Class 12

2017-18

By Top 100 IIT-JEE rankers & Senior Faculty of Premier Institutes.

# PHYSICS

## FOR JEE MAIN & ADVANCED

SECOND EDITION

Exhaustive Theory
(Now Revised)

Formula Sheet

9000+ Problems
based on latest JEE pattern

2500 + 1000 (New) Problems
of previous 35 years of
AIEEE (JEE Main) and IIT-JEE (JEE Adv)

5000+Illustrations and Solved Examples

Detailed Solutions
of all problems available

Topic Covered

Electric Charges,
Forces and Fields

Plancess Concepts

Tips & Tricks, Facts, Notes, Misconceptions,
Key Take Aways, Problem Solving Tactics

PlancEssential

Questions recommended for revision

PLANCESS®
WHERE PLAN MEETS SUCCESS

# 18. ELECTRIC CHARGES, FORCES AND FIELDS

## 1. INTRODUCTION

You must have felt the attraction of hair of your hand when you bring it near to your Television screen. Did you ever think of cause behind it? These all are the electric charges and their properties. Now we will extend our concept to electric charges and their effects.

### 1.1 Nature of Electricity

The atomic structure shows that matter is electrical in nature i.e. matter contains particles of electricity viz. protons and electrons. Whether a given body shows electricity (i.e. charge) or not depends upon the relative number of these particles in the body.

**(a)** If the number of protons is equal to the number of electrons in a body, the resultant charge is zero and the body will be electrically neutral. For example, the paper of this book is electrically neutral (i.e. exhibits no charge) because it has the same number of protons and electrons.

**(b)** If from a neutral body, some *electrons are removed, the protons outnumber the electrons. Consequently, the body attains a positive charge. Hence, a positively charged body has deficit of electrons from the normal due share.

## 2. TYPES OF CHARGES

Depending upon whether electrons are removed or added to a body, there are two types of charges viz

(i) Positive charge (ii) Negative charge

If a glass rod is rubbed with silk, some electrons pass from glass rod to silk. As a result, the glass rod becomes positively charged and silk attains an equal negative charge as shown in Fig. 18.1. It is because silk gains as many electrons as lost by the glass rod. It can be shown experimentally that like charges repel each other while unlike charges attract each other. In other words, if the two charges are of the same nature (i.e., both positive or both negative), the force between them is of repulsion. On the other hand, if one charge is positive and the other is negative, the force between them is of attraction. The following points may be noted:

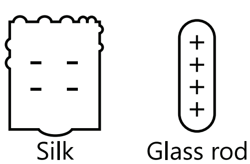



**Figure 18.1**

**(a)** The charges are not created by the rubbing action. There is merely transfer of electrons from one body to another.

**(b)** Electrons are transferred from glass rod to silk due to rubbing because we have done external work. Thus law of conservation of energy holds.

**(c)** The mass of negatively charged silk will increase and that of glass rod will decrease. It is because silk has gained electrons while glass rod has lost electrons.

## 3. PROPERTIES OF CHARGE

**(a)** Charge is a scalar quantity

**(b)** Charge is transferable

**(c)** Charge is conserved

**(d)** Charge is quantized

**(e)** Like point charges repel each other while unlike point charges attract each other.

**(f)** A charged body may attract a neutral body or an oppositely charged body but it always repels a similarly charged body

**(g)** Note: Repulsion is a sure test of electrification whereas attraction may not be.

**(h)** Charge is always associated with mass, i.e. charges cannot exist without mass though mass can exist without charge.

**(i)** Charge is relatively invariant: This means that charge is independent of frame of reference, i.e, charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

**(j)** A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself while a charge having acceleration emits electromagnetic radiation also in addition to producing electric and magnetic fields.

## 4. ELECTROSTATICS

The branch of physics which deals with charges at rest is called electrostatics. When a glass rod is rubbed with silk and then separated, the former becomes positively charged and the latter attains equal negative charge. It is because during rubbing, some electrons are transferred from glass to silk. Since glass rod and silk are separated by an insulating medium (i.e. air), they retain the charges. In other words, the charges on them are static or stationary. Note that the word 'electrostatic' means charges at rest.

## 5. CONDUCTORS AND INSULATORS

In general, the substances are divided into the following two classes on the basis of their ability to conduct electric charges:

**(a) Conductors:** Those substances through which electric charges can flow easily are called conductors e.g., silver, copper, aluminum, mercury, etc. In a metallic conductor, there are a large number of free electrons which act as charge carries. However, in a liquid conductor, both positive and negative ions are the charge carries. When a positively charged body is brought close to or touches a neutral conductor (metallic), the free electrons (charge carriers) in the conductor move quickly towards this positive charge. On the other hand, if a negatively charged body is brought close to or touches a neutral conductor, the free electrons in the conductor move away from the negative charge that is brought close.

**(b) Insulators:** Those substances through which electric charges cannot flow are called insulators e.g., glass, rubber, mica etc. When such materials are charged by rubbing, only the area that is rubbed becomes charged and there is no tendency of the charge to move into other regions of the substance. It is because there are practically no free electrons in an insulator.

## 6. CHARGING OF A BODY

A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermionic ionization, (e) photoelectric effect and (f) field emission.

**(a) Charging by Friction:** When a neutral body is rubbed with another neutral body (at least one of them should be insulator) then some electrons are transferred from one body to another. The body which gains electrons becomes negatively charged and the other becomes positively charged.

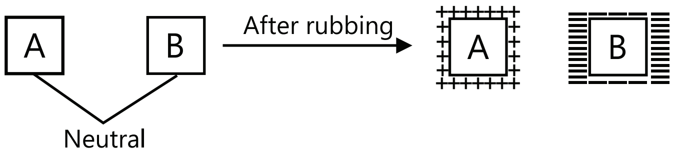



**Figure 18.2**

**(b) Conduction (flow):** There are two types of materials in nature.

**(i)** Conductor: Materials which have large number of free electrons.

**(ii)** Insulator or Dielectric or Nonconductors: Materials which do not have free electrons.

When a charged conductor is connected with a neutral conductor, then charge flows from one body to another body. In case of two charged conductors, charge flows from higher potential to lower potential. The charge stops flowing when the potential of the two bodies become same.

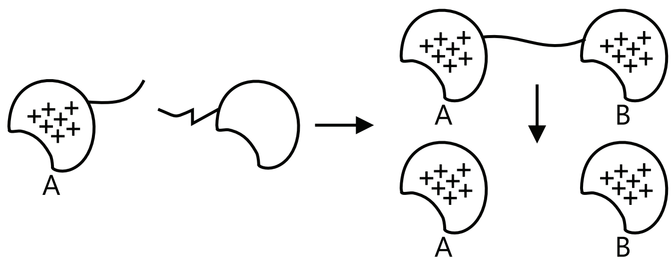



**Figure 18.3**

**Note:** If two identical shaped conductors kept at large distance are connected to each other, then they will have equal charges finally.

**(c) Induction:** When a charged particle is taken near to a neutral object, then the electrons move to one side and there is excess of electrons on that side making it negatively charged and deficiency on the other side making that side positively charged. Hence charges appear on two sides of the body (although total charge of the body is still zero). This phenomenon is called induction and the charge produced by it is called induced charge.

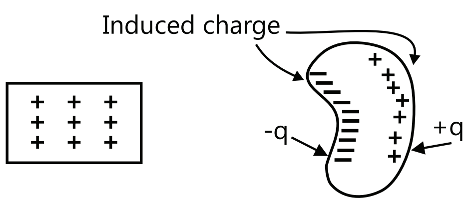



**Figure 18.4**

A body can be charged by induction in following two ways.

**Method-I:** The potential of conductor A becomes zero after earthing. To make potential zero some electrons flow from the Earth to the conductor A and now connection is removed making it negatively charged.

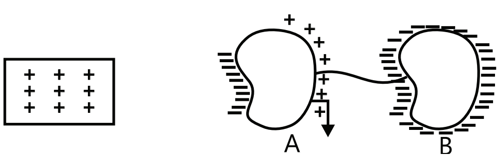



**Figure 18.5**

**Method-II:** The conductor which has included charge on it, is connected to a neutral conductor which makes the flow of charge such that their potentials become equal and now they are disconnected making the neutral conductor charged.

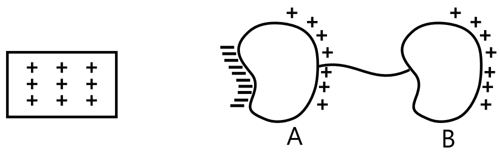



**Figure 18.6**

**(d) Thermo-ionic emission:** When the metal is heated at a high temperature then some electrons of metals are ejected and the metal gets ionized. It becomes positively charged.

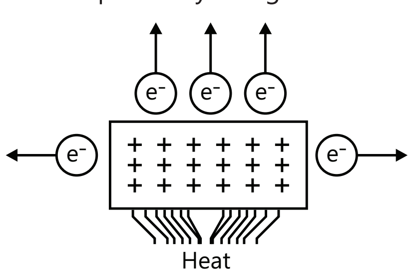



**Figure 18.7**

**(e) Photoelectric effect:** When light of sufficiently high frequency is incident on metal surface then some electrons come out and metal gets ionized.

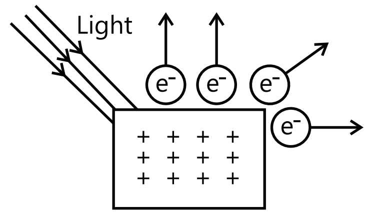



Figure 18.8

**(f) Field emission:** When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.

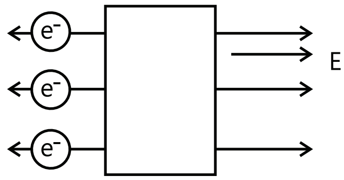



Figure 18.9

## 7. UNIT OF ELECTRIC CHARGE

We know that a positively charged body has deficit of electrons and a negatively charged body has excess of electrons from normal due share. Since the charge on an electron is very small, it is not convenient to select it as the unit of charge. In practice, coulomb is used as the unit of charge, i.e., SI unit of charge is coulomb abbreviated as C.

The charge on one electron in coulomb is $=-1.6\times10^{-19}$C

Note that charge on an electron has been found experimentally.

## 8. QUANTIZATION OF ELECTRIC CHARGE

The charge on an electron ($-e=1.6\times10^{-19}$C) or on a proton ($+e=+1.6\times10^{-19}$C) is minimum. We know that charge on a body is due to loss or gain of electrons by the body. Since a body cannot lose or gain a fraction of an electron, the charge on a body must be an integral multiple of electronic charge $\pm e$. In other words, charge on a body can only be $q=\pm ne$ where n=1, 2, 3, 4, and $e=1.6\times10^{-19}$C. This is called quantization of charge.

The fact that all free charges are integral multiple of electronic charge ($e=1.6\times10^{-19}$C) is known as quantization of electric charge.

