Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-MAINS PAPER-4

Max.Marks:360

IMPORTANT INSTRUCTIONS:

1) Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2) The test is of 3 hours duration.
3) The Test Booklet consists of 90 questions. The maximum marks are 360.
4) There are three parts in the question paper A, B, C consisting of **Chemistry, Physics** and **Mathematics** having 30 questions in each part of equal weight age. Each question is allotted 4 (four) marks for correct response.
5) Candidates will be awarded marks as stated above in instruction No. 4 for correct response of each question. (1/4) (One fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6) There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

SYLLABUS

CHEMISTRY:

STOICHIOMETRY-I (8×2=16): Mole, significant figures, laws of chemical combination, Chemical calculations based upon weight, volume relations of chemical equations, percentage composition of mixtures, empirical and molecular formula, Concept of redox reactions - oxidation number - Types of redox reactions, Balancing Redox reactions, Equivalent weight, (70%)

De-Broglie Equation, Heisenberg's Uncertainity principle, Schrodinger's wave equation; Quantum numbers, Pauli's exclusion Principle; Hund's rule; Electronic configuration of the elements, (30%)

PHYSICS:

Lenses and combination of lenses + experiments involving concave mirror, convex lens, concave lens (including optical bench) (Reference: JEE mains syllabus) (50%) (Exclude problems involving relative motion and differential and integral calculus)

Refraction at plane surface, TIR and Prism, Refraction at curved surface (50%) (Exclude problems involving relative motion and differential and integral calculus)

MATHS:

Quadratic equation (30%); Properties of AP,AM,GP,GM; Sum of ‘n’ Terms of AP & GP, Properties of HP & HM; Sum of infinite GP & AGP, method of differences/Vn method (70%)

CHEMISTRY

1. A, B and C are three elements forming a compound in oxidation states of +2,+5 and -2 respectively. What could be the compound?

   1) $A_2(BC)_2$ 2) $A_2(BC_4)_3$

   3) $A_3(BC_4)_2$ 4) ABC

2. For the redox reaction:

   $MnO_4^- + C_2O_4^{2-} + H^+ \rightarrow Mn^{2+} + CO_2 + H_2O$,

   the correct coefficients of the reactants for the balanced reaction are

	MnO_4^-	$C_2O_4^{2-}$	H^+
1)	2	5	16
2)	16	5	2
3)	5	16	2
4)	2	16	5

3. $Ba(MnO_4)_2$ can oxidise FeC_2O_4 into Fe^{+3} ions and CO_2 in acid medium. What is the equivalent weight of FeC_2O_4 is

   1) M 2) $\frac{M}{4}$

   3) $\frac{M}{2}$ 4) $\frac{M}{3}$

4. Calculate n-factor of the reductant in the following reaction.

   $Fe_3O_4 + KMnO_4 \rightarrow Fe_2O_3 + MnO_2$

   1)1 2)8/3

   3)2 4)3

5. 4 grams of hydrocarbon (C_xH_y) on complete combustion gave 12 grams of CO_2. What is the empirical formula of the hydrocarbon? (C = 12, H = 1)

   1) C_3H_8 2) CH_3

   3) C_4H_9 4) CH

6. How many moles of magnesium phosphate $Mg_3(PO_4)_2$ will contain 0.25mole of oxygen atoms?

   1) 0.02 2) 3.125×10^{-2}

   3) 1.25×10^{-2} 4) 2.5×10^{-2}

7. The law of multiple proportions is illustrated by the pair of compounds:

   1) sodium chloride and sodium bromide

   2) water and heavy water

   3) sulphur dioxide and sulphur trioxide

   4) magnesium hydroxide and magnesium oxide

8. Suppose X and Y combine to form two compounds XY_2 and X_3Y_2. When 0.1 mole of XY_2 weighs 10g and 0.05mole of X_3Y_2 weighs 9g, the atomic weights of X and Y are, respectively

1) 40, 30 2) 60, 40

3) 20, 30 4) 30,20

9. In one type of breath analyser used to detect alcohol in a driver's breath, an orange solution of potassium dichromate ($K_2Cr_2O_7$) in dilute sulfuric acid reacts with alcohol (if present) to form green $Cr_2(SO_4)_3$, and the intensity of the green colour is measured. In this reaction, what is the change in the oxidation number of each chromium atom?

