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12. SOUND WAVES

1. INTRODUCTION

This chapter discusses the nature of sound waves. We will apply concepts learned in the chapter on waves on a string are applied to understand the phenomena related to sound waves. We will learn about what all parameters the speed of sound in a medium depends. Reflection, transmission and interference are important phenomena associated with sound. The study of sound waves enables us to design musical instruments and auditoriums. We will understand the properties of sound waves in air columns and the phenomena of echo. Phenomena of beats and doppler effect have been discussed.

2. NATURE AND PROPAGATION OF SOUND WAVES

Sound is a mechanical wave that results from the back and forth vibration of the particles of a medium through which the sound wave is traveling. Further, if a sound wave is traveling from left to right in air, then particles in air will be displaced in both rightward and leftward directions due to the energy of the sound wave passing through it. However, the motion of the particles is parallel (and antiparallel) to the direction of the energy transport. This unique property characterizes sound waves in air as longitudinal waves.

A typical case of propagation of sound waves in air is shown in the Fig. 12.1.

We know that as the prong vibrates in simple harmonic motion, the pressure variations in the layer close to the prong also change in a simple harmonic fashion. Thus, the increase in pressure above its normal value may, therefore, be written as $\delta P = P - P_0 = \delta P_0 \sin\omega t$,

where δP_0 is the maximum increase in pressure above its normal value. As this disturbance, due to the traveling of the sound wave, moves towards right with the speed u (the above speed and not the particle speed), the equation for the excess pressure at any point x at any time t is given by $\delta P = \delta P_0 \sin\omega\left(t - x / v\right)$.

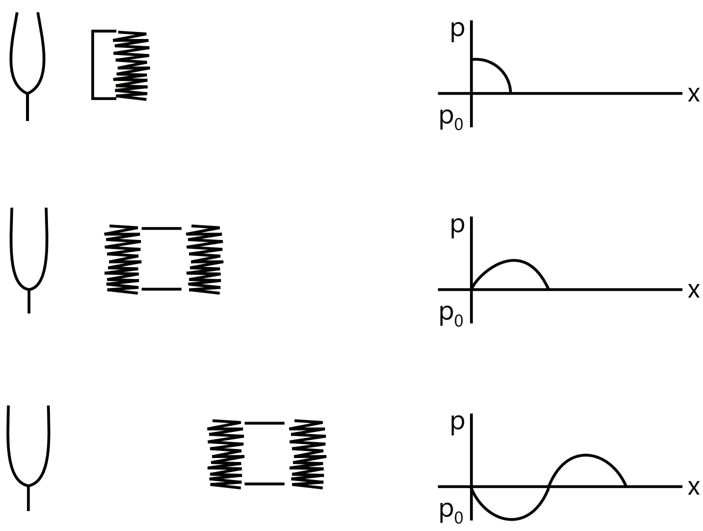


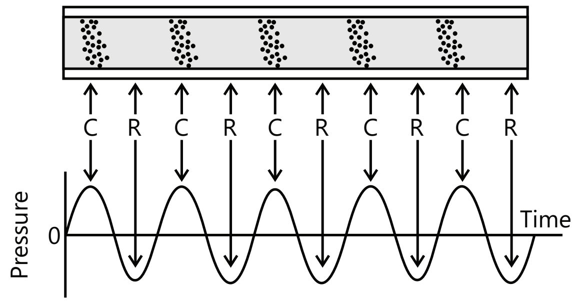
Figure 12.1

2.1 Compression and Rarefaction

Due to the phenomenon of longitudinal motion of the air particles, we observe that there are regions in the air where the air particles are compressed together and other regions where the is air spread apart. These regions are respectively known as compression and rarefaction. The formation of these regions is due to back and forth motion of the particles of the medium.

Compression is the result of increase in density and pressure in a medium, such as air, due to the passage of a sound wave. However, rarefaction is quite just the opposite of compression, i.e., decrease in density and pressure in a medium due to the passage of a sound wave.

Sound is a pressure wave



Note: "C" stands for compression and "R" stands for rarefaction

Figure 12.2

2.2 Wavelength

The wavelength of a wave is just the distance that a disturbance is carried along the medium when one wave cycle is completed. A longitudinal wave typically consists of a repeating pattern of compressions and rarefactions.

Hence, the wavelength is commonly measured either as the distance from one compression point to the next adjacent compression or the distance from one rarefaction point to the next adjacent rarefaction.

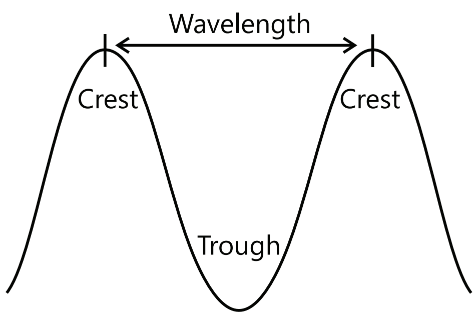


Figure 12.3

2.3 Polarization of Sound Waves

It is to be noted that all directions perpendicular to the propagation of sound waves are equivalent and therefore sound waves cannot be polarized.

2.4 Wave Front

A wave front usually is the locus of point that is having the same phase, i.e., a line or curve in 2d, or a surface for a wave propagating in 3d. Further, the sound observed at some point by a vibrating source travels virtually in all directions of the medium only if the medium is extended. However, for a homogenous and isotropic medium, the wave fronts are usually normal to the direction of propagation.

2.5 Infrasonic and Ultrasonic Sound Waves

Sound waves are audible only if the frequency of alternation of pressure is in the range of 20 Hz and 20,000Hz. In other words, beyond this upper limit they are not audible. The waves are classified based on their frequency range, i.e., below the audible range they are called infrasonic waves, whereas those with frequency greater than the audible range are termed ultrasonic waves.

Illustration 1: A wave of wavelength of 0.60 cm is produced in air and it travels at a speed of 300 ms^{-1}. Will it be audible? **(JEE MAIN)**

Sol: The frequency of the sound wave is given as $\nu = \frac{V}{\lambda}$. The audible range is 20 Hz to 20 KHz.

From the relation $V = \nu\lambda$, we calculate the frequency of the wave as $\nu = \frac{V}{\lambda}$

$$= \frac{300\ ms^{-1}}{0.60 \times 10^{-2}\ m} = 50000\ Hz$$

This is clearly very much above the audible range. Therefore, it is an ultrasonic wave and hence will not be audible.

2.6 Displacement Wave and Pressure Wave

A longitudinal wave can be described either in terms of longitudinal displacement of the particle of the medium or in terms of excess pressure generated due to phenomena of compression and rarefaction.

3. EQUATION OF SOUND WAVE

As we have already noted above, a longitudinal wave in a fluid (liquid or gas) can be described either in terms of the longitudinal displacement suffered by the particles of the medium or in terms of the excess pressure generated due to the compression or rarefaction. Let us now verify how these two representations are related to each other.

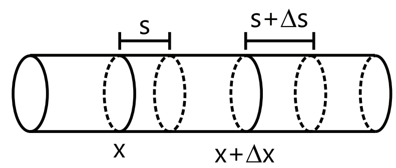


Figure 12.4

Consider a wave traveling in the x-direction in a fluid. Further, now suppose that at time t, the particle at the undisturbed position x suffers a displacement s in the x-direction. The wave can then be described by the equation

$$s = s_0 \sin\omega(t - x/v) \quad \text{... (i)}$$

Now, consider the element of the material which is within x and $x + \Delta x$ (see the fig. 12.4) in the undisturbed state. Therefore, by considering a cross-sectional area A, the volume of the element in the undisturbed state is $A\Delta x$ and its mass is $\rho A\Delta x$. As the wave passes through, the ends at x and $x + \Delta x$ are displaced by amount s and $s + \Delta s$ according to Eq. (i). Thus, the increase in volume of the element at time t is given as

$$\Delta V = A\,\Delta s = A\frac{\delta s}{\delta x}\Delta x = As_0\left(-\omega / v\right)\cos\omega\left(t - x / v\right)\Delta x$$

(where Δs has been obtained by differentiating Eq. (i) with respect to x. The element is, therefore, under a volume strain. $\frac{\Delta v}{v} = \frac{-As_0\omega\cos\left(t - x/v\right)}{vA\Delta x} = \frac{-s_0\omega}{v}\cos\omega\left(t - x/v\right)$)

However, the corresponding stress, i.e., the excess pressure developed in the element at x at time t is $p = B\left(\frac{-\Delta v}{v}\right)$, where B is the bulk modulus of the material.

$$\text{Thus, } p = B\frac{s_0\omega}{v}\cos\omega\left(t - x/v\right) \quad \text{... (ii)}$$

Comparing with standard wave equation, we see that the amplitude p_0 and the displacement amplitude s_0 are related as $p_0 = \frac{B\omega}{v}s_0 = Bks_0$ (where k is the wave number)

Also, we observe from (i) and (ii) that the pressure wave differs in phase by $\pi / 2$ from the displacement wave. Further, the pressure maxima observed is at the point where the displacement is zero and displacement maxima occur where the pressure is at its normal level.

