

15.

CALORIMETRY AND THERMAL EXPANSION

1. INTRODUCTION

Have not been we dealing with the temperature and thermal energy in our daily life? Such as, we store our perishable food in refrigerator, switch on the heater of the car if we ever feel cold, and always handle hot utensils with thermal glove. To make a cup of cold coffee, ice cubes are used by our mother and how can of coke kept out of refrigerator comes to the room temperature.

2. DEFINITION OF HEAT

Heat is energy in transit. Heat energy flows from one body to another body due to their temperature difference. It is measured in units of calories. The SI unit is Joule. 1 calorie = 4.2J

Illustration 1: What is the difference between heat and temperature?

(JEE MAIN)

Sol: Temperature is associated with kinetic energy of atoms/molecule while heat is energy in transit. Temperature is a measure of the motion of the molecules or atoms within a substance; more specifically, it is the measure of the average kinetic energy of the molecules or atoms in a substance. Heat is the flow of energy from one body to another as a result of a temperature difference. It is important to point out that matter does not contain heat; it contains molecular kinetic energy and not heat. Heat flows and it is the energy that is being transferred. Once heat has been transferred to an object, it ceases to be heat. It becomes internal energy.

3. DEFINITION OF CALORIE

The amount of heat needed to increase the temperature of 1 g of water from 14.5°C to 15.5°C at a pressure of 1 atm is called 1 calorie.

1 kilo calorie = 10^3 calories; 1 calorie = 4.186 Joule

If the temperature of a body a mass m is raised through a temperature ΔT , then the heat, ΔQ , given to the body is $\Delta Q = m \cdot s \cdot \Delta T$ where s is the specific heat of the body which is defined as the amount of the heat required to raise the temperature of a unit mass of the body through 1°C. Its unit is cal/gm/°C or J/kg/K.

Thermal capacity of a body is the quantity of heat required to raise its temperature through 1°C and is equal to the product of mass and specific heat of the body. $Q = m \int_{T_1}^{T_2} s dt$ (be careful about unit of temperature, use units according to the given units of s)

PLANCESS CONCEPTS

Historically, first calorie was defined and hence such a weird unit conversion is used between calorie and Joule.

Chinmay Spurandare (JEE 2012, AIR 698)

4. PRINCIPLE OF CALORIMETRY

When two bodies at different temperatures are mixed, heat will pass from the body at a higher temperature to the body at a lower temperature until the temperature of the mixture becomes constant. The principle of calorimetry implies that heat lost by the body at a higher temperature is equal to the heat gained by the other body at a lower temperature assuming that there is no loss of heat in the surroundings.

5. TEMPERATURE SCALES

5.1 Kelvin Temperature Scale

Kelvin is a temperature scale designed such that zero K is defined as absolute zero (at absolute zero, a hypothetical temperature, all molecular movement stops- all actual temperatures are above absolute zero) and the size of one unit is the same as the size of one degree Celsius. Water freezes at 273.15K; water boils at 373.15K. $[K = C + 273.15^{\circ}, F = (9/5)C + 32^{\circ}]$. For calculation purposes, we take $0^{\circ}\text{C} = 273\text{K}$.

5.2 Celsius Temperature Scale

Celsius Temperature Scale - Temperature Scale according to which the temperature difference between the reference temperature of the freezing and boiling of water is divided into 100 degrees. The freezing point is taken as zero degree Celsius and the boiling point as 100 degrees Celsius. The Celsius scale is widely known as the centigrade scale because it is divided into 100 degrees.

5.3 Fahrenheit Scale

Fahrenheit temperature scale is a scale based on 32 for the freezing point of water and 212 for the boiling point of water, the interval between the two being divided into 180 parts.

The 18th-century German physicist Daniel Gabriel Fahrenheit originally took as the zero of his scale the temperature of an ice-salt mixture and selected the value of 30 and 90 for the point of water and normal body temperature, respectively; these later were revised to 32 and 96, but the final scale required an adjustment to 98.6 for the latter value

	Kelvin	Celsius	Fahrenheit
Water boils	373.16K	100°C	212°F
Water freezes	273.16K	0°	32°F
Absolute zero	0 k	-273.16°C	-459.7°F

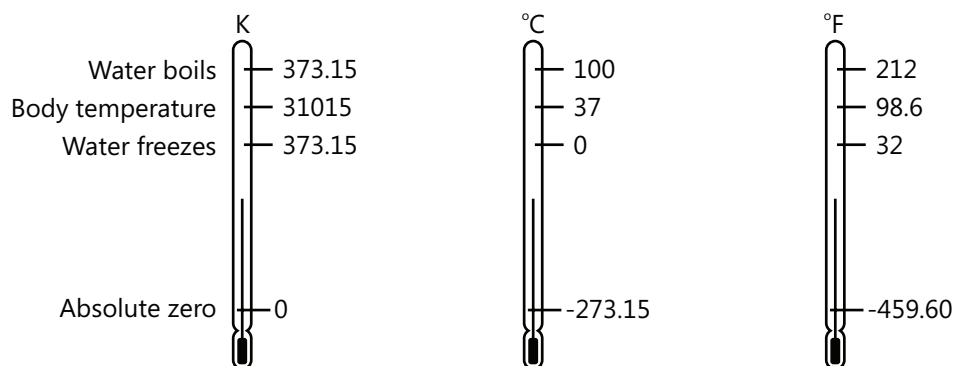


Figure 15.1

PLANCESS CONCEPTS

For easy conversion of temperature units, remember the following equation

$$\left(\frac{C - 0}{100 - 0} \right) = \left(\frac{F - 32}{212 - 32} \right) = \left(\frac{K - 273}{373 - 273} \right)$$

Where C, F and K are respectively temperatures in Celsius, Fahrenheit and Kelvin scale. Note the values used in denominator, are actually the boiling and melting points of water in respective scales, so quite easy to remember.

Nitin Chandrol (JEE 2012, AIR 134)

Illustration 2: Express a temperature of 60° F in degree Celsius and in Kelvin.

(JEE MAIN)

$$(i) T_C = T - 273.15 \quad (ii) T_F = 32 + \frac{9}{5} T_C$$

Sol: (Using above formulas) Find the temp in Celsius first, then in Kelvin as kelvin and Celsius have more simple relation. Substituting $T_F = 60^\circ\text{C}$ in Eq. (ii); $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(60^\circ\text{C} - 32^\circ\text{C}) = 15.55^\circ\text{C}$

From Eq. (i) $T = T_C + 273.15 = 15.55^\circ\text{C} + 273.15 = 288.7\text{K}$

Illustration 3: Calculate the temperature which has the same value on (i) the Celsius and Fahrenheit (ii) Fahrenheit and Kelvin scales.

