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24. MODERN PHYSICS

1. INTRODUCTION

The developments in the post-classical/Newtonian physics, also known as modern physics, has given us revelatory insights into the structure and nature of fundamental forces/particles in the universe. The wave-particle duality/paradox, which postulates that every elementary particle exhibits the properties of not only particles, but also waves is one such insight. For example, when electromagnetic radiation is absorbed by matter, it predominantly displays particle-like properties. It was de-Broglie who propounded the concept of matter waves, i.e. the particles exhibiting wave properties. We will be dealing here with the energy, wavelength, and frequency of electromagnetic waves and the relationship between these quantities. We will also be dealing with the photoelectric effect on which Einstein's based his photon theory of light. We will be discussing the Bohr atomic model, the hydrogen spectra, and the laws describing the characteristics of X-rays.

2. DUAL NATURE OF ELECTROMAGNETIC WAVES

Classical physics always defined motions in terms of particles or waves i.e., it treated particle and waves as distinct entities. An electron is considered as a particle because it possess mass (9.109×10^{-31} kilograms), electric charge (-1.602×10^{-19} coulomb) and they behave according to the laws of particle mechanics. However, we shall see that an electron in motion is as much a particle as it is a wave manifestation.

Electromagnetic radiation has properties in common with other forms of waves such as reflection, refraction, diffraction, and interference. It, however, also has particle-like properties in addition to those associated with wave motion (Photoelectric effect and Einstein's theory, black body radiation, Compton effect). Therefore, we can say that they a have dual nature.

Einstein's Equation: $E = hf = \dfrac{hc}{\lambda}$

2.1 Electromagnetic Spectrum

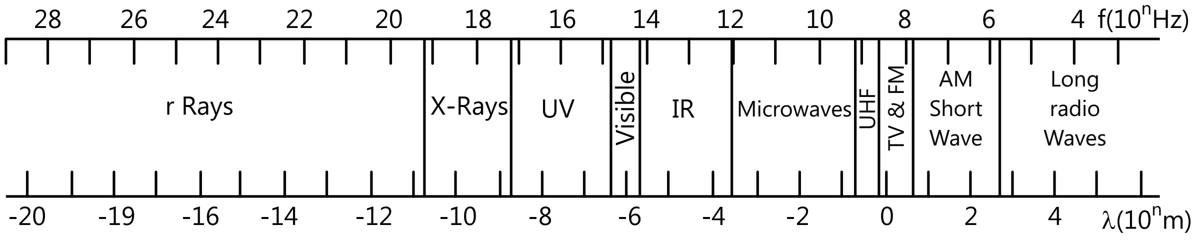


Figure 24.1 Electromagnetic spectrum

2.2 Electron Emission

Electrons are negatively charged particles. Therefore, though they move about arbitrarily in a conductor at room temperature, they cannot leave the surface of the conductor due to attraction of positively charged particles(protons). Therefore, some external energy has to be provided so that the electrons can be ejected from the atoms on the surface of the conductor. To eject the electrons which are just on the surface of the conductor, only minimal energy is required. This minimal energy or thermodynamic work that is needed to remove an electron from a conductor/ solid body to a point in the vacuum immediately outside the surface of the solid body/conductor is called the work function (denoted by W) of the conductor. Work function is the property of the metallic surface.

Heat, light, electric energy etc., can be employed to liberate an electron from a metal surface. Depending on the source of energy, the following methods are possible:

(a) **Thermionic emission:** In this method, the metal/conductor is heated to overcome binding potential of the conductor, and consequentially, free the electrons.

(b) **Field emission:** The emission of electrons induced by an electrostatic field is called field emission. In this process, the high electric field acting on the conductor exerts an electric force on the free electrons in the conductor in the opposite direction of field. This force overcomes the binding potential of the conductor and the electrons start coming out of the metal's surface

(c) **Secondary emission:** Ejection of electrons from a solid that is bombarded by a beam of charged particles (e.g., electrons or ions) is known as secondary emission.

(d) **Photoelectric emission:** The photoelectric effect refers to the emission/ejection of electrons from the surface of a metal in response to incident light (or electromagnetic wave). This happens when the incident light or electromagnetic wave has greater energy then the work function of the metal. The electrons emitted in this process are called **photoelectrons**.

3. PHOTOELECTRIC EFFECT

(a) The photoelectric effect was discovered by Wilhelm Ludwig Franz Hallwachs in 1888, the experimental verification which was done by Hertz.

(b) The photoelectric effect refers to the emission/ejection of electrons from the surface of a metal in response to incident light (or electromagnetic wave).

(c) The electron ejected due to photoelectric effect is is called a photoelectron and is denoted by e^-.

(d) Current produced as a result of the ejected electrons is called photoelectric current.

(e) Photoelectric effect proves quantum nature of light.

(f) Photoelectric effect can not be explained by the classical wave theory of light. The wave theory is incable of explaining the first 3 obserations of the photoelectric effect.

(g) Photoelectrons, generally, refer to the free electrons that are in the inter-molecular spaces in the metal.

(h) Explanation for Photoelectric effect was successfulyy explained given by Albert Einstein as being the result of light energy being carried in discrete quantized packets. For this excellent work he was honored with the Nobel prize in 1921.

(i) The law of conservation of energy forms the basis for photoelectric effect.

Threshold Frequency (v_0): The minimum frequency of the incident light or radiation that will produce a photoelectric effect i.e., ejection of photoelectrons from a metal surface is known as threshold frequency for that metal. Its value, though constant for a specific metal, may be different for different metals.

If v = frequency of incident photon & v_0 = Threshold Frequency

Then

(a) If $v < v_0$, there will be no ejection of photoelectron and, therefore, no photoelectric effect.

(b) If $v = v_0$, photoelectrons are just ejected from metal surface, in this case the kinetic energy of electron is zero.

(c) If $v > v_0$, then photoelectrons will come out of the surface along with kinetic energy.

Threshold Wavelength (λ_0): The greatest wavelength of the incident light or radiation for a specified surface for the emission of photoelectrons is known as threshold wavelength $\lambda_0 = \frac{c}{v_0}$. For wavelengths above this threshold, there will be no photoelectron emission.

For λ = wavelength of incident photon, then

(a) If $\lambda < \lambda_0$, then photoelectric effect will take place and ejected electron will posses kinetic energy.

(b) If $\lambda = \lambda_0$, then just photoelectric effect will take place and kinetic energy of ejected photoelectron will be zero.

(c) If $\lambda > \lambda_0$, there will be no photoelectric effect.

3.1 Work Function or Threshold Energy (ϕ)

(a) The minimal energy or thermodynamic work that is needed to remove an electron from a conductor/solid body to a point in the vacuum immediately outside the surface of the solid body/conductor is called the work function or threshold energy for the conductor.

$$\phi = hv_0 = \frac{hc}{\lambda_0}$$

(b) Work function is the characteristic of given metal

(c) If E = energy of incident photon, then

  (i) If $E < \phi$, no photoelectric effect will take place.

  (ii) If $E = \phi$, just photoelectric effect will take place but the kinetic energy of ejected photoelectron will be zero.

  (iii) If $E > \phi$, photoelectric effect will take place along with possession of the kinetic energy by ejected electron.

3.2 Laws of Photoelectric Effect

Lenard postulated the following laws regarding photo emission on the basis of his experiments:

(a) For a given substance, there is a minimum value of frequency of incident light called threshold frequency below which no photoelectric emission is possible, howsoever, the intensity of incident light may be. It is given by $v_0 = \frac{c}{\lambda_0}$

(b) The number of photoelectrons emitted per second (i.e. photoelectric current) is directly proportional to the intensity of incident light provided the frequency is above the threshold frequency.

(c) The maximum kinetic energy of the photoelectrons is directly proportional to the frequency provided the frequency is above the threshold frequency. However, the relationship between the wavelength and kinetic energy is inversely proportional. With increasing frequency of incident light, the kinetic energy of photoelectron increases but with increasing wavelength it decreases. So $v \uparrow \lambda \downarrow$ K.E. of emitted electrons $\uparrow$ $v \downarrow \lambda \uparrow$ K.E. of emitted electrons $\downarrow$

(d) The maximum kinetic energy of the photoelectrons is independent of the intensity of the incident light.

(e) The process of photoelectric emission is instantaneous, i.e., as soon as the photon of suitable frequency falls on the substance, it emits photoelectrons.

(f) The photoelectric emission is one-to-one. i.e. for every photon of suitable frequency one electron is emitted.

(g) Value of threshold frequency or threshold wavelength depends upon photo sensitive nature of metal.

Illustration 1: The work function of silver is 5.26×10^{-19} J. Calculate its threshold wavelength. **(JEE MAIN)**

Sol: For any metal to eject photo electron the work function of surface is given as $\phi = \frac{hc}{\lambda_0}$

Threshold wavelength = $\lambda_0 = \frac{hc}{\phi}$; $\therefore \lambda = \frac{6.6\times10^{-34}\times3\times10^{8}}{5.26\times10^{-19}} = 3.764\times10^{-7}\,\text{m}$; $\lambda = 3764\,\mathring{A}$

Illustration 2: The work function of Na is 2.3 eV. What is the maximum wavelength of light that will cause photo electrons to be emitted from sodium? **(JEE MAIN)**

Sol: For any metal to eject photo electron the work function of surface is given as $\phi = \frac{hc}{\lambda_0}$

The threshold wavelength $\lambda_0 = \frac{hc}{\phi}$; ($\because \phi = hv_0 = \frac{hc}{\lambda_0}$) ;& hc = 1.24 × 10^{-6} (eV) m

$\lambda_0 = \frac{1.24\times10^{-6}}{2.3}$ m ; λ_0 = 0.539 × 10^{-6} m = 539 nm; $\lambda_o = 5930\,\mathring{A}$

3.3 Failure of Wave Theory to Explain Photoelectric Effect

Note - The assumptions of the classical wave theory could not explain some observations of the photoelectric effect. These aspects of the photoelectric effect were later explained by Albert Einstein's photon theory. The failures of the classical wave theory in explaining the photoelectric effect are enumerated below:

(a) The wave theory suggests that the intensity of the radiation should have a proportional relationship with the resulting maximum kinetic energy. However $K_{max} = eV_0$ suggests that it is independent of the intensity of light.

(b) According to the wave theory, the photoelectric effect should occur for any intense light, regardless of frequency or wavelength. However, the equation suggests that photo emission is possible only when frequency of incident light is either greater than or equal to the threshold frequency f_0.

(c) The wave theory states that there should be a delay on the order of seconds between the radiation's contact with the metal and the initial release of photoelectrons. It was assumed that between the impinging of the light on the surface and the ejection of the photoelectrons, the electron should be "soaking up" energy from the beam until it had accumulated enough energy to escape. However, no detectable time lag has ever been measured.

In reality, due to collision between atoms inside the metal, some energy is lost. Hence kinetic energy emitted by electrons is K< K_{max}. Hence the term K_{max} is used for the actual total kinetic energy.

3.4 Einstein's Photon Theory

Albert Einstein worked his way around the limitations of the classical wave theory by explaining that lights exists and travels as tiny packets/bundles called photons. The energy E of a single photon is given by E = hf

Applying the photon concept to the photoelectric effect, Einstein wrote:

$hf = W + K_{max}$ (Already discussed)

Discussed below is how Einstein's photon hypothesis overcomes the three objections raised against the wave theory interpretation of the photoelectric effect.

Objection 1: Intensity of the radiation should have a proportional relationship with the resulting maximum kinetic energy. This objection is overcome by Einstein's photon theory because, doubling the light intensity merely doubles the number of photons, thereby doubling the photoelectric current. It does not, however, change the energy of the individual photons.

Objection 2: Photoelectric effect should occur for any intense light, regardless of frequency or wavelength. The existence of a minimum frequency level (in Einstein's photon theory) follows from equation hf = W + K_{max}. If K_{max} equals zero, then $hf_0 = W$, which implies that the photon's energy will be barely adequate to eject the photoelectron and that there will be no residual energy to manifest as kinetic energy. The quantity W is the work function of the metal/substance. If the frequency f is reduced below f_0, the individual photons, irrespective of how numerous they are(in other words, no matter what the intensity of the incident light/radiation is), will not have enough energy to eject photoelectrons.

Objection 3: There should be a delay on the order of seconds between the radiation's contact with the metal and the initial release of photoelectrons. The absence of a time lag follows from the photon theory because the required energy is supplied in packets/bundles. Unlike in the wave theory, the energy is not spread uniformly over a large area.

Therefore, as far as photoelectricity goes, the photon/particle theory seems to be in total contradiction of the wave theory of light. Modern physicists have reconciled this apparent paradox by postulating the dual nature of light, i.e., light behaves as a wave under some circumstances and like a particle, or photon, under others.

3.5 Einstein's Equation of Photoelectric Effect

Einstein (1905) explained photoelectric effect on the basis of quantum theory.

According to Einstein, when photons fallon a metal surface, they transfer their energy to the electrons of metal. When the energy of photon is larger than the minimum energy required by the electrons to leave the metal surface, the emission of electrons take place instantaneously.

He proposed that after absorbing the photon, an electron either leaves the surface or dissipates its energy within the metal in such a short interval that it has almost no chance to absorb second photon

The energy supplied to the electrons is used in two ways:

(a) Removes the electron from the surface of metal

(b) Supplies some part of kinetic energy to the photoelectron. Therefore, Einstein's equation of photoelectric effect can be written as:

If v_{max} is the maximum velocity of emitted electrons then by law of conservation of energy

$hv = \phi + \frac{1}{2}mv^2$. If v_0 : Threshold frequency $\therefore\ \phi_0 = hv_0$, So $\Rightarrow$ $hv = hv_0 + \frac{1}{2}mv_{max}^2$.

Einstein's equation explains the following concepts

(a) The frequency of the radiation/incident light is directly proportional to the kinetic energy of the electrons and the wavelength of radiation/incident light is inversely proportional to the kinetic energy of the electrons.

If v_0 is threshold frequency then maximum kinetic energy $E_{max} = hv - hv_0 \Rightarrow \frac{1}{2}mv_{max}^2 = h(v - v_0)$

So maximum velocity of photoelectrons: $\Rightarrow$ $v_{max} = \sqrt{\frac{2h(v - v_0)}{m}}$

m - mass of electron; v - frequency of incident light; v_0 - threshold frequency;

λ_0 - threshold wavelength λ - incident wavelength $\Rightarrow E_{max} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) \Rightarrow \frac{1}{2}mv_{max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

(b) If $v = v_0$ or $\lambda = \lambda_0$ then v = 0

(c) $v < v_0$ or $\lambda > \lambda_0 \Rightarrow$ There will be no emission of photoelectrons.