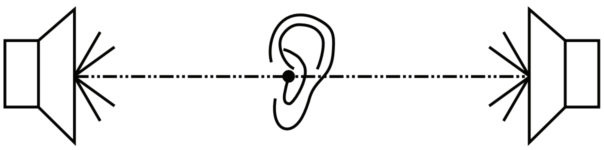


Figure 12.5

The assertion here being that displacement is zero where the pressure change is maximum and vice versa, and therefore sets the two descriptions on different footings. Naturally, the human ear or an electronic detector responds only to the change in pressure and not to the displacement. Let us suppose that two audio speakers are driven by the same amplifier and are placed facing each other. Further, a detector is placed midway between them. Now, it is clear that the displacement of the air particles near the detector will be zero as the two sources drive these particles in opposite directions. However, both the sources send compression waves and rarefaction waves together.

PLANCESS CONCEPTS

The human ear or an electronic detector responds to the pressure change and not the displacement in a straightforward way.

Vaibhav Krishnan (JEE 2009, AIR 22)

Illustration 2: Suppose that a sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given a point is $1.0\times10^{-3}\ Nm^{-2}$, then find the amplitude of vibration of the particles of the medium. The bulk modulus of air is $1.4\times10^{5}\ Nm^{-2}$. **(JEE MAIN)**

Sol: The amplitude of pressure at a point is given by $P_o = \frac{P_{max} - P_{min}}{2}$. As the bulk modulus of the air is given, the amplitude of the vibration is given by $S_0 = \frac{P_0}{Bk}$ where k is wave number.

The pressure amplitude is $P_0 = \frac{1.0\times10^{-3}\ Nm^{-2}}{2} = 0.5\times1.0\times10^{-3}\ Nm^{-2}$

The displacement amplitude s_0 is given by $P_0 = Bks_0$

or $s_0 = \frac{P_0}{Bk} = \frac{P_0\lambda}{2\pi B} = \frac{0.5\times1.0\times10^{-3}\ Nm^{-2}\times\left(40\times10^{-2}m\right)}{2\times3.14\times1.4\times10^{5}Nm^{-2}} = 2.2\times10^{-10}m.$

Illustration 3: Assume that a wave is propagating on a long stretched string along its length taken as the positive x-axis. The wave equation is given as $y = y_0 \exp\left(\frac{t}{T} - \frac{x}{\lambda}\right)^2$ where y_0 = 4 mm, T =1 s, and λ = 4 cm. Now, (a) Find the velocity of the wave.

(b) Find the function $f(t)$ giving the displacement of the particle at x = 0.

(c) Find the function $g(x)$ giving the shape of the string at t = 0.

(d) Plot the shape $g(x)$ of the string at t = 0.

(e) Plot the shape of the string at t = 5 s. **(JEE MAIN)**

Sol: The wave moves having natural frequency of ν and wavelength λ has velocity $V = \nu\lambda$. As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation may be written as $y = y_0 \exp\left[\frac{1}{T}\left(t - \frac{x}{\lambda / T}\right)^2\right]$

Comparing with the general equation, $y = f(t - x/v)$ we see that $\nu = \frac{\lambda}{T} = \frac{4cm}{1.0s}/\sec$

(b) Substituting x = 0 in the given equation, we have $f(t) = y_0 e^{-(t/T)^2}$ ... (i)

(c) Substituting t = 0 in the given equation, we have $g(x) = y_0 e^{-(x/\lambda)^2}$ ... (ii)

Figure 12.6

4. VELOCITY OF SOUND WAVES

We know that the sound waves travel in air or in gaseous media as longitudinal waves. Further, when these waves travel longitudinally, then compression and rarefaction are produced in the layers of air in such a way that the particles in layers of the air move in a to and fro fashion about their mean position in the direction exactly as that of the direction of propagation of sound waves. Therefore, the speed v of longitudinal waves in an elastic medium of modulus of elastic E and density ρ is given by $v = \sqrt{\frac{E}{\rho}}$.

For both liquids and gases, E is the bulk modulus of elasticity. For a thin solid rod, E is Young's modulus. However, for large solids, E depends upon the bulk modulus and shear modulus.

Newton assumed that the changes produced due to propagation of sound in gases are isothermal; this implies that a compressed layer of air at higher temperature loses heat immediately to the surroundings, whereas a rarefied layer of at lower temperature gains heat from the surroundings so that temperature of air remains constant. As the modulus of elasticity for isothermal change is equal to the pressure P according to Newton's formula for change,

$$v = \sqrt{\frac{P}{\rho}}.$$

Laplace showed that the sound is propagated in air or gases under adiabatic change. This is because the compression and rarefactions produced due to the propagation of sound follow each other so rapidly that there is no time available for the compressed layer at a higher temperature and rarefied layer at a lower temperature to equalize their temperature with the surroundings. Thus, the velocity v of sound travelling under adiabatic conditions in a gas is given by Laplace's formula as:

$$v = \sqrt{\frac{E_{adiabatic}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} \text{ ; because } E_{adiabatic} = \gamma P \text{ and } \gamma = \frac{C_p}{C_v}$$

By substituting γ = 1.41 for air, density of air = 1.293 kg/m^3, atmospheric pressure = $1.013 \times 10^5 N/m^2$, the velocity of sound in air, v = 332 m/s. However, in general, the velocity of sound in solid is greater than the velocity of sound in liquids and the velocity of sound in liquids is greater than the velocity of sound in gases.

4.1 Sound Wave in Solids

Usually, sound waves travel in solids just like they travel in fluids. The speed of longitudinal sound waves in a solid rod can be shown to be $v = \sqrt{Y/\rho}$,

where Y is the Young's modulus of the solids and ρ its density.

However, for extended solids, the speed is a more complicated function of bulk modulus and shear modulus. The table provided hereunder gives the speed of sound in some common materials.

Medium	**Speed (m/s)**	**Medium**	**Speed (m/s)**
Air (dry 0^0C)	332	Copper	3810
Hydrogen	1330	Aluminum	5000
Water	1486	Steel	5200

4.2 Sound Wave in Fluids

A sound wave in air is a typical longitudinal wave. As a sound wave passes through air, its potential energy is usually associated with periodic compression and expansion of small volume element of the air. The unique property that determines the extent to which an element in the medium changes its volume as the pressure applied to it increases or decreases is the bulk modulus, B. $B = \frac{-\Delta P}{\Delta V / V}$

where $\frac{\Delta V}{V}$ is the fractional change in volume produced by a change in pressure ΔP.

Let us now suppose that air of density ρ is filled inside a tube of cross-sectional area A under a pressure P. Initially, the air is at rest.

At t = 0, the piston at the left end of the tube (as shown in the Fig. 12.7) is set to motion toward the right with a speed μ. After a time interval Δt, all portions of the air to the left of section 1 are moving with speed u, whereas all portions to the right of the section are at rest. Further, the boundary between the moving and the stationary portion travels to the right with v, the speed of the elastic wave (or sound wave). In the time interval Δt, the piston has moved $u\,\Delta t$ and the elastic disturbance has moved across a distance of $v\,\Delta t$.