$\therefore$ Charge on a body, $q=\pm ne$

Where n=1, 2, 3........ and $e=1.6\times10^{-19}$C

Suppose you measure the charge on a tiny body as $+4.5\times10^{-19}$C. This measurement is not correct because measured value is not an integral multiple of minimum charge (i.e., $1.6\times10^{-19}$C).

**Note:** **(i)** The quantization of charge shows that charge is discrete in nature and not of continuous nature.

**(ii)** Since the charge on an electron is so small ($e=1.6\times10^{-19}$C), we normally do not notice its discreteness in macroscopic charge (1μC charge requires about $10^{13}$ electrons) which thus seems continuous.

## 9. CONSERVATION OF ELECTRIC CHARGE

Just as total linear momentum of an isolated system always remains constant, similarly, the total electric charge of an isolated system always remains constant. This is called law of conservation of charge and may be stated as under: The total electric charge of an isolated system always remains constant.

In any physical process, the charges may get transferred from one part of the system to the other but total or net charge remains the same. In other words, charges can neither be created nor destroyed. No violation of this law has ever been found and it is as firmly established as the laws of conservation of linear momentum and energy.

| Electrostatic Force-Coulomb's Law | |
|---|---|
| F = Electrostatic force<br>q = Electric charge<br>r= Distance between charge centers<br>k= Coulomb constant $9.0\times10^{9}N.m^2/C^2$ | $F_s = k\frac{q_1q_2}{r^2}$ |

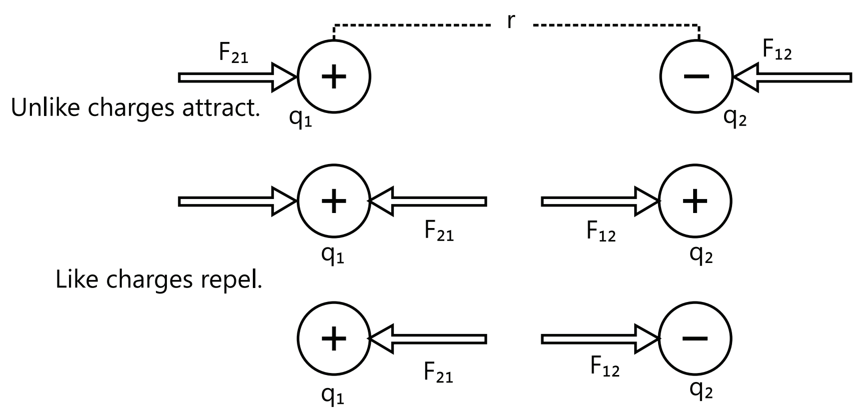



**Figure 18.10**

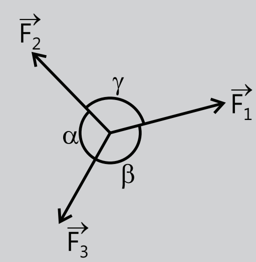
$F_{21}$ is the force on charge 1 due to 2 and $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -\vec{r}_{21}$

## PLANCESS CONCEPTS

In few problems of electrostatics, Lami's theorem is very useful.

According to this theorem, "if three concurrent forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ as shown in Fig. 18.11 are in equilibrium or if $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$, then

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$




**Nivvedan (JEE 2009 AIR 113)**

**Figure 18.11**

# 10. RELATIVE PERMITTIVITY OR DIELECTRIC CONSTANT

Permittivity is the property of a medium and affects the magnitude of force between two point charges. Air or vacuum has a minimum value of permittivity. The absolute (or actual) permittivity of air or vacuum is $8.854\times10^{-12}C^2N^{-1}m^{-2}$. The absolute permittivity $\varepsilon$ of all other insulating materials is greater than $\varepsilon_0$. The ratio $\varepsilon/\varepsilon_0$ is called relative permittivity of the material and is denoted by K or ($\varepsilon_r$).

$K = \frac{\varepsilon}{\varepsilon_0} = \frac{\text{Absolute permittivity of medium}}{\text{Absolute permittivity of air (or vacuum)}}$ It may be noted that the relative permittivity is also called dielectric constant.

**Another Definition**. Force between two charges in air (or vacuum) is $F_{air} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ [See Fig. 18.12]

Force between the same two charges held same distance apart in a medium of absolute permittivity $\varepsilon$ is

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1q_2}{r^2} \quad \text{[see Fig. 18.12]}$$

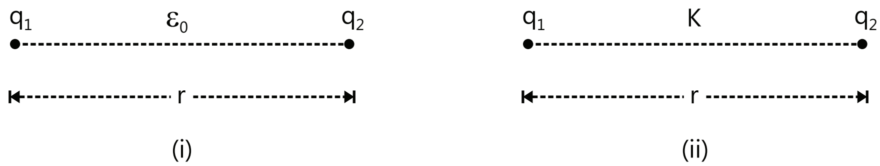



**Figure 18.12**

$$\therefore \frac{F_{air}}{F_m} = \frac{\varepsilon}{\varepsilon_0} = K = \text{Relative permittivity of the medium}$$

Hence, relative permittivity (or dielectric constant) of a medium may be defined as the ratio of force between two charges separated by a certain distance in air (or vacuum) to the force between the same charges separated by the same distance in the medium.

**Discussion**. The following points may be noted:

**(a)** For air or vacuum, $K = \varepsilon_0 / \varepsilon_0 = 1$. For all other insulating materials, the value of K is more than 1.

**(b)** $F_m = F_{air} / K$. This implies that force between two charges is decreased when air is replaced by other insulating medium. For example, K for water is 80. It means that for the same charges $(q_1, q_2)$ and same distance (r), the force between two charges in water is 1/80$^{th}$ of that in air.

**(c)** K is number; being the ratio of two absolute permitivities. $\boxed{K = \frac{F_{air}}{F_{med}}}$ $K = \frac{\varepsilon}{\varepsilon_0}$

**Comparison of Electrical Force with the Gravitational Force.**

**(a)** Both electrical and gravitational forces follow the inverse square law.

**(b)** Both can act in vacuum also.

**(c)** Electrical forces may be attractive or repulsive but gravitational force is always attractive.

**(d)** Electrical forces are much stronger than gravitational forces.

**(e)** Both are central as well as conservative forces.

**(f)** Both the forces obey Newton's third law.

## 11. SUPERPOSITION OF ELECTROSTATIC FORCE

If in a region, more than 2 charges are present, then the net force acting on a particular charge will be the vector sum of the individual contribution of all other charges present in region, presence of any other charge in space cannot affect the force applied by a particular charge.

$$\vec{F}_{1.net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \text{.........} + \vec{F}_{1n},$$

**Illustration 1:** Two identical balls each having a density $\rho$ are suspended from a common point by two insulating strings of equal length. Both the balls have equal mass and charge. In equilibrium, each string makes an angle $\theta$ with vertical. Now, both the balls are immersed in a liquid. As a result, the angle $\theta$ does not change. The density of the liquid is $\sigma$. Find the dielectric constant of the liquid. **(JEE ADVANCED)**

**Sol:** Inside the liquid, up thrust would act but simultaneously, electric force would also weaken due to dielectric of the liquid.

In vacuum each ball is in equilibrium under the following three forces:

(i) Tension, (ii) Electric force and (iii) Weight.

So, Lami's theorem can be applied.

In the liquid, $F_e^{'} = \frac{F_e}{K}$ Where, K=dielectric constant of liquid and W'=W-up thrust

Applying Lami's theorem in vacuum

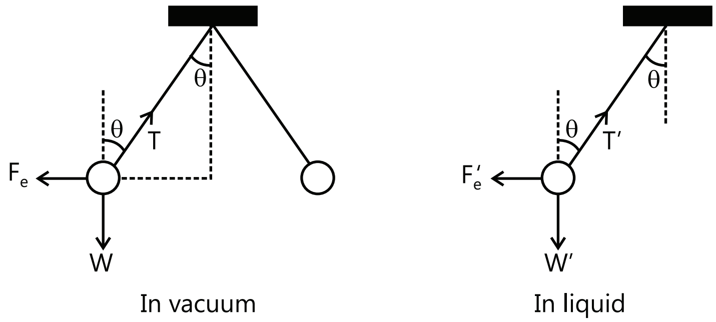



**Figure 18.13**

$$\frac{W}{\sin\left(90^o+\theta\right)} = \frac{F_e}{\sin\left(180^o-\theta\right)} \text{ or } \frac{W}{\cos\theta} = \frac{F_e}{\sin\theta} \quad \text{... (i)}$$

Similarly in liquid $\frac{W'}{\cos\theta} = \frac{F_e^{'}}{\sin\theta}$ ... (ii)

Dividing Eq.(i) by Eq.(ii), we get $\frac{W}{W'} = \frac{F_e}{F_e^{'}}$ or $K = \frac{W}{W - \text{upthrust}}$ $\left(\text{as } \frac{F_e}{F_e^{'}} = k\right)$

$\frac{V\rho g}{V\rho g - V\sigma g}$ (V=volume of ball) Or $K = \frac{\rho}{\rho - \sigma}$

**Note:** In the liquid $F_e$ and W have been changed. Therefore, T will also change.

**Illustration 2:** A non-conducting rod of length L with a uniform positive charge density $\lambda$ and a total charge Q is lying along the x-axis, as illustrated in Fig. 18.14. **(JEE ADVANCED)**

Calculate the force at a point P located along the axis of the rod and a distant $x_0$ from one end of the rod.

**Sol:** Consider rod as large number of small charges and apply principle of superposition of forces.

The linear charge density is uniform and is given by $\lambda = Q/L$. The amount of charge contained in a small segment of length $dx'$ is $dq = \lambda dx'$.

Since the source carries a positive charge Q, the force at P points in the negative x direction, and the unit vector that points from the source to P is $\hat{r} = \hat{i}$. The contribution to the electric field due to dq is

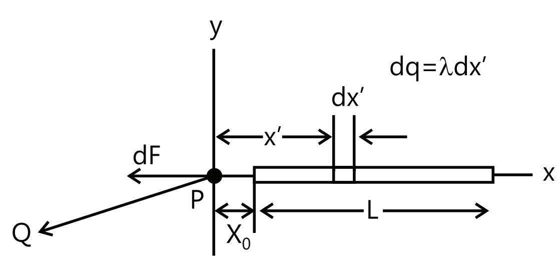



**Figure 18.14**

$$d\vec{F} = \frac{Q}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r} = \frac{Q}{4\pi\varepsilon_o}\frac{\lambda dx'}{x'^2}\left(-\hat{i}\right) = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2 dx'}{Lx'^2}\hat{i}$$

Integrating over the entire length leads to

$$\vec{F} = \int d\vec{F} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{L}\int_{x_0}^{x_0+L}\frac{dx'}{x'^2}\hat{i} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{L}\left(\frac{1}{x_o} - \frac{1}{x_o+L}\right)\hat{i} = -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{x_o\left(L+x_o\right)}\hat{i}$$

Notice that when P is very far away from the rod, $x_o >> L$ and the above expression becomes $\vec{F} \approx -\frac{1}{4\pi\varepsilon_o}\frac{Q^2}{x_o^2}\hat{i}$

The result is to be expected since at sufficiently far distance away, the distinction between a continuous charge distribution and a point charge diminishes.

