We choose our Gaussian surface to be a sphere of radius $r \le a$, as shown in Fig. 18.41 (a).

Fig. 18.41 (b) shows Gaussian surface for uniformly charged solid sphere, for (a) $r \le a$, and (b) $r > a$.

The flux through the Gaussian surface is $\Phi_E = \oiint_S \vec{E} \cdot d\vec{A} = EA = E\left(4\pi r^2\right)$ With uniform charge distribution, the charge enclosed is $q_{enc} = \int_V \rho dV = \rho V = \rho\left(\frac{4}{2}\pi r^3\right) = Q\left(\frac{r^3}{a^3}\right)$

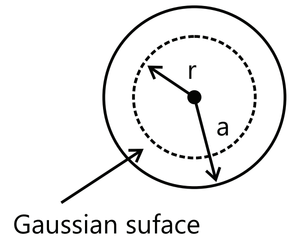


Figure 18.43

Which is proportional to the volume enclosed by the Gaussian surface. Applying Gauss's law

$\Phi_E = q_{enc} / \varepsilon_o$, we obtain $E\left(4\pi r^2\right) = \frac{\rho}{\varepsilon_o}\left(\frac{4}{3}\pi r^3\right)$ or $\boxed{E = \frac{\rho r}{3\varepsilon_o} = \frac{Qr}{4\pi\varepsilon_o a^3} \qquad r \le a}$

Case 2: $r \ge a$

In this case, our Gaussian surface is a sphere of radius $r \ge a$, as shown in Fig. 18.44 . Since the radius of the Gaussian surface is greater than the radius of the sphere all the charge is enclosed in our Gaussian surface: $q_{enc} = Q$. With the electric flux through the Gaussian surface given by $\Phi_E = E\left(4\pi r^2\right)$, upon applying Gauss's law, we obtain

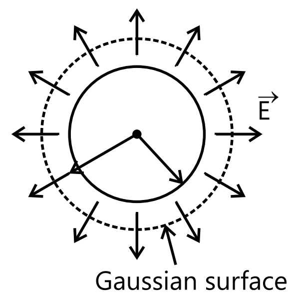


Figure 18.44

$E\left(4\pi r^2\right) = Q / \varepsilon_o$, or $\boxed{E = \frac{Q}{4\pi\varepsilon_o r^2} = k_e \frac{Q}{r^2}, \qquad r > a}$

The field outside the sphere is the same as if all the charges were concentrated at the center of the sphere. The qualitative behavior of E as a function of r is plotted in Fig. 18.45.

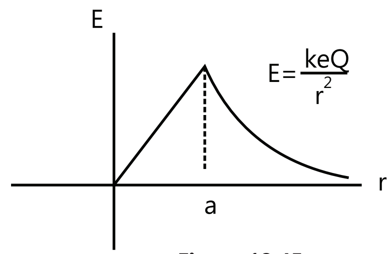


Figure 18.45

PROBLEM-SOLVING TACTICS

The following steps may be useful when applying Gauss's law:

(a) Identify the symmetry associated with the charge distribution.

(b) Determine the direction of the electric field, and a "Gaussian surface" on which the magnitude of the electric field is constant over portions of the surface.

(c) Divide the space into different regions associated with the charge distribution. For each region, calculate q_{enc}, the charge enclosed by the Gaussian surface.

(d) Calculate the electric flux Φ_E through the Gaussian surface for each region.

(e) Equate Φ_E with q_{enc} / ε_o, and deduce the magnitude of the electric field.

In this chapter, we have discussed how electric field can be calculated for both the discrete and continuous charge distributions. For the former, we apply the superposition principle: $\vec{E} = \frac{1}{4\pi\varepsilon_o}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$

For the latter, we must evaluate the vector integral $\vec{E} = \frac{1}{4\pi\varepsilon_o}\int \frac{dq}{r^2}\hat{r}$

Where r is the distance from dq to the field point P and $\hat{r}$ is the corresponding unit vector. To complete the integration, we shall follow the procedure outlined below:

(a) Start with $d\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r^2}\hat{r}$

(b) Rewrite the charge element dq as $dq = \begin{cases} \lambda d\ell & (\text{length}) \\ \sigma dA & (\text{area}) \\ \rho dV & (\text{volume}) \end{cases}$

Depending on whether the charge is distributed over a length, an area, or a volume.

(c) Substituting dq into the expression for $d\vec{E}$.

(d) Specify an appropriate coordinate system (Cartesian, cylindrical or spherical) and express the differential element $(d\ell, dA \text{ or } dV)$ and r in terms of the coordinates (see table below for summary.)