(d) Intensity of the radiation or incident light refers to the number of photons in the light beam. More intensity means more number of photons and vice-versa. Intensity has no bearing on the energy of photons. Therefore, intensity of the radiation is increased, the rate of emission increases but there will be no change in kinetic energy of electrons. With increasing number of emitted electrons, value of photoelectric current increases.

Illustration 3: A light beam of wavelength 4000 Å is directed on a metal whose work function is 2 eV. Calculate the maximum possible kinetic energy of the photoelectrons. **(JEE MAIN)**

Sol: According to photoelectric equation the maximum kinetic energy of photoelectron after being ejected from metal is $E_K = h\nu - \phi$

Energy of the incident photon = $\frac{hc}{\lambda}$. Energy of the incident photon in eV = $\frac{19.8\times10^{-19}}{4\times1.6\times10^{-19}}$ = 3.09 eV ;

Kinetic energy of the emitted electron $E_K = h\nu - \phi$ = 3.09 – 2.00 = 1.09 eV

Illustration 4: Calculate the maximum kinetic energy of photoelectrons emitted from a metal with a threshold wavelength of 5800 Å, if the wavelength of the incident light is 4500 Å. **(JEE ADVANCED)**

Sol: The maximum kinetic energy of photoelectron with which it is ejected from metal is $E_K = h\nu - \phi$.

Therefore maximum velocity of photoelectron is $v_{max} = \sqrt{\frac{2E_{K_{max}}}{m_e}}$

$$E_{k_{max}} = \frac{hc[\lambda_0 - \lambda]}{\lambda_0\lambda} = 6.62\times10^{-34}\times3\times10^{8}\,\frac{[5800\times10^{-10} - 4500\times10^{-10}]}{5800\times4500\times10^{-20}} = 9.9\times10^{-20}\ \text{J}$$

$$E_{k_{max}} = \frac{9.9\times10^{-20}}{1.6\times10^{-19}} = 0.62\ \text{eV};\ \Rightarrow\ v_{max} = \sqrt{\frac{2hc(\lambda_0-\lambda)}{m_e\lambda\lambda_0}} = \sqrt{\frac{2\times0.62\times1.6\times10^{-19}}{9.31\times10^{-31}}} = 4.67\times10^{5}\ \text{m/s}$$

3.6 Photoelectric Current

(a) When light/radiation is directed on a cathode, photoelectrons are emitted and these are attracted by an anode. The electric current, thus generated, flows in the circuit. This is called a photoelectric current.

(b) Value of photoelectric current depends upon following parameters:

(i) Potential difference between electrodes. **(ii)** Intensity of incident light.

3.6.1 Intensity of Light (I)

(a) It is quantity of light energy falling normally on a uniform surface area in unit time.

or $I = \frac{E}{A.t}$ where I = Intensity of light in $\frac{W}{m^2}$ E = total energy incident = nhv = $n\frac{hc}{\lambda}$

n = no. of photons; A = cross sectional area; T = time of exposure

(b) Intensity of light is proportional to saturation current

(c) For point source of light $I_r \propto \frac{I}{r^2}$

(d) For the Linear source of light $I_r \propto \frac{I}{r}$

Where r is the distance of the point from the light source.

3.7 Stopping Potential and Maximum Kinetic Energy

When the frequency f of the light/radiation is greater than the threshold frequency of the metal on which the light is directed, some photoelectrons are emitted from the metal with substantial initial speeds. Let us assume that E is the energy of light incident on a metal surface and W (<E) the work function of metal. In this case, as minimum energy is required to extract electrons from the surface, the emitted electrons will have the maximum kinetic energy which is E – W. $K_{max} = E - W$

As the potential V is increased, the electrons experience greater resistance/repulsion, and consequentially, less number of electrons reach the plate Q. This leads to a decrease in the flow of current in the circuit. At a certain value V_0, the electrons having maximum kinetic energy (K_{max}) also stop flowing and current in the circuit becomes zero. This is called the stopping potential.

(a) In photoelectric cell, when (+)ve voltage is applied on cathode and negative voltage is applied on anode applied, then the magnitude of photoelectric current decreases as the potential difference between the two points (cathode and anode) increases.

(b) The stopping potential is the negative potential (V_0) applied to the anode where the current gets reduced to zero or stops flowing in the circuit.

(c) When the magnitude of negative potential on anode is greater than or equal to magnitude of stopping potential the current in the circuit becomes zero.

(d) If emitted electrons do not reach from cathode to anode then stopping potential is given by

$$eV_0 = \frac{1}{2}mv_{max}^2 \text{ or } E_{max} = eV_0 \text{ ; } eV_0 = h(v - v_0) \text{ ; } V_0 = \frac{h(v - v_0)}{e}$$

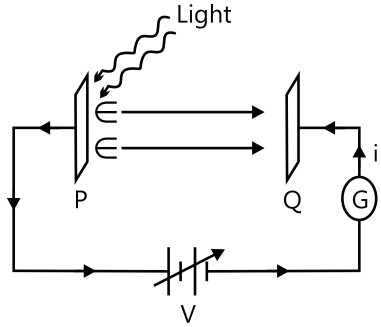


Figure 24.2: Photoelectric effect

(e) Value of stopping potential depends upon frequency of incident light.

(f) Stopping potential also depends upon nature of metal (or work function)

(g) Stopping potential does not depend upon intensity of light

(h) **Example**: Suppose stopping potential = -3 V, then $\frac{1}{2}mv_{max}^2 = 3eV$

If we apply – 5 V, then also there will be zero current in the circuit but $\frac{1}{2}mv_{max}^2 \neq 5eV$

Because stopping potential is not equal to 5V which cannot be used in Einstein's equation.

3.7.1 Graphs

(a) Kinetic energy V/s frequency: At $v = v_0$, $E_{max} = 0$

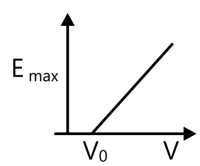


Figure 24.3

(b) V_{max} V/s v : At $v = v_0$ $V_{max} = 0$

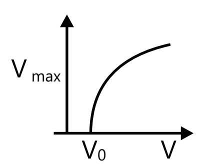


Figure 24.4

(c) Saturated Current V/s Intensity:

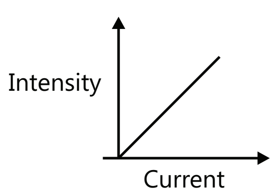


Figure 24.5

(d) Stopping potential V/s frequency:

$\because eV_0 = hv - hv_0$

$\tan\theta$ = slope

$= \frac{h}{e}$ (constant for all type of metals)

Intercept on x-axis = v_0

Intercept on y-axis = v

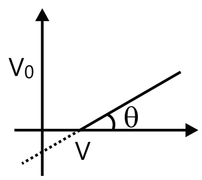


Figure 24.6

(e) Potential V/s current: (v : constant)

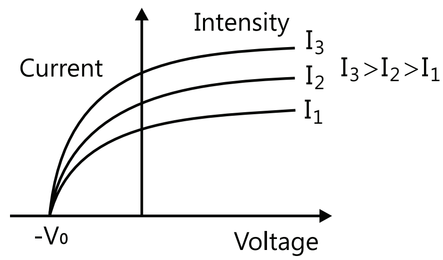


Figure 24.7

$\Rightarrow$ Stopping potential does not depend upon intensity of light.

(f) Photoelectric current V/s Retarding potential:

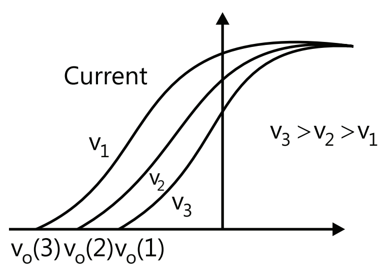


Figure 24.8

Illustration 5: Calculate the value of the stopping potential if one photon has 25 eV energy and the work function of material is 7 eV. **(JEE MAIN)**

Sol: The stopping potential required to stop the photoelectrons to reach cathode is $V_0 = \frac{E - \phi_0}{e}$

Stopping potential is $V_0 = \frac{E-\phi_0}{e} = \frac{25-7}{e} = \frac{18\text{ eV}}{e} \Rightarrow V_0 = 18\text{ V}$

3.8 Derivation of de-Broglie Wavelength

De broglie equation is given by: $\lambda = \frac{h}{p}$

Derivation: Let us start with the energy of a photon in terms of its frequency v, E = hv

Albert Einstein's special theory of relativity gives a new expression with reference to the velocity of light. This expression is $E = mc^2$, where m refers to the relativistic mass of light which is non-zero as it is travelling with velocity c. If it were at rest, it's mass would be zero.

Now, by equating both the energy equations we get $E = h\nu = mc^2$. Also, as seen earlier $\nu = \frac{c}{\lambda}$

Wavelength of a photon. $\therefore \frac{h}{\lambda} = mc$ and $\lambda = \frac{h}{mc}$

Analogously, de Broglie argued that a particle with non-zero rest mass m and velocity v would have a wavelength given $\lambda = \frac{h}{mv}$

Also, mv = p, where p is the particle's momentum. Substituting p for mv we get $\lambda = \frac{h}{p}$

3.8.1 Criterion for Type of Behaviour

Like electromagnetic waves, moving bodies also exhibit the wave-particle duality and the wave and particle aspects of moving bodies cannot be observed simultaneously. A moving body will exhibt particle behavior if the wavelength of the body is negligible in comparison to its dimensionwhereas it will exhibit a wave nature if its wavelength is in order of the dimension of body.

Illustration 6: Determine Broglie wavelengths of (a) a 46g golf ball with a velocity of 30 m/s and (b) an electron with a velocity of 10^7 m/s. **(JEE ADVANCED)**

Sol: The de-Broglie wavelength of the particle of mass m and moving with velocity v is given by $\lambda = \frac{h}{mv}$, where h is Planck's constant

(a) Since v<<c, we can let (effective mass = Rest mass) m =m_0 .

Hence $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}\,\text{J.s}}{(0.04\,\text{kg})(30\,\text{m/s})} = 4.8 \times 10^{-34}$ m.

Thus, we see that the wavelength of the golf ball is so negligible compared with its dimensions that we would not be able to observe expect to find any wave aspects in its behavior.

(b) Again v << c, so with m = m_0 = 9.1 × 10^{-31} kg,

we have $\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}\,\text{J.s}}{9.1 \times 10^{-31} \times 10^{7}\,(\text{kg m/s})} = 7.3 \times 10^{-11}\,\text{m} = 0.73\,\text{Å}$

The dimensions of atoms are comparable with the radius of the hydrogen atom which in reality is 5.93 × 10^{-11} m. So, it it is clear that an electron with a wavelength of 7.3 × 10^{-11} m would demonstrate a wave behavior. Also, we can see that the wave character of moving electrons is facilitates the understanding atomic structure and behavior.

4. ENERGY, MOMENTUM, AND WAVELENGTH OF PHOTONS

(a) The quantum theory states that the light photons are undivided energy packets.

(b) Energy of photons is denoted by E = hv, where h is Planck constant, v is the frequency of photons, and E is the energy of photons.

(c) The velocity of photons and the velocity of light are equal (c). Therefore, c = $v\lambda$

Here, λ is the wavelength of wave connected to photon. ∴ $E = hv = \frac{hc}{\lambda}$

(d) The mass of photons at rest is zero but it will be non-zero if the photons are moving.. Assuming m to be the effective mass of photons, energy of photon according to Einstein:

$E = mc^2 \Rightarrow E = hv = \frac{hc}{\lambda} = mc^2$

(e) Momentum of moving photon $p = mc = \frac{mc^2}{c} = \frac{E}{c} = \frac{1}{c}\left(\frac{hc}{\lambda}\right) = \frac{h}{\lambda} \Rightarrow p = \frac{h}{\lambda}$

(f) Effective mass $m = \frac{E}{c^2} = \frac{hv}{c^2} = \frac{h}{c\lambda} = \frac{p}{c}$

(g) Wavelength connected to moving photons $\lambda = \frac{h}{p} = \frac{h}{mc} = \frac{hc}{E}$

(h) From Point (e) and (f):- Momentum of photon $p \propto m$ $p \propto E$ Energy of photons $E \propto m$

Wavelength of wave connected to photons $\lambda \propto \frac{1}{p}$; $\lambda \propto \frac{1}{m}$; $\lambda \propto \frac{1}{E}$

(i) Graphs

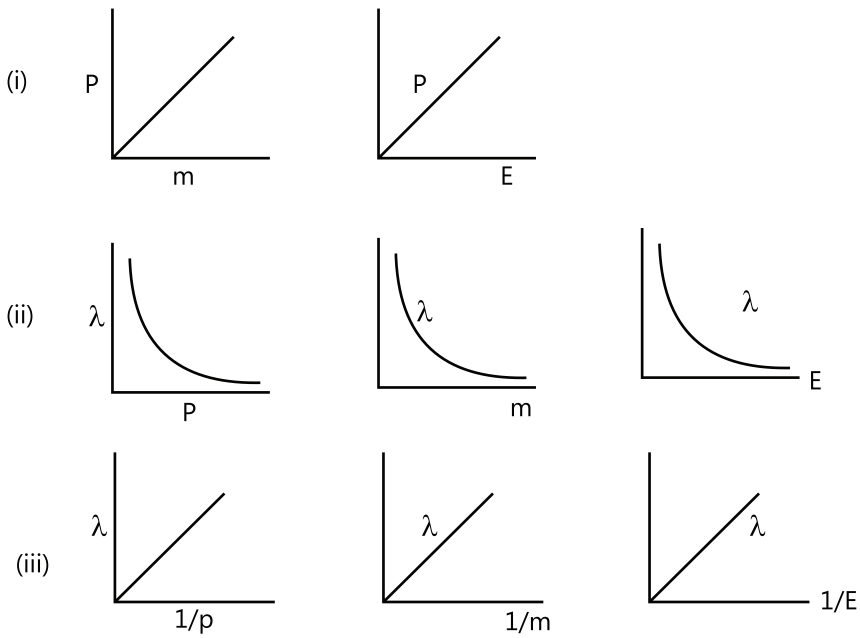


Figure 24.9

(j) There is no charge on photons

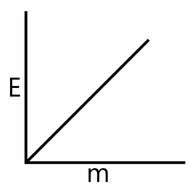


Figure 24.10

Illustration 7: A Determine the velocity of a light wave, given that frequency of the photon is ν, energy is $h\nu$, and momentum is $p = \frac{h}{\lambda}$ **(JEE MAIN)**

Sol: For light of frequency ν, the energy is $E = h\nu$ and the frequency of light wave is $\nu = \frac{c}{\lambda}$. Hence speed of light is easily determined.

