on the convex side. The pressure on the concave side is greater than the pressure on the convex side. This result is true in all cases. If we have an air bubble inside a liquid (figure 14.20), a single surface is formed. There is air on the concave side and liquid on the convex side. The pressure in the concave side (that is in the air) is greater than the pressure in the convex side (that is in the liquid) by an amount $2S/R$.

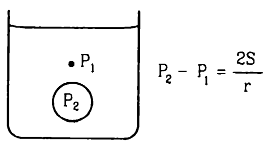


Figure 14.20

Example 14.6

Find the excess pressure inside a mercury drop of radius 2·0 mm. *The surface tension of mercury* = 0·464 N/m.

Solution : The excess pressure inside the drop is

$$P_2 - P_1 = 2S/R$$

$$= \frac{2 \times 0·464 \text{ N/m}}{2·0 \times 10^{-3} \text{ m}} = 464 \text{ N/m}^2.$$

14.12 EXCESS PRESSURE IN A SOAP BUBBLE

Soap bubbles can be blown by dipping one end of a glass tube in a soap solution for a short time and then blowing air in it from the other end. Such a bubble has a small thickness and there is air both inside the bubble and outside the bubble. The thickness of the bubble may look small to eye but it still has hundreds of thousands of molecular layers. So it has two surface layers, one towards the outside air and the other towards the enclosed air. Between these two surface layers there is bulk soap solution.

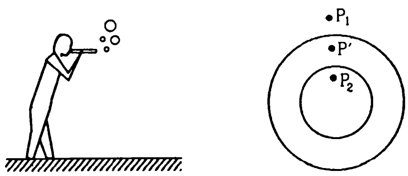


Figure 14.21

Let the pressure of the air outside the bubble be P_1, that within the soap solution be P' and that in the air inside the bubble be P_2. Looking at the outer surface, the solution is on the concave side of the surface, hence

$$P' - P_1 = 2S/R$$

where R is the radius of the bubble. As the thickness of the bubble is small on a macroscopic scale, the difference in the radii of the two surfaces will be negligible.

Similarly, looking at the inner surface, the air is on the concave side of the surface, hence

$$P_2 - P' = 2S/R.$$

Adding the two equations,

$$P_2 - P_1 = 4S/R. \quad \dots (14.16)$$

The pressure inside a bubble is greater than the pressure outside by an amount $4S/R$.

Example 14.7

A 0·02 cm *liquid column balances the excess pressure inside a soap bubble of radius* 7·5 mm. *Determine the density of the liquid. Surface tension of soap solution* = 0·03 N/m.

Solution : The excess pressure inside a soap bubble is

$$\Delta P = 4S/R = \frac{4 \times 0·03 \text{ N/m}}{7·5 \times 10^{-3} \text{ m}} = 16 \text{ N/m}^2.$$

The pressure due to 0·02 cm of the liquid column is

$$\Delta P = h\rho g$$

$$= (0·02 \times 10^{-2} \text{ m}) \, \rho \, (9·8 \text{ m/s}^2).$$

Thus, $16 \text{ N/m}^2 = (0·02 \times 10^{-2} \text{ m}) \, \rho \, (9·8 \text{ m/s}^2)$

or, $\rho = 8·2 \times 10^3 \text{ kg/m}^3.$

14.13 CONTACT ANGLE

When a liquid surface touches a solid surface, the shape of the liquid surface near the contact is generally curved. When a glass plate is immersed in water, the surface near the plate becomes concave as if the water is pulled up by the plate (figure 14.22). On the other hand, if a glass plate is immersed in mercury, the surface is depressed near the plate.

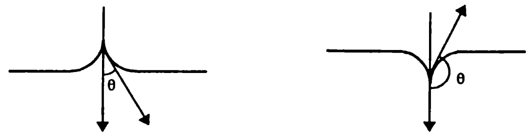


Figure 14.22

The angle between the tangent planes at the solid surface and the liquid surface at the contact is called the *contact angle*. In this the tangent plane to the solid surface is to be drawn towards the liquid and the tangent plane to the liquid is to be drawn away from the solid. Figure (14.22) shows the construction of contact angle. For the liquid that rises along the solid surface, the contact angle is smaller than 90°. For the liquid that is depressed along the solid surface, the contact angle is greater than 90°. Table (14.4) gives the contact angles for some of the pairs of solids and liquids.

Table 14.4 : ***Contact angles***

Substance	Contact angle	Substance	Contact angle
Water with glass	0	Water with paraffin	107°
Murcury with glass	140°	Methylene iodide with glass	29°

Let us now see why the liquid surface bends near the contact with a solid. A liquid in equilibrium cannot sustain tangential stress. The resultant force on any small part of the surface layer must be perpendicular to the surface there. Consider a small part of the liquid surface near its contact with the solid (figure 14.23).

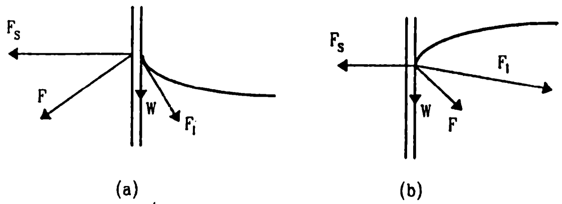


Figure 14.23

The forces acting on this part are

(a) F_s, attraction due to the molecules of the solid surface near it,

(b) F_l, the force due to the liquid molecules near this part, and

(c) W, the weight of the part considered.

The force between the molecules of the same material is known as *cohesive force* and the force between the molecules of different kinds of material is called *adhesive force*. Here F_s is adhesive force and F_l is cohesive force.

As is clear from the figure, the adhesive force F_s is perpendicular to the solid surface and is into the solid. The cohesive force F_l is in the liquid, its direction and magnitude depends on the shape of the liquid surface as this determines the distribution of the molecules attracting the part considered. Of course, F_s and F_l depend on the nature of the substances especially on their densities.

The direction of the resultant of F_s, F_l and W decides the shape of the surface near the contact. The liquid rests in such a way that the surface is perpendicular to this resultant. If the resultant passes through the solid (figure 14.23a), the surface is concave upward and the liquid rises along the solid. If the resultant passes through the liquid (figure 14.23b), the surface is convex upward and the liquid is depressed near the solid.

If a solid surface is just dipped in liquid (figure 14.24) so that it is not projected out, the force F_s will not be perpendicular to the solid. The actual angle between the solid surface and the liquid surface may be different from the standard contact angle for the pair.

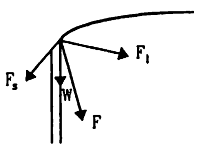


Figure 14.24

14.14 RISE OF LIQUID IN A CAPILLARY TUBE

When one end of a tube of small radius (known as a capillary tube) is dipped into a liquid, the liquid rises or is depressed in the tube. If the contact angle is less than 90°, the liquid rises. If it is greater than 90°, it is depressed.

Suppose a tube of radius r is dipped into a liquid of surface tension S and density ρ. Let the angle of contact between the solid and the liquid be θ. If the radius of the tube is small, the surface in the tube is nearly spherical. Figure (14.25) shows the situation.

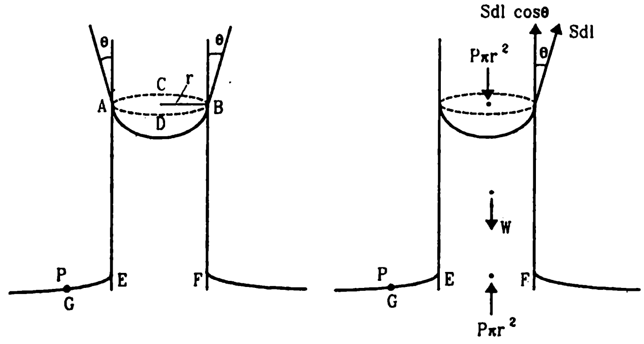


Figure 14.25

Consider the equilibrium of the part of liquid raised in the tube. In figure (14.25) this liquid is contained in the volume $ABEF$. Forces on this part of the liquid are

(a) F_1, by the surface of the tube on the surface $ABCD$ of the liquid,

(b) F_2, due to the pressure of the air above the surface $ABCD$,

(c) F_3, due to the pressure of the liquid below EF and

(d) the weight W of the liquid $ABEF$.

$ABCD$ is the surface of the liquid inside the capillary tube. It meets the wall of the tube along a circle of radius r. The angle made by the liquid surface with the surface of the tube is equal to the contact angle θ.

Consider a small part dl of the periphery $2\pi r$ along which the surface of the liquid and the tube meet. The liquid surface across this pulls the tube surface by a force $S\,dl$ tangentially along the liquid surface. From Newton's third law, the tube surface across this small part pulls the liquid surface by an equal force $S\,dl$ in opposite direction. The vertical component of this force is $S\,dl\cos\theta$. The total force exerted on the liquid surface by the tube surface across the contact circle is

$$
\begin{aligned}
F_1 &= \int S\,dl\cos\theta \\
&= S\cos\theta \int dl \\
&= 2\pi r S\cos\theta \qquad \dots \text{(i)}
\end{aligned}
$$

The horizontal component $Sdl \sin\theta$ adds to zero when summed over the entire periphery.

The force F_2 due to the pressure of the air outside the surface $ABCD$ is $P.\pi r^2$ where P is the atmospheric pressure. (This result was derived for hemispherical surface while deducing the excess pressure inside a drop. Same derivation works here.)

This force acts vertically downward. The presure at EF is equal to the atmospheric pressure P. This is because EF is in the same horizontal plane as the free surface outside the tube and the pressure there is P. The force due to the liquid below EF is, therefore, $P\pi r^2$ in vertically upward direction.

