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25. NUCLEAR PHYSICS AND RADIOACTIVITY

NUCLEAR PHYSICS

1. INTRODUCTION

Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei.

2. PROPERTIES OF ATOMIC NUCLEUS

Nuclear physics is the field of physics that studies the constituents and interactions of atomic nuclei. The most commonly known applications of nuclear physics are nuclear power generation and nuclear weapons technology, but the research has provided application in many fields, including those in nuclear medicine and magnetic resonance imaging, ion implantation in materials engineering and radiocarbon dating in geology and archaeology. The field of particle physics evolved out of nuclear physics and is typically taught in close association with nuclear physics.

Properties: Atomic nuclei have following properties:

2.1 Composition

All nuclei contain protons and neutrons except ordinary hydrogen atom which has only single proton. Proton has change $+e$ and neutron is neutral.

Mass no. of nuclei $(A) = Z + N$

Where $Z =$ no. of protons in the nucleus; $N =$ no. of neutrons

Symbolically atomic nuclei is represented as ${}_{Z}^{A}X$

Illustration 1: How many electrons, protons, and neutrons are there in nucleus of atomic number 11 and mass number 24? **(JEE MAIN)**

Sol: The atomic number Z of atom represents the number of protons present in the nucleus. The number of electrons in an atom are same as the number of protons. The Atomic mass number A is sum of proton number Z and neutron number N.

Number of protons in nucleus = Atomic number = 11

Number of electrons = Number of protons = 11

Number of neutrons = Mass number A – atomic number Z N = 24 – 11 = 13

2.2 Mass

Nuclear mass has been measured accurately by using mass spectrometer. It is convenient to express mass in terms of amu which is defined as $\frac{1}{12}$ the mass of carbon isotope ${}^{12}_{6}C$

$1amu = 1.66 \times 10^{-27} kg$

According to Einstein's equation $E = mc^2$ 1amu can be expressed as energy

Energy equivalence of 1 amu $= \dfrac{1.66 \times 10^{-27} \times \left(3 \times 10^{8}\right)^{2}}{1.6 \times 10^{-19}} eV = 931 MeV$

2.3 Nuclear Radius

The nuclear radius (R) is considered to be one of the basic quantities that any model must predict. For stable nuclei (not halo nuclei or other unstable distorted nuclei) the nuclear radius is roughly proportional to the cube root of the mass number (A) of the nucleus, and particularly in nuclei containing many nucleons, as they arrange in more spherical configurations:

The stable nucleus has approximately a constant density and therefore the nuclear radius R can be approximated by the following formula, $R = r_0 A^{1/3}$

Where A=Atomic mass number (the number of protons Z, plus the number of neutrons N) and $r_0 = 1.25 fm = 1.25 \times 10^{-15} m$.

Illustration 2: The ratio of the radii of the nuclei ${}^{27}_{13}Al$ and ${}^{125}_{52}Te$ is approximately. **(JEE MAIN)**

Sol: The radius of the atomic nuclei is directly proportional to the cube root of atomic mass number.

$$R_{Al} / R_{Te} = \frac{(27)^{1/3}}{(125)^{1/3}} = \frac{3}{5} = \frac{6}{10}$$

Illustration 3: The radius of the ${}_{30}Zn^{64}$ nucleus is nearly (in fm) **(JEE MAIN)**

Sol: The radius of any atomic nucleus is given by $R = R_0 A^{1/3}$ where Ro = 1.2×10^{-15} is the Fermi radius.

$$R = R_0 A^{1/3} = 1.2 \times 10^{-15} \times (64)^{1/3} = 1.2 \times 10^{-15} \times 4 = 4.8 fm \quad A \propto R^3$$

$A =$ Nucleon number or mass number

Any element X with mass number A and charge number Z can be represented by ${}^{A}_{Z}X$ or ${}^{A}_{Z}X$.

Number of neutron $= A - Z$ Mass number $= A = P + N$

1 amu $= \frac{1}{12}$th Mass of 12gm of ${}^{12}_{6}C$ atom.

2.4 Nuclear Density

Nuclear density is the density of the nucleus of an atom. The nuclear density for an atom with radius R and molar mass A(mass number) is $n = \dfrac{A}{\frac{4}{3}\pi R^3}$

Typical nucleus can be approximately calculated from the size of the nucleus, which itself can be approximated based on the number of protons and neutrons in it. The radius of a typical nucleus, in terms of number of nucleons, is $R = A^{1/3} r_0$ where A is the mass number and r_0 is 1.25 fm, with deviations of 0.2 fm from this value.

Illustration 4: Nuclear radius of ${}^{16}_{8}O$ is 3×10^{-15} m. Find the density of nuclear matter. **(JEE MAIN)**

Sol: Considering the nucleus of the oxygen as a sphere of the uniform density ρ, the density can be given as $\rho=\frac{M}{V}$ where M is the atomic mass number (convert it from amu to kg) of the oxygen and V is the volume of the sphere.

Use $\rho=\text{mass / volume }=\dfrac{1.66\times10^{-27}\times16}{(4/3)\pi\left(3\times10^{-15}\right)^{2}}=2.35\times10^{17}\,\text{kgm}^{-3}$

2.5 Nuclear Spin and Magnetism

Many nuclides have an intrinsic nuclear angular momentum or spin and an associated intrinsic nuclear magnetic moment. Although nuclear angular momenta are roughly of the same magnitude as the angular momenta of atomic electrons, nuclear magnetic moments are much smaller than typical atomic magnetic moments.

2.6 Types of Nuclei

(a) Isotopes: Nuclei having same atomic number Z but different mass no. are called isotopes.

Ex. ${}^{1}_{1}H,{}^{2}_{1}H,{}^{3}_{1}H$

(b) Isobars: Nuclei having same mass number A but different atomic number Z are called isobars.

Ex. ${}^{14}_{6}C$ and ${}^{14}_{7}N$.

(c) Isotones: Nuclei having same number of neutrons are called isotones Ex. ${}^{3}_{1}H,{}^{4}_{2}He$.

3. NUCLEAR STABILITY AND RADIOACTIVITY

Nuclear Stability means that nucleus is stable meaning that it does not spontaneously emit any kind of radioactivity (radiation). On the other hand, if the nucleus is unstable (not stable), it has the tendency of emitting some kind of radiation, i.e., it is radioactive. Therefore the radioactivity is associated with unstable nucleus:

Stable nucleus -> non-radioactive, Unstable nucleus –> radioactive

PLANCESS CONCEPTS

Keep in mind that less stable means more radioactive and more stable means less radioactive.

We want to know why there is radioactivity. What makes the nucleus a stable one? There are no concrete theories to explain this but there are only general observations based on the available stable isotopes. It appears that neutron to proton (n/p) ratio is the dominant factor in nuclear stability. This ratio is close to 1 for atoms of elements with low atomic number and increase as the atomic number increases. Then how do we predict the nuclear stability? One of the simplest ways of predicting the nuclear stability is based on whether nucleus contains odd/even number of protons and neutrons:

Protons	**Neutrons**	**Number of stable Nuclides**	**Stability**
Odd	Odd	4	
Odd	Even	50	least stable
Even	Odd	57	↓
Even	Even	167	most stable

- Nuclides containing odd numbers of both protons and neutrons are the least stable means more radioactive.

PLANCESS CONCEPTS

- Nuclides containing even numbers of both proton and neutrons are most stable means less radioactive.
- Nuclides contain odd number of protons and even numbers of neutrons are less stable than nuclides containing even numbers of protons and odd numbers of neutrons.

In general, nuclear stability is greater for nuclides containing even numbers of protons and neutrons or both.

Yashwanth Sandupatla (JEE 2012, AIR 821)

4. NUCLEAR FORCE

The force that controls the motions of atomic electrons is the familiar electromagnetic force. To bind the nucleus together, however, there must be a strong attractive nuclear force of a totally different kind, strong enough to overcome the repulsive force between the (positively charged) nuclear protons and to bind both protons and neutrons into the tiny nuclear volume. The nuclear force must also be of short range because its influence does not extend very far beyond the nuclear "surface". Its range is of the order of 2fm. The present view is that the nuclear force that binds neutrons and protons in the nucleus is not a fundamental force of nature but is a secondary, or "spillover", effect of the strong force that binds quarks together to form neutrons and protons. In much the same way, the attractive force between certain neutral molecules is a spillover effect of the Coulomb electric force that acts within each molecule to bind it together. This strong force is independent of the charge. This means that the strong force of proton-proton, neutron-neutron, proton-neutron interactions is the same, apart from the additional repulsive Coulomb force for the proton-proton interaction. It is customary to talk of the potential energy when we talk of nuclear forces. Here, the potential energy of interaction of a proton and a neutron is shown in Fig 25.1.

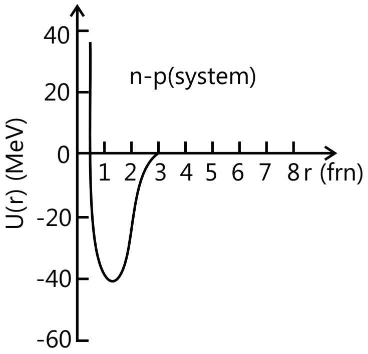


Figure 25.1

4.1 Properties of Nuclear Forces

(a) These forces are attractive by nature. At very short distance s (< 0.7 fm) these become repulsive.

(b) The nuclear force is short range force. It means that it exist only when particles are very-very close to each other. In nucleus the separation between particles is 10^{-15} m or I Fermi. At this infinitesimal small separation, the nuclear force becomes 100 times stronger than the repulsive than the electric forces between the nucleons. In the short range force, the force between the particles rapidly decreases. Thus the nuclear force only exists in the nucleus.