As $E = h\nu$ and $P = \frac{h}{\lambda}$,

$$E = \frac{hc}{\lambda} = Pc \Rightarrow c = \frac{E}{P}$$

Illustration 8: Determine the mass of a photon witha wavelength of $0.01\ \mathring{A}$. **(JEE MAIN)**

Sol: Using equation of equivalent mass of photon, $m = \frac{h}{c\lambda}$, we can find the mass of proton.

$$m = \frac{E}{c^2} = \frac{h}{c\lambda} = \frac{6.62 \times 10^{-34}}{3 \times 10^8 \times 10^{-12}}\ ;\ m = 2.21 \times 10^{-30}\ \text{kg}$$

Illustration 9: Determine the momentum of a photon with a of frequency 10^9 Hz. **(JEE MAIN)**

Sol: The momentum of photon is $p = \frac{h\nu}{c}$

$$p = \frac{h}{\lambda} = \frac{h\nu}{c} = \frac{6.62\times10^{-34}\times10^{9}}{3\times10^{8}}\ ;\ p = 2.2\times10^{-33}\ \text{kg m/s}$$

Illustration 10: Determine the energy and momentum of a γ-ray photon with a wavelength of 0.01 Å. **(JEE MAIN)**

Sol: For wave of wavelength λ the energy and momentum is given by $E = \frac{hc}{\lambda}$ and $p = \frac{h}{\lambda} = \frac{E}{c}$

$$E = \frac{hc}{\lambda} = \frac{1240(\text{eV})\times1\times10^{-9}}{0.01\text{Å}}\ ;$$

$$E = \frac{1240\times10^{-9}}{10^{-2}\times10^{-10}}(\text{eV}) = 1.24\times10^{6}\ \text{eV}\ ;\ E = 1.24\ \text{MeV}\ ;\ \text{The momentum is } P = \frac{E}{c} = 1.24\ \frac{\text{MeV}}{c}$$

$$P = \frac{1.24\times10^{6}\times1.6\times10^{-19}}{3\times10^{8}} = 6.62\times10^{-22}\ \text{kg m/s}$$

5. ENERGY, MOMENTUM, AND WAVELENGTH OF A MOVING PARTICLE

Suppose the mass of a particle at rest is m and it is moving with velocity v.

(a) Mass at rest = m

(b) Effective mass (or relativistic mass) $= \sqrt{\frac{m}{1-v^2/c^2}}$

(c) Momentum or $\vec{p} = m\vec{v}$ or p = mv $\therefore$ $p = mv = \sqrt{2mE}$ Here E : Kinetic energy

(d) Kinetic energy $E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

(e) If λ is the wavelength of connected wave to the moving particle, then $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

Illustration 11: A body of 10 gm is moving with velocity 2×10^{3} m/s. Determine the value of its associated de-Broglie wavelength. **(JEE ADVANCED)**

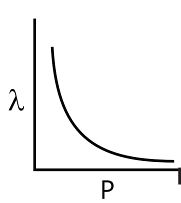
Sol: The de-Broglie wavelength associated with particle moving with speed v is calculated as $\lambda = \frac{h}{mv}$.

de-Broglie wavelength $\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{10\times10^{-3}\times2\times10^{3}}$;

$\lambda = 3.3\times10^{-35}$ m

So $\lambda \propto \frac{1}{p}$; $\lambda \propto \frac{1}{\sqrt{E}}$

$\lambda \propto \frac{1}{v}$ $\lambda \propto \frac{1}{\sqrt{E}}$



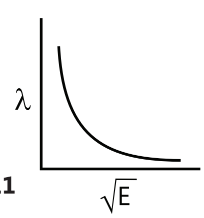
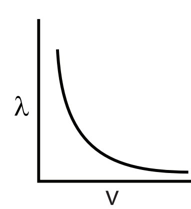


Figure 24.11



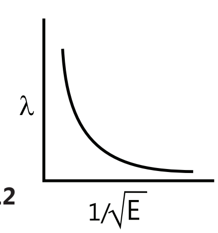
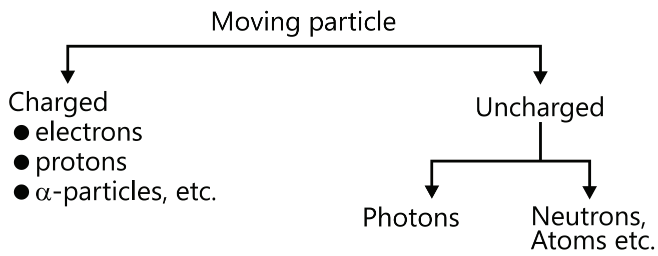


Figure 24.12



5.1 Energy, Momentum, and Wavelength of Charged Particle Accelerated by V-volt

(a) Potential difference or electric field can be used to accelerate a charged.

(b) The kinetic energy of a charged particle having charge q, mass m, accelerated by V volt, and a velocity v is denoted by $E = \frac{1}{2}mv^2 = qV$

(c) Velocity V = $\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2E}{m}}$

(d) Momentum p = $\sqrt{2mE} = \sqrt{2mqV}$

(e) Wavelength $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$

(Here it is assumed that initial potential given to electron is zero)

If the particle is given some initial potential V_i and if final potential is V_f then, $\lambda = \frac{h}{\sqrt{2mq(V_f - V_i)}}$

From above Relation

$\lambda \propto \frac{1}{\sqrt{V}}$

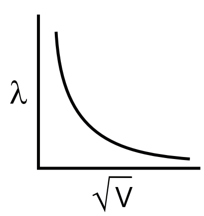


Figure 24.13

$\lambda^2 \propto \frac{1}{V}$

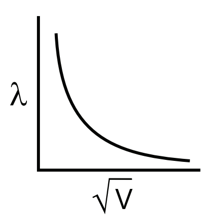


Figure 24.14

$\lambda \propto \frac{1}{\sqrt{V}}$

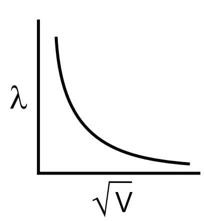


Figure 24.15

Cases:

(a) If the moving charged particle is an electron, then

(i) $v_e = \sqrt{\frac{2eV}{m_e}}$ **(ii)** $p_e = \sqrt{2\,m_e eV}$ **(iii)** $\lambda_e = \frac{h}{\sqrt{2m_e eV}} = \frac{12.27}{\sqrt{V}}$ Å

(b) If the moving charged particle is a proton, then

(i) $v_P = \sqrt{\frac{2eV}{m_p}}$ **(ii)** $p_P = \sqrt{2m_e pV}$ **(iii)** $\lambda_P = \frac{h}{\sqrt{2m_e pV}} = \frac{0.286}{\sqrt{V}}$ Å

(c) If the charged particle is an α-particle, then

(i) $v_\alpha = \sqrt{\frac{2(2e)V}{m_\alpha}} = \frac{eV}{m_P} = \frac{1}{\sqrt{2}} v_P$ **(ii)** $P_\alpha = \sqrt{2m_\alpha e_\alpha V} = \sqrt{2 \times 8m_p \times eV} = 2\sqrt{2}\,p_p$

(iii) $\lambda_\alpha = \frac{h}{\sqrt{16m_p eV}} = \frac{0.101}{\sqrt{V}}$ Å

Illustration 12: Determine the potential to be applied to accelerate an electron such that its de-Broglie wavelength becomes 0.4 Å. **(JEE MAIN)**

Sol: The de-Broglie wavelength of an electron in terms of accelerating potential difference is $\lambda_e = \frac{12.27}{\sqrt{V_0}}$ Å

Where V is the applied potential on electron to accelerate it. $\lambda = \frac{12.27}{\sqrt{V_0}}$ Å ; $0.4 = \frac{12.27}{\sqrt{V_0}}$

Squaring on both the sides we get $0.16 = \frac{(12.27)^2}{V_0} \Rightarrow V_0 = \frac{12.27 \times 12.27}{16 \times 10^{-2}}$;$\Rightarrow V_0 = 941.0$ V

5.2 Wavelength of Wave Connected to Uncharged Particle

(Like neutron atoms, molecules etc.)

(a) If m is the mass and v is the velocity of particle, then kinetic energy $E = \frac{1}{2}mv^2$, momentum p = mv

(b) Wavelength $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$

(c) If λ is the wavelength of wave connected to matter particle then particle energy E will be

$$E = \frac{h^2}{2m\lambda^2}\ J = \frac{h^2}{(2m\lambda^2)e} eV$$

(d) Energy E of particle (e.g., electron, neutron, or atom) at equilibrium temperature TE = (3/2) KT

Here K = Boltzman constant

(e) $\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mKT}}$ here m : mass of a single atom.

Illustration 13: Determine the associated de-Broglie wavelength if the energy of a thermal neutron is 0.02 eV, **(JEE MAIN)**

Sol: For neutron having kinetic energy K, the associated de-Broglie wavelength is found to be $\lambda = \frac{h}{\sqrt{2mK}}$

de-Broglie Wavelength $\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 0.02 \times 1.6 \times 10^{-19}}}$; $\lambda = 2 \times 10^{-10} m = 2Å$

6. EXPERIMENTAL VERIFICATION OF MATTER WAVES

The Davisson–Germer experiment conducted by American physicists Clinton Davisson and Lester Germer confirmed the De Broglie hypothesis which says that the particles of matter such as electrons have wave-like properties (see Fig. 24.16).

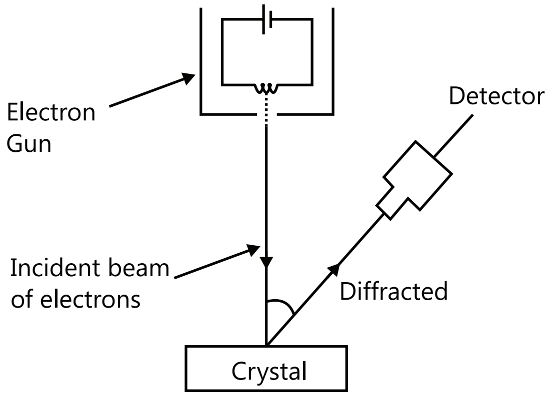


Figure 24.16: Diffraction of matter waves

(a) Davisson–Germer's experiment

(i) Experimental confirmation of De Broglie waves was done by scientist Davisson and Germer by firing slow-moving electrons at a crystalline nickel target. The diffraction pattern of electron beam through the nickel crystal was same as those predicted by Bragg for X-rays.

(ii) Since diffraction is the property of waves, the diffraction of electronic beam confirmed that a wave is connected to moving electron beam.

(iii) The electron gun was used to obtain electrons with different energies. This was done by accelerating the e^- by V volt in the electron gun.

(iv) When these accelerated electrons fall on a crystal, they are diffracted in various directions.

(v) Electrons were collected by a detector of Faraday cup which was connected to an electrometer.

(vi) Electron beams with different energies produced different intensities of diffracted electrons.

(vii) Results of the Davisson–Germer's experiment

- Intensity at any angle is proportional to the distance of the curve at that angle from the point of scattering.

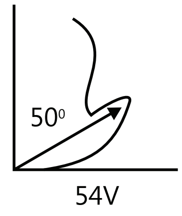


Figure 24.17

- Intensity is maximum at 54 V potential difference and 50° diffraction angle.

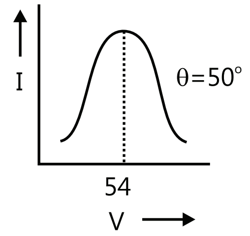


Figure 24.18

(viii) From Bragg's Law :- $D\sin\theta = n\lambda$ (constructive interference)

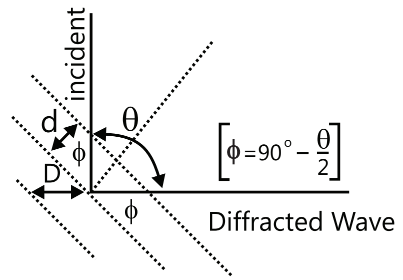


Figure 24.19

θ : Angle between incident ray and nth maxima.

n: Diffraction order

D: Distance between atoms or 2d $\sin\phi = n\lambda$

D: Distance between lattice planes

ϕ : Angle between diffraction plane and incident ray.

(ix) The critical value of wavelength of an accelerated electron at 54 V = 1.67Å

Experimental value = 1.65 Å

(x) Any wave or particle is diffracted by crystal plane only when wavelength is in order of distance between lattice planes of an atom.

Illustration 14: In a Davisson-Germer experiment, a, electron beam of wavelength 1.5 Å is normally incident on a crystal, having 3Å distance between atoms. Determine the angle at which first maximum occurs. **(JEE MAIN)**

Sol: According to Davison–Germer's experiment, when electrons accelerating at some potential difference V are incident on a crystal, they diffract. The angle at which the first maxima of diffraction pattern occurs can be found by Bragg's law i.e., $D\sin\theta = n\lambda$

$$D\sin\theta = n\lambda \;\therefore\; \sin\theta = \frac{n\lambda}{D} = \frac{1\times 1.5}{3} = \frac{1}{2},\ \theta = 30^\circ$$

7. ATOMIC MODELS

Model : A model is simply a testable idea or hypothesis based on logical and scientific facts.

Theory : A model becomes a theory when it is verified by rigorous scientific analysis and experiments. . Otherwise, the model is simply not accepted.

7.1 Dalton's Atomic Model

(a) All matter is made of tiny particles called atoms. Atoms are indivisible and indestructible.

(b) All atoms of a given element are identical in mass and properties, while atoms of different elements differ in mass and properties.

(c) All matter is made up of hydrogen atoms. The mass and radius of heaviest atom is about 250X and 10X of the than that of the hydrogen atom, respectively.

(d) Atoms are stable and electrically neutral.

Reason of Failure of model: The discovery of electron by J.J. Thomson (1897) proved that atoms are not indivisible. Hence, the model is no longer valid.

7.2 Thomson's Atomic Model (or Plum-Pudding Model)

In this model, the atom is composed of electrons (which Thomson still called "corpuscles") surrounded by a soup of positive charge to balance the electrons' negative charges, like negatively charged "raisins" surrounded by positively charged "pudding".