(JEE ADVANCED)

Sol: The value of temp which shows same reading on the Celsius as well as on the Fahrenheit (i part) and on the Kelvin and Fahrenheit (ii part).

$$(i) \text{ Let the required temperature be } x^\circ, \text{ now } T_F = \frac{9}{5} T_C + 32$$

$$\text{or } 5T_F = 9T_C + 160 \text{ or } 5X = 9X + 160 \therefore X = \frac{160}{-4} = -40^\circ \Rightarrow -40^\circ\text{C} = -40^\circ\text{F}$$

$$(ii) \text{ Let the required temperature be } x^\circ \frac{T_F - 32}{180} = \frac{T_k - 273.15}{100}$$

$$\therefore \frac{X - 32}{180} = \frac{T_k - 273.15}{100}$$

On solving we get, $X = 574.6$

5.4 Triple Point of Water

The triple point of water is that unique temperature and pressure at which water can coexist in equilibrium between the solid, liquid and gaseous states. The pressure at the triple point of water is 4.58 mm of Hg and the temperature is 273.16 K (or 0.01°C). The absolute or Kelvin temperature T at any point is then defined, using a constant volume

gas thermometer for an ideal gas as: $T = 273.16 \times \frac{P}{P_{tp}}$ [ideal gas; constant volume]

In this relation, P_{tp} is the pressure in the thermometer at the triple point temperature of water and P is the pressure in the thermometer when it is at the point where T is being measured. Note that if we let $P = P_{tp}$ in this relation, $T = 273.16$ K as it must.

Illustration 4: When in thermal equilibrium at the triple point of water, the pressure of Hg in a constant volume gas thermometer is 1020 Pa. The pressure of He is 288 Pa when the thermometer is in thermal equilibrium with liquid nitrogen at its normal boiling point. What is the normal boiling point of nitrogen as measured using this thermometer? **(JEE MAIN)**

Sol: As we consider volume of the fluid to be constant, and hence T/P ratio remains constant, Normal boiling point

of nitrogen is $T = 273.16 \times \frac{P}{P_{tp}}$; Here $P = 288$ Pa; $P_{tp} = 1020$ Pa

$$\therefore T = 273.16 \times \frac{288}{1020} = 77.1 \text{ K}$$

6. HEAT CAPACITY

The heat capacity of a body is defined as the amount of heat required to raise its temperature by 1°C. It is also known as the thermal capacity of the body. Suppose a body has mass m and specific heat c . Heat capacity = Heat required to raise the temperature of the body by 1°C = $mc \times 1 = mc$

$$\therefore \text{Heat capacity} = mc$$

Hence heat capacity of a body (solid or liquid) is equal to the product of its mass and specific heat. Clearly, the SI unit of heat capacity is J/°C or J/K. The greater the mass of a body, the greater is its heat capacity.

7. SPECIFIC HEAT CAPACITY

When we supply heat to a solid substance (or liquid), its temperature increases. It is found that the amount of heat Q absorbed by the solid substance (or liquid), is

(i) Directly proportional to the mass (m) of the substance i.e., $Q \propto m$

(ii) Directly proportional to the rise in temperature (ΔT) i.e., $Q \propto \Delta T$

Combining the two factors, we have, $Q \propto m \Delta T$ (i)

$$\text{or } Q = cm \Delta T$$

Where C is constant of proportionality and is called specific heat capacity or simply specific heat of the substance.

$$\text{From eq. (i), we have } c = \frac{Q}{m \Delta T} \text{ (ii)}$$

If $m = 1$ kg and $\Delta T = 1^\circ\text{C}$, then $c = Q$.

Hence the specific heat of a solid (or liquid) may be defined as the amount of heat required to raise the temperature of 1 kg of solid (or liquid) through 1°C (or 1 K). It is clear from eq. (ii) that SI unit of specific heat is $\text{J kg}^{-1} ^\circ\text{C}^{-1}$ or $\text{J kg}^{-1} \text{K}^{-1}$.

PLANCESS CONCEPTS

Don't get confused here with the terminology of heat capacity and specific heat capacity. Always remember that Specific heat capacity is the property of material and heat capacity is property of a given body.

B Rajiv Reddy (JEE 2012, AIR 11)

Illustration 5: A geyser heats water flowing at the rate of 3.0 liters per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of combustion of the fuel if its heat of combustion is $4.0 \times 10^4 \text{ J g}^{-1}$?
(JEE ADVANCED)

Sol: The total heat required to increase the temperature of the water is equal to the heat supplied by the combustion of gas per minute. Mass of 3 liters of water = 3kg \therefore Mass of water flowing per minute, $m = 3 \text{ kg} = 3000 \text{ g min}^{-1}$

Rise of temperature, $\Delta \theta = 77 - 27 = 50^\circ\text{C}$; Heat absorbed by water per minute = $mc \Delta \theta = 3000 \times 1 \times 50 \text{ cal}$

$$= 3000 \times 1 \times 50 \times 4.2 \text{ J min}^{-1} = 630000 \text{ J min}^{-1}$$

\therefore Heat supply by gas burner = $630000 \text{ J min}^{-1}$ and heat of combustion of fuel = $4.0 \times 10^4 \text{ J g}^{-1}$

$$\therefore \text{Rate of combustion of fuel} = \frac{630000}{4.0 \times 10^4} = 15.75 \text{ g min}^{-1}$$

Illustration 6: A copper block of mass 60 g is heated till its temperature is increased by 20°C. Find the heat supplied to the block. Specific heat capacity of copper = $0.09 \text{ cal g}^{-1}^\circ\text{C}^{-1}$.
(JEE MAIN)

Sol: Here the heat is utilized to increase the temperature of the block only.

The heat supplied is $Q = ms \Delta \theta = (60 \text{ g}) (0.09 \text{ cal g}^{-1}^\circ\text{C}^{-1}) (20^\circ\text{C}) = 108 \text{ cal}$.

The quantity ms is called the heat capacity of the body. Its unit is J K^{-1} . The mass of water having the same heat capacity as given body is called the water equivalent of the body.

8. MOLAR SPECIFIC HEAT CAPACITY FOR SOLIDS OR LIQUIDS

The molar specific heat of a solid (or liquid) is defined as the amount of heat required to raise the temperature of 1 mole of the solid (or liquid) through 1°C (or 1K). It is denoted by the symbol C . Therefore, the amount of heat Q required to raise the temperature of n moles of a solid (or liquid) through a temperature change ΔT is given by;
 $Q = n C \Delta T$

It is clear that SI unit of C is $\text{J mol}^{-1} \text{K}^{-1}$. For any material of mass m and molecular mass M , the number of moles

$$n = m/M. \therefore Q = \frac{m}{M} C \Delta T \text{ also } \therefore Q = m C \Delta T \therefore \frac{m}{M} C \Delta T = m c \Delta T \text{ or } C = M c \quad \dots(i)$$

Eq. (i) gives the relation between molar specific heat C and the ordinary specific heat.

9. MOLAR SPECIFIC HEAT CAPACITY FOR THE GASES

The amount of heat required to increase the temperature of 1 mole of a gas through 1°C is called molar heat capacity.