(c) These forces do not obey inverse square law.

(d) Nuclear forces are not central forces. It means that these forces do not depend upon the center of one particle to another particle.

(e) Strong nuclear forces are the strongest force in nature. In the given range of distance, the nuclear forces are 10^{38} times stronger than the gravitational forces.

5. MASS DEFECT

It has been observed that actual mass of the nucleus (determined by mass spectrometer of high resolving power) is always less than the sum of masses of proton and neutrons in Free State.

$\Delta m = \left[Zm_p + \left(A - Z\right)m_n\right] - M$, where M_p is mass of proton; m_n is mass neutron; M is mass of nucleus

Illustration 5: Consider the decay of radium (A=226) atom into an alpha particle and radon (A=222). Then, what is the mass defect of the reaction.

Mass of radium -226 atom = 226.0256u; Mass of radon -222 atom = 222.0715u and Mass if helium – 4 atom = 4.0026u **(JEE MAIN)**

Sol: Mass defect is the difference in masses of parent and daughter nuclei. Mass defect is given by $\Delta m = M(Ra^{226}) - M(Rn^{222}) - M(\alpha)$

Mass defect $\Delta m = M(Ra^{226}) - M(Rn^{222}) - M(\alpha) = 226.0256 - 222.0175 - 4.00026 = 0.0053u$

6. BINDING ENERGY

It is defined as energy released during formation nucleus as a result of disappearance of mass i.e., mass defect.

Binding energy = $(\Delta m)c^2$; Binding energy per nucleon= $\frac{\left(\Delta m\right)c^2}{A}$

Illustration 6: If mass equivalent to one mass of proton is completely converted into energy then determine the energy produced? **(JEE MAIN)**

Sol: When one proton is converted into its equivalent energy, the energy released during this conversion is given by $E = mc^2$

$$E = mc^2 = \left(1.66\times10^{-27}\right)\left(3\times10^{8}\right)^2 J = 1.49\times10^{-10}\ J = \frac{1.49\times10^{-10}}{1.6\times10^{-13}}\ MeV = 931.49\ MeV \quad \because 1amu = 931.49MeV$$

Variation of B.E. per nucleon with mass no. A

If the average binding energy per nucleon is calculated for all nuclides and the results are plotted against A, the mass number, a graph shown in Fig. 25.2 is obtained.

It is observed from the graph that binding energy per nucleon (except for He^4, C^{12} and O^{16}) rises first sharply and reaches a maximum value 8.8 MeV in the neighborhood of $A = 50$. The curve falls very slowly after $A = 50$ and reaches at 8.4 MeV at about $A = 140$. For higher mass number, the energy decreases to about 7.6 MeV.

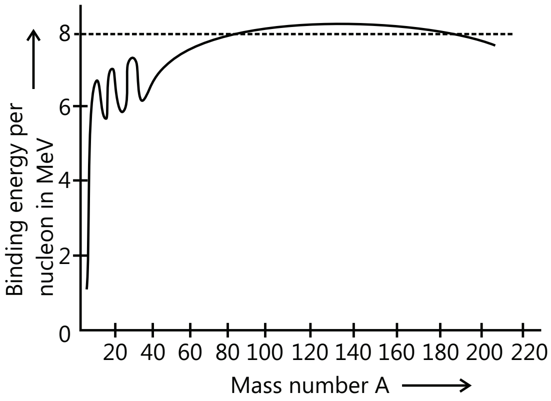


Figure 25.2

Illustration 7: Binding energy per nucleon of an α-particle from the following data:

Mass of the helium nucleus = 4.001265amu; Mass of proton = 1.007277amu

Mass of neutron = 1.00866amu; (1amu=931.4812MeV) **(JEE MAIN)**

Sol: The binding energy is given by $B.E. = \Delta m\times c^2\ J = \Delta m\times 931.5\ Mev$

Mass of two protons = $2\times1.007277 = 2.014554amu$

Mass of two neutron = $2\times1.008666 = 2.017332amu$

Total initial mass of two proton and neutrons = 2.014554 + 2.017332 = 4.031886amu

Mass defect $\Delta m = 4.031816 - 4.001265, \Delta m = 0.030621amu$

$\therefore$ Binding energy of α particle $= 0.030621 \times 931.4812 = 28.5221$MeV

Binding energy of nucleon = 28.5221/4=7.10525Mev

PLANCESS CONCEPTS

The energy differences in allowed energy levels of a nucleus are generally large of the order of MeVs. Hence, it is difficult to excite the nucleus by usual method of supplying energy as heat.

GV Abhinav (JEE 2012, AIR 329)

7. NEUTRON TO PROTON RATIO

According to Pauli Exclusion Principle, each quantum state can contain at most two protons or two neutrons that too with opposite spin. Hence nuclear forces favor pairing of two protons and two neutrons together. In lighter nuclei nuclear forces are dominant over coulomb repulsion and hence number of protons and number of neutrons are nearly the same. In heavier nuclei the case is different, the interaction between nucleon pairs through nuclear forces is not that effective and Coulomb repulsion dominates. Stability is achieved by having more neutrons as they are neutral and don't participate in Coulomb repulsion. That is why N/Z increases with atomic number for stable nuclides. The heaviest stable nuclide is ${}^{209}_{83}\mathrm{Bi}$. Bismuth in fact is of radioactive nature but the decay rate is so less that it can be considered stable.

PLANCESS CONCEPTS

Having too many neutrons do not account for higher stability as many of these neutrons won't have pairing with protons. It will in fact decrease the stability.

The fact that the binding energy curve "drops" at both high and low mass numbers has very important practical consequences.

Anurag Saraf (JEE 2011, AIR 226)

RADIOACTIVITY

1. INTRODUCTION

The phenomenon of spontaneous disintegration of nuclei of unstable atoms is defined as radioactivity. Generally it is exhibited by atoms with A>192 and Z>82. It was discovered by Henry Becqurel. Lead isotope is the stable end product of any natural radioactive series Radio activity is a nuclear process and not an atomic process Radioactivity is not associated with the electron configuration of the atom.

Becquerel, in 1896, discovered accidently that uranium salt crystals emit an invisible radiation which affected a photographic plate even though it was properly covered. Further investigations by Marie and Pierre Curie and other workers showed that many other substances also emitted similar radiations. This property of spontaneous emission of radiation is called radioactivity. Subsequent works, notably of Rutherford, suggested that radioactivity was, in fact, due to decay or disintegration of unstable nuclei.

Emission of α particles: During α-particle emission atomic no. reduces by 2 while mass no. reduces by 4 i.e. ${}_{Z}^{A}x \rightarrow {}_{Z-2}^{A-4}y + {}_{2}^{4}He$

Emission of β-particle: When Nuclei has excess neutrons, it emits β-particle to bring n/p ratio into stable region. A neutron gets converted into proton and β-particle, therefore atomic mass remains constant white atomic number increases by 1.

Y-Radiation: After emission of α or β particle nuclei are left in excited state, Nucleus comes to stable state by emitting electromagnetic radiation known as γ radiation. There is no change in A or Z during this process, α and β emission don't take place simultaneously while γ radiation can emit along with any of them.

1.1 Properties of Alpha, Beta and Gamma Rays

The comparison of the properties of α , β and γ rays are shown below in the table:

Properties	**α-rays**	**β-rays**	**γ-rays**
Nature photons	Helium nucleus	Fast moving electrons	Electromagnetic waves
Nature of charge	Positive	Negative	No change
Magnitude of charge	3.2×10^{-19} coulomb	1.6×10^{-19} coulomb	Zero
Mass	6.6×10^{-27} kg	3.1×10^{-31} kg	Rest mass zero
Velocity	Between 1.4×10^{7} m/sec to 2.2×10^{7} m/sec	1% to 99% velocity of light	3×10^{8} m/sec.
Effect of electric & magnetic fields	Deflected	Deflected	Not deflected
Range	2.7to 8.62 cm in air or 1/100 mm of Al	5mm of Al or 1mm of lead	30 cm of iron
Penetrating power	Minimum	100 times of α-rays	1000 times of α-rays
Ionising power	Maximum	Lesser	Minimum

1.2 Natural Radioactivity

Natural Radioactivity is the spontaneous disintegration of an unstable atomic nucleus and the emission of particles or electromagnetic radiation. All naturally occurring elements with atomic numbers greater than 83 as well as some isotopes of lighter elements show natural radioactivity.

1.3 Artificial Radioactivity

Radioactivity produced in a substance by bombardment with high-speed particles (as protons or neutrons), also called as induced radioactivity.

1.4 Parent and Daughter Nuclei

Nucleus which decays in a radioactive decay is called parent nucleus. This parent nucleus transforms to an atom with a nucleus in a different state, or to a different nucleus containing different numbers of protons and neutrons. Either of these products is named the daughter nucleus.

1.5 Law of Radioactive Disintegration

(a) Radioactivity is a process in which nuclei of certain elements undergo spontaneous disintegration without excitation by any external means.

(b) The radioactivity results the emission of powerful radiations known as Alpha (α), Beta (β) and Gamma (γ) rays.

(c) Radioactivity is a nuclear phenomenon i.e. it is not depend upon no. of electrons present in outer shell.

It was studied by Rutherford and Soddy in 1902. The disintegration of nuclei is purely statistical which means all nuclei take different time to disintegrate and are independent for radioactive decay. Rate of disintegration is directly proportional to no. of not decayed nuclei present at that time, i.e. $-\frac{dN}{dt} \propto N = \lambda N$ ... (i)

Where λ is disintegration or decay constant.

