16	Normal modes of stretched string	Frequency of transverse waves in a stretched string of length L and fixed at both the ends is given by f = nv /2L where n = 1, 2, 3........ The above relation gives a set of frequencies called normal modes of oscillation of the system. Mode n=1 is called the fundamental mode with frequency f_1= v/2L. Second harmonic is the oscillation mode with n = 2 and so on. Thus the string has infinite number of possible frequency of vibration which are harmonics of fundamental frequency f_1 such that $f_n = nf_1$.

Solved Examples

JEE Main/Boards

Example 1: The length of a wave propagated on a long stretched string is taken as the positive x axis. The wave equation is given by

$y = y_0 e^{-\left(\frac{t}{T}-\frac{x}{\lambda}\right)^2}$ where $y_0 = 4\,mm$, $T = 1.0\,s$ and $\lambda = 4cm$.

(a) Find the velocity of the wave.

(b) Find the function finding the displacement of the particle at x = 0.

(c) Find the function giving the shape of the string at = 0.

(d) Plot the shape of the string at t =0.

(e) Plot the shape of the string at t=5s.

Sol: The wave moves having natural frequency of ν and wavelength λ has velocity $V = \nu\lambda$. As the frequency is $\nu = \frac{1}{T}$ the velocity of the wave is then $V = \frac{\lambda}{T}$.

(a) The wave equation may be written as

$y = y_0 e^{-\frac{1}{T}\left(t-\frac{x}{\lambda/t}\right)^2}$

Comparing with the general equation we see that

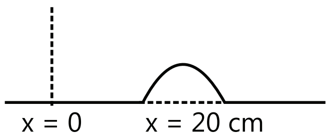
$\nu = \frac{\lambda}{1.0s} = 4\,cm = 4cm/sec$

(b) Putting x = 0 in the given equation $f(t) = y_0\,e^{-(t/T)^2}$ … (i)

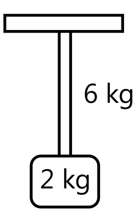
(c) Putting t = 0 in the given equation $g(t) = y_0\,e^{-(x/\lambda)^2}$ ... (ii)

(d)

(e)



Example 2: The dimensions of a uniform rope are as follows: length 12 m, mass 6 kg. The rope hangs vertically from a rigid support with a slab of a mass of 2 kg is attached to the free end of the rope. If a transverse pulse of wavelength 0.06 m is transmitted from the free end of the rope, what is the wavelength of the pulse when it reaches the top of the rope?



Sol: The wave velocity will be $V = \nu\lambda = \sqrt{\frac{F}{\mu}}$ where F is the tension in string at a point and μ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant λ will vary.

We have, $V = \nu\lambda$

Or, $\sqrt{\frac{F}{\mu}} = \nu\lambda$ or $\frac{\sqrt{F}}{\lambda} = \nu\sqrt{\mu}$

Since the frequency of the wave pulse is dependent only on the frequency of the source, it will be consistent across the length of the rope. The mass per unit length will also be consistent for the entire rope as the rope is uniform. Thus,

By (i) $\frac{\sqrt{F}}{\lambda}$ is constant.

Hence, $\frac{\sqrt{(2kg)g}}{0.06}=\frac{\sqrt{(8kg)g}}{\lambda_1}$

where λ_1 is the wavelength at the top of the rope. This gives λ_1 = 0.12m.

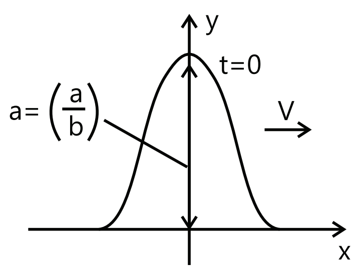
Example 3: A traveling wave pulse is given by $y=\frac{10}{5+(x+2t)^2}$. What is the direction, velocity and amplitude of the pulse?

Sol: The wave equation given above is of form $y=\frac{a}{b+(x\mp\upsilon t)^2}$ where 'a' is the amplitude of the disturbance.

A wave pulse is a disturbance confined to only in a small part of the medium at a given instant [see figure] and its shape does not change during propagation. It is usually expressed by the form $y=\frac{a}{b+(x\mp\upsilon t)^2}$

Comparing the above with the given pulse we find that

$f(x\mp\upsilon t)=(x+2t)^2$



i.e, the pulse is traveling along negative x axis with velocity 2 m/s.

Further, amplitude is the maximum value of wave function which will be when $(x+2t)^2=0$

So, $A=y_{max}=\frac{10}{5}=2$

Example 4: Consider a tube that is closed at one end and has a vibrating diaphragm at the other end. The diaphragm, which may be assumed to be the displacement node, produces a stationary wave pattern at the frequency of 2000 Hz, in which the distance between adjacent nodes is 8 cm. When the frequency is gradually reduced, the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate

(i) The speed of sound in air.

(ii) The distance between adjacent nodes at a frequency of 1600 Hz,

(iii) The distance between diaphragm and the closed end

(iv) The next lower frequencies at which stationary wave patterns will be obtained.

Sol: The standing waves generated inside the tube closed at one end, have the wavelength $n\lambda=2L$ where L is length of the tube. The velocity of the wave in air is given by v = fλ, where n is the frequency of the sound wave.

Since the node-to node distance is $\lambda/2$, $\lambda/2=0.08$ or λ = 0.16m

(i) c = n λ; ∴ c= 2000 × 0.16 = 320ms^{-1}

(ii) 320 = 1600 × $\lambda/2$ or $\lambda=0.2$m

∴ Distance between nodes = 0.2/2 = 0.1 m = 10cm.

(iii) Since there are nodes at the ends, the distance between the closed end and the membrane must be exact integrals of $\lambda/2$.

∴ $0.4=n\lambda/2=v'\times0.2/2\Rightarrow\frac{n}{n'}=\frac{5}{4}$

When n = 5, n' = 4 l = n × 0.16/2 = 0.4μ = 40cm

(iv) For the next lower frequency n = 3, 2, 1

∴ 0.4 = 3 λ / 2 or λ = 0.8/3

Since c=n λ, $n=\frac{320}{0.8/3}=1200\text{Hz}$

Again 0.4 = 1. λ / 2 or λ = 0.4M

∴ n = 320/0.4 = 800 Hz

Again 0.4 = 1. λ / 2 or λ = 0.8M

∴ n = 320/0.8 = 400 Hz

Example 5: Consider a tuning fork of frequency 256 Hz and an open organ pipe of slightly lower frequency. Both are at 17°C temperature. When sounded together, they produce 4 beats per second. When the temperature of air in the pipe is altered, the number of beats per second first diminishes to zero and then increases again to 4. Determine the quantum of temperature change in the pipe? Also, in what direction has the temperature of the air in the pipe been altered?

Sol: In an open organ pipe the frequency of the wave is $n=\frac{V_t}{\lambda}$ where V_t is the velocity of wave at temperature t and λ=2L is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $V \propto \sqrt{T}$.

$n=\frac{V_{17}}{2l}$ where l = length of the pipe;

$\therefore \quad 256-\frac{V_{17}}{2l}=4 \quad$ or $\quad \frac{V_{17}}{2l}=252$

Since beats decreases first and then increases to 4, the frequency of the pipe increases. This can happen only if the temperature increases.

Let t be the final temperature, in Celsius,

Now $\frac{V_t}{2l}$ =256 + 4 or $\frac{V_t}{2l}$ = 260

Dividing $\frac{V_t}{V_{17}}=\frac{260}{252}$ or $\sqrt{\frac{273+t}{273+17}}=\frac{260}{252}$

$(\therefore V<\sqrt{T})$ or $t=308.7-273=35.7^0C.$

$\therefore$ Rise in temperature =35.7 – 17 = 18.7 ^{0}C.

Example 6: Determine the fundamental frequency and the first four overtones of a 15 cm pipe

(a) If the pipe is closed at one ends,

(b) If the pipe is open at both ends

(c) How many overtones are within the human auditory range in each of the above cases? Velocity of sound in air = 330 ms^{-1}.

Sol: For the organ pipe closed at one end, the fundamental frequency of the wave of wavelength λ is given by, $n_0=\frac{v}{4L}$. The frequency of i^{th} over tone is given by $n_i=(i+1)\times n_0$ where i=1,2,3.... etc.

(a) $n_0=\frac{V}{4l}$

Wher n_0 = frequency of the fundamental node

$\Rightarrow n_0=\frac{330}{4\times 0.15}=550Hz$

The first four overtones are $3n_0$, $5n_0$, $7n_0$ and $9n_0$

$\therefore$ So, the required frequencies are 550, 1650, 2750, 3850, and 4950 Hz.

(b) $n_0=\frac{V}{2l}=\frac{330}{2\times 0.15}=1100Hz$

The first overtones are $2n_0$, $3n_0$, $4n_0$ and $5n_0$

So, the required frequency are 1100, 2200, 3300, 4400, and 5500 Hz

The frequency of the n^{th} overtone is (n +1) n_0.

$\therefore$ (n +1) n_0 =20000 or (n+1)100 = 20000

Or n = 17.18

The acceptable value is 17.

Example 7: The displacement of a particle of a string carrying a traveling wave is given by

$y=(3.0\,cm)\sin 6.28(0.50x-50t)$,

where x is in centimeter and t in second. Find (a) the amplitude, (b) the wavelength, (c) the frequency and (d) the speed of the wave.

Sol: In a open organ pipe the frequency of the wave is $n=\frac{V_t}{\lambda}$ where V_t is the velocity of wave at temperature t and λ=2L is the wavelength of the vibrating wave. If temperature of air inside the organ pipe changes, the velocity of wave also changes, since $V \propto \sqrt{T}$.

On comparing with the standard wave equation

$y=A\sin(kx-\omega t)=A\sin 2\pi(\frac{x}{\lambda}-\frac{t}{T})$

we see that, Amplitude = A = 3.0 cm,

Wavelength = $\lambda=\frac{1}{0.50}$ cm = 2.0 cm, and the frequency

$=\nu=\frac{1}{T}=50Hz$

The speed of the wave is $V=\nu\lambda$

= (50 s^{-1}) (2.0cm) = 100 cm s^{-1}

Example 8: The equation for a wave traveling in the direction x on a string is

y = (3.0 cm) sin [(3.14 cm^{-1}) x – (314 s^{-1}) t].

(a) Find the maximum velocity of a particle of the string.

(b) Find the acceleration of a particle at x = 6.0 cm at time t = 0.11 s

Sol: The maximum velocity is $v=\frac{\partial y}{\partial t}$ While the acceleration $a=\frac{\partial v}{\partial t}$

(a) The velocity of the particle at x at time t is $v=\frac{\partial y}{\partial t}$

$$=(3.0\text{ cm})(-314s^{-1})\cos[(3.14cm^{-1})x-(314s^{-1})t]$$

$$=(-9.4ms^{-1})\cos[((3.14cm^{-1})x-(314s^{-1})t]$$

The maximum velocity of a particle will be v = 9.4 ms^{-1}.

(b) The acceleration of the particle at x at time t is

$$a=\frac{\partial v}{\partial t}=-(9.4ms^{-1})(314s^{-1})\sin[(3.14cm^{-1})x-(314s^{-1})t]$$

$$=-(2952ms^{-2})\sin[(3.14ms^{-1})x-(3.14s^{-1})].$$

The acceleration of the particle at x = 6.0 cm at time t = 0.11 s = 1(2952ms-²) sin [6π - 11 π] = 0.

Example 9: One end of a long string is attached to an oscillator moving in transverse direction at a frequency of 20 Hz. The string has a cross-section area of 0.80 mm^2 and a density of 12.5 g cm^{-3} . It is subjected to a tension of 64 N along the X axis. At t= 0, the source is at a maximum displacement y = 1.0 cm. (a) Find the speed of the wave traveling on the string. (b) Write the equation for the wave. (c) What is the displacement of the particle of the string at x = 50 cm at time t = 0.05 s? (d) What is the velocity of this particle at this instant?

Sol: As the wave is under tension F, the maximum wave velocity of wave is $v=\sqrt{\frac{F}{\mu}}$ where μ is the mass per unit length of the string. The wave equation is y=A cos (ωt) where ωis angular frequency and A is amplitude of wave.

(a) The mass of 1 m long part of the string is m= (0.80mm²) × (1 m) × (12.5gcm⁻³)

$$=(0.80\times10^{-6}\,m^3)\times(12.5\times10^3kg\,m^{-3})=0.01\text{ kg}$$

The linear mass density is μ 0.01 kg m^{-1}. The wave speed is

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{64N}{0.01kgm^{-1}}}=80ms^{-1}$$

(b) The amplitude of the source is A = 1.0cm and the frequency is =20 Hz. The angular frequency is $\omega=2\pi\nu=40\pi s^{-1}$. Also at t =0, the displacement is equal to its amplitude, i.e., at t=0, x = A. The equation of motion of the source is, therefore, $y=(1.0cm)\cos[(40\pi s^{-1})t]$ ... (i)

The equation of the wave traveling on the string along the position X – axis is obtained by replacing t with $t-x/v$ in equation (i). It is, therefore,

$$y=(1.0cm)\cos\left[(40\pi s^{-1})\left(t-\frac{x}{v}\right)\right]$$

$$=(1.0cm)\cos\left[(40\pi s^{-1})t-\left(\frac{x}{2}m^{-1}\right)\right] \quad \text{... (ii)}$$

Where the value of v has been put from part (a).

(c) The displacement of the particle at x = 50 cm at time t = 0.05 s is by equation (ii),

$$y=(1.0cm)\cos[(40\pi s^{-1})(0.05s)-\left(\frac{\pi}{2}m^{-1}\right)(0.5m)]$$

$$=(1.0cm)\cos\left[2\pi-\frac{\pi}{4}\right]=\frac{1.0}{\sqrt{2}}=0.71cm$$

(d) The velocity of the particle at position x at time t is, by equation (ii),

$$\upsilon=\frac{\partial y}{\partial t}=-(1.0cm)(40\pi s^{-1})\sin\left[(40\pi s^{-1})t-\left(\frac{\pi}{2}m^{-1}\right)x\right]$$

Putting the values of x and t,

$$\upsilon=-(40\pi cms^{-1})\sin\left(2\pi-\frac{\pi}{2}\right)=\frac{40\pi}{\sqrt{2}}cms^{-1}\approx 89cms^{-1}$$

Example 10: The speed of a transverse wave traveling through a wire is 80 m s^{-1}. The length of the wire is 50 cm, the mass is 5.0 g, the area of cross-section of the wire is 1.0 mm^2, and its Young modulus is 16×10^{11} Nm^{-2}. Find the extension of the wire over its natural length.

Sol: The maximum velocity of the wave is $v=\sqrt{F/\mu}$ where F is the tension in the string and μ is mass per unit length of string. And the Young's modulus of the string is $Y=\frac{F/A}{\Delta L/L}$.

The linear mass density is

$$\mu=\frac{5\times10^{-3}kg}{50\times10^{-2}m}=1.0\times10^{-2}kgm^{-1}$$

The wave speed is $\upsilon=\sqrt{F/\mu}$.

Thus, the tension is

$$F=\mu v^2=(1.0\times10^{-2}\,kgm^{-1})\times6400\,m^2s^{-2}=64\text{ N}$$

The Young modulus is given by $Y=\frac{F/A}{\Delta L/L}$

The extension is, therefore,

$$\Delta L=\frac{FL}{AY}=\frac{(64N)(0.50m)}{(1.0\times10^{-6}m^2)\times(16\times10^{11}Nm^{-2})}=0.02mm$$

JEE Advanced/Boards

Example 1: The interference of two traveling waves of equal amplitude and frequency moving in opposite directions along a string produces a standing wave having the equation

$y = A\cos kx \sin\omega t$ in which A = 1.0 mm, $k = 1.57 cm^{-1}$ and $\omega = 78.5 s^{-1}$.

(a) Find the velocity of the component traveling waves.

(b) Find s the node closest to the origin in the region x > 0.

(c) Find the antinode closest to the origin in the region x>0

(d) Find the amplitude of the particle at x= 2.33 cm.