Thus, F_2 and F_3 cancel each other and the force $F_1 = 2\pi r S \cos\theta$ balances the weight W in equilibrium. If the height raised in the tube is h and if we neglect the weight of the liquid contained in the meniscus, the volume of the liquid raised is $\pi r^2 h$. The weight of this part is then

$$W = \pi r^2 h \rho g. \quad \text{... (ii)}$$

Thus, $\pi r^2 h \rho g = 2\pi r S \cos\theta$

so that
$$h = \frac{2S \cos\theta}{r \rho g}. \quad \text{... (14.17)}$$

We see that the height raised is inversely proportional to the radius of the capillary. If the contact angle θ is greater than 90°, the term $\cos\theta$ is negative and hence h is negative. The expression then gives the depression of the liquid in the tube.

The correction due to the weight of the liquid contained in the meniscus can be easily made if the contact angle is zero. This is the case with water rising in a glass capillary. The meniscus is then hemispherical (Figure 14.26).

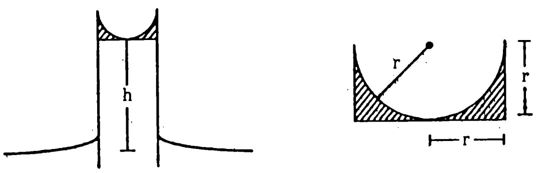


Figure 14.26

The volume of the shaded part is

$$(\pi r^2)r - \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{1}{3}\pi r^3.$$

The weight of the liquid contained in the meniscus is $\frac{1}{3}\pi r^3 \rho g$. Equation (ii) is then replaced by

$$\pi r^2 h \rho g + \frac{1}{3}\pi r^3 \rho g = 2\pi r S$$

or,
$$h = \frac{2S}{r \rho g} - \frac{r}{3}. \quad \text{... (14.18)}$$

Example 14.8

A capillary tube of radius 0·20 mm *is dipped vertically in water. Find the height of the water column raised in the tube. Surface tension of water* = 0·075 N/m *and density of water* = 1000 kg/m³. *Take* g = 10 m/s².

Solution : We have,

$$h = \frac{2S \cos\theta}{r \rho g}$$

$$= \frac{2 \times 0{\cdot}075 \text{ N/m} \times 1}{(0{\cdot}20 \times 10^{-3} \text{ m}) \times (1000 \text{ kg/m}^3)(10 \text{ m/s}^2)}$$

$$= 0{\cdot}075 \text{ m} = 7{\cdot}5 \text{ cm}.$$

Tube of Insufficient Length

Equation (14.17) or (14.18) gives the height raised in a capillary tube. If the tube is of a length less than h, the liquid does not overflow. The angle made by the liquid surface with the tube changes in such a way that the force $2\pi r S \cos\theta$ equals the weight of the liquid raised.

14.15 VISCOSITY

When a layer of a fluid slips or tends to slip on another layer in contact, the two layers exert tangential forces on each other. The directions are such that the relative motion between the layers is opposed. This property of a fluid to oppose relative motion between its layers is called *viscosity*. The forces between the layers opposing relative motion between them are known as the *forces of viscosity*. Thus, viscosity may be thought of as the internal friction of a fluid in motion.

If a solid surface is kept in contact with a fluid and is moved, forces of viscosity appear between the solid surface and the fluid layer in contact. The fluid in contact is dragged with the solid. If the viscosity is sufficient, the layer moves with the solid and there is no relative slipping. When a boat moves slowly on the water of a calm river, the water in contact with the boat is dragged with it, whereas the water in contact with the bed of the river remains at rest. Velocities of different layers are different. Let v be the velocity of the layer at a distance z from the bed and $v + dv$ be the velocity at a distance $z + dz$. (figure 14.27).

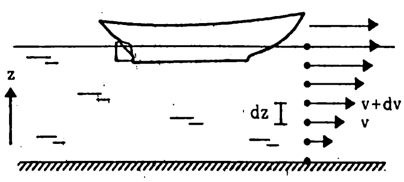


Figure 14.27

Thus, the velocity differs by dv in going through a distance dz perpendicular to it. The quantity dv/dz is called the *velocity gradient.*

The force of viscosity between two layers of a fluid is proportional to the velocity gradient in the direction perpendicular to the layers. Also the force is proportional to the area of the layer.

Thus, if F is the force exerted by a layer of area A on a layer in contact,

$$F \propto A \text{ and } F \propto dv/dz$$

or, $$F = -\eta A\, dv/dz. \qquad \text{... (14.19)}$$

The negative sign is included as the force is frictional in nature and opposes relative motion. The constant of proportionality η is called the *coefficient of viscosity.*

The SI unit of viscosity can be easily worked out from equation (14.19). It is N-s/m^2. However, the corresponding CGS unit dyne-s/cm^2 is in common use and is called a *poise* in honour of the French scientist Poiseuille. We have

$$1 \text{ poise} = 0{\cdot}1 \text{ N-s/m}^2.$$

Dimensions of the Coefficient of Viscosity

Writing dimensions of different variables in equation (14.19),

$$MLT^{-2} = [\eta]\, L^2 \cdot \frac{L/T}{L}$$

or, $$[\eta] = \frac{MLT^{-2}}{L^2T^{-1}}$$

or, $$[\eta] = ML^{-1}T^{-1}. \qquad \text{... (14.20)}$$

The coefficient of viscosity strongly depends on temperature. Table(14.5)gives the values for some of the commom fluids.

Table 14.5 : *Coefficient of viscosity*

Temperature °C	Viscosity of castor oil, poise	Viscosity of water, centi-poise	Viscosity of air, micro-poise
0	53	1·792	171
20	9·86	1·005	181
40	2·31	0·656	190
60	0·80	0·469	200
80	0·30	0·357	209
100	0·17	0·284	218

14.16 FLOW THROUGH A NARROW TUBE : POISEUILLE'S EQUATION

Suppose a fluid flows through a narrow tube in steady flow. Because of viscosity, the layer in contact with the wall of the tube remains at rest and the layers away from the wall move fast. Poiseuille derived a formula for the rate of flow of viscous fluid through a cylindrical tube. We shall try to obtain the formula using dimensional analysis.

Suppose a fluid having coefficient of viscosity η and density ρ is flowing through a cylindrical tube of radius r and length l. Let P be the pressure difference in the liquid at the two ends. It is found that the volume of the liquid flowing per unit time through the tube depends on the pressure gradient P/l, the coefficient of viscosity η and the radius r. If V be the volume flowing in time t, we guess that

$$\frac{V}{t} = k\left(\frac{P}{l}\right)^a \eta^b r^c \qquad \text{... (i)}$$

where k is a dimensionless constant.

Taking dimensions,

$$L^3T^{-1} = \left(\frac{ML^{-1}T^{-2}}{L}\right)^a (ML^{-1}T^{-1})^b L^c$$

or, $$L^3T^{-1} = M^{a+b}L^{-2a-b+c}T^{-2a-b}.$$

Equating the exponents of M, L and T we get,

$$0 = a + b$$
$$3 = -2a - b + c$$
$$-1 = -2a - b.$$

Solving these equations,

$$a = 1,\ b = -1 \text{ and } c = 4.$$

Thus,

$$\frac{V}{t} = k\frac{Pr^4}{\eta l}.$$

The dimensionless constant k is equal to $\pi/8$ and hence the rate of flow is

$$\frac{V}{t} = \frac{\pi P r^4}{8\eta l}. \qquad \text{... (14.21)}$$

This is *Poiseuille's formula.*

14.17 STOKES' LAW

When a solid body moves through a fluid, the fluid in contact with the solid is dragged with it. Relative velocities are established between the layers of the fluid near the solid so that the viscous forces start operating. The fluid exerts viscous force on the solid to oppose the motion of the solid. The magnitude of the viscous force depends on the shape and size of the solid body, its speed and the coefficient of viscosity of the fluid.

Suppose a spherical body of radius r moves at a speed v through a fluid of viscosity η. The viscous force F acting on the body depends on r, v and η. Assuming that the force is proportional to various powers of these quantities, we can obtain the dependence through dimensional analysis.

Let $F = k r^a v^b \eta^c$ ... (i)

where k is a dimensionless constant. Taking dimensions on both sides,

$$MLT^{-2} = k L^a (LT^{-1})^b (ML^{-1}T^{-1})^c.$$

Comparing the exponents of M, L and T.

$$1 = c$$
$$1 = a + b - c$$
$$-2 = -b - c.$$

Solving these equations, $a = 1$, $b = 1$ and $c = 1$.

Thus, by (i), $F = k r v \eta$.

The dimensionless constant k equals 6π, so that the equation becomes

$$F = 6\pi \eta r v. \quad \dots (14.22)$$

This equation is known as *Stokes' law*.

Example 14.9

An air bubble of diameter 2 mm *rises steadily through a solution of density* 1750 kg/m^3 *at the rate of* 0·35 cm/s. *Calculate the coefficient of viscosity of the solution. The density of air is negligible.*

Solution : The force of buoyancy B is equal to the weight of the displaced liquid. Thus,

$$B = \frac{4}{3}\pi r^3 \sigma g.$$

This force is upward. The viscous force acting downward is

$$F = 6\pi \eta r v.$$

The weight of the air bubble may be neglected as the density of air is small. For uniform velocity,

$$F = B$$

or, $6\pi \eta r v = \frac{4}{3}\pi r^3 \sigma g$

or, $\eta = \dfrac{2r^2 \sigma g}{9v}$

or, $= \dfrac{2 \times (1 \times 10^{-3}\,\mathrm{m})^2 \times (1750\,\mathrm{kg/m^3})(9{\cdot}8\,\mathrm{m/s^2})}{9 \times (0{\cdot}35 \times 10^{-2}\,\mathrm{m/s})}$

≈ 11 poise.