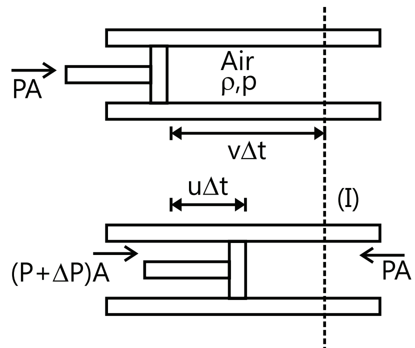


Figure 12.7

The mass of air that has attained a velocity u in time Δt is taken as $v = \sqrt{\frac{B}{\rho}}\; P(\Delta x)$ A. Therefore, now the momentum imparted is $\left[Pv(\Delta t)A\right]u$ and the net impulse $= (\Delta PA).\Delta t$.

Thus, impulse = change in momentum $(\Delta PA).\Delta t = \left[Pv(\Delta t)A.\right]u$ or $\Delta P = Pvu$ ...(xv)

Since $B = \frac{\Delta P}{\Delta V / V}$ $\therefore$ $\Delta P = B\left(\frac{\Delta V}{V}\right)$ where $V = Av\Delta t$ and $\Delta V = Au\,\Delta t$

$\therefore \frac{\Delta V}{V} = \frac{Au\,\Delta t}{Av\Delta t} = \frac{u}{v}$ thus, $\Delta P = B\frac{u}{v}$ ...(xvi)

From (xv) and (xvi) $v = \sqrt{\frac{B}{P}}$.

4.3 Speed of Sound in a Gas: Newton's Formula and Laplace Correction

The speed of sound in a gas can be expressed in terms of its pressure and density. We now summarize these properties hereunder:

(a) For a given mass of an ideal gas, the pressure, volume and the temperature are related as $\frac{PV}{T}$ = constant.

However, if the temperature remains constant (called an isothermal process), then the pressure and the volume of a given mass of a gas satisfy PV = constant. Here, T is the absolute temperature of the gas. This is known as Boyle's law.

(b) If no heat is supplied to a given mass of a gas (called an adiabatic process), then its pressure and volume satisfy PV^{γ}= constant where γ is a constant for the given gas. It is, in fact, the ratio C_p / C_V of two specific heat capacities of the gas.

Newton assumed that when a sound wave is propagated through a gas, the temperature variation in the layer of compression and rarefaction is negligible. Hence, the condition here is isothermal and hence Boyle's law will be applicable.

$PV = \text{constant}$ or, $P\Delta V + V\Delta P = 0$ or, $B = -\frac{\Delta P}{\Delta V / V} = P$ ...... (i)

Using the above result, the speed of sound in the gas is given by $v = \sqrt{P/\rho}$.

Laplace, however, suggested that the compression or rarefaction takes place too rapidly and the gas element being compressed or rarefied is hardly left with enough time to exchange heat with the surroundings. It is hence an adiabatic process and therefore one should use the equation $PV^{\gamma} = \text{constant}$. Taking logarithms, in P+in V = constant.

Now, by taking differentials, $\frac{\Delta P}{P} + \gamma \frac{\Delta V}{V} = 0$ or $B = -\frac{\Delta P}{\Delta V / V} = \gamma P$

Thus, the speed of sound is $v = \sqrt{\frac{\gamma P}{\rho}}$.

5. EFFECT OF PRESSURE, TEMPERATURE AND HUMIDITY AND SPEED OF SOUND IN AIR

(a) Effect of temperature as PV = nRT and $\rho = \frac{m}{V}$ $\therefore$ $v = \sqrt{\frac{\gamma PV}{m}} = \sqrt{\frac{\gamma RT}{M_m}}$ where M_m is mass of one mole of gas.

Thus, the velocity of sound is directly proportional to the square root of the absolute temperature. If v_t and v_0 are velocities of sound at t^0C and 0 ^{0}C, respectively, then $\frac{V_t}{V_0} = \sqrt{\frac{T_t}{T_0}} = \sqrt{\frac{273+t}{273}}$

where T_t and T_0 are respective absolute temperatures...

(b) Effect of pressure. If the temperature of the gas remains constant, then the velocity of sound does not change with the change of pressure because $\frac{p}{\rho}$ is a constant quantity. PM = ρRT

(c) Effect of humidity. As the density of water vapor at STP, 0.8kg/m^3, is lower than the density of dry air, 1.29km/m^3, the speed of sound in air increases when the humidity increases in the moist air.

6. INTENSITY OF SOUND

Normally, the intensity of sound I at any point is the quantum of energy transmitted per second across a unit area normal to the direction of propagation of sound waves. The intensity follows the pattern of an inverse square law of distance, i.e., $I \propto 1/R^2$ and I is proportional to the square of amplitude. Further, the level of intensity of sound as perceived by humans is called loudness. Thus, the intensity level or loudness L is quantitatively measured as compared to a minimum intensity of sound audible to human ear. Hence, the intensity level or loudness, measured in unit of decibel, dB, is given as $L = 10 \log_{10}\left(\frac{I}{I_0}\right)$..

where I_0 is the minimum audible intensity which is equal to 10^{-12} watt/m^2. Thus, the intensity of sound increases by a factor of 10 when the intensity level or loudness increases by 10 decibels.

Now, let us consider again a sound wave travelling along the x-direction. Let the equation for the displacement of the particles and the excess pressure developed by the wave be given as

$$s = s_0 \sin\omega(t - x/v) \text{ and } P = P_0 \cos\omega(t - x/v) \quad \text{... (i)}$$

where $P_0 = \frac{B\omega s_0}{v}$

Now, consider a cross section of area 'A' perpendicular to x-direction. The power W, transmitted by the wave across the section considered is $W=(PA)\frac{\delta s}{\delta t}$; $W=AP_0\cos\omega(t-x/v)\omega s_0\cos\omega(t-x/v)$ $=\frac{A\omega^2 s_0^2 B}{n}\cos^2\omega(t-x/v)$

The intensity 'I' is thus $I=\frac{1}{2}\frac{\omega^2 s_0^2 B}{u}=\frac{2\pi^2 B}{u}s_0^2 V^2.$; $I=\frac{P_0^2 u}{2B}$

As $B=Pv^{-2}$, the intensity can also be written as $I=\frac{v}{2pv^2}P_0^2=\frac{P_0^2}{2pv}$.

Loudness: Our ear is sensitive for an extremely large range of intensity. Therefore, a logarithmic rather than a linear scale in this regard is convenient. Accordingly, the intensity level β of t = a sound wave is defined by the equation $\beta=10\log\left(\frac{I}{I_0}\right)$ Decibel , where $I_0=10^{-12}\,W/m^2$ is the reference or threshold intensity level to which any intensity I is compared.

PLANCESS CONCEPTS

Intensity is directly proportional to the square of the pressure amplitude.

Nivvedan (JEE 2009, AIR 113)

Illustration 4: Assume that the pressure amplitude in a sound wave from a radio receiver is $2.0\times10^{-2}Nm^{-2}$ and the intensity at a point is $5.0\times10^{-7}Wm^{-2}$. If by turning the "volume" knob the pressure amplitude is increased to $2.5\times10^{-2}Nm^{-2}$, then evaluate the intensity. **(JEE MAIN)**

Sol: The intensity of the wave is proportional to square of the pressure amplitude of wave. If we increase the pressure amplitude then the intensity of sound will accordingly.

The intensity is proportional to the square of the pressure amplitude.

Thus, $\frac{I'}{I}=\left(\frac{P'_0}{P_0}\right)^2$ or $I'=\left(\frac{P'_0}{P_0}\right)^2 I=\left(\frac{2.5}{2.0}\right)^2\times5.0\times10^{-7}Wm^{-2}=7.8\times10^{-7}Wm^{-2}$

7. PERCEPTION OF SOUND TO HUMAN EAR

There are three parameters which govern the perception of sound to human ear. They are listed hereunder.

(a) Pitch and frequency,

(b) Loudness, and

(c) Quality and waveform.

7.1 Pitch and Frequency

The frequency of a wave generally signifies how often the particles of the medium vibrate when a wave travels through the medium. It is measured as the number of complete back-and-forth vibrations of a particle of the medium per unit of time. Further, the sensation of frequency is commonly referred to as the pitch of a sound. Therefore, a high pitch sound generally corresponds to a frequency sound wave and a low pitch sound corresponds to a low frequency sound wave. Our ability to perceive pitch is associated with the frequency of the sound wave that impinges upon our ear. This is because sound waves travelling through air are longitudinal waves that produce high- and low-pressure disturbances of the particles of the air at a given frequency. Therefore, our ear has an ability to detect such frequencies and associate them with the pitch of the sound.