The number of moles, n , in mass m of the gas is given by $n = \frac{\text{Mass of the gas}}{\text{Molecular weight}}$

9.1 Molar Specific Heat at Constant Volume, C_v :

If $(\Delta Q)_v$ is the heat required to raise the temperature of mass m gm or n moles of gas of molecular weight M at constant volume through temperature ΔT , $(\Delta Q)_v = mc_v \Delta T = nMc_v \Delta T = nC_v \Delta T$

Where C_v molar specific is heat at constant volume and is equal to Mc_v .

9.2 Molar specific heat at Constant Pressure, C_p :

If $(\Delta Q)_p$ is the heat required to raise the temperature of mass m gm or n moles of gas of molecular weight M at constant pressure through temperature ΔT , $(\Delta Q)_p = mc_p \Delta T = nMc_p \Delta T = nC_p \Delta T$

Where C_p molar specific is heat at constant volume and is equal to Mc_p .

For monatomic gases, $C_p = \frac{5R}{2}$, $C_v = \frac{3R}{2}$; $\gamma = \frac{C_p}{C_v} = \frac{5}{3} = 1.67$; for diatomic gases, $C_p = \frac{7R}{2}$, $C_v = \frac{5R}{2}$ $\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1.4$;

Mayer's relation gives, $C_p - C_v = R$; where $C_v = \frac{R}{\gamma - 1}$, $C_p = \frac{\gamma R}{\gamma - 1}$

Illustration 7: How much heat is required to raise the temperature of an ideal monoatomic gas by 10 K if the gas is maintained at constant pressure? **(JEE MAIN)**

Sol: The process is at constant pressure here. Formula for heat capacity of gas at constant pressure is used.

The heat required is given by $Q = n C_p \Delta T$ Here $n=1$ & $\Delta T = 10$ K;

$$C_p = \frac{5}{2}R = \frac{5}{2} \times 8.3 \text{ J mol}^{-1} \text{ K}^{-1}; \quad \therefore Q = 1 \times \frac{5}{2} \times 8.3 \times 10 = 207.5 \text{ J}$$

PLANCESS CONCEPTS

Without calculation, one can tell that C_p is always greater than C_v . Think of a situation in which we need to raise the temperature of same amount of gas in constant pressure conditions and constant volume conditions. It is quite obvious that in constant volume conditions all the heat will be used up to raise internal energy of gas. We see that the rise in internal energy of gas is same in both cases as increase in temperature is same. However, we see that for constant pressure conditions, more heat is required as some of it will also be used to expand the volume. This condition requires that C_p must be greater than C_v .

Anand K (JEE 2011, AIR 47)

Illustration 8: Calculate the amount of heat necessary to raise the temperature of 2 moles of He gas from 20°C to 50°C using (i) constant –volume process and (ii) constant-pressure process.

For He $C_v = 1.5 R$ and $C_p = 2.49 R$.

(JEE ADVANCED)

Sol: Heat capacity at constant volume and constant pressure are applicable here.

(i) The amount of heat required for constant –volume process is $Q_v = C_v \Delta T$; Here $n=2$ moles;

$$C_v = 1.5 R = 1.5 \times 8.314 \text{ J mol}^{-1} \text{ } ^\circ\text{C}^{-1}; \quad \Delta T = 50 - 20 = 30^\circ\text{C}; \quad Q_v = 2 \times (1.5 \times 8.314) \times 30 = 748 \text{ J}$$

(ii) The amount of heat required for constant –pressure process is $Q_p = nC_p \Delta T$

$$\text{Here } n=2 \text{ moles; } C_p = 2.49 R = 2.49 \times 8.314 \text{ J mol}^{-1} \text{ } ^\circ\text{C}^{-1}; \quad \Delta T = 30^\circ\text{C}$$

$$\therefore Q_p = (2.49 \times 8.314) \times 30 = 1242 \text{ J}$$

Since the temperature rise is the same in the two cases, the change in internal energy is same i.e, 748 J. however, in constant-pressure excess heat supplied = $1242 - 748 = 494$ J. This extra heat of 494 J went into the work of expansion of the gas.

10. LATENT HEAT

The amount of heat required to change a unit mass of a substance completely from one state to another at constant temperature is called the latent heat of the substance.

If a substance of mass m required heat Q to change completely from one state to another at constant temperature, then, the latent heat $L = \frac{Q}{m}$. The SI unit of latent heat of a substance is J kg^{-1} . There are two types of latent heats viz. latent heat of fusion and latent heat of vaporization.

(a) Latent heat of fusion. We know that a solid changes into liquid at a constant temperature which is called the melting point.

The amount of heat required to change the unit mass of solid mass into its liquid state at constant temperature is called the latent heat of fusion of the solid.

For example, the latent heat of fusion of ice is 334 J/kg . It means to change 1 kg of ice at 0°C into liquid water at 0°C , we must supply 334 KJ of heat.

(b) Latent heat of vaporization. We know that a liquid changes into gaseous state at a constant temperature which called the boiling point. The amount of heat required to change the unit mass of a liquid into its gaseous state at constant temperature is called latent heat of vaporization of the liquid.

Illustration 9: A piece of ice of mass 100 g and at temperature 0°C is put in 200 g of water at 25°C . How much ice will melt as the temperature of the water reaches 0°C ? The specific heat capacity of water = 4200 JK^{-1} and the specific latent heat of fusion of ice = $3.4 \times 10^5 \text{ JK}^{-1}$. **(JEE MAIN)**

Sol: Total heat lost by the water equal to the total heat gained by the ice.

The heat released as the water cools down from 25°C to 0°C is

$$Q = ms\Delta\theta = (0.2 \text{ kg})(4200 \text{ Jk}^{-1} \text{ K}^{-1})(25\text{K}) = 21000 \text{ J}.$$

The amount of ice melted by this much heat is given by $m = \frac{Q}{L} = \frac{21000 \text{ J}}{3.4 \times 10^5 \text{ Jkg}^{-1}} = 62 \text{ g}$

11. WATER EQUIVALENT

The water equivalent of a body is defined as the mass of water that will absorb or lose the same amount of heat as the body for the same rise or fall in temperature. The water equivalent of a body is measured in kg in SI unit and in g in C.G.S. units. Suppose the water equivalent of a body is 10 kg . It means that if the body is heated through, say 10°C , it will absorb the same amount of heat as absorbed by 10 Kg of water when heated through 10°C . Consider a body of mass m and specific heat required to raise the temperature of the body through ΔT is $Q = cm\Delta T$... (i)

Suppose w is water equivalent of this body. Then, by definition, Q is given by:

$$Q = w \Delta T \text{ From eqs. (ii) And (i), we have, } w \Delta T = cm \Delta T \text{ or } w = m c$$

Thus the water equivalent of a body is numerically equal to the product of the mass of the body and its specific heat. Note that mc is the heat capacity of the body. Therefore, we may conclude that water equivalent and heat capacity of a body are numerically equal.