# 12. ELECTRIC FIELD

A charged particle cannot directly interact with another particle kept at a distance. A charge produces something called an electric field in the space around it and this electric field exerts a force on any other charge (except the source charge itself) placed in it.

Thus, the region surrounding a charge or distribution of charge in which its electrical effects can be observed is called the electric field of the charge or distribution of charge. Electric field at a point can be defined in terms of either a vector function $\vec{E}$ called 'electric field strength' or a scalar function V called 'electric potential'. The electric field can also be visualized graphically in terms of 'lines of force'. The field propagates through space with the speed of light, c. Thus, if a charge is suddenly moved, the force it exerts on another charge a distance r away does not change until a time r/c later. In our forgoing discussion we will see that electric field strength $\vec{E}$ and electric potential V are interrelated. It is similar to a case where the acceleration, velocity and displacement of a particle are related to each other.

## 12.1 Electric Field Strength $(\vec{E})$

Like its gravitational counterpart, the electric field strength (often called electric field) at a point in an electric field is defined as the electrostatic force $\vec{F}_e$ per unit positive charge. Thus, if the electrostatic force experienced by a small test charge $q_0$ is $\vec{F}_e$, then field strength at that point is defined as, $\vec{E} = \lim\limits_{q_0 \to 0} \frac{\vec{F}_e}{q_0}$

($q_0 \to 0$ so that it doesn't interfere with the electrical field)

The electric field is a vector quantity and its direction is the same as the direction of the force $\vec{F}_e$ on a positive test charge. The SI unit of electric field is N/C. Here it should be noted that the test charge $q_0$ does not disturb other charges which produces $\vec{E}$. With the concept of electric field, our description of electric interactions has two parts. First, a given charge distribution acts as a source of electric field. Second, the electric field exerts a force on any charge that is present in this field.

**An electric field leads to a force**

Suppose there is an electric field strength $\vec{E}$ at some point in an electric field, then the electrostatic force acting on a charge $+q$ is qE in the direction of $\vec{E}$, while on the charge –q it is qE in the opposite direction of $\vec{E}$.

The electric field at a point is a vector quantity. Suppose $\vec{E}_1$ is the field at a point due to a charge $q_1$ and $\vec{E}_2$ is the field at the same point due to a charge $q_2$. The resultant field when both the charges are present is $\vec{E} = \vec{E}_1 + \vec{E}_2$

If the given charges distribution is continuous, we can use the technique of integration to find the resultant electric field at a point.

**Illustration 3:** A uniform electric field E is created between two parallel charged plates as shown in Fig. 18.15. An electron enters the field symmetrically between the plates with a speed $v_o$. The length of each plate is l. Find the angle of deviation of the path of the electron as it comes out of the field. **(JEE MAIN)**

**Sol:** Electron gains velocity in the vertical direction due to field between the plates.

The acceleration of the electron is $a = \frac{eE}{m}$ in the upward direction. The horizontal velocity remains $v_o$ as there is no acceleration in this direction. Thus, the time taken in crossing the field is $t = \frac{l}{v_o}$. ... (i)

The upward component of the velocity of the electron as it emerges from the field region is

$v_y = at = \frac{eEl}{mv_o}$

The horizontal component of the velocity remains $v_x = v_o$.

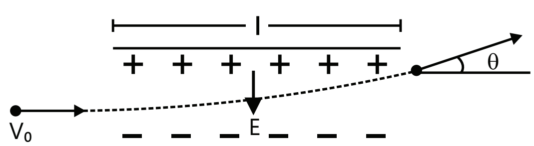



Figure 18.15

The angle $\theta$ made by the resultant velocity with the original direction is given by

$\tan\theta = \frac{v_y}{v_x} = \frac{eEl}{mv_o^2}$. Thus, the electron deviates by an angle $\theta = \tan^{-1}\frac{eEl}{mv_o^2}$.

## PLANCESS CONCEPTS

**Charge Densities**

It is of three types:

**(i) Linear charge density:** It is defined as charge per unit length, i.e.

$\lambda = \frac{q}{l}$ its S.I. unit is coulomb/ metre and dimensional formula is $\left[ATL^{-1}\right]$

**(ii) Surface charge density:** It is defined as charge per unit area, i.e.

$\sigma = \frac{q}{A}$ its S.I. unit is $\text{coulomb}/\text{metre}^2$ and dimensional formula is $\left[ATL^{-2}\right]$

**(iii) Volume charge density:** It is defined as charge per unit volume i.e.

$\rho = \frac{q}{V}$ its S.I. unit is $\text{coulomb}/\text{metre}^3$ and dimensional formula is $\left[ATL^{-3}\right]$

**Nitin Chandrol (JEE 2012 AIR 134)**

## 12.2 Electric Fields Due to Continuous Charge Distributions

The electric field at a point P due to each charge element dq is given by Coulomb's law: $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r}$

Where r is the distance from dq to P and $\hat{r}$ is the corresponding unit vector. Using the superposition principle, the total electric field $\vec{E}$ is the vector sum (integral) of all these infinitesimal contributions: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\int_V \frac{dq}{r^2}\hat{r}$

This is an example of a vector integral which consists of three separate integrations, one for each component of the electric field.

## 12.3 Electric Field Due to a Point Charge

The electric field produced by a point charge q can be obtained in general terms from Coulomb's law. First note that the magnitude of the force exerted by the charge q on a test charge $q_0$ is,

$$F_e = \frac{1}{4\pi\varepsilon_o}\cdot\frac{qq_0}{r^2}$$

Then divide this value by $q_0$ to obtain the magnitude of the field: $E = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{r^2}$

If q is positive, $\vec{E}$ is directed away from q. On the other hand, if q is negative, then $\vec{E}$ is directed towards q.

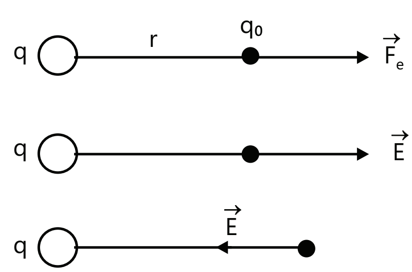



**Figure 18.16**

## 12.4 Electric Field Due to a Ring of Charge

A conducting ring of radius R has a total charge q uniformly distributed over its circumference. We are interested in finding the electric field at point P that lies on the axis of the ring at a distance x from its center. We divide the ring into infinitesimal segments of length dl. Each segment has a charge dq and acts as a point charge source of electric field.

Let $d\vec{E}$ be the electric field from one such segment; the net electric field at p is then the sum of all contributions $d\vec{E}$ from all the segments that make up the ring. If we consider two ring segments at top and bottom of the ring, we see that the contributions $d\vec{E}$ to the field at P from these segments have the same x-component but opposite y-components. Hence, the total y-component of field due to this pair of segments is zero. When we add up the contributions from all such pairs of segments, the total field $\vec{E}$ will have only a component along the ring's symmetry axis (the x-axis) with no component perpendicular to that axis (i.e. no y or z component). So the field at P is described completely by its x component $E_x$.

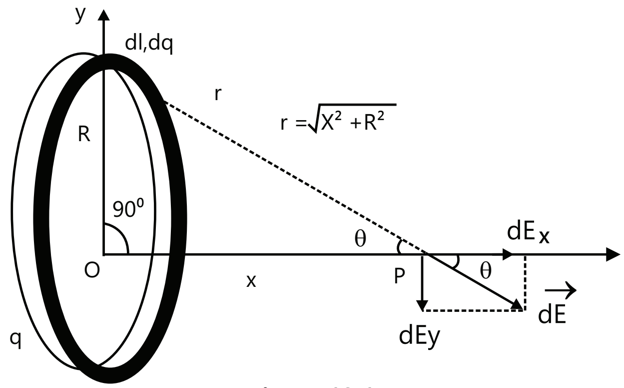



Figure 18.17

**Calculation of** $E_x$ $dq = \left(\frac{q}{2\pi R}\right).dl$; $\quad dE = \frac{1}{4\pi\varepsilon_o}.\frac{dq}{r^2}$

$$\therefore dE_x = dE\cos\theta = \left(\frac{1}{4\pi\varepsilon_o}\right)\left(\frac{dq}{x^2+R^2}\right)\left(\frac{x}{\sqrt{x^2+R^2}}\right) = \left(\frac{1}{4\pi\varepsilon_o}\right).\frac{(dq)x}{\left(x^2+R^2\right)^{3/2}}$$

$$\therefore E_x = \int dE_x = \frac{x}{4\pi\varepsilon_o\left(x^2+R^2\right)^{3/2}}\int dq \text{; or } E_x = \left(\frac{1}{4\pi\varepsilon_o}\right)\frac{qx}{\left(x^2+R^2\right)^{3/2}}$$

From the above expression, we can see that

**(a)** $E_x = 0$ at x=0, i.e., field is zero at the center of the ring. We should expect this, charges on opposite sides of the ring would push in opposite directions on a test charge at the center, and the forces would add to zero.

**(b)** $E_x = \frac{1}{4\pi\varepsilon_o}.\frac{q}{x^2}$ for x >> R i.e., when the point P is much farther from the ring, its field is the same as that of a point charge. To an observer far from the ring, the ring would appear like a point, and the electric field reflects this.

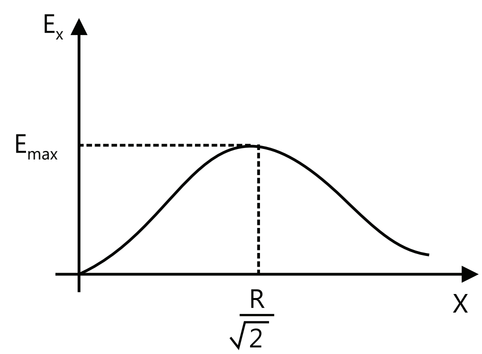



Figure 18.18

**(c)** $E_x$ will be maximum where $\frac{dE_x}{dx} = 0$. Differentiating $E_x$ w.r.t. x and putting it equal to zero we get $x = \frac{R}{\sqrt{2}}$ and $E_{max}$ comes out to be, $\frac{2}{\sqrt[3]{3}}\left(\frac{1}{4\pi\varepsilon_o}.\frac{q}{R^2}\right)$.

## 12.5 Electric Field Due to a Line Charge

Positive charge q is distributed uniformly along a line with length 2a, lying along the y-axis between y=-a and y=+a. We are here interested in finding the electric field at point P on x-axis.

