The expression $\left|\frac{d\nu}{dt}\right|$ represents the magnitude of tangential acceleration. The differential $\frac{d\theta}{dt}$ represents the magnitude of angular velocity. The expression $r\frac{d\theta}{dt}$ represents the magnitude of tangential velocity and the expression $\frac{d^2r}{dt^2}$ is second-order differentiation of position vector (r). This is the actual expression of acceleration of a particle under motion. Hence, the expression $\left|\frac{d^2r}{dt^2}\right|$ represents the magnitude of total or resultant acceleration.

Hence, option (d) alone is correct.

Illustration 18: A particle is executing circular motion. But the magnitude of velocity of the particle changes from zero to (0.3i + 0.4j) m/s in a period of 1 second. The magnitude of average tangential acceleration is:

(A) 0.1 m/s^2 (B) 0.2 m/s^2 (C) 0.3 m/s^2 (D) 0.5 m/s^2 **(JEE MAIN)**

Sol: Tangential acceleration is equal to the rate of change of speed. Average tangential acceleration is change in speed divided by total time.

The magnitude of average tangential acceleration is the ratio of change in speed and time as given by: $a_T = \frac{\Delta\nu}{\Delta t}$

Now, $\Delta\nu = \sqrt{\left(0.3^2 + 0.4^2\right)} = \sqrt{0.25} = 0.5 m/s$; $a_T = 0.5 m/s^2$

Hence, option (d) alone is correct.

PLANCESS CONCEPTS

Radial acceleration contributes in changing the direction of velocity of an object, but it does not affect the magnitude of velocity. However, tangential acceleration affects the speed of the object in motion.

Vaibhav Krishan (JEE 2009, AIR 22)

FORMULAE SHEET

(a) Projectile Motion

Time of flight: $T = \frac{2u\sin\theta}{g}$

Horizontal range: $R = \frac{u^2\sin 2\theta}{g}$

Maximum height: $H = \frac{u^2\sin^2\theta}{2g}$

Trajectory equation (equation of path):

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$$

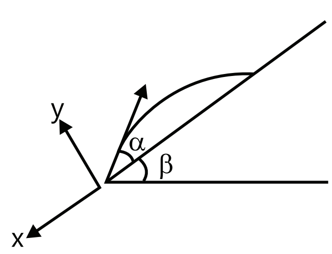


Figure 3.23

Projection on an inclined plane

(b) Relative Motion

v_{AB} (velocity of A with respect to B) $= v_A - v_B$

a_{AB} (acceleration of A with respect to B) $= a_A - a_B$

Relative motion along straight line $= x_{BA} = x_B - x_A$

(c) Crossing River: A boat or man in a river always moves in the direction of resultant velocity of velocity of boat (or man) and velocity of the river flow.

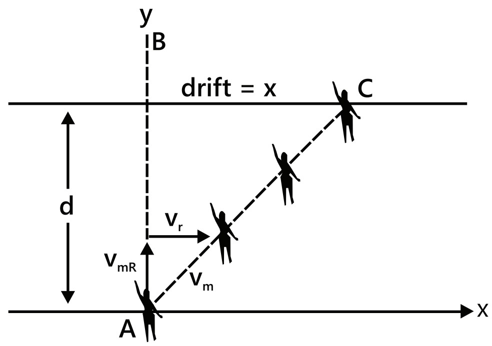


Figure 3.24

(d) Shortest Time: Velocity along the river, $V_X = V_R$

Velocity perpendicular to the river, $V_f = V_{mR}$

The net speed is given by $V_m = \sqrt{V_{mR}^2 + V_R^2}$

(e) Shortest Path: Velocity along the river, $V_x = 0$

and velocity perpendicular to river $V_y = \sqrt{V_{mR}^2 - V_R^2}$

The net speed is given by $V_m = \sqrt{V_{mR}^2 - V_R^2}$

at an angle of $90°$ with the river direction.

velocity V_y is used only to cross the river, therefore time to cross the river,

$t = \frac{d}{v_y} = \frac{d}{\sqrt{v_{mR}^2 - v_R^2}}$ and velocity v_x is zero, therefore, in

this case the drift should be zero.