Integrating equation (i)

$\log_e N = -\lambda t + C$

$\because$ at, $t = 0$, $N = N_0$ $\Rightarrow$ $C = \log_e N$

$\Rightarrow \log_e N = -\lambda t + \log_e N_0$ or $N = N_0 e^{-\lambda t}$ ... (ii)

Equation (ii) shows that no. of nuclei of given radioactive substance decreases exponentially with time. It also shows that decays occurs rapidly initially and rate of decay decreases with time.

Half-life $\left(T_{1/2}\right)$: The time in which half of radioactive substance decays is known as half-life.

or $t = T_{1/2}$, $N = \frac{N_0}{2}$; $\Rightarrow$ $\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}}$; $\Rightarrow$ $T_{1/2} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda}$ ... (iii)

If $t = nT_{1/2}$ where n is integer, equation (ii) reduces to $N = N_0\left(1/2\right)^n$

Illustration 8: A count-rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute. Find:

(a) Decay constant (b) the half-life of the sample. **(JEE MAIN)**

Sol: The decay constant of radioactive element is given by $\lambda = \frac{\log_e 10}{t} \log \frac{N_0}{N_t}$ where N0 is the number of radioactive nuclei at t=0 and Nt is the number of radioactive nuclei at time t. The half-life of the radioactive element is $t_{1/2} = \frac{0.693}{\lambda}$.

Initial activity, $A_0 = dN/dt$ at $t = 0$

Final activity, $A_t = dN/dt$ at $t = t$

$$\left.\frac{dN}{dt}\right|_{t=0} = IN_0 \quad \& \quad \left.\frac{dN}{dt}\right|_{t=5} = IN_t; \quad \frac{4750}{2700} = \frac{N_0}{N_t}$$

Using $\lambda t = 2.303 \log\frac{N_0}{N_t}$; $\lambda(5) = 2.303 \log\frac{4750}{2700}$; $\lambda = \frac{2.303}{5}\log\frac{4750}{2700} = 0.1129 \text{ min}^{-1}$

$t_{1/2} = \frac{0.693}{0.1129} = 6.14 \text{ min}$

Mean life (τ): Mean life of radioactive substance is defined as sum of life times of all radioactive nuclei divided by total no. of nuclei.

or $\tau = \frac{\int t\,dN}{\int dN} = \frac{\int t dN}{N_0}$ or $\tau = \frac{1}{\lambda}$ ... (iv)

if $t = \tau$; $N = N_0 e^{-\lambda(1/\lambda)} = 0.37 N_0$

i.e., In mean life radioactive substance decays by nearly 63%.

From (iii) and (iv) $T_{1/2} = 0.693\,\tau$ ... (v)

Illustration 9: The mean lives of a radioactive substance are 1620 and 405 years for α emission and β emission respectively. Find out the time during which three fourth of a sample will decay if it its decaying both the α emission and β emission simultaneously. **(JEE ADVANCED)**

Sol: When substance decays by α and β emission simultaneously, the average rate of λ_{av} disintegration is given by $\lambda_{av} = \lambda_\alpha + \lambda_\beta$; Where λ_α and λ_β are disintegration constant for α emission and β emission respectively. The average time of the disintegration is given by $\lambda_{av} t_{av} = 2.303 \log \frac{N_o}{N_t}$ where N_o is the number of atoms present at time t=0 s. And N_t is the number of disintegration atoms present at the time t s.

Mean life is given by: $\tau_m = 1/\lambda$; $\lambda_{av} = \lambda_\alpha + \lambda_\beta$; $\frac{1}{\tau_{av}} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta} = \frac{1}{1620} + \frac{1}{405} = 3.08 \times 10^{-3}$

$\Rightarrow \lambda_{av} t = 2.303 \log \frac{100}{25}$; $(3.08 \times 10^{-3})\ t = 2.303\ \log \frac{100}{25}$

$\Rightarrow t = 2.303 \times \frac{1}{3.08 \times 10^{-3}} \log 4 = 499.24$ years

Activity of a Radioactive Isotope: The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by A. $\Rightarrow A = \frac{dN}{dt}$

If a time t=0 sec, the activity of a radioactive substance is A_o and after time t=t sec it is observed to be A_t, then:

$$A_o = \left|\frac{dN}{dt}\right|_{t=0} = \lambda N_0 \quad A_t = \left|\frac{dN}{dt}\right|_{t=t} = \lambda N_t$$

Units of Rate of Decay or Activity: A number of units have been used to express the activity of a radioactive sample. The more commonly used ones are the following:

(a) Curie (Ci): The activity of a radioactive sample is said to be one curie when 3.7×10^{10} decays take place per second. Thus $1\text{Ci} \equiv 3.7 \times 10^{10} \text{ decays / s}$

This is the approximate activity of 1 g of radium. In practice, the smaller units milli curie and micro curie are used. $1\,\text{mCi} \equiv 3.7 \times 10^7 \text{ decays / s}$; $1\,\mu\text{ Ci} \equiv 3.7 \times 10^4 \text{ decays / s}$

(b) Becquerel (Bq): The Si unit of activity is called the Becquerel and it represents 1 decay per second. Thus 1 Bq = 1 decay/s We thus have $1\text{Ci} \equiv 3.7 \times 10^{10}\text{Bq}$

(c) Rutherford (Rd): Another unit for activity is Rutherford and it represents 10^6 decays per second. $1\text{Rd} = 10^6 \text{decays} / \text{s}$

Illustration 10: Radioisotopes of phosphorus P^{32} and P^{38} are mixed in the ratio 2:1 of atoms. The activity of the sample is 2 m Ci. Find the activity of the sample after 30 days, $t_{1/2}$ of P^{32} is 14 days and, $t_{1/2}$ of P^{38} is 25 days.

(JEE ADVANCED)

Sol: When the radio isotopes are mixed in the proportion 2:1, the compound activity of mixture over time t is given by $A_t = A_{1t} + A_{2t}$. The activity A of any radioactive substance with half-life τ is defined as $A = \lambda N = \frac{0.693 \times N}{\tau}$.

Let A_o be the initial activity of the sample,

Let A_{1o} be initial activity of isotope 1 and A_{2o} be the initial activity of sample 2

$A_o = A_{1o} + A_{2o}$

Similarly for final activity (Activity after time t), $A_t = A_{1t} + A_{2t}$ $\qquad A_t = A_{1o}e^{-\lambda_1 t} + A_{2o}e^{-\lambda_2 t}$

Now in the given equation $A_o = 2$ m Ci $\Rightarrow A_0 = A_{1o} + A_{2o} = 2\text{ m Ci}$ ... (i)

Initial ratio of atoms of isotopes=2:1

We know from definition of activity, $A = \lambda N$ here λ is the decay constant and N is number of radioactive nuclei present at time instant t s.

$\frac{A_{1o}}{A_{2o}} = \frac{N_{1o}}{N_{2o}} \times \frac{T_2}{T_1}$ where T represents half-life; $\frac{A_{1o}}{A_{2o}} = \frac{2}{1} \times \frac{25}{14} = \frac{25}{7}$ ... (ii)

On solving equation (i) and (ii), we get, $A_{1o} = 25/16$ and $A_{2o} = 7/16$; $\quad A_t = A_{1o}e^{-\lambda_1 t} + A_{2o}e^{-\lambda_2 t}$

How to solve expression like this? For example, consider the first exponential term $\exp\left(-\frac{0.693 \times 30}{14}\right) = e^{-1.485}$

Let $y = e^{-1.485}$ Therefore, $\text{In}y = -1.485$; $\quad \log y = -(1.485/2.303)$ $y = \text{antilog}\,(-1.485/2.303)$

So, from above calculations you can derive a general result i.e. $e^{-x} = \text{antilog}\left|\frac{-x}{2.303}\right|$

$$A_t = \frac{25}{16} \times 0.2265 + \frac{7}{16} \times 0.4353 = 0.5444 \text{ Ci}.$$

Important Formulae

(a) $N = N_o e^{-\lambda t}$

(b) $A = A_o e^{-\lambda t}$

(c) $M = M_o e^{-\lambda t}$

(d) $\lambda = \frac{2.3027 \log_{10}\left(\frac{N_o}{N}\right)}{t}$

(e) $\lambda = \frac{2.3027 \log_{10}\left(\frac{A_o}{A}\right)}{t}$

(f) $\lambda = \frac{2.3027 \log_{10}\left(\frac{M_o}{M}\right)}{t}$

(g) $\lambda = \lambda_\alpha + \lambda_\beta$

(h) $\tau = \frac{\tau_\alpha \tau_\beta}{\tau_\alpha + \tau_\beta}$ (when two particles decay simultaneously)

(i) $N = \frac{N_o}{2^n} + \frac{N_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

(j) $A = \frac{A_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

(k) $M = \frac{M_o}{2^{\left(\frac{T}{T_{1/2}}\right)}}$

2. ALPHA DECAY

In alpha decay, the unstable nucleus emits an alpha particle reducing its proton number Z as well as its neutron N by 2. The alpha decay process may be represented as

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He.$$

As the proton number Z is changed, the element itself is changed and hence the chemical symbol of the residual nucleus is different from that of the original nucleus. The nucleus before the decay is called the parent nucleus and resulting after the decay is called the daughter nucleus. An example of alpha decay is $^{212}_{83}Bi \rightarrow {}^{208}_{81}Tl + {}^{4}_{2}He$.