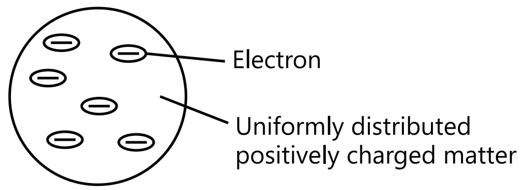


Figure 24.20: Thomson's Atomic Model

Achievements of model: Explained successfully the phenomenon of thermionic emission, photoelectric emission, and ionization.

Failure of the model:

(a) It could not explain the line spectrum of H-atom.

(b) It could not explain the Rutherford's α – particle scattering (Rutherford gold foil) experiment.

7.3 Rutherford's Experiment and Atomic Model

α-Scattering Experiment:

Results of Experiment:

(a) It was seen that in the experiment that when the α - particles were fired at the gold foil, some of the particles (<1 in 8000) bounced off the metal foil in all directions, some right back at the source. This should have been impossible according to Thomson's model; the alpha particles should have all gone straight through. Obviously, those particles had encountered an electrostatic force far greater than Thomson's model suggested they would, which in turn implied that the atom's positive charge was concentrated in a much tinier volume than Thomson imagined. This was possible only in the case when there exists a solid positive mass confining in a very narrow space.

(b) However, most of the α - particles just flew straight through the foil. This suggested that those tiny spheres of intense positive charge were separated by vast gulfs of empty space.

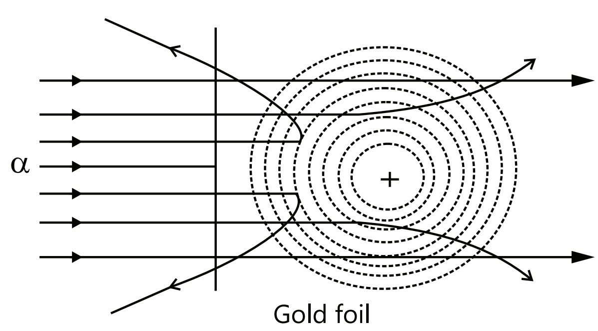


Figure 24.21: Scattering of alpha particles by gold nucleus

(c) $N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} \Rightarrow$ If $\theta\uparrow$ then $N\downarrow$, N = No. of particles scattered per unit time

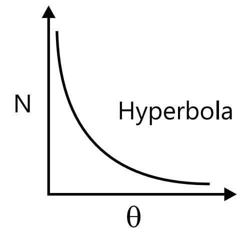


Figure 24.22

Equation indicates that at larger deflection (scattering) angle, number of particles deflected are very-very less.

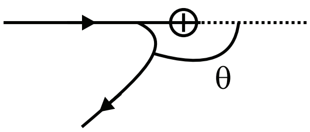


Figure 24.23

Graph for N & θ show that coulomb's law holds for atomic distances also.

(d) $N \propto$ (Nuclear charge)^2

Illustration 15: In an α - particle scattering experiment using gold foil, the number of particles scattered at 60° is 1000 per minute. What will be the number of particles per minute scattered at 90° angle? **(JEE ADVANCED)**

Sol: In Rutherford's experiment, the number of particles deflected at an angle θ by the gold atoms per minute are best represented by relation $N \propto \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$

Let N = No. of α - particles scattered per minute at an angle 90°.

$$\therefore \quad N \propto \frac{1}{\sin^4\left(\frac{90}{2}\right)} \qquad \text{... (i)}$$

Given that $1000 \propto \frac{1}{\sin^4\left(\frac{60}{2}\right)}$ ... (ii)

Taking ratio of (i) to (ii) we get $N = 1000 \times \frac{\sin^4\left(\frac{60}{2}\right)}{\sin^4\left(\frac{90}{2}\right)} = 250$ /min

Rutherford's Atomic Model

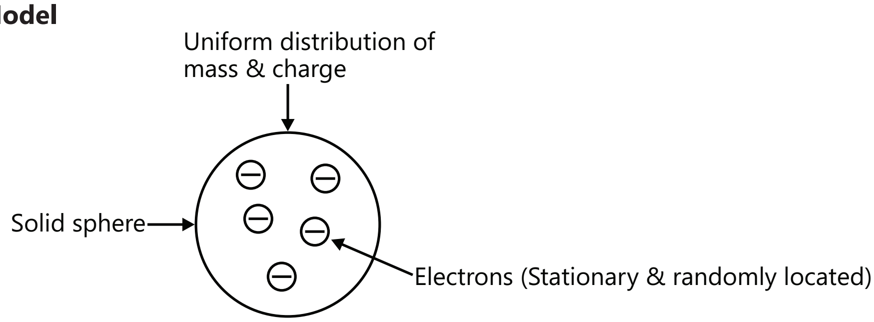


Figure 24.24: Thomson's Atomic Model

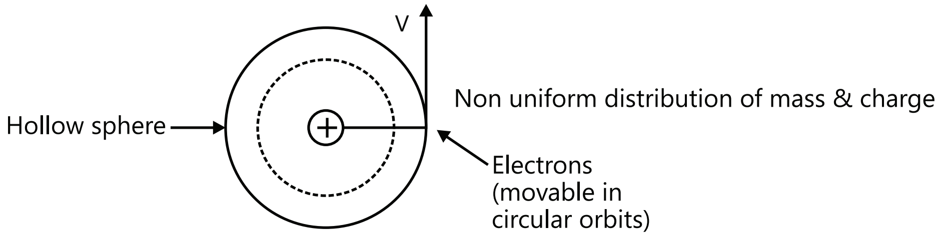


Figure 24.25: Ruthorford's Atomic Model

(a) The whole positive charge and almost whole mass of an atom (leaving aside the mass of revolving e^- in various circular orbits) remains concentrated in nucleus of radius of the order of 10^{-15} m.

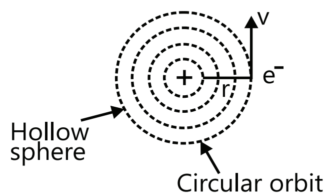


Figure 24.26: Motion of electron in atom

(b) $\Sigma q(+)ve$ on proton in a nucleus = $\Sigma q(-)ve$ on e^- in various circular orbits & hence, the atom is electrically neutral.

(c) The necessary centripetal force for revolving round the nucleus in circular orbit is provided by coulomb's electrostatic force of attraction $\frac{mv^2}{r} = \frac{k(ze)(e)}{r^2}$

Reason of failure of model

(a) It could not explain the line spectrum of H-atom.

Justification: Asper Maxwell's electromagnetic theory, every accelerated moving charged particle emits energy in the form of electromagnetic waves and, therefore, the frequency of an Modern Physics - Solution (1) while moving in a circular orbit around the nucleas will steadily decline, resulting in the continuous emission of lines thereby mandating that the spectrum of an atom be continuous, but in reality, one obtains line spectrum for atoms.

(b) It could not explain the stability of atoms.

Justification : Since revolving electron continuously radiates energy, the radii of circular path will continuously decrease and in a time of about 10^{-8} s the revolving electron must fall down in a nucleus by adopting a spiral path.

Application of Rutherford's model

Determintion of distance of closest approach: When a positively charged particle approaches a stationary nucleus (which is the positively charged core of the atom), then due to repulsion between the two (like charges repel), the kinetic energy of positively charged particle gradually decreases, reaching a stage where its kinetic energy becomes zero and from where it again starts retracing its original path.

Definition: The distance of closest approach is the minimum distance of a stationary nucleus from a point where the kinetic energy of a positively charged particle approaching the nucleas for a head-on collision becomes zero. Suppose a positively charged particle A of charge q_1 ($=z_1e$) approaches from infinity towards a stationary nucleus of charge z_2e then,

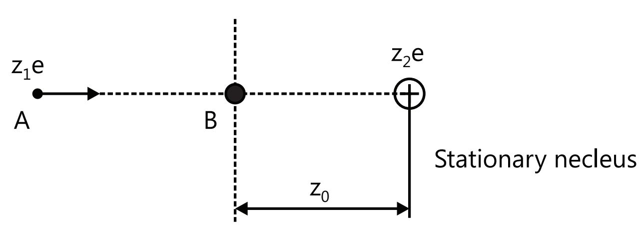


Figure 24.27: Distance of closest approach

Let at point B, kinetic energy of particle A becomes zero then by the law of conservation of energy at point A & B.

$TE_A = TE_B$; $KE_A + PE_A = KE_B + PE_B$; $E + 0 = 0 + \frac{k(z_1e)(z_2e)}{r_0}$ (in joule) $\therefore\ r_0 = \frac{k(z_1e)(z_2e)}{E}$ m

Illustration 16: Calculate the distance of closest approach where an α-particle with kinetic energy 10 MeV is heading towards a stationary point-nucleus of atomic number 50. **(JEE MAIN)**

Sol: The nucleus of tin (atomic number 50) being more massive than the alpha particle, remains stationary. So the kinetic energy of the alpha particle is converted into electric potential energy at the distance of closest approach.

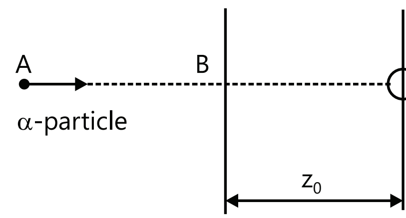


Figure 24.28

The electric potential energy of alpha particle is $TE_\alpha = \frac{K \times (Z_1e) \times (Z_2e)}{r_0}$

where $K = \frac{1}{4\pi\varepsilon_0}$ and r_0 is the distance of closest approach of alpha particle from nucleus of tin.

$TE_A = TE_B$; $\therefore\ 10 \times 10^6\,eV = \frac{K \times (2e)(50e)}{r_0}$

$r_0 = 1.44 \times 10^{-14}\,m$; $r_0 = 1.44 \times 10^{-4}\,Å$

Illustration 17: Find the distance of closest approach for a proton moving with a speed of 7.45×10^5 m/s towards a free proton originally at rest. **(JEE MAIN)**

Sol: As the moving proton approaches the free proton originally at rest, it exerts an electric force of repulsion on the proton at rest. At the distance of closest approach, both the protons move with same velocity along the line of impact. The initial kinetic energy of moving proton is equal to the final kinetic energy of both the protons plus the electric potential energy at the distance of closest approach, given by $\frac{Ke^2}{r_0}$. Here r_0 is the distance of closest approach.

$\rightarrow V = 7.45 \times 10^5$ m/s u = 0

O Proton O Free proton

Originally

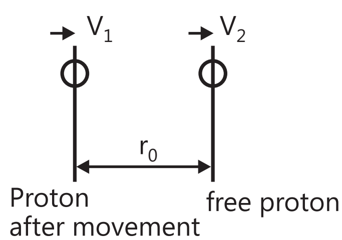


Figure 24.29

At the time of distance of closest approach

By the law conservation of energy

$$\frac{1}{2}mv^2 + 0 = \frac{ke^2}{r_0} + \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 \quad \text{... (i)}$$

By the conservation of momentum $mv + 0 = mv_1 + mv_1$ $\therefore\ v_1 = \frac{v}{2}$

From equation (i) $\frac{1}{2}mv^2 = \frac{ke^2}{r_0} + m\left(\frac{v}{2}\right)^2$; $r_0 = \frac{4}{mv^2} \times ke^2 = \frac{4 \times (9 \times 10^9)(1.6 \times 10^{-19})^2}{(1.66 \times 10^{-27})(7.45 \times 10^5)^2}$ $r_0 = 1.0 \times 10^{-12}\,m$

7.4 Bohr's Model

Bohr combined the concepts of classical physics with quantum mechanics to propose his model for H or H-like atoms. This model is based on law of conservation of angular momentum.

(a) According to de Broglie, in a stationary orbit the circumference of Bohr's orbit must be an integral multiple of the wavelength associated with the moving particle

or $2\pi r = n\lambda$ (Constructive interference)

or $2\pi r = \frac{nh}{mv}$ or $mvr = \frac{nh}{2\pi}$ which is Bohr's quantum condition.

(b) In an orbit, waves are always formed in whole numbers..

7.4.1 Concept of Stable, Stationary, Quantized, Fixed Allowed Radii Orbit, or Maxwell's Licensed Orbits

According to Bohr, if an electron revolves in these orbits the electron neither radiates nor absorbs any energy.

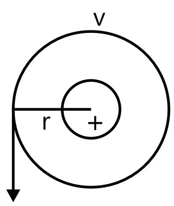


Figure 24.30: Bohr radius

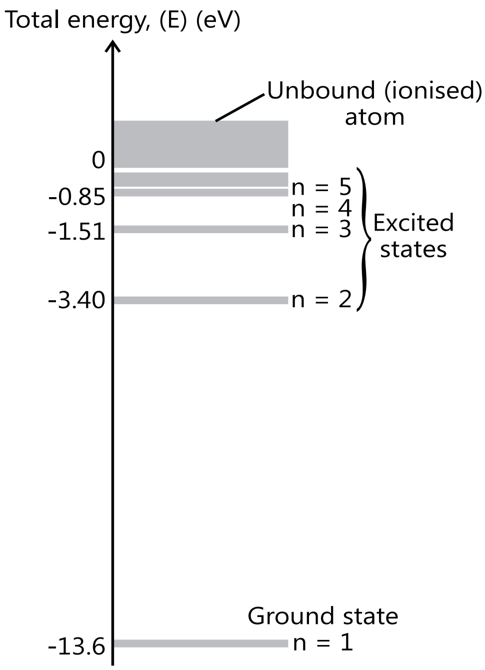


Figure 24.31: Energy level diagram

(b) **Emission of energy**

Where n = principle quantum no.