$\lambda$ =charge per unit length= $\frac{q}{2a}$ $dq = \lambda dy = \frac{q}{2a}dy$; $dE = \frac{1}{4\pi\varepsilon_o}.\frac{dq}{r^2} = \frac{q}{4\pi\varepsilon_o}\frac{dy}{2a\left(x^2+y^2\right)}$

$$dE_x = dE\cos\theta = \frac{q}{4\pi\varepsilon_o}.\frac{xdy}{2a\left(x^2+y^2\right)^{3/2}}$$

$$dE_y = -dE\sin\theta = \frac{q}{4\pi\varepsilon_o}\cdot\frac{ydy}{2a\left(x^2+y^2\right)^{3/2}}$$

$$\therefore E_x = \frac{1}{4\pi\varepsilon_o}\cdot\frac{qx}{2a}\int_{-a}^{a}\frac{dy}{\left(x^2+y^2\right)^{3/2}} = \frac{q}{4\pi\varepsilon_o}\cdot\frac{1}{x\sqrt{x^2+a^2}}$$

and $E_y = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{2a}\int_{-a}^{a}\frac{ydy}{\left(x^2+y^2\right)^{3/2}} = 0$

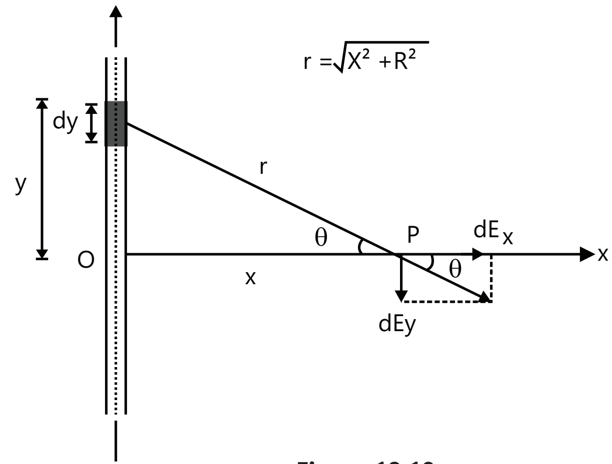



**Figure 18.19**

Thus, electric field is along x-axis only and which has a magnitude,

$$E_x = \frac{q}{4\pi\varepsilon_o x\sqrt{x^2+a^2}} \quad \text{... (i)}$$

From the above expression, we can see that:

**(a)** If $x >> a$, $E_x = \frac{1}{4\pi\varepsilon_o}\cdot\frac{q}{x^2}$, i.e., if point P is very far from the line charge, the field at P is the same as that of a point charge.

**(b)** Now assume that, we make the line of charge longer and longer, adding charge in proportion to the total length so that $\lambda$, the charge per unit length remains constant. In this case Eq(i) can be written as,

$$E_x = \frac{1}{2\pi\varepsilon_o}\cdot\left(\frac{q}{2a}\right)\cdot\frac{1}{x\sqrt{x^2/a^2+1}} = \frac{\lambda}{2\pi\varepsilon_o x\sqrt{x^2/a^2+1}}$$

Now, $= x^2/a^2 \to 0$ as $a >> x$, $E_x = \frac{\lambda}{2\pi\varepsilon_o x}$

## 13. ELECTRIC FIELD LINES

An electric line of force is an imaginary smooth curve in an electric field along which a free, isolated unit positive charge moves.

**Properties**

**(a)** Electric lines of force start at a positive and terminate at a negative charge.

**(b)** A tangent to a line of force at any point gives the direction of the force on positive charge and hence direction of electric field at that point.

**(c)** No two lines of force can intersect one another.

**(d)** The lines of force are crowded in the region of larger intensity and further apart in the region of weak field.

**(e)** Lines of force leave the surface of a conductor normally.

**(f)** Electric lines of force do not pass through a closed conductor.

**Field of some special classes**

We here highlight the following charge distributions.

**(a)** Single positive or negative charge (Fig. 18.20 (a) and (b))- The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward.

**(b)** Two equal positive charges (Fig18.20 (c))- the field lines around a system of two positive charges (q, q) give a vivid pictorial description of their mutual repulsion.

**(c)** Two equal and opposite charges (Fig18.20 (d))-The field around the configuration of two equal and opposite charges (q,-q), a dipole, show clearly the mutual attraction between the charges.

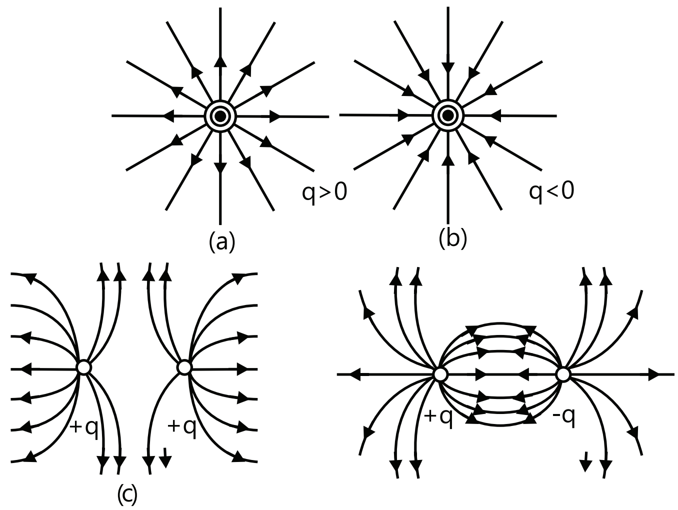



**Figure 18.20**

**Properties:**

**(a)** Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge then lines start from positive charge and terminate at $\infty$. If there is only one negative charge then lines start from $\infty$ and terminate at negative charge.

**(b)** The electric intensity at a point is the number of lines of force streaming through per unit area normal to the direction of the intensity at that point. The intensity will be more where the density of lines is more.

**(c)** Number of lines originating (terminating) from (on) is directly proportional to the magnitude of the charge.

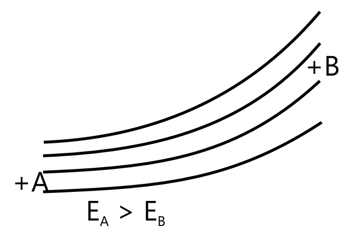



**Figure 18.21**

**Note:** A charge particle need not follow an Electric field lines.

**(a)** Electric field lines of resultant electric field can never intersect with each other.

**(b)** Electric field lines produced by static charges do not form close loop.

**(c)** Electric field lines end or start perpendicularly on the surface of a conductor.

**(d)** Electric field lines never enter in to conductors.

**Illustration 4:** Consider the situation shown in Fig. 18.22. What are the signs of $q_1$ and $q_2$? If the lines are drawn in proportion to the charge, what is the ratio $q_1 / q_2$? **(JEE MAIN)**

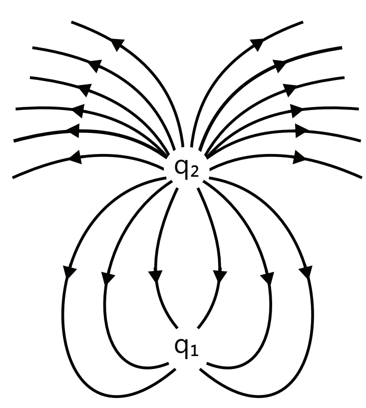



**Figure 18.22**

**Sol:** Use properties of field lines.

The basic concept of this question is that number density is directly proportional to electric field. If we take the entire area of the sphere around the charge, then area will be the same. Now, we just have to count the number of lines originating from the two charges.

In case of point charges, $E \propto q$

Thus, $E_1 / E_2 = q_1 / q_2 = n_1 / n_2 = 6/18 = 1/3$

However, this problem can also be seen by flux. Why don't you try it as an exercise?Plus, $q_1$ has to be negative, while $q_2$ would be positive.

## 14. ELECTRIC FLUX

The strength of an electric field is proportional to the number of field lines per unit area. The number of electric field lines that penetrates a given surface is called an "electric flux," which we denote as $\Phi_E$. The electric field can therefore be thought of as the number of lines per unit area.

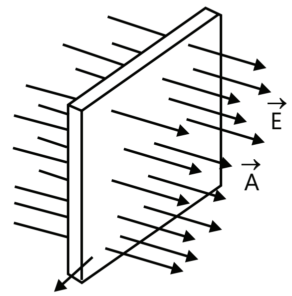



**Figure 18.23**

In Fig. 18.23 shows Electric field lines passing through a surface of area A.

Consider the surface shown in Fig. 18.24. Let $\vec{A} = A\hat{n}$ be defined as the area vector having a magnitude of the area of the surface, A, and pointing in the normal direction, $\hat{n}$. If the surface is placed in a uniform electric field $\vec{E}$ that points in the same direction as $\hat{n}$, i.e., perpendicular to the surface A, the flux through the surface is

$$\Phi_E = \vec{E}.\vec{A} = \vec{E}.\hat{n}A = EA$$

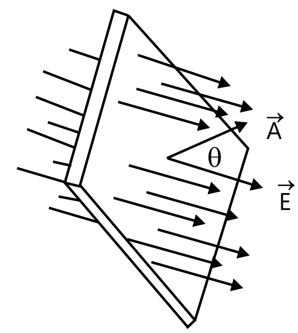



**Figure 18.24**

On the other hand, if the electric field $\vec{E}$ makes an angle $\theta$ with $\hat{n}$, the electric flux becomes $\Phi_E = \vec{E}.\vec{A} = \vec{E}.A\cos\theta = E_n A$

Where $E_n = \vec{E}.\hat{n}$ is the component of $\vec{E}$ perpendicular to the surface.

Note that with the definition for the normal vector $\hat{n}$, the electric flux $\Phi_E$ is positive if the electric field lines are leaving the surface, and negative if entering the surface.

In general, a surface S can be curved and the electric field $\vec{E}$ may vary over the surface. We shall be interested in the case where the surface is closed. A closed surface is a surface which completely encloses a volume. In order to compute the electric flux, we divide the surface into a large number of infinitesimal area elements $\Delta\vec{A}_i = \Delta A_i \hat{n}_i$, as shown in Fig. 18.25. Note that for a closed surface, the unit vector $\hat{n}_i$ is chosen to point in the outward normal direction.

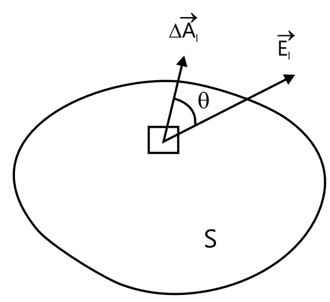



**Figure 18.25**

Electric field is passing through an area element $\Delta\vec{A}_i$, making an angle $\theta$ with the normal of the surface.

The electric flux through $\Delta\vec{A}_i$ is $\Delta\Phi_E = \vec{E}_i.\Delta\vec{A}_i = E_i \Delta A_i \cos\theta$

The total flux through the entire surface can be obtained by summing over all the area elements. Taking the limit $\Delta\vec{A}_i \to 0$ and the number of elements to infinity, we have $\Delta\Phi_E = \lim_{\Delta A_i \to 0} \sum \vec{E}_i.d\vec{A}_i = \int \vec{E}.d\vec{A}$

In order to evaluate the above integral, we must first specify the surface and then sum over the dot product $\vec{E}.d\vec{A}$..