Sol: Here the two waves of same amplitude and frequency interfere with each other to form the standing waves, the velocity of the resultant wave will be $V = \frac{\omega}{K}$ where ω is the angular frequency of the wave and K is the wave number. At the node the waves are 90° opposite in phase, so that the amplitude of resulting wave is zero at the node.

(a) The standing wave is formed by the superposition of the waves

$$y_1 = \frac{A}{2}\sin(\omega t - kt) \text{ and } y_2 = \frac{A}{2}\sin(\omega t + kt)$$

The wave velocity (magnitude) of either of the waves is

$$v = \frac{\omega}{k} = \frac{78.5 s^{-1}}{1.57 cm^{-1}} = 50 cm/s.$$

(b) For a node, cos kx = 0

The smallest position value of x satisfying this relation is given by $kx = \frac{\pi}{2}$

Or $x = \frac{\pi}{2k} = \frac{3.14}{2\times 1.57 cm^{-1}} = 1cm$

(c) For an antinode, |coskx| = 1

The smallest positive x satisfying this relation is given by $kx = \pi$ or $x = \frac{\pi}{k} = 2cm$

(d) The amplitude of vibration of the particle at x is given by |Acoskx|. For the given point,

$$kx = (1.57 cm^{-1})(2.33cm) = \frac{7}{6}\pi = \pi + \frac{\pi}{6}$$

Thus, the amplitude will be (1.0mm)|

$$\cos(\pi + \pi/6)| = \frac{\sqrt{3}}{2} mm = 0.86 mm$$

Example 2: A tuning fork of frequency 500 Hz is used to generate a transverse harmonic wave of amplitude 0.01 m at one end (x = 0) of a long, horizontal string. At a given instant of time the displacement of the particle at x = 0.1 m is –0.005m and that of the particle at x = 0.2 m is +0.005m. Calculate the wavelength and wave velocity. Assuming that the wave is traveling along the positive direction x and that the end x = 0 is at equilibrium position at t = 0, obtain the equation of the wave.

Sol: The fork is the source to generate the transverse wave on string whose frequency is also 500 Hz. The equation of this wave is given by $y = A\sin(kx - \omega t)$ where k is the wave number and x is the displacement of particle. The wave velocity is given by $V = \nu\lambda$ where υ is the frequency of source

Since the wave is traveling along positive direction x and the displacement of the end x = 0 is at time t = 0, the general equation of this wave is

$$y(x,t) = A\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x)\right\} \quad \text{... (i)}$$

Where A = 0.01 m. When x = 0.1m, Y = -0.005m

$$\therefore \quad -0.005 = 0.01\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x_1)\right\}$$

Where $x_1 = 0.1m$ or $\sin\left\{\frac{2\pi}{\lambda}(\upsilon t - x_1)\right\} = -\frac{1}{2}$

$$\therefore \text{Phase } \sin\phi_1 = \frac{2\pi}{\lambda}(\upsilon t - x_1) = \frac{7\pi}{6} \quad \text{... (ii)}$$

When x = 0.2m y= + 0.005. Therefore, we have +0.005

$$= 0.01 \sin\left\{\frac{2\pi}{\lambda}(Vt - x_2)\right\}$$

Where $x_2 = 0.2$ m

$$\therefore \phi_2 = \frac{2\pi}{\lambda}(Vt - x_1) = \frac{2\pi}{6} \quad \text{... (iii)}$$

From eqs. (ii) and (iii)

$$\therefore \quad \Delta\phi = \phi_1 - \phi_2 = \pi$$

Now, $\Delta\phi = -\frac{2\pi}{\lambda}\Delta x$ thus,

$$\pi = -\frac{2\pi}{\lambda}(x_1 - x_2) = \frac{2\pi}{\lambda}(0.1 - 0.2) \quad \text{or} \quad \lambda = 0.2m$$

Now, frequency n of the wave = frequency of the tuning fork = 500 Hz. Hence, wave velocity

$$\upsilon = n\lambda = 500 \times 0.2 = 100 ms^{-1}$$

Substituting for A, λ, and v in equation. (i) We get $y(x,t) = 0.01\sin\{10\pi(100t - x)\}$

This is the equation of the wave where y and x are in meters and t in seconds.

Example 3: Two tuning forks A and B sounded together produce 6 beats per second. With the introduction of an air resonance tube closed at one end, the two forks give resonance when the two air columns are 24 cm and 25 cm, respectively. Calculate the frequencies of forks.

Sol: Beats are produced when the two waves of similar amplitude but different frequency are interacting with each other. For vibrating air column $\frac{\ell_1}{\ell_2} = \frac{f_2}{f_1}$ where ℓ is length of vibrating air column and f is frequency of tuning fork.

Let the frequency of the first fork be f_1 and that of second be f_2.

We then have, $f_1 = \frac{v}{4\times 24}$ and $f_2 = \frac{v}{4\times 25}$

We also see that $f_1 > f_2$

$\therefore \quad f_1 - f_2 = 6$ ... (i)

And $\frac{f_1}{f_2} = \frac{25}{24}$ ... (ii)

Solving (i) and (ii), we get $f_1 = 150$ Hz and $f_2 = 144$ Hz

Example 4: The oscillation of a string of length 60 cm fixed at both ends is represented by the equation: $y = 4\sin\left(\frac{\pi x}{15}\right)\cos(96\pi t)$ where x and y are in cm and *t* is in seconds.

(a) What is the maximum displacement of a point at x = 5 cm?

(b) Where are the nodes located along the string?

(c) What is the velocity of a particle at x = 7.5 cm & at t = 0.25 sec.

(d) Write down the components waves which give the above wave on superposition.

Sol: The wave equation oscillating string is written in form of $y = 2a\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi vt}{\lambda}\right)$ where x is displacement and v is velocity of wave. The maximum velocity of wave is $v = \frac{\partial y}{\partial t}$ and the distance of nodes from any fixed end of string found using relation $kx = \frac{\pi}{2}$

Comparing given equation with equation of standing wave, $y = 2a\cos\left(\frac{2\pi x}{\lambda}\right)\sin\left(\frac{2\pi vt}{\lambda}\right)$

$\frac{2\pi}{\lambda} = \frac{\pi}{15},$; $\lambda = 30$ cm; $a = 2$ cm

$\frac{2\pi v}{\lambda} = 96\pi \quad \Rightarrow v = 1440$ cm / sec

(a) $x = 5$ cm, $y_{max} = 4\sin\left(\frac{\pi\times 5}{15}\right) = \frac{4\sqrt{3}}{2} = 2\sqrt{3}$ cm

(b) As $\lambda = 30$ cm; nodes are at 0, 15, 30, 45, 60 cm

(c) $\frac{\partial y}{\partial t} = -4\sin\left(\frac{\pi x}{15}\right)\sin(96\pi r)\times 96\pi$

x = constant For x = 7.5 cm, t = 0.25 sec.

$\frac{\partial y}{\partial t} = -4\sin\left(\frac{\pi\times 7.5}{15}\right)\sin(96\pi\times 0.25)\times 96\pi = 0$

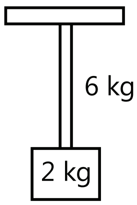
(d) Component waves $y = -4\sin\left(\frac{\pi x}{15}\right)\sin(96\pi t)$

$= 2\sin\left(\frac{\pi x}{15} + 96\pi t\right) + 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$

$\Rightarrow$ Component waves are $y_1 = 2\sin\left(\frac{\pi x}{15} + 96\pi t\right)$;

$y_2 = 2\sin\left(\frac{\pi x}{15} - 96\pi t\right)$

Example 5: A uniform rope hangs vertically from a rigid support with a slab of mass 2 kg attached to the free end of the rope. The rope has a length of 12 m and a mass of 6 kg. A transverse pulse of wavelength 0.06 m is produced at the free end of the rope. Determine the wavelength of the pulse when it reaches the top of the rope?



Sol: The wave velocity will be $V = \nu\lambda = \sqrt{\frac{F}{\mu}}$ where F is the tension in rope at a point and μ is mass per unit length of the string. As F is varying along the length of the rope so the velocity will vary along the length of the rope. As source frequency is constant λ will vary.

As the rope is stretched using a slab, its tension will be different at different points along the length of the rope. The tension at the free end will be (2 kg) g while at the upper end it will be (8kg) g.

We have, $v = \nu\lambda \Rightarrow \sqrt{F/\mu} = \nu\lambda$ or $\sqrt{F}/\lambda = \nu\sqrt{\mu}$ ... (i)

Since the frequency of the wave pulse is dependent only on the frequency of the source, it will be consistent across the length of the rope. The mass per unit length will also be consistent for the entire rope as the rope is uniform. Thus, by

(i) $\frac{\sqrt{F}}{\lambda}$ is constant. Hence, $\frac{\sqrt{(2kg)g}}{0.06m} = \frac{\sqrt{(8kg)g}}{\lambda_1}$,

Where λ_1 is the wavelength at the top of the rope, this gives $\lambda_1 = 0.12m$.

Example 6: Two waves passing through a region are represented by

$y = (1.0\,cm)\sin[(3.14\,cm^{-1})x - (157s^{-1})t]$

and $y = (1.5\,cm)\sin[(1.57\,cm^{-1})x - (314\,s^{-1})t]$

Find the displacement of the particle at x = 4.5 cm at time t = 5.0 ms.

Sol: As the waves are superimposed on each other, the resultant displacement is $Y = y_1 + y_2$.

According to the principle of superposition, each wave produces its own disturbance and the resultant disturbance is equal to the vector sum of the individual disturbances. The displacements of the particle at x = 4.5cm at time t = 5.0 due to the two waves are,

$$y_1 = (1.0\,cm)\sin[(3.14\,cm^{-1})(4.5\,cm) - (157s^{-1})](5.0\times10^{-3}s)$$

$$= (1.0\,cm)\sin\left(4.5\pi - \frac{\pi}{4}\right) = (1.0\,cm)\sin[4\pi + \pi/4]$$

$$= \frac{1.0\,cm}{\sqrt{2}} \text{ and}$$

$$y_2 = (1.5\,cm)\sin[(1.57\,cm^{-1})(4.5\,cm) - (314s^{-1})] \times (5.0\times10^{-3}s)$$

$$= (1.5\,cm)\sin\left(2.25\pi - \frac{\pi}{2}\right) = (1.5\,cm)\sin[2\pi + \pi/4]$$

$$= -(1.5\,cm)\sin\frac{\pi}{4} = -\frac{1.5\,cm}{\sqrt{2}}$$

The net displacement is

$$y = y_1 + y_2 = \frac{-0.5cm}{\sqrt{2}} = -0.35\,cm.$$

Example 7: The vibrations of a string fixed at both ends are described by the equation

$y = (5.00mm)\sin[(1.57cm^{-1})x]\sin[(314\,s^{-1})t]$

(a) What is the maximum displacement of the particle at x = 5.66cm?

(b) What are the wavelengths and the wave speeds of the two transverse waves that combine to give the above vibration?

(c) What is the velocity of the particle at x = 5.66 cm at time t = 2.00s?

(d) If the length of the string is 10.0 cm, locate the nodes and the antinodes. How many loops are formed in the vibration?

Sol: The transverse velocity of particle of string is $u = \frac{\partial y}{\partial t}$. The wave velocity is $V = \nu\lambda$. Comparing wave equation with y=A sin kx sin ωt, we get the amplitude A and angular frequency of the wave.

(a) The amplitude of the vibration of the particle at position x is

$$A = |(5.00\text{ mm})\sin[(1.57cm^{-1})x]$$

For $x = 5.66\,cm$,

$$A = |(5.00\text{ mm})\sin\left[\frac{\pi}{2}\times5.66\right]$$

$$= \left|(5.00\text{ mm})\sin\left(2.5\pi + \frac{\pi}{3}\right)\right|$$

$$= \left|(5.00\text{ mm})\cos\frac{\pi}{3}\right| = 2.50\text{ mm}$$

(b) From the given equation, the wave number k = 1.57 cm^{-1} and the angular frequency ω = 314 s-1. Thus, the wavelength is

$$\lambda = \frac{2\pi}{k} = \frac{2\times3.14}{1.57\,cm^{-1}} = 4.00\text{ cm}$$

and Frequency is $\nu = \frac{\omega}{2\pi} = \frac{314s^{-1}}{2\times3.14} = 50\,s^{-1}$

The wave speed is

$\upsilon = \nu\lambda = (50s^{-1})(4.00\text{ cm}) = 2.00\,ms^{-1}$.

(c) The velocity of the particle at position x at time t is given by

$$\upsilon = \frac{\partial y}{\partial t} = (5.00\text{ mm})\sin[(1.57\,cm^{-1})x]\,[314s^{-1} \times\cos(314s^{-1})t]$$

$= (157 \text{ m s}^{-1}) \text{ in } (1.57 \text{ cm}^{-1})\, x \cos(314 \text{ s}^{-1})t$

(d) The nodes occur where the amplitude is zero, i.e., sin (1.57cm-1) x = 0 or $\left(\frac{\pi}{2}\text{cm}^{-1}\right)x = n\pi$

Where n is an integer. Thus, x = 2n cm.

The nodes, therefore, occur at x = 0, 2 cm, 4 cm, 6 cm, 8cm and 10 cm. Antinodes occur in between them, i.e., at x = 1 cm, 3 cm, 5cm, 7 cm and 9 cm. The string vibrates in 5 loops.

Example 8: A guitar of 90 cm length has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz?

Sol: As wires of guitar resemble the sonometer wire, thus the fundamental frequency of the guitar wire fixed at both ends is $\nu = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$. And for two vibrating strings, the ratio of their vibrating lengths is $\frac{\ell_1}{\ell_2} = \frac{\nu_2}{\nu_2}$.

The fundamental frequency of a string fixed at both ends is given by $\nu = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

As F and μ are fixed,

$$\frac{\nu_1}{\nu_2} = \frac{L_2}{L_1} \text{ or } \quad L_2 = \frac{\nu_1}{\nu_2}L_1$$

$$= \frac{124\text{Hz}}{186\text{ Hz}}(90\text{ cm}) = 60\text{ cm}$$

Thus, the string should be pressed at 60 cm from an end.

Example 9: The total length of a sonometer wire is 1 m between the fixed ends. Where the two bridges should be placed in the sonometer so that the three segments of the wire have their fundamental frequencies in the ratio 1:2:3?

Sol: For sonometer the ratio of length of wires is $L \propto \frac{1}{\nu}$ where v is the frequency of the wave and L is length of vibrating string.

Suppose the lengths of the three segments are L_1, L_2, and L_3, respectively. The fundamental frequencies are

$$\nu_1 = \frac{1}{2L_1}\sqrt{F/\mu}$$

$$\nu_2 = \frac{1}{2L_2}\sqrt{F/\mu} \; ; \; \nu_3 = \frac{1}{2L_3}\sqrt{F/\mu}$$

So that $\nu_1 L_1 = \nu_2 L_2 = \nu_3 L_3$. ... (i)

As $\nu_1 : \nu_2 : \nu_3 = 1:2:3$ we have

$\nu_2 = 2\nu_1$ and $\nu_3 = 3\nu_1$ so that by (i)

$$L_2 = \frac{\nu_1}{\nu_2}L_1 = \frac{L_1}{2} \text{ and } \nu_3 = \frac{\nu_1}{\nu_3}L_1 = \frac{L_1}{3} \text{ and}$$

$L_1 + L_2 + L_3 = 1\text{m}$

We get $L_1\left(1 + \frac{1}{2} + \frac{1}{3}\right) = 1\text{m}$

$$L_1 = \frac{6}{11}\text{m} \qquad \text{Thus,} \quad L_2 = \frac{L_1}{2} = \frac{6}{11}\text{m}$$

$$L_3 = \frac{\nu_1}{3} = \frac{2}{11}\text{m}$$

One bridge should be placed at $\frac{6}{11}$m from one end and the other should be placed at $\frac{2}{11}$m from other end.

Example 10: A wire having a linear mass density 5.0×10^{-3} kg m^{-1} resonates at a frequency of 420 Hz when it is stretched between two rigid supports with a tension of 450 N.. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.

