Illustration 39: Find minimum value of ℓ so that truck can avoid the dead end, without toppling the block kept on it. **(JEE ADVANCED)**

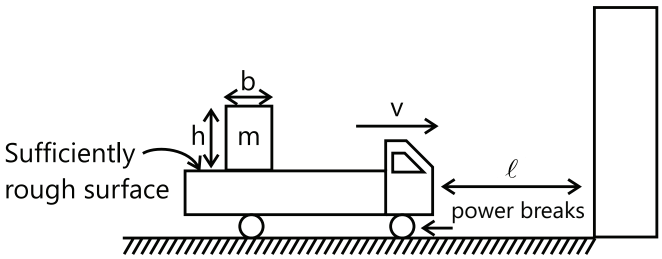


Figure 7.63

Sol: The block kept on truck will experience pseudo force in forward direction and friction force due to the floor of the truck in backward direction. We assume the case of toppling before sliding. In extreme case the normal reaction N = mg will pass through the edge.

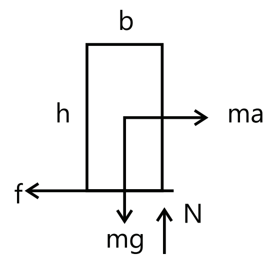


Figure 7.64

$$ma\frac{h}{2} \le mg\frac{b}{2} \Rightarrow a \le \frac{b}{h}g$$

Final velocity of truck is zero. So that 0 = $v^2 - 2(\frac{b}{h}g)\ell$

$$\ell = \frac{h}{2b}\frac{v^2}{g}$$

PROBLEM-SOLVING TACTICS

- Most of the problems involving incline and a rigid body can be solved by using conservation of energy during pure rolling. In case of non-conservative forces, work done by them also has to be taken into consideration in the equation. Care has to be taken in writing down the Kinetic energy. Rotational Kinetic Energy term has to be taken into consideration. And while writing the rotational energy, the axis about which the moment of inertia is taken should pass through the COM.
- The motion of a body in pure rolling can be viewed as pure rotation about the bottommost point of the body or the point of contact with the ground. Hence an axis passing through the point of contact and tangential to the point would be the Instantaneous axis of rotation. So problems on pure rolling can be solved easily by using the concept of instantaneous axis of rotation.
- Problems on toppling can be easily solved by writing the moments on the body and visualizing them as forces acting on the body. If the net moment is tending to stabilize the body, then the body doesn't topple. For any condition else it may get toppled.
- Problems which include the concept of sliding and rolling can be solved easily by using the concept of conservation of angular momentum. But care has to be taken in selecting the proper axis so that net moment about that axis vanishes.

FORMULAE SHEET

S. No	Term	Description	Linear Motion	Rotational motion & relation
1	Displacement	Displacement (linear or angular) is the physical change in the position of the body when a body moves linearly or angular in position. **(a)** The linear displacement Δs is difference between final and initial position measured in linear direction. **S.I. unit**: meter **m** **(b)** The angular displacement of the body while rotating about a fixed axis is the displacement $\Delta\theta$ it swept out with respect to its initial position in sense of rotation. It can be positive (anti clockwise) or negative (clockwise) **S.I. unit:** radians **rad**,	s	θ $(s = r\theta)$
2	Velocity	Velocity of any moving object is the time rate of change of position. The velocity is the vector quantity. Linear velocity is in the plane of motion. Angular velocity can be positive or negative & its direction is perpendicular to the plane of rotation Linear velocity is categorized as - Average velocity= $\Delta s / \Delta t$ - Instantaneous velocity= ds/dt. **S.I. unit: m/s** Angular velocity is categorized as - Average angular velocity $\Delta\theta / \Delta t$ - Instantaneous angular velocity $\omega = d\theta / dt$ **S.I. unit: rad/s**	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$ $(v = r\omega)$
3	Acceleration	Acceleration is the time rate change of velocity of a body. It's a vector quantity. Linear acceleration can be positive or negative and related to direction of motion. Linear acceleration is categorized as - Average acceleration= $\Delta v / \Delta t$ - Instantaneous acceleration = dv/dt.	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$ $(a = r\alpha)$