Let $\Delta\vec{A}_1 = \Delta A_1 \hat{r}$ be

An area element on the surface of a sphere $S_1$ of radius $r_1$, as shown in Fig. 18.26.

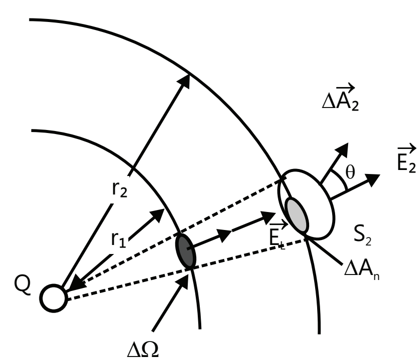



**Figure 18.26**

The area element $\Delta A$ subtends a solid angle $\Delta\Omega$. The solid angle $\Delta\Omega$ subtended by $\Delta\vec{A}_1 = \Delta A_1 \hat{r}$ at the center of the sphere is defined as

$$\Delta\Omega \equiv \frac{\Delta A_1}{r_1^2}$$

Solid angles are dimensionless quantities measured in steradians (sr). Since the surface area of the sphere $S_1$ is $4\pi r_1^2$, the total solid angle subtended by the sphere is

$$\Omega = \frac{4\pi r_1^2}{r_1^2} = 4\pi$$

In Fig. 18.26, the area element $\Delta\vec{A}_2$ makes an angle $\theta$ with the radial unit vector $\hat{r}$, then the solid angle subtended by $\Delta A_2$ is

$$\Delta\Omega = \frac{\Delta\vec{A}_2.\hat{r}}{r_2^2} = \frac{\Delta A_2 \cos\theta}{r_2^2} = \frac{\Delta A_{2n}}{r_2^2}$$

**Illustration 5:** A non - uniform electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 18.28 (E is in newton per coulomb and x is in meters.) What is the electric flux through the right face, the left face, and the top face? **(JEE ADVANCED)**

**Sol:** We can find the flux through the surface by integrating the scalar product $\vec{E}.d\vec{A}$ over each face.

Right face: An area vector $\vec{A}$ is always perpendicular to its surface and always points away from the interior of a Gaussian surface. Thus, the vector $d\vec{A}$ for the right face of the cube must point in the positive direction of the x axis. In unit-vector notation,

$d\vec{A} = dA\hat{i}$. The flux $\Phi$, through the right face is then

$$\Phi = \int \vec{E}.d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}).(dA\,\hat{i}) = \int \left[(3.0x)(dA)\hat{i}\cdot\hat{i} + (4.0)(dA)\hat{j}\cdot\hat{i}\right] = \int (3.0xdA + 0) = 3.0\int xdA.$$

We are about to integrate over the right face, but we note that x has the same value everywhere on that face-namely, x=3.0m. This means we can substitute that constant value for x. Then

$$\Phi_r = 3.0\int (3.0)dA = 9.0\int dA.$$

The integral $\int dA$ merely gives us the area A=4.0 $m^2$ of the right face; so

$$\Phi_r = (9.0N/C)(4.0m^2) = 36N.m^2/C.$$

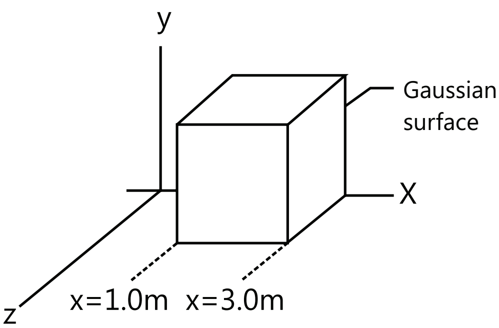


**Figure 18.27**

**Left face:** The procedure for finding the flux through the left face is the same as that for the right face. However, two factors change. (i) The differential area vector $d\vec{A}$ points in the negative direction of the x axis, and thus $d\vec{A} = -dA\hat{i}$. (ii) The term x again appears in our integration, and it is again constant over the face being considered. However, on the left face, x=1.0m. With these two changes, we find that the flux $\Phi_l$ through the left face is

$$\Phi_1 = -12N\cdot m^2/C.$$

**Top face:** The differential area vector $d\vec{A}$ points in the positive direction of the y axis, and thus $d\vec{A} = +dA\hat{j}$. The flux $\Phi_l$ through the top face is then

$$\Phi_1 = \int (3.0x\hat{i} + 4.0\hat{j})\cdot(dA\hat{j}) = \int \left[(3.0x)(dA)\hat{i}\cdot\hat{j} + (4.0)(dA)\hat{j}.\hat{j}\right] = \int (0 + 4.0dA) = 4.0\int dA = 16\ N\cdot m^2/C.$$

## 15. GAUSS' LAW

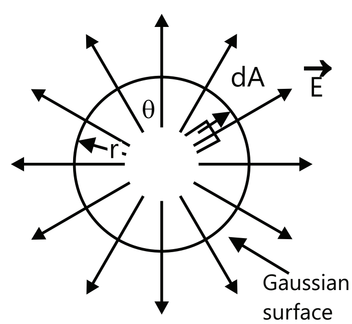


**Figure 18.28**

Consider a positive point charge Q located at the center of a sphere of radius r, as shown in Fig. 18.28. The electric field due to the charge Q is $\vec{E} = (Q/4\pi\varepsilon_0 r^2)\hat{r}$, which points in the radial direction. We enclose the charge by an imaginary sphere of radius r called the "Gaussian surface".

A spherical Gaussian surface enclosing a charge Q.

In spherical coordinates, a small surface area element on the sphere is given by
$d\vec{A} = r^2 \sin\theta d\theta d\phi \hat{r}$

A small area element on the surface of a sphere of radius r.

Thus the net electric flux through the area element is

$$d\Phi_E = \vec{E}\cdot d\vec{A} = EdA = \left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)\left(r^2 \sin\theta d\theta d\phi\right) = \frac{Q}{4\pi\varepsilon_o}\sin\theta d\theta d\phi\sqrt{a^2+b^2}$$

The total flux through the entire surface is

$$\Phi_E = \oiint_s \vec{E}\cdot d\vec{A} = \frac{Q}{4\pi\varepsilon_o}\int_o^z \sin\theta d\theta \int_o^{2\pi} d\phi = \frac{Q}{\varepsilon_o}$$

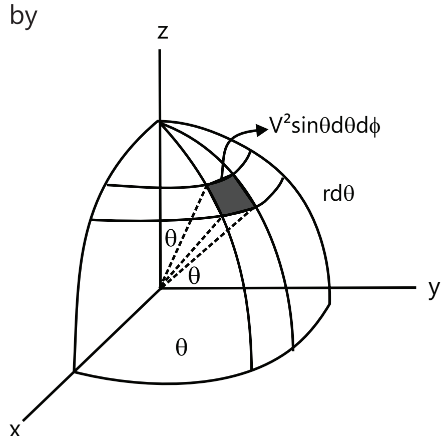



Figure 18.29

The same result can also be obtained by noting that a sphere of radius r has a surface area $A = 4\pi r^2$, and since the magnitude of the electric field at any point on the spherical surface is $E = Q/4\pi\varepsilon_o r^2$, the electric flux through the surface is

$$\Phi_E = \oiint_s \vec{E}\cdot d\vec{A} = E\oiint_s dA = EA = \left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}\right)4\pi r^2 = \frac{Q}{\varepsilon_o}$$

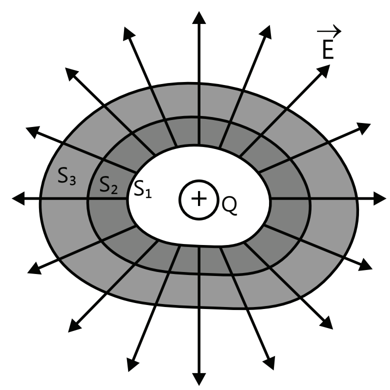



Figure 18.30

In the above, we have chosen a sphere to be the Gaussian surface. However, it turns out that the shape of the closed surface can be arbitrarily chosen. For the surfaces shown in Fig. 18.30, the same result $\left(\Phi_E = Q/\varepsilon_o\right)$ is obtained. Whether the choice is $S_1, S_2$ or $S_3$.

The statement that the net flux through any closed surface is proportional to the net charge enclosed is known as Gauss's law. Mathematically, Gauss's law is expressed as

$$\Phi_E = \oiint_s \vec{E}\cdot d\vec{A} = \frac{q_{ene}}{\varepsilon_o} \qquad \text{(Gauss's law)}$$

Where $q_{ene}$ is the net charge inside the surface. One way to explain why Gauss's law holds is that the number of field lines that leave the charge is independent of the shape of the imaginary Gaussian surface we choose to enclose the charge.

**Illustration 6:** Fig. 18.31 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1\,nC$, $q_2 = q_5 = -5.9\,nC$, and $q_3 = -3.1\,nC$? Five plastic objects, each with an electric charge, and a coin, which has no net charge. A Gaussian surface, shown in cross section, encloses three of the charged objects and the coin. **(JEE MAIN)**

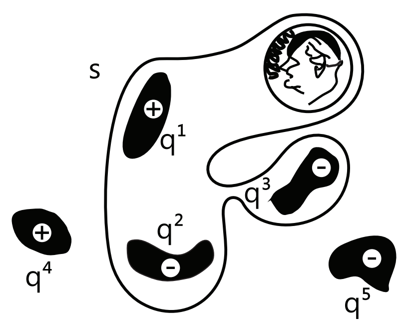



Figure 18.31

**Sol:** In Gauss's law, only enclosed charges used to calculate the flux.

The net flux $\Phi$ through the surface depends on the net charge $q_{enc}$ enclosed by surface S.

The coin does not contribute to $\Phi$ because it is neutral and thus contains equal amounts of positive and negative charge. Charges $q_4$ and $q_5$ do not contribute because they are outside surface S. Thus, $q_{enc}$ is $q_1 + q_2 + q_3$ and gives us

$$\Phi = \frac{q_{enc}}{\varepsilon_o} = \frac{q_1+q_2+q_3}{\varepsilon_o} = \frac{+3.1\times10^{-9}C - 5.9\times10^{-9}C - 3.1\times10^{-9}C}{8.85\times10^{-12}C^2/N\cdot m^2} = -670 N.m^2/C.$$

**Conclude:** The minus sign shows that the net flux through the surface is inward and thus that the net charge within the surface is negative.