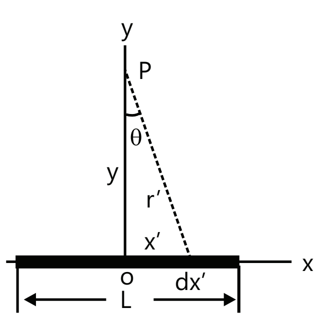
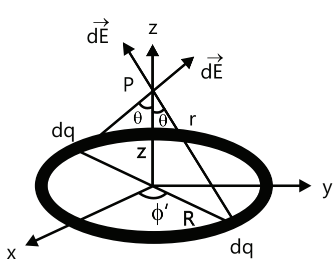
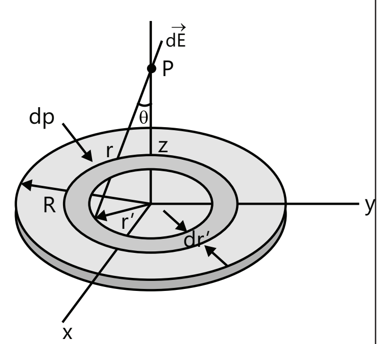
	Cartesian (x,y,z)	Cylindrical (ρ, ϕ, z)	Spherical (r, θ, ϕ)
Dl	dx, dy, dz	$d\rho,\ \rho d\phi,\ dz$	$dr, rd\theta, r\sin\theta d\phi$
dA	dxdy, dydz, dzdx	$d\rho dz,\ \rho d\phi dz,\ \rho d\phi d\rho$	$rdrd\theta, r\sin\theta drd\phi, r^2\sin\theta d\theta d\phi$
dV	dxdydz	$\rho d\ \rho d\phi dz$	$r^2\sin\theta dr\,d\theta\,d\phi$

Differential elements of length, area and volume in different coordinates

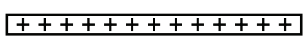
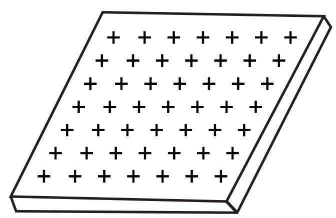
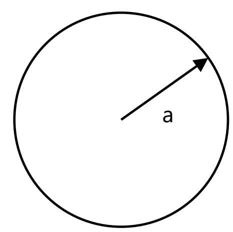
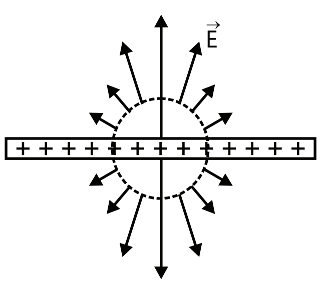
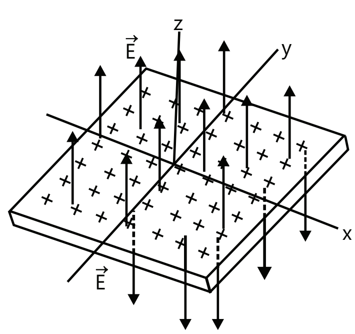
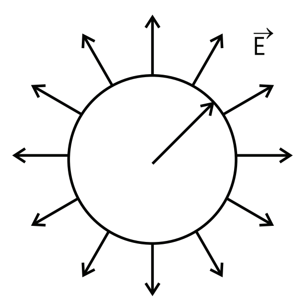
(a) Rewrite $d\vec{E}$ in terms of the integration variable(s), and apply symmetry argument to identify non-vanishing component(s) of the electric field.

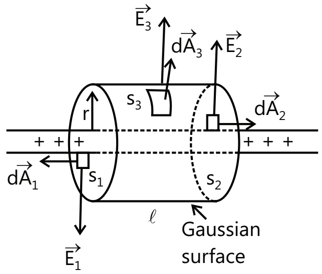
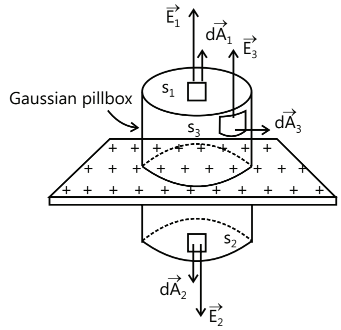
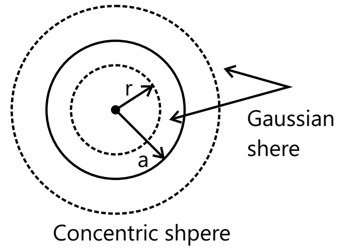
(b) Complete the integration to obtain $\vec{E}$.

In the Table below we illustrate how the above methodologies can be utilized to compute the electric field for an infinite line charge, a ring of charge and a uniformly charged disk.