E_n = energy of e^- in nth orbit

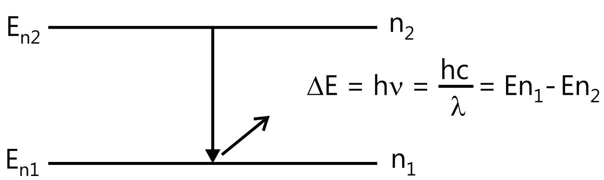


Figure 24.32: Emission of energy by electron

(c) Absorption of energy

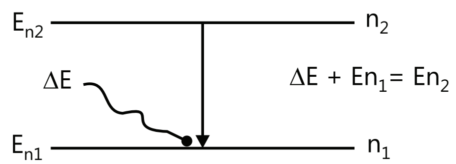


Figure 24.33: Absorption of energy by electron

Electron revolves only in those orbits in which its angular momentum is integer multiple of $\frac{h}{2\pi}$

$$mvr = I\omega = n\frac{h}{2\pi}$$

$$\frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

7.4.2 Determination of Radius, Velocity & Energy of e⁻ in Bohr's Orbit

(a) Determination of radius of circular path (orbit)

$$\because mvr = \frac{nh}{2\pi} \quad \text{... (i)}$$

$$\therefore v = \frac{nh}{2\pi mr} \quad \text{... (ii)}$$

and $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$; $\quad \therefore m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{kZe^2}{r}$; $\quad r_n = v\left(\frac{n^2h^2}{4\pi^2 mkZe^2}\right)$; $\quad r_n = \frac{n^2}{Z} \times \frac{h^2}{4\pi^2 mke^2}$

$$r_n = \frac{n^2}{Z} \times 0.529\ \overset{\circ}{A}$$

Results:

(i) $\because \quad r_1 = \frac{(1)^2}{z} \times 0.529\ \text{Å}$; $\therefore \ r_n = n^2 r_1$

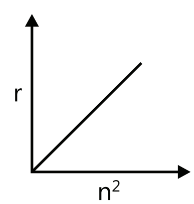


Figure 24.34

(ii) $\therefore \ r \propto n^2$

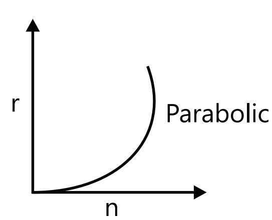


Figure 24.35

Illustration 18: The radius of the shortest orbit of a single-electron system is 18 pm. This system can be represented as **(JEE MAIN)**

Sol: According to Bohr's model, the radius of orbit of electron is directly proportional to square of principle quantum number i.e., $r_n \propto n^2$. When the electron is in ground state (i.e., for principle quantum number =1) $r_1 = \frac{0.529}{Z}\overset{\circ}{A}$

For shortest orbit n = 1; $r_n = n^2 r_1$; $\frac{(1)^2}{Z} \times 0.529$ Å = 18×10^{-2} Å

$\Rightarrow Z = 3$ system is Li^{2+} since only single e is present.

Illustration 19: What will be the ratio of the area of circular orbits in doubly ionized lithium atom in 2nd & 3rd Bohr orbit? **(JEE MAIN)**

Sol: According to Bohr's theory as $r_n \propto n^2$, but $A \propto r^2$. Therefore $A \propto n^4$.

$$\therefore \frac{A_2}{A_3} = \frac{(2)^4}{(3)^4} = \frac{16}{81}$$

7.4.3 Determination of Velocity of Electron in Circular Orbit

$$\therefore mvr = \frac{nh}{2\pi} \qquad ...(i)$$

$$r = \frac{nh}{2\pi mv}; \Rightarrow \frac{mv^2}{r} = \frac{kZe^2}{r^2} \Rightarrow v = \frac{2\pi kZe^2}{nh}; \Rightarrow \quad v = \frac{Z}{n} \times \frac{2\pi ke^2}{h} \Rightarrow v = 2.18 \times 10^6 \frac{Z}{n} \text{ m/s}$$

$v = \frac{c}{137}\frac{Z}{n}$ m/s; where c = velocity of light in vacuum = 3×10^8 m/s

Results:

(i) $v \propto \frac{1}{n}$ (Z = constant)

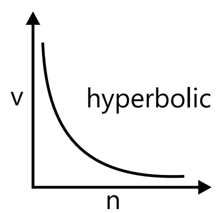


Figure 24.36

Illustration 20: What will be the ratio of speed of electrons in hydrogen atom in its 3rd & 4th orbit? **(JEE MAIN)**

Sol: According to the Bohr's theory $v \propto \frac{Z}{n}$ where v is the speed of electron in its orbit, n is the principle quantum number and Z is the atomic number of the element.

$$\because \; v \propto \frac{Z}{n} \quad \therefore \frac{v_3}{v_4} = \frac{4}{3}$$

Illustration 21: What will be the the ratio of speed of electron in 3rd orbit of He^+ to 4th orbit of Li^{++} atom? **(JEE MAIN)**

Sol: According to the Bohr's theory $v \propto \frac{Z}{n}$, where v is the speed of electron in its orbit n is the principle quantum number and Z is the atomic number of the element.

Here the element in consideration differs in atomic number, i.e., Z(He) = 2 and Z (Li) =3

$$\therefore \frac{(v_3)_{He^+}}{(v_4)_{Li^{2+}}} = \frac{\frac{2}{3}}{\frac{3}{4}} = \frac{8}{9}.$$

7.4.4 Determination of Energy of Electron in Bohr's Circular Orbit

(a) Kinetic energy of electron $KE = \frac{1}{2}mv^2$ **;** $KE = \frac{kZe^2}{2r}$

Results:

(i) KE of an e^- = positive quantity

(ii) $r\uparrow$, $KE\downarrow$

(iii) when, $r = \infty$, KE = 0

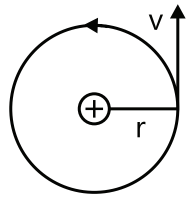


Figure 24.37: Bohr's Orbit

(b) Potential energy of an electron $PE = \frac{K(+Ze)(-e)}{r}$; $PE = -\frac{KZe^2}{r}$

Results:

(i) Potential energy (PE) of an e^- = negative quantity **(ii)** $r\uparrow$, $PE\uparrow$ (c) If $r = \infty$, PE = 0

(c) Total energy of electron: The total energy of an electron in any orbit equals the sum of its kinetic and potential energy in that orbit. TE = KE + PE = $\frac{KZe^2}{2r} - \frac{KZe^2}{r}$; $TE = -\frac{KZe^2}{2r}$

Results:

(i) TE of an electron in atom = (-)ve quantity. (-)ve sign indicates that electron is in bound state.

(ii) If $r\uparrow$, $TE\uparrow$

(iii) if $r = \infty$, TE = 0

(iv) TE = -KE = $\frac{PE}{2}$ in any H-like atom

Total energy of terms of n

$$TE = -\frac{kZe^2}{2\times\left(\frac{n^2h^2}{4\pi^2mkZe^2}\right)}$$

$$TE = -\frac{2\pi^2mk^2Z^2e^4}{n^2h^2} \Rightarrow TE = -R\,ch\,\frac{Z^2}{n^2} \Rightarrow TE = -13.6\frac{Z^2}{n^2}ev$$

where R = Rydberg constant = $\frac{2\pi^2mk^2e^4}{ch^3} = \frac{me^4}{8\epsilon_0^2\,ch^3} = 1.097\times10^7\ m^{-1}$

Note: Rydberg constant is not a universal constant. In Bohr calculation, it is determined by assuming the nucleus to be stationary

For Bohr Rydberg constant, $R_\infty = 1.097\times10^7 m^{-1}$, if nucleus is not assumed stationary then

$R = \frac{R}{1+\left(\frac{m_e}{m_N}\right)}$, m_N = mass of nucleus

7.4.5 Results Based on Total Energy Equation

(a) With the increase in principal quantum number n (relative overall energy of each orbital), both total energy and potential energy of an electron increases, whereas the kinetic energy decreases.

(b) With the increase in principal quantum number, the difference between any two consecutive energy level decreases.

(c) Total energy of an electron in any orbit in H-like atom = (Total energy of an electron in that orbit in H-atom $\times Z^2$)

(d) PE of an electron in any orbit in H-like atom = (PE of an electron in that orbit in H-atom) $\times Z^2$ (v) KE of an electron in any orbit in H-like atom= (KE of an electron in that orbit in H-atom) $\times Z^2$ (vi) $\Delta E_{n_1 n_2}$ in any H-like atom= ($\Delta E_{n_1 n_2}$ in H-atom) $\times Z^2$

7.4.6 Success of Bohr's Theory

(a) Bohr successfully combined Rutherford's model with the Planck hypothesis on the quantified energy states at atomic level

(b) Bohr's theory explained the atomic emission and absorption spectra

(c) It explained the general characteristics of the periodic table

(d) Bohr's theory offered the first "working" model for the atom

7.4.7 Short Coming of Bohr's Model

(a) Bohr's model holds true only for atoms with one electron. E.g, H, He^{+}, Li^{+2}, Na^{+1}

(b) Bohr's model posits circular orbits whereas according to Somerfield these are elliptical.

(c) The model could not explain the intensity of spectral lines.

(d) It assume the nucleus to be stationary, but it also rotates on its own axis.

(e) It failed to account for the minute structure in spectrum line.

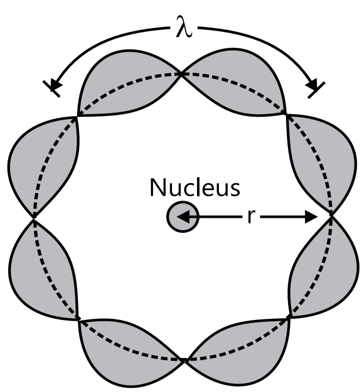
(f) The model offered no explanation for the Zeeman effect (splitting up of spectral lines in magnetic field) and Stark effect (splitting up of spectral lines in electric field)

(g) Doublets observed in the spectrum of some of the atoms like sodium (5890 Å & 5896 Å) could not be explained by Bohr's model.

7.4.8 de Broglie's Explanation of Bohr's Second Postulate of Quantization

In Bohr's model of the atom, it is stated that the angular momentum of the electron orbiting around the nucleus is quantized (that is, $L_n = \frac{nh}{2\pi}$; n = 1, 2, 3,.....). Why is it that the values of angular momentum are only integral multiples of $\frac{h}{2\pi}$

De Broglie, speculated that nature did not single out light as being the only matter which exhibits a wave-particle duality. He proposed that ordinary ``particles" such as electrons, protons, or bowling balls could also exhibit wave characteristics in certain circumstances.C.J. Davisson and L.H. German later experimentally verified the wave nature of electron in 1927. It was De Broglie's contention (like Bohr) was that an electron in motion around the nucleas must be seen as a particle wave. Analogous to waves travelling on a string, particle waves too can lead to standing waves under resonant conditions.



A standing wave is shown in a circular orbit where four de Broglie wavelength fit into the circumference of the orbit.

Figure 24.38: De broglie model

We know that when a string is peturbed, it generates a number of wavelengths along the length of the string. Of these, only those wavelengths that have nodes at either ends and form standing waves survive, while other wavelengths get reflected upon themselves resulting in their amplitudes quickly dropping to zero. Therefore, standing waves are formed when a wave travels the along the enrire length of the string and back in one, two, or any integral number of wavelengths. For an electron moving in nth circular orbit of radius r_n, the total distance is the circumference of the orbit, $2\pi r_n$.

Thus, $2\pi r_n = n\lambda$, n = 1, 2, 3 .... We have, $\lambda = \frac{h}{p}$, where p is the magnitude of the electron's momentum.

If the speed of the electron is much less than the speed of light, the momentum is mv_n.

Thus, $\lambda = \frac{h}{mv_n}$.; $2\pi r_n = \frac{nh}{mv_n}$ or $mv_n r_n = \frac{nh}{2\pi}$

This is the quantum condition proposed by Bohr for the angular momentum of the electron. Thus de Broglie hypothesis provided an explanation for Bohr's second postulate for the quantization of angular momentum of the orbiting electron by postulating the wave nature of matter particles like electrons. The quantized electron orbits and energy states are due to the wave nature of the electron and only resonant standing waves can persist.

7.4.9 Limitations

(a) The Bohr model is applicable to hydrogenic atoms with a single electron. All attempts to use Bohr's Model to analyze atoms with more than one electron failed as Bohr's model deals only with interaction between the electron and the positively charged nucleus but does not account for the interaction of an electron with other electrons as would be the case with multi-electron atoms.(ii) While the Bohr's model correctly predicts the frequencies of the light emitted by hydrogenic atoms, it cannot predict the relative intensities of spectral lines. Some frequencies in the hydrogen emission spectrum, for example, have weak intensity while others have strong intensity. Bohr's model is unable to account for the intensity variations.

7.4.10 Some Important Definitions and their Meaning

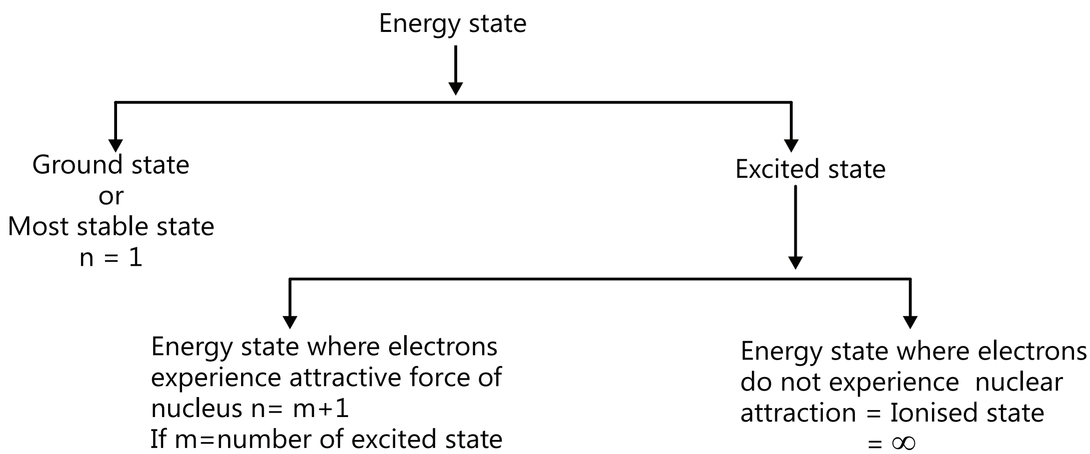


Figure 24.39: Energy level classification

(a) **Ionization energy and ionization potential:** The ionization energy is the energy necessary to remove an electron from the neutral atom. It is a minimum for the alkali metals which have a single electron outside a closed shell. The ionization potential is the potential through which an electron is accelerated for removal an electron from the neutral atom is called ionization potential.

I.E. = $E_\infty - E_1 = -E_1$ = Binding energy of e^- (e_∞ assumed to be zero)

(b) **Excitation energy and excitation potential:** The minimum energy required to excite an atom i.e., alteration from the condition of lowest energy (ground state) to one of higher energy (excited state) is called excitation energy of the particular excited state and corresponding potential is called excitation potential.