**Illustration 7:** Find the flux through the disk shown in Fig. 18.32. The line joining the charge to the center of the disk is perpendicular to the disk. **(JEE MAIN)**

**Sol:** The electric flux through the disk cannot be found by the equation

$\phi = \vec{E}\cdot\vec{A} = EA\cos\theta$ If we wish to use the basic formula, we can divide the disk into small rings as shown in Fig. 18.33 and find the electric field due to charge at all the rings:

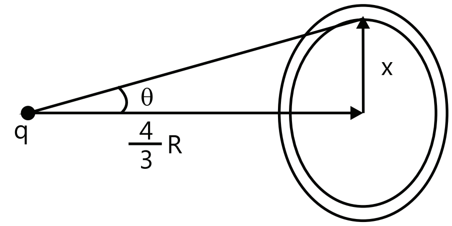


Figure 18.32

$\phi = \int \vec{E}.\vec{ds}$. Here we divide the entire disk into thin ring and find the flux due to the charge through the thin ring.

the electric field due to the point charge at the location of the ring shown is given by

$$E = \frac{kq}{(16/9)R^2 + x^2}.$$

As we discussed before, the area of the ring is $2\pi x dx$. But the electric field is not normal to the ring. The angle can be found as shown:

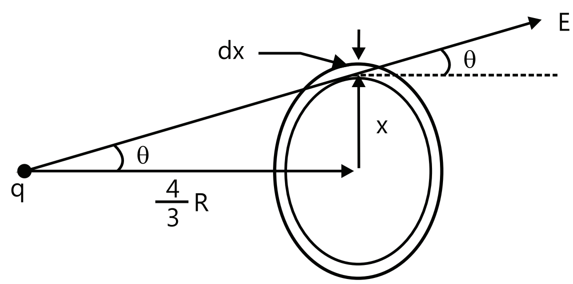


Figure 18.33

$$\cos\theta = \frac{4R/3}{\sqrt{(16R^2/9)+x^2}}, \qquad \phi = \int \vec{E}\cdot\vec{ds},$$

$$\phi = \int_0^{0.75R} \frac{kq\times(4R/3)\times 2\pi x dx}{\left[x^2+(16R^2/9)\right]\times\sqrt{x^2+(16R^2/9)}} = \frac{q}{10\varepsilon_o}.$$

**Illustration 8:** An infinitely long rod of negligible radius has a uniform charge density $\lambda$. Calculate the electric field at a distance r from the wire. **(JEE MAIN)**

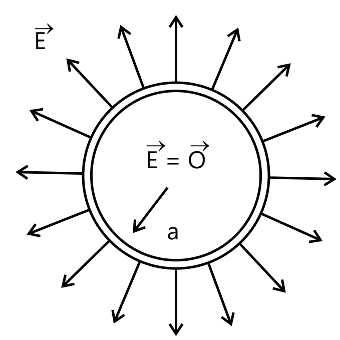


Figure 18.34

**Sol:** We shall solve the problem by following the steps outlined above.

**(a)** An infinitely long rod possesses cylindrical symmetry.

**(b)** The charge density is uniformly distributed throughout the length, and the electric field $\vec{E}$ must point radially away from the symmetry axis of the rod (Fig. 18.34). The magnitude of the electric field is constant on cylindrical surface of radius r. Therefore, we choose a coaxial cylinder as our Gaussian surface.

**(c)** Field lines for an infinite uniformly charged rod (the symmetry axis of the rod and the Gaussian cylinder are perpendicular to plane of the page.)

**(d)** The amount of charge enclosed by the Gaussian surface, a cylinder of radius r and length $\ell$ (Fig. 18.35), is $q_{enc} = \lambda\ell$.

**(e)** As indicated in Fig. 18.36, the Gaussian surface consists of three parts: a two ends $S_1$ and $S_2$ plus the curved side wall $S_3$. The flux through the Gaussian surface is

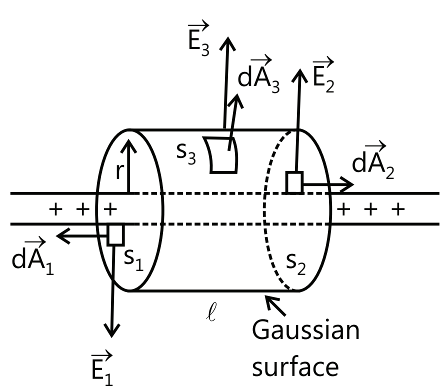


Figure 18.35

$$\Phi_E = \oiint_S \vec{E}.d\vec{A} = \iint_{S_1}\vec{E}_1\cdot d\vec{A}_1 + \iint_{S_2}\vec{E}_2\cdot d\vec{A}_2 + \iint_{S_3}\vec{E}_3\cdot d\vec{A}_3 = 0+0+E_3A_3 = E(2\pi r\ell)$$

Where we have set $E_3 = E$. As can be seen from the Fig. 18.35, no flux passes through the ends since the area vectors $d\vec{A}_1$ . and $d\vec{A}_2$ are perpendicular to the electric field which points in the radial direction.

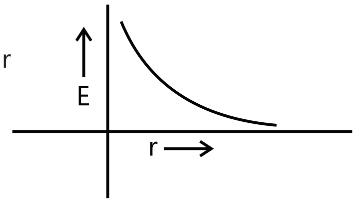



**Figure 18.36**

**(f)** Applying Gauss's Law gives $E(2\pi r\ell) = \lambda\ell / \varepsilon_o$, or $E = \dfrac{\lambda}{2\pi\varepsilon_o r}$

The result is in complete agreement with that obtained in equation using Coulomb's law. Notice that the result is independent of the length $\ell$ of the cylinder, and only depends on the inverse of the distance r from the symmetry axis. The qualitative behavior of E as a function of r is plotted in Fig. 18.36.

**Illustration 9:** Consider an infinitely large non-conduction plane in the xy-plane with uniform surface charge density $\sigma$. Determine the electric field everywhere in space. **(JEE MAIN)**

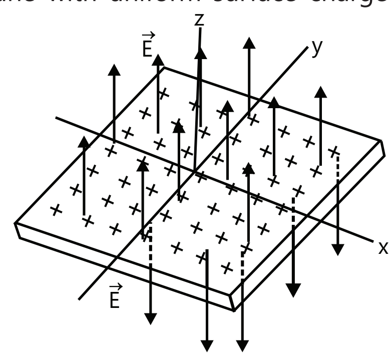



**Figure 18.37**

**Sol:** (i) An infinitely large plane possesses a planar symmetry.

(ii) Since the charge is uniformly distributed on the surface, the electric field $\vec{E}$ must point perpendicularly away from the plane, $\vec{E} = E\hat{k}$. The magnitude of the electric field is constant on planes parallel to the non-conducting plane.

We choose our Gaussian surface to be a cylinder, which is often referred to as a "pillbox"

The pillbox also consists of three parts: two end-caps $S_1$ and $S_2$, and a curved side $S_3$.

(ii) Since the surface charge distribution is uniform, the charge enclosed by the Gaussian "pillbox" is $q_{enc} = \sigma A$, where $A=A_1=A_2$ is the area of the end-caps.

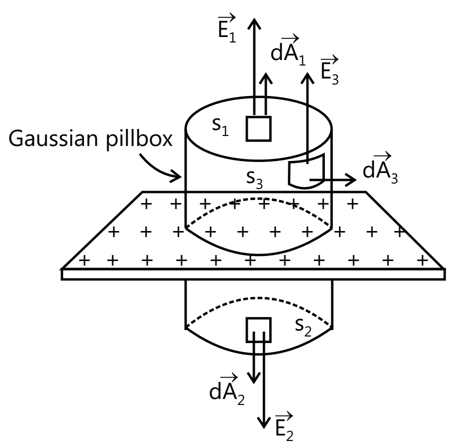



**Figure 18.38**

(iv) The total flux through the Gaussian pillbox flux is

$$\Phi_E = \oiint_S \vec{E}\cdot d\vec{A} = \oiint_{S_1} \vec{E}_1 \cdot d\vec{A}_1 + \oiint_{S_2} \vec{E}_2 \cdot d\vec{A}_2 + \oiint_{S_3} \vec{E}_3 \cdot d\vec{A}_3$$

$$= E_1A_1 + E_2A_2 + 0 = (E_1 + E_2)A$$

Since the two ends are at the same distance from the plane, by symmetry, the magnitude of the electric field must be the same: $E_1 = E_2 = E$. Hence, the total flux can be rewritten as

$$\Phi_E = 2EA$$

(v) By applying Gauss's law, we obtain $2EA = \dfrac{q_{enc}}{\varepsilon_o} = \dfrac{\sigma A}{\varepsilon_o}$ Which gives $E = \dfrac{\sigma}{2\varepsilon_o}$

In unit-vector notation, we have $\vec{E} = \begin{cases} \dfrac{\sigma}{2\varepsilon_o}\hat{k}, & z > 0 \\ -\dfrac{\sigma}{2\varepsilon_o}\hat{k}, & z < 0 \end{cases}$

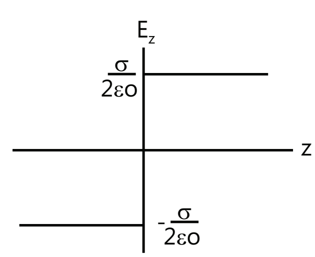



**Figure 18.39**

Thus, we see that the electric field due to an infinite large non-conducting plane is uniform in space. The result, plotted in Fig. 18.39, is the same as that obtained using Coulomb's law. Note again the discontinuity in electric field as we cross the plane:

$$\Delta E_z = E_{z+} - E_{z-} = \frac{\sigma}{2\varepsilon_o} - \left(-\frac{\sigma}{2\varepsilon_o}\right) = \frac{\sigma}{\varepsilon_o}$$

**Illustration 10:** A thin spherical shell of radius a has a charge +Q evenly distributed over its surface. Find the electric field both inside and outside the shell. **(JEE MAIN)**

**Sol:** Apply Gauss's law, as the charge distribution is symmetric.

The charge distribution is spherically symmetric, with a surface charge density $\sigma = Q/A_s = Q/4\pi a^2$, where $A_s = 4\pi a^2$ is the surface area of the sphere. The electric field $\vec{E}$ must be radially symmetric and directed outward (Fig. 18.40). We treat the regions $r \le a$ and $r \ge a$ separately.

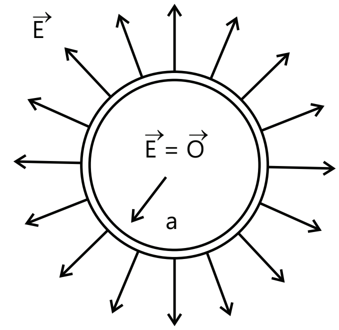



**Figure 18.40**

Electric field for uniform spherical shell of charge

**Case 1:** $r \le a$ We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in Fig. 18.41 (a).

The charge enclosed by the Gaussian surface is $q_{enc} = 0$ since all the charge is located on the surface of the shell. Thus, from Gauss's law, $\Phi_E = q_{enc}/\varepsilon_o$, we conclude E=0, r<a

**Case 2:** $r \ge a$ In this case, the Gaussian surface is a sphere of radius $r \ge a$, as shown in Fig. 18.42 (b). Since the radius of the "Gaussian sphere" is greater than the radius of the spherical shell, all the charge is enclosed: $q_{enc} = Q$

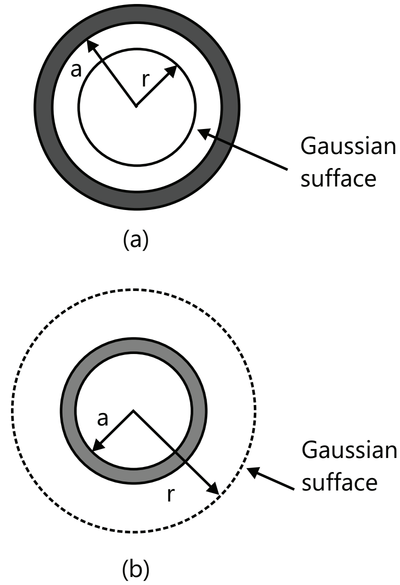



**Figure 18.41**

Since the flux through the Gaussian surface is $\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = EA = E\left(4\pi r^2\right)$

By applying Gauss's law, we obtain $E = \dfrac{Q}{4\pi\varepsilon_o r^2} = k_e \dfrac{Q}{r^2}$, $r \ge a$

Note that the field outside the sphere is the same as if all the charges were concentrated at the center of the sphere. The qualitative behavior of E as a function of r is plotted in Fig. 18.42 showing electric field as a function of r due to a uniformly charged spherical shell.