Illustration 10: A calorimeter of water equivalent 15g contains 165 g of water at 25°C. Steam at 100° is passed through the water for some time. The temperature is increased to 30°C and the mass of calorimeter and its contents are increased by 1.5 g. Calculate the specific latent heat of vaporization of water. Specific heat capacity of water is $1 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}$. **(JEE ADVANCED)**

Sol: The change in mass of the content of calorimeter is due to formation of more water from condensation of steam and all comes to the same temperature.

let L be the specific latent heat of vaporization of water. The mass of the steam condensed is 1.5 g. Heat lost in condensation of steam is $Q_1 = (1.5\text{g}) L$. The condensed water cools from 100°C to 30 °C. Heat lost in the process is

$$Q_2 = (1.5\text{g}) (1 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}) (70^{\circ}\text{C}) = 105 \text{ cal.}$$

Heat supplied to the calorimeter and to the cold water during the rise in temperature from 25°C to 30°C is

$$Q_3 = (15\text{g} + 165\text{g}) (1 \text{ cal g}^{-1} \text{ }^{\circ}\text{C}^{-1}) (5^{\circ}\text{C}) = 900 \text{ cal.}$$

If no heat is lost to the surrounding.

$$(1.5\text{g})L + 105 \text{ cal} = 900 \text{ cal} \text{ or } L = 530 \text{ cal g}^{-1}$$

Illustration 11: The water equivalent of a body is 10 kg. What does it mean? **(JEE MAIN)**

Sol: It means that if a body is heated through say 5°C, it will absorb the same amount of heat as absorbed by 10 kg of water when heated through 5°C.

12. MECHANICAL EQUIVALENT OF HEAT

In early days, heat was not recognized as a form of energy. Heat was supposed to be something needed to raise the temperature of a body or to change its phase. Calorie was defined as the unit of heat. A number of experiments were performed to show that the temperature may also be increased by doing mechanical work on the system. These experiments established that heat is equivalent to mechanical energy and measured how much mechanical energy is equivalent to a calorie. If mechanical work W produces the same temperature change as heat H , we write, $W = JH$. Where J is called mechanical equivalent of heat. It is clear that if W and H are both measured in the same unit then $J = 1$. If W is measured in joule (work done by a force of 1 N in displacing an object by 1 m in its direction) and H in calorie (heat required to raise the temperature of 1 g of water by 1°C) then J is expressed in joule per calorie. The value of J gives how many joules of mechanical work is needed to raise the temperature of 1 g of water by 1°C.

Illustration 12: Assuming that the density of air at N.T.P. = 0.0013 g/cc, $C_p = 0.239 \text{ cal g}^{-1} \text{ K}^{-1}$ and the ratio $C_p/C_v = 1.40$, calculate the mechanical equivalent of heat. **(JEE MAIN)**

Sol: Compare the value of Gas Constant (R) by calculating in different unit (Calorie and Joule). $R = C_p - C_v$. And $R = PV/T$, then find the ratio (in joule/in calorie).

$$\text{Now, } C_p = 0.239 \text{ cal g}^{-1} \text{ K}^{-1}; C_p/C_v = 1.40; \therefore C_v = \frac{C_p}{1.40} = \frac{0.239}{1.40} = 0.171 \text{ cal g}^{-1} \text{ K}^{-1}$$

$$\text{Volume of 1 g of air at N.T.P.} = \frac{1}{0.0013} \text{ cc} = \frac{10^{-6}}{0.0013} \text{ m}^3$$

$$\text{Volume of 1 kg (=1000g) of air at N.T.P., } V = \frac{10^{-6}}{0.0013} \times 1000 \frac{10^{-3}}{0.0013} \text{ m}^3$$

$$\text{Normal pressure, } p = hpg = 0.76 \times 13600 \times 9.8 = 101292.8 \text{ Nm}^{-2}$$

Normal temperature, $T = 273 \text{ K}$

Gas constant r for 1 kg of air is given by; $R = \frac{PV}{T} = 101292.8 \times \frac{10^{-3}}{0.0013} \times \frac{1}{273} = 285.4 \text{ J kg}^{-1} \text{ K}^{-1}$

Note that $R = C_p - C_v$. It means that if 1 kg of air is heated through 1°C (or 1K) first at constant pressure and then at constant volume, then extra heat needed for constant-pressure process to do this work of 285.4 J i.e., $W = 285.4 \text{ J}$

Heat supplied to do work is $Q = 1000\text{g} \times C_p \times 1\text{K} - 1000\text{g} \times C_v \times 1\text{K}$

$$= 1000\text{g} \times 0.239 \times 1\text{K} - 1000\text{g} \times 0.171 \times 1\text{K}$$

$$= 239 - 171 = 68 \text{ cal; Now } W = JQ; \quad \therefore J = \frac{W}{Q} = \frac{285}{68} = 4.2 \text{ J / cal}$$

13. LAW OF HEAT EXCHANGE

When a hot body is mixed or kept in contact with a cold body, the hot body loses heat and its temperature falls. On the other hand, the cold body gains heat and its temperature rises. The final temperature of the mixture will lie between the original temperatures of the hot body and the cold body. If a system is completely isolated, no energy can flow into and out of the system. Therefore according to the law of conservation of energy, the heat lost by one body is equal to the heat gained by other body i.e. Heat lost = heat gained

This is known as law of heat exchange.

THERMAL EXPANSION

1. DEFINITION OF THERMAL EXPANSION

It is the expansion due to increase in temperature. Most substances expand when they are heated. Thermal expansion is a consequence of the change in average separation between the constituent atoms of an object. Atoms of an object can be imagined to be connected to one another by stiff springs as shown in figure. At ordinary temperatures, the atoms in a solid oscillate about their equilibrium positions with an amplitude of approximately 10^{-11} m . The average spacing between the atoms is about 10^{-10} m . As the temperature of the solid increases, the atoms oscillate with greater amplitudes, as a result the average separation between them increases, and consequently the object expands. More precisely, thermal expansion arises from the asymmetrical nature of the potential energy curve.

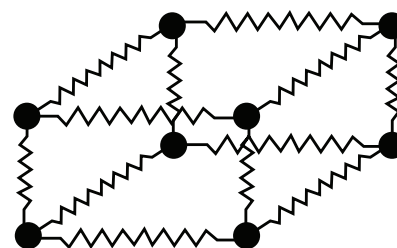


Figure 15.2

2. THERMAL EXPANSION OF SOLIDS

2.1 Linear Expansion

When a solid substance is heated, most of them generally expand. If a solid has a length L_0 and has a very small area of cross-section, at a temperature T_0 , its length increases to L_T when its temperature is increased by ΔT . The increase in length, ΔL , is then given by,

$\Delta L = L_T - L_0 = L_0 \times \alpha \times \Delta T$ Where α is the coefficient of linear expansion which is given by

$$\alpha = \frac{L_T - L_0}{L_0 \Delta T}; \quad L_T = L_0(1 + \alpha \Delta T)$$

The coefficient of linear expansion is equal to the increase in length per unit length per degree rise of temperature.