Sol: For vibrating string the n^{th} harmonic of fundamental frequency is $f = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$. Here L is the length of vibrating string and F is the tension in the string. The two given frequencies correspond to two consecutive values n and (n+1).

Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its $(n+1)^{th}$ harmonic.

$$420\text{s}^{-1} = \frac{n}{2L}\sqrt{F/\mu} \quad \text{... (i)}$$

$$\text{and} \quad 490\text{s}^{-1} = \frac{(n+1)}{2L}\sqrt{F/\mu} \quad \text{... (ii)}$$

This gives $\frac{490}{420} = \frac{(n+1)}{n}$ or n = 6

Putting the value in (i),

$$420\text{s}^{-1} = \frac{6}{2L}\sqrt{\frac{450\text{N}}{5.0 \times 10^{-3}\text{kgm}^{-1}}} = \frac{900}{L}\text{ms}^{-1}$$

$$\text{Or} \quad L = \frac{900}{420}\text{m} = 2.1\text{m}$$

JEE Main/Boards

Exercise 1

Q.1 Audible frequencies have a range 40 hertz to 30,000 hertz. Explain this range in terms of

(i) Period T

(ii) Wavelength λ in air, and

(iii) Angular frequency

Give velocity of sound in air is 350 ms^{-1}

Q.2 From a radio station, the frequency of waves is 15 Mega cycle/sec. Calculate their wavelength.

Q.3 The velocity of sound in air at N.T.P is 331 ms^{-1}. Find its velocity when the temperature rises to 91^0C and its pressure is doubled.

Q.4 A displacement wave is represented by = 0.25 × 10^{-3} sin (500 t – 0.025 x). Deduce (i) amplitude (ii) period (iii) angular frequency (iv) wavelength (v) amplitude of particle velocity (vi) amplitude of particle acceleration. , the t and x are in cm, sec and meter respectively.

Q.5 The length of a sonometer wire between two fixed ends is 110 cm. Where should be two bridges be placed so as to divide the wire into three segments, whose fundamental frequencies are in the ratio 1: 2: 3?

Q.6 Calculate the velocity of sound in a gas, in which two wave lengths 2.04 m and 2.08 m produce 20 beats in 6 seconds.

Q.7 A tuning fork of unknown frequency gives 6 beats per second with a tuning fork of frequency 256. It gives same number of beats/ sec when loaded with wax. Find the unknown frequency.

Q.8 Is it possible to have longitudinal waves on a string? A transverse wave in a steel rod?

Q.9 What type of mechanical waves do you expect to exist in (a) vacuum (b) air (c) inside the water (d) rock (e) on the surface of water?

Q.10 What will be the speed of sound in a perfect rigid rod?

Q.11 What is the distance between compression and its nearest rarefaction in a longitudinal wave?

Q.12 What is the distance between a node and an adjoining antinode in a stationary wave?

Q.13 Explain why waves on strings are always transverse.

Q.14 What is a wave function? Give general form of wave function. What is a periodic function?

Q.15 Distinguish between harmonics and overtones.

Q.16 A stone is dropped into a well in which water is 78.4 m deep. After how long will the sound of splash be heard at the top? Take velocity of sound in air = 332 ms^{-1}

Q. 17 From a cloud at an angle of 30^0 to the horizontal, we hear the thunder clap 8s after seeing the lightening flash. What is the height of the cloud above the ground if the velocity of sound in air is 330 m/s?

Q.18 A steel wire 0.72m long has a mass of 5.0×10^{-3} kg. If the wire is under a tension of 60 N, what is the speed of transverse wave on the wire?

Q.19 For a metal, bulk modulus of elasticity is 7.5×10^{10} Nm^{-2}, and density is 2.5×10^3 m^{-3}. Deduce the velocity of longitudinal waves.

Q.20 A steel wire 70 cm long has mass of 7g. If the wire is under a tension of 100 N, what is the speed of transverse waves in the wire?

Q.21 Two waves of angular frequencies 50 and 5000 rad s^{-1} have the same displacement amplitude, 3×10^{-5} cm. Deduce the acceleration amplitude for them.

Q.22 The equation of a wave traveling in x- direction on a string is y = (3.0 cm) sin [($3.14 cm^{-1}$) x – ($314\ s^{-1}$)t]

(a) Find the max. Velocity of a particle of the string.

(b) Find the acceleration of a particle at x = 6.0 cm and at time t = 00.11 s.

Q.23 A fork of frequency 250 Hz is held over and maximum sound is obtained when the column of air is 31 cm or 97 cm. Determine (i) velocity of sound (ii) the end correction (iii) the radius of the tube.

Q.24 In an experiment, it was found that a tuning fork and a sonometer gave 5 beats/sec, both when length of wire was 1 m and 1.05m. Calculate the frequency of the fork.

Exercise 2

Single Correct Choice Type

Q.1 A wave is propagating along x–axis. The displacement of particle of the medium in z – direction at t = 0 is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At t = 1s, the same wave disturbance is given by: $z = \exp[-2(2-x)^2]$. Then, the wave propagation velocity is

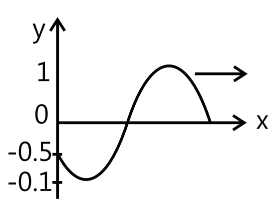
(A) 4 m/s in + x direction

(B) 4 m/s in - x direction

(C) 2 m/s in + x direction

(D) 2 m/s in - x direction

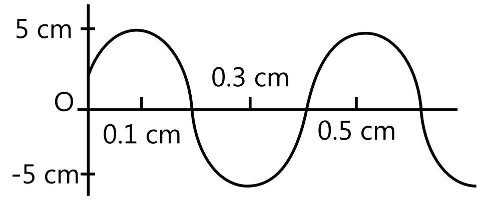
Q.2 The equation of a wave traveling along the positive x – axis, as shown in figure at t = 0 is given by



(A) $\sin\left(kx-\omega t+\frac{\pi}{6}\right)$ (B) $\sin\left(kx-\omega t-\frac{\pi}{6}\right)$

(C) $\sin\left(\omega t-kx+\frac{\pi}{6}\right)$ (D) $\sin\left(\omega t-kx-\frac{\pi}{6}\right)$

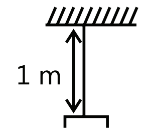
Q.3 In the figure shown the shape of part of a long string in which transverse wave are produced by attaching one end of the string to tuning fork of frequency 250 Hz. What is the velocity of the waves?



(A) 1.0 ms^{-1} (B) 1.5 ms^{-1}

(C) 2.0 ms^{-1} (D) 2.5 ms^{-1}

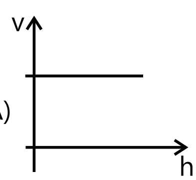
Q.4 A block of mass 1 kg is hanging vertically from a string of length 1 m and mass/ length = 0.001 Kg/m. A small pulse is generated at its lower end. The pulse reaches the top end in approximately

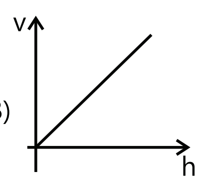


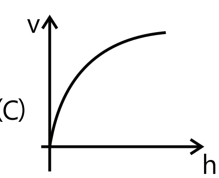
(A) 0.2 sec (B) 0.1 sec

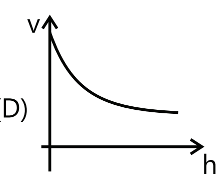
(C) 0.02 sec (D) 0.01 sec

Q. 5 A uniform rope having some mass hangs vertically from a rigid support. A transvers wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A) 

(B) 

(C) 

(D) 

Q. 6 A wire of 10^{-2} kgm^{-1} passes over a frictionless light pulley fixed on the top of a frictionless inclined plane which makes an angle of 30^0 with the horizontal. Masses m and M are tied at two ends of wire such that m rests on the plane and M hangs freely vertically downwards. The entire system is in equilibrium and a transverse wave propagates along the wire with a velocity of 100 ms^{-1}. Then,

(A) M = 5kg (B) $\frac{m}{M}=\frac{1}{4}$

(C) m=20kg (D) $\frac{m}{M}=4$

Q.7 Consider a function $y = 10 \sin^2 (100\pi t + 5\pi z)$ where y, z are in cm and t is in second.

(A) The function represents a traveling, periodic wave propagating in (-z) direction with speed 20m/s.

(B) The function does not represent a traveling wave.

(C) The amplitude of the wave is 5 cm.

(D) The amplitude of the wave is 10 cm.

Q. 8 The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment t = T/ 6 (T is the time period). Assume that the source was at mean position at t = 0. The wavelength of the running wave is

(A) 0.96m (B) 0.48m (C) 0.24m (D) 0.12m

Q. 9 The period of oscillations of a point is 0.04 sec. and the velocity of propagation of oscillation is 300m/sec. The difference of phases between the oscillations of two points at distance 10 and 16m respectively from the source of oscillations is

(A) 2π (B) $\pi/2$ (C) $\pi/4$ (D) π

Q.10 A motion is described by $y = \frac{3}{a^2 + (x+3t)^2}$ where y, x are in meter and t is in second.

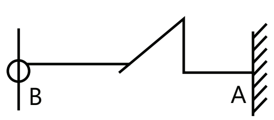
(A) This represents equation of progressive wave propagation along – x direction with 3 ms^{-1}.

(B) This represents equation of progressive wave propagation along + x direction with 3 ms^{-1}.

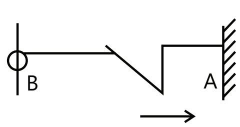
(C) This does not represent a progressive wave equation.

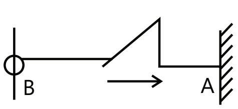
(D) Data is insufficient to arrive at any conclusion.

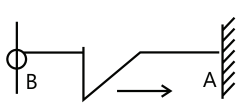
Q.11 A pulse shown here is reflected from the rigid wall A and then from free end B. The shape of the string after these 2 reflection will be

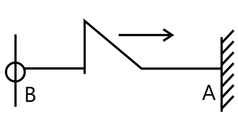


Q.12 A composition string is made up by joining two strings of different masses per unit length →μ and 4μ. The composite string is under the same tension.

(A)


(B)


(C)


(D)


A transverse wave pulse: Y = (6 mm) sin (5t + 40x), where 't' is in seconds and 'x' in meters, is sent along the lighter string towards the joint. The joint is at x = 0. The equation of the wave pulse reflected from the joint is

(A) (2 mm) sin (5t – 40x)

(B) (4 mm) sin (40t – 5x)

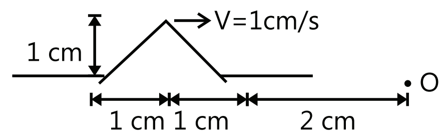
(C) - (2 mm) sin (5t – 40x)

(D) (2 mm) sin (5t – 10x)

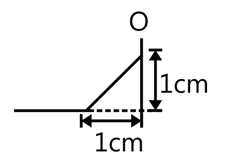
Q. 13 In the previous question, the percentage of power transmitted to the heavier string through the joint is approximately

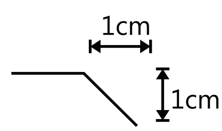
(A) 33% (B) 89% (C) 67% (D) 75%

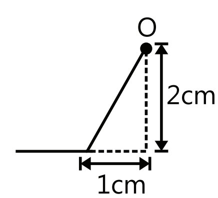
Q.14 A wave pulse on a string has the dimension shown in figure. The waves speed is V= 1 cm/s. If point O is a free end. The shape of wave at time t = 3 s is:



(A)


(B)


(C)


(D)


Q.15 A string 1 m long is drawn by a 300 Hz vibrator attached to its end. The string vibrates in 3 segments. The speed of transverse waves in the string is equal to

(A) 100 m/s (B) 200 m/s

(C) 300 m/s (D) 400 m/s

Q.16 The resultant amplitude due to superposition of two waves $y_1 = 5\sin(\omega t - kx)$ and $y_2 = -5\cos(\omega t - kx - 150^0)$

(A) 5 (B) $5\sqrt{3}$

(C) $5\sqrt{2-\sqrt{3}}$ (D) $5\sqrt{2+\sqrt{3}}$

Q.17 A wave represented by the equation $y = A\cos(kx - \omega t)$ is superimposed with another wave to from a stationary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) $-A\sin(kx+\omega t)$ (B) $-A\cos(kx+\omega t)$

(C) $A\sin(kx+\omega t)$ (D) $A\cos(kx+\omega t)$

Q.18 A taut string at both ends vibrates in its n^{th} overtone. The distance between adjacent Node and Antinode is found to be 'd'. If the length of the string is L, then

(A) $L=2d(n+1)$ (B) $L=d(n+1)$

(C) $L=2dn$ (D) $L=2d(n-1)$

Q.19 A metallic wire of length L is fixed between two rigid supports. If the wire is cooled through a temperature difference ΔT (Y = young's modulus, ρ = density, α = coefficient of linear expansion) then the frequency of transverse vibration is proportional to:

(A) $\frac{\alpha}{\sqrt{\rho Y}}$ (B) $\frac{\sqrt{Y\alpha}}{\rho}$ (C) $\frac{\rho}{\sqrt{Y\alpha}}$ (D) $\sqrt{\frac{\rho\alpha}{\sqrt{Y}}}$

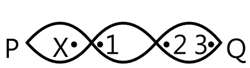
Q.20 A standing wave $Y = A\sin\left(\frac{20}{3}\pi x\right)\cos(1000\pi t)$ is maintained in a taut string where y and x are expressed in meters. The distance between the successive points oscillating with the amplitude A/2 across a node is equal to

(A) 25 cm (B) 2.5 cm (C) 5 cm (D) 10 cm

Q.21 A string of length 0.4m & mass 10^{-2}kg is tightly clamped at its ends. The tension in the string is 1.6 N. Identical wave pulses are produced at one end at equal intervals of time, Δt. The minimum value of Δt which allows constructive interference between successive pulses is:

(A) 0.05s (B) 0.10s (C) 0.20s (D) 0.40s

Q. 22 Fig 11.46, show a stationary wave between two fixed points P and Q. which points (s) of 1, 2 and 3 are in phase with the point X?