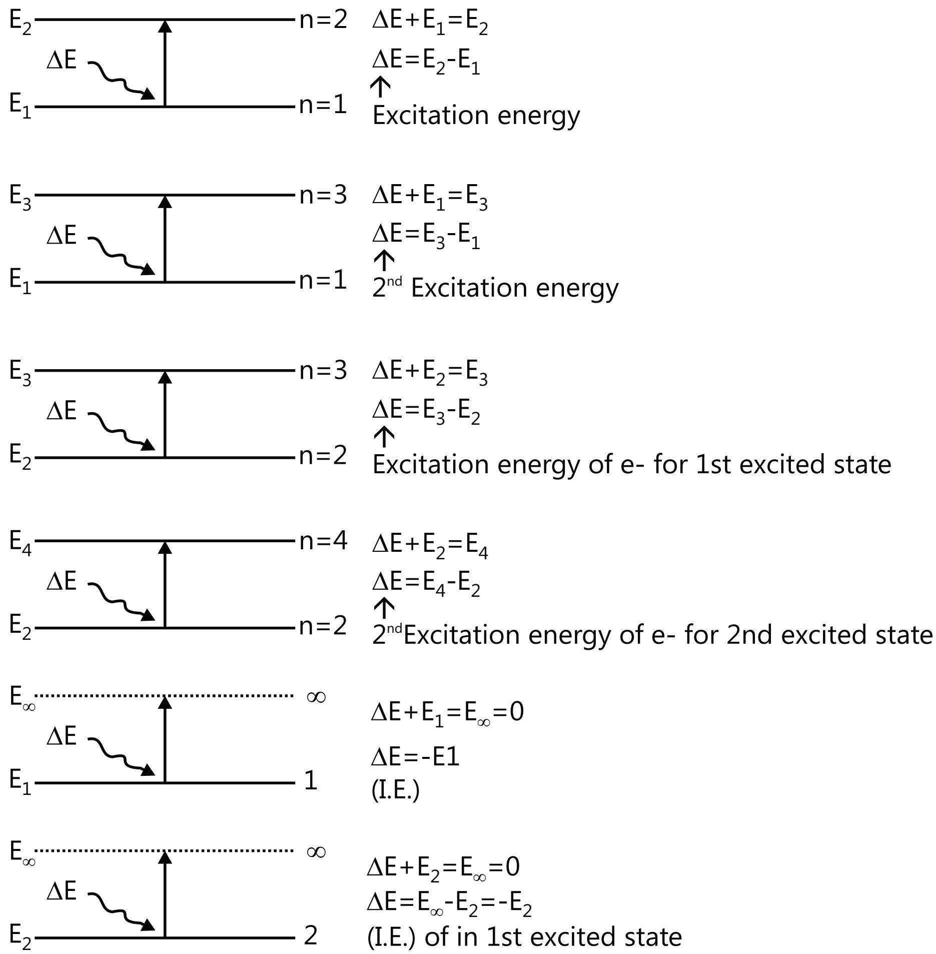


Figure 24.40

If excitation energy and ionization energy are represented in eV, then corresponding value in volt is termed as excitation potential and ionization potential, respectively.

For Example: Excitation energy and ionization energy for H-atom are 10.2 eV and 13.6 eV, respectively and, therefore, 10.2V and 13.6V are excitation and ionization potential, respectively.

PLANCESS CONCEPTS

Reduced mass: Both the proton and electron revolve in circular orbits about their common centre of mass. However, we can account for the motion of the nucleus simply by replacing the mass of electron m by the reduced mass μ of the electron and the nucleus.

Here $\mu = \frac{Mm}{M+m}$ ....(i)

Where M = mass of nucleus. The reduced mass can also be written as, $\mu = \frac{m}{1+\frac{m}{M}}$

Note: If motion of the nucleus is also considered, then m is replaced by μ, where μ = reduced mass of electron – nucleus system = $\frac{mM}{m+M}$. In this case, $E_n = (-13.6\text{eV})\frac{Z^2}{n^2}.\frac{\mu}{m_e}$

Vaibhav Krishnan (JEE 2009, AIR 22)

8. SPECTRUM

8.1 Types of Line Spectrum

Emission line spectrum: When an electric current passes through a gas which is at less than atmospheric pressure, it gives energy to the gas. This energy is then given out as light of several definite wavelengths (colours). This is called a emission line spectrum. These are caused when an electron hops from excited states to lower states. Different The wavelength of emission lines of different elements have emissions of different wavelengths. For one element the emission spectrum are unique for each element.

Absorption line spectrum: It is the electromagnetic spectrum, broken by a specific pattern of dark lines or bands, observed when radiation traverses a particular absorbing medium and through a spectroscope. The absorption pattern of an element is unique and can be used to identify the substance of the medium. When white light is passed through a gas, the gas is absorbs light of certain wavelength. The bright background on the photographic plate is then crossed by dark lines that corresponds to those wavelengths which are absorbed by the gas atoms, resulting in transition of an atom from lower energy states to higher energy states.

(The emission spectrum consists of bright lines on dark background.)

The spectrum of sunlight has dark lines called **Fraunhoffer lines**. These lines are produced when the light emanating from the core of the sun passes through the layer of cooler gas. This layer absorbs light of certain wavelengths corresponding to the elements present in the cooler gas. This results in dark lines (absorption of certain wavelengths) on a brighter background. Fraunhoffer lines reveal the composition of the star.

8.2 Time Period and Frequency of Electron's Motion

(a) **Time period** of revolution of an electron in the nth Bohr orbit is $T_n = \frac{2\pi r_n}{v_n} = \frac{n^3}{Z^2}\frac{h^3}{4\pi^2 mk^2e^4} = 1.5\times10^{-16}\frac{n^3}{Z^2}$ sec

For H-atom, Z = 1 ; then for n = 1, $T_1 = 1.5\times10^{-16}$ sec, $T_1 : T_2 : T_3 = 1 : 8 : 27$

(b) **Frequency of revolution** $v_n = \frac{1}{T_n}$ $v_n \propto \frac{Z^2}{n^3}$

For H-atom $v_1 = 6.6\times10^{15}$ Hz, $v_1 : v_2 : v_3 = 1 : \frac{1}{8} : \frac{1}{27}$

(c) Current and Magnetic field Due to Electron's Motion: The motion of an electron in a circular orbit, gives rise to some equivalent current in the orbit. It is equal to (in the nth orbit) M = current × area; $M_n = I_n.\pi r_n^2$;

$M_n = \frac{nhe}{4\pi m}$; $M_n = \frac{eL}{2m}$

Where L = $\frac{nh}{2\pi}$, angular momentum of the electron in its orbit.

What you must memorise is their dependence on Z and n and order of magnitudes in first Bohr orbit.

$T_n \propto \frac{n^3}{Z^2}$; $T_1 \approx 1.5\times10^{-16}$ s

$v_n \propto \frac{Z^2}{n^3}$; $v_1 \approx 6.6\times10^{15}$ Hz

$\omega_n = 2\pi v_n$; $\omega_n \propto \frac{Z^2}{n^3}$

$L_n = \frac{nh}{2\pi}$; $L_n \propto n$

PLANCESS CONCEPTS

Total energy of an electron in an atom = $\frac{1}{2}\times$ potential energy of electron = – kinetic energy of electron

Nivvedan (JEE 2009, AIR 113)

8.3 Determination of Number of Spectral Lines (Theoretical) in Emission and in Absorption Transitions

8.3.1 Number of Emission Spectra Lines

When an electron is in an excited state with principal quantum number n, then the electron may go to $(n-1)^{th}$ state, ..., 2^{nd} state or 1^{st} state from the n^{th}state. Therefore, there could be (n – 1) possible transitions starting from the n^{th} state. The electron reaching $(n-1)^{th}$ state may make (n – 2) different transitions. Similarly for other lower states, the total number of possible transitions is

$$(n-1)+(n-2)+(n-3)+\ldots\ldots\ldots 2+1=\frac{n\,(n-1)}{2}$$

8.3.2 Number of Absorption Spectral Line

At ordinary temperatures almost all the atoms remain in their lowest energy level (n = 1) and, therefore. absorption transition can start only from the lowest energy level i.e., n = 1 level (not from n = 2, 3, 4, ... levels). Hence, only Lyman series is found in the absorption spectrum of hydrogen atom (which as in the emission spectrum, all the series are found)

Number of absorpton spectral lines = (n – 1)

Remember: The absorpton spectrum of sun has Balmer series also besides the Lyman series. Many H-atoms remain in n = 2 also due to very high temperature.

8.4 Explanation of H-Spectrum and Spectral Line Formula

In a single-electron atom, the transition of an electron from any higher energy state n_2 to any lower energy state n_1 causes a photon of frequency v or wavelength λ to be emitted.

Then $\Delta E = h\nu = \frac{hc}{\lambda} = E_{n_2} - E_{n_1}$;∵ $E = -Rch\frac{Z^2}{n^2}J = -13.6\frac{Z^2}{n^2}$ eV

$$\therefore \Delta E = -\frac{RchZ^2}{n_2^2} - \left(\frac{RchZ^2}{n_1^2}\right) \quad\Rightarrow\quad \Delta E = Rch\,Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$\Rightarrow\quad h\nu = \frac{hc}{\lambda} = RchZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right) \Rightarrow\ \nu = \frac{1}{\lambda} = R\,Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

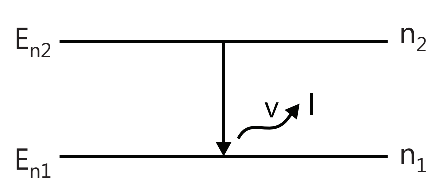


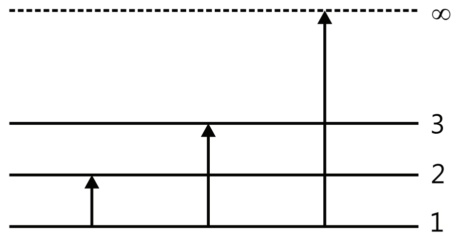
Figure 24.41

$\bar{\nu}$ = wave number = number of wave in unit length $\nu = c\bar{\nu}$

For H-atom, Z= 1 & there for, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

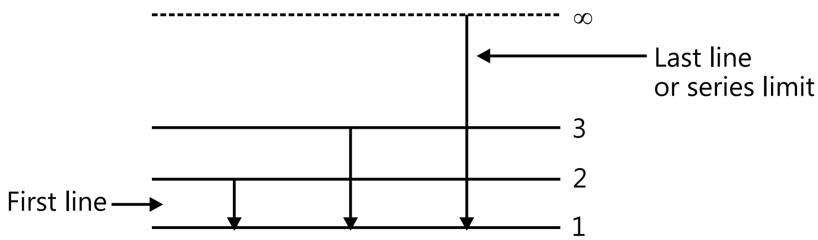
8.5 Hydrogen Spectral Series

(a) Lyman series: $n_1 = 1$, $n_2 = 2$, 3, 4, ...∞



Absorption line

Figure 24.42



Emission line

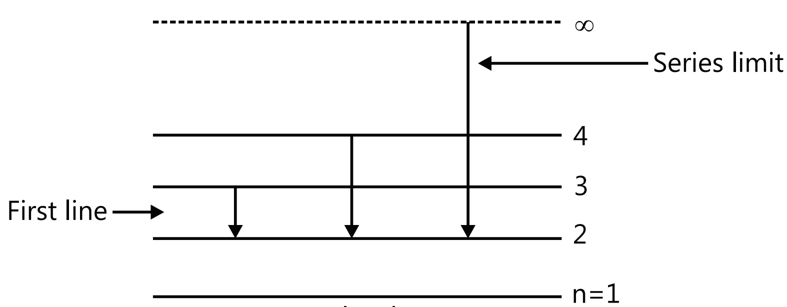
Figure 24.43

For 1st line or series beginning $n_1 = 1,\ n_2 = 2;\ \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right];\ \lambda_{max} = \frac{4}{3R} = 1216\,\text{Å}$

For series limit or last line $n_1 = 1,\ n_2 = \infty;\ \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{1}{R} = 912.68\,\text{Å}$

***** Remember – Lyman series is found in UV region of electromagnetic spectrum

(b) Balmer series:



Energy level

Figure 24.44

$n_1 = 2,\ n_2 = 3, 4, 5, 6, \ldots\ldots\infty$ Wavelength of first line

i.e. maximum wavelength $\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right];\ \therefore\ \lambda_{max} = 6563\,\text{Å}$

Wavelength of last line or series limit i.e. minimum wavelength

$\lambda_{min} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{4}{R} = 3646\ \text{Å}$

***** Balmer series is found only in emission spectrum.

***** Balmer series lies in the visible region of electromagnetic spectrum. Only the first four lines of Balmer series lies in visible region. Rest of them lie in the infrared region of EM spectrum.

(c) Paschen series: $n_1 = 3, n_2 = 4, 5, 6 \ldots \infty$

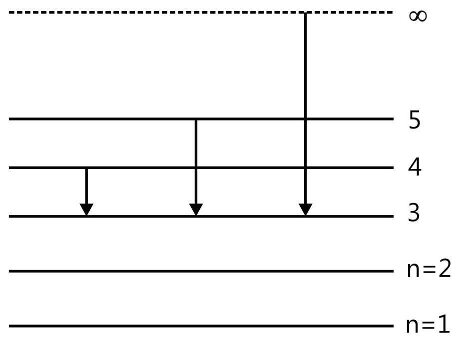


Figure 24.45

For first line $n_1 = 3, n_2 = 4$, then $\frac{1}{\lambda_{max}} = R \times \left[\frac{1}{3^2} - \frac{1}{4^2}\right]$

$\lambda_{max} = 18751$ Å For last line or series limit

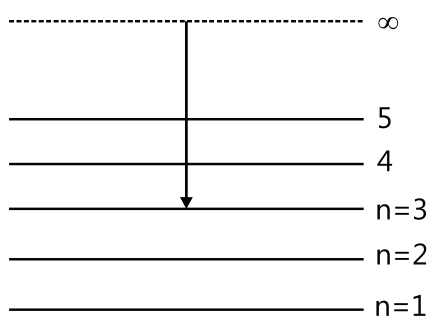


Figure 24.46

$n_1 = 3,\ n_2 = \infty;\ \frac{1}{\lambda_{min}} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right];\ \lambda_{min} = \frac{9}{R} = 8107$ Å

* Paschen series is also found only in emission spectrum.

* Paschen series is obtained in infrared region of electromagnetic spectrum.