1) increases by 3

2) decreases by 6

3) increases by 6

4) decreases by 3

10. Two oxides of metal 'M' contain 27.6% and 30.0% of oxygen respectively. If the formula of the first oxide is M_3O_4, find that of the second.

1) M_2O_3 2) M_2O

3) MO_2 4) M_3O_2

11. Oxidation state of Cr in CrO_5 is

1) +6 2) +8

3) +10 4) +4

12. A reaction mixture is prepared containing 0.6mol of aluminum and 1.2mol of manganese dioxide. The mixture is heated until one of the reactants has been completely consumed according to the equation. $2Al + 3MnO_2 \rightarrow 3Mn + Al_2O_3$.

What is the amount of the left over reagent?

1) 0.20 mol Al 2) 0.40 mol Al

3) 0.30 mol MnO_2 4) 0.60mol MnO_2

13. A mixed oxide has the formula X_7O_8. If element X exists as both X^{+2} and X^{+3} in the compound, what is the ratio of X^{+2}/X^{+3}?

1) 0.438 2) 0.875

3) 1.14 4) 2.5

14. When an object is silver plated, cyanide ions are added to the electrolyte to keep the silver ions in solution as soluble silver cyanide complexes. The unbalanced oxidation and reduction half-reactions are given below:

Oxidation:

$CN^-_{(aq)} + Ag_{(s)} \rightarrow [Ag(CN)_2]^-_{(aq)}$

Reduction: $O_{2\,(g)} + 4H^+_{(aq)} \rightarrow 2H_2O_{(l)}$

In acidic solution, the balanced chemical reaction is:

1) $2CN^-_{(aq)} + Ag_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow [Ag(CN)_2]^-_{(aq)} + 2H_2O_{(l)}$

2) $4CN^-_{(aq)} + 2Ag_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 2[Ag(CN)_2]^-_{(aq)} + 2H_2O_{(l)}$

3) $8CN^-_{(aq)} + 4Ag_{(s)} + O_{2(g)} + 4H^+_{(aq)} \rightarrow 4[Ag(CN)_2]^-_{(aq)} + 2H_2O_{(l)}$

4) $6CN^-_{(aq)} + 3Ag_{(s)} + \frac{1}{2}O_{2(g)} + 2H^+_{(aq)} \rightarrow 3[Ag(CN)_2]^-_{(aq)} + H_2O_{(l)}$

15. 20.0 g of magnesium carbonate sample decomposes on heating to give carbon dioxide and 8.0g of magnesium oxide. What will be the percentage purity of magnesium carbonate in the sample?

1) 60 2) 84

3) 75 4) 96

16. An unused flashbulb contains magnesium and oxygen. After use, the contents are changed to magnesium oxide but the total mass does not change. This observation can best be explained by the

1) Law of Constant Composition.

2) Law of Multiple Proportions.

3) Avogadro's Law.

4) Law of Conservation of Mass.

17. Balance the following equation with the smallest whole number coefficients. Choose the answer that is the sum of the coefficients in the balanced equation.

$PtCl_4 + XeF_2 \rightarrow PtF_6 + ClF + Xe$

1) 16 2) 22

3) 24 4) 26

18. Which of the following is not an example of disproportionation reaction?

1) $\left[Ag(NH_3)_2\right]^+ + 2H^+ \rightarrow Ag^+ + 2NH_4^+$

2) $Cl_2 + OH^- \rightarrow ClO^- + Cl^- + H_2O$

3) $Cu_2O + 2H^+ \rightarrow Cu + Cu^{2+} + H_2O$

4) $2HCuCl_2 \rightarrow Cu + Cu^{+2} + 4Cl^- + 2H^+$

19. An aqueous solution contains 0.10 mol of NaI and 0.10 mol of CaI_2. What is the minimum number of moles of $Pb(NO_3)_2$ that must be added to the solution in order to precipitate all of the I^- as $PbI_{2(s)}$?