(A) 1, 2 and 3 (B) 1 and 2 only

(C) 2 and 3 only (D) 3 only

Q.23 A wave travels uniformly in all directions from a point source in an isotropic medium. The displacement of the medium at any point at a distance r from the source may be represented by (A is a constant representing strength of source)

(A) $[A/\sqrt{r}]\sin(kr-\omega t)$ (B) $[A/r]\sin(kr-\omega t)$

(C) $[Ar]\sin(kr-\omega t)$ (D) $[A/r^2]\sin(kr-\omega t)$

Q.24 A sinusoidal progressive wave is generated in a string. Its equation is given by $Y = (2mm)\sin(2\pi x - 100\pi t + \pi/3)$. The time when particle at x = 4 m first passes through mean position, will be

(A) $\frac{1}{150}$sec (B) $\frac{1}{12}$sec

(C) $\frac{1}{300}$sec (D) $\frac{1}{100}$sec

Q.25 A transverse wave is described by the equation $Y = A\sin[2\pi x(ft - x/\lambda)]$. The maximum particle velocity is equal to four times the wave velocity if:

(A) $\lambda = \pi A/4$ (B) $\lambda = \pi A/2$

(C) $\lambda = \pi A$ (D) $\lambda = 2\pi A$

Previous Years' Questions

Q. 1 A transverse wave is described by the equation $y = y_0 \sin 2\pi\left(ft - \frac{x}{\lambda}\right)$. The maximum particle velocity is equal to four time the wave velocity if ***(1984)***

(A) $\lambda = \pi$ (B) $\lambda = \pi y_0/2$

(C) $\lambda = 2\pi$ (D) $\lambda = 2\pi y_0$

Q.2 A wave represented by the equation $y = a\cos(kx - \omega t)$ is superimposed with another wave to from a stationary wave such that point x = 0 is a node. The equation for the other wave is ***(1988)***

Q.3 The displacement y of a particle executing periodic motion is given by $y = 4\cos^2\left(\frac{1}{2}t\right)\sin(1000t)$. This expression may be considered to be a result of the superposition of .... Independent harmonic motions. ***(1992)***

(A) Two (B) Three (C) Four (D) Five

Q.4 The extension in a string, obeying Hooke's law, is x. The speed of transverse wave in the stretched string is. If the extension in the string is increased to 1.5 x, the speed of transverse wave will be ***(1996)***

(A) 1.22 (B) 0.61 (C) 1.50 (D) 0.75

Q. 5 A traveling wave in a stretched string is described by the equation; Y = A sin (kx – ωt)

The maximum particle velocity is ***(1997)***

(A) Aω (B) ω/κ (C) dω/dκ (D) x/ω

Q.6 Two vibrating strings of the same material but of lengths L and 2L have radii 2r and r respectively. They are stretched under the same tension. Both the strings vibrate in their fundamental modes. The one of length L with frequency V_1 and the other with frequency V_2. The ratio V_1/V_2 is given by ***(2000)***

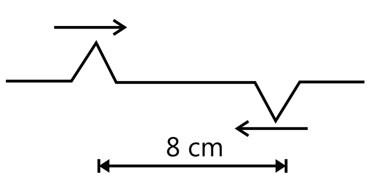
(A) 2 (B) 4 (C) 8 (D) 1

Q.7 The ends of a stretched wire of length L are fixed at x = 0 and x = L. In one experiment the displacement of the wire is $y_1 = A\sin\left(\frac{\pi x}{L}\right)\sin\omega t$ and energy is E_1 and in other experiment its displacement is $y_2 = A\sin\left(\frac{2\pi x}{L}\right)\sin 2\omega t$ and energy is E_2. Then ***(2011)***

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$

(C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$

Q.8 Two pulses in a stretched string, whose centers are initially 8 cm apart, are moving towards each other as shown in the figure. The speed of each pulse is 2 cm/s. After 2 s the total energy of the pulses will be ***(2001)***



(A) Zero

(B) Purely kinetic

(C) Purely potential

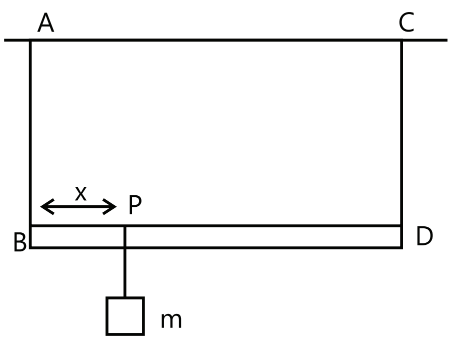
(D) Partly kinetic and partly potential

Q.9 A sonometer wire resonates with a given tuning fork forming standing waves with five antinodes between the two bridges when a mass of 9 kg is suspended from the wire. When this mass is replaced by mass M. The wire resonates with the same tuning fork forming three antinodes for the same positions of the bridges. The value of M is ***(2002)***

(A) 25 kg (B) 5kg

(C) 12.5 kg (D) 1/25 kg

Q.10 A massless rod BD is suspended by two identical massless strings AB and CD of equal lengths. A block of mass m is suspended from point P such that BP is equal to x. If the fundamental frequency of the left wire is twice the fundamental frequency of right wire, then the value of x is ***(2006)***



(A) l/5 (B) l/4 (C) 4l/5 (D) 3l/4

Q.11 A hollow pipe of length 0.8 m is closed at one end. At its open end, a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is $320 ms^{-1}$, the mass of the string is ***(2010)***

(A) 5 kg (B) 10kg (C) 20 kg (D) 40 kg

Q.12 The displacement of particles in a string stretched in the x – direction is represented by y. Among the following expressions for y, those describing wave motion is (are) ***(1987)***

(A) $\cos kx \sin\omega t$ (B) $k^2x^2 - \omega^2t^2$

(C) $\cos^2(kx + \omega t)$ (D) $\cos\left(k^2x^2 - \omega^2t^2\right)$

Q.13 A wave is represented by the equation;
$y = A \sin\left(10\pi x + 15\pi t + \pi/3\right)$

Where x is in meter and t is in second. The expression represents ***(1990)***

(A) A wave traveling in the position x – direction with a velocity 1.5 m/s

(B) A wave traveling in the negative x – direction with a velocity 1.5 m/s

(C) A wave traveling in the negative x – direction with a wavelength 0.2 m

(D) A wave traveling in the position x – direction with a wavelength 0.2 m

Q.14 Two identical straight wires are stretched so as to produce 6 beats/ s when vibrating simultaneously. On changing the tension slightly in one of them, the beat frequency remains unchanged. Denoting by T_1, T_2 the higher and the lower initial tension in the strings, then it could be said that while making the above changes in tension ***(1991)***

(A) T_2 was decreased (B) T_2 was increased

(C) T_1 was decreased (D) T_1 was increased

Q. 15 A wave disturbance in a medium is described by

$$y(x,\ t) = 0.02\cos\left(50\pi t+\frac{\pi}{2}\right)\cos(10\pi x),$$

Where x and y are in meter and t is in second. ***(1995)***

(A) A node occurs at x = 0.15m

(B) An antinode occurs at x = 0.3 m

(C) The speed of wave is 5 ms^{-1}

(D) The wavelength of wave is 0.2 m

Q.16 The (x, y) coordinates of the corners of a square plate are (0, 0), (L, 0), (L, L) and (0, L). The edges of the plates are clamped and transverse standing waves are set-up in it. If $u(x,\ y)$ denotes the displacement of the plate at the point (x, y) at some instant of time, the possible expression (s) for u is (are) (a = positive constant) ***(1998)***

(A) $a\cos(\pi x/2L)\cos(\pi y/2L)$

(B) $a\sin(\pi x/L)\sin(\pi y/L)$

(C) $a\sin(\pi x/L)\sin(2\pi y/L)$

(D) $a\cos(2\pi x/L)\cos(\pi y/L)$

Q.17 A transverse sinusoidal wave of amplitude a, wavelength λ and frequency is traveling on a stretched string. The maximum speed of any point on the string is $v/10$, where is the speed of propagation of the wave. If $a = 10^{-3}$m and v = 10m/s, then λ and f are given by ***(1998)***

(A) $\lambda = 2\pi\times10^{-2}$ m (B) $\lambda = 10^{-3}$ m

(C) $f = \frac{10^{-3}}{2\pi}$Hz (D) f = 10^4Hz

Q.18 In a wave motion, y = a sin(kx – wt), y can represent ***(1999)***

(A) Electric field (B) Magnetic field

(C) Displacement (D) Pressure

Q.19 Standing waves can be produced ***(1999)***

(A) On a string clamped at both ends

(B) On a string clamped at one end and free at the other

(C) When incident wave gets reflected from a wall

(D) When two identical waves with a phase difference of π are moving in the same direction

Q.20. A wave travelling along the x-axis is described by the equation y(x, t) = 0.005 cos ($\alpha x - \beta t$). If the wavelength and the time period of the wave are 0.08 m and 2.0 s, respectively, then α and β in appropriate units are ***(2008)***

(A) α = 25.00 π, $\beta = \pi$ (B) $\alpha = \frac{0.08}{\pi}, \frac{2.0}{\pi}$

(C) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (D) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

Q.21 The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(t)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is ***(2010)***

(A) 4.0 N (B) 12.5 N

(C) 0.5 N (D) 6.25 N

Q.22 The transverse displacement y (x,t) of a wave on a string is given by $y(a,t) = e^{-(ax^2+bt^2+2\sqrt{ab}xt)}$.This represents a ***(2011)***

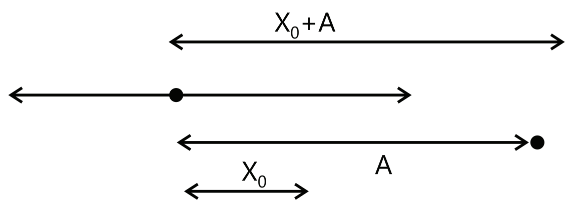
(A) Wave moving in – x direction with speed $\sqrt{\frac{b}{a}}$

(B) Standing wave of frequency $\sqrt{b}$

(C) Standing wave of frequency $\frac{1}{\sqrt{b}}$

(D) Wave moving in + x direction with $\sqrt{\frac{a}{b}}$

Q.23 Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 (X_0 > A). If the maximum separation between them is (X_0 + A), the phase difference between their motion is: ***(2011)***



(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

Q.24 A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is: ***(2011)***

(A) $\frac{M+m}{M}$ (B) $\left(\frac{M+m}{M}\right)^{1/2}$

(C) $\left(\frac{M}{M+m}\right)^{1/2}$ (D) $\frac{M}{M+m}$

Q.25 A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%.What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively? ***(2013)***

(A) 188.5 Hz (B) 178.2 Hz

(C) 200.5 Hz (D) 770 Hz

Q.26 The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is: ***(2015)***

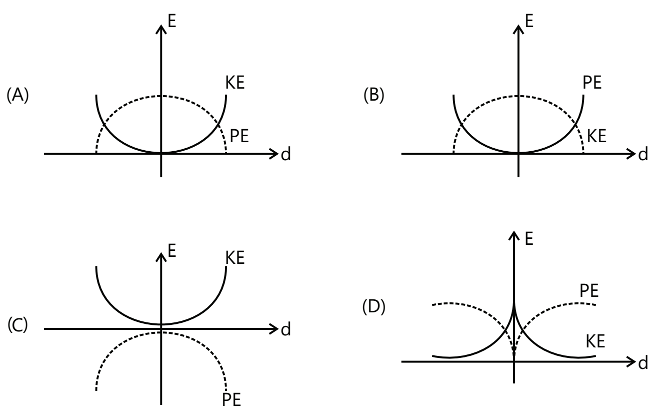
(A) 2% (B) 3% (C) 1% (D) 5%

Q.27 A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M. If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to : (g = gravitational acceleration) ***(2015)***

(A) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$ (B) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$

(C) $\left[1-\left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$ (D) $\left[1-\left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Q.28 For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly? (Graphs are schematic and not drawn to scale) ***(2015)***



Q.29 A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is: (take g = 10 ms^{-2}) ***(2016)***

(A) 2s (B) $2\sqrt{2}$ s (C) $\sqrt{2}$ s (D) $2\pi\sqrt{2}$ s

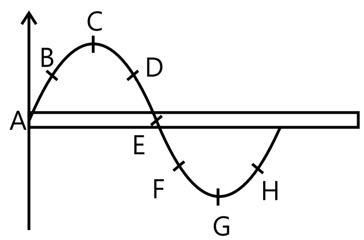
Q.30 A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is. ***(2016)***

(A) 3A (B) $A\sqrt{3}$ (C) $\frac{7A}{3}$ (D) $\frac{A}{3}\sqrt{41}$

JEE Advanced/Boards

Exercise 1

Q.1 A transverse wave is traveling along a string from left to right. The figure. represents the shape of the string (snap - shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity?

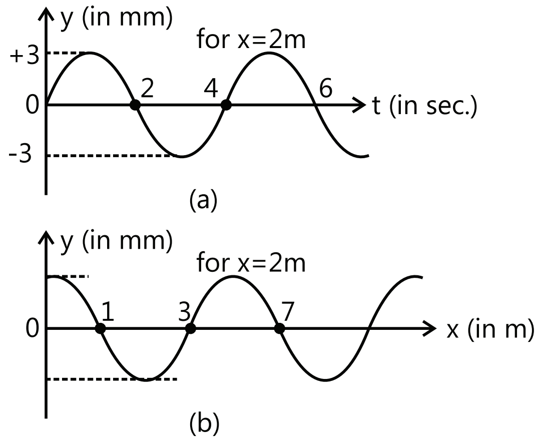


Q.2 A sinusoidal wave propagates along a string. In figure (a) and (b). 'y' represents displacement of particle from the mean position. 'x' & 't' have usual meanings. Find:

(a) Wavelength, frequency and speed of the wave.

(b) Maximum velocity and maximum acceleration of the particles

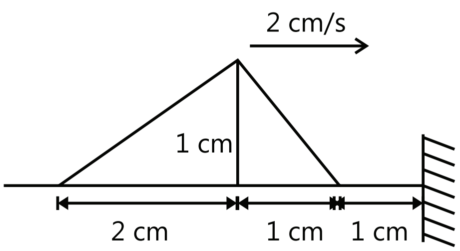
(c) The magnitude of slope of the string at x = 2 at t = 4 sec.



Q.3 The extension in a string, obeying Hook's low is x. the speed of wave in the stretched string is v. If the extension in the string increased to 1.5x find the new the speed of wave.

Q.4 A steel wire has a mass of 5gm and is under tension 450N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength.

Q.5 The figure shown a triangle pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s



(a) Draw the pulse at t = 2sec.

(b) The particle speed on the leading edge at the instant depicted is_______.

Q.6 Two strings A and B with μ =2 kg/m and μ = 8 kg/m respectively are joined in series and kept on a horizontal table with both the ends fixed. The tension in the string is 200 N. If a pulse of amplitude 1 cm travels in A towards the junction, then find the amplitude of reflected and transmitted pulse.

Q.7 A parabolic pulse given by equation y (in cm) = $0.3 - 0.1\,(x - 5t)^2$ ($y \geq 0$) x in meter and t in second traveling in a uniform string. The pulse passes through a boundary beyond which its velocity becomes 2.5 m/s. What will be the amplitude of pulse in this medium after transmission?

Q.8 A 40 cm long wire having a mass 3.2 gm and area of c.s 1 mm^2 is stretched between the support 40.05 cm apart. In its fundamental mode, it vibrates with a frequency 1000/64 Hz. Find the young's modulus of the wire.

Q.9 A string of mass 0.2 kg/m and length L = 0.6m is fixed at both ends and stretched such that is has a tension of 80 N. The string is vibrating in its third normal mode, has an amplitude of 0.5 cm. What is the frequency of oscillation? What is the maximum transverse velocity amplitude?

Q.10 A rope, under tension of 200N and fixed at both ends, oscillates in a second – harmonic standing wave pattern. The displacement of the rope is given by:

$$Y = (0.10\text{m})\ (\sin \pi x / 2) \sin 12\pi t$$

Where x = 0 at one end of the rope, x is in meters and t is in seconds. What are

(a) The length of the rope

(b) The speed of the progressive waves on the rope, and

(c) The mass of the rope

(d) If the rope oscillates in a third – harmonic standing wave pattern, what will be the period of oscillation?

Q.11 A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork.

Q.12 A string fixed at both ends has consecutive standing wave modes for which the distances between adjacent nodes are 18 cm and 16 cm respectively.

(a) What is the length of the string?

(b) If the tension is 10 N and the linear mass density is 4kg/m, what is the fundamental frequency?

Q.13 In a mixture of gases, the average number of degree of freedom per molecules is 6. The rms speed of the molecules of the gas is c. find the velocity of sound in the gas.

Exercise 2

Single Correct Choice Type

Q.1 A wave is represented by the equation $Y = 10\sin 2\pi(100t - 0.02x) + 10\sin 2\pi(100t + 0.02x)$ The maximum amplitude and loop length are respectively

(A) 20 units and 30 units

(B) 20 units and 25 units

(C) 30 units and 20 units

(D) 25 units and 20 units

Q.2 A string of length 1 m and linear mass density 0.01 kgm^{-1} is stretched to a tension of 100N. When both ends of the string are fixed, the three lowest frequencies for standing wave are f_1, f_2 and f_3. When only one end of the string is fixed, the three lowest frequencies for standing wave are n_1, n_2 and n_3. Then

(A) $n_3 = 5n_1 = f_3 = 125$ Hz

(B) $f_3 = 5f_1 = n_3 = 125$ Hz

(C) $f_3 = n_2 = 3f_1 = 150$ Hz

(D) $n_2 = \frac{f_1 + f_2}{2} = 75\text{Hz}$

Q.3 A chord attached to a vibrating string from divides it into 6 loops, when its tension is 36N. the tension at which it will vibrate in 4 loops is

(A) 24N (B) 36N (C) 64N (D) 81N

Q.4 A wave equation is given as y = cos (500t – 70x), where y in mm and t is in sec.

(A) The wave is not a transverse propagating wave.

(B) The speed of wave is 50/7 m/s

(C) The frequency of oscillation 1000π Hz

(D) Two closest points which are in same phase have separation 45 $\pi/7$ cm.