As in the case of a non-conducting charged plane, we again see a discontinuity in E as we cross the boundary at r=a. The change, from outer to the inner surface, is given by

$$\Delta E = E_+ - E_- = \frac{Q}{4\pi\varepsilon_o a^2} - 0 = \frac{\sigma}{\varepsilon_o}$$

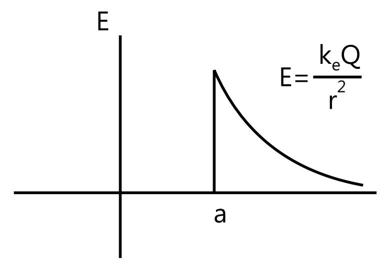



**Figure 18.42**

**Illustration 11: Non-Conducting Solid Sphere**

An electric charge +Q is uniformly distributed throughout a non-conducting solid sphere of radius a. Determine the electric field everywhere inside and outside the sphere. **(JEE MAIN)**

**Sol:** For non-conducting object. Charge distributed thoughout the mass.

The charge distribution is spherically symmetric with the charge density given by

$$\rho = \frac{Q}{V} = \frac{Q}{\left(4/3\right)\pi a^3}$$

Where V is the volume of the sphere. In this case, the electric field $\vec{E}$ is radially symmetric and directed outward. The magnitude of the electric field is constant on spherical surfaces of radius r. The regions $r \le a$ and $r \ge a$ shall be studied separately.

**Case 1:** $r \le a$

We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in Fig. 18.41 (a).

Fig. 18.41 (b) shows Gaussian surface for uniformly charged solid sphere, for (a) $r \le a$, and (b) $r > a$.

The flux through the Gaussian surface is $\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = EA = E\left(4\pi r^2\right)$ With uniform charge distribution, the charge enclosed is $q_{enc} = \int_V \rho dV = \rho V = \rho\left(\frac{4}{2}\pi r^3\right) = Q\left(\frac{r^3}{a^3}\right)$

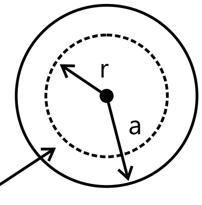


**Figure 18.43**

Which is proportional to the volume enclosed by the Gaussian surface. Applying Gauss's law

$\Phi_E = q_{enc} / \varepsilon_o$, we obtain $E\left(4\pi r^2\right) = \frac{\rho}{\varepsilon_o}\left(\frac{4}{3}\pi r^3\right)$ or $\boxed{E = \frac{\rho r}{3\varepsilon_o} = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad r \le a}$

**Case 2:** $r \ge a$

In this case, our Gaussian surface is a sphere of radius $r \ge a$, as shown in Fig. 18.44 . Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface: $q_{enc} = Q$. With the electric flux through the Gaussian surface given by $\Phi_E = E\left(4\pi r^2\right)$, upon applying Gauss's law, we obtain

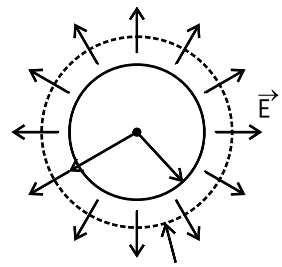


**Figure 18.44**

$E\left(4\pi r^2\right) = Q / \varepsilon_o$, or $\boxed{E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}, \qquad r > a}$

The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere. The qualitative behavior of E as a function of r is plotted in Fig. 18.45.

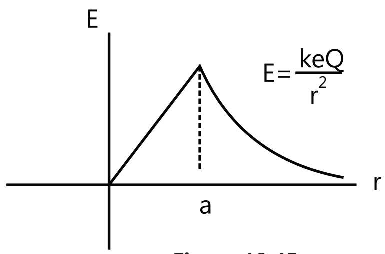


**Figure 18.45**

# PROBLEM-SOLVING TACTICS

The following steps may be useful when applying Gauss's law:

**(a)** Identify the symmetry associated with the charge distribution.

**(b)** Determine the direction of the electric field, and a "Gaussian surface" on which the magnitude of the electric field is constant over portions of the surface.

**(c)** Divide the space into different regions associated with the charge distribution. For each region, calculate $q_{enc}$ , the charge enclosed by the Gaussian surface.

**(d)** Calculate the electric flux $\Phi_E$ through the Gaussian surface for each region.

**(e)** Equate $\Phi_E$ with $q_{enc} / \varepsilon_o$, and deduce the magnitude of the electric field.

In this chapter, we have discussed how electric field can be calculated for both the discrete and continuous charge distributions. For the former, we apply the superposition principle: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$

For the latter, we must evaluate the vector integral $\vec{E} = \frac{1}{4\pi\varepsilon_o}\int \frac{dq}{r^2}\hat{r}$

Where r is the distance from dq to the field point P and $\hat{r}$ is the corresponding unit vector. To complete the integration, we shall follow the procedure outlined below:

**(a)** Start with $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r}$

**(b)** Rewrite the charge element dq as $dq = \begin{cases} \lambda d\ell & (\text{length}) \\ \sigma dA & (\text{area}) \\ \rho dV & (\text{volume}) \end{cases}$

Depending on whether the charge is distributed over a length, an area, or a volume.

**(c)** Substituting dq into the expression for $d\vec{E}$.

**(d)** Specify an appropriate coordinate system (Cartesian, cylindrical or spherical) and express the differential element $(d\ell, dA\,\text{or}\,dV)$ and r in terms of the coordinates (see table below for summary.)

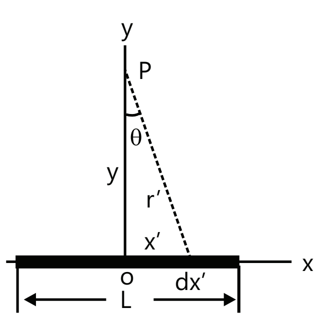
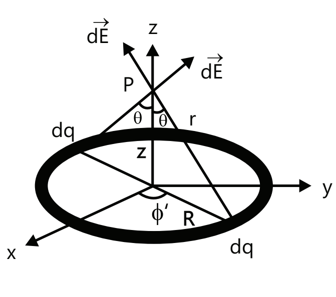
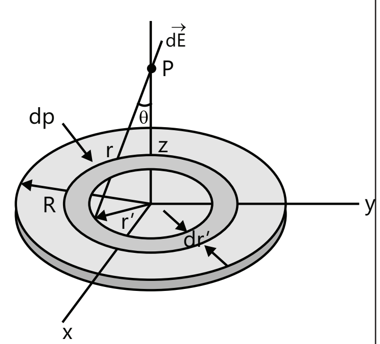
| | Cartesian (x,y,z) | Cylindrical $(\rho,\phi,z)$ | Spherical $(r,\theta,\phi)$ |
|---|---|---|---|
| Dl | dx, dy, dz | $d\rho,\ \rho d\phi,\ dz$ | $dr, rd\theta, r\sin\theta d\phi$ |
| dA | dxdy, dydz, dzdx | $d\rho dz,\ \rho d\phi dz,\ \rho d\phi d\rho$ | $rdrd\theta, r\sin\theta drd\phi, r^2\sin\theta d\theta d\phi$ |
| dV | dxdydz | $\rho d\ \rho d\phi dz$ | $r^2\sin\theta dr\,d\theta\,d\phi$ |

**Differential elements of length, area and volume in different coordinates**

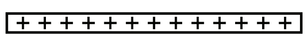
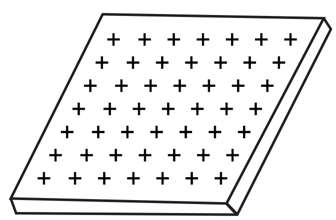
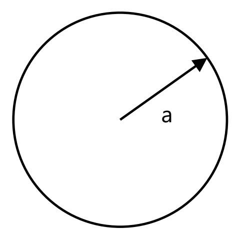
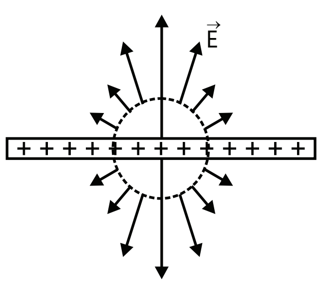
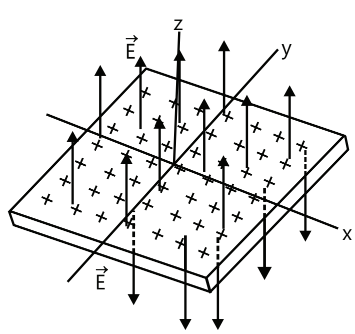
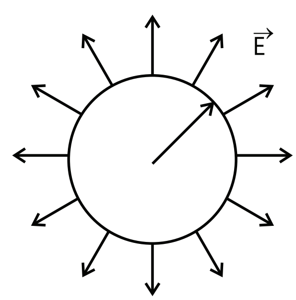
**(a)** Rewrite $d\vec{E}$ in terms of the integration variable(s), and apply symmetry argument to identify non-vanishing component(s) of the electric field.

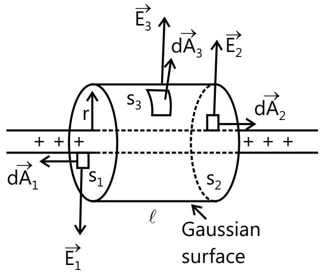
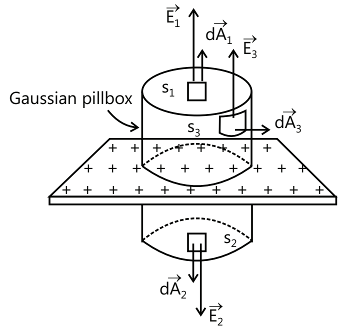
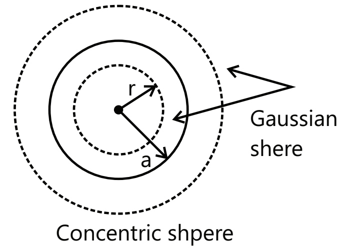
**(b)** Complete the integration to obtain $\vec{E}$.