(a) Characteristics of α-decay:

- **(i)** The spectrum of α-particles is a discrete line spectrum.
- **(ii)** Spectrum of α-particles has fine structure i.e. every spectral line consists of a number of fine lines.
- **(iii)** The α- emitting nuclei have discrete energy levels i.e., energy levels in nuclei are analogous to discrete energy levels in atoms.
- **(iv)** α-decay is explained on the basis of tunnel effect.
- **(v)** Geiger-Muller law- $\log_e \lambda = A + B\log_e R$ For radioactive series B is same whereas A is different

(b) Size of the nucleus decreases by α emission

3. BETA DECAY

Beta Decay: Beta decay is a process in which either a neutron is converted into a proton or a proton is converted into a neutron. Thus, the ratio N/Z is altered in beta decay. If a nucleus is formed with more number of neutrons than needed for stability, a neutron will convert itself into a proton to move towards stability. Similarly, if a nucleus is formed with more number of protons than needed for stability, a proton will convert itself into a neutron. Such transformations take place because of weak forces operating within a neutron or a proton. When a neutron is converted into a proton, an electron and a new particle named antineutrino are created and emitted from the nucleus $n \rightarrow p + e + \overline{v}$

3.1 Characteristics β-Decay

(a) The energy spectrum of β-particles is continuous i.e. β-particles of all energies up to a certain maximum are emitted.

(b) The number of such β-particles is maximum whose energy is equal to the maximum probable energy i.e. at $E = E_{mp}$, N_B =maximum.

(c) There is a characteristic maximum value of energy in the spectrum of β-particles which is known as the end point energy (E_0).

(d) In β-decay process, a neutron is converted into proton or proton is converted into neutron.

$$^{1}_{0}n = {}^{1}_{1}p + {}^{0}_{-1}e \;\; \beta \text{ particle } \;\; {}^{1}_{1}p = {}^{1}_{0}n + {}^{0}_{1}e \;\; \left(\beta^{+}\text{particle}\right)$$

(e) The energy of β-particles emitted by the same radioactive material may be same or different.

(f) The number of β-particles with energy $E = E_0$ (end point energy) is zero.

4. GAMMA DECAY

In gamma decay, a nucleus changes from a higher energy state to a lower energy state through the emission of electromagnetic radiation (photons). The number of protons (and neutrons) in the nucleus does not change in this process, so the parent and daughter atoms are the same chemical element. In the gamma decay of a nucleus, the emitted photon and recoiling nucleus each have a well-defined energy after the decay. The characteristic energy is divided between only two particles. The process is similar to that in a hydrogen atom when an electron jumps from a higher energy orbit to a lower energy orbit emitting a photon.

4.1 Characteristics γ-Decay

(a) The spectrum of γ -rays is a discrete line spectrum.

(b) Whenever α or β -particles is emitted by a nucleus then the daughter nucleus is left in the excited state. It suddenly makes transition in the ground state thereby emitting γ -rays.

(c) Knowledge about nuclear energy levels is obtained by γ -spectrum.

(d) γ -rays interact with matter as a consequence of which the phenomena of photoelectric effect, Compton Effect and pair production happen. (At low energy photoelectric effect and at high energy pair-production is effective).

5. RADIOACTIVE SERIES

(a) Elements beyond Bismuth are all radioactive in nature. These radioactive elements disintegrate to give new elements which further disintegrate to form other elements and so on. The process is continued till a non-radioactive end product is reached.

(b) The whole chain of such elements starting from the parent radioactive elements to the end non-radioactive element is called "radioactive series or a family."

$\left(4n+1\right)$ is artificial series & 4n, $\left(4n+2\right)$, $\left(4n+3\right)$ are natural series.

S.No.	Series	Name of the series	Initial element	Final element	Nature of series	No of α & β particles emitted
1.	4n+2	Uranium series	$_{92}^{238}U$	$_{82}^{206}pb$	Natural	$8\alpha, 6\beta$
2.	4n+3	Actinium series	$_{92}^{235}U$	$_{82}^{207}pb$	Natural	$7\alpha, 4\beta$
3.	4n	Thorium series	$_{90}^{232}Th$	$_{82}^{208}Pb$	Natural	$6\alpha, 4\beta$
4.	4n+1	Neptunium series	$_{93}^{237}Np$	$_{83}^{209}Bi$	Artificial	$7\alpha, 4\beta$

5.1 Thorium Series

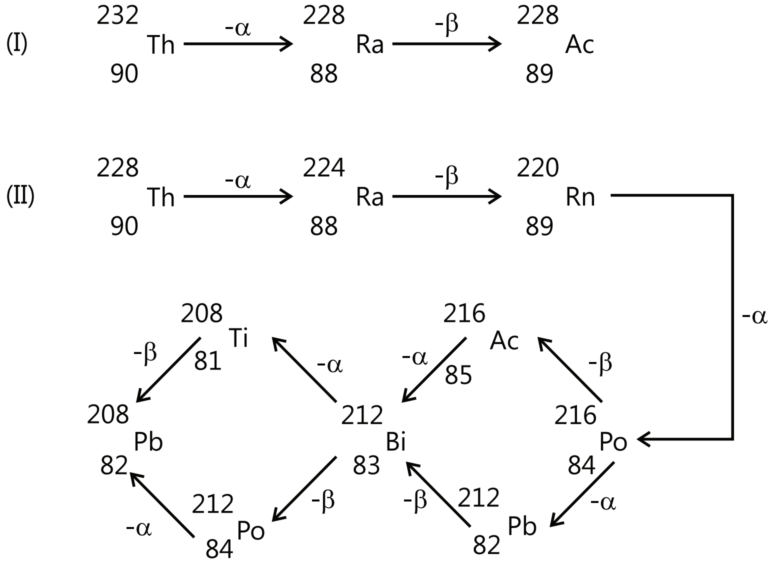


Figure 25.3

5.2 Uranium Series

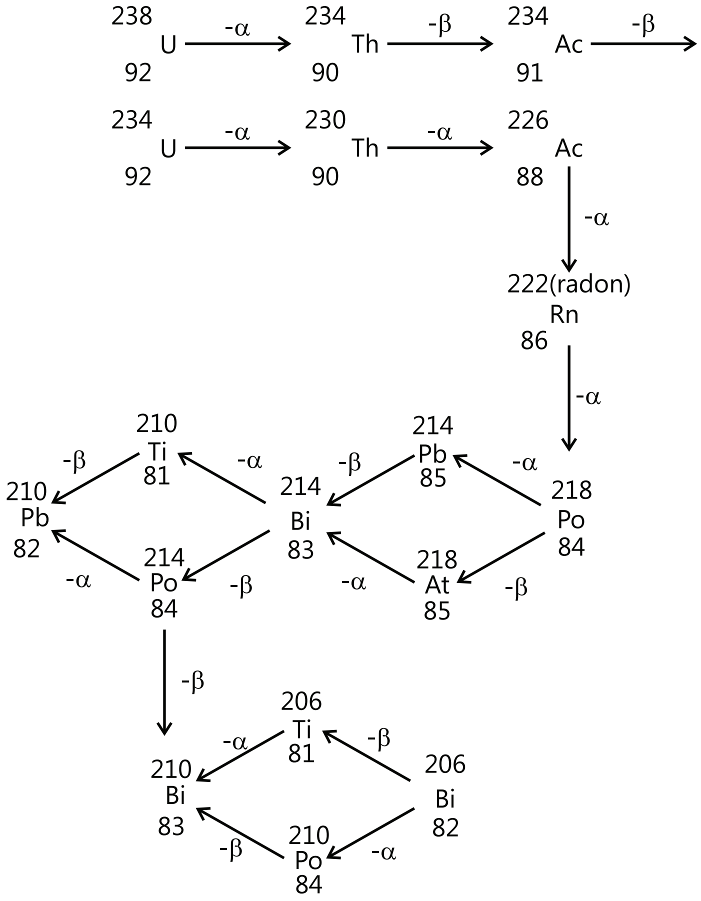


Figure 25.4

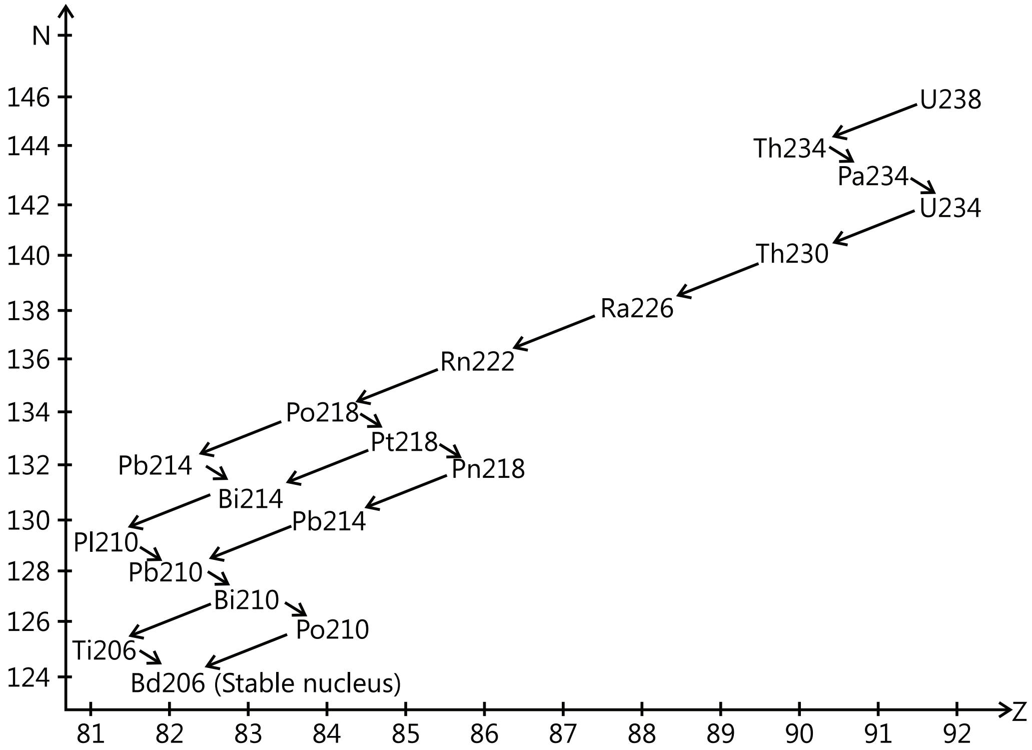


Figure 25.5: Uranium series

The mass number of each element in the series is equal to 4n+3. Where n is a positive integer.