	Line charge	Ring of charge	Uniformly charged disk
(1) Figure	 Figure 18.46	 Figure 18.47	 Figure 18.48
(2) Express dq in terms of charge density	$dq = \lambda dx'$	$dq = \lambda d\ell$	$dq = \sigma dA$
(3) write down dE	$dE = k_e\frac{\lambda dx'}{r'^2}$	$dE = k_e\frac{\lambda dl}{r^2}$	$dE = k_e\frac{\sigma dA}{r^2}$

	Line charge	**Ring of charge**	**Uniformly charged disk**
(4) Rewrite r and the differential element in terms of the appropriate coordinates	dx' $\cos\theta = \frac{y}{r'}$ $r' = \sqrt{x'^2 + y^2}$	$d\ell = Rd\phi'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{R^2 + z^2}$	$dA = 2\pi r'dr'$ $\cos\theta = \frac{z}{r}$ $r = \sqrt{r'^2 + z^2}$
(5) Apply symmetry argument to identify non-vanishing component(s) of dE	$dE_y = dE\cos\theta$ $= k_e \frac{\lambda y dx'}{\left(x'^2 + y^2\right)^{3/2}}$	$dE_y = dE\cos\theta$ $= k_e \frac{\lambda Rzd\phi'}{\left(R^2 + z^2\right)^{3/2}}$	$dE_y = dE\cos\theta$ $= k_e \frac{2\pi\sigma zr'dr'}{\left(r'^2 + z^2\right)^{3/2}}$
(6) Integrate to get E	$E_y = k_e\lambda y\int_{-\ell/2}^{+\ell/2} \frac{dx}{\left(x^2 + y^2\right)^{3/2}}$ $= \frac{2k_e\lambda}{y} \frac{\ell/2}{\sqrt{(\ell/2)^2 + y^2}}$	$E_z = k_e \frac{R\lambda z}{\left(R^2 + z^2\right)^{3/2}} \oint d\phi'$ $= k_e \frac{(2\pi R\lambda)z}{\left(R^2 + z^2\right)^{3/2}}$ $k_e \frac{Qz}{\left(R^2 + z^2\right)^{3/2}}$	$E_z = 2\pi\sigma k_e z\int_0^R \frac{r'dr'}{\left(r'^2 + z^2\right)^{3/2}}$ $= 2\pi\sigma k_e \left(\frac{z}{\lvert z \rvert} - \frac{z}{\sqrt{z^2 + R^2}}\right)$

System	**Infinite line of charge**	**Infinite plane of charge**	**Uniformly Charged solid sphere**
Figure	 **Figure 18.49**	 **Figure 18.50**	 **Figure 18.51**
Identify the symmetry	Cylindrical	Planar	Spherical
Determine the direction of $\vec{E}$	 **Figure 18.52**	 **Figure 18.53**	 **Figure 18.54**
Divide the space into different regions	r>0	Z >0 and z<0	$r \le a$ and $r \ge a$

Choose Gaussian surface	 **Figure 18.55**	 **Figure 18.56**	 **Figure 18.57**
Calculate electric flux	$\Phi_E = E(2\pi rl)$	$\Phi_E = EA + EA = 2EA$	$\Phi_E = E(4\pi r^2)$
Calculate enclosed charge q_{in}	$q_{enc} = \lambda l$	$q_{enc} = \sigma A$	$q_{enc} = \begin{cases} Q^{(r/a)^3} & r \le a \\ Q & r \ge a \end{cases}$
Apply Gauss's law $\Phi_E = q_{in} / \varepsilon_0$ to find E	$E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$	$E = \dfrac{\sigma}{2\varepsilon_0}$	$E = \begin{cases} \dfrac{Qr}{4\pi\varepsilon_0 a^3}, & r \le a \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & r \ge a \end{cases}$

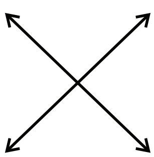
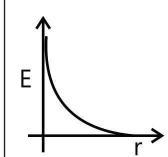
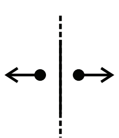
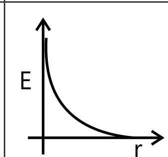
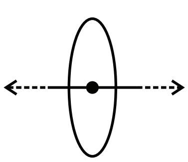
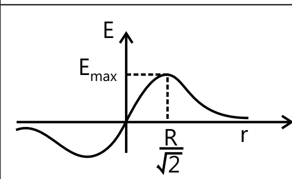
FORMULAE SHEET