This appears to be a highly viscous liquid.

14.18 TERMINAL VELOCITY

The viscous force on a solid moving through a fluid is proportional to its velocity. When a solid is dropped in a fluid, the forces acting on it are

(a) weight W acting vertically downward,

(b) the viscous force F acting vertically upward and

(c) the buoyancy force B acting vertically upward.

The weight W and the buoyancy B are constant but the force F is proportional to the velocity v. Initially, the velocity and hence the viscous force F is zero and the solid is accelerated due to the force $W - B$. Because of the acceleration, the velocity increases. Accordingly, the viscous force also increases. At a certain instant the viscous force becomes equal to $W - B$. The net force then becomes zero and the solid falls with constant velocity. This constant velocity is known as the *terminal velocity.*

Consider a spherical body falling through a liquid. Suppose the density of the body = ρ, density of the liquid = σ, radius of the sphere = r and the terminal velocity = v_0. The viscous force is

$$F = 6\pi \eta r v_0.$$

The weight $W = \frac{4}{3}\pi r^3 \rho g$

and the buoyancy force $B = \frac{4}{3}\pi r^3 \sigma g.$

We have

$$6\pi \eta r v_0 = W - B = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

or, $$v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}. \quad \dots (14.23)$$

14.19 MEASURING COEFFICIENT OF VISCOSITY BY STOKES' METHOD

Viscosity of a liquid may be determined by measuring the terminal velocity of a solid sphere in it. Figure (14.28) shows the apparatus. A test tube A contains the experimental liquid and is fitted into a water bath B. A thermometer T measures the temperature of the bath. A tube C is fitted in the cork of the test tube A. There are three equidistant marks P, Q and R on the test tube well below the tube C.

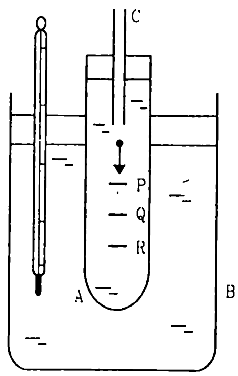


Figure 14.28

A spherical metal ball is dropped in the tube C. The time interval taken by the ball to pass through the length PQ and through the length QR are noted with the help of a stop watch. If these two are not equal, a smaller metal ball is tried. The process is repeated till the two time intervals are the same. In this case the ball has achieved its terminal velocity before passing through the mark P. The radius of the ball is determined by a screw guage. Its mass m is determined by weighing it. The length $PQ = QR$ is measured with a scale.

Let r = radius of the spherical ball
m = mass of the ball
t = time interval in passing through the length PQ or QR
d = length $PQ = QR$
η = coefficient of viscosity of the liquid
and σ = density of the liquid.

The density of the solid is $\rho = \frac{m}{\frac{4}{3}\pi r^3}$ and the terminal velocity is $v_0 = d/t$. Using equation (14.23),

$$\eta = \frac{2}{9}\frac{(\rho - \sigma)gr^2}{d/t}.$$

This method is useful for a highly viscous liquid such as Castor oil.

14.20 CRITICAL VELOCITY AND REYNOLDS NUMBER

When a fluid flows in a tube with a small velocity, the flow is steady. As the velocity is gradually increased, at one stage the flow becomes turbulent. The largest velocity which allows a steady flow is called the *critical velocity*.

Whether the flow will be steady or turbulent mainly depends on the density, velocity and the coefficient of viscosity of the fluid as well as the diameter of the tube through which the fluid is flowing. The quantity

$$N = \frac{\rho v D}{\eta} \quad \ldots (14.24)$$

is called the *Reynolds number* and plays a key role in determining the nature of flow. It is found that if the Reynolds number is less than 2000, the flow is steady. If it is greater than 3000, the flow is turbulent. If it is between 2000 and 3000, the flow is unstable. In this case it may be steady and may suddenly change to turbulent or it may be turbulent and may suddenly change to steady.

Worked Out Examples

1 *One end of a wire* 2 m *long and* 0·2 cm^2 *in cross-section is fixed in a ceiling and a load of* 4·8 kg *is attached to the free end. Find the extension of the wire. Young's modulus of steel* = $2{\cdot}0 \times 10^{11}$ N/m^2. *Take* $g = 10$ m/s^2.

Solution : We have

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L}$$

with symbols having their usual meanings. The extension is

$$l = \frac{TL}{AY}.$$

As the load is in equilibrium after the extension, the tension in the wire is equal to the weight of the load

$$= 4{\cdot}8\ \text{kg} \times 10\ \text{m/s}^2 = 48\ \text{N}.$$

Thus, $$l = \frac{(48\ \text{N})(2\ \text{m})}{(0{\cdot}2 \times 10^{-4}\ \text{m}^2) \times (2{\cdot}0 \times 10^{11}\ \text{N/m}^2)}$$
$$= 2{\cdot}4 \times 10^{-5}\ \text{m}.$$

2. *One end of a nylon rope of length* 4·5 m *and diameter* 6 mm *is fixed to a tree-limb. A monkey weighing* 100 N *jumps to catch the free end and stays there. Find the elongation of the rope and the corresponding change in the diameter. Young's modulus of nylon* = $4{\cdot}8 \times 10^{11}$ N/m^2 *and Poisson's ratio of nylon* = 0·2.

Solution : As the monkey stays in equilibrium, the tension in the rope equals the weight of the monkey. Hence,

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{T/A}{l/L}$$

or, $$l = \frac{TL}{AY}$$

or, elongation = $$l = \frac{(100\ \text{N}) \times (4{\cdot}5\ \text{m})}{(\pi \times 9 \times 10^{-6}\ \text{m}^2) \times (4{\cdot}8 \times 10^{11}\ \text{N/m}^2)}$$
$$= 3{\cdot}32 \times 10^{-5}\ \text{m}.$$

Again, Poisson's ratio = $\frac{\Delta d/d}{l/L} = \frac{(\Delta d)L}{ld}$

or, $$0{\cdot}2 = \frac{\Delta d \times 4{\cdot}5\ \text{m}}{(3{\cdot}32 \times 10^{-5}\ \text{m}) \times (6 \times 10^{-3}\ \text{m})}$$

or, $$\Delta d = \frac{0{\cdot}2 \times 6 \times 3{\cdot}32 \times 10^{-8}\ \text{m}}{4{\cdot}5}$$
$$= 8{\cdot}8 \times 10^{-9}\ \text{m}.$$

3. *Two blocks of masses* 1 kg *and* 2 kg *are connected by a metal wire going over a smooth pulley as shown in figure (14-W1). The breaking stress of the metal is* 2×10^9 N/m^2. *What should be the minimum radius of the wire used if it is not to break? Take* $g = 10$ m/s^2

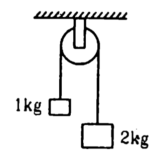


Figure 14-W1

Solution : The stress in the wire $= \dfrac{\text{Tension}}{\text{Area of cross-section}}$.

To avoid breaking, this stress should not exceed the breaking stress.

Let the tension in the wire be T. The equations of motion of the two blocks are,

$$T - 10\text{ N} = (1\text{ kg})\,a$$

and $$20\text{ N} - T = (2\text{ kg})\,a.$$

Eliminating a from these equations,

$$T = (40/3)\text{ N}.$$

$$\text{The stress} = \frac{(40/3)\text{ N}}{\pi r^2}.$$

If the minimum radius needed to avoid breaking is r,

$$2 \times 10^9 \frac{\text{N}}{\text{m}^2} = \frac{(40/3)\text{ N}}{\pi r^2}.$$

Solving this,

$$r = 4{\cdot}6 \times 10^{-5}\text{ m}.$$

4. *Two wires of equal cross-section but one made of steel and the other of copper, are joined end to end. When the combination is kept under tension, the elongations in the two wires are found to be equal. Find the ratio of the lengths of the two wires. Young's modulus of steel* $= 2{\cdot}0 \times 10^{11}\text{ N/m}^2$ *and that of copper* $= 1{\cdot}1 \times 10^{11}\text{ N/m}^2$.

Solution : As the cross-sections of the wires are equal and same tension exists in both, the stresses developed are equal. Let the original lengths of the steel wire and the copper wire be L_s and L_c respectively and the elongation in each wire be l.

$$\frac{l}{L_s} = \frac{\text{stress}}{2{\cdot}0 \times 10^{11}\text{ N/m}^2} \quad \text{... (i)}$$

and $$\frac{l}{L_c} = \frac{\text{stress}}{1{\cdot}1 \times 10^{11}\text{ N/m}^2}. \quad \text{... (ii)}$$

Dividing (ii) by (i),

$$L_s / L_c = 2{\cdot}0 / 1{\cdot}1 = 20 : 11.$$

5. *Find the decrease in the volume of a sample of water from the following data. Initial volume* $= 1000\text{ cm}^3$, *initial pressure* $= 10^5\text{ N/m}^2$, *final pressure* $= 10^6\text{ N/m}^2$, *compressibility of water* $= 50 \times 10^{-11}\text{ N}^{-1}\text{m}^2$.

Solution : The change in pressure

$$= \Delta P = 10^6\text{ N/m}^2 - 10^5\text{ N/m}^2$$
$$= 9 \times 10^5\text{ N/m}^2.$$

$$\text{Compressibility} = \frac{1}{\text{Bulk modulus}} = -\frac{\Delta V/V}{\Delta P}$$

or, $$50 \times 10^{-11}\text{ N}^{-1}\text{m}^2 = -\frac{\Delta V}{(10^{-3}\text{ m}^3) \times (9 \times 10^5\text{ N/m}^2)}$$

or, $$\Delta V = -50 \times 10^{-11} \times 10^{-3} \times 9 \times 10^5\text{ m}^3$$
$$= -4{\cdot}5 \times 10^{-7}\text{ m}^3 = -0{\cdot}45\text{ cm}^3.$$

Thus the decrease in volume is $0{\cdot}45\text{ cm}^3$.