Q.5 A wave pulse passing on a string with a speed of 40 cm s^{-1} in the negative x – direction has its maximum at x = 0 at t = 0. Where will this maximum be located at t = 5s?

(A) 2 m (B) 3 m (C) 1 m (D) 2.5 m

Q.6 A steel wire of length 64 cm weights 5 g. If it is stretched by a force of 8 N, what would be the speed of a transverse wave passing on it?

(A) 10 m/s (B) 38 m/s (C) 32 m/s (D) 22 m/s

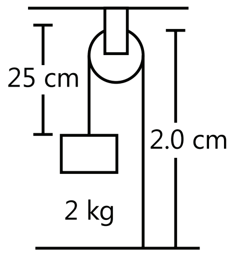
Q.7 Two blocks each having a mass of 3.2 kg are connected by a wire CD and the system is suspended from the ceiling by another wire AB. The linear mess density of the wire AB is 10 g m^{-1} and that of CD is 8 gm^{-1}. Find the speed of a transverse wave pulse produced in AB and in CD.

(A) 80 m/s, 63 m/s (B) 75 m/s, 54 m/s

(C) 82 m/s, 33 m/s (D) 87 m/s , 60 m/s

Q.8 In the arrangement shown in figure, the string has a mass of 4.5 g. How much time will it take for a transverse disturbance produced at the floor to reach the pulley?

(Take g = 10 ms^{-2})



(A) 0.03 s (B) 0.02 s (C) 0.01 s (D) 0.04 s

Assertion Reasoning Type

Q.9 Statement-I: In a sinusoidal traveling wave on a string potential energy of deformation of string element at extreme position is maximum

Statement-II: The particle in sinusoidal traveling wave perform SHM.

(A) Statement-I is true, statement-II is true, statement-II is a correct explanation for statement-I

(B) Statement-I is true, statement-II is true, statement-II is NOT correct explanation for statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Q. 10 Statement-I: When a pulse on string reflects from free end, the resultant pulse is formed in such a way that slope of string at free end is zero.

Statement-II: Zero resultant slope ensures that there is no force components perpendicular to string.

(A) Statement-I is true, statement-II is true, statement-II is a correct explanation for Statement-I

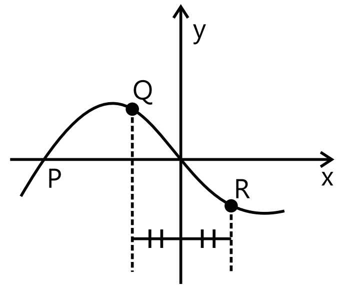
(B) Statement-I is true, statement-II is true, statement-II is NOT correct explanation for Statement-I

(C) Statement-I is true, statement-II is false

(D) Statement-I is false, statement-II is true

Multiple Correct Choice Type

Q.11 At a certain moment, the photograph of a string on which a harmonic wave is traveling to the right is shown. Then, which of the following is true regarding the velocities of the points P, Q and R on the string.

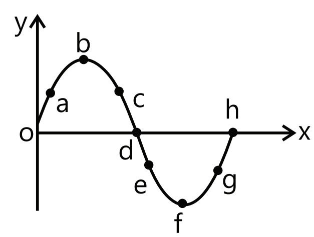


(A) V_P is upwards (B) $V_Q = -V_R$

(C) $|V_P| > |V_Q| = |V_R|$ (D) $V_Q = V_R$

Comprehension Type

The figure represents the instantaneous picture of a transverse harmonic wave traveling along the negative X – axis. Choose the correct alternative (s) related to the movement of the mine points shown in the figure.



Q.12 The point/s moving upward is/are

(A) a (B) c (C) f (D) g

Q.13 The point/s moving downwards is/are

(A) o (B) b (C) d (D) h

Q.14 The stationary points is/ are

(A) o (B) b (C) f (D) h

Q.15 The point/s moving with maximum velocity is/are

(A) b (B) c (C) d (D) h

Previous Years' Questions

Q. 1 An object of specific gravity ρ is hung from a thin steel wire. The fundamental frequency for transverse standing waves in the wire is 300 Hz. The object is immersed in water, so that one half of its volume is submerged. The new fundamental frequency (in Hz) is ***(1995)***

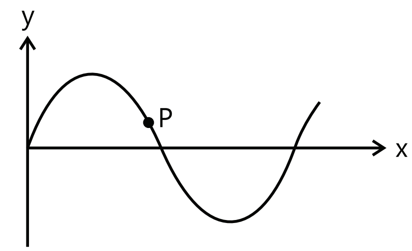
(A) $300\left(\frac{2\rho-1}{2\rho}\right)^{1/2}$ (B) $300\left(\frac{2\rho}{2\rho-1}\right)^{1/2}$

(C) $300\left(\frac{2\rho}{2\rho-1}\right)$ (D) $300\left(\frac{2\rho-1}{2\rho}\right)$

Q.2 A string of length 0.4m and mass 10^{-2} kg is tightly clamped at its ends. The tension in the string is 1.6N. Identical wave pulses are produced at one end at equal intervals of time Δt, The minimum value of Δt, which allows constructive interference between successive pulses, is ***(1998)***

(A) 0.05 s (B) 0.10 s (C) 0.20 s (D) 0.40s

Q. 3 A transverse sinusoidal wave moves along a string in the positive x – direction at a speed of 10 cm/s. The wavelength of the wave is 0.5 m and its amplitude is 10 cm. At a particular time t, the snap – shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is ***(2008)***



(A) $\frac{\sqrt{3}\pi}{50}\hat{j}\,m/s$ (B) $-\frac{\sqrt{3}\pi}{50}\hat{j}\,m/s$

(C) $\frac{\sqrt{3}\pi}{50}\hat{i}\,m/s$ (D) $-\frac{\sqrt{3}\pi}{50}\hat{i}\,m/s$

Q.4 A vibrating string of certain length l under a tension T resonates with a mode corresponding to the first overtone (third harmonic) of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats/s when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is ***(2008)***

(A) 344 (B) 336 (C) 117.3 (D) 109.3

Paragraph 1:

Two plane harmonic sound waves are expressed by the equations.

$y_1(x,t) = A\cos(\pi x - 100\pi t)$

and $y_2(x,t) = A\cos(0.4\pi x - 92\pi t)$

(All parameters are in MKS) ***(2006)***

Q.5 How many times does an observer hear maximum intensity in one second?

(A) 4 (B) 10 (C) 6 (D) 8

Q.6 What is the speed of sound?

(A) 200 m/s (B) 180 m/s (C) 192 m/s (D) 96 m/s

Q.7 At x = 0 how many times the amplitude of $y_1 + y_2$ is zero in one second?

(A) 192 (B) 48 (C) 100 (D) 96

Q.8 A wave equation which gives the displacement along the y – direction is given by; $y = 10^{-4}\sin(60t + 2x)$. Where x and y are in meter and t is time in second. This represents a wave ***(1981)***

(A) Traveling with a velocity of 30 m/s in the negative x – direction

(B) Of wavelength π m

(C) Of frequency 30/ π Hz

(D) Of amplitude 10^{-4} m

Q.9 As a wave propagates ***(1999)***

(A) The wave intensity remains constant for a plane wave

(B) The wave intensity decreases as the inverse of the distance from the source for a spherical wave

(C) The wave intensity decreases as the inverse square of the distance from the source for a spherical wave

(D) Total intensity of the spherical wave over the spherical surface centered at the source remains constant at all times

Q.10 $Y(x,t) = \frac{0.8}{[(4x+5t)^2+5]}$ represents a moving pulse where x and y are in meter and t is in second. Then, ***(1999)***

(A) Pulse is moving in positive x – direction

(B) In 2 s it will travel a distance of 2.5 m

(C) Its maximum displacement is 0.16 m

(D) It is a symmetric pulse

Q.11 A copper wire is held at the two ends by rigid supports. At 30^{0}C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10^{0}C. ***(1998)***

Given: Young modulus of copper = 1.3×10^{11} N/m^2

Coefficient of linear expansion of copper = $1.7 \times 10^{-5}\,{}^{o}C^{-1}$

Density of copper = 9×10^3 kg/m^2

Q.12 A string 25 cm long and having a mass of 2.5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats/ s are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s find the tension in the string. ***(1999)***

Q.13 A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope? ***(1984)***

Q.14 A steel wire of length 1 m, mass 0.1 kg and uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20^{0}C. If transverse waves are set – up by plucking the string in the middle, calculate the frequency of the fundamental mode of vibration. ***(2009)***

Given : $Y_{steel} = 2\times10^{11}\,N/m^2$ and

$\alpha_{steel} = 1.21\times10^{-5}\,/\,{}^{o}C.$

Q.15 When two progressive waves y_1 = 4 sin (2x - 6t) and $y_2 = 3\sin\left(2x - 6t - \frac{\pi}{2}\right)$ are superimposed, the amplitude of the resultant wave is: ***(2010)***

Q.16 A horizontal stretched string fixed at two ends, is vibrating in its fifth harmonic according to the equation y(x, t) = 0.01m sin [(62.8m^{-1})x] cos[(628s^{-1})t]. Assuming π = 3.14, the correct statement(s) is (are): ***(2013)***

(A) The number of nodes is 5.

(B) The length of the string is 0.25 m.

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01m.

(D) The fundamental frequency is 100 Hz.

Q.17 One end of a taut string of length 3m along the x axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y direction so that stationary waves are set up in the string. The possible waveform(s) of these stationary waves is (are): ***(2014)***

(A) $y(t) = A\sin\frac{\pi x}{6}\cos\frac{50\pi t}{3}$

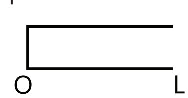
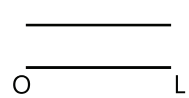
(B) $y(t) = A\sin\frac{\pi x}{3}\cos\frac{100\pi t}{3}$

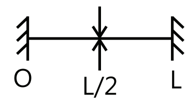
(C) $y(t) = A\sin\frac{5\pi x}{6}\cos\frac{250\pi t}{3}$

(D) $y(t) = A\sin\frac{5\pi x}{6}\cos 250\pi t$

Q.18 A metal rod AB of length 10x has its one end A in ice at 0°C and the other end B in water at 100°C. If a point P on the rod is maintained at 400°C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of λx from the ice end A, find the value of λ. [Neglect any heat loss to the surrounding.] ***(2009)***

Q.19. Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in column II describing the nature and wavelength of the standing waves. ***(2011)***

Column I	Column II
(A) Pipe closed at one end 	(p) Longitudinal waves
(B) Pipe open at both ends 	(q) Transverse waves

Column I	Column II
(C) Stretched wire clamped at both ends 	(r) $\lambda_f = L$
(D) Stretched wire clamped at both ends and at mid-point 	(s) $\lambda_f = 2L$
	(t) $\lambda_f = 4L$

PlancEssential Questions

JEE Main/Boards

Exercise 1

Q. 5 Q. 17 Q. 22

Exercise 2

Q.1 Q.4 Q.5
Q.6 Q.10 Q.14
Q.21

JEE Advanced/Boards

Exercise 1

Q. 2 Q.10 Q.12

Exercise 2

Q.1 Q.2 Q.4
Q.9 Q.10

Answer Key

JEE Main/Boards

Exercise 1

Q. 2 20 m

Q. 3 382.2 ms^{-1}

Q. 4 (i) 0.25×10^{-3} cm

(ii) $\pi/250$ sec (iii) 500 rad/sec

(iv) $80\ \pi$ meters

(v) 0.125 cm/s (vi) 62.5 cm/sec^2

Q. 6 353.6 ms^{-1}

Q. 7 250 Hz

Q. 16 4.23 s

Q. 17 1.320 km

Q. 18 92.95 ms^{-1}

Q. 19 5.27×10^3 ms^{-1}

Q. 20 100 ms^{-1}

Q. 21 7.5×10^{-2} cms^{-2}; 7.5×10^{2} cms^{-2}

Q. 22 (a) 9.42 m/s (b) zero

Q. 23 330 ms^{-1}, 0.02 m, 0.033 m

Q. 24 205 Hz

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** D	**Q. 3** A	**Q. 4** D	**Q. 5** C	**Q. 6** C
Q.7 C	**Q.8** B	**Q. 9** D	**Q. 10** A	**Q. 11** A	**Q. 12** C
Q.13 B	**Q.14** D	**Q.15** B	**Q.16** A	**Q. 17** B	**Q. 18** B
Q.19 B	**Q.20** C	**Q.21** B	**Q.22** C	**Q. 23** B	**Q.24** C
Q.25 B					

Previous Years' Questions

Q. 1 B	**Q. 2** C	**Q. 3** B	**Q. 4** A	**Q. 5** A	**Q. 6** D
Q. 7 C	**Q. 8** B	**Q. 9** A	**Q. 10** A	**Q. 11** B	**Q. 12** A, C
Q. 13 B, C	**Q. 14** D	**Q. 15** A, B, C, D	**Q. 16** B,C	**Q. 17** A	**Q. 18** A,B
Q. 19 A	**Q. 20** A	**Q. 21** D	**Q. 22** A	**Q. 23** D	**Q. 24** C
Q. 25 B	**Q. 26** B	**Q. 27** A	**Q. 28** B	**Q. 29** B	**Q. 30** C

JEE Advanced/Boards

Exercise 1

Q. 1 (a) D,E,F, (b) A,B,H, (c) C,G, (d) A,E

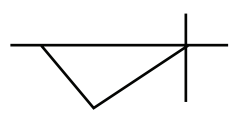
(b) $\frac{3\pi}{2}mm/s, \frac{3\pi^2}{4}mm/s^2$, (c) $\frac{3\pi}{2}$

Q. 2 (a) λ=4m, $f=\frac{1}{4}Hz$, 1 m/s

Q. 3 1.22 v

Q. 4 106.59 kW

Q. 5 (a)



(b) 2 cm/s

Q.6 $A_1 = -\frac{1}{3}cm, A_1 = \frac{2}{3}cm$

Q. 7 0.2 cm

Q. 8 1×10^9 Nm^{-2}

Q. 9 50Hz, 50π cm/sec

Q. 10 4 m, 24 m/s, 25/18 kg, 1/9 sec

Q. 11 1.007%

Q. 12 (a) 144 cm; (b) 17.36 Hz

Q. 13 2/3c

Exercise 2

Single Correct choice type

Q. 1 B	**Q. 2** D	**Q. 3** D	**Q. 4** B	**Q. 5** A	**Q. 6** C
Q. 7 A	**Q. 8** B				

Assertion Reasoning Type

Q. 9 D **Q. 10** A

Multiple Correct Choice Type

Q.11 C, D

Comprehension Type

Q. 12 A, D **Q.13** C **Q.14** B, C **Q.15** C, D

Previous Years' Questions

Q.1 A **Q.2** B **Q.3** A **Q.4** A **Q.5** A **Q.6** A

Q.7 C **Q.8** A, B, C, D **Q.9** A **Q.10** A **Q.11** 70.1 m/s **Q.12** 27.04 N

Q.13 0.12 m **Q.14** 11 Hz **Q.15** 5 **Q. 16** B, C **Q. 17** A, C, D **Q. 18** 9

Q. 19 A $\rightarrow$ p, t; B $\rightarrow$ p, s; C $\rightarrow$ q, s; D $\rightarrow$ q, r

Solutions

JEE Main/Boards

Exercise 1

Sol 1: (i) $\frac{1}{30,000}\text{ s} \le T \le \frac{1}{40}$

(ii) $\frac{350}{30000}\text{ m/s} \le \lambda \le \frac{350}{40}\text{ m/s}$

(iii) $80\pi\text{ rads}^{-1} \le \omega \le 60000\pi\text{ rads}^{-1}$

Sol 2: f = 15 × 10^6 Hz

$\lambda = \frac{V}{f} = \frac{3\times10^8}{15\times10^6} = 20\text{ m}$

Sol 3: $V' = \sqrt{\frac{4}{3}}\,V = \sqrt{\frac{4}{3}} \times 331 = 382.2\text{ ms}^{-1}$

Sol 4: (i) A = 0.25 × 10^{-3} cm

(ii) $T = \frac{2\pi}{500} = \frac{\pi}{250}\text{ s}$

(iii) ω = 500 rad/s

(iv) $\lambda = \frac{2\pi}{0.025}\text{ m} = 80\pi\text{ m}$

(v) $V_{max} = 0.25 \times 10^{-3} \times 500\text{ cm s}^{-1}$

$V_{max.} = 0.125\text{ cms}^{-1}$

(vi) $a_{max.} = V_{max}\,\omega = 0.125 \times 500$

$a_{max.} = 6.25\text{ cms}^{-2}$

Sol 5:

$f \propto \frac{1}{\ell}$

$f \rightarrow 1 : 2 : 3$

$\ell \rightarrow 1 : \frac{1}{2} : \frac{1}{3}$

$\ell \rightarrow 6 : 3 : 2$

Bridges must be placed at 60 cm from one end and 20 cm from another end

Sol 6: $\frac{V}{2.04} - \frac{V}{2.08} = \frac{20}{6}$; $V\left(\frac{0.04}{2.04\times2.05}\right) = \frac{20}{6}$

$V = 353.\ 6\text{ ms}^{-1}$

Sol 7: On loading with wax frequency decreases

f – 256 = ± 6

f = 256 ± 6 Hz

f = 262 Hz

Sol 8: No because string is not stretchable yes transverse waves are possible in a steel rod.