1) 0.3 mol 2) 0.2 mol
3) 0.15 mol 4) 0.4mol

20. Balance the half-reaction, $C_8H_{10} \rightarrow C_8H_4O_4^{2-}$, taking place in basic media. How many electrons are needed to balance the half reaction?

1) 4 electrons, left side
2) 8 electrons, right side
3) 8 electrons, left side
4) 12 electrons,right side

21. A gaseous anesthetic with an unknown molecular formula is 85.63% carbon and 14.37% hydrogen by mass. What is the molecular formula of the unknown if 0.45 L of the compound combusts with excess oxygen at 120 ^{O}C at 72.93 kPa to form 2.70 L of an equimolar mixture of carbon dioxide and water vapour?

1) C_3H_6 2) C_4H_8
3) C_5H_{10} 4) C_6H_{12}

22. For an e^- in a hydrogen atom, the wave function ψ is proportional to $e^{\frac{-r}{a_0}}$ where a_0 as Bohr's radius; what is the ratio of probability density of e^- at the nucleus to the probability density at a_0, the wave function is $\psi = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$

1) e 2) e^2
3) $1/e^2$ 4) Zero

23. Given the following sets of quantum numbers, which one of these sets is not a possible set for an electron in an atom for n,l, m and s respectively?

1). 3 1 -1 0 2) 3 2 2 -½
3) 4 3 2 ½ 4) 4 3 -2 -½

24. The wave functions which are solutions to the Schrodinger's wave equation which describes the behavior of the electron in the hydrogen atom are described by how many quantum numbers?

1) 1 2) 2

3) 3 4) 4

25. The correct order of spin-only magnetic moment of the following is

1) $Mn^{+2} > V^{+2} > Cr^{+2}$

2) $V^{+2} > Cr^{+2} > Mn^{+2}$

3) $Mn^{+2} > Cr^{+2} > V^{+2}$

4) $Cr^{+2} > V^{+2} > Mn^{+2}$

26. If the nitrogen atom had electronic configuration $1s^7$, it would have energy lower than that of the normal ground state configuration $1s^2\ 2s^2\ 2p^3$, because the electrons would be closer to the nucleus, yet $1s^7$ is not observed because it violates:

1) Heisenberg uncertainity principle

2) Hund's rule

3) Pauli's exclusion principle

4) Bohr's postulate of stationary orbit

27. In an atom, an electron is moving with a speed of 600 m/s with an accuracy of 0.005%. Certainity with which the position of the electron can be located is ($h = 6.6 \times 10^{-34}$ kg $m^2 s^{-1}$, mass of electron, $e_m = 9.1 \times 10^{-31}$ kg)

1) 5.10×10^{-3}m

2) 1.92×10^{-3}m

3) 3.84×10^{-3}m

4) 1.52×10^{-4}m

28. The maximum number of electrons that can have principal quantum number n=3 and m = 0

1) 2 2) 4

3) 6 4) 8

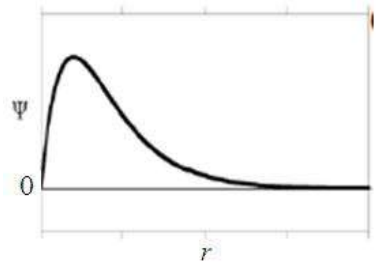
29. If the radius of first orbit of H atom is a_0 the deBroglie wavelength of electron in 4^{th} orbit is

1) $8\pi a_0$ 2) $\frac{a_0}{4}$

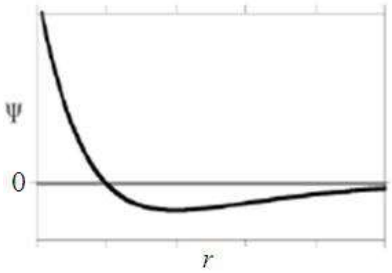
3) $16a_0$ 4) $2\pi a_0$

30. Which graph best describes the radial wavefunction of a 2p orbital?

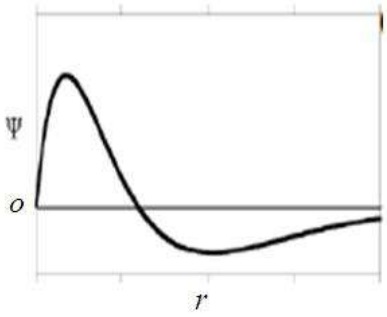
1)