6. *One end of a metal wire is fixed to a ceiling and a load of* 2 kg *hangs from the other end. A similar wire is attached to the bottom of the load and another load of* 1 kg *hangs from this lower wire. Find the longitudinal strain in both the wires. Area of cross-section of each wire is* $0{\cdot}005\text{ cm}^2$ *and Young's modulus of the metal is* $2{\cdot}0 \times 10^{11}\text{ N/m}^2$. *Take* $g = 10\text{ m/s}^2$.

Solution : The situation is described in figure (14-W2). As the 1 kg mass is in equilibrium, the tension in the lower wire equals the weight of the load.

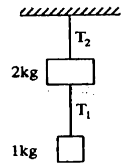


Figure 14-W2

Thus $$T_1 = 10\text{ N}$$

$$\text{Stress} = 10\text{ N}/0{\cdot}005\text{ cm}^2$$
$$= 2 \times 10^7\text{ N/m}^2.$$

$$\text{Longitudinal strain} = \frac{\text{stress}}{Y} = \frac{2 \times 10^7\text{ N/m}^2}{2 \times 10^{11}\text{ N/m}^2} = 10^{-4}.$$

Considering the equilibrium of the upper block, we can write,

$$T_2 = 20\text{ N} + T_1, \quad \text{or,} \quad T_2 = 30\text{ N}.$$

$$\text{Stress} = 30\text{ N}/0{\cdot}005\text{ cm}^2$$
$$= 6 \times 10^7\text{ N/m}^2.$$

$$\text{Longitudinal strain} = \frac{6 \times 10^7\text{ N/m}^2}{2 \times 10^{11}\text{ N/m}^2} = 3 \times 10^{-4}.$$

7. *Each of the three blocks P, Q and R shown in figure (14-W3) has a mass of* 3 kg. *Each of the wires A and B has cross-sectional area* $0{\cdot}005\text{ cm}^2$ *and Young's modulus* $2 \times 10^{11}\text{ N/m}^2$. *Neglect friction. Find the longitudinal strain developed in each of the wires. Take* $g = 10\text{ m/s}^2$.

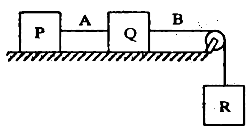


Figure 14-W3

Solution : The block R will descend vertically and the blocks P and Q will move on the frictionless horizontal table. Let the common magnitude of the acceleration be a. Let the tensions in the wires A and B be T_A and T_B respectively.

Writing the equations of motion of the blocks P, Q and R, we get,

$$T_A = (3\text{ kg})\,a \quad \text{... (i)}$$

$$T_B - T_A = (3\text{ kg})\,a \qquad \dots \text{(ii)}$$

and $(3\text{ kg})\,g - T_B = (3\text{ kg})\,a$. ... (iii)

By (i) and (ii),

$$T_B = 2\,T_A.$$

By (i) and (iii),

$$T_A + T_B = (3\text{ kg})\,g = 30\text{ N}.$$

or, $3\,T_A = 30\text{ N}$

or, $T_A = 10\text{ N}$ and $T_B = 20\text{ N}$.

$$\text{Longitudinal strain} = \frac{\text{Longitudinal stress}}{\text{Young's modulus}}$$

$$\text{Strain in wire } A = \frac{10\text{ N}/0{\cdot}005\text{ cm}^2}{2\times 10^{11}\text{ N/m}^2} = 10^{-4}$$

$$\text{and strain in wire } B = \frac{20\text{ N}/0{\cdot}005\text{ cm}^2}{2\times 10^{11}\text{ N/m}^2} = 2\times 10^{-4}.$$

8. *A wire of area of cross-section* $3{\cdot}0\text{ mm}^2$ *and natural length* 50 cm *is fixed at one end and a mass of* 2·1 kg *is hung from the other end. Find the elastic potential energy stored in the wire in steady state. Young's modulus of the material of the wire* $= 1{\cdot}9\times 10^{11}\text{ N/m}^2$. *Take* $g = 10\text{ m/s}^2$.

Solution : The volume of the wire is

$$V = (3{\cdot}0\text{ mm}^2)(50\text{ cm})$$
$$= (3{\cdot}0\times 10^{-6}\text{ m}^2)(0{\cdot}50\text{ m}) = 1{\cdot}5\times 10^{-6}\text{ m}^3.$$

Tension in the wire is

$$T = mg$$
$$= (2{\cdot}1\text{ kg})(10\text{ m/s}^2) = 21\text{ N}.$$

The stress $= T/A$

$$= \frac{21\text{ N}}{3{\cdot}0\text{ mm}^2} = 7{\cdot}0\times 10^6\text{ N/m}^2.$$

The strain = stress/Y

$$= \frac{7{\cdot}0\times 10^6\text{ N/m}^2}{1{\cdot}9\times 10^{11}\text{ N/m}^2} = 3{\cdot}7\times 10^{-5}.$$

The elastic potential energy of the wire is

$$U = \frac{1}{2}(\text{stress})(\text{strain})(\text{volume})$$
$$= \frac{1}{2}(7{\cdot}0\times 10^6\text{ N/m}^2)(3{\cdot}7\times 10^{-5})(1{\cdot}5\times 10^{-6}\text{ m}^3)$$
$$= 1{\cdot}9\times 10^{-4}\text{ J}.$$

9. *A block of weight* 10 N *is fastened to one end of a wire of cross-sectional area* 3 mm^2 *and is rotated in a vertical circle of radius* 20 cm. *The speed of the block at the bottom of the circle is* 2 m/s. *Find the elongation of the wire when the block is at the bottom of the circle. Young's modulus of the material of the wire* $= 2\times 10^{11}\text{ N/m}^2$.

Solution : Forces acting on the block are (a) the tension T and (b) the weight W. At the lowest point, the resultant force is $T - W$ towards the centre. As the block is going in a circle, the net force towards the centre should be mv^2/r with usual symbols. Thus,

$$T - W = mv^2/r$$

or, $T = W + mv^2/r$

$$= 10\text{ N} + \frac{(1\text{ kg})(2\text{ m/s})^2}{0{\cdot}2\text{ m}} = 30\text{ N}.$$

We have $Y = \dfrac{T/A}{l/L}$

or, $l = \dfrac{TL}{AY}$

$$= \frac{30\text{ N}\times(20\text{ cm})}{(3\times 10^{-6}\text{ m}^2)\times(2\times 10^{11}\text{ N/m}^2)}$$
$$= 5\times 10^{-5}\times 20\text{ cm} = 10^{-3}\text{ cm}.$$

10. *A uniform heavy rod of weight W, cross-sectional area A and length L is hanging from a fixed support. Young's modulus of the material of the rod is Y. Neglect the lateral contraction. Find the elongation of the rod.*

Solution : Consider a small length dx of the rod at a distance x from the fixed end. The part below this small element has length $L - x$. The tension T of the rod at the element equals the weight of the rod below it.

$$T = (L - x)\frac{W}{L}.$$

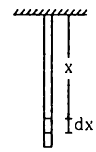


Figure 14-W4

Elongation in the element is given by

$$\text{elongation} = \text{original length}\times\text{stress}/Y$$
$$= \frac{T\,dx}{AY} = \frac{(L-x)\,W\,dx}{LAY}.$$

$$\text{The total elongation} = \int_0^L \frac{(L-x)\,W\,dx}{LAY}$$
$$= \frac{W}{LAY}\left(Lx - \frac{x^2}{2}\right)_0^L = \frac{WL}{2AY}.$$

11. *There is an air bubble of radius* 1·0 mm *in a liquid of surface tension* 0·075 N/m *and density* 1000 kg/m^3. *The bubble is at a depth of* 10 cm *below the free surface. By what amount is the pressure inside the bubble greater than the atmospheric pressure? Take* $g = 9{\cdot}8\text{ m/s}^2$.

Solution :

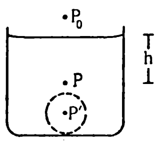


Figure 14-W5

Let the atmospheric pressure be P_0. The pressure of the liquid just outside the bubble is (figure 14-W5)

$$P = P_0 + h\rho g.$$

The pressure inside the bubble is

$$P' = P + \frac{2S}{r} = P_0 + h\rho g + \frac{2S}{r}$$

or,

$$P' - P_0 = (10 \text{ cm}) (1000 \text{ kg/m}^3) (9{\cdot}8 \text{ m/s}^2) + \frac{2 \times 0{\cdot}075 \text{ N/m}}{1{\cdot}0 \times 10^{-3} \text{ m}}$$

$$= 980 \text{ N/m}^2 + 150 \text{ N/m}^2$$

$$= 1130 \text{ Pa}.$$

12. *A light wire AB of length* 10 cm *can slide on a vertical frame as shown in figure (14-W6). There is a film of soap solution trapped between the frame and the wire. Find the load W that should be suspended from the wire to keep it in equilibrium. Neglect friction. Surface tension of soap solution* = 25 dyne/cm. *Take* g = 10 m/s^2.