7.2 Loudness

Loudness is that characteristic of a sound that is primarily a psychological correlate of physical strength (amplitude). However, more formally it is defined as "that attribute of auditory sensation in terms of which sounds can be ordered on a scale extending from quiet to loud." Further, loudness is also affected by parameters other than sound pressure, including frequency, bandwidth and duration.

7.3 Quality of Waveform

The quality of sound is typically an assessment of the accuracy, enjoyability, or intelligibility of audio output from an electronic device. Therefore, quality of sound can be measured objectively, such as when tools are used to gauge the accuracy with which the device reproduces an original sound; or it can be measured subjectively, such as when we respond to the sound or gauge its perceived similarity to another sound. Thus, we differentiate between the sound from a table and that from a mridang on the basis of their quality alone.

Illustration 5: Suppose that a source emitting sound of frequency 180 Hz is placed in front of a wall at a distance of 2 m from it. Further, a detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum loudness. Speed of sound in air = 360 m/s. **(JEE ADVANCED)**

Sol: As there is a wall at a distance of 2 m from the source, the wave will reflect from the wall and interfere with the wave directly from the source. If constructive interference takes place between the reflected wave and original wave then the maximum loudness is heard. The condition of constrictive interference is $\Delta = n\lambda$.

The situation is visualized in the Fig. 12.8. Now, suppose that the detector is placed at a distance of x meter from the sources. Then, the wave received from the source after reflection from the wall has travelled a distance of $2\left[(2)^2 + x^2/4\right]^{1/2}$ m. Therefore, the difference between the two waves is $\Delta = \left\{2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x\right\}$ m.

However, constructive interference will take place when $\Delta = \lambda, 2\lambda$. Thus, the minimum distance x for a maximum loudness corresponds to $\Delta = \lambda$ ... (i)

The wavelength is $\lambda = \frac{u}{\nu} = \frac{360\text{m}/\text{s}}{180}\text{s}^{-1} = 2\text{m}$

Thus, by (i), $2\left[(2)^2 + x^2/4\right]^{1/2} - x = 2$ or, $\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$

Or, $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$ or $3 = x$.

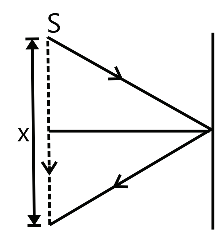


Figure 12.8

Thus, condition here is that the detector should be placed at a distance of 3 m from the source.
Note, however, that there is no abrupt phase change.

8. INTERFERENCE OF SOUND WAVES

Superposition of waves: When two or more waves travelling in the same direction act on the particles simultaneously, then the intensity of the resultant wave is modified due to superposition of the wave according to the principle discussed hereunder.

Superposition principle: If two or more waves arrive at a point simultaneously, then displacement at any point is equal to the vector sum of the displacement due to individual waves:

$\therefore\ y = y_1 + y_2 + ... + y_n$

where y is the resultant displacement due to the superposition of displacement y_1, y_2 .... y_n

Superposition can, in turn, give rise to the following phenomena:

Interference: When two waves of the same frequency and of constant phase difference travelling in the same direction superpose, then they can effect modification in intensity in the form of alternate maximum and minimum intensities which is called the interference phenomenon.

If the waves $y_1 = a_1 \sin(\omega t - kx)$ and $y_2 = a_2 \sin(\omega t - kx + \phi)$ superimpose, then by applying the principle of superposition, $y = y_1 + y_2 = R \sin(\omega t - kx + \phi)$

where the resultant amplitude, $R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2 \cos\phi}$ and phase angle $\theta = \tan^{-1}\left[\frac{a_1 \sin\phi}{a_1 + a_2 \cos\phi}\right]$

When $\phi = 2\pi n$ where n = 0, 1, 2... it produces constructive interference which gives $R = R_{max} = a_1 + a_2$.

However, when $\phi = (2n+1)\pi$ where n = 0, 1, 2..., $R = R_{min} = a_1 - a_2$ or amplitude is minimum due to destructive interference.

As intensity is proportional to the square of amplitude, the ratio of maximum intensity, I_{max}, to minimum intensity , I_{min}, is given by $\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$.

8.1 Coherent and Incoherent Sources

Two sources are called coherent sources only when their phase difference remains constant in time. In case if the phase difference does not remain constant in time, then the sources are incoherent.

The Fig. 12.9 here shows two tuning forks s_1 and s_2, placed side by side, which vibrate with equal frequency and equal amplitude. The point p is situated at a distance x from s_1 and $x + \Delta x$ from S_2.

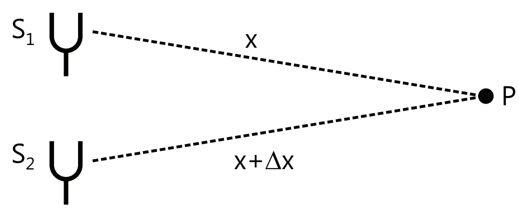


Figure 12.9

Now, suppose that the two forks are vibrating in phase so that $\delta_0 = 0$. Also, let p_{01} and p_{02} be the amplitudes of the wave from s_1 and s_2, respectively. Now, let us examine the resultant change in pressure at a point p. The pressure change at A due to the two waves are described by

$$P_1 = P_{01} \sin(kx - \omega t);\ P_2 = P_{02} \sin\left[k(x + \Delta x) - \omega t\right] = P_{02} \sin\left[(kx - \omega t) + \delta\right],$$

where $\delta = k\Delta x = \frac{2\pi \Delta x}{\lambda}$ ... (i)

is the phase difference between two waves reaching P.

The resultant wave is thus given by $p = p_0 \sin\left[(kx - \omega t) + \delta\right]$ where $p_0^2 = p_{01}^2 + p_{02}^2 + 2p_{01}p_{02} \cos\delta$

And $\tan\varepsilon = \frac{p_{02} \sin\delta}{p_{01} + p_{02} \cos\delta}$

The resultant amplitude is maximum when $\delta = 2n\pi$ and minimum when $\delta = (2n+1)\pi$ where n is an integer. These are correspondingly the conditions for constructive and destructive interference:

$\delta = 2n\pi$ constructive interference

$\delta = (2n+1)\pi$ destructive interference ... (ii)

Using Eq. (i), i.e., $\delta = \frac{2\pi}{\lambda} \Delta x$, these conditions may be written in terms of the path difference as $\Delta x = n\lambda$ (constructive) or $\Delta x = (n + 1/2)\lambda$ (destructive) ... (iii)

At constructive interference, $p_0 = p_{01} + p_{02}$.

And at destructive interference, $p_0 = |p_{01} + p_{02}|$.

However, if the sources have an initial phase difference δ_0 between them, then the wave reaching 'p' at time t is represented by $p = p_{01}\sin[kx - \omega t]$ and $p = p_{02}\sin[k(x+\Delta x) - \omega t + \delta_0]$

The phase difference between these waves, therefore, is $\delta = \delta_0 + k\Delta x = \delta_0 + \frac{2\pi\Delta x}{\lambda}$.

Illustration 6: Two sound waves, originating from the same source, travel along different paths in air and then meet at a point. Now, if the source vibrates at frequency of 1.0 KHz and one path is 83 cm longer than the other, what will be the nature of interference? The speed of sound in air is 33 ms^{-1} **(JEE ADVANCED)**

Sol: The phase difference between the sound waves, is given by $\delta = \frac{2\pi}{\lambda}\Delta x$ where λ is the wavelength of the wave and Δx is the path difference between the waves

The wavelength of sound wave is $\lambda = \frac{u}{\nu}$; $= \frac{332\ ms^{-1}}{1.0\times 10^3\ Hz} = 0.332\ m$

The phase difference between the waves arriving at the point of observation is

$$\delta = \frac{2\pi}{\lambda}\Delta x = 2\pi \times \frac{0.83m}{0.332m} = 2\pi \times 2.5 = 5\pi$$

As this is an odd multiple of π, the waves interfere destructively.