The SI unit of α is $^{\circ}\text{C}$ or $/\text{K}$. Its value is different for different solid materials. For example α for aluminum is $2.4 \times 10^{-5} /^{\circ}\text{C}$ whereas for brass, its value is $2.0 \times 10^{-5} /^{\circ}\text{C}$. Note that the change in temperature ΔT will be the same whether it is measured in Celsius scale or on the Kelvin scale: $\Delta T ^{\circ}\text{C} = \Delta T \text{ K}$.

2.2 Superficial Expansion

If a solid plate of area A_0 and of very small thickness is heated through a temperature ΔT so that its area increases to A_T then the increase in area ΔA is given by

$$\Delta A = A_T - A_0 = \beta A_0 \Delta T \quad \text{or} \quad \beta = \frac{A_T - A_0}{A_0 \Delta T}$$

Where β is called the coefficient of superficial expansion. $\beta = 2\alpha$

Hence the coefficient of superficial expansion of a solid may be defined as the fractional change in surface area ($\Delta S / S$) per degree change in temperature. Its SI unit is also $/^{\circ}\text{C}$ or $/\text{K}$.

Note that change in temperature ΔT will be the same whether it is measured on the Celsius scale or on the Kelvin scale.

2.3 Volume Expansion

If a solid of initial volume V_0 at any temperature is heated so that its volume is increased to V_T with increase of temperature ΔT , the increase in volume, ΔV , is given by

$$\Delta V = V_T - V_0 = \gamma V_0 \Delta T; \gamma = \frac{V_T - V_0}{V_0 \Delta T} = \frac{\Delta V}{V_0 \Delta T}$$

Where γ is called the coefficient of volume or cubical expansion. $\gamma = 3\alpha$

As the temperature of solid increases, the amplitude of oscillation of atoms increases which results in an increase of average distance between atoms with increase of temperature due to which the volume increases. If ρ_0 is the density of a solid at 0°C and ρ_T is its density $T^{\circ}\text{C}$, then for a constant mass m of the solid,

$$\rho_0 = \frac{m}{V_0} \text{ And } \rho_T = \frac{m}{V_T} \text{ where } V_0 \text{ and } V_T \text{ are its respective volume at } 0^{\circ}\text{C} \text{ \& } T^{\circ}\text{C}$$

$$\therefore \frac{\rho_0}{\rho_T} = \frac{V_T}{V_0} = \frac{V_0 (1 + \gamma T)}{V_0} = 1 + \gamma T \quad \therefore \rho_T = \frac{\rho_0}{1 + \gamma T}$$

Hence coefficient of cubical expansion of a solid may be defined as the fractional change in volume ($\Delta V / V$) per degree change in temperature. Its SI unit is $/^{\circ}\text{C}$ or $/\text{K}$.

PLANCESS CONCEPTS

For anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$.

Here α_1, α_2 and α_3 are coefficients of linear expansion in X, Y and Z directions.

For solid value of are generally small so we can write density, $d = d_0 (1 - \gamma \Delta T)$ (Using binomial expansion). γ is not always positive. It can have a negative value.

E.g. For water, density increases from 0 to 4°C so γ is -ve (0 to 4°C) and for 4°C to higher temperature γ is -ve. At 4°C density is maximum. Coefficients of thermal expansion are generally not independent of temperature. But for JEE purpose you are supposed to assume it as a constant if not mentioned.

If α is not constant

(i) (α varies with distance) Let $\alpha = ax + b$; Total expansion = $\int \text{expansion of length } dx = \int_0^1 (ax + b) dx \Delta t$

PLANCESS CONCEPTS

(ii) (α Varies with temperature) ; Let $\alpha = f(T)$; $\Delta\ell = \int_{T_1}^{T_2} \alpha \ell_0 dT$

Caution: If α is in $^{\circ}\text{C}$,

Then put T_1 and T_2 in $^{\circ}\text{C}$. Similarly if α is in K then put T_1 and T_2 in K.

If you have a difficulty in remembering the definition of different capacity then just look at the units given heat capacity and figure out whether it's per unit mass/mole/ or for entire mass.

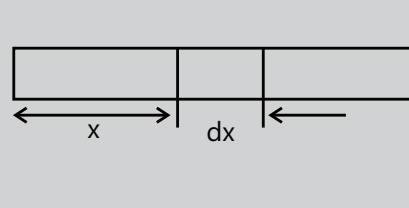


Figure 15.3

Yashwanth Sandupatla (JEE 2012, AIR 821)

3. PRACTICAL APPLICATION OF THERMAL EXPANSION OF SOLIDS

There are a large number of important practical applications of thermal expansion of solids. However, we shall brief only a few of them by way of illustration.

- While laying the railway tracks, a small gap is left between the successive lengths of the rails. This gap is provided to allow for the expansion of the rails during summer. If no gap is left, these expansions cause the rails to buckle.
- When the iron tyre of a wheel to be put on the wheel, the tyre is made slightly smaller in diameter than that of wheel. The iron tyre is first heated uniformly till its diameter becomes more than that of the wheel and is then slipped over the wheel. On cooling the tyre contracts and makes a tight fit on the wheel.
- In bridges, one end is rigidly fastened to its abutment while the other rests on rollers. This provision allows the expansion and contraction to take place during changes in temperature.
- The fact that a solid expands on heating and contracts on cooling is utilized in riveting e.g., riveting two metal plates together, joining steel girders etc. For joining two steel plates, holes are drilled between them. The rivets (small rods) are made red hot and inserted in the holes in the plates. The ends of the rivets are hammered into the head. After some time, the rivets contract on being and hold the plates very tightly.
- The concrete roads and floor are always made in sections and enough space is provided between the sections. This provision allows expansion and contraction to take place due to change in temperature.

PLANCESS CONCEPTS

- If a solid object has a hole in it, what happens to size of the hole, when the temperature of the object increases? A common misconception is that if the object expands, the hole will shrink because material expands into the hole. But the, truth is that if the Object expands the hole will expand too, because every linear dimension of an object change in the same way when the temperature changes.
- Effect of temperature on the time period of a pendulum:

The time Period of a simple pendulum is given by.

$$T = 2\pi\sqrt{\frac{\ell}{g}}; \quad \text{or} \quad T \propto \sqrt{\ell}$$

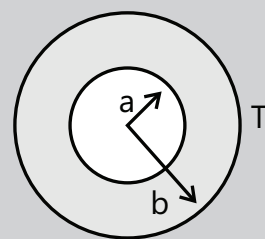


Figure 15.4

PLANCESS CONCEPTS

As the temperature is increased length of the pendulum and hence, time period gets increased or a pendulum clock becomes slow and it's loses the time. Time lost in time t (by a pendulum clock whose

actual time period is T and changed time period at some higher

temperature is T') is $\Delta t = \left(\frac{\Delta T}{T'} \right) t$.