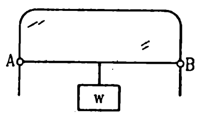


Figure 14-W6

Solution : Soap solution film will be formed on both sides of the frame. Each film is in contact with the wire along a distance of 10 cm. The force exerted by the film on the wire

$$= 2 \times (10 \text{ cm}) \times (25 \text{ dyne/cm})$$

$$= 500 \text{ dyne} = 5 \times 10^{-3} \text{ N}.$$

This force acts vertically upward and should be balanced by the load. Hence the load that should be suspended is 5×10^{-3} N. The mass of the load should be

$$\frac{5 \times 10^{-3} \text{ N}}{10 \text{ m/s}^2} = 5 \times 10^{-4} \text{ kg} = 0{\cdot}5 \text{ g}.$$

13. *The lower end of a capillary tube is dipped into water and it is seen that the water rises through* 7·5 cm *in the capillary. Find the radius of the capillary. Surface tension of water* = $7{\cdot}5 \times 10^{-2}$ N/m. *Contact angle between water and glass* = 0°. *Take* g = 10 m/s^2.

Solution : We have,

$$h = \frac{2S\cos\theta}{r\rho g}$$

or,

$$r = \frac{2S\cos\theta}{h\rho g}$$

$$= \frac{2 \times (7{\cdot}5 \times 10^{-2} \text{ N/m}) \times 1}{(0{\cdot}075 \text{ m}) \times (1000 \text{ kg/m}^3) \times (10 \text{ m/s}^2)}$$

$$= 2 \times 10^{-4} \text{ m} = 0{\cdot}2 \text{ mm}.$$

14. *Two mercury drops each of radius r merge to form a bigger drop. Calculate the surface energy released.*

Solution :

Surface area of one drop before merging $= 4\pi r^2$.

Total surface area of both the drops $= 8\pi r^2$.

Hence, the surface energy before merging $= 8\pi r^2 S$.

When the drops merge, the volume of the bigger drop

$$= 2 \times \frac{4}{3}\pi r^3 = \frac{8}{3}\pi r^3.$$

If the radius of this new drop is R,

$$\frac{4}{3}\pi R^3 = \frac{8}{3}\pi r^3$$

or,

$$R = 2^{1/3} r$$

or,

$$4\pi R^2 = 4 \times 2^{2/3} \times \pi r^2.$$

Hence, the surface energy $= 4 \times 2^{2/3} \times \pi r^2 S$.

The released surface energy $= 8\pi r^2 S - 4 \times 2^{2/3} \pi r^2 S$

$$\approx 1{\cdot}65\, \pi r^2 S.$$

15. *A large wooden plate of area* 10 m^2 *floating on the surface of a river is made to move horizontally with a speed of* 2 m/s *by applying a tangential force. If the river is* 1 m *deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river* = 10^{-2} poise.

Solution : The velocity decreases from 2 m/s to zero in 1 m of perpendicular length. Hence, velocity gradient

$$= dv/dx = 2 \text{ s}^{-1}.$$

Now,

$$\eta = \left| \frac{F/A}{dv/dx} \right|$$

or,

$$10^{-3} \frac{\text{N-s}}{\text{m}^2} = \frac{F}{(10 \text{ m}^2)(2 \text{ s}^{-1})}$$

or,

$$F = 0{\cdot}02 \text{ N}.$$

16. *The velocity of water in a river is* 18 km/hr *near the surface. If the river is* 5 m *deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water* = 10^{-2} poise.

Solution : The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18 \text{ km/hr}}{5 \text{ m}} = 1 \cdot 0 \text{ s}^{-1}.$$

The magnitude of the force of viscosity is

$$F = \eta A \frac{dv}{dx}.$$

The shearing stress is

$$F/A = \eta \frac{dv}{dx} = (10^{-2} \text{poise})(1 \cdot 0 \text{ s}^{-1}) = 10^{-3} \text{ N/m}^2.$$

17. *Find the terminal velocity of a rain drop of radius* 0·01 mm. *The coefficient of viscosity of air is* $1 \cdot 8 \times 10^{-5}$ N-s/m^2 *and its density is* 1·2 kg/m^3. *Density of water* = 1000 kg/m^3. *Take* g = 10 m/s^2.

Solution : The forces on the rain drop are

(a) the weight $\frac{4}{3} \pi r^3 \rho g$ downward,

(b) the force of buoyancy $\frac{4}{3} \pi r^3 \sigma g$ upward,

(c) the force of viscosity $6\pi\eta r v$ upward.

Here ρ is the density of water and σ is the density of air. At terminal velocity the net force is zero. As the density of air is much smaller than the density of water, the force of buoyancy may be neglected.

Thus, at terminal velocity

$$6\pi\eta r v = \frac{4}{3} \pi r^3 \rho g$$

or,

$$v = \frac{2 r^2 \rho g}{9\eta}.$$

$$= \frac{2 \times (0 \cdot 01 \text{ mm})^2 \times (1000 \text{ kg/m}^3)(10 \text{ m/s}^2)}{9 \times (1 \cdot 8 \times 10^{-5} \text{ N-s/m}^2)}$$

$$\approx 1 \cdot 2 \text{ cm/s}.$$

□

QUESTIONS FOR SHORT ANSWER

1. The ratio stress/strain remains constant for small deformation of a metal wire. When the deformation is made larger, will this ratio increase or decrease ?
2. When a block of mass M is suspended by a long wire of length L, the elastic potential energy stored in the wire is $\frac{1}{2}$ × stress × strain × volume. Show that it is equal to $\frac{1}{2} Mgl$, where l is the extension. The loss in gravitational potential energy of the Mass-earth system is Mgl. Where does the remaining $\frac{1}{2} Mgl$ energy go ?
3. When the skeleton of an elephant and the skeleton of a mouse are prepared in the same size, the bones of the elephant are shown thicker than those of the mouse. Explain why the bones of an elephant are thicker than proportionate. The bones are expected to withstand the stress due to the weight of the animal.
4. The yield point of a typical solid is about 1%. Suppose you are lying horizontally and two persons are pulling your hands and two person are pulling your legs along your own length. How much will be the increase in your length if the strain is 1% ? Do you think your yield point is 1% or, much less than that ?
5. When rubber sheets are used in a shock absorber, what happens to the energy of vibration ?
6. If a compressed spring is dissolved in acid, what happens to the elastic potential energy of the spring ?
7. A steel blade placed gently on the surface of water floats on it. If the same blade is kept well inside the water, it sinks. Explain.
8. When some wax is rubbed on a cloth, it becomes waterproof. Explain.
9. The contact angle between pure water and pure silver is 90°. If a capillary tube made of silver is dipped at one end in pure water, will the water rise in the capillary ?
10. It is said that a liquid rises or is depressed in a capillary due to the surface tension. If a liquid neither rises nor depresses in a capillary, can we conclude that the surface tension of the liquid is zero ?
11. The contact angle between water and glass is 0°. When water is poured in a glass to the maximum of its capacity, the water surface is convex upward. The angle of contact in such a situation is more than 90°. Explain.
12. A uniform vertical tube of circular cross-section contains a liquid. The contact angle is 90°. Consider a diameter of the tube lying in the surface of the liquid. The surface to the right of this diameter pulls the surface on the left of it. What keeps the surface on the left in equilibrium ?
13. When a glass capillary tube is dipped at one end in water, water rises in the tube. The gravitational potential energy is thus increased. Is it a violation of conservation of energy ?
14. If a mosquito is dipped into water and released, it is not able to fly till it is dry again. Explain.
15. The force of surface tension acts tangentially to the surface whereas the force due to air pressure acts perpendicularly on the surface. How is then the force due to excess pressure inside a bubble balanced by the force due to the surface tension ?
16. When the size of a soap bubble is increased by pushing more air in it, the surface area increases. Does it mean

that the average separation between the surface molecules is increased ?

17. Frictional force between solids operates even when they do not move with respect to each other. Do we have viscous force acting between two layers even if there is no relative motion ?

18. Water near the bed of a deep river is quiet while that near the surface flows. Give reasons.

19. If water in one flask and castor oil in other are violently shaken and kept on a table, which will come to rest earlier ?

OBJECTIVE I

1. A rope 1 cm in diameter breaks if the tension in it exceeds 500 N. The maximum tension that may be given to a similar rope of diameter 2 cm is
(a) 500 N (b) 250 N (c) 1000 N (d) 2000 N.

2. The breaking stress of a wire depends on
(a) material of the wire (b) length of the wire
(c) radius of the wire (d) shape of the cross-section.

3. A wire can sustain the weight of 20 kg before breaking. If the wire is cut into two equal parts, each part can sustain a weight of
(a) 10 kg (b) 20 kg (c) 40 kg (d) 80 kg.

4. Two wires A and B are made of same material. The wire A has a length l and diameter r while the wire B has a length $2l$ and diameter $r/2$. If the two wires are stretched by the same force, the elongation in A divided by the elongation in B is
(a) 1/8 (b) 1/4 (c) 4 (d) 8.

5. A wire elongates by 1·0 mm when a load W is hanged from it. If this wire goes over a pulley and two weights W each are hung at the two ends, the elongation of the wire will be
(a) 0·5 m (b) 1·0 mm (c) 2·0 mm (d) 4·0 mm.

6. A heavy uniform rod is hanging vertically from a fixed support. It is stretched by its own weight. The diameter of the rod is
(a) smallest at the top and gradually increases down the rod
(b) largest at the top and gradually decreases down the rod
(c) uniform everywhere
(d) maximum in the middle.

7. When a metal wire is stretched by a load, the fractional change in its volume $\Delta V/V$ is proportional to
(a) $\frac{\Delta l}{l}$ (b) $\left(\frac{\Delta l}{l}\right)^2$ (c) $\sqrt{\Delta l/l}$ (d) none of these.