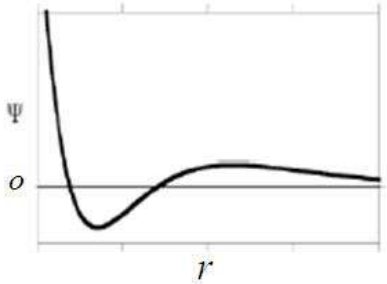
2)



3)



4)



PHYSICS

31. An object is placed in front of a slab $(\mu = 1.5)$ of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image

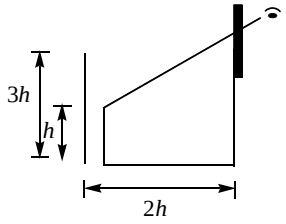
1) 30cm behind the silvered surface
2) 30cm in front of the silvered surface
3) 32 cm behind the silvered surface
4) 32 cm in front of the silvered surface

32. A prism has refracting angle equal to $\pi/2$. It is given that γ is the angle of minimum deviation and β is the deviation of the ray entering at grazing incidence.

1) $\cos\gamma = \cos^2\beta$
2) $\cos\gamma = \sin^2\beta$
3) $\sin\gamma = \sin^2\beta$
4) $\sin\gamma = \cos^2\beta$

33. A observer can see through a pin hole, the top end of a thin rod of height h, placed as shown in figure. The beaker's height is 3h and radius is h. When the beaker is filled with a liquid up to a height 2h, he can see the lower end of the

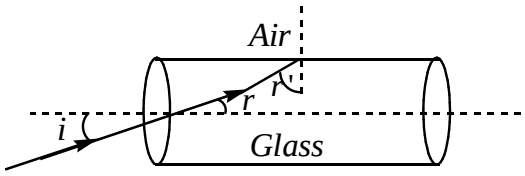
rod. Find the refractive index of the liquid.



1) $\sqrt{\frac{3}{2}}$ 2) $\sqrt{\frac{5}{2}}$

3) $\sqrt{\frac{9}{2}}$ 4) $\sqrt{\frac{1}{2}}$

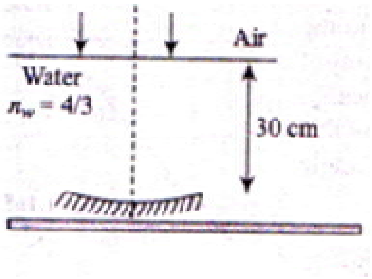
34. What should be the value of refractive index μ of a glass rod placed in air, So that the light entering through the flat surface of the rod does not cross the curved surface of the rod?



1) $\mu > \sqrt{3}$ 2) $\mu > \sqrt{5}$

3) $\mu > \sqrt{2}$ 4) $\mu > \sqrt{7}$

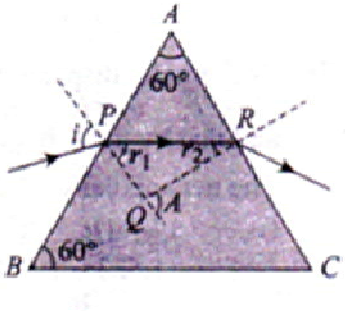
35. A concave mirror of focal length 20 cm is placed inside water with its shining surface upwards and principal axis vertical as shown in Fig. Rays are incident parallel to the principle axis of concave mirror. Find the position of final image



1) 7.5 cm 2) 5 cm

3) 6 cm 4) 9 cm

36. A ray of light undergoes a deviation of 30^0 when incident on an equilateral prism of refractive index $\sqrt{2}$.What is the angle subtended by the ray inside the prism with the base of the prism?