Similarly, if the temperature is decreased the length and hence, the time period gets decreased. A pendulum clock on this case runs fast

and it gains the time. Time gained in time t is the same, i.e., $\Delta t = \left(\frac{\Delta T}{T'} \right) t$.

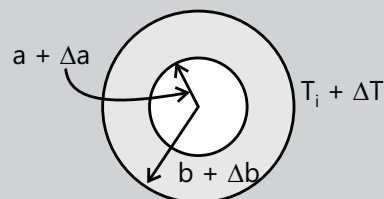


Figure 15.5

Gv Abhinav (JEE 2012, AIR 329)

Illustration 13: A steel ruler exactly 20 cm long is graduated to give correct measurements at 20°C.

(a) What happens to the reading if the temperature decreases below 20°C?

(b) What is the actual length of the ruler at 40°C?

(JEE MAIN)

Sol: Lowering the temperature, shorten the scale from 1 m of original length. It'll show length of 1m lengthier than its length. And hence will show 1m to be more than 1m. It will now measure more. And reverse, in case of increasing the temperature.

(a) If the temperature decreases, the length of the ruler also decreases through thermal contraction. Below 20°C, each centimeter division is actually somewhat shorter than 1.0 cm, so the steel ruler gives reading that are too long.

(b) At 40°C, the increase in length of the ruler is

$$\Delta \ell = \ell \alpha \Delta T = (20)(1.2 \times 10^{-5})(40^\circ - 20^\circ) = 0.48 \times 10^{-2} \text{ cm}$$

\therefore The actual length of the ruler is, $\ell' = \ell + \Delta \ell = 20.0048 \text{ cm}$

Illustration 14: A second pendulum clock has a steel wire. The clock is calibrated at 20°C. How much time does the clock lose or gain in one week when the temperature is increased to 30°C?

$$\alpha_{\text{steel}} = 1.2 \times 10^{-5} (^\circ\text{C})^{-1}$$

(JEE ADVANCED)

Sol: Increment in length increase the time period of oscillation.

The time period of second's pendulum is 2 seconds. As the temperature increases, length and hence, time period increases, clock becomes slow and it loses the time. The change in time period is $\Delta T = \frac{1}{2} T \alpha \Delta \theta =$

$$\left(\frac{1}{2} \right) (2) (1.2 \times 10^{-5}) (30^\circ - 20^\circ) = 1.2 \times 10^{-4} \text{ s} \therefore \text{New time period is, } T' = T + \Delta T = (2 + 1.2 \times 10^{-4}) = 2.00012 \text{ s}$$

$$\therefore \text{Time lost in one week } \Delta t = \left(\frac{\Delta T}{T'} \right) t = \left(\frac{1.2 \times 10^{-4}}{2.0012} \right) (7 \times 24 \times 3600) = 36.28 \text{ s}$$

4. THERMAL EXPANSION OF LIQUIDS

As a liquid in a vessel acquires the shape of the vessel, its heating increases the volume of the vessel initially due to expansion of the vessel which decreases the level of the liquid initially. When the temperature of the liquid is increased further, it increased the volume of the liquid. Thus the observed or apparent expansion of the liquid is lesser than the real level of the liquid when the temperature increases. Thus the apparent expansion of the liquid is lesser than the real expansion of the liquid which gives a value of coefficient of real expansion more than that for the coefficient of apparent expansion.

The coefficient of real expansion, γ_r of a liquid is defined as the real increase in volume per degree rise of temperature per unit original volume of the liquid.

$$\gamma_r = \frac{\text{Real increase in volume}}{\text{Original volume} \times \text{rise in temperature}}$$

The coefficient of apparent expansion, γ_a of a liquid is defined as the ratio of the observed increase in volume of the liquid with respect to the original level before heating per degree rise of temperature to the original volume of the liquid.

$$\gamma_a = \frac{\text{Observed increased in volume}}{\text{Original volume} \times \text{rise in temperature}}$$

If γ_g is the coefficient of cubical expansion of the material of the vessel, then $\gamma_r = \gamma_a + \gamma_g$; $\gamma_g = 3\alpha$

Then density of the liquid ρ_T at temperature T is related to density ρ_0 at $^{\circ}\text{C}$ as $\rho_T = \frac{\rho_0}{1 + \gamma T}$

Where γ is the coefficient of real expansion of the liquid and T is the increase in temperature.

It is clear that $\gamma_r > \gamma_a$ and both are measured unit $^{\circ}\text{C}^{-1}$. It can be shown that: $\gamma_r = \gamma_a + \gamma_g$

Where γ_g is the coefficient of cubical expansion of glass (or material of the container).

Illustration 15: Find the coefficient of volume expansion for an ideal gas at constant pressure.

(JEE MAIN)

Sol: Recall the formula for coefficient of volume expansion for ideal gas.

For an ideal gas $PV = nRT$

As P is constant, we have $P \cdot dV = nRdt \therefore \frac{dV}{dT} = \frac{nR}{P}$ or $\gamma = \frac{1}{V} \cdot \frac{dV}{dT} = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T} \therefore \gamma = \frac{1}{T}$

5. THERMAL EXPANSION OF GASES

The molecules in an ideal gas have only kinetic energy due to their motion but do not possess any potential energy. The thermodynamic state of any gas is defined in terms of its pressure, volume and temperature denoted as P, V and T respectively. A change in one of these quantities produces a corresponding change in the other quantities depending upon the condition under which the transformation take place. Such changes are governed by the following gas laws:

5.1 Boyle's Law

The pressure of given mass of a gas is inversely proportional to its volume if temperature T remains constant

$$P \propto \frac{1}{V} \text{ or } PV = \text{constant};$$

If the pressure P_1 and volume V_1 changes to the respective values P_2, V_2 when the temperature remains

Constant, then $P_1 V_1 = P_2 V_2$.

5.2 Charles's Law of Volume

The volume V of a given mass of a gas is directly proportional to its absolute temperature, T , when its pressure remains constant. $V \propto T$ $\frac{V}{T} = \text{constant}$

If the volume V_1 and temperature T_1 are respectively changed to V_2 , T_2 at constant pressure, then $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.

Where temperatures T_1 and T_2 are in Kelvin scale.

If V_0 and V_t are volume of the gas at 0°C and $t^\circ\text{C}$ respectively,

$$\frac{V_t}{273+t} = \frac{V_0}{273}; V_t = V_0 \left[1 + \frac{t}{273} \right] V_0 = [1 + \alpha_v t]$$

Where α_v is the volume coefficient of a gas and is equal to $1/273$.