Sol 9: (a) No wave possible as there is no particle.

(b) Longitudinal waves (direction of motion of particles parallel to direction of propagation of wave)

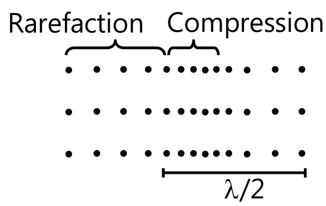
(c) Longitudinal

(d) Both are possible

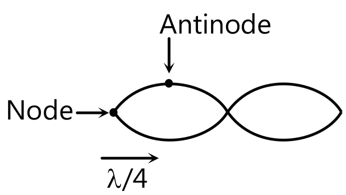
(e) Combined longitudinal & transverse (ripples)

Sol 10: Infinite as young's modulus of a rigid body is infinite

Sol 11: Half the wavelength ($\lambda/2$)



Sol 12:



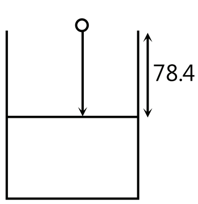
Sol 13: Strings cannot be compressed or extended hence there won't be regions of compression and rarefaction. Strings have elasticity of shape. Hence wave on strings are transverse.

Sol 14: Refer theory

Sol 15: A harmonic of a wave is a component frequency of the signal that is an integer multiple of the fundamental frequency. If f is the fundamental frequency the harmonics have frequency 2f, 3f, 4f ..... etc.

An overtone is any frequency higher than the fundamental frequency of a sound.

Sol 16:

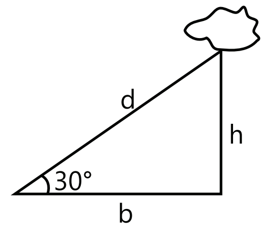


Time to reach water = $\sqrt{\dfrac{2\times 78.4}{9.8}}$ = 45

Time for sound to reach top = $\dfrac{78.4}{332}$ = 0.23 s

Total time = 4.23 s

Sol 17:



$d\left(\dfrac{1}{330}-\dfrac{1}{3\times 10^8}\right) = 8 \times 330 \times 3 \times 10^8$

$d\,(3\times 10^8 - 330)$

$d \cong 8 \times 330$

d = 264 m

height of cloud = 1320 m = 1.32 Km

Sol 18: $m = \dfrac{5\times 10^{-3}}{0.72} = \dfrac{1}{144}$

T = 60 N

$V = \sqrt{\dfrac{t}{\mu}} = \sqrt{\dfrac{60}{1/144}} = 24\sqrt{15} = 92.95\ ms^{-1}$

Sol 19: $B = 7.5 \times 10^{10}\ N\ m^{-2}$

$E = 2.7 \times 10^3\ kg\ m^{-3}$

$V = \sqrt{\dfrac{B}{e}} = \sqrt{\dfrac{7.5\times 10^{10}}{2.7\times 10^3}} = 5270.46\ ms^{-1}$

$V = 5.27 \times 10^3\ ms^{-1}$

Sol 20: $\mu = \dfrac{7\times 10^{-3}}{0.7} = 10^{-2}$ kg/m

T = 100 N

$V = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{100}{10^{-2}}} = 100\ ms^{-1}$

Sol 21: ω, = 50 $rads^{-1}$ CO_2 = 5000 $rads^{-1}$

$a = 3 \times 10^{-5} \times (50)^2\ cms^{-2}$

$a_1 = 7.5 \times 10^{-2}\ cms^{-2}$

$a_2 = 3 \times 10^{-5} \times (5000)^2\ cms^{-2}$

$a_2 = 750\ cms^{-2}$

$a_2 = 7.5\ ms^{-2}$

Sol 22: y = (3.0 cm) sin [(3.14 cm^{-1})x – (314 s^{-1})t]

(a) $V_{max.} = 3.0\ cm \times 314\ s^{-1} = 942\ cm\ s^{-1}$

$= 9.42\ ms^{-1}$

(b) $a = -3 \times (314)^2 \sin(\pi \times 6 - 100\pi \times 0.11)$

$a = -3 \times (314)^2 \sin(-5\pi)$

$a = 0$

Sol 23: f = 250 Hz

$(31 + h) = \frac{\lambda}{2}$

$(97 + h) = \frac{3\lambda}{4} \Rightarrow 66 = \frac{\lambda}{2}$

$\lambda = 132$ cm

$V = f\lambda = 250 \times 1.32\ ms^{-1} \Rightarrow V = 330\ ms^{-1}$

$H = \frac{132}{4} - 31 = 2$ cm $= 0.02$ m

Radius of tube $= \frac{\text{End Cross section}}{0.6}$

$= \frac{0.02}{0.6} = \frac{0.2}{6} = \frac{0.1}{3} = 0.033$ m

Sol 24: $\frac{V}{2} - F = 5$

$F - \frac{V}{2.1} = 5$

$\frac{V}{2} - \frac{V}{2.1} = 10$

$V = 420\ ms^{-1}$

$F = 5 + \frac{420}{2.1} = 205$ Hz

Exercise 2

Single Correct Choice Type

Sol 1: (A) $z = e^{-(x-vt+c)^2}$

$c - V \times 0 = 2$

$c - V \times 1 = -2$

$V = +4$ m/s

Sol 2: (D) $y = -\sin\left(kx - \omega t + \frac{\pi}{6}\right)$

$\Rightarrow y = \sin\left(\omega t - kx - \frac{\pi}{6}\right)$

Sol 3: (A) $V = f\lambda \Rightarrow V = 250 \times \frac{0.4}{100} = 1\ ms^{-1}$

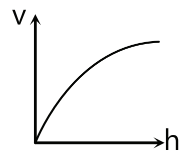
Sol 4: (D) $V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.001}} = 100\ ms^{-1}$

$T = \frac{1\,m}{100ms^{-1}} = 0.01$ sec

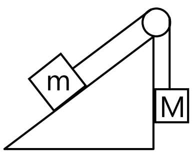
Sol 5: (C) $V \propto T^{1/2}$

T decreases linearly with height

$\therefore$ Parabolic curve



Sol 6: (C)



$T = V^2M = 100$ N; $T = mg \sin\theta$

$Mg = 100$ N ; $100 = m \times 10 \times \frac{1}{2}$

M = 10 kg, m = 20 kg

$\frac{m}{M} = 2$

Sol 7: (C) $y = 5(1 - \cos(200\pi t + 10\pi z))$

Amplitude = 5 cm

Sol 8: (B) $y = A \sin(kx + wt)$

$A \sin\left(4k + \frac{\pi}{3}\right) = \frac{A}{2} \Rightarrow 4k + \frac{\pi}{3} = \frac{\pi}{6}$

$\Rightarrow K = \frac{2\pi}{\lambda} = \frac{\pi}{24}$

$\lambda = 48$ cm ; $\lambda = 0.48$ m

Sol 9: (D) T = 0.04 sec; $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.04} = 50\pi$

V = 300 m/s, $k = \frac{\omega}{V} = \frac{50\pi}{300} = \frac{\pi}{6}$

$\Delta\phi = \frac{\pi}{6} \times 6 = \pi$

Sol 10: (A) $y = \frac{3}{a^2 + (x-3t)^2}$; $y = \frac{3}{a^2 + (x+vt)^2}$

$V = -3$ m/s

Sol 11: (A) Phase change of π due to reflection from rigid wall.

Sol 12: (C) $V_1 \to$ speed in light string

$V_2 \to$ speed in heavy string

$V_2 = \frac{V_1}{2}$

$A_r = \frac{V_2}{V_1 + V_2} A = \frac{\frac{1}{2}}{1+\frac{1}{2}}$ 6 mm = 2 mm

y= (2mm) sin (kx – wt)

y = (2mm) sin(40x – 5t)

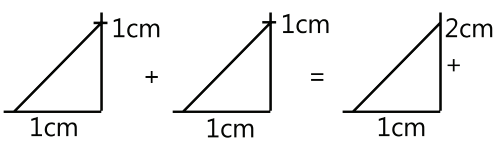
Sol 13: (B) For a given string Power $\propto A^2$

Power reflected = $\left(\frac{1}{3}\right)^2 P = P/9$

Power transmitted = $\frac{8P}{9}$

$\therefore$ 89% power transmitted

Sol 14: (D) By superposition



Sol 15: (B) ℓ = 1m f = 300 Hz

$\frac{3}{2}\lambda = \ell$; $f = \frac{v}{\ell}$; $\lambda = \frac{2\ell}{3}$; $v = f\lambda$

$v = 300 \times \frac{2}{3} \times 1 = 200$ m/s

Sol 16: (A) $y_1 = 5 \sin(\omega t - kx)$

$y_2 = -5 \cos(\omega t - kx - 150°)$

$y_1 + y_2 = 5(\sin(\omega t - kx) - \sin(\omega t - kx - 60°))$

$= 10 (\sin 30° \cos(\omega t - kx - 30°))$

$= 5 \cos(\omega t - kx - 30°)$

Sol 17: (B) $y_1 = A \cos(kx - \omega t)$

$y_2 = A \cos(kx + \omega t + \phi)$

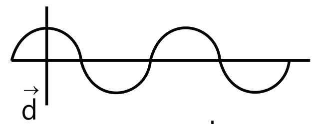
$y_1 + y_2 = 2A \cos 20\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t + \frac{\phi}{2}\right)$

$\cos\frac{\phi}{2} = 0$; $\phi = \pi$

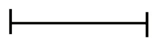
$y_2 = -A \cos(kx + \omega t)$

Sol 18: (B) $(n + 1) = \frac{L}{2d}$

$L = (n + 1)d$



No. of loops $= \frac{L}{2d}$

Sol 19: (B) 

$T = Y \alpha \Delta T A$

$f \propto \frac{v}{\ell}$; $\frac{v}{\ell} = \frac{1}{2\ell^2}\sqrt{\frac{T}{\mu}}$

$f \propto \frac{1}{r^2}\left(\frac{T}{\mu}\right)^{1/2}$

$\propto \frac{1}{\ell^2}\left(\frac{y\alpha\Delta T_A}{\frac{m}{A\ell}}\right)^{1/2}$; $\propto \frac{1}{\ell^2}\left(\frac{y\alpha\Delta T}{\rho}\right)^{1/2}$

Sol 20: (C) $y = A \sin\left(\frac{20}{3}\pi x\right)\cos(1000\,\pi t)$

$\sin\left(\frac{20\pi}{3}x\right) = \frac{1}{2}$

$\frac{20x}{3} = \frac{\pi}{6}$; $x = \frac{1}{40}$ m

Distance = $2x = \frac{1}{20}$ m = 5 cm

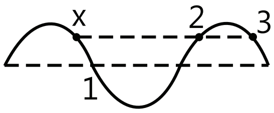
Sol 21: (B) $\ell = 0.4$ m, $m = 10^{-2}$ kg, T = 1.6 N

$\mu = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2}$ kg/m

$f = \frac{1}{2 \times 0.4}\sqrt{\frac{1.6}{2.5 \times 10^{-2}}} = \frac{1}{0.8} \times \frac{4}{0.5} = 10$ Hz

T = 0.1 s

Sol 22: (C) $\sin\phi = \sin(\pi - \phi)$



Sol 23: (B) Refer theory $y = \frac{A}{r}\sin(kr - \omega t)$

Sol 24: (C) At x = 4

$\Rightarrow y = (2mm)\sin\left(8\pi + \frac{\pi}{3} - 100\pi t\right)$

$\Rightarrow y = (2mm)\sin\left(\frac{\pi}{3} - 100\pi t\right)$

$\frac{\pi}{3} = 100\pi t$; $t = \frac{1}{300}$ sec

Sol 25: (B) $V_{pmax} = 2\pi fA$

$V_{wave} = \frac{2\pi f}{2\pi/\lambda} = f\lambda$

$2\pi fA = 4f\lambda$

$\lambda = \frac{\pi A}{2}$

Previous Years' Questions

Sol 1: (B) We velocity $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{2\pi f}{2\pi/\lambda}$

$= \lambda f$

Maximum particle velocity $v_{pm} = \omega A = 2\pi f y_0$

Given, $V_{pm} = 4V$ or $2\pi f\, y_0 = 4\lambda f$

$\therefore \lambda = \frac{\pi y_0}{2}$

Sol 2: (C) For a stationary wave to form, two indentical waves should travel in opposite direction. Further at x = 0, resultant y (from both the waves) should be zero at all instant.

Sol 3: (B) The given equation can be written as

$y = 2(2\cos^2\frac{t}{2})\sin(1000\,t)$

$y = 2(\cos t + 1)\sin(1000\,t)$

$= 2\cos t \sin 1000\,t + 2\sin(1000\,t)$

$= \sin(1001t) + \sin(999t) + 2\sin(1000t)$

i.e., the given expression is a result of superposition of three independent harmonic motions of angular frequencies 999, 1000 and 1001 rad/s.

Sol 4: (A) From Hooke's law

Tension in a string (T) $\propto$ extension (x) and speed of sound in string $v = \sqrt{T/\mu}$ or $v \propto \sqrt{T}$

Therefore, $v \propto \sqrt{x}$

x is increased to 1.5 times i.e., speed will increase by $\sqrt{1.5}$ times of 1.22 times. Therefore speed of sound in new position will be 1.22 v.

Sol 5: (A) This is an equation of a travelling wave in which particles of the medium are in SHM and maximum particle velocity in SHM is $A\omega$, where A is the amplitude and ω the angular velocity.

Sol 6: (D) Fundamental frequency is given by

$V = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$ (with both the ends fixed)

$\therefore$ Fundamental frequency

$V \propto \frac{1}{\ell\sqrt{\mu}}$ (for same tension in both strings)

Where μ = mass per unit length of wire

$= \rho.A$ (ρ = density)

$= \rho(\pi r^2)$ or $\sqrt{\mu} \propto r \therefore v \propto \frac{1}{r\ell}$

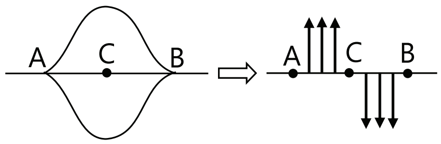
$\therefore \frac{V_1}{V_2} = \left(\frac{r_2}{r_1}\right)\left(\frac{\ell_2}{\ell_1}\right) = \left(\frac{r}{2r}\right)\left(\frac{2L}{L}\right) = 1$

Sol 7: (C) Energy $E \propto (\text{amplitude})^2(\text{frequency})^2$

Amplitude (A) is same in both the cases, but frequency 2ω in the second case is two times the frequency (ω) in the first case

Therefore, $E_2 = 4E_1$

Sol 8: (B) After two seconds both the pulses will move 4 cm towards each other. So by their superposition, the resultant displacement at every point will be zero. Therefore total energy will be purely in the form of kinetic. Half of the particles will be moving upwards and half downwards.