S. No	Term	Description	Linear Motion	Rotational motion & relation
		S.I. unit: m/s⁻² Angular acceleration is categorized as - Average angular acceleration $\Delta\omega / \Delta t$ - Instantaneous angular acceleration $\alpha = d\omega / dt$ **S.I. unit: rad/s⁻²**		
4	Mass	Mass is the basic entity of any body by virtue of which the body gains weight. In linear kinematics the mass of whole body is constant. **S.I. unit:** kilogram **kg** In angular kinematics mass of body is distributed among various tiny rigid points so mass is measured about inertia of rotating body- moment of inertia **I**	M	I ($I = \sum mr^2$)
5	Momentum	Momentum of body is product of mass and its velocity of motion. It's a vector quantity. Linear momentum= mv **S.I. unit: kg m/s** Angular momentum of body is a vector in direction perpendicular to plane of rotation given by $\vec{L}$ **S.I. unit: kg m²/s**	p = mv	$\vec{L}$ = I $\vec{L} = \vec{r} \times \vec{p}$
6	Impulse	Impulse is the product of force and time period And it is categorized as -Linear impulse -Angular impulse	$\int F\, dt$	$\int \tau dt$
7	Force (Newton's second law of motion)	From the newton second law of motion, force is time rate of change of momentum. It's a vector quantity. Linear force $F = \frac{dp}{dt} = ma$ **S.I. unit:** Newton **N** Angular force $\vec{\tau} = I \times \vec{\alpha}$ **Laws of conservation of momentum** - Linear momentum is said to be conserved if $\frac{dp}{dt}$ = 0, than P remains constant - Angular momentum is said to be conserved if $\frac{d\vec{L}}{dt}$ = 0 than L remains constant	F = ma If = 0 the body is in equilibrium with its surrounding	$\vec{\tau} = r \times \vec{F} = I \times \vec{\alpha}$ $= \frac{d\vec{L}}{dt}$ If = 0 the body is in equilibrium with its surrounding

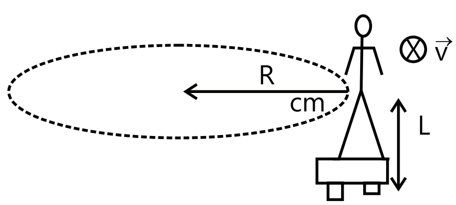
S. No	Term	Description	Linear Motion	Rotational motion & relation
8	Work	Work is the product of displacement of body under action of external applied force.	$W = \int F\,ds$	$W = \int \tau d\theta$
9	Power	Power is the time rate change of work done	P =F	$P = \tau\,\omega$
10	Kinetic energy	The phenomenon associated with the moving bodies	$K.E._{tran} = \frac{1}{2}mv^2$	$K.E._{rot} = \frac{1}{2}I\omega^2$
11	Kinematics of Motion	Kinematical equation are the interrelation of displacement, velocity, acceleration and time and are categorized as follows: -Linear kinematical equation -Angular kinematical equation	$v = u + at$ $s = ut + \frac{1}{2}at^2$ $v^2 = u^2 + 2as$	$\omega = \omega_0 + \alpha t$ $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha\theta$
12	Parallel Axis Theorem	$I_{xx} = I_{cc} + Md^2$ where I_{cc} is the moment of inertia about the center of mass		
13	Perpendicular Axis Theorem	$I_{xx} + I_{yy} = I_{zz}$ It is valid for plane laminas only.		
14	Work energy principle	Work energy principle is used to determine the change in the kinetic energy of moving body	$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$	$W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2$

Solved Examples

JEE Main/Boards

The first five Examples discussed below show us the strategy to tackle down any problem in the rigid body motion. Hence follow them up properly! They may be lengthy but are very learner friendly!!

Example 1: A person of mass M is standing on a railroad car, which is rounding an unbanked turn of radius at speed v. His center of mass is at a height of L above the car midway between his feet, which are separated by a distance of d. The man is facing the direction of motion. What is the magnitude of the normal force on each foot?



Sol: The frictional forces acting on the feet of man will provide the necessary centripetal acceleration to move in a circular path. Apply the Newton's second law of motion at the center of mass of the man to get the equation of motion along the circular path. In the vertical plane the man is in rotational and translational equilibrium under the action of its weight acting vertically downwards and the normal reactions at its feet acting vertically upwards. Get one equation each