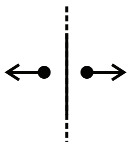
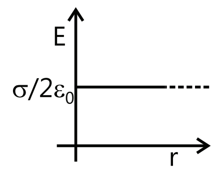
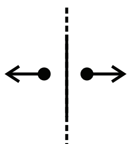
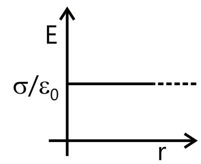
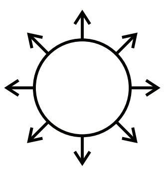
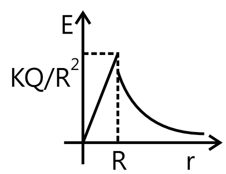
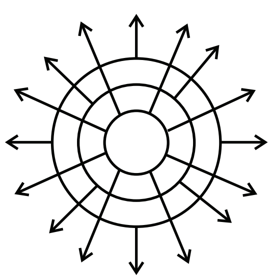
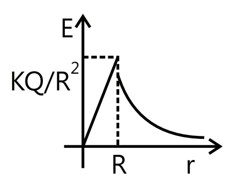
Electric Charges, Forces and Fields

S. No	Term	Description
1	Charge	Charges are of two types (a) Positive charge (b) Negative charge Like charges repel each other and unlike charges attract each other.
2	Properties of charge	1. Quantization:-q=ne where n=0, 1, 2...... and e is charge of an electron. 2. Additive: $-q_{net} = \sum q$ 3. Conservation: - total charge of an isolated system is constant
3	Coulomb's law	The mutual electrostatic force between the charges q_1 and q_2 separated by a distance r is given by Force on the charge q_1 $F_1 = Kq_1q_2r_{12} / r^2$ Where $\hat{r}_{12}$ is the unit vector in the direction from q_2 and q_1. For more than two charges in the system, the force acting on any charge is vector sum of the coulomb force from each of the other charges. This is called principle of superposition for $q_1, q_2, q_3 \ldots\ldots q_n$ Charges are present in the system.

S. No	Term	Description
4	Electric Field	-The region around a particular charge in which its electrical effects can be observed is called the electric field of the charge -Electric field has its own existence and is present even if there is no charge to experience the electric force.
5	Electric field Intensity	$E=F/q_0$ Where F is the electric force experienced by the test charge q_0 at this point. It is a vector quantity. Some points to note on this 1. Electric field lines extend away from the positive charge and towards the negative charge. 2. Electric field produces the force so if a charge q is placed in the electric field E, the force experienced by the charge is F=qE 3. Principle of superposition also applies to electric field so $E = E_1 + E_2 + E_3 + E_4 + ...$ Electric field intensity due to point charge $\vec{E} = \frac{KQ\,\vec{r}}{r^2}$ Where r is the distance from the point charge and r is the unit vector along the direction from source to point.

Electric Field Intensities due to various Charge Distributions

Name/Type	Formula	Note	Graph
Point Charge 	$\frac{Kq}{\lvert\vec{r}\rvert^2}\hat{r} = \frac{Kq}{r^3}\vec{r}$	• q is source charge • $\vec{r}$ is vector drawn from source charge to the test point.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{2K\lambda}{r}\hat{r}$	• λ is linear charge density (assumed uniform) • r is perpendicular distance of point from line charge • $\hat{r}$ is radial unit vector drawn from the charge to test point	
Uniformly Charged Ring 	$E = \frac{KQx}{(R^2 + X^2)^{3/2}}$ $E_{centre} = 0$	• Q is total charge of the ring • x=distance of point on the axis from centre of the ring. • Electric field is always along the axis.	

Infinitely large non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	• σ is surface charge density (assumed uniform) • $\hat{n}$ is unit normal vector • Electric field intensity is independent of distance	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	• σ is surface charge density (assumed uniform) • $\hat{n}$ is unit normal vector • Electric field intensity is independent of distance	
Uniformly charged hollow conducting/non conducting sphere or solid conducting sphere 	(i) for $r \geq R$ $\vec{E} = \frac{KQ}{\|\vec{r}\|^2}\hat{r}$ (ii) for $r < R$ $\vec{E} = 0$	• R is radius of the sphere • $\vec{r}$ is vector drawn from centre of the sphere to the test point. • Sphere acts like a point charge placed at the centre for point outside the sphere. • $\vec{E}$ is always along radial direction. • Q is total charge $(= \sigma 4\pi R^2)$. (σ = Surface charge density)	
Uniformly charged solid non conducting sphere (insulating material) 	(i) for $r \geq R$ $\vec{E} = \frac{KQ}{\|\vec{r}\|^2}\hat{r}$ (ii) for $r \leq R$ $\vec{E} = \frac{KQ}{R^3}\vec{r} = \frac{\rho}{3\varepsilon_0}\vec{r}$	• $\vec{r}$ is vector drawn from centre of the sphere to the test point. • Sphere acts like a point charge placed at the centre for points outside the sphere. • $\vec{E}$ is always along radial direction • Q is total charge $(= \rho\frac{4}{3}\pi R^3)$. ($\rho$ =volume charge density) • Inside the sphere $E \propto r$ • Outside the sphere $E \propto 1/r^2$	

Note: (i) Net charge on a conductor remains only on the outer surface of a conductor.

(ii) On the surface of spherical conductors charge is uniformly distributed.