Sol 9: (A) Let f_0 = frequency of tuning fork

Then, $f_0 = \frac{5}{2\ell}\sqrt{\frac{9g}{\mu}} = \frac{3}{2\ell}\sqrt{\frac{Mg}{\mu}}$

(μ = mass per unit length of wire)

Solving this, we get M = 25 kg

In the first case, frequency corresponds to fifth harmonic while in the second case it corresponds to third harmonic.

Sol 10: (A) $f \propto v \propto \sqrt{T}$

$f_{AB} = 2f_{CD}$

$\therefore T_{AB} = 4T_{CD}$ ...(i)

Further $\Sigma\tau_p = 0$

$\therefore T_{AB}(x) = T_{CD}(l - x)$ or $4x = l - x$

($T_{AB} = 4T_{CD}$)

or $x = l/5$

Sol 11: (B) The fundamental mode in a pipe closed at one end and the second harmonic in a string are shown in figure. It can be seen that $\lambda_p / 4 = Lp$ and $\lambda_s = L_s$.

For the pipe closed at one end,

$$v_p = \frac{\nu_p}{\lambda_p} = \frac{\nu_p}{4L_p} = \frac{320}{4(0.8)} = 100 \text{ Hz}$$

Where ν_p =320m/s is the velocity of sound in the pipe and L_p=0.8m is length of the pipe. For string of mass m, length L_s and having tension T, velocity of the string is given by,

$$v_s = \frac{\nu_s}{\lambda_s} = \sqrt{\frac{T/(m/L_S)}{L_s}} = \sqrt{\frac{T}{mL_s}} = \sqrt{\frac{50}{m(0.5}} = \frac{10}{\sqrt{m}}$$

At resonance $\nu_p = \nu_s$ substitute ν_p and ν_s from first and second equation to get m= 0.01kg=10 gram.

Sol 12: (A, C) options satisfy the condition;

$$\frac{\partial^2 y}{\partial x^2} = \text{(constant)} \frac{\partial^2 y}{\partial t^2}$$

Sol 13: (B, C) $\omega = 15\pi$, k = 10 p

Speed of wave, $v = \frac{\omega}{k} = 1.5$ m/s

Wavelength of wave $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ m

10π x and 15πt have the same sign. Therefore, wave is traveling in negative x-direction.

Sol 14: (D) $T_1 > T_2$

$\therefore v_1 > v_2$

or $f_1 > f_2$

and $f_1 - f_2 = 6$ Hz

Now, if T_1 is increased, f_1 will increase or $f_1 - f_2$ will increase. Therefore, (d) option is wrong.

If T_1 is decreased, f_1 will decrease and it may be possible that now $f_2 - f_1$ become 6 Hz. Therefore, (C) option is correct. Similarly, when T_2 is increased, f_2 will increase and again f_2-f_1 may become equal to 6 Hz. So, (B) is also correct. But (A) is wrong.

Sol 15: (A, B, C, D) It is given that

$y(x, t) = 0.02\cos(50\pi t + \pi/2)\cos(10\pi x)$

$\cong A\cos(\omega t + \pi/2)\cos kx$

Node occurs when $kx = \frac{\pi}{2}, \frac{3\pi}{2}$ etc.

$10\pi x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow$ x = 0.05m, 0.15 m option (a)

Antinode occurs when $kx = \pi, 2\pi, 3\pi$ etc.

$10\pi x = \pi, 2\pi, 3\pi$ etc.

$\Rightarrow$ x = 0.1m, 0.2, 0.3 m option (b)

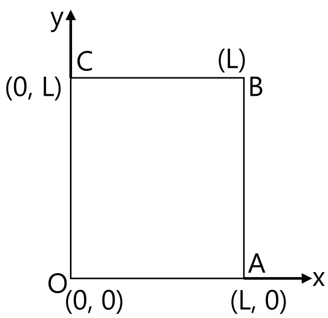
Speed of the wave is given by,

$v = \frac{\omega}{k} = \frac{50\pi}{10\pi} = 5$ m/s option (c)

Wavelength is given by,

$\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = \left(\frac{1}{5}\right)$ m = 0.2 m

Sol 16: (B, C) Since, the edges are clamped, displacement of the edges u(x, y) = 0 for



Line, OA i.e., y = 0; $0 \le x \le L$

AB i.e., x = L; $0 \le y \le L$

BC i.e., y = L; $0 \le x \le L$

OC i.e., x = 0; $0 \le y \le L$

The above conditions are satisfied only in alternatives (B) and (C).

Note that u (x, y) = 0, for all four values eg, in alternative (D), u(x, y) = 0 for y = 0, y = L but it is not zero for x = 0 or x = L. Similarly, in option (A) u (x, y) = 0 at x = L, y = L but it is not zero for x = 0 or y = 0 while in option (B) and (C), u (x, y) = 0 for x = 0, y = 0, x = L and y = L.

Sol 17: (A) Maximum speed of any point on the string = $a\omega = a(2\pi f)$

$\therefore = \frac{v}{10} = \frac{10}{10} = 1$ (Given: v = 10 m/s)

$\therefore 2\pi af = 1$; $f = \frac{1}{2\pi a}$

a = 10^{-3} m (Given)

$\therefore f = \frac{1}{2\pi \times 10^{-3}} = \frac{10^3}{2\pi}$ Hz

Speed of wave v = f l

$\therefore (10\text{ m/s}) = \left(\frac{10^3}{2\pi} S^{-1}\right)\lambda$; $\lambda = 2\pi \times 10^{-2}$m

Sol 18: (A, B) In case of sound wave, y can represent pressure and displacement, while in case of an electromagnetic wave it represents electric and magnetic fields.

Note: In general, y is general physical quantity which is made to oscillate at one place and these oscillations are propagated to other places also.

Sol 19: (A) Standing waves can be produced only when two similar type of waves (same frequency and speed, but amplitude may be different) travel in oposite directions.

Sol 20: (A) y = 0.005 cos ($\alpha x - \beta t$)

Comparing the equation with the standard form,

$$y = A\cos\left[\left(\frac{x}{\lambda} - \frac{t}{T}\right)2\pi\right]$$

$2\pi/\lambda = \alpha$ and $2\pi/T = \beta$

$\alpha = 2\pi/0.08 = 25.00\ \pi$

$\beta = \pi$

Sol 21: (D) $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} = 6.25\text{N}$

Sol 22: (A) $y_{(x,t)} = e^{-}(\sqrt{a}x + \sqrt{b}t)^2 V = \sqrt{\frac{b}{a}}$

Wave moving in − ve x –direction.

Sol 23: (D) $\phi_1 = 0$; $\phi_2 = \frac{\pi}{2}$

Sol 24: (C) Energy of simple harmonic oscillator is constant.

$$\Rightarrow \frac{1}{2}M\omega^2 A_1^2 = \frac{1}{2}(m+M)\omega^2 A_2^2$$

$$\frac{A_1^2}{A_2^2} = \frac{M+m}{M}$$

$$\therefore \frac{A_1}{A_2} = \sqrt{\frac{M+m}{M}}$$

Sol 25: (B) $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{Ad}}$

Also, $Y = \frac{T\ell}{A\Delta\ell}$ $\Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell}$ $\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{Y\Delta\ell}{\ell d}}$

$\ell = 1.5\text{m}, \frac{\Delta\ell}{\ell} = 0.01, d = 7.7\times 10^3\text{ kg/m}^3$

$y = 2.2\times 10^{11}\text{N/m}^2$

After solving

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}\text{Hz}$$

$f \approx 178.2\text{Hz}$

Sol 26: (B) Given $\frac{\Delta L}{L} = \frac{0.1}{20}$

$T = \frac{90}{100}\text{sec.} \qquad \Delta T = \frac{1}{100}\text{sec.}$

$$\frac{\Delta T}{T} = \frac{1}{90}$$

$$g = \left(\frac{1}{4\pi^2}\right)\frac{L}{T^2}$$

$$\Rightarrow \frac{\Delta g}{g}\times 100\% = \frac{\Delta L}{L}\times 100 + \frac{2\Delta T}{T}\times 100$$

$$\frac{\Delta g}{g}\times 100\% = \left(\frac{0.1}{20}\right)100 + 2\left(\frac{1}{90}\right)100 = 2.72\%$$

So, nearest option is 3%.

Sol 27: (A)

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T_M = 2\pi\sqrt{\frac{\ell+\Delta\ell}{g}} \qquad \Delta\ell = \frac{Mg\ell}{AY}$$

$$\frac{T_M}{T} = \sqrt{\frac{\ell+\Delta\ell}{\ell}}$$

$$\left(\frac{T_M}{T}\right)^2 = 1+\frac{\Delta\ell}{\ell}$$

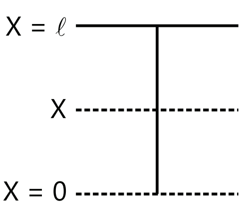
$$\left(\frac{T_M}{T}\right)^2 = 1+\frac{Mg}{AY}$$

$$\frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$$

Sol 28: (B) K.E. is maximum at mean position, whereas P.E. is minimum.

At extreme position, K.E. is minimum and P.E. is maximum.

Sol 29: (B) Let mass per unit length be λ



$T = \lambda g x \qquad v = \sqrt{\frac{T}{\lambda}} = \sqrt{gx}$

$$v^2 = gx$$

$$a = \frac{vdv}{dx} = \frac{g}{2}$$

$$\ell = \frac{1}{2}\frac{g}{2}t^2 \Rightarrow t = \sqrt{\frac{4\ell}{g}} = 2\sqrt{2}\,\text{sec}$$

Sol 30: (C)

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$$

$$v = \sqrt{5}\frac{A\omega}{3}$$

$$v_{new} = 3v = \sqrt{5}A\omega$$

So the new amplitude is given by

$$v_{new} = \omega\sqrt{{A_{new}}^2 - x^2} \Rightarrow \sqrt{5}A\omega = \omega\sqrt{{A_{new}}^2 - \left(\frac{2A}{3}\right)^2}$$

$$A_{new} = \frac{7A}{3}$$

JEE Advanced/Boards

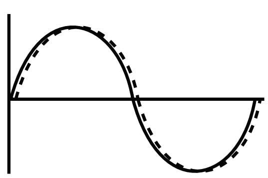
Exercise 1

Sol 1: (a) D, E, F

(b) A, B, H

(c) C, G

(d) A, E



Sol 2: (a) λ = 4 m

$f = \frac{1}{T} = 0.25$ Hz

$V = f\lambda = 1\ ms^{-1}$ in –ve n-direction

(b) $V_{max.} = 0.5\,\pi \times 3\ mm\ s^{-1} = 1.5\pi\ mm\ s^{-1}$

$a_{max.} = 0.75\pi^2$ mm s^{-2}

(c) $y = (3 \text{ mm}) \sin\left(\frac{\pi}{2}x - \frac{\pi}{2}t + \pi\right)$

$= (3 \text{ mm}) \sin\left(\frac{\pi}{2}(x - t + 2)\right)$

$\left.\frac{dy}{dx}\right|_t = \left(\frac{3\pi}{2}\right)\cos\frac{\pi}{2}(x - t + 2)$

Slope at x = 2m & t = 4 sec

$= \frac{3\pi}{2}\cos\frac{\pi}{2}(2 - 4 + 2) = \frac{3\pi}{2}$

Sol 3: $V \propto \sqrt{T}$, $V' = \sqrt{1.5}\,V$

Sol 4: $\mu = 5 \times 10^{-3}$ kg/m ; $V = \sqrt{\frac{450}{5 \times 10^{-3}}} = 300$ m/s

T = 450 N

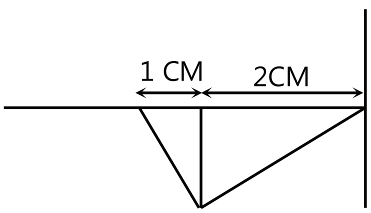
$A \le \frac{\lambda}{5}$

$P_{avg\ max.} = \frac{1}{2}\frac{\omega^2 A^2 F}{V}$

$= \frac{1}{2} \times 4\pi^2 \frac{f^2\lambda^2}{25} \times \frac{450}{300}$

$= \frac{2\pi^2}{25} \times 450 \times \frac{(300)^2}{300} = 106.59$ kW

Sol 5:



$V_p = -\text{slope} \times V_{wave} = -(-1) \times -2$ cm/s

$V_p = -2$ cm/s

–ve sign represents particle moving down

Sol 6: $\mu_A = 2$ kg/m$\mu_B = 8$ kg/m

T = 200 N

$V_A = \sqrt{\frac{200}{2}} = 10$m/s $V_B = \sqrt{\frac{200}{8}} = 5$m/s

$A_r = -\frac{V_B}{V_A + V_B}A$; $A_T = \frac{V_A}{V_A + V_B}A$

$A_r = -\frac{1}{3}$cm; $A_T = \frac{2}{3}$cm

Sol 7: $V_1 = 5$ m/s

$V_2 = 2.5$ m/s

$A_T = \frac{5}{7.5}A = \frac{2}{3} \times 0.3 = 0.2$ cm

Sol 8: $\ell = 0.4$ m ; m = 3.2 g ; A = 1 mm^2

$m = \frac{3.2}{0.4} \times 10^{-3} = 8 \times 10^{-3}$

$\frac{100}{64} = \frac{1}{2 \times 0.4}\sqrt{\frac{T}{8 \times 10^{-3}}}$

$\left(\frac{800}{64}\right)^2 \times 8 \times 10^{-3} = T$

$\frac{10^4}{8 \times 8} \times 5 \times 10^{-3} = T$

T = 1.25 N

$Y = \frac{T\ell}{A\Delta\ell} = \frac{1.25 \times 0.4}{10^{-6} \times 5 \times 10^{-4}} = \frac{0.5}{5} \times 10^{10}\ Nm^{-2}$

$Y = 10^9\ Nm^{-2}$

Sol 9: $\mu = 0.2$ kg/m

L = 0.6 m

T = 80 N

$f = \frac{3}{2 \times 0.6}\sqrt{\frac{80}{0.2}} = \frac{3}{1.2} \times 20 = 50$ Hz

$V_{max} = 2\pi fA$

$= 2\pi \times 50 \times 0.5\ cms^{-1} = 50\pi\ cms^{-1}$

Sol 10: T = 200 N

$k = \frac{\pi}{2}\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/2} = 4$ m

(a) Second harmonic

$\therefore \ell = \lambda = 4$ m

(b) $V = \frac{12\pi}{\pi/2} = 24\ ms^{-1}$

(c) $\mu = \frac{T}{V^2} = \frac{200}{24 \times 24}$

mass $= \mu \times \ell = \frac{200}{24 \times 24} \times 4 = 1.39$ kg

(d) $f = \frac{3}{8} \times 24 = 9$ Hz

$T = \frac{1}{f}; T = \frac{1}{9}$ sec

Sol 11: $\sqrt{\frac{T'}{T}}$ f= $\frac{200}{199}$

$\frac{T'}{T} = \left(\frac{200}{199}\right)^2$

T′ = 1.01007

ΔT = T′ – T = 0.01007

% change in tension = 1.007%

Sol 12: λ_1 = 36 cm λ_2 = 32 cm

$\lambda_1 = \frac{2\ell}{n} = 36; \lambda_2 = \frac{2\ell}{(n+1)} = 32$

$\Rightarrow \frac{n+1}{n} = \frac{36}{32} = \frac{9}{8}$

⇒ 8n + 8 = 9n

⇒ n = 8

$\ell = \frac{36\times 8}{2}$ = 144 cm

ℓ = 1.44 m

(b) $f_0 = \frac{1}{2\times 1.44}\sqrt{\frac{10}{4\times 10^{-3}}}$

$\Rightarrow f_0 = \frac{50}{2\times 1.44}$

f_0 = 17.36 Hz

Sol 13: $V = \sqrt{\frac{\gamma RT}{M}}$

$\sqrt{\frac{RT}{M}} = \frac{c}{\sqrt{3}}$

$\gamma = 1 + \frac{2}{6} = \frac{4}{3}$

$V = \sqrt{\gamma} \times \frac{c}{\sqrt{3}} = \sqrt{\frac{4}{3}} \times \frac{c}{\sqrt{3}}$