8. The length of a metal wire is l_1 when the tension in it is T_1 and is l_2 when the tension is T_2. The natural length of the wire is
(a) $\frac{l_1 + l_2}{2}$ (b) $\sqrt{l_1 l_2}$ (c) $\frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$ (d) $\frac{l_1 T_2 + l_2 T_1}{T_2 + T_1}$.

9. A heavy mass is attached to a thin wire and is whirled in a vertical circle. The wire is most likely to break
(a) when the mass is at the highest point
(b) when the mass is at the lowest point
(c) when the wire is horizontal
(d) at an angle of $\cos^{-1}(1/3)$ from the upward vertical.

10. When a metal wire elongates by hanging a load on it, the gravitational potential energy is decreased.
(a) This energy completely appears as the increased kinetic energy of the block.
(b) This energy completely appears as the increased elastic potential energy of the wire.
(c) This energy completely appears as heat.
(d) None of these.

11. By a surface of a liquid we mean
(a) a geometrical plane like $x = 0$
(b) all molecules exposed to the atmosphere
(c) a layer of thickness of the order of 10^{-8} m
(d) a layer of thickness of the order of 10^{-4} m.

12 An ice cube is suspended in vacuum in a gravity-free hall. As the ice melts it
(a) will retain its cubical shape
(b) will change its shape to spherical
(c) will fall down on the floor of the hall
(d) will fly up.

13. When water droplets merge to form a bigger drop
(a) energy is liberated (b) energy is absorbed
(c) energy is neither liberated nor absorbed
(d) energy may either be liberated or absorbed depending on the nature of the liquid.

14. The dimension $ML^{-1}T^{-2}$ can correspond to
(a) moment of a force (b) surface tension
(c) modulus of elasticity (d) coefficient of viscosity.

15. Air is pushed into a soap bubble of radius r to double its radius. If the surface tension of the soap solution is S, the work done in the process is
(a) $8\pi r^2 S$ (b) $12\pi r^2 S$ (c) $16\pi r^2 S$ (d) $24\pi r^2 S$.

16. If more air is pushed in a soap bubble, the pressure in it
(a) decreases (b) increases
(c) remains same (d) becomes zero.

17. If two soap bubbles of different radii are connected by a tube,
(a) air flows from bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air.

18. Figure (14-Q1) shows a capillary tube of radius r dipped into water. If the atmospheric pressure is P_0, the pressure at point A is
(a) P_0 (b) $P_0 + \frac{2S}{r}$ (c) $P_0 - \frac{2S}{r}$ (d) $P_0 - \frac{4S}{r}$.

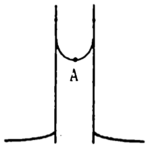


Figure 14-Q1

19. The excess pressure inside a soap bubble is twice the excess pressure inside a second soap bubble. The volume of the first bubble is n times the volume of the second where n is
(a) 4 (b) 2 (c) 1 (d) 0·125.

20. Which of the following graphs may represent the relation between the capillary rise h and the radius r of the capillary ?

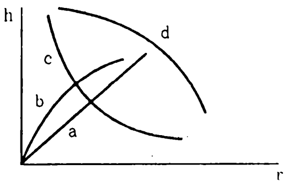


Figure 14-Q2

21. Water rises in a vertical capillary tube upto a length of 10 cm. If the tube is inclined at 45°, the length of water risen in the tube will be
(a) 10 cm (b) $10\sqrt{2}$ cm
(c) $10/\sqrt{2}$ cm (d) none of these.

22. A 20 cm long capillary tube is dipped in water. The water rises up to 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be
(a) 8 cm (b) 6 cm (c) 10 cm (d) 20 cm.

23. Viscosity is a property of
(a) liquids only (b) solids only
(c) solids and liquids only (d) liquids and gases only.

24. The force of viscosity is
(a) electromagnetic (b) gravitational (c) nuclear (d) weak.

25. The viscous force acting between two layers of a liquid is given by $\frac{F}{A} = -\eta \frac{dv}{dz}$ This F/A may be called
(a) pressure (b) longitudinal stress
(c) tangential stress (d) volume stress.

26. A raindrop falls near the surface of the earth with almost uniform velocity because
(a) its weight is negligible
(b) the force of surface tension balances its weight
(c) the force of viscosity of air balances its weight
(d) the drops are charged and atmospheric electric field balances its weight.

27. A piece of wood is taken deep inside a long column of water and released. It will move up
(a) with a constant upward acceleration
(b) with a decreasing upward acceleration
(c) with a deceleration
(d) with a uniform velocity.

28. A solid sphere falls with a terminal velocity of 20 m/s in air. If it is allowed to fall in vacuum,
(a) terminal velocity will be 20 m/s
(b) terminal velocity will be less than 20 m/s
(c) terminal velocity will be more than 20 m/s
(d) there will be no terminal velocity.

29. A spherical ball is dropped in a long column of a viscous liquid. The speed of the ball as a function of time may be best represented by the graph
(a) A (b) B (c) C (d) D.

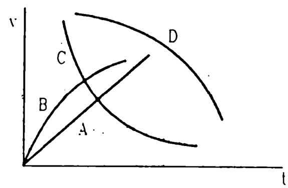


Figure 14-Q3

OBJECTIVE II

1. A student plots a graph from his readings on the determination of Young's modulus of a metal wire but forgets to put the labels (figure 14-Q4). The quantities on X and Y-axes may be respectively
(a) weight hung and length increased
(b) stress applied and length increased
(c) stress applied and strain developed
(d) length increased and the weight hung.

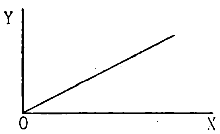


Figure 14-Q4

2. The properties of a surface are different from those of the bulk liquid because the surface molecules
(a) are smaller than other molelcules
(b) acquire charge due to collision from air molecules
(c) find different type of molecules in their range of influence
(d) feel a net force in one direction.

3. The rise of a liquid in a capillary tube depends on
(a) the material (b) the length
(c) the outer radius (d) the inner radius of the tube.

4. The contact angle between a solid and a liquid is a property of
(a) the material of the solid
(b) the material of the liquid

(c) the shape of the solid
(d) the mass of the solid.

5. A liquid is contained in a vertical tube of semicircular cross-section (figure 14-Q5). The contact angle is zero. The forces of surface tension on the curved part and on the flat part are in ratio
(a) 1 : 1 (b) 1 : 2 (c) $\pi : 2$ (d) $2 : \pi$.

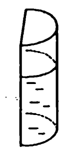


Figure 14-Q5

6. When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary.
(a) The surface tension of the liquid must be zero.
(b) The contact angle must be 90°.
(c) The surface tension may be zero.
(d) The contact angle may be 90°.

7. A solid sphere moves at a terminal velocity of 20 m/s in air at a place where $g = 9.8 \text{ m/s}^2$. The sphere is taken in a gravity free hall having air at the same pressure and pushed down at a speed of 20 m/s.
(a) Its initial acceleration will be 9.8 m/s^2 downward.
(b) Its initial acceleration will be 9.8 m/s^2 upward.
(c) The magnitude of acceleration will decrease as the time passes.
(d) It will eventually stop.

EXERCISES

1. A load of 10 kg is suspended by a metal wire 3 m long and having a cross-sectional area 4 mm^2. Find (a) the stress (b) the strain and (c) the elongation. Young's modulus of the metal is $2.0 \times 10^{11} \text{ N/m}^2$.

2. A vertical metal cylinder of radius 2 cm and length 2 m is fixed at the lower end and a load of 100 kg is put on it. Find (a) the stress (b) the strain and (c) the compression of the cylinder. Young's modulus of the metal $= 2 \times 10^{11} \text{ N/m}^2$.

3. The elastic limit of steel is $8 \times 10^8 \text{ N/m}^2$ and its Young's modulus $2 \times 10^{11} \text{ N/m}^2$. Find the maximum elongation of a half-meter steel wire that can be given without exceeding the elastic limit.

4. A steel wire and a copper wire of equal length and equal cross-sectional area are joined end to end and the combination is subjected to a tension. Find the ratio of (a) the stresses developed in the two wires and (b) the strains developed. Y of steel $= 2 \times 10^{11} \text{ N/m}^2$. Y of copper $= 1.3 \times 10^{11} \text{ N/m}^2$.

5. In figure (14-E1) the upper wire is made of steel and the lower of copper. The wires have equal cross-section. Find the ratio of the longitudinal strains developed in the two wires.

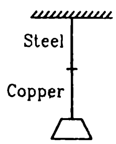


Figure 14-E1

6. The two wires shown in figure (14-E2) are made of the same material which has a breaking stress of $8 \times 10^8 \text{ N/m}^2$. The area of cross-section of the upper wire is 0.006 cm^2 and that of the lower wire is 0.003 cm^2. The mass $m_1 = 10 \text{ kg}$, $m_2 = 20 \text{ kg}$ and the hanger is light. (a) Find the maximum load that can be put on the hanger without breaking a wire. Which wire will break first if the load is increased? (b) Repeat the above part if $m_1 = 10 \text{ kg}$ and $m_2 = 36 \text{ kg}$.

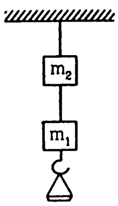


Figure 14-E2

7. Two persons pull a rope towards themselves. Each person exerts a force of 100 N on the rope. Find the Young's modulus of the material of the rope if it extends in length by 1 cm. Original length of the rope = 2 m and the area of cross-section = 2 cm^2.

8. A steel rod of cross-sectional area 4 cm^2 and length 2 m shrinks by 0.1 cm as the temperature decreases in night. If the rod is clamped at both ends during the day hours, find the tension developed in it during night hours. Young's modulus of steel $= 1.9 \times 10^{11}$ N/m 2.