9. REFLECTION OF SOUND

When there is discontinuity in the medium, sound waves obviously gets reflected. Therefore, when a sound wave gets reflected from a rigid boundary, then the particles at the boundary are unable to vibrate. Thus, a reflected wave is generated which interferes with the incoming wave to produce zero displacement at the boundary. At these points, however, the pressure variation is maximum. Thus, a reflected pressure wave has the same phase as the indicated wave.

Alternatively, a sound wave can also get reflected if it encounters a low pressure region. The reflected pressure wave interferes destructively with the incoming waves in this case. Thus, there is a phase change of $\prod$ in this case.

10. STANDING/LONGITUDINAL WAVES

When two progressive waves of the same frequency moving in the opposite direction superpose, then stationary waves are formed.

Let us now consider the superposition of two such waves along a stretched string having fixed ends where a harmonic wave travels toward right as $y_1 = a\sin\frac{2\pi}{\lambda}(vt - x)$. This wave is reflected from the second point and due to the reflection, its amplitude becomes $-a$ due to phase change of π. Further, the reflected wave y_2 = a sin $\frac{2\pi}{\lambda}(vt - x)$ travels toward left and these waves superpose due to a phase change of pi, and hence give the resultant displacement y.

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) - a\sin\frac{2\pi}{\lambda}(vt + x) = -2a\cos\left(\frac{2\pi vt}{\lambda}\right)\sin\frac{2\pi x}{\lambda} = -A\cos\left(\frac{2\pi vt}{\lambda}\right) \text{ where } A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right)$$

The strings here apparently execute harmonic motion such that the particles of the string vibrate with the same frequency but with different amplitudes. Such a resultant wave is called a standing or stationary wave. The portion along the string where the amplitude is zero is called a node and where the amplitude is maximum is called an antinode.

For nodes: $A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right) = 0; \Rightarrow \quad \frac{2\pi vt}{\lambda} = n\pi$ where n = 0, 1, 2, 3,

The relation $x = \frac{n\lambda}{2}$ gives the position of the nth node and the distance between successive nodes is $\frac{\lambda}{2}$.

For antinodes, $A = 2a\sin\left(\frac{2\pi vt}{\lambda}\right) = 2a, \frac{2\pi vt}{\lambda} = (2n-1)\frac{\pi}{2}$ where n = 1, 2, 3...

$x = (2n-1)\frac{\lambda}{4},$ i.e., $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots\ldots\ldots$

Such points are called antinodes with maximum amplitude of $2a$.

The distance between the successive nodes and antinodes is $\frac{\lambda}{4}$.

11. MODE OF VIBRATION IN AIR COLUMNS

Longitudinal/stationary waves can be generated in both open- and closed pipes like organ pipes having both open ends and one closed end, respectively. If a tuning fork produces a sound wave at the open end, then it is reflected from the second end such that the incident and reflected wave superpose to generate stationary waves. Further, the closed end is always a node and the open end is always an antinode.

11.1 Open pipe

The first three modes of vibration of an open pipe are given as follows:

For fundamental or first harmonic mode in the Fig. 12.10

(a) $l = \frac{\lambda_1}{2}; \lambda_1 = 2l; n_1 = \frac{v}{2l}$

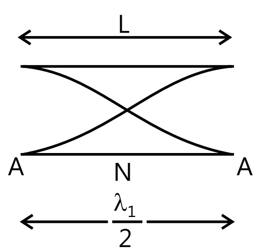


Figure 12.10

For the first overtone or the second harmonic in the Fig. 12.11

(b) $l = \lambda_2; n_2 = \frac{v}{2}$

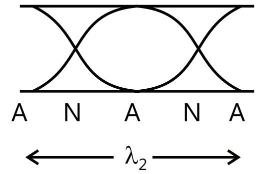


Figure 12.11

For the second overtone or the third harmonic mode in Fig. 12.12

$l = \frac{3\lambda_3}{2}; \lambda_3 = \frac{21}{3};$ or $\lambda_3 = \frac{3v}{2l};$ for path harmonic, $n_p = \frac{pv}{2l}$

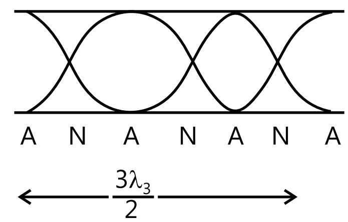


Figure 12.12

11.2 Closed pipe

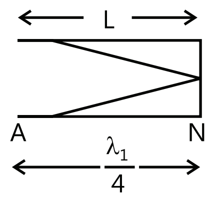


Figure 12.13

The first three modes of vibration of a closed pipe are given as follows:

For fundamental or first harmonic mode in Fig. 12.13.

$$l = \frac{\lambda_1}{4};\ \lambda_1 = 4l;\ n_1 = \frac{v}{4l}$$

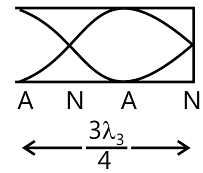


Figure 12.14

For the first overtone or third harmonic in Fig. 12.14

$$l = \frac{3\lambda_2}{4};\ \lambda_2 = \frac{4l}{3};\ n_2 = \frac{3v}{4l}$$

For the second overtone or the fifth harmonic,

$$l = \frac{5\lambda_2}{4};\ \lambda_3 = \frac{4l}{5};\ n_3 = \frac{5v}{4l}$$

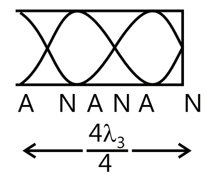


Figure 12.15

For the pth overtone or (2p+1)th harmonic, $n = \frac{(2p+1)v}{2l}$.

At the open end of the pipe, the antinode is formed at a small distance outside the open end. Thus, the correct length of the closed pipe is $l + e$ and that for an open pipe, it is $l + 2e$ and e is equal to 0.3D where D is the internal diameter of the pipe.

12. DETERMINATION OF SPEED OF SOUND IN AIR

12.1 Resonance Column Method

Generally, systems have one or more natural vibrating frequencies. Further, when a system is driven at a natural frequency, then there is a maximum energy transfer and the vibrating amplitude steadily increases till up to a maximum. However, when a system is driven at a natural frequency, we say that the system is in resonance (with the driven source) and refer to the particular frequency at which this occurs as a resonance frequency. Moreover, from the relationship between the frequency f, the wavelength λ, and the wave speed v, which is $\lambda f = v$, it is very obvious that if both the frequency and wavelength are known, then the wave speed can be easily determined. Further, if the wavelength and speed are known, then the frequency can be determined.

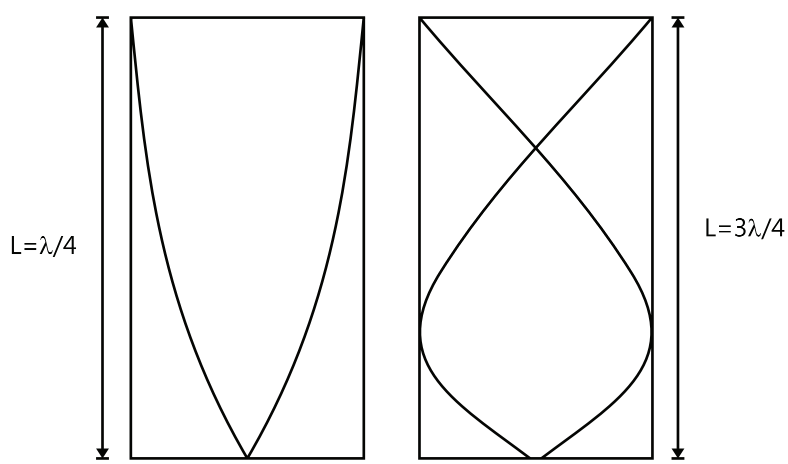


Figure 12.16

We know that air column in pipes or tubes of fixed length has particular resonant frequencies. Moreover, the interference of the waves travelling down the tube and the reflected waves traveling up the tube produces (longitudinal) standing wave which must have a node at the closed end of the tube and an antinode at the open end of the tube.

The resonance frequencies of a pipe or tube usually depend on its length L. As we observe from the Fig. 12.16, only a certain number of lengths or “loops” can be “fitted” into the tube length with the node–antinode requirements.