$v_R = v_{mR}\sin\theta = 0$ or $v_R = v_{mR}\sin\theta$ or $\theta = \sin^{-1}\frac{v_R}{v_{mR}}$

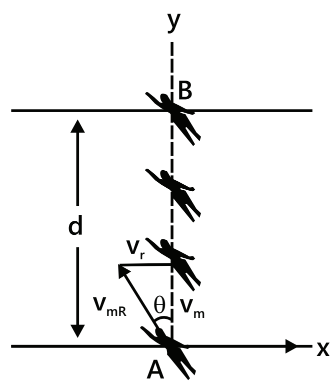


Figure 3.25

(f) Rain Problems: $v_{Rm} = \vec{v}_R - \vec{v}_m$ or $v_{Rm} = \sqrt{v_R^2 + v_m^2}$

(g) Circular Motion

i. Average angular velocity $\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$

ii. Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$

iii. Average angular acceleration $\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$

iv. Instantaneous angular acceleration $\alpha = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta}$

v. Relation between speed and angular velocity $v = r\omega$ and $v = \omega\ r$

vi. Tangential acceleration (rate of change of speed) $a_t = \frac{dV}{dt}$

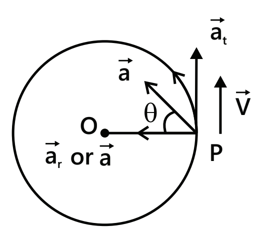


Figure 3.26

vii. Radial or normal or centripetal acceleration $a_r = \frac{V^2}{r} = \omega^2 r$

viii. Total acceleration $\vec{a} = \vec{a}_t + \vec{a}_r$, $a = \left(a_t^2 + a_r^2\right)^{1/2}$

ix. Angular acceleration $\alpha = \frac{d\omega}{dt}$ (non-uniform circular motion)

x. Radius of curvature $R = \frac{v^2}{a_\perp} = \frac{mv^2}{F_\perp}$

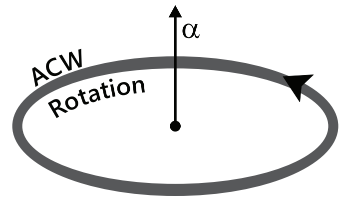


Figure 3.27

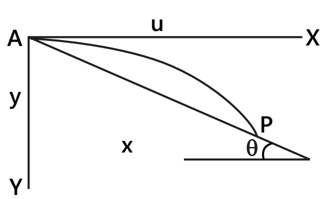
Solved Examples

JEE Main/Boards

Example 1: A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far from the point of projection will the particle strike the plane?

Sol: Take the x-axis parallel to the horizontal. Take the y-axis along the vertical. Along x-axis velocity is uniform. Along y-axis initial velocity is zero and acceleration is uniform.

Take, X–Y axes as shown in Figure. Suppose that the particle strikes the plane at point P with coordinates (x and y). Consider the motion between A and P.



Motion in x-direction: initial velocity = u

Acceleration = 0; X = ut ... (i)

Motion in y-direction: initial velocity = 0

Acceleration = g; $y = \frac{1}{2} g t^2$ ... (ii)

Eliminating t from (i) and (ii)

$y = \frac{1}{2} g \frac{x^2}{u^2}$ Also, $y = x\tan\theta$.

Thus, $\frac{gx^2}{2u^2} = x\tan\theta$ giving x = 0, or, $\frac{2u^2\tan\theta}{g}$

Clearly the point P corresponds to x = $\frac{2u^2\tan\theta}{g}$

Then, $y = x\tan\theta = \frac{2u^2\tan^2\theta}{g}$

The distance $AP = l = \sqrt{x^2 + y^2}$

$= \frac{2u^2}{g}\tan\theta\sqrt{1+\tan^2\theta} = \frac{2u^2}{g}\tan\theta\sec\theta$

Examples 2: A projectile is projected at an angle 60° from the horizontal with a speed of $\left(\sqrt{3}+1\right)$ m/s. The time (in seconds) after which the inclination of the projectile with horizontal becomes 45° is:

Sol: Take the x-axis along the horizontal. Take the y-axis vertically upwards. Along x-axis velocity is uniform. Along y-axis initial velocity is positive and acceleration is uniform and negative.

Let "u" and "v" be the speed at the two specified angles. The initial components of velocities in horizontal and vertical directions are:

$u_x = u\cos 60^\circ$

$u_y = u\sin 60^\circ$