5.3. Gay Lussac's Law of Pressure

The pressure of a given mass of a gas is directly proportional to its absolute temperature provided the volume of the gas is kept constant. $P \propto T$ or $\frac{P}{T} = \text{constant}$

If the pressure and temperature P_1, T_1 is change respectively to P_2, T_2 at constant volume, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}.$$

If P_t and P_0 are pressure of the gas at $t^\circ\text{C}$ & 0° respectively, then $P_t = P_0 (1 + \alpha_p t) = P_0 \left(1 + \frac{t}{273} \right)$

Where α_p is equal to the pressures coefficient of the gas which also equal to $1/273$.

5.4 Gas Equation

If the above mentioned three laws are combined, then $\frac{PV}{T} = \text{constant}; \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \text{constant}.$

The value of the constant depends on the mass of the gas.

If the gas has n moles, $PV = nRT$ which is called the equation of the state of an ideal gas. R is called the universal or molar gas constant and its value in S.I. units is $8.314 \text{ J. mol}^{-1} \text{ K}^{-1}$.

6. RELATION BETWEEN COEFFICIENTS OF EXPANSION

We shall now show that for solid, the approximate relations between α , β and γ are:

$$\beta = 2\alpha \text{ and } \gamma = 3\alpha;$$

(a) Relation between β and α . Consider a square plate of side ℓ_0 at 0°C and ℓ_1 at $t^\circ\text{C}$.

$$\ell_1 = \ell_0 (1 + \alpha t);$$

$$\text{Area of plate at } 0^\circ\text{C}, A_0 = \ell_0^2;$$

$$\text{Area of plate at } t^\circ\text{C}, A_1 = \ell_1^2 = \ell_0^2 (1 + \alpha t)^2 = A_0 (1 + \alpha t)^2$$

$$\text{Also Area of plate at } t^\circ\text{C}, A_1 = A_0 (1 + \beta t)$$

$$\therefore A_0 (1 + \alpha t)^2 = A_0 (1 + \beta t) \text{ or } \therefore 1 + \alpha^2 t^2 + 2\alpha t = 1 + \beta t$$

Since the value of α is small, the term $\alpha^2 t^2$ may be neglected. $\therefore \beta = 2\alpha$

The result is altogether general because any flat surface can be regarded as a collection of small squares.

(b) Relation between γ and α . Consider a cube of side ℓ_0 at 0°C and ℓ_1 at $t^\circ\text{C}$.

$$\therefore \ell_1 = \ell_0 (1 + \alpha t) ; \text{ Volume of cube at } 0^\circ\text{C}, V_0 = \ell_0^3 ;$$

$$\text{Volume of cube at } t^\circ\text{C}, V_1 = \ell_1^3 = \ell_0^3 (1 + \alpha t)^3 = V_0 (1 + \alpha t)^3$$

$$\text{Also Volume of cube at } t^\circ\text{C}, V_1 = V_0 (1 + \gamma t) ; \therefore V_0 (1 + \alpha t)^3 = V_0 (1 + \gamma t)$$

$$\text{Or } 1 + 3\alpha t + 3\alpha^2 t^2 + \alpha^3 t^3 = 1 + \gamma t$$

Since the value of α is small, we can neglect the higher power of α .

$$\therefore 3\alpha t = \gamma t \text{ or } \gamma = 3\alpha$$

Again, result is general because any solid can be regarded as a collection of small cubes.

7. VARIATION OF DENSITY WITH TEMPERATURE

Variation of Density with temperature: Most substances expand when they are heated, i.e.. Volume of a given mass of a substance increases on heating, so the density should decrease (as $\rho \propto \frac{1}{V}$)

Let us see how the density (ρ) varies with increase in temperature. $\rho = \frac{m}{V}$ or $\rho \propto \frac{1}{V}$ (for a given mass)

$$\therefore \frac{\rho'}{\rho} = \frac{V}{V'} = \frac{V}{V + \Delta V} = \frac{1}{1 + \gamma \Delta T} ; \quad \therefore \rho' = \frac{\rho}{1 + \gamma \Delta T}$$

This expression can also be written as, $\rho' = \rho(1 + \gamma \Delta T)^{-1}$

As γ is small. $(1 + \gamma \Delta T)^{-1} \approx 1 - \gamma \Delta T$ $\therefore \rho' \approx \rho(1 - \gamma \Delta T)$

Illustration 16: A glass flask of volume 200 cm^3 is just filled with mercury at 20°C . How much mercury will overflow when the temperature of the system is raised to 100°C ? The coefficient of volume expansion of glass is $1.2 \times 10^{-5}/^\circ\text{C}$ and that of mercury is $18 \times 10^{-5}/^\circ\text{C}$. **(JEE MAIN)**

Sol: Increase in temperature, increase the volume of both, mercury as well as flask but mercury expands more than flask because the coefficient of volume expansion of mercury is more than of flask.

$$\text{The increase in the volume of the flask is } \Delta V = \gamma_R V \Delta T = (1.2 \times 10^{-5}) \times (200) \times (100 - 20) = 0.19 \text{ cm}^3$$

$$\text{The increase in the volume of the mercury is } \Delta V' = \gamma_m V \Delta T = (18 \times 10^{-5}) \times (200) \times (100 - 20) = 2.88 \text{ cm}^3$$

$$\therefore \text{The volume of the mercury that will overflow } \Delta V' - \Delta V = 2.88 - 0.19 = 2.69 \text{ cm}^3$$

Illustration 17: A sheet of brass is 40 cm long and 8 cm broad at 0°C . If the surface area at 100°C is 320.1 cm^2 , find the coefficient of linear expansion of brass. **(JEE MAIN)**

Sol: Calculate the coefficient of area expansion, coefficient of linear exp. Equal to half of coeff. of area expansion.

$$\text{Surface area of sheet at } 0^\circ\text{C}, A_0 = 40 \times 8 = 320 \text{ cm}^2$$

$$\text{Surface area of sheet at } 100^\circ\text{C}, A_{100} = 320.1 \text{ cm}^2$$

$$\text{Rise in temperature, } \Delta T = 100 - 0 = 100^\circ\text{C}$$

$$\text{Increase in surface area } \Delta A = A_{100} - A_0 = 320.1 - 320 = 0.1 \text{ cm}^2$$

$$\text{Coefficient of surface expansion } \beta \text{ is given by; } \beta = \frac{\Delta A}{A_0 \times \Delta T} = \frac{0.1}{320 \times 100} = 31 \times 10^{-7} / ^\circ\text{C}$$

$$\therefore \text{Coefficient of linear expansion, } \alpha = \frac{\beta}{2} = \frac{31 \times 10^{-7}}{2} = 15.5 \times 10^{-7} / ^\circ\text{C}$$

8. THERMAL STRESS

If the ends of rods of length L_0 are rigidly fixed and it is heated, its length L_0 tends to increase due to increase in temperature ΔT , but it is prevented from expansion. It results in setting up compressive or tensile stress in the rod which is called the thermal stress.

As $Y = \frac{\text{Stress}}{\text{Strain}}$, $\text{Stress} = Y \times \text{Strain} = \frac{Y\Delta L}{L_0} = \frac{Y\alpha L_0 \Delta T}{L_0} = Y\alpha \Delta T$ The force, F , on rigid support is given by.