(d) Brackett series – $n_1 = 4, n_2 = 5, 6, 7 \ldots \infty$

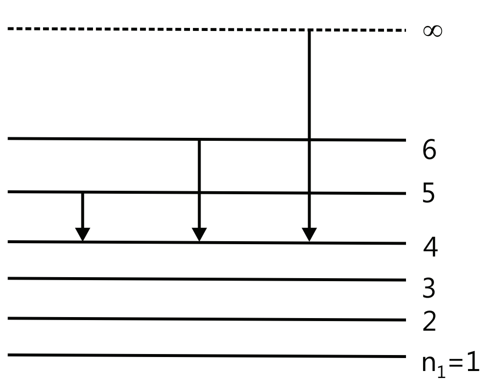


Figure 24.47

For first list $\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right];\ \lambda_{max} = 40477$ Å

For last line or series limit $\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right]$; $\lambda_{min} = \frac{16}{R} = 14572$ Å

* Brackett series is also found only in emission spectrum.

* Brackett series is also obtained in infrared region of electromagnetic spectrum.

(e) Pfund series-

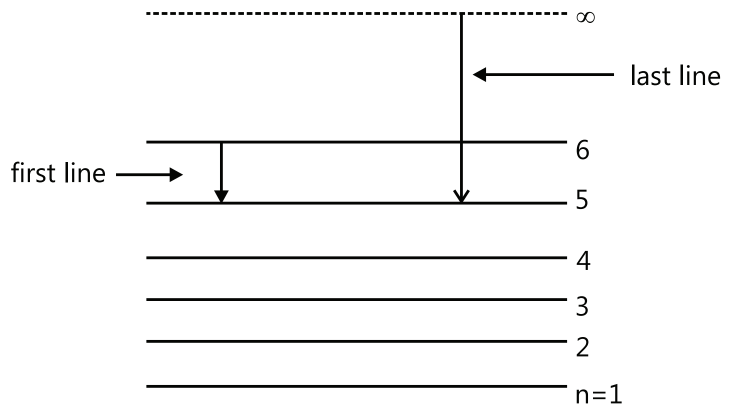


Figure 24.48

$n_1 = 5$, $n_2 = 6, 7, 8, \ldots\ldots\infty$

For first line $\frac{1}{\lambda_{max}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right]$; $\lambda_{max} = 74515$ Å For last line or series limit

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{5} - \frac{1}{\infty^2}\right] \Rightarrow \lambda_{min} = \frac{25}{R} = 22768 \text{ Å}$$

* Pfund series is also obtained only in emission spectrum.

* Pfund series is situated in the infrared region of electromagnetic spectrum.

PLANCESS CONCEPTS

The minimum wavelength of a series (Lyman, Balmer, Paschen, Brackett etc.) correlates with the ionization potential of the electron from that shell.

Chinmay S Purandare (JEE 2012, AIR 698)

General Point for Spectral Lines in Every Spectral Series

(a) Wavelength of first line is maximum and last line is minimum.

(b) As the order of spectral series increases, wavelength also usually increases

$\lambda_{PF} > \lambda_{BR} > \lambda_P > \lambda_B > \lambda_L$

(c) Frequency of energy emission in Lyman transitions are highest among all other series.

PLANCESS CONCEPTS

Total energy of an electron in an atom = $\frac{1}{2}$ * Potential energy of electron

= – Kinetic energy of electron

* If an electron jumps from then $\Delta E = E_{high} - E_{low}$

Where E_{low} is the low-energy state from where the jump begins and E_{high} is the high-energy state where the jump ends.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 22: What will be the two highest wavelengths of the radiation emitted when hydrogen atoms make transitions from higher states to n = 2 states? **(JEE ADVANCED)**

Sol: For electronic transition from energy state $E_n > E_2$ (where n = 3, 4, 5...) to E_2, the spectral series corresponds to Balmer series. Therefore the wavelength of this transition is $\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n^2}\right]$ where $n = 3, 4, 5....\infty$ and R is Rydberg's constant.

The highest wavelength corresponds to the lowest energy of transition. This will be the case for the transition n = 3 to n = 2. The second highest wavelength corresponds to the transition n = 4 to n = 2.

The energy of the state n is $E_n = \frac{E_1}{n^2}$

Thus, $E_2 = -\frac{13.6\text{eV}}{4} = -3.4\,\text{eV}$; $E_3 = -\frac{13.6\text{eV}}{9} = -1.5\,\text{eV}$; and $E_4 = -\frac{13.6\text{eV}}{16} = -0.85$ eV

The highest wavelength is $\lambda_1 = \frac{hc}{\Delta E} = \frac{1242\,\text{eV} \times 1\text{ nm}}{(3.4\text{eV} - 1.5\text{eV})} = 654$ nm

The second highest wavelength is $\lambda_2 = \frac{1242\,\text{eV} \times 1\text{ nm}}{(3.4\,\text{eV} - 0.85\,\text{eV})} = 487$ nm.

Illustration 23: The particle μ-meson has a charge equal to that of an electron and a mass that is 208 times that of the electron. It moves in a circular orbit around a nucleus of charge +3e. Assume that the mass of the nucleus is infinite. Supposing that Bohr's model is applicable to this system, (a) derive an equation for the radius of the nth Bohr orbit, (b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom (c) find the wavelength of the radiation emitted when the μ-meson jumps from the third orbit to the first orbit. **(JEE ADVANCED)**

Sol: According to Bohr's theory, the radius of nth Bohr's orbit is $r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2 Z}$ and energy of μ-meson in nth orbit is $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2n^2h^2}$. If μ-meson jumps from a higher energy orbit to a lower energy orbit, the energy emitted is $\Delta E = Z^2 \times 13.6 \times \left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$eV. To derive the expression for the nth orbit we have to keep in mind that the electrostatic force of attraction between μ-meson and the nucleus provides the required centripetal force for circular orbit. According to Bohr's postulate, the magnitude of angular momentum of μ-meson must be integral multiple of $\frac{h}{2\pi}$.

(a) We have, $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$ or $v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m}$ ... (i)

The quantization rule is $vr = \frac{nh}{2\pi m}$

The radius is $r = \frac{(vr)^2}{v^2 r} = \frac{n^2h^2}{4\pi^2 m^2} \frac{4\pi\varepsilon_0 m}{Ze^2} = \frac{n^2h^2\varepsilon_0}{Z\pi me^2}$ ... (ii)

For the given system, Z = 3 and m = 208 m_e.;Thus $r_\mu = \frac{n^2h^2\varepsilon_0}{624\pi m_e e^2}$.

(b) From (ii), the radius of the first Bohr orbit for the hydrogen atom is $r_h = \frac{h^2\varepsilon_0}{\pi m_e e^2}$.

For $r_\mu = r_h$, $\frac{n^2h^2\varepsilon_0}{624\pi m_e e^2} = \frac{h^2\varepsilon_0}{\pi m_e e^2}$ or, $n^2 = 624$ or, n = 25

(c) From (i), the kinetic energy of the atom is $\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$ and the potential energy is $-\frac{Ze^2}{4\pi\varepsilon_0 r}$.

The total energy is $E_n = -\frac{Ze^2}{8\pi\varepsilon_0 r}$ Using (ii),

$$E_n = -\frac{Z^2\pi me^4}{8\pi\varepsilon_0^2 n^2h^2} = -\frac{9\times 208 m_e e^4}{8\varepsilon_0^2 n^2h^2} = \frac{1872}{n^2}\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right) \quad \text{... (iii)}$$

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2 h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to $-13.6\,eV$.

From (iii), $E_n = -\frac{1872}{n^2} \times 13.6\text{ eV} = \frac{-25459.2\,eV}{n^2}$

Thus, $E_1 = -25459.2$ eV and $E_3 = \frac{E_1}{9} = -2828.8$ eV, The energy difference is $E_3 - E_1 = 22630.4$ eV.

$$E_3 - E_1 = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{E_3 - E_1} = \frac{12375\,eV - \overset{o}{A}}{22630.4\,eV} = 0.5468\,\overset{o}{A}$$

Illustration 24: A neutron moving with speed v makes a head-on collision with a stationary hydrogen atom in ground state. Determine the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron $\approx$ mass of hydrogen = 1.67×10^{-27} kg. **(JEE ADVANCED)**

Sol: It is important to remember the hydrogen atom will absorb the kinetic energy lost in an inelastic collision, causing the atom to reach one of its excited states. The quantum of energy thus absorbed by hydrogen atom will be equal to what is required to reach a possible excited state, and not more. Since the hydrogen atom is initially in ground state (n = 1), the minimum energy it can absorb will be equal to that required to reach the first excited state (n = 2). If the colliding neutron's kinetic energy is less than this minimum energy, no energy will be absorbed, i.e., inelastic collision may not take place.

Let us assume that the neutron and the hydrogen atom move at speeds v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way.

Considering collision to be inelastic, using conservation of linear momentum and energy,

$mv = mv_1 + mv_2$ ...(i)

And $\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E$ ...(ii)

From (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$; From (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$ Thus, $2v_1v_2 = \frac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$; As $v_1 - v_2$ must be real, ; $v^2 - \frac{4\Delta E}{m} \geq 0$;

or $\frac{1}{2}mv^2 > 2\Delta E$.

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is $\frac{1}{2}mv_{min}^2 = 2 \times 10.2\,eV = 20.4\,eV$

Illustration 25: The potential energy U of a small moving particle of mass m is $\frac{1}{2}m\omega^2r^2$, where ω is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, show that radius of the n^{th} allowed orbit is proportional to $\sqrt{n}$. **(JEE ADVANCED)**

Sol: The force acting on the particle in the radial direction $F_r = -\frac{dU}{dr}$ provides the necessary centripetal acceleration for the particle to move in a circular orbit.

The force at a distance r is $F_r = -\frac{dU}{dr} = -m\omega^2 r$ ... (i)

Suppose the particle moves along a circle of radius r. The net force on it should be $\frac{mv^2}{r}$ along the radius.

Comparing with (i), $\frac{mv^2}{r} = m\omega^2 r \Rightarrow v = r\omega$ ... (ii)

The quantization of angular momentum gives $mvr = \frac{nh}{2\pi}$ or, $v = \frac{nh}{2\pi mr}$ ... (iii)

From (ii) and (iii), $r = \left(\frac{nh}{2\pi m\omega}\right)^{1/2}$.

Thus, the radius of the n^{th} orbit is proportional to $\sqrt{n}$.

9. BINDING ENERGY

Binding energy,is amount of energy required to separate a particle from a system of particles or to disperse all the particles of the system. Conversely it also defined as the energy released when particles are brought together to form a system of particles. For example, if an electron and a proton are initially at rest and brought from large distances to form a hydrogen atom, 13.6 eV energy will be released. The binding energy of a hydrogen atom is, therefore, 13.6 eV, same as its ionization energy.

10. CONCEPT OF RECOILING OF AN ATOM DETERMINATION OF MOMENTUM & ENERGY FOR RECOIL ATOMS

When a nuclear particle is emitted or ejected at high velocity from an atom the remainder of the atom recoils with a velocity inversely proportional to its mass. This happens when an electron makes transition from any higher energy state to any lower energy state. The atom is recoiled by sharing some energy from the energy evolved during electronic transition.

If m = mass of recoiled atom, V = velocity of recoiled atom

Then $\frac{1}{2}mv^2 + \frac{hc}{\lambda} = E_{n_2} - E_{n_1} = \Delta E$

Recoil momentum of atom = $\frac{h}{\lambda}$ = momentum of photon

Recoil energy of atom = $\frac{p^2}{2m}$

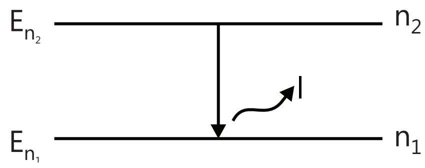


Figure 24.49

Illustration 26: Given that the excitation energy of a hydrogen-like ion in its first excited state is 40.8 eV, determine the energy needed to remove the electron from the ion. **(JEE MAIN)**

Sol: The excitation energy for hydrogen like ion for (n-1)th excited state (nth orbit) is $E = hc \times R \times Z^2\left(1 - \frac{1}{n^2}\right)$ where n = 2, 3, 4, ...etc. The energy needed to remove the electron from the ion is $E = hc \times R \times Z^2$.

The excitation energy in the first excited state (n=2) is

$E = RhcZ^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ = (13.6 eV) × Z^2 × $\frac{3}{4}$.

Equating this to 40.8 eV, we get Z = 2. So, the ion in question is He^+.

The energy of the ion in the ground state is E = $-\frac{RhcZ^2}{1^2}$ = $-4 \times (13.6\,eV)$ = –54.4 eV

Thus 54.4 eV is required to remove the electron from the ion.

PLANCESS CONCEPTS

The energy of a photon and its wavelength are inversely proportional.

B Rajiv Reddy JEE 2012, AIR 11

11. THE WAVE FUNCTION OF AN ELECTRON

Quantum mechanics has enabled physicists to develop a mathematically and logically rigorous theory which describes the spectra in a much better way. The following is a very brief introduction to this theory.

We have already seen that to understand the behavior of light, we understand it as both a wave (the electric field $\vec{E}$ (as well as a particle (the photon). The energy of a particular 'photon' is related to the 'wavelength' of the $\vec{E}$ wave. Light going in x direction is represented by the wave function. $E(x,t) = E_0 \sin(kx - \omega t)$ In general, if light can go in any direction, the wave function is $\vec{E}(\vec{r}, t) = \vec{E}_0 \sin(\vec{k}.\vec{r} - \omega t)$ ... (i)

Where $\vec{r}$ is the position vector; $\vec{k}$ is the wave vector.

11.1 Quantum Mechanics of the Hydrogen Atom

The wave function $\Psi(\vec{r}, t)$ of the electron and the possible energies E of a hydrogen atom or a hydrogen-like ion are obtained from the Schrodinger's equation.

$$\frac{-h^2}{8\pi^2 m}\left[\frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}\right] - \frac{Ze^2\Psi}{4\pi\varepsilon_0 r} = E\Psi \quad \text{... (ii)}$$

Here (x, y, z) refers to a point with the nucleus as the origin and r is the distance of this point from the nucleus. E refers to energy. The constant Z is the number of protons in the nucleus. For hydrogen, we have to put Z = 1. There are infinite number of functions $\Psi(\vec{r})$ which satisfy equation (ii). These functions, which are solutions of equation (ii), may be characterized in terms of three parameters n, l and m_l With each solution Ψ_{nlm_l}, there is associated a unique value of the energy E of the atom or the ion. The energy E corresponding to the wave function Ψ_{nlm_l} depends only on n and may be written as $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2h^2n^2}$

12. LASER

12.1 Basic Process of Laser

The basic strategy to get light amplification by stimulated emission is as follows:

A system is chosen which has a metastable state at having an energy E_2 (See Fig. 24.50). There is another allowed energy E_1 which is less than E_2. The system could be any of the following: a gas or a liquid in a cylindrical tube or a solid in the shape of a cylindrical rod. Let us assume that the number of atoms in the metastable state E_2 is increased to more than that in E_1. Let us also assume that a photon of light of energy $E_2–E_1$ is incident on one of the atoms in the metastable state E_2. Then this atom drops to the state E_1 i.e., emitting a photon in the same phase, energy, and direction as the first one. Then these two photons interact with two more atoms in the state E_2 and so on. Therefore, the number of photons keeps on increasing. All these photons will have the same phase, the same energy, and the same direction. Thus, the amplification of light is achieved.