However, since each loop corresponds to one-half wavelength, resonance occurs when the tube is nearly equal to an odd number of quarter wavelength, i.e., $L = \lambda / 4, L = 3\lambda / 4, L = 5\lambda / 4$, etc

or in general, $L = (2n+1)\lambda / 4;\ \lambda = 4L / (2n+1);\ f_n = (2n+1)v / 4L$

Hence, an air column (tube) of length L has particular resonance frequencies and therefore will be in resonance with the corresponding odd harmonic driving frequencies.

As we can observe from the above equation, the three experimental parameters involved in the resonance condition of an air column are f, V, and L. However, to study the resonance in this experiment, the length L of an air column will be varied for a given driven frequency. The length of the air column achieved by changing the position of the movable piston in the tube is as seen in the Fig. 12.17.

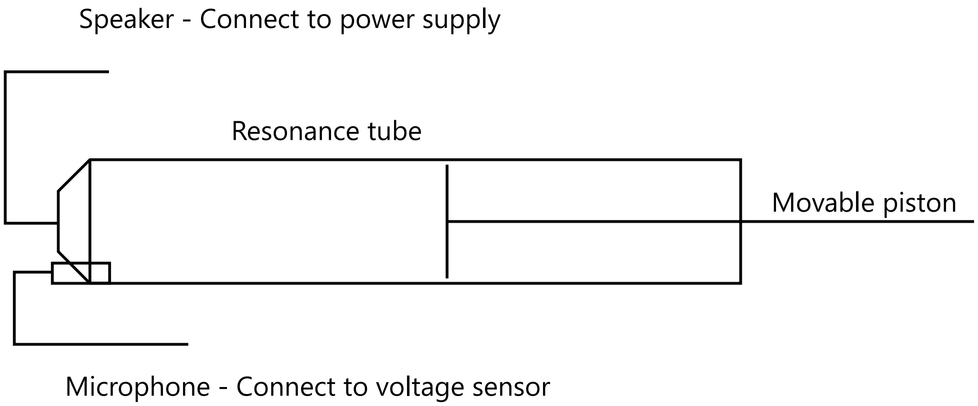


Figure 12.17

Further, as the piston is removed, increasing the length of the air column, more wavelength segments will fit into the tube, consistent with the node–antinode requirements at the ends. Thus, the difference in the tube lengths when successive antinodes are at the open end of the tube and resonance occurs is equal to a half wavelength; for example: $\Delta L = L_2 - L_1 = 3\lambda / 4 - \lambda / 4 = \lambda / 2$

Further, when an antinode is at the open end of the tube, a loud resonance tone is heard. Hence, the tube length for antinodes to be at the open end of the tube can be determined by moving the piston away from the open end of the tube and "listening" for resonances. However, no end correction is needed for the antinode occurring slightly above the end of the tube since in this case, difference in tube lengths for successive antinodes is equal to λ/2. Further, if we know the frequency of the driving source, then the wavelength is determined by measuring difference in tube length between successive antinodes, $\Delta L = \lambda / 2$ or $\lambda = 2\Delta L$, the speed of sound in air, $v_s = \lambda f$.

12.2 Kundt's Tube Method

In the Kundt's method, a gas is filled in a long cylindrical tube closed at both the ends, one by disk and the other by a movable piston. A metal rod is welded with the disk and is clamped exactly at the middle point. The length of the tube in this method can be varied by moving the movable piston. Some powder is sprinkled in the tube along its length.

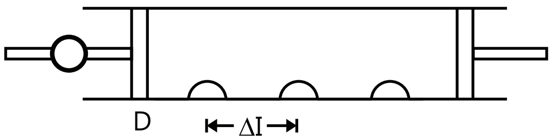


Figure 12.18

The rod in the setup is set into longitudinal vibrations either electronically or by rubbing it with some cloth or otherwise. Further, if the length of the gas column is such that one of its resonant frequency is equal to the frequency of the longitudinal vibration of the rod, then standing waves originate in the gas. Moreover, the powder particles at the displacement antinodes fly apart due to the inherent violent disturbance there, whereas the powder at the displacement nodes remain undisturbed because the particles here do not vibrate. Thus, the powder which was initially dispersed along the whole length of the tube gets collected in a heap at the displacement nodes. By measuring the seperation Δl between the successive heaps, we can find the wavelength of the sound in the enclosed gas. $\lambda = 2\Delta l$

However, it should be noted that the length of the column is adjusted by moving the piston such that the gas resonates and wavelength λ is obtained.

The speed of sound is given by $\nu = \lambda v = 2\Delta l * v$.

Further, if the frequency of the longitudinal vibration in the rod is not known, then the experiment is repeated with air filled in the tube. Now, the length between the heaps of the powder, $\Delta l'$ is measured. The speed of sound in air is then $\nu = 2\Delta l' v$ . ... (i)

Now, $\frac{\nu}{\nu} = \frac{\Delta l}{\Delta l'}$ or $\nu = \nu' \frac{\Delta l}{\Delta l'}$

By calculating the speed of ν' of sound in air, we can find the speed of sound in the gas.

13. BEATS

It two sources of slightly different frequencies produce sound waves in the same direction at the same point, these waves then superpose to produce alternate loud and feeble sounds. Such variations in loudness are called beats. The number of times such a fluctuation in loudness from maxima to minima takes place per second is called the beat frequency.

If two waves $y_1 = a\sin(2\pi n_1 t)$ and $y_2 = a\sin(2\pi n_2 t)$ of respective frequencies n_1 and n_2 superpose at the same place x = 0, then $y = y_1 + y_2 = a[\sin(2\pi n_1 t) + (\sin 2\pi n_2 t)]$

$$\therefore\ y = 2\ a\cos\left[\frac{2\pi(n_1 - n_2)t}{2}\right] \times \sin\left[\frac{2\pi(n_1 + n_2)t}{2}\right] = 2\ a\cos\left[2\pi(n_1 - n_2)t\right] \times \sin\left[2\pi(n_1 + n_2)t\right]$$

$$y = A\ \sin\left[\pi(n_1 + n_2)t\right]\ ;\ A = 2a\cos\left[\pi(n_1 - n_2)t\right]$$

The resultant wave is a harmonic wave with a frequency $\left(\frac{n_1 + n_2}{2}\right)$ but its amplitudes vary harmonically as a function of the difference in the frequency $n_1 - n_2$. The beat frequency n_B is $n_B = n_1 - n_2$.

If $n_1 - n_2$ is small, i.e., the number of times the intensity of sound fluctuates between maxima and minima per second is small, i.e., less than about 10 to 15, then the beats can be heard distinctly.

Illustration 7: Suppose that a string of length 25 cm and 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, then 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s, then find the tension in the string **(JEE ADVANCED)**

Sol: The fundamental frequency of the string and the closed organ pipe are $\nu_s = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$ and $\nu_p = \frac{v}{4\ell}$. When two waves of equal amplitude and slightly different frequencies superimpose with each other, phenomenon called beats take place. Number of beats n = $\Delta\nu$ where $\Delta\nu$ is the difference in the frequencies of superimposed waves.

Fundamental of the string $\nu_s = \frac{1}{2\ell}\sqrt{\frac{T}{m}} = \frac{1}{2 \times 0.25}\sqrt{\frac{T}{10^{-2}}} = 20\sqrt{T}$

The fundamental frequency of a closed pipe $\nu_p = \frac{v}{4\ell} = \frac{320}{4 \times 0.40} = 200$ Hz

The frequency of the first overtone of the string = $2\nu_s = 40\sqrt{T}$

Since there are 8 beat per second, $2\nu_s - \nu_p = 8$ or $40\sqrt{T} - 200 = 8$

Since on decreasing the tension, the beat frequency decreases, $2\nu_s$ is definitely greater than ν_p

$\therefore 40\sqrt{T} - 200 = 8$ or $T = 27.04\,N$

Illustration 8: A sonometer wire of 100 cm in length has a fundamental frequency of 330 Hz. Find

(a) The velocity of propagation of transverse waves along the wire and

(b) The wavelength of the resulting sound in air if velocity of sound in air is 330 ms^{-1}. **(JEE ADVANCED)**

Sol: As the wave travelling on the sonometer wire is the standing wave, the wavelength of the wire is $\lambda = 2L$. And the velocity of the wave is given by $v = f\lambda = 2fL$.