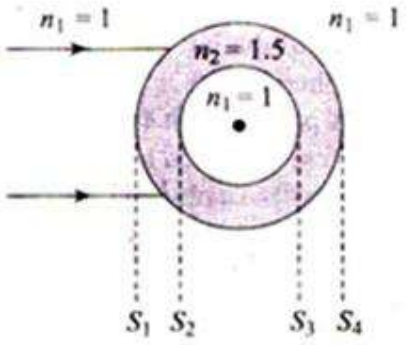


1) 30^0 2) 60^0

3) 0^0 4) 45^0

37. A glass sphere, refractive index 1.5 and radius 10 cm, has a spherical cavity of

radius 5 cm concentric with it. A narrow beam of parallel light is directed into the sphere. Locate the final image.



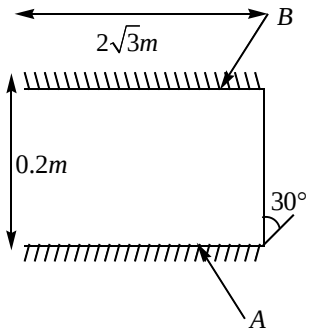
1) 25 cm right of the vertex of surface S_4

2) 25 cm left of the vertex of surface S_4

3) 30 cm right of vertex of surface S_4

4) 30 cm left of vertex of surface S_4

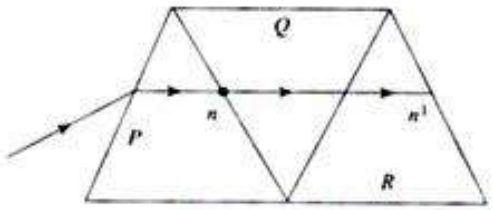
38. Two plane mirrors A and B are aligned parallel to each other, as shown in Fig. A light ray is incident at an angle of 30^0 at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflection (including the first one) before it emerges out is



1) 28 2) 30

3) 32 4) 34

39. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prism Q and R of identical shape and of the same material as P are now added as shown Fig. The ray will now suffer



1) Greater deviation
2) No deviation
3) Same deviation as before
4) Total internal reflection

40. A cylindrical glass rod of radius 0.1m and refractive index $\sqrt{3}$ lies on a horizontal plane mirror. A horizontal ray of light moving perpendicular to the axis of the rod is incident on it. At what height from the mirror should the ray be incident so that leaves the rod at a height of 0.1 m above the plane mirror?

1) 0.176 m 2) 0.186 m

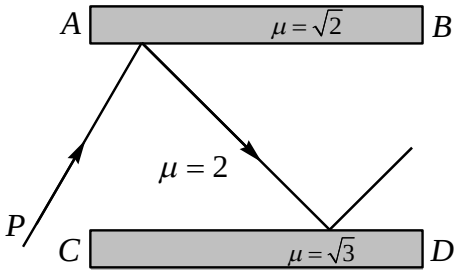
3) 0.156 m 4) 0.166 m

41. The bottom of a tub has a black spot. A glass slab of thickness 4.5 cm is placed over it and then water is filled to the height of 8 cm above the glass slab. Looking from top, find the effective refractive index of the combination of glass slab and water layer. (Refractive index of glass is $\frac{3}{2}$ and of water is $\frac{4}{3}$).

1) 1.39 2) 1.52

3) 1.45 4) 1.71

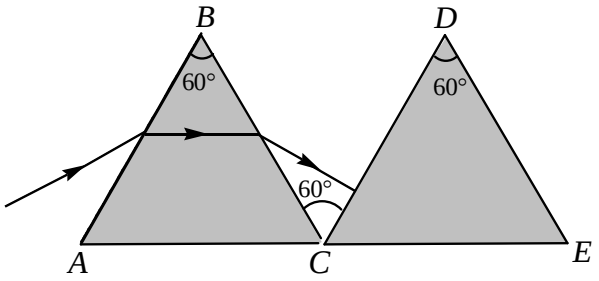
42. AB and CD are two slabs. The medium between the slabs has refractive index 2. Find the minimum angle of incidence at Q, so that the ray is totally reflected by both the slabs.



1