5.3 Actinum Series

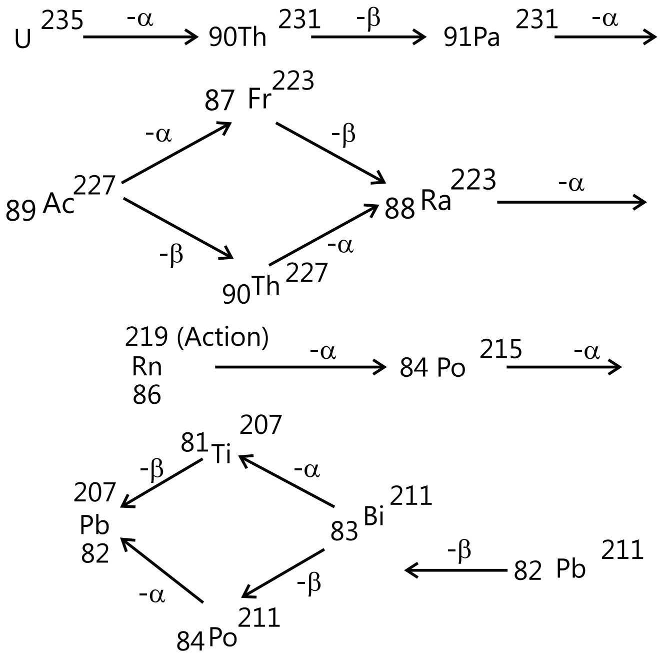


Figure 25.6

PLANCESS CONCEPTS

(a) In all series one element of zero group is present (atomic no=86) in gaseous state which is called emanation.

(b) In all series last product is an isotopes of lead ${}^{208}Pb$, ${}^{206}Pb$, ${}^{207}Pb$ respectively.

Pb is found in nature as a mixture of these three isotopes.

(c) The (4n+1) series (Neptunium series):-

- Except the last member all other members of this series have been obtained by artificial means.
- The series does not contain gaseous emanation.
- The last member of the series is an isotope of Bi and not an isotope of Pb.

Vijay Senapathi (JEE 2011, AIR 71)

6. ELECTRON CAPTURE

Electron capture is a process in which a proton-rich nuclide absorbs an inner atomic electron, thereby changing a nuclear proton to a neutron and simultaneously causing the emission of an electron neutrino. Various photon emissions follow, as the energy of the atom falls to the ground state of the new nuclide.

Electron capture is the primary decay mode for isotopes with a relative superabundance of protons in the nucleus, but with insufficient energy difference between the isotope and its prospective daughter (the isobar with one less positive charge) for the nuclide to decay by emitting a positron. Electron capture is an alternate decay mode for radioactive isotopes with sufficient energy to decay by positron emission. It is sometimes called inverse beta decay, through this term can also refer to the interaction of an electron anti-neutron with a proton.

A free proton cannot normally be changed to a free neutron by this process the proton and neutron must be part of a larger nucleus. In the process of electron capture, one of the orbital electrons, usually from the K or L electron shell (K-electron capture, also K-capture, or L-electron capture, L-capture) is captured by a proton in the nucleus forming a neutron and emitting an electron neutrino.

6.1 Calculation of Number of Alpha and Beta Particles Emitted

Consider the following general reaction. ${}^{m}_{n}X \to {}^{m'}_{n'}Y + a{}^{4}_{2}\alpha + b{}^{0}_{-1}\beta$

Then, $m = m' + 4a + 0b$ (ii) $n = n' + 2a - b$

Solve for a and b

Where a is the number of ${}^{4}_{2}He$ emitted and b is the number of ${}^{0}_{-1}\beta$ emitted ${}^{A}_{Z}X \to {}^{A_1}_{Z_1}Y + x\,{}^{4}_{2}\alpha + y\beta$

x : no of α-particles emitted y : not of β-particles emitted

$X_Z^A \to Y_{Z^1}^{A^1} + xHe_2^4 + ye_{-1}^0$; $A = A^1 + 4x$; $x = \frac{A - A^1}{4}$

$Z = Z^1 + 2x - y$ $y = Z^1 - Z + 2x$ $y = \left(\frac{A - A^1}{2}\right) - \left(Z - Z^1\right)$

eg: $U_{92}^{238} \to Pb_{82}^{206} + xHe_2^4 + ye_{-1}^0$; $x = \left(\frac{A - A^1}{4}\right) = \left(\frac{238 - 206}{4}\right) = 8\alpha -$ particles

$$y = \left(\frac{A - A^1}{2}\right) - \left(Z - Z^1\right) = \left(\frac{238 - 206}{2}\right) - \left(92 - 82\right) = 16 - 10 = 6\beta - \text{particles}$$

7. GROUP DISPLACEMENT LAW

Law is given by Fajan, Soddy and Russel.

The law is basically given for the position of daughter elements in periodic table.

(a) **α-particle emission:** When an alpha particle emits the position of daughter element is two places to the left in the periodic table from parent element.

Example:

$$\begin{array}{ccccccc} {}^{214}_{84}\text{Po} & \rightarrow & {}^{210}_{82}\text{Pb} & \quad & {}^{213}_{83}\text{Bi} & \rightarrow & {}^{209}_{81}\text{Tl} \\ 16 & & 14 & & 15 & & 13 \\ (\text{VIA}) & & (\text{IVA}) & & (\text{VA}) & & (\text{IIIA}) \end{array}$$

(b) **β-particle emission:** When an β-particle emits the position of daughter element is one place right in the periodic table from parent element.

Examples:

$$\begin{array}{ccccccc} {}^{27}_{12}\text{Mg} & \rightarrow & {}^{27}_{13}\text{Al} + {}^{0}_{-1}\text{e} & \quad & {}^{14}_{6}\text{C} & \rightarrow & {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} \\ (\text{IIA}) & & (\text{IIIA}) & & (\text{IV A}) & & (\text{VA}) \end{array}$$

8. RADIOACTIVE ISOTOPES

The isotopes of elements which spontaneously decay by emitting radioactivity radiations are defined as radioactive isotopes.

They are two types.

(a) Natural radioactive isotopes (b) Artificial radioactive isotopes

(b) Natural radioactive isotopes: Those radioactive isotopes which exist naturally are known as natural radioactive isotopes. e.g. Th^{232}, Pu^{240} etc.

(c) Artificial radioactive isotopes: Those isotopes, which are prepared artificially by bombarding fundamental particles like α, β, γ, p, n etc. no matter, are known as artificial isotopes.

8.1 Uses of Radioactive Isotopes

(a) **In Medicine:**

- **(i)** For testing blood chromium-51
- **(ii)** For testing blood circulation-Sodium-24
- **(iii)** For detecting brain tumor-Radio mercury-203
- **(iv)** For detecting fault in thyroid gland-Radio iodine-131
- **(v)** For cancer-Cobalt-60
- **(vi)** For blood-Gold_189
- **(vii)** For skin diseases-Phosphorous-31

(b) **In Archaeology**:

- **(i)** For determining age of archaeological sample (Carbon dating) - C^{14}

- **(ii)** For determining age of meteorites - K^{40}
- **(iii)** For determining age of earth and isotopes

(c) **In Agriculture:**

- **(i)** For protecting potato crop from earthworm-Cobalt-60
- **(ii)** For artificial rains AgI
- **(iii)** As fertilizers-Phosphorous-32

(d) **As Tracers:**

Very small quantity of radio isotopes present in mixture is known as tracer. Tracer technique is used for studying biochemical reactions in trees and animals.

(e) **In Industries:**

- **(i)** For detecting leakage in oil or water pipe lines.
- **(ii)** For testing machine parts.

(f) **In Research:**

- **(i)** In the study of carbon-nitrogen cycle.
- **(ii)** For determining the age of planets.

8.2 Radioactive Dating

Radioactive dating also called carbon dating is used to estimate the age of organic samples. The technique is based on the β-activity of the ratio-isotope C^{14}. $\quad {}^{14}_{6}C \rightarrow {}^{14}_{7}N + \beta^- + \bar{\nu}$

High energy particles from outer space, called cosmic rays, induce nuclear reactions in the upper atmosphere and create carbon-14. The carbon dioxide molecule of the earth's atmosphere has a constant ratio ($\approx 1.3 \times 10^{-12}$) of C^{14} and C^{12} isotope. All living organisms also show the same the same ratio as they continuously exchange CO_2 with their surroundings. However, after its death, an organism can no longer absorb CO_2 and the ratio C^{14}/C^{12} decrease due to the β-decay of C^{14} Thus by measuring the β-activity per unit mass, it is possible to estimate the age of a material.

