He. Helium obtained as a result of decay of $^{238}_{92}U$ has almost certainly been formed from α-particles. Thus, if ^{238}U and He contents are known in a rock we can determine the age of rock sample (1 g of ^{238}U in equilibrium with its decay products produces about 10^{-7} g He in a year). Also, by assuming that initially the rock does not contain ^{206}Pb and it is present in rock due to decay of ^{238}U, we can calculate the age of rocks and mineral by measuring the ratio of ^{238}U and ^{206}Pb. The amount of ^{206}Pb is supposed to be obtained by decay of ^{238}U Thus, $^{238}_{92}U \longrightarrow {}^{206}_{82}Pb + 8\,^{4}_{2}He + 6\,^{0}_{-1}e$

Mole of ^{238}U left = N at time t i.e. N_t ; Mole of ^{206}Pb formed = N′ at time t

$\because$ Initial mole of ^{238}U = N + N′ (at time 0) i.e., (N_0)

Thus, time t can be evaluated by $t = \dfrac{2.303}{\lambda}\log\dfrac{N_0}{N_t}$

Illustration 31: On analysis, a sample of uranium are was found to contain 0.277 g of $^{206}_{82}Pb$ and 1.667 g of $^{238}_{92}U$. The half-life period of ^{238}U is 4.51×10^9 years. If all the lead were assumed to have come from decay of $^{238}_{92}U$, what is the age of earth? **(JEE ADVANCED)**

Sol: Given, at time t; $^{238}_{92}U$ = 1.667 g = (1.667/238) mole ; $^{206}_{82}Pb$ =0.277 g = (0.277/206) mole

Since, all lead has been formed from ^{238}U, therefore moles of U decayed = Moles of Pb formed = (0.277 / 206)

$\therefore$ Total moles of U before decay (N_0) = Moles of U at time t (N) + Moles of U decayed

$$= \frac{1.667}{238} \times \frac{0.277}{206} \quad \because t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 4.51\times 10^9}{0.693}\log\frac{(1.667/238)+(0.277/206)}{(1.667/238)} \quad \therefore t = 1.147 \times 10^9 \text{ years}$$

Illustration 32: The activity of the hair of an Egyptian mummy is 7 disintegrations minute^{-1} of ^{14}C. Find the age of the mummy, given $t_{0.5}$ of ^{14}C is 5770 years and disintegration ratio of fresh sample of ^{14}C is 14 disintegration minute^{-1} **(JEE MAIN)**

Sol: r_0 = 14 dpm and r_1 = 7 dpm $\therefore \dfrac{r_0}{r} = \dfrac{14}{7} = 2 = \dfrac{N_0}{N}$ $(\because r_0 \propto N_0)$

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 5770}{0.693}\log 2 = 5771 \text{ year}$$

Illustration 33: The half period of ^{14}C is 5760 years. A piece of wood when buried in the earth had 1% ^{14}C. Now as charcoal it has only 0.25% ^{14}C. How long has the piece of wood been buried? **(JEE MAIN)**

Sol: Given, $N_{0_{^{14}C}}$ = 1% ; $N_{^{14}C}$ = 0.25% and $t_{1/2}$ = 5760 year

$$t = \frac{2.303}{\lambda}\log\frac{N_0}{N} = \frac{2.303\times 5760}{0.693}\log\frac{1}{0.25} = 11524 \text{ year}$$

PROBLEM SOLVING TACTICS

Nuclear radius (r)= $R_0A^{1/3}$, where A= Mass no. , = 1.4×10^{-15}m

For calculation of geological dating :

(i) Calculation λ from $t_{1/2}, \lambda = \dfrac{0.693}{t_{1/2}}$

(ii) Calculate uranium converted into lead

(iii) Calculate total initial amount of uranium initially present

(iv) Apply , $t = \dfrac{2.3030}{\lambda}\log\dfrac{N_0}{N}$

For calculation in carbon dating method

(i) Calculated from $t_{1/2}$

(ii) m% activity of C-14 now present means $\frac{N_0}{N} = \frac{m}{100}$

(iii) Apply, $\lambda = \frac{2.3030}{t}\log\frac{N_0}{N}$

POINTS TO REMEMBER

Kinetics of Radioactive Disintegration: All radioactive isotopes decays spontaneously following first order kinetics, i.e, rate of decay (-dN/dt) is directly proportional to the amount of radioactive isotope (N).	$-\frac{dN}{dt} \propto N \quad \Rightarrow \quad -\frac{dN}{dt}\lambda N$ Where, 'λ' is decay constant. Integrating the above rate law gives $\lambda t = \text{In}\left(\frac{N_0}{N}\right)$; N_0= Initial number of nuclides N= Number of nuclides remaining after time t. Also $N=N_0e^{-\lambda t}$.
Half-life $(t_{1/2})$: Time in which half of the nuclides are decayed	$t_{1/2} = \frac{1}{\lambda}\text{In}\left(\frac{N_0}{N_0/2}\right) = \frac{\text{In}2}{\lambda}$
Activity (A) It is the instantaneous rate of decay.	$A = -\frac{dN}{dt} = \lambda N \Rightarrow$ Initial activity $(A_0) = \lambda N_0$ Also $A = A_0e^{-\lambda t}$
Units of Radioactivity: Curie (Ci) and Rutherford (Rd) Gray (Gy): 1Gy = 1 kg tissue receiving 1 J energy. If w0 gram of a radioisotope decay for 'n' half-lives, the amount of radio-isotope remaining undecayed (w) is given by the expression. It is a derived unit of ionizing radiation.	$1Ci=3.7 \times 10^{10}$ dps 1Rd = 10^6 dps $w = w_0\left(\frac{1}{2}\right)$

Total Binding Energy (BE) : It is the total energy released when a nucleus is formed from nucleons. BE is determined from mass defect (Δm) as BE = $(\Delta m)C^2$

$\Delta m=\sum$(Mass of nucleons – Mass of nucleus) (Δm = 1u correspond to BE=931 MeV)

Unstable nuclei decay by spontaneous emission of radioactive rays. Stability of a nucleus is accounted qualitatively by its N/P ratio (N=Number of neutrons and P=number of protons).

Up to Z=20, for stable nuclei, N/P=1 is required.

Above Z=20, more neutrons are required to shield the strong electrostatic repulsion between large number of like charged protons in a small nuclear volume, hence N/P> 1 is required for stability in case unstable nuclei, if N/P ratio is greater than that required for stability, β–emission takes place, eg,

$${}_7N^{16} \rightarrow {}_8O^{16} + \beta\left({}_{-1}e^0\right)$$

If N/P ratio is less than that required for stability, radio nuclide may decay by one of the following modes:

(i) Positron emission $\quad {}_5B^8 \rightarrow {}_4Be^8 + {}_{+1}\beta^0\left(\text{Positron} + {}_{+1}e^0\right)$

(ii) Electron capture $\quad {}_{20}Ca^{38} + {}_{-1}e^0 \rightarrow {}_{19}K^{38}$

Alpha (α) emission occurs when Z>82.