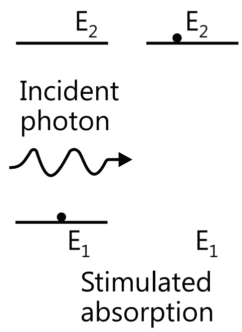


Figure 24.50: Laser

12.2 Working

When power is suppliedy and the electric field is established, some of the atoms of the mixture get ionized. These ionized atoms release some electrons which are accelerated by the high electric field. Consequentially, these electrons collide with helium atoms to take them to the metastable state at energy E_3. These atoms collide with a neon atom and transfer the extra energy to it. As a result, the helium atom returns to its ground state and the neon atom is excited to the state at energy E_2. This process keeps looping so that the neon atoms are continuously pumped to the state at energy E_2, keeping the population (of atoms) of this state large.

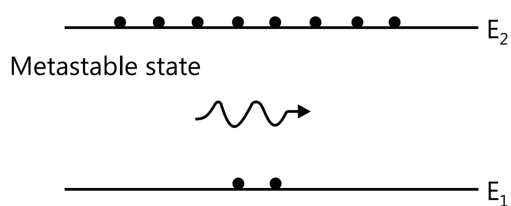


Figure 24.51: metastable state of electron

12.3 Uses of Laser

(a) Spectroscopy: Most lasers being inherently pure source of light, emit near monochromatic light with a very clear range of wavelengths. This makes the laser ideal for spectroscopy.

(b) Heat Treatment: In laser heat treating, energy is transmitted to the material's surface in order to create a hardened layer by metallurgical transformation. The use of lasers results in little or no distortion of the component and, as such, eliminates much of part reworking that is currently done. Therefore, the laser heat treatment system is cost-effective.

(c) Lunar laser ranging: The Apollo astronauts planted retroreflector arrays on te moon to make possible the Lunar Laser Ranging Experiment. In this experiment laser beams are focused, through large telescopes on Earth, on the arrays, and the time taken for the beam to be reflected back to Earth is measured to determine the distance between the Earth and Moon with high accuracy.

(d) Photochemistry: Extremely brief pulses of light – as short as picoseconds or femtoseconds (10^{-12} to 10^{-15} s) – produced by some laser systems are used to initiate and analyze chemical reactions. This technique is known as photochemistry.

(e) Laser Cooling: This technique involves atom trapping, wherein a number of atoms are enclosed in a specially shaped arrangement of electric and magnetic fields.

(f) Nuclear Fusion: Powerful and complex arrangements of lasers and optical amplifiers are used to produce extremely high-intensity pulses of light of extremely short duration. These pulses are arranged to impact pellets of tritium-deuterium, simultaneously, from all directions, hoping that the compression effect of the impacts will induce atomic fusion in the pellets.

13. X-RAYS

X-radiation is a form of electromagnetic radiation. Most X-rays have a wavelength ranging from 0.01 to 10 nanometers, corresponding to frequencies in the range 30 petahertz to 30 exahertz (3×10^{16} Hz to 3×10^{19} Hz) and energies in the range 100 eV to 100 keV. X-radiation is also referred to as Röntgen radiation, after Wilhelm Röntgen, who is usually credited as its discoverer, and who had named it X-radiation to signify an unknown type of radiation produced when electron collided with the walls of the tube.

The wave nature of X-rays, was established by Laue who demonstrated that they are diffracted by crystals.

13.1 Production

The modern X-ray tube, called Coolidge tube, is shown in the Fig. 24.52. A heated element emits electronswhich are accelerated towards a cooled copper anode under a high potential difference. A target metal of high atomic number and high melting point is lodged on the anode.

The intensity of the X-ray beam is controlled by The filament current controls the intensity of the X-ray beam by regulating the number of electrons striking the target per unit time. The potential difference between the cathode and anode controls the penetrating power of the beam.

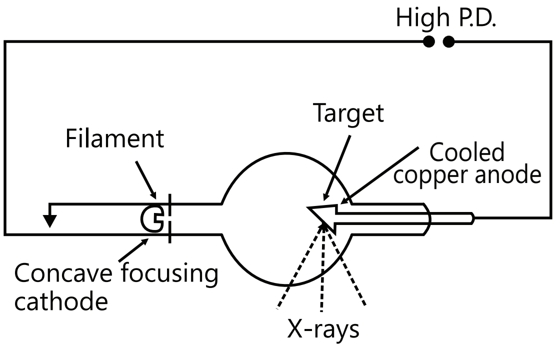


Figure 24.52: X- Ray tube

13.2 X-Ray Spectra

A typical X-ray spectrum given by a target is shown in the Fig.24.53. The spectrum is basically continuous range of wavelengths starting from a minimum value. A line spectrum having sharp wavelengths is superimposed on this.

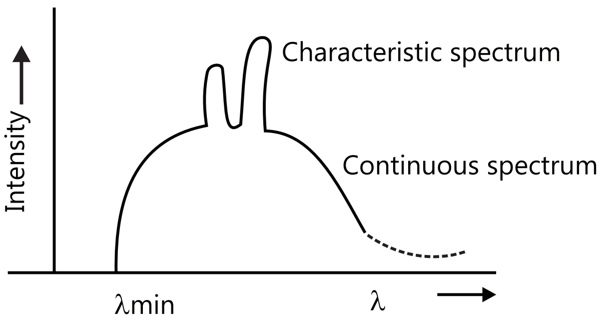


Figure 24.53: X-Ray spectra

13.3 Origin Of Characteristic Spectrum

If an incoming electron knocks out an electron in one of the inner shells, the exiting electron creates a vacancy in that shell. This vacancy gets fille by another electron from a higher shell that makes a transition to this shell, creating another vacancy in the higher shell. This hopping of electrons from higher to lower shells continues till the inner shells are filled up. This process produces a series of radiations, some of which pertain to the X-ray region. These radiations are typical of the target element. The X-ray spectrum of a substance is classified into K-series, L-series, M-series etc.

13.4 Moseley's Experiment and the Concept of Atomic Number

Moseley's experiment involved the analysis of the X-ray spectra of 38 different elements, ranging from aluminum to gold. He measured the frequency of principal lines of a particular series (the α-lines in the K-series) of the spectra and was able to show that the frequencies of certain characteristic X-rays emitted from chemical elements are proportional to the square of a number which was close to the element's atomic number (Z). He presented the following relationship: $\sqrt{\nu} = a\,(Z-b)$

where v = frequency of X-rays, Z = atomic number, a and b are constants. On plotting the values of square root of the frequency against atomic numbers of the elements producing X-rays on a graph, a straight line was obtained.

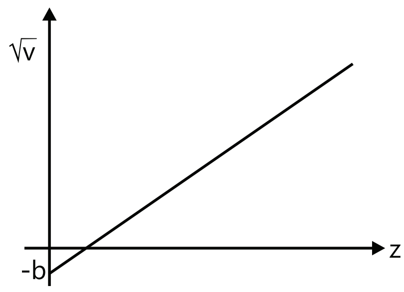


Figure 24.54

When electron is knocked out from n_1 energy state and it is filled with electron from n_2 energy state wavelength of X-ray emitted is $\frac{1}{\lambda} = R(Z-b)^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For K-series b = 1, $n_1 = 1$ and $n_2 = 2, 3, \ldots\ldots$ For L-series : b = 7.4, $n_1 = 2$, $n_2 = 3, 4, \ldots\ldots$

For K_α -line, the electron jumps from L-level to a vacancy in the K-kevel. So for the L electron, there are Z protons in the nucleus and an electron in the K-shell which screens off the positive charge. So the net charge the L electron faces can be taken as (Z – 1)e.

Now, E = Rhc $(Z-1)^2$ $= \Rightarrow h\nu = Rhc(Z-1)^2 \times \left[\frac{3}{4}\right]$

$$\sqrt{v} = \sqrt{\frac{3Rc}{4}}\,(Z-1)$$

13.5 Origin of Continuous Spectrum

When an incident electron comes very close to a target nucleus, it is suddenly accelerated due to the electrostatic field around the nucleus (Coulomb field) and emits electromagnetic radiation. This radiation is referred to as breaking radiation and is continuous. The minimum wavelength (and maximum frequency) correlates with an electron losing all its energy in a single collision with a target atom.

If V is the acceleration p.d., then $h\nu_{max} = \frac{hc}{\lambda_{min}} = eV$ or $\lambda_{min} = \frac{hc}{eV} = \frac{12420}{V}$ Å

13.6 Properties of X-rays

(a) X-rays ionize the material which they penetrate.

(b) They produce the same effect on photographic plates as visible light.

(c) They cause fluorescence when they act on certain chemical compounds like zinc sulphide.

(d) X-rays penetrate matter and get absorbed as they pass through it. If I_0 is the intensity of incident relation and I is the intensity after travelling through a distance x, then $I = I_0 e^{-\mu x}$ where μ is called the absorption coefficient of the material. The atomic number of the material and its absorption coefficient are directly proportional. This is the basis of radiography.

(e) They cause photoelectric emission.

(f) Electric and magnetic fields have no effect on X-rays as they contain no charged particles.

Note: X-Rays are not affected by electric or magnetic fields. Intensity of X-rays depends on number of electrons in the incident beam.

PROBLEM-SOLVING TACTICS

(a) This section of Physics is more fact-based. The key to answering questions of these sections is establish a lonk between the known and asked quantities

(b) One has to be very conversant with the formulae and standard scientific constants.

(c) In this section, graphical questions seeking relationship between various fundamental quantities are usually asked. Assign the dependent variable as y and the independent variable as x and then look for a relation between them.

(d) One must not get confused about approaching the questions from a wave nature or particle nature or try to combine both. Just solve questions on the basis of the known and asked quantities and the relationship between the two.

(e) It is important to learn the scientific constants in various units to avoid unnecessary unit conversion. (e.g., if energy of a photon is in eV units and wavelength asked in angstrom, one can directly use the relation = 12400/E, here 12400 is the product of Planck's constant and speed of light.)

(f) Analytical questions pertaining to H-atom can be solved easily if one knows proportionality relation between quantities. They need not be learnt by heart. They can be derived without bothering about constants appearing in these relations. (e.g., radius of nth shell is directly proportional to n^2, keeping Z constant.)

FORMULAE SHEET

Speed of E.M.W. in vacuum c = $3\times10^8\,m/s = \nu\lambda$

Each photon having a frequency ν and energy $E = h\nu = \frac{hc}{\lambda}$ where h = 6.63×10^{-34} Js is Planck's Constant

Einstein's Photo Electric Equation:

Photon energy = K.E. of electron + work function.

$$h\nu = \frac{1}{2}mv^2 + \phi$$

ϕ = Work function = energy needed by the electron in freeing itself from the atoms of the metal.

$$\phi = hv_0$$

The minimum value of the retarding potential to prevent electron emission is:

$$eV_{cut\ off} = (KE)_{max}$$

De Broglie wave length given by $\lambda = \frac{h}{p}$ (wave length of a particle)

The electron in a stable orbit does not radiate energy i.e. $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

A stable orbit is that in which the angular momentum of the electron about nucleus is an integral (n) multiple of $\frac{h}{2\pi}$ i.e. mvr = $n\frac{h}{2\pi}$; n = 1, 2, 3, ... $(n \neq 0)$.

For Hydrogen atom : (Z = atomic number = 1)

(i) L_n = angular momentum in the nth orbit = $n\frac{h}{2\pi}$

(ii) r_n = radius of nth circular orbit = $r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$ (0.529 Å) n^2 ; (1 Å = 10^{-10} m) ; $r_n \propto n^2$

(iii) E_n energy of the electron in the nth orbit = $\frac{-13.6\text{eV}}{n^2}$ i.e. $E_n \propto \frac{1}{n^2}$

(iv) n^{th} orbital speed $v_n = \frac{e^2}{2\varepsilon_0 nh}$

Note: Total energy of the electron in an atom is negative, indicating that it is bound.

Binding Energy $(BE)_n = -E_n = \frac{13.6\text{eV}}{n^2}$

(iv) $E_{n_2} - E_{n_1}$ = Energy emitted when an electron jumps from n_2 orbit to n_1 orbit $(n_2 > n_1)$.

$$\Delta E = (13.6\text{ eV})\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$\Delta E = h\nu$; ν= frequency of spectral line emitted.

Wave number= $\bar{\nu} = \frac{1}{\lambda}$ = [no. of waves in unit length (1m)] = $R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Where R = Rydberg's constant for hydrogen = $1.097 \times 10^7 m^{-1}$

(v) For hydrogen like atoms of atomic number Z:

$$r_{nz} = \frac{\text{Bohr radius}}{Z} \times n^2 = (0.529\text{Å})\frac{n^2}{Z} ;$$

$$E_{nZ} = (-13.6)\frac{Z^2}{n^2}\text{ev}$$

Note: If motion of the nucleus is also considered, then m is replaced by μ.

Where μ = reduced mass of electron – nucleus system = $\frac{mM}{m+M}$

In this case, $E_n = (-13.6\text{eV})\ \frac{Z^2}{n^2} \cdot \frac{\mu}{m_e}$

Excitation potential for quantum jump from $n_1 \rightarrow n_2$ = $\frac{E_{n_2} - E_{n_1}}{\text{electron charge}}$

From Mosley's Law $\sqrt{\nu} = a(z-b)$ where b (shielding factor) is different for different series.

For x-rays $\frac{1}{\lambda} = R \times (Z-b)^2 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$R = R_0 A^{1/3}$. Where R_0 = empirical constant = 1.1×10^{-15}m ; A = Mass number of the atom.