$V = \frac{2}{3}c$

Exercise 2

Single Correct Choice Type

Sol 1: (B) y = 20 sin 2π(100t) cos(2π(0.02x))

$A_{max.}$ = 20 units

$\frac{2\pi}{\lambda} = 2\pi(0.02)$

λ = 50 units

Maximum loop length = $\frac{\lambda}{2} = \frac{50}{2}$ = 25 units

Sol 2: (D) M = 0.01 kg m^{-1}

T = 100N; $f_1 = \frac{1}{2}\sqrt{\frac{100}{0.01}}$ = 50Hz

$f_1 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$; $f_2 = 2f_1$; $f_3 = 3f_1$

$n_1 = \frac{f_1}{2}$; $n_2 = \frac{3f_1}{2}$; $n_3 = \frac{5f_1}{2}$

Sol 3: (D) $\nu = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$

$\nu_1 = \nu_2$

$6 \times \sqrt{36} = 4 \times \sqrt{T}$

$T = \frac{36^2}{4^2}$ = 81 N

Sol 4: (B) y = cos (70x – 500 t)

Transverse wave as particle oscillate perpendicular to the direction of motion

$V = \frac{500}{70} = \frac{50}{7}$ m/s

$f = 500 = \frac{250}{\pi}$

$\lambda = \frac{2\pi}{70}$ m $= \frac{20\pi}{7}$ cm

Sol 5: (A) $y = A\cos(kx - \omega t)$

For this the maximum occurs at x = 0 and at t = 0

For getting the maximum at t = 5 sec

$kx = 5\omega \Rightarrow x = 5\omega / k$

x = 5 v = 200 cm = 2m

Sol 6: (C) The speed of the transverse is given as

$$V = \sqrt{\frac{T}{\mu}}$$

Here, T is tension and is Mass per unit length. Now putting values in above equation, we get

$$V = \sqrt{8 \times 64 \times \frac{1000}{100} \times 5} = 32\,m/s$$

Sol 7: (A) $\nu = \sqrt{\frac{T}{\mu}} \Rightarrow \nu_{AB} = \sqrt{\frac{6.4 \times 10}{10 \times 10^{-3}}} = 80ms^{-1}$

$$\nu_{CD} = \sqrt{\frac{3.2 \times 10}{8 \times 10^{-3}}} = 63ms^{-1}$$

Sol 8: (B) $\nu = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20}{2 \times 10^{-3}}} = 100ms^{-1}$

$$\mu = \frac{4.5}{2.25} = 2gm^{-1}$$

$$\Rightarrow s = \mu t - \frac{1}{2}gt^2$$

$\Rightarrow 2 = 100t - 5t^2$

$\Rightarrow t^2 - 20t + 0.4 = 0$

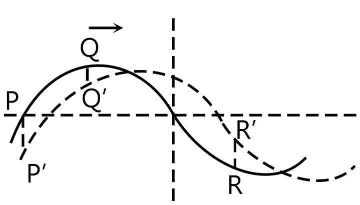
$$\Rightarrow t = \frac{+20 \pm \sqrt{400 - 1.6}}{2} = 0.02s$$

Assertion Reasoning Type

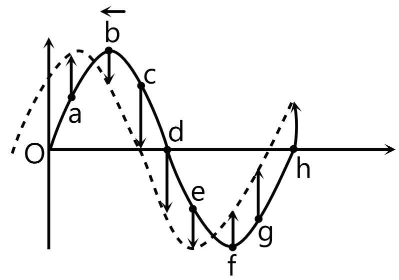
Sol 9: (D) Potential energy is maximum at the extremes and particle oscillate in SHM.

Sol 10: (A) There cannot be a perpendicular force to a string.

Sol 11: (C, D)



Comprehension Type



Sol 12: (A, D) Upward

a, g , h

Sol 13: (C) Downward

c, d, e

Sol 14: (B, C) Stationary

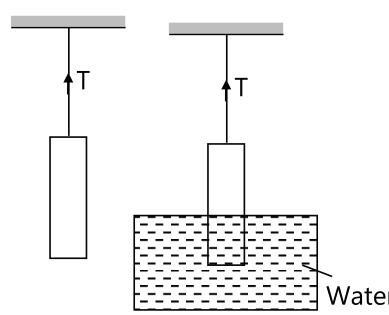
b, f

Sol 15: (C, D) Maximum velocity

o, d, h

Previous Years' Questions

Sol 1: (A) The diagramatic representation of the given problem is shown in figures. The expression of fundamental frequency is $V = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$



In air T = mg (Vρ)g

$$\therefore v = \frac{1}{2\ell}\sqrt{\frac{A\rho g}{\mu}} \quad ...(i)$$

When the object is half immersed in water

$$T' = mg - \text{upthrust} = V\rho g - \left(\frac{V}{2}\right)\rho_w g$$

$= \left(\frac{V}{2}\right) g\,(2\rho - \rho_w)$

The new fundamental frequency is

$$V' = \frac{1}{2\ell} \times \sqrt{\frac{T'}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{(Vg/2)(2\rho - \rho_w)}{\mu}} \qquad \text{...(ii)}$$

$$\therefore \frac{v'}{v} = \left(\sqrt{\frac{2\rho - \rho_w}{2\rho}}\right)$$

$$\text{or } v' = v\left(\frac{2\rho - \rho_w}{2\rho}\right)^{1/2} = 300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

Sol 2: (B) Mass per unit length of the string.

$$m = \frac{10^{-2}}{0.4} = 2.5 \times 10^{-2} \text{ kg/m}$$

$\therefore$ Velocity of wave in the string,

$$V = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5 \times 10^{-2}}}$$

$v = 8$ m/s

For constructive interference between successive pulses

$$\Delta t_{min} = \frac{2\ell}{v} = \frac{(2)(0.4)}{8} = 0.10\text{s}$$

(After two reflections, the wave pulse is in same phase as it was produced, since in one reflection its phase changes by π, and if at this moment next identical pulse is produced, then constructive interference will be obtained.)

Sol 3: (A) Particle velocity $v_p = -v$ (slope of y-x graph)

Here, v = + ve, as the wave is traveling in positive x-direction.

Slope at P is negative.

$\therefore$ Velocity of particle is in positive y (or $\hat{j}$) direction.

Sol 4: (A) With increase in tension, frequency of vibrating string will increase. Since number of beats are decreasing. Therefore, frequency of vibrating string or third harmonic frequency of closed pipe should be less that the frequency tuning fork by 4.

$\therefore$ Frequency of tuning fork

= Third harmonic frequency of closed pipe +4

$$= 3\left(\frac{v}{4\ell}\right) + 4 = 3\left(\frac{340}{4 \times 0.75}\right) + 4 = 344 \text{ Hz}$$

Sol 5: (A) In one second number of maximas is called the beat frequency. Hence,

$$f_b = f_1 - f_2 = \frac{100\pi}{2\pi} - \frac{92\pi}{2\pi} = 4\text{Hz}$$

Sol 6: (A) Speed of wave $v = \frac{\omega}{k}$

$$\text{or } v = \frac{100\pi}{0.5\pi} \text{ or } \frac{92\pi}{0.46\pi} = 200 \text{ m/s}$$

Sol 7: (C) At x = 0, $y = y_1 + y_2$

$= 2A \cos 96\pi t \cos 4\pi t$

Frequency of cos (96πt) function is 48 Hz and that of cos (4πt) function is 2 Hz.

In one second, cos function becomes zero at 2f times, where f is the frequency. Therefore, first function will become zero at 96 times and the second at 4 times. But second will not overlap with first. Hence, net y will become zero 100 times in 1 s.

Sol 8: (A, B, C, D) $y = 10^{-4} \sin (60t + 2x)$

$A = 10^{-4}$ m, $\omega = 60$ rad/s, $k = 2\text{m}^{-1}$

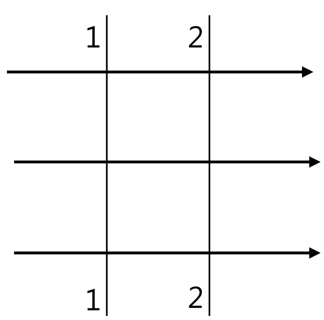
Speed of wave, $v = \frac{\omega}{k} = 30$ m/s

Frequency, $f = \frac{\omega}{2\pi} = \frac{30}{\pi}$ Hz.

Wavelength $\lambda = \frac{2\pi}{k} = \pi$m

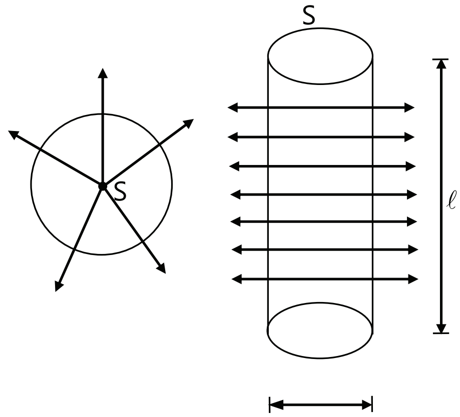
Further, 60 t and 2x are of same sign. Therefore, the wave should travel in negative x-direction.

Sol 9: (A) For a plane wave intesity (energy crossing per unit area per unit time) is constant at all points.



But for a spherical wave, intensity at a distance r from a point source of power P (energy transmitted per unit time) is given by

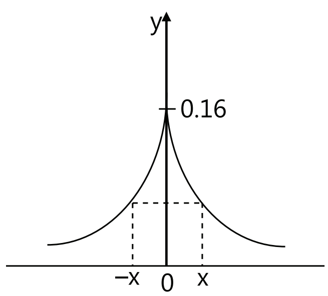
$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2}$$



Note: for a line source $I \propto \frac{1}{r}$

Because, $I = \frac{P}{\pi r \ell}$

Sol 10: (A) The shape of pulse at x = 0 t = 0 would be as shown, in figure(a).



t= 0(a)

$Y(0, 0) = \frac{0.8}{5} = 0.16\text{m}$

From the figure it is clear that $y_{max} = 0.16$ m

Pulse will be symmetric (Symmetry is checked about y_{max}) if at t = 0

y(x) = y(–x)

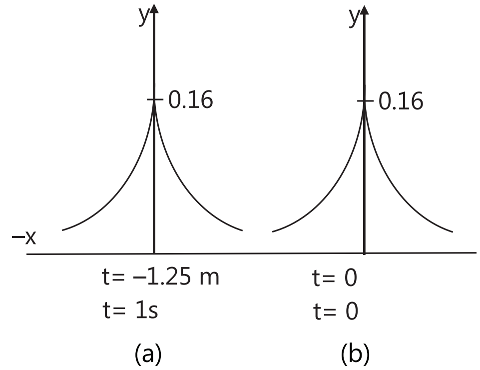
From the given equation

$$\text{And} \left.\begin{aligned} y(x) &= \frac{0.8}{16x^2+5} \\ y(-x) &= \frac{0.8}{16x^2+5} \end{aligned}\right\} \text{at } t = 0$$

or y (x) = y (– x)

Therefore, pulse is symmetric.

Speed of pulse, at t = 1s, and x = – 1.25 m



Value of y is again 0.16 m, i.e., pulse has traveled a distance of 1.25 m in 1 s in negative x-direction or we can say that the speed of pulse is 1.25 m/s and it is traveling in negative x-direction. Therefore, it will travel a distance of 2.5 m in 2 s. The above statement can be better understood from figure (b)

Sol 11: Tension due to thermal stresses,

$T = YA\,\alpha\,.\,\Delta\theta$

$v = \sqrt{\frac{T}{\mu}}$

Hence, μ = mass per unit length = ρA

$\therefore v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{YA\alpha.\Delta\theta}{\rho A}} = \sqrt{\frac{Y\alpha\,\Delta\theta}{\rho}}$

Substituting the values we have,

$v = \sqrt{\frac{1.3\times10^{11}\times1.7\times10^{-5}\times20}{9\times10^{3}}} = 70.1\text{ m/s}$

Sol 12: By decreasing the tension in the string beat frequency is decreasing, it means frequency of string was greater than frequency of pipe. Thus,

First overtone frequency of string–Fundamental frequence of closed pipe = 8

$\therefore 2\left(\frac{v_1}{2\ell_1}\right) - \left(\frac{v_2}{4\ell_2}\right) = 8$

or $v_1 = \ell_1\left[8 + \frac{v_2}{4\ell_2}\right]$

Substituting the value, we have

$v_1 = 0.25\left[8 + \frac{320}{4\times0.4}\right] = 52\text{ m/s}$

Now, $v_1 = \sqrt{\frac{T}{\mu}}$

$\therefore T = \mu v_1^2 = \left(\frac{m}{\ell}\right)v_1^2 = \left(\frac{2.5\times10^{-3}}{0.25}\right)(52)^2 = 27.04$ N

Sol 13: v = $\sqrt{T/\mu}$

$\frac{v_{top}}{v_{bottom}} = \sqrt{\frac{T_{top}}{T_{bottom}}} = \sqrt{\frac{6+2}{2}} = 2$ ...(i)

Frequency will remain unchanged. Therefore, equation (i) can be written as, $\frac{f\lambda_{top}}{f\lambda_{bottom}} = 2$

Or $\lambda_{top} = 2(\lambda_{bottom}) = 2 \times 0.06 = 0.12$ m

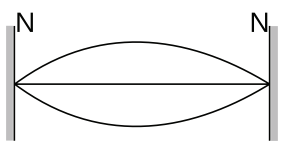
Sol 14: The temperature stress is $\sigma = Ya\Delta q$

or tension in the steel wire $T = \sigma A = YA\alpha\Delta q$

Substituting the values, we have

$T = (2 \times 10^{11})(10^{-6})(1.21 \times 10^{-5})(20) = 48.4$ N

Speed of transverse wave on the wire, $v = \sqrt{\frac{T}{\mu}}$



$\lambda/2 = l$

Hence, μ = mass per unit length of wire = 0.1 kg/m

$\therefore v = \sqrt{\frac{48.4}{0.1}} = 22$ m/s

Fundamental frequency $f_0 = \frac{v}{2\ell} = \frac{22}{2\times1} = 11$ Hz

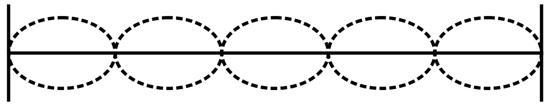
Sol 15:

$A_{eq} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{eq} = \sqrt{4^2 + 3^2 + 2(4)(3)\cos\frac{\pi}{2}}$

$A_{eq} = 5$

Sol 16: (B, C) y = 0.01 m sin (20 πx) cos 200 πt



No. of nodes is 6

$20\pi = \frac{2\pi}{\lambda}$

$\therefore \lambda = \frac{1}{10}m = 0.1m$

Length of the spring $= 0.5 \times \frac{1}{2} = 0.25$

Mid point is the antinode

Frequency at this mode is $f = \frac{200\pi}{2\pi} = 100Hz$

$\therefore$ Fundamental frequency $= \frac{100}{5} = 20Hz$

Sol 17: (A, C, D) Taking y(t) = A f(x) g(t) & Applying the conditions:

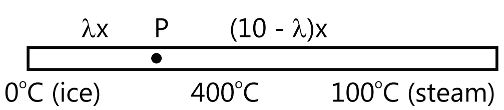
1; here x = 3m is antinode & x = 0 is node

2; possible frequencies are odd multiple of fundamental frequency.

where, $\nu_{fundamental} = \frac{v}{4\ell} = \frac{25}{3}Hz$

The correct options are A, C, D.

Sol 18:



$\frac{dm_{ice}}{dt} = \frac{dm_{vapour}}{dt}$

$\frac{400kS}{\lambda x L_{ice}} = \frac{300kS}{(100-\lambda)xL_{vapour}}$

$\lambda = 9$

Sol 19: A $\rightarrow$ p, t; B $\rightarrow$ p, s; C $\rightarrow$ q, s; D $\rightarrow$ q, r