9. Consider the situation shown in figure (14-E3). The force F is equal to the $m_2 g/2$. If the area of cross-section of the string is A and its Young's modulus Y, find the strain developed in it. The string is light and there is no friction anywhere.

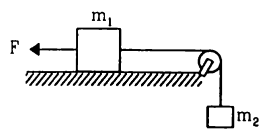


Figure 14-E3

10. A sphere of mass 20 kg is suspended by a metal wire of unstretched length 4 m and diameter 1 mm. When in equilibrium, there is a clear gap of 2 mm between the sphere and the floor. The sphere is gently pushed aside so that the wire makes an angle θ with the vertical and is released. Find the maximum value of θ so that the sphere does not rub the floor. Young's modulus of the metal of the wire is $2.0 \times 10^{11} \text{ N/m}^2$. Make appropriate approximations.

11. A steel wire of original length 1 m and cross-sectional area $4{\cdot}00\ mm^2$ is clamped at the two ends so that it lies horizontally and without tension. If a load of 2·16 kg is suspended from the middle point of the wire, what would be its vertical depression ?

Y of the steel = $2{\cdot}0 \times 10^{11}\ N/m^2$. Take $g = 10\ m/s^2$.

12. A copper wire of cross-sectional area $0{\cdot}01\ cm^2$ is under a tension of 20 N. Find the decrease in the cross-sectional area. Young's modulus of copper = $1{\cdot}1 \times 10^{11}\ N/m^2$ and Poisson's ratio = 0·32.

[**Hint** : $\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$]

13. Find the increase in pressure required to decrease the volume of a water sample by 0·01%. Bulk modulus of water = $2{\cdot}1 \times 10^9\ N/m^2$.

14. Estimate the change in the density of water in ocean at a depth of 400 m below the surface. The density of water at the surface = $1030\ kg/m^3$ and the bulk modulus of water = $2 \times 10^9\ N/m^2$.

15. A steel plate of face-area $4\ cm^2$ and thickness 0·5 cm is fixed rigidly at the lower surface. A tangential force of 10 N is applied on the upper surface. Find the lateral displacement of the upper surface with respect to the lower surface. Rigidity modulus of steel = $8{\cdot}4 \times 10^{10}\ N/m^2$.

16. A 5·0 cm long straight piece of thread is kept on the surface of water. Find the force with which the surface on one side of the thread pulls it. Surface tension of water = 0·076 N/m.

17. Find the excess pressure inside (a) a drop of mercury of radius 2 mm (b) a soap bubble of radius 4 mm and (c) an air bubble of radius 4 mm formed inside a tank of water. Surface tension of mercury, soap solution and water are 0·465 N/m, 0·03 N/m and 0·076N/m respectively.

18. Consider a small surface area of $1\ mm^2$ at the top of a mercury drop of radius 4·0 mm. Find the force exerted on this area (a) by the air above it (b) by the mercury below it and (c) by the mercury surface in contact with it. Atmospheric pressure = $1{\cdot}0 \times 10^5$ Pa and surface tension of mercury = 0·465 N/m. Neglect the effect of gravity. Assume all numbers to be exact.

19. The capillaries shown in figure (14-E4) have inner radii 0·5 mm, 1·0 mm and 1·5 mm respectively. The liquid in the beaker is water. Find the heights of water level in the capillaries. The surface tension of water is $7{\cdot}5 \times 10^{-2}$ N/m.

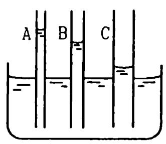


Figure 14-E4

20. The lower end of a capillary tube is immersed in mercury. The level of mercury in the tube is found to be 2 cm below the outer level. If the same tube is immersed in water, upto what height will the water rise in the capillary ?

21. A barometer is constructed with its tube having radius 1·0 mm. Assume that the surface of mercury in the tube is spherical in shape. If the atmospheric pressure is equal to 76 cm of mercury, what will be the height raised in the barometer tube. The contact angle of mercury with glass = 135° and surface tension of mercury = 0·465 N/m. Density of mercury = $13600\ kg/m^3$.

22. A capillary tube of radius 0·50 mm is dipped vertically in a pot of water. Find the difference between the pressure of the water in the tube 5·0 cm below the surface and the atmospheric pressure. Surface tension of water = 0·075 N/m.

23. Find the surface energy of water kept in a cylindrical vessel of radius 6·0 cm. Surface tension of water = $0{\cdot}075\ J/m^2$.

24. A drop of mercury of radius 2 mm is split into 8 identical droplets. Find the increase in surface energy. Surface tension of mercury = $0{\cdot}465\ J/m^2$.

25. A capillary tube of radius 1 mm is kept vertical with the lower end in water. (a) Find the height of water raised in the capillary. (b) If the length of the capillary tube is half the answer of part (a), find the angle θ made by the water surface in the capillary with the wall.

26. The lower end of a capillary tube of radius 1 mm is dipped vertically into mercury. (a) Find the depression of mercury column in the capillary. (b) If the length dipped inside is half the answer of part (a), find the angle made by the mercury surface at the end of the capillary with the vertical. Surface tension of mercury = 0·465 N/m and the contact angle of mercury with glass = 135°.

27. Two large glass plates are placed vertically and parallel to each other inside a tank of water with separation between the plates equal to 1 mm. Find the rise of water in the space between the plates. Surface tension of water = 0·075 N/m.

28. Consider an ice cube of edge 1·0 cm kept in a gravity free hall. Find the surface area of the water when the ice melts. Neglect the difference in densities of ice and water.

29. A wire forming a loop is dipped into soap solution and taken out so that a film of soap solution is formed. A loop of 6·28 cm long thread is gently put on the film and the film is pricked with a needle inside the loop. The thread loop takes the shape of a circle. Find the tension in the thread. Surface tension of soap solution = 0·030 N/m.

30. A metal sphere of radius 1 mm and mass 50 mg falls vertically in glycerine. Find (a) the viscous force exerted by the glycerine on the sphere when the speed of the sphere is 1 cm/s, (b) the hydrostatic force exerted by the glycerine on the sphere and (c) the terminal velocity with which the sphere will move down without acceleration. Density of glycerine = $1260\ kg/m^3$ and its coefficient of viscosity at room temperature = 8·0 poise.

31. Estimate the speed of vertically falling raindrops from the following data. Radius of the drops = 0·02 cm, viscosity of air = $1{\cdot}8 \times 10^{-4}$ poise, $g = 9{\cdot}9\ m/s^2$ and density of water = $1000\ kg/m^3$.

32. Water flows at a speed of 6 cm/s through a tube of radius 1 cm. Coefficient of viscosity of water at room termperature is 0·01 poise. Calculate the Reynolds number. Is it a steady flow ?

□

ANSWERS

OBJECTIVE I

1. (d)	2. (a)	3. (b)	4. (a)	5. (b)	6. (a)
7. (a)	8. (c)	9. (b)	10. (d)	11. (c)	12. (b)
13. (a)	14. (c)	15. (d)	16. (a)	17. (c)	18. (c)
19. (d)	20. (c)	21. (b)	22. (d)	23. (d)	24. (a)
25. (c)	26. (c)	27. (b)	28. (d)	29. (b)	

OBJECTIVE II

1. all	2. (c), (d)	3. (a), (b), (d)
4. (a), (b)	5. (c)	6. (c), (d)
7. (b), (c), (d)		

EXERCISES

1. (a) $2·5 \times 10^{7}$ N/m^{2} (b) $1·25 \times 10^{-4}$ (c) $3·75 \times 10^{-4}$ m
2. (a) $7·96 \times 10^{5}$ N/m^{2} (b) 4×10^{-6} (c) 8×10^{-6} m
3. 2 mm
4. (a) 1 (b) $\dfrac{\text{strain in copper wire}}{\text{strain in steel wire}} = \dfrac{20}{13}$
5. $\dfrac{\text{strain in copper wire}}{\text{strain in steel wire}} = 1·54$
6. (a) 14 kg, lower (b) 2 kg, upper
7. 1×10^{8} N/m^{2}
8. $3·8 \times 10^{4}$ N
9. $\dfrac{m_2 g(2m_1 + m_2)}{2AY(m_1 + m_2)}$
10. 36·4°
11. 1·5 cm
12. $1·164 \times 10^{-6}$ cm^{2}
13. $2·1 \times 10^{6}$ N/m^{2}
14. 2 kg/m^{3}
15. $1·5 \times 10^{-9}$ m
16. $3·8 \times 10^{-3}$ N
17. (a) 465 N/m^{2} (b) 30 N/m^{2} (c) 38 N/m^{2}
18. (a) 0·1 N (b) 0·10023 N (c) 0·00023 N
19. 3 cm in A, 1·5 cm in B, 1 cm in C
20. 5·73 cm
21. 75·5 cm
22. 190 N/m^{2}
23. $8·5 \times 10^{-4}$ J
24. 23·4 μ J
25. (a) 1·5 cm (b) 60°
26. (a) 5·34 mm (b) 112°
27. 1·5 cm
28. $(36\pi)^{1/3}$ cm^{2}
29. 3×10^{-4} N
30. (a) $1·5 \times 10^{-4}$ N (b) $5·2 \times 10^{-5}$ N (c) 2·9 cm/s
31. 5 m/s
32. 120, yes.