(a) In the case of transverse vibration of a string for fundamental mode: $L = (\lambda/2) \Rightarrow \lambda = 2\,L = 2\times 1 == 2m$

i.e., the wavelength of transverse wave propagating on the string is 2 m. Now, as the frequency of the wire is given to be 330 Hz, so from $v = f\lambda$, the velocity of transverse wave along the wire will be $v = 330 \times 2 = 660 m/s$

(b) Here, the vibrating wire will act as a source and produce sound, i.e., longitudinal waves in air: Now, as the frequency does not change with change in medium so $f = 330 Hz$, and as velocity in air is given to be = 330 m/s

so from relation $v = f\lambda$ we get $\lambda_{air} = \frac{v_{air}}{f} = \frac{330}{330} = 1m$

i.e., for sound (longitudinal mechanical waves) in air produced by vibration of wire (body),

$f = 330\ s^{-1}, \lambda_{air} = 1\,m$ and $v = f \times \lambda = 330 m/s$

14. DOPPLER EFFECT

We are familiar with the fact that when a source of sound or an observer or both are moving relative to each other, then there is an apparent change in the frequency of sound as heard by an observer and this is called Doppler Effect. Further, the apparent frequency increases if the source is moving toward the observer or the observer is moving toward the source. On the contrary, the apparent frequency decreases if either the source is moving away from the observer or the observer is moving away from the source. This apparent change in the frequency is principally due to the basic effect of motion of source to change the effective wavelength and the basic effect of motion of observer is the change in the number of waves received per second by the observer.

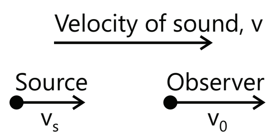


Figure 12.19

However, if both the source and the observer are moving in the positive direction of x-axis, then sound of frequency 'n' propagating in air with velocity in still air will result in an apparent frequency n' heard by observed as $n' = \left(\frac{v - v_0}{v - v_s}\right) n$

Moreover, if the direction of motion of source or observer is changed, then the signs of v_0 and v_s are accordingly changed from negative to positive. Thus, the frequency n', in still air for the different cases is obtained as follows:

(a) Both the source and observer are moving toward right when the source is approaching a receding observer toward right $n' = \left(\frac{v - v_0}{v - v_s}\right) n$

Figure 12.20

(b) Both the source and observer are receding from each other $n' = \left(\frac{v - v_0}{v + v_s}\right) n$

Figure 12.21

(c) Both the source and observer are moving toward each other $n' = \left(\frac{v + v_0}{v - v_s}\right) n$

Figure 12.22

(d) When the observer is approaching a receding source, $n' = \left(\frac{v+v_0}{v+v_s}\right)n$ If the wind is blowing with a velocity ω in the direction of sound, then ω is added to ν and if the wind is blowing with velocity ω opposite to direction of wind, then ω is subtracted from ν. The general formula for the apparent frequency n′ due to Doppler effect is, $n' = \left(\frac{\nu \pm \omega \mp \nu_0}{\nu \pm \omega \mp \nu_s}\right)n$

Figure 12.23

Illustration 9: Assume that a siren emitting a sound of frequency 2000 Hz moves away from you toward a cliff at a speed of 8 m/s.

(a) What is the frequency of the sound you hear coming directly from the siren?

(b) What is the frequency of sound you hear reflected off the cliff? Speed of sound in air is 330 m/s. **(JEE MAIN)**

Sol: As the siren being source is moving away from you the observer on cliff, the apparent frequency is given by $f' = f_0\left(\frac{v}{v+v_s}\right)$. Where f_0 is natural frequency of the sound wave. The intensity of the sound wave appears to be decreasing. When sound reflects from cliff it moves towards observer (cliff) and hence the frequency of the sound wave is $f' = f_o\left(\frac{v}{v-v_s}\right)$. When source moves towards the observer, the intensity of sound wave appears to be increasing.

(a) The frequency of sound heard directly is given by

$$f_1 = f_0\left(\frac{v}{v+v_s}\right);\ v_s = 8m/s;\ \therefore f_1 = \left(\frac{330}{330+8}\right)\times 2000$$

(b) The frequency of the reflected sound is given by

$$f_2 = f_1\left(\frac{v}{v-v_s}\right);\ \therefore\ f_2 = \left(\frac{330}{330-8}\right)\times 2000;\ \ f_2 = \frac{330}{322}\times 2000 = 2050\text{Hz}.$$

Illustration 10: Let us suppose that a sound detector is placed on a railway platform. A train, approaching the platform at a speed of 36 km h^{-1}, sounds its horn. The detector detects 12.0 kHz as the most dominant frequency in the horn. If the train stops at the platform and sounds the horn, what would be the most dominant frequency detected? The speed of sound in air is 340 ms^{-1}. **(JEE MAIN)**

Sol: In the first case, when train is moving towards the stationary observer on the platform, the intensity of the wave appears to be increasing. And the frequency is given by $f' = f_o\left(\frac{v}{v-v_s}\right)$. In the second case both the train and the observer are stationary so we hear the natural frequency f_0 of the sound wave.

Here, the observer (detector) is at rest with respect to the medium (air). Suppose that dominant frequency as emitted by the train is v_0. When the train is at rest at the platform, the detector will select the dominant frequency as v_0. When this same train was approaching the observer, then frequency detected would be

$$v' = \frac{\nu}{\nu - u_s}v_0\ ;\ \text{ or }\ v_0 = \frac{\nu - u_s}{\nu}v' = \left(1-\frac{u_s}{\nu}\right)v'$$

The speed of the source is $u_s = 36\text{kmh}^{-1} = \frac{36\times 10^3\text{m}}{3600\text{s}} = 10\text{ms}^{-1}$

Thus $v_0 = \left(1-\frac{10}{340}\right)\times 12.0\text{kHz} \quad = 11.6\text{kHz}$

14.1 Change in Wavelength

If a source moves with respect to the medium, then wavelength becomes different from the wavelength observed when there is no relative motion between the source and the medium. Thus, the formula for calculation of apparent wavelength may be derived immediately from the relation $\lambda = \nu / v$. It is given as

$$\lambda = \frac{\nu - u}{\nu}\lambda.$$

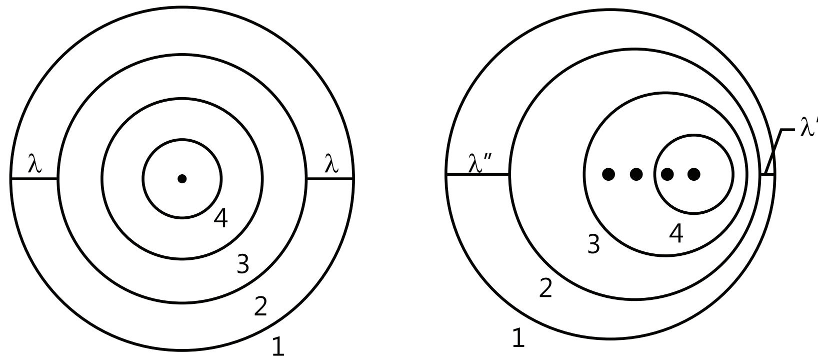


Figure 12.24

15. SONIC BOOM AND MACH NUMBER

A sonic boom is basically the sound associated with the shock waves created by an object travelling through air faster than the speed of sound. This boom generates a huge amount of energy, sounding much like an apparent explosion. The crack of a supersonic bullet passing overhead is an excellent example of a sonic boom in miniature.

Mach number is purely a dimensionless quantity representing the ratio of speed of an object moving through a fluid and the local speed of sound, $M = \frac{\nu}{\nu_{sound}}$ where M is the Mach number,

ν is the velocity of the source relative to the medium, and ν_{sound} is the speed of sound in the medium.

16. MUSICAL SCALE

A musical scale is a sequence of frequencies which has a particularly pleasing effect on our ear. A widely used musical scale, called diatonic scale, has eight frequencies covering an octave. We call each frequency as a note.

17. ACOUSTICS OF A BUILDING

Good concert halls: Good concert halls are so designed to eliminate unwanted reflection and echoes and to optimize the quality of the sound perceived by the audience. This is accomplished by suitably engineering the shape of the room and the walls, as well as to include sound-absorbing materials in areas that may cause echoes.