Using such techniques samples of wood, sample of wood, charcoal, bone, etc., have been identified to have lived from 1000 to 25000 years ago.

9. PROPERTIES AND USES OF NUCLEAR RADIATION

9.1 Alpha Ray

- **(a)** It is a stream of alpha particles, each particle containing two protons and two neutrons. An alpha particle is nothing but a helium nucleus.
- **(b)** Being made of positively charged particles, alpha ray can be deflected by an electric field as well as by a magnetic field.
- **(c)** Its penetrating power is low. Even in air, its intensity falls down to very small values within a few centimeters.
- **(d)** Alpha rays coming from radioactive materials travel at large speeds of the order of $10^6\ ms^{-1}$
- **(e)** All the alpha particles coming from a particular decay scheme have the same energy.

(f) Alpha ray produces scintillation (flashes of light) when it strikes certain fluorescent materials, such as barium platinocyanide.

(g) It causes ionization in gases.

9.2 Beta Ray

(a) It is a stream of electrons coming from the nuclei. Thus, the properties of beta ray, cathode ray, thermions, photoelectrons, etc., are all identical except for their origin. Beta particles are created at the time of nuclear transformation, whereas, in cathode ray, thermions, etc., the electrons are already present and get ejected.

(b) Being made of negatively charged particles, beta ray can be deflected by an electric field as well as by a magnetic field.

(c) Its penetrating power is greater than that of alpha ray. Typically, it can travel several meters in air before its intensity drops to small values.

(d) The ionizing power is less than that of alpha rays.

(e) Beta ray also produces scintillation in fluorescent materials, but the scintillation is weak.

(f) The energy of the beta particles coming from the same decay scheme are not equal. This is because the available energy is shared by antineutrinos. The energy of beta particles thus varies between zero and a maximum.

9.3 Beta-Plus Ray

Beta-plus ray has all the properties of beta ray, except that it is made of positively charged particles.

9.4 Gamma Ray

(a) Gamma ray is an electromagnetic radiation of short wavelength. Its wavelength is, in general, smaller than X-rays. Many of its properties are the same as those of X-rays.

(b) Being charge less, it is not deflected by electric or magnetic field.

(c) It has the least ionizing power and the largest penetrating power among different types of nuclear radiation.

(d) All the photons coming from a particular gamma decay scheme have the same energy.

(e) Being an electromagnetic wave, gamma ray travels in vacuum with the velocity c.

10. NUCLEAR FISSION

It was first observed by German Scientist Otto Hahn and Fritz Strassmann in 1983 in nuclear fission heavy nucleus splits into two smaller nuclei with liberation of energy. When uranium with Z=92 is bombarded with neutrons, it splits into two fragments namely barium (Z=56) and krypton (Z=36) and a large amount of energy is released which appears due to decrease in the mass. The reaction is represented as ${}^{238}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{138}_{56}\text{Ba} + {}^{88}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n} + \text{energy}$

The disintegration process in which heavy nucleus after capturing a neutron splits up into nuclei of nearly equal mass is called nuclear fission.

Energy released in nuclear fission: The amount of energy released in nuclear fission may be obtained by the method of mass defect. For example consider the fission of U^{235} (Z=92) into Ba^{141} (Z=56) and Kr^{92} (Z=36) by slow neutrons. The reaction is given by

$$ {}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \to {}^{236}_{92}\text{U} \to {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + 3\,{}^{1}_{0}\text{n} + \text{Q} $$

Let us estimate the actual masses before and after the fission reactions.

Actual mass before fission reaction

Mass of uranium nucleus	235.124 a. m. u
Mass of neutron	1.009 a. m. u
$\therefore$ Total mass	236.133 a. m. u

Actual mass after the fission reaction

Mass of barium nucleus	140.958 a. m. u
Mass of krypton nucleus	91.926 a. m. u
Mass of three neutrons	3.026 a. m. u
$\therefore$ Total mass	235.910 a. m. u

Now mass decrease during nuclear reaction = 236.133-235.910=0.223 a. m. u.

$\therefore$ Corresponding energy release $=0.223 \times 931 = 200$ MeV

If we calculate the energy produced by one gm of uranium its comes out to be

2.28×10^4 k. w. h. = 22.8 M watt.

This shows that 1 kg of U^{236} would give power of 1 M watt for more than two years.

PLANCESS CONCEPTS

The drooping of the binding energy curve at high mass numbers tells us that nucleons are more tightly bounded when they are assembled into two middle-mass nuclei rather than a single high-mass nucleus. In other words, energy can be released in the nuclear fission.

Shrikant Nagori (JEE 2009, AIR 30)

10.1 Chain Reaction

A nuclear chain reaction occurs when one nuclear reaction causes an average of one or more nuclear reactions, thus leading to a self-propagation series of these reactions. The specific nuclear chain reaction releases several million times more energy per reaction that any chemical reaction.

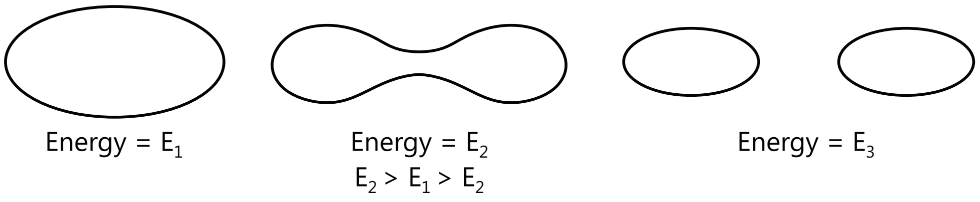


Figure 25.7

10.1.1 Fission Chain Reaction

Fission chain reaction occurs because of interactions between neutrons and fissile isotopes (such as ^{235}U). The chain reaction requires both the release of neutrons from fissile isotopes undergoing nuclear fission and the subsequent absorption of some of these neutrons in nuclear fission, a few

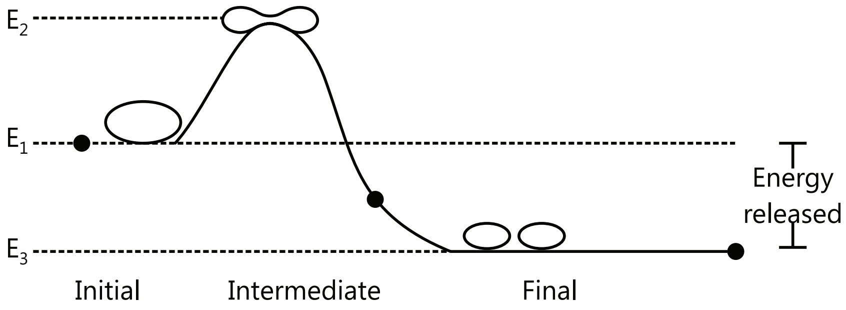


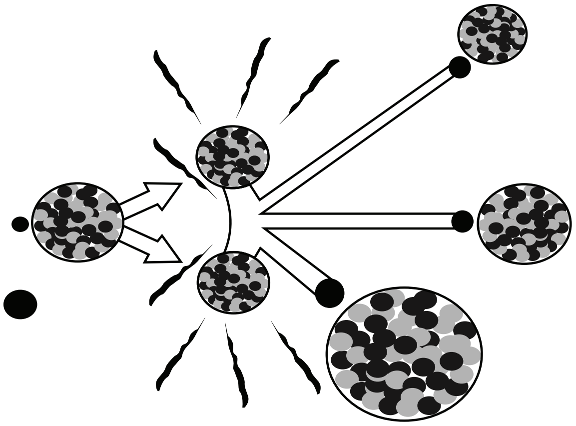
Figure 25.8

neutrons (the exact number depends on several factors) are ejected from the reaction. These free neutrons will then interact with the surrounding medium, and if more fissile fuel is present, some may be absorbed and cause more fission. Thus, the cycle repeats to give a reaction that is self-sustaining.

10.2 Nuclear Reactor

A nuclear reactor is a device to initiate and control a sustained nuclear chain reaction. Nuclear reactors are used at nuclear power plants for electricity generation and in propulsion of ships. Heat from nuclear fission is passed to a working fluid (water or gas), which runs through turbines. These either drive a ship's propellers or turn electrical generators. Nuclear generated steam in principle can be used for industrial process heat or for district heating. Let us see the working of a typical Uranium nuclear reactor. The volume in the core is filled with low-Z material like, D_2O graphite, beryllium etc. This material is called moderator.

Chain reaction



1. A neutron is about to hit the nucleus of a uranium atom.
2. The uranium nucleus splits (fissions) into several smaller atoms, releasing heal and several more neutrons.
3. The chain reaction begins : those neutrons hit other nuclei, casing them to fission. And so on.

Figure 25.9

When fission takes place in a uranium rod, most of the fast neutrons produced escape from the rod and enter into the moderator. These neutrons make collisions with the particles of the moderator and thus slow down. About 25 collisions with deuteron (present in heavy water) or 100 collisions with carbon or beryllium are sufficient to slow down a neutron from 2 MeV to thermal energies. The distances between the rods are adjusted in such a way that a neutron coming from one rod is generally slowed down to thermal energies before entering the other rod. This eliminates the possibility of a neutron being absorbed by U^{238} in 1-100 eV regions. The geometry of the core is such that out of the average 2.5 neutrons produced per fission, 1 neutron is used to trigger the next fission and the remaining are lost without triggering any fission. The reaction is then sustained at a constant rate. If the rate of the loss of neutrons is decreased further, the fission rate will keep on increasing which may lead to explosion. If the rate of loss of neutrons is increased, the rate of fission will keep on decreasing and ultimately the chain reaction will stop. The finer control of fission rate is made by the control rods which are made of cadmium and are inserted up to a certain depth in the moderator. Cadmium is a very good neutron absorber. If the stage is set for stable chain reaction and the cadmium rods are pushed into the moderator, the reactor will be shut off. Pulling the cadmium rods out will start the reactor.