Where A is area of cross-section of the rod.

If ΔT represent a decrease in temperature, then F/A and F are tensile stress and tensile force respectively.

Note: When the temperature of a gas enclosed in a vessel of rigid material is increased, then thermal stress is equal to the increase in pressure (ΔP) and is given by: $\Delta P = K\gamma\Delta T$

Where K = bulk modulus of gas; γ = coefficient of cubical expansion; ΔT = increase in temperature

Proof. $V = V(1 + \gamma\Delta T)$ or $V - V = V\gamma\Delta T$ or $\Delta V = V\gamma\Delta T$ now $K = \frac{V\Delta P}{\Delta V} = \frac{V\Delta P}{\gamma V\Delta T} \therefore \Delta P = \gamma K\Delta T$

Illustration 18: A steel wire of 2.0mm cross-section is held straight (but under no tension) by attaching it firmly to two rigid walls at a distance 1.50 m apart, at 30°C . If the temperature now decreases to -10°C , and if the end points remain fixed, what will be the tension in the wire? For steel, $Y = 200\,000\text{ MPa}$ **(JEE MAIN)**

Sol: Here the concept of strain is applicable with linear expansion. Decreased temp. tends to decrease the length of wire but strain keep it intact.

Conceptualize: If free to do so, the wire would contract but since we have tied its ends, it will not contract and maintain its original length.

Classify: Until now we have seen when the length of a wire is changed, it produces strain and hence stress. This situation is different as strain will be produced because of wire maintaining its length. At a lower temperature the wire would have an unstrained length smaller than the original length. However since its ends are tied, it will maintain its length but develop strain. Or in other words it has longer length than what it would have had at this temperature if not tied at its ends.

Compute: If free to do so, the wire would contract a distance ΔL as it cooled, where

$$\Delta L = \alpha L \Delta T = (1.2 \times 10^{-5} \text{ } ^\circ\text{C}^{-1})(1.5\text{m})(40^\circ\text{C}) = 7.2 \times 10^{-4} \text{ m}$$

But the ends are fixed. As a result, forces at the ends must, in effect, stretch the wire this same length, ΔL . Therefore,

$$\text{from } Y = (F/A) / (\Delta L/L), \text{ we have tension } F = \frac{YA\Delta L}{L} = \frac{(2 \times 10^{11} \text{ N/m}^2)(2 \times 10^{-6} \text{ m}^2)(7.2 \times 10^{-4} \text{ m})}{1.5\text{m}} = 192 \text{ N}$$

Conclude: Strictly, we should have substituted $(1.5 \pm 7.2 \times 10^{-4})$ m for L in the expression of tension. However. The error incurred in not doing so, is negligible.

PROBLEM-SOLVING TACTICS

While solving a problem of heat transfer in these cases, do look for state changes because that's where students generally make a mistake. State changes cause some of the energy to be used up as latent heat and hence must be taken care of always.

FORMULAE SHEET

1. Type of thermal expansion

	Coefficient of expansion	For temperature change Δt change in
(i) Linear	$\alpha = \lim_{\Delta t \rightarrow 0} \frac{1}{\ell_0} \frac{\Delta \ell}{\Delta t}$	Length $\Delta \ell = \ell_0 \alpha \Delta t$
(ii) Superficial	$\beta = \lim_{\Delta t \rightarrow 0} \frac{1}{A_0} \frac{\Delta A}{\Delta t}$	Area $\Delta A = A_0 \beta \Delta t$
(iii) Volume	$\gamma = \lim_{\Delta t \rightarrow 0} \frac{1}{V_0} \frac{\Delta V}{\Delta t}$	Volume $\Delta V = V_0 \gamma \Delta t$

- For isotropic solids $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$ (let) so $\beta = 2\alpha$ and $\gamma = 3\alpha$
- For anisotropic solids $\beta = \alpha_1 + \alpha_2$ and $\gamma = \alpha_1 + \alpha_2 + \alpha_3$ Here α_1, α_2 and α_3 are coefficient of linear expansion in X, Y, and Z directions.

Variation in density: With increase of temperature volume increases so density decreases and vice-versa.

$$\rho = \frac{\rho_0}{(1 + \gamma \Delta t)} \approx \rho_0(1 - \gamma \Delta t)$$

Thermal Stress: A rod of length ℓ_0 is clamped between two fixed walls with distance ℓ_0 .

If temperature is changed by amount Δt then stress = $\frac{F}{A}$ (area assumed to be constant)

$$\text{Strain} = \frac{\Delta \ell}{\ell_0}; \text{ so, } Y = \frac{F/A}{\Delta \ell / \ell_0} = \frac{F \ell_0}{A \Delta \ell} \Rightarrow F = Y A \alpha \Delta t$$

- $\Delta Q = mc\Delta T$ where c : Specific heat capacity
- $\Delta Q = nC\Delta T$ C : Molar heat capacity
- Heat transfer in phase change : $\Delta Q = mL$ L : latent heat of substance
- 1 Calorie = 4.18 joules of mechanical work
- Law of Calorimetry: heat released by one of the substances = Heat absorbed by other substances.

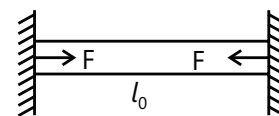


Figure 15.6

Solved Examples

JEE Main/Boards

Example 1: Calculate the amount of heat required to convert 1.00kg of ice at -10°C into steam at 100°C at normal pressure. Specific heat capacity of ice = $2100 \text{ J kg}^{-1} \text{ K}^{-1}$, latent heat of fusion of ice = $3.36 \times 10^5 \text{ J kg}^{-1}$, specific heat capacity of water = $4200 \text{ J kg}^{-1} \text{ K}^{-1}$ and latent heat of vaporization of water = $2.25 \times 10^6 \text{ J kg}^{-1}$.

Sol: Here the temperature of ice and water changes along with change in phases. i. e. ice to water and then water to steam.

Heat required to take the ice from -10°C to

$$0^\circ\text{C} = (1\text{kg})(2100 \text{ J kg}^{-1} \text{ K}^{-1})(10\text{K}) = 21000 \text{ J.}$$

$$\text{Heat required to melt the ice at } 0^\circ\text{C} \text{ to water} = (1\text{kg})(3.36 \times 10^5 \text{ J kg}^{-1}) = 336000 \text{ J.}$$

$$\text{Heat required to take 1 kg of water from } 0^\circ\text{C} \text{ to } 100 = (1\text{kg})(4200 \text{ J kg}^{-1} \text{ K}^{-1})(100\text{K}) = 420000 \text{ J.}$$

$$\text{Heat required to convert 1kg of water at } 100^\circ\text{C} \text{ into steam} = (1\text{kg})(2.25 \times 10^6 \text{ J kg}^{-1}) = 2.25 \times 10^6 \text{ J.}$$