Lecture hall: Similar consideration such as the one made in the above must be made particularly in a college lecture hall, so that the professor can be heard by all of the students in the session. Although the sound quality need not be as good as in a concert hall where music is being played, it still must be good enough to prevent echoes and other things that will distort the audio quality of the speech delivered by the professor.

Work buildings: In an office building where there are cubicles with a divider in a large work area, there is often the problem of noise from conversation and activities. However, in this case the quality of the sound is not an issue as much as suppressing unwanted noise.

17.1 Echo

An echo (plural echoes) is a reflection of sound, arriving back at the listener particularly sometime after the direct sound.

17.2 Reverberation and Reverberation Time

Reverberation is the persistence of sound in a particular space after the original sound is produced. A reverberation, or reverb, is generated when sound is produced in an enclosed space causing a large number of echoes to build up and then slowly decay as the sound is absorbed by the wall and air.

Reverberation time: The interval between the initial direct arrival of a sound wave and the last reflected wave is called the reverberation time.

18. APPLICATION OF ULTRASONIC WAVES

Biomedical application: Ultrasound has very good therapeutic applications, which can be highly beneficial when used with appropriate dosage precautions. Relatively high power ultrasound can eliminate stony deposits or tissue, accelerate the effect of drugs in a targeted area, assist in the measurement of the elastic properties of tissue, and can also be used to sort cells or small particles for research.

Ultrasonic impact treatment: Ultrasonic impact treatment (UIT) is a technique wherein ultrasound is used to enhance the mechanical and physical properties of metals. Basically, it is a metallurgical processing technique in which ultrasonic energy is applied to a metal object.

Ultrasonic welding: In ultrasonic welding of plastics, high frequency (15 kHz to 40 kHz) low amplitude vibration is used to create heat by way of friction between the materials to be joined. The interface of the two parts is specially designed so as to concentrate the energy for the maximum weld strength.

Sonochemistry: Power ultrasound in the 20–100 kHz range alone is used in chemistry. The ultrasound does not interact directly with molecules to induce the chemical change, as its typical wavelength (in the millimeter range) is too long compared to the molecules. Instead, the energy causes cavitation, which generates extremes of temperature and pressure in the liquid where the reaction takes place.

19. SHOCK WAVES

A shock wave is one type of propagating disturbance. Similar to an ordinary wave, it carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases even in the absence of a material medium, through a field such as an electromagnetic field. Generally, shock waves are characterized by an abrupt, nearly discontinuous change in the characteristics of the medium.

PROBLEM-SOLVING TACTICS

1. Most of the questions are naturally related with the concepts of wave on a string. Therefore, one must be thorough with the concept of that particular topic. (E.g., standing waves formed in open pipe here are analogous to string tied at both ends. Further, many of the cases can be related in the same way.)

2. Questions dealing with physical experiments form another set of questions. Therefore, one must be familiar with usual as well as unusual (or specific) terminology of each experiment. Mostly, it happens that if we do not know the term, then we are usually stuck (E.g., end correction is one term used with the resonance column method, which is directly related with the radius of the tube.)

3. Path difference between two sources form another set of questions and this is the only place where some mathematical complexity can be involved. Hence, one must take care of them.

4. Questions related to Doppler effect and beats are generally formulae specific; therefore, one must carefully use the formulae. (It is, however, also advised that one must know about the derivation of these formulae.)

FORMULAE SHEET

S. No.	Term	Description
1.	Wave	It is a disturbance, which travels through the medium due to repeated periodic motion of particles of the medium about their equilibrium position. Examples include sound waves travelling through an intervening medium, water waves, light waves, etc.
2.	Mechanical waves	Waves requiring material medium for their propagation. These are basically governed by Newton's laws of motion. Sound waves are mechanical waves in the atmosphere between source and the listener and hence require medium for their propagation.
3.	Non-mechanical waves	These waves do not require material medium for their propagation. Examples include waves associated with light or light waves, radio waves, X-rays, micro waves, UV light, visible light and many more.
4.	Transverse waves	These are waves in which the displacements or oscillations are perpendicular to the direction of propagation of wave.
5.	Longitudinal waves	These are those waves in which displacement or oscillations in medium are parallel to the direction of propagation of wave, for example, sound waves.
6.	Equation of harmonic wave	At any time t, displacement y of the particle from its equilibrium position as a function of the coordinate x of the particle is y(x,y) = A $\sin(\omega t - kx)$ where A is the amplitude of the wave, k is the wave number, ω is the angular frequency of the wave, And $(\omega t - kx)$ is the phase.
7.	Wave number	Wave length λ and wave number k are related by the relation k = 2n/λ.
8.	Frequency	Wavelength λ and wave number k are related by the relation $v = \omega / k = \lambda / T = \lambda f$.
9.	Speed of a wave	Speed of a wave is given by $v = \omega / k = \lambda / T = \lambda f$.
10.	Speed of a transverse wave	Speed of a transverse wave on a stretched string depends only on tension and the linear mass density of the string but not on frequency of the wave, i.e., $v = \sqrt{T / \mu}$.
11.	Speed of a longitudinal wave	Speed of longitudinal waves in a medium is given by v = B = bulk modulus; ρ = density of medium; Speed of longitudinal waves in an ideal gas is $v = \sqrt{\gamma p / \rho}$ P = pressure of the gas , ρ = density of the gas and y = C_p/C_v.
12.	Principle of super position	When two or more waves traverse through the same medium, then the displacement of any particle of the medium is the sum of the displacements that the individual waves would give it, i.e., $y = \sum y_i(x,t)$.

S. No.	Term	Description
13.	Interference of waves	If two sinusoidal waves of the same amplitude and wavelength travel in the same direction, then they interfere to produce a resultant sinusoidal wave travelling in the direction with resultant wave given by the relation $y'(x,t) = [2A_m \cos(u/2)]\sin(\omega t - kx + u/2)$ where u is the phase difference between the two waves. If u = 0, then interference would be fully constructive. If u = π , then waves would be out of phase and their interference would be destructive.
14.	Reflection of waves	When a pulse or travelling wave encounters any boundary, it gets reflected. However, if an incident wave is represented by $y_i(x,t) = A\sin(\omega t - kx)$, then the reflected wave at rigid boundary is $y_r(x,t) = A\sin(\omega t + kx + n) = -A\sin(\omega t + kx)$ and for reflection at open boundary, reflected waves is given by $y_r(x,t) = A\sin(\omega t + kx)$.
15.	Standing waves	The interference of two identical waves moving in opposite directions produces standing waves. The particle displacement in a standing wave is given by $y(x,t) = [2A\cos(kx)]\sin(\omega t)$. In standing waves, amplitude of waves is different at different points, i.e., at nodes, amplitude is zero and at antinodes, amplitude is maximum which is equal to sum of amplitudes of constituting waves.
16.	Normal modes of stretched string	Frequency of transverse motion of stretched string of length L fixed at both the ends is given by f = nv/2L where n = 1, 2, 3, 4. The set of frequencies given by the above relation is called normal modes of oscillation of the system. Mode n = 1 is called the fundamental mode with the frequency f_1 = v/2L. Second harmonic is the oscillation mode with n = 2 and so on. Thus, a string has infinite number of possible frequencies of vibration which are harmonics of fundamental frequency f_1 such that $f_n = nf_1$.
17.	Beats	Beats arise when two waves having slightly differing frequency V_1 and V_2 and comparable amplitudes are superposed.
18.	Doppler effect	Doppler effect is a change in the observed frequency of the wave when the source S and the observer O move relative to the medium. There are three different ways where we can analyze this change in frequency as listed hereunder. (1) when observer is stationary and source is approaching observer v = $v_0(1+V_s/V)$ where V_s = velocity of the source relative to the medium v = velocity of wave relative to the medium V = observed frequency of sound waves in terms of source frequency V_0 = source frequency Change in the frequency when source recedes from stationary observer is v = $V_0(1–V_S/V)$ Observer at rest measures higher frequency when source approaches it and it measures lower frequency when source recedes from the observer. (2) observer is moving with a velocity V_0 toward a source and the source is at rest is V = $V_0(1+V_0/V)$ (3) both the source and observer are moving, then frequency observed by observer is V = V_0 $(V+V_0)/(V+V_S)$ and all the symbols have respective meanings as discussed earlier