PROBLEM-SOLVING TACTICS

1. Understanding and remembering all formulae is the key to solving problems in these sections. If the relation between the given quantities and the questions asked is known, it will be easy to solve most of the problems. All the quantities discussed in this topic are in some sense related to each other.

2. The concept of reflection (of waves) can be encapsulated in a single point: "Inversion- Reflected wave will invert only when it encounters a denser medium. And transmitted wave will never invert." If this much is clear, one can easily identify the case in every question.

3. Waves must always be understood in the context of transfer of energy rather than as just some function of x and t for better understanding of physics.

4. For questions pertaining to the derivation of the wave equation, one can begin easily with only the x part and subsequently add or subtract vt from x depending on the direction of velocity.

5. Most questions related to velocity and energy appear complicated due to the introduction of the usual Newton mechanics. This should, however, be treated just as some additional information to calculate tension in the string (e.g., Pulley systems).

FORMULAE SHEET

S. No	Term	Description
1	Wave	It is a disturbance or variation traveling through a medium due to the repeated undulating motion of particles of the medium through their equilibrium position. Examples are sound waves travelling through an intervening medium, water waves etc.
2	Mechanical waves	Waves that are propagated through a material medium are called MECHANICAL WAVES. These are governed by Newton's Law of Motion. Sound waves are mechanical waves propagated through the atmosphere from a source to the listener and it requires a medium for its propagation.
3	Non mechanical waves	Waves which are not propagated through a material medium. Eg: light waves, EM waves.
4	Transverse wave	These are waves in which the displacements or oscillations are perpendicular to the direction of propagation of the wave.
5	Longitudinal wave	Longitudinal wave waves in which the displacement or oscillations in medium are parallel to the direction of propagation of wave. Example: sound waves
6	Equation of harmonic wave	At any time t, displacement y of the particle from its equilibrium position as a function of the coordinate x of the particle is $y(x, t) = A\sin(\omega t - kx)$ where, A is the amplitude of the wave, K- is the wave number ω is angular frequency of the wave and (ωt-kx) is the phase
7	Wave number	Wavelength λ and wave number k are related by the relation $k = 2\pi/\lambda$

8	Frequency	Time period T and frequency f of the wave are related to ω by $\omega/2\pi = f = 1/T$
9	Speed of wave	Speed of the wave is given by $v = \omega/k = \lambda/T = \lambda f$
10	Speed of a transverse wave	The tension and the linear mass density of a stretched string, and not the frequency, determines the speed of a transverse wave i.e., $v=\sqrt{\frac{T}{\mu}}$ T = Tension in the string μ = Linear mass density of the string.
11	Speed of longitudinal waves	Speed of longitudinal waves in a medium is given by $v = \sqrt{\frac{B}{\rho}}$; B = bulk modulus; ρ = Density of the medium speed of longitudinal waves in ideal gas is $v = \sqrt{\frac{\gamma P}{\rho}}$ P = Pressure of the gas, ρ = Density of the gas and $\gamma = C_p / C_V$
12	Principle of superposition	It states that when two or more waves of same type come together at a single point in space, the total displacement at that point is equal to the sum of the displacements of the individual waves. It is given by $y=\sum y_i(x,t)$
13	Interference of waves	Two sinusoidal waves traveling in the same direction interfere to produce a resultant sinusoidal wave traveling in that direction if they have the same amplitude and frequency, with resultant wave given by the relation $y'(x,t)=[2A_m\cos(u/2)]\sin(\omega t-kx+u/2)$ where u is the phase difference between two waves. If u = 0, then interference would be fully constructive. If u = π, then waves would be out of phase and the interference would be destructive.
14	Reflection of waves	An incident wave encountering a boundary gets reflected. If an incident wave is represented by $y_i(x,t) = A\sin(\omega t-kx)$ then reflected wave at rigid boundary is $y_r(x,t) = A\sin(\omega t+kx+\pi)=-A\sin(\omega t+kx)$ And for reflections at open boundary, the reflected wave is given by $y_r(x,t) = A\sin(\omega t+kx)$
15	Standing waves	When two identical waves moving in opposite directions meet, the interference produces standing waves. The particle displacement in standing wave is given by $y(x,t) = [2A\sin(kx)]\sin(\omega t)$. The amplitude of standing waves is different at different point i.e., at nodes amplitude is zero and at antinodes amplitude is maximum or equal to sum of amplitudes of constituting waves.

16	Normal modes of stretched string	Frequency of transverse waves in a stretched string of length L and fixed at both the ends is given by f = nv /2L where n = 1, 2, 3........ The above relation gives a set of frequencies called normal modes of oscillation of the system. Mode n=1 is called the fundamental mode with frequency f_1= v/2L. Second harmonic is the oscillation mode with n = 2 and so on. Thus the string has infinite number of possible frequency of vibration which are harmonics of fundamental frequency f_1 such that $f_n = nf_1$.

Solved Examples

JEE Main/Boards

Example 1: The length of a wave propagated on a long stretched string is taken as the positive x axis. The wave equation is given by

$y = y_0 e^{-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2}$ where $y_0 = 4\,mm$, $T = 1.0\,s$ and $\lambda = 4cm$.

(a) Find the velocity of the wave.

(b) Find the function finding the displacement of the particle at x = 0.

(c) Find the function giving the shape of the string at = 0.

(d) Plot the shape of the string at t =0.

(e) Plot the shape of the string at t=5s.

Sol: The wave moves having natural frequency of ν and wavelength λ has velocity $V = \nu\lambda$. As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation may be written as

$y = y_0 e^{-\frac{1}{T}\left(t-\frac{x}{\lambda/t}\right)^2}$

Comparing with the general equation we see that

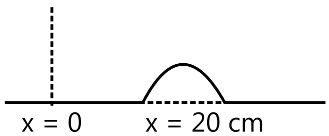
$$\nu = \frac{\lambda}{1.0\,s} = 4\,cm = 4cm/sec$$

(b) Putting x = 0 in the given equation $f(t) = y_0\, e^{-(t/T)^2}$ … (i)

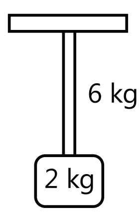
(c) Putting t = 0 in the given equation $g(t) = y_0\, e^{-(x/\lambda)^2}$ ... (ii)

(d)

(e)



Example 2: The dimensions of a uniform rope are as follows: length 12 m, mass 6 kg. The rope hangs vertically from a rigid support with a slab of a mass of 2 kg is attached to the free end of the rope. If a transverse pulse of wavelength 0.06 m is transmitted from the free end of the rope, what is the wavelength of the pulse when it reaches the top of the rope?



Sol: The wave velocity will be $V = \nu\lambda = \sqrt{\frac{F}{\mu}}$ where F is the tension in string at a point and μ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant λ will vary.

We have, $V = \nu\lambda$

Or, $\sqrt{\frac{F}{\mu}} = \nu\lambda$ or $\frac{\sqrt{F}}{\lambda} = \nu\sqrt{\mu}$

Since the frequency of the wave pulse is dependent only on the frequency of the source, it will be consistent