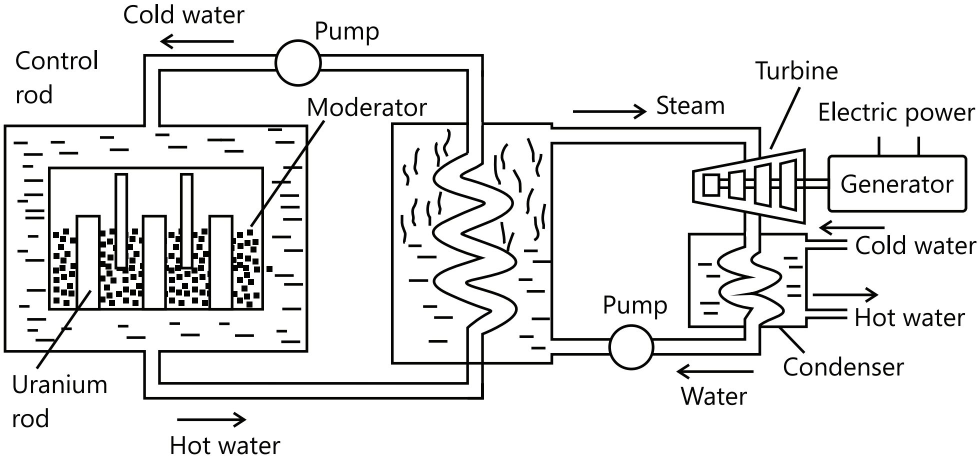


Figure 25.10

10.3 Uranium Fission Reactor

The most attractive bid, from a practical point of view, to achieve energy from nuclear fission is to use ${}^{236}_{92}U$ as the fission material. This nuclide is highly fissionable and hence is not found in nature. Natural uranium contains about 99.3% of ${}^{238}_{92}U$ and 0.7% of ${}^{235}_{92}U$. The technique is to hit a uranium sample by slow-moving neutrons (kinetic energy≈0.04eV, also called thermal neutrons). A ${}^{235}_{92}U$ nucleus has large probability of absorbing a slow neutron and forming ${}^{236}_{92}U$ nucleus. This nucleus then fissions into two parts. A variety of combinations of the middle-weight nuclei may be formed due to the fission. For example, one may have ${}^{236}_{92}U \to {}^{137}_{53}I + {}^{97}_{39}Y + 2n$,

And a number of the other combination ${}^{236}_{92}U \to {}^{140}_{56}Ba + {}^{94}_{36}Kr + 2n$

10.4 Breeder Reactors

Although fission generates large amount of energy and the world is heavily depending on fission for its energy requirement, uranium resources are also limited. The following Table shows that fission can easily take place with ${}^{240}Pu$ besides ${}^{236}U$. But ${}^{239}Pu$ is not a naturally occurring isotope. However, ${}^{238}U$ can capture a neutron to produce ${}^{239}Pu$ which can be used as a nuclear fuel.

$${}^{238}_{92}U + n \to {}^{239}_{92}U \xrightarrow{\beta} {}^{239}_{93}Np \xrightarrow{\beta} {}^{239}_{94}Pu$$

Suppose, used uranium rods, which contain only ${}^{238}U$, are kept in or around a uranium-reactor core. Also suppose, the geometry is such that out of the average 2.5 neutrons produced in fission, one neutron is absorbed by a ${}^{238}U$ nucleus in these rods resulting in ${}^{239}Pu$. Then we produce as much nucleus in these rods resulting in ${}^{239}Pu$. Then we produce as much nuclear fuel in the form of ${}^{239}Pu$ as we consume in the form of ${}^{235}U$. If more than one neutron can be absorbed by these ${}^{238}U$ rods per fission then we produce more fuel than what we consume. Thus, apart from nuclear energy, these reactors give us fresh nuclear fuel which often exceeds the nuclear fuel used. Such a reactor is called a breeder reactor.

11. NUCLEAR FUSION

Binding energy vs. Mass Number a graph shows that when lighter nuclei with A<20 combine to form bigger nuclei binding energy per nucleon increases. The total binding energy of the product is less than total binding energy of reactants resulting in release of energy. This process of combining of two lighter nuclei into bigger one is known as nuclear fusion. i.e.

$${}^{2}_{1}H + {}^{2}_{1}H \to {}^{4}_{2}He + 24\,MeV$$

The following points deserve particular attention concerning nuclear fusion.

(a) The energy 21.62 MeV released in one fusion event is much smaller than about 200 MeV released in one fission event. This does not mean that fusion is a weaker source of energy than fission. If we compare the energy released per unit mass, we find that one fusion event is accompanied by a release of $\frac{21.62\text{ MeV}}{6\,\text{amu}}$ or 3.6 MeV Per amu as against $\frac{200\text{ MeV}}{235}$ or 0.85 MeV per amu released in one fission.

(b) For fusion, positively charged nuclei have to come very close to each other. This requires a very high energy to be provided to the fusing nuclei to enable them to overcome the strong electrostatic repulsion between them. Calculations show that the necessary energy can be provided by raising the temperature to about 10^8

K. Such high temperature can be produced by first inducing a fission event. A fusion reaction is therefore also called a thermonuclear reaction. This is the basic hydrogen bomb.

(c) Unlike the highly radioactive fission fragments, the end product of the fusion of hydrogen nuclei is safe, non-radioactive helium.

(d) Unfortunately a sustained and controllable fusion reactor that can deliver a net power output is not yet a reality. A great deal of effort is currently under way to resolve various difficulties in the development of a successful device. Nevertheless controlled fusion is regarded as the ultimate energy source because of the abundant availability of its main fuel: water.

PLANCESS CONCEPTS

The drooping of the binding energy curve at low mass number tells us that energy will be released if two nuclei of small mass numbers combine to form a single middle-mass nucleus. This is nuclear fusion.

Ankit Rathore (JEE Advanced 2013, AIR 158)

Illustration 11: In the nuclear fusion reaction: ${}_1^2\text{H} + {}_2^4\text{He} \Rightarrow 2\,{}_1^3\text{H}$ in a nuclear reactor, of 200 MW rating. If the energy from above reaction is used with a 25% efficiency in the reactor, how many grams of deuterium will be needed per day? (The masses of ${}_1^3\text{H}$ and ${}_2^4\text{He}$ are 2.0141 and 4.0026 amu respectively) **(JEE ADVANCED)**

Sol: The energy absorbed during the nuclear fusion reaction is calculated using Q value equation i.e. $Q = -mc^2 = -m \times (931)\ \text{MeV}$. The number of deuterium atoms required during this reaction is obtained by

$$N = \frac{\text{Power Required}}{\text{Efficiency} \times (\text{energy released from one fusion reaction})}.$$

Let us first calculate the Q value of nuclear function, $Q = -mc^2 = -m \times (931)\ \text{MeV}$

$Q = (2 \times 2.0141 - 4.0026) \times 931\,\text{MeV} = 23.834\,\text{MeV} = 23.834 \times 10^6\ \text{eV}$. Now efficiency of reactor is 25%

So effective energy used $= 0.25 \times 23.834 \times 10^6 \times 1.6 \times 10^{-19}\,\text{J} = 9.534 \times 10^{-13}\,\text{J}$

Now $9.534 \times 10^{-13}\,\text{J}$ energy is released by fusion of 2 deuterium.

Requirement is $200\,\text{MW} = 200 \times 10^6\ \text{J/s}$ per second$= 200 \times 10^6 \times 86400\,\text{J/s}$ for 1 days.

No. of deuterium nuclei required$= \dfrac{200 \times 10^6 \times 86400}{\dfrac{9.534}{2} \times 10^{-13}} = 3.624 \times 10^{25}$

Number of deuterium nuclei $= \dfrac{g}{2} \times 6 \times 10^{23}$; $\quad 3.624 \times 10^{25} = \dfrac{g}{2} \times 6 \times 10^{23}$

$$g = \frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83\ \text{gm/day}.$$

11.1 Thermonuclear Fusion

To generate useful amount of energy, nuclear fusion must occur in bulk matter. The best hope for bringing this about is to raise the temperature of the material until the particles have enough energy-due to their thermal motions alone-to penetrate the Coulomb barrier. We call this process thermonuclear fusion.

In thermonuclear studies, temperatures are reported in terms of the kinetic energy K of interacting particles via the relation K=3kT/2

In which K is the average kinetic energy of the interacting particles, k is the Boltzmann constant, and the temperature T is in kelvins. Thus, rather than saying,

Fusion in Sun: Among the celestial bodies in which energy is produced, the sun is relatively cooler. There are stars with temperature around 10^8K inside. In sun and other stars, where the temperature is less than or around 10^7K, fusion takes place dominantly by proton-proton cycle as follows:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$$

$$^2H + {}^1H \rightarrow {}^3He + \gamma$$

$$^3He + {}^3He \rightarrow {}^4He + 2\,{}^1H$$

$$4\,{}^1H \rightarrow {}^4He + 2e^+ + 2\nu + 2\gamma$$

Note that the first two reactions should occur twice to produce two ^3He nuclei and initiate the third reaction. As a result of this cycle, effectively, four hydrogen nuclei combine to form a helium nucleus. About 26.7 MeV energy is released in the cycle. Thus, hydrogen is fuel which 'burns' into helium to release energy. The sun is estimated to have been radiating energy for the last 4.5×10^9 years and will continue to do so till all the hydrogen in it is used up. It is estimated that the present store of hydrogen in the sun is sufficient for the next 5×10^9 years.