□

SOLUTIONS TO CONCEPTS
CHAPTER 14

1. F = mg

   Stress = $\frac{F}{A}$

   Strain = $\frac{\Delta L}{L}$

   Y = $\frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{YA}$

2. ρ = stress = mg/A

   e = strain = ρ/Y

   Compression ΔL = eL

3. y = $\frac{F}{A}\frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$

4. $L_{steel} = L_{cu}$ and $A_{steel} = A_{cu}$

   a) $\frac{\text{Stress of cu}}{\text{Stress of st}} = \frac{F_{cu}}{A_{cu}}\frac{A_g}{F_g}$ = $\frac{F_{cu}}{F_{st}} = 1$

   b) Strain = $\frac{\Delta Lst}{\Delta lcu} = \frac{F_{st}L_{st}}{A_{st}Y_{st}} \cdot \frac{A_{cu}Y_{cu}}{F_{cu}l_{cu}}$ ($\because$ $L_{cu} = l_{st}$; $A_{cu} = A_{st}$)

5. $\left(\frac{\Delta L}{L}\right)_{st} = \frac{F}{AY_{st}}$

   $\left(\frac{\Delta L}{L}\right)_{cu} = \frac{F}{AY_{cu}}$

   $\frac{\text{strain steel wire}}{\text{Strain om copper wire}} = \frac{F}{AY_{st}} \times \frac{AY_{cu}}{F} (\because A_{cu} = A_{st}) = \frac{Y_{cu}}{Y_{st}}$

6. Stress in lower rod = $\frac{T_1}{A_1} \Rightarrow \frac{m_1g + \omega g}{A_1}$ $\Rightarrow$ w = 14 kg

   Stress in upper rod = $\frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + wg}{A_u} \Rightarrow$ w = .18 kg

   For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

   $\frac{T_1}{A_1} = \frac{m_1g + \omega g}{A_1}$ = $8 \times 10^8 \Rightarrow$ w = 14 kg

   $\frac{T_2}{A_u} \Rightarrow \frac{m_2g + m_1g + \omega g}{A_u}$ = $8 \times 10^8 \Rightarrow \omega_0$ = 2 kg

   The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \frac{F}{A}\frac{L}{\Delta L}$

8. $Y = \frac{F}{A}\frac{L}{\Delta L} \Rightarrow F = \frac{YA\ \Delta L}{L}$

9. $m_2g - T = m_2a$ …(1)

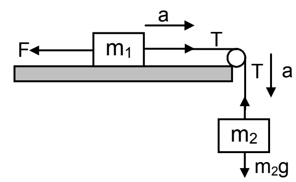
   and $T - F = m_1a$ …(2)

   $\Rightarrow a = \frac{m_2g - F}{m_1 + m_2}$

From equation (1) and (2), we get $\frac{m_2 g}{2(m_1+m_2)}$

Again, T = F + m_1a

$\Rightarrow T = \frac{m_2 g}{2} + m_1 \frac{m_2 g}{2(m_1+m_2)} \Rightarrow \frac{m_2^2 g + 2m_1 m_2 g}{2(m_1+m_2)}$



Now Y = $\frac{FL}{A\ \Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{AY}$

$\Rightarrow \frac{\Delta L}{L} = \frac{(m_2^2 + 2m_1 m_2)g}{2(m_1+m_2)AY} = \frac{m_2 g(m_2 + 2m_1)}{2AY(m_1+m_2)}$

10. At equilibrium ⇒ T = mg

When it moves to an angle θ, and released, the tension the T′ at lowest point is

$\Rightarrow T' = mg + \frac{mv^2}{r}$

The change in tension is due to centrifugal force $\Delta T = \frac{mv^2}{r}$ …(1)

⇒ Again, by work energy principle,

$\Rightarrow \frac{1}{2}mv^2 - 0 = mgr(1 - \cos\theta)$

$\Rightarrow v^2 = 2gr(1 - \cos\theta)$ …(2)

So, $\Delta T = \frac{m[2gr(1-\cos\theta)]}{r} = 2mg(1-\cos\theta)$

⇒ F = ΔT

$\Rightarrow F = \frac{YA\ \Delta L}{L} = 2mg - 2mg\cos\theta \Rightarrow 2mg\cos\theta = 2mg - \frac{YA\ \Delta L}{L}$

$= \cos\theta = 1 - \frac{YA\ \Delta L}{L(2mg)}$

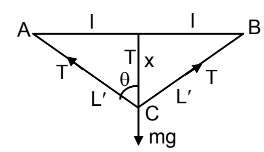
11. From figure $\cos\theta = \frac{x}{\sqrt{x^2+l^2}} = \frac{x}{l}\left[1+\frac{x^2}{l^2}\right]^{-1/2}$

$= x / l$ … (1)

Increase in length ΔL = (AC + CB) – AB

Here, AC = $(l^2 + x^2)^{1/2}$

So, ΔL = $2(l^2 + x^2)^{1/2} - 100$ …(2)



Y = $\frac{F}{A}\frac{l}{\Delta l}$ …(3)

From equation (1), (2) and (3) and the freebody diagram,

2l cosθ = mg.

12. $Y = \frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{Ay}$

$\sigma = \frac{\Delta D / D}{\Delta L / L} \Rightarrow \frac{\Delta D}{D} = \frac{\Delta L}{L}$

Again, $\frac{\Delta A}{A} = \frac{2\Delta r}{r}$

$\Rightarrow \Delta A = \frac{2\Delta r}{r}$

13. B = $\frac{Pv}{\Delta v}$ $\Rightarrow$ P = $B\left(\frac{\Delta v}{v}\right)$

14. $\rho_0 = \frac{m}{V_0} = \frac{m}{V_d}$

so, $\frac{\rho_d}{\rho_0} = \frac{V_0}{V_d}$ …(1)

vol.strain = $\frac{V_0 - V_d}{V_0}$

B = $\frac{\rho_0 gh}{(V_0 - V_d)/V_0}$ $\Rightarrow 1 - \frac{V_d}{V_0} = \frac{\rho_0 gh}{B}$

$\Rightarrow \frac{vD}{v_0} = \left(1 - \frac{\rho_0 gh}{B}\right)$ …(2)

Putting value of (2) in equation (1), we get

$\frac{\rho_d}{\rho_0} = \frac{1}{1-\rho_0 gh/B}$ $\Rightarrow$ $\rho_d = \frac{1}{(1-\rho_0 gh/B)} \times \rho_0$

15. $\eta = \frac{F}{A\theta}$

Lateral displacement = lθ.

16. F = T l

17. a) $P = \frac{2T_{Hg}}{r}$ b) $P = \frac{4T_g}{r}$ c) $P = \frac{2T_g}{r}$

18. a) F = P_0A
b) Pressure = P_0 + (2T/r)
F = P′A = (P_0 + (2T/r)A
c) P = 2T/r
F = PA = $\frac{2T}{r}$A

19. a) $h_A = \frac{2T\cos\theta}{r_A - \rho g}$ b) $h_B = \frac{2T\cos\theta}{r_B \rho g}$ c) $h_C = \frac{2T\cos\theta}{r_C \rho g}$

20. $h_{Hg} = \frac{2T_{Hg}\cos\theta_{Hg}}{r\rho_{Hg} g}$

$h_\omega = \frac{2T_\omega \cos\theta_\omega}{r\rho_\omega g}$ where, the symbols have their usual meanings.

$\frac{h_\omega}{h_{Hg}} = \frac{T_\omega}{T_{Hg}} \times \frac{\rho_{Hg}}{\rho_\omega} \times \frac{\cos\theta_\omega}{\cos\theta_{Hg}}$

21. $h = \frac{2T\cos\theta}{r\rho g}$

22. P = $\frac{2T}{r}$

P = F/r

23. A = πr^2

24. $\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \times 8$

$\Rightarrow$ r = R/2 = 2

Increase in surface energy = TA′ – TA

25. $h = \dfrac{2T\cos\theta}{r\rho g}$, $h' = \dfrac{2T\cos\theta}{r\rho g}$

$\Rightarrow \cos\theta = \dfrac{h'r\rho g}{2T}$

So, $\theta = \cos^{-1}(1/2) = 60°$.

26. a) $h = \dfrac{2T\cos\theta}{r\rho g}$

b) $T \times 2\pi r \cos\theta = \pi r^2 h \times \rho \times g$

$\therefore \cos\theta = \dfrac{hr\rho g}{2T}$

27. $T(2l) = [1 \times (10^{-3}) \times h]\rho g$

28. Surface area = $4\pi r^2$

29. The length of small element = $r\, d\,\theta$

$dF = T \times r\, d\,\theta$

considering symmetric elements,

$dF_y = 2T\, rd\theta \,.\, \sin\theta$ $[dF_x = 0]$

so, $F = 2Tr\int_0^{\pi/2} \sin\theta d\theta = 2Tr[\cos\theta]_0^{\pi/2} = T \times 2\,r$

Tension $\Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$

30. a) Viscous force = $6\pi\eta rv$

b) Hydrostatic force = $B = \left(\dfrac{4}{3}\right)\pi r^3\sigma g$

c) $6\pi\eta\, rv + \left(\dfrac{4}{3}\right)\pi r^3\sigma g = mg$

$$v = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow \frac{2}{9}r^2\frac{\left(\dfrac{m}{(4/3)\pi r^3} - \sigma\right)g}{n}$$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi\, r^3\, \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi\, r^3\, \sigma g$ upward.

iii) Force of viscosity $6\,\pi\,\eta\, r\, v$ upward.

Because, σ of air is very small, the force of buoyancy may be neglected.

Thus,

$$6\,\pi\,\eta\, r\, v = \left(\frac{4}{3}\right)\pi r^2\rho g \quad \text{or} \quad v = \frac{2r^2\rho g}{9\eta}$$

32. $v = \dfrac{R\eta}{\rho D} \Rightarrow R = \dfrac{v\rho D}{\eta}$

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