In the Table below we illustrate how the above methodologies can be utilized to compute the electric field for an infinite line charge, a ring of charge and a uniformly charged disk.

| | **Line charge** | **Ring of charge** | **Uniformly charged disk** |
|---|---|---|---|
| (1) Figure |  **Figure 18.46** |  **Figure 18.47** |  **Figure 18.48** |
| (2) Express dq in terms of charge density | $dq = \lambda dx'$ | $dq = \lambda d\ell$ | $dq = \sigma dA$ |
| (3) write down dE | $dE = k_e\frac{\lambda dx'}{r'^2}$ | $dE = k_e\frac{\lambda dl}{r^2}$ | $dE = k_e\frac{\sigma dA}{r^2}$ |

| | Line charge | Ring of charge | Uniformly charged disk |
|---|---|---|---|
| (4) Rewrite r and the differential element in terms of the appropriate coordinates | $dx'$<br>$\cos\theta = \frac{y}{r'}$<br>$r' = \sqrt{x'^2 + y^2}$ | $d\ell = Rd\phi'$<br>$\cos\theta = \frac{z}{r}$<br>$r = \sqrt{R^2 + z^2}$ | $dA = 2\pi r'dr'$<br>$\cos\theta = \frac{z}{r}$<br>$r = \sqrt{r'^2 + z^2}$ |
| (5) Apply symmetry argument to identify non-vanishing component(s) of dE | $dE_y = dE\cos\theta$<br>$= k_e \frac{\lambda y dx'}{\left(x'^2 + y^2\right)^{3/2}}$ | $dE_y = dE\cos\theta$<br>$= k_e \frac{\lambda Rz d\phi'}{\left(R^2 + z^2\right)^{3/2}}$ | $dE_y = dE\cos\theta$<br>$= k_e \frac{2\pi\sigma z r' dr'}{\left(r'^2 + z^2\right)^{3/2}}$ |
| (6) Integrate to get E | $E_y = k_e \lambda y \int_{-\ell/2}^{+\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}}$<br>$= \frac{2k_e\lambda}{y} \frac{\ell/2}{\sqrt{\left(\ell/2\right)^2 + y^2}}$ | $E_z = k_e \frac{R\lambda z}{\left(R^2 + z^2\right)^{3/2}} \oint d\phi'$<br>$= k_e \frac{\left(2\pi R\lambda\right)z}{\left(R^2 + z^2\right)^{3/2}}$<br>$k_e \frac{Qz}{\left(R^2 + z^2\right)^{3/2}}$ | $E_z = 2\pi\sigma k_e z \int_0^R \frac{r'dr'}{\left(r'^2 + z^2\right)^{3/2}}$<br>$= 2\pi\sigma k_e \left(\frac{z}{\lvert z \rvert} - \frac{z}{\sqrt{z^2 + R^2}}\right)$ |

| System | Infinite line of charge | Infinite plane of charge | Uniformly Charged solid sphere |
|---|---|---|---|
| Figure | <br>**Figure 18.49** | <br>**Figure 18.50** | <br>**Figure 18.51** |
| Identify the symmetry | Cylindrical | Planar | Spherical |
| Determine the direction of $\vec{E}$ | <br>**Figure 18.52** | <br>**Figure 18.53** | <br>**Figure 18.54** |
| Divide the space into different regions | r>0 | Z >0 and z<0 | $r \le a$ and $r \ge a$ |

| | | | |
|---|---|---|---|
| Choose Gaussian surface | <br>**Figure 18.55** | <br>**Figure 18.56** | <br>**Figure 18.57** |
| Calculate electric flux | $\Phi_E = E(2\pi rl)$ | $\Phi_E = EA + EA = 2EA$ | $\Phi_E = E(4\pi r^2)$ |
| Calculate enclosed charge $q_{in}$ | $q_{enc} = \lambda l$ | $q_{enc} = \sigma A$ | $q_{enc} = \begin{cases} Q^{(r/a)^3} & r \le a \\ Q & r \ge a \end{cases}$ |
| Apply Gauss's law $\Phi_E = q_{in}/\varepsilon_0$ to find E | $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ | $E = \dfrac{\sigma}{2\varepsilon_0}$ | $E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 a^3}, & r \le a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r \ge a \end{cases}$ |

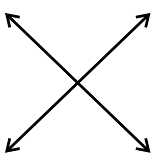
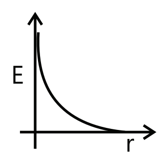
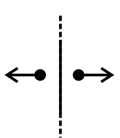
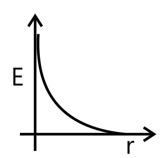
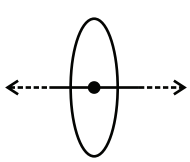
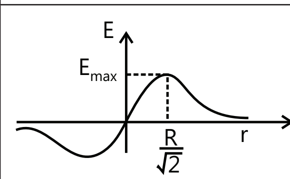
# FORMULAE SHEET

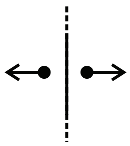
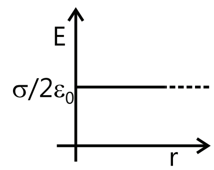
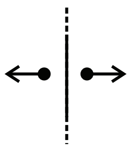
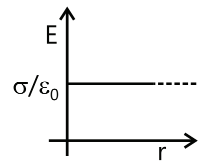
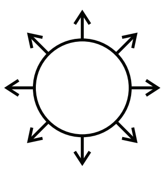
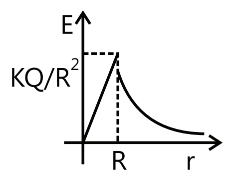
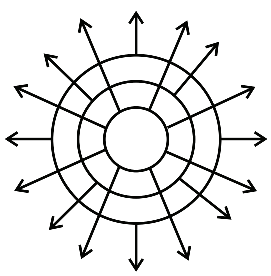
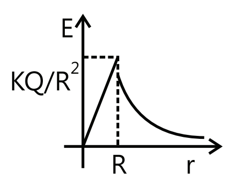
**Electric Charges, Forces and Fields**

| S. No | Term | Description |
|---|---|---|
| 1 | Charge | Charges are of two types<br>(a) Positive charge (b) Negative charge<br>Like charges repel each other and unlike charges attract each other. |
| 2 | Properties of charge | 1. Quantization:-q=ne where n=0, 1, 2...... and e is charge of an electron.<br>2. Additive: $-q_{net} = \sum q$<br>3. Conservation: - total charge of an isolated system is constant |
| 3 | Coulomb's law | The mutual electrostatic force between the charges $q_1$ and $q_2$ separated by a distance r is given by Force on the charge $q_1$ $F_1 = Kq_1q_2r_{12}/r^2$<br>Where $\hat{r}_{12}$ is the unit vector in the direction from $q_2$ and $q_1$.<br>For more than two charges in the system, the force acting on any charge is vector sum of the coulomb force from each of the other charges. This is called principle of superposition for $q_1, q_2, q_3 ..... q_n$ Charges are present in the system. |

| S. No | Term | Description |
|---|---|---|
| 4 | Electric Field | -The region around a particular charge in which its electrical effects can be observed is called the electric field of the charge<br>-Electric field has its own existence and is present even if there is no charge to experience the electric force. |
| 5 | Electric field Intensity | $E=F/q_0$ Where F is the electric force experienced by the test charge $q_0$ at this point. It is a vector quantity.<br>Some points to note on this<br>1. Electric field lines extend away from the positive charge and towards the negative charge.<br>2. Electric field produces the force so if a charge q is placed in the electric field E, the force experienced by the charge is F=qE<br>3. Principle of superposition also applies to electric field so<br>$E = E_1 + E_2 + E_3 + E_4 + .....$<br>Electric field intensity due to point charge $\vec{E} = \frac{KQ\,\vec{r}}{r^2}$<br>Where r is the distance from the point charge and r is the unit vector along the direction from source to point. |

**Electric Field Intensities due to various Charge Distributions**

| Name/Type | Formula | Note | Graph |
|---|---|---|---|
| Point Charge  | $\frac{Kq}{\lvert\vec{r}\rvert^2}\hat{r} = \frac{Kq}{r^3}\vec{r}$ | • q is source charge<br>• $\vec{r}$ is vector drawn from source charge to the test point. |  |
| Infinitely long line charge  | $\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda}{r}\hat{r}$ | • $\lambda$ is linear charge density (assumed uniform)<br>• r is perpendicular distance of point from line charge<br>• $\hat{r}$ is radial unit vector drawn from the charge to test point |  |
| Uniformly Charged Ring  | $E = \frac{KQx}{(R^2 + X^2)^{3/2}}$<br>$E_{centre} = 0$ | • Q is total charge of the ring<br>• x=distance of point on the axis from centre of the ring.<br>• Electric field is always along the axis. |  |

| | | | |
|---|---|---|---|
| Infinitely large non-conducting thin sheet  | $\frac{\sigma}{2\varepsilon_0}\hat{n}$ | • $\sigma$ is surface charge density (assumed uniform)<br>• $\hat{n}$ is unit normal vector<br>• Electric field intensity is independent of distance |   |
| Infinitely large charged conducting sheet  | $\frac{\sigma}{\varepsilon_0}\hat{n}$ | • $\sigma$ is surface charge density (assumed uniform)<br>• $\hat{n}$ is unit normal vector<br>• Electric field intensity is independent of distance |   |
| Uniformly charged hollow conducting/non conducting sphere or solid conducting sphere  | (i) for $r \geq R$<br>$\vec{E} = \frac{KQ}{\lvert\vec{r}\rvert^2}\hat{r}$<br>(ii) for $r<R$<br>$\vec{E} = 0$ | • R is radius of the sphere<br>• $\vec{r}$ is vector drawn from centre of the sphere to the test point.<br>• Sphere acts like a point charge placed at the centre for point outside the sphere.<br>• $\vec{E}$ is always along radial direction.<br>• Q is total charge $(= \sigma 4\pi R^2)$.<br>($\sigma$ = Surface charge density) |   |
| Uniformly charged solid non conducting sphere (insulating material)  | (i) for $r \geq R$<br>$\vec{E} = \frac{KQ}{\lvert\vec{r}\rvert^2}\hat{r}$<br>(ii) for $r \leq R$<br>$\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{\rho}{3\varepsilon_0}\vec{r}$ | • $\vec{r}$ is vector drawn from centre of the sphere to the test point.<br>• Sphere acts like a point charge placed at the centre for points outside the sphere.<br>• $\vec{E}$ is always along radial direction<br>• Q is total charge $(= \rho\frac{4}{3}\pi R^3)$.<br>($\rho$ =volume charge density)<br>• Inside the sphere $E \propto r$<br>• Outside the sphere $E \propto 1/r^2$ |   |

**Note:** (i) Net charge on a conductor remains only on the outer surface of a conductor.

(ii) On the surface of spherical conductors charge is uniformly distributed.