11.2 Lawson Criterion

J. D. Lawson showed that in order to get an energy output greater than the energy input, a fusion reactor should achieve $n\tau > 10^{14}\,s\,cm^{-3}$

Where n is the density of the interacting particles and τ is the confinement time. The quantity $n\tau$ in $s\,cm^{-3}$ is called Lawson number.

The ratio of the energy output to the energy input is known as Q of the fusion machine. For a viable fusion machine, Q should be greater than 1.

11.3 Tokamak Design

In one of the method receiving serious attention, one uses the so-called Tokamak design. The deuterium plasma is contained in a toroidal region by specially designed magnetic field. The directions and magnitudes of the magnetic field are so managed in the toroidal space that whenever a charged plasma particle attempts to go out, the $q\vec{v}\times\vec{B}$ force tends to push it back into the toroidal volume. It is a difficult task to design a magnetic field which will push the particles moving in random directions with random speeds into a specified volume, but it is possible and has been done. The plasma is, therefore, confined by the magnetic field. Such confinement has been achieved for short durations (≈few microseconds) in which some fusion occur. Fusion thus proceeds in bursts or pulse. The heating is accomplished by passing high frequency oscillating current through the plasma gas. A schematic design is shown in Fig. 25.11.

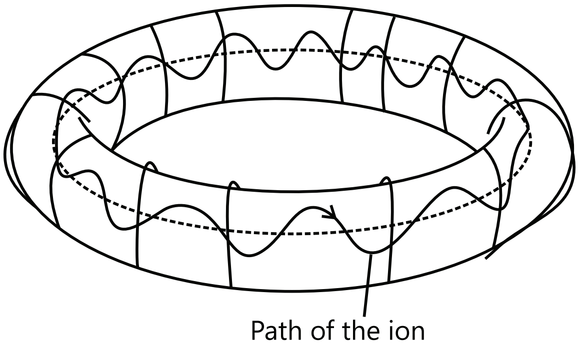


Figure 25.11

11.4 Inertial Confinement

In another method known as inertial confinement, laser beams are used to confine the plasma. A small solid pellet is made which contains deuterium and tritium. Intense laser beams are directed on the pellet from many directions distributed over all sides. The laser first vaporizes the pellet converting it into plasma and then compresses it from all directions because of the large pressure exerted. The density increases by 10^3 to 10^4 times the initial density and the temperature rises to high values. The fusion occurs in this period. The α-particles (He Nuclei) generated by the fusion are also forced to remain inside the plasma. Their kinetic energy is lost into the plasma itself contributing further rise in temperature. Again the lasers are operated in pulses of short duration.

The research in fusion energy is going on. Fusion is the definite and ultimate answer to our energy problems. The 'fuel' used for fusion on earth is deuterium which is available in natural water (0.03%). And with oceans as the almost unlimited source of water, we can be sure of fuel supply for thousands of years. Secondly, fusion reactions are neat and clean. Radioactive radiation accompanying fission reactors will not be there with fusion reactors.

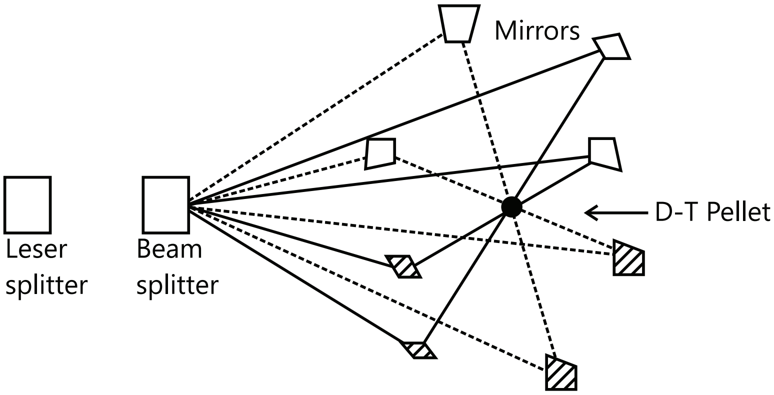


Figure 25.12

11.5 Nuclear Holocaust

Nuclear holocaust refers to a possible nearly complete annihilation of human civilization by nuclear warfare. Under such a scenario, all or most of the Earth is made uninhabitable by nuclear weapons in future word wars.

PROBLEM-SOLVING TACTICS

1. Problems from this section do not need any mathematically difficult involvement. One only needs to focus on exponential functions and its properties.

2. Questions related to energy can easily be solved by thinking.

3. For e.g. consider energy as money and think of it in terms of loss and gain, But overall total money is conserved i.e. total energy is conserved; only it is exchanged. One must not be worried with the relation $E = mc^2$ at this stage and just consider mass and energy as equivalent. So, if more clearly stated this equivalent quantity is conserved in every process.

4. Mostly, questions related to basic understanding of Nuclear force are asked rather than which involve complicated calculations.

5. Statistics must always be kept in mind while solving a problem of radioactive decay.

FORMULAE SHEET

1. After n half-lives

(a) Number of nuclei left $= N_0\left(\frac{1}{2}\right)^n$

(b) Fraction of nuclei left $= \left(\frac{1}{2}\right)^n$ and

(c) Percentage of nuclei left $= 100\left(\frac{1}{2}\right)^2$

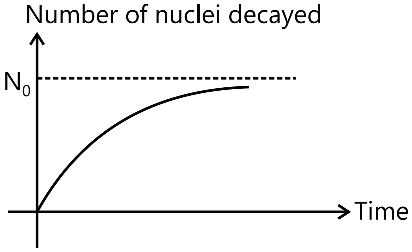


Figure 25.13

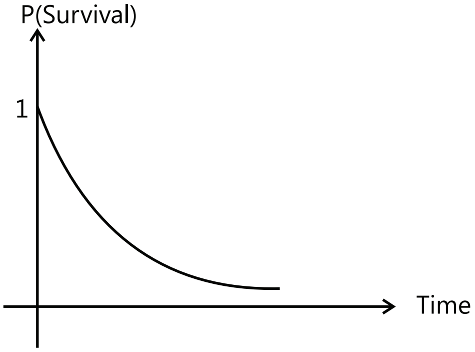


Figure 25.14

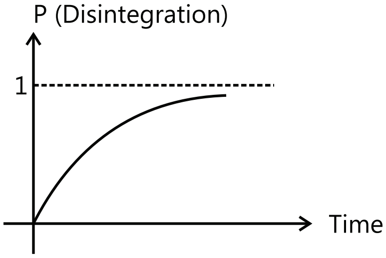


Figure 25.15

2. Number of nuclei decayed after time $t = N_0 - N$

$= N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t})$

The corresponding graph is as shown in Fig. 25.13.

3. Probability of a nucleus for survival of time t,

$$\text{P(survival)} = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

The corresponding graph is shown in Fig. 25.14.

4. Probability of a nucleus to disintegrate in time t is,

$$\text{P(disintergration)} = 1 - \text{P(survival)} = 1 - e^{-\lambda t}$$

The corresponding graph is as shown.

5. Half-life and mean life are related to each other by the relation, $t_{1/2} = 0.693\,t_{av}$ or $t_{av} = 1.44\,t_{1/2}$

6. As we said in point number (2), number of nuclei decayed in time t are $N_0(1 - e^{-\lambda t})$. This expression involves power of e. So to avoid it we can use, $\Delta N = \lambda N \Delta t$ where, ΔN are the number of nuclei decayed in time Δt, at the instant when total number of nuclei are N. But this can be applied only when $\Delta t << t_{1/2}$.

7. In same interval of time, equal percentage (or fraction) of nuclei are decayed (or left un decayed).

1. $R = R_0 A^{1/3}$
2. $\Delta E_{be} = \sum\left(mc^2\right) - Mc^2$ (binding energy)
3. $\Delta E_{ben} = \frac{\Delta E_{be}}{A}$ (binding energy per nucleon.)
4. $\frac{dN}{N} = -\lambda dt$
5. $N = N_0 e^{-\lambda t}$ (radioactive delay),
6. $\tau = \frac{1}{\lambda}$
7. $T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2.$

Solved Examples

JEE Main/Boards

Example 1: Sun radiates energy in all direction. The average energy received at earth is 1.4 kW/m^2. The average distance between the earth and the sun is 1.5×10^{11} m. If this energy is released by conversion of mass into energy, then the mass lost per day by sun is approximately (use 1 day = 86400 sec)

Sol: The sun produces energy by fusion reaction of hydrogen atoms. The loss in mass of sun is calculated using $\Delta m = \frac{\Delta E}{c^2}$ where ΔE is the amount of energy released during the day.

The sun radiates energy in all directions in a sphere. At a distance R, the energy received per unit area per second is 1.4 KJ (given). Therefore the energy released in area $4\pi R^2$ per sec is $1400 \times 4\pi R^2$ J the energy released per day = $1400 \times 4\pi R^2 \times 86400$ J

Where $R = 1.5 \times 10^{11}$m, thus

$$\Delta E = 1400 \times 4 \times 3.14 \times (1.5 \times 10^{11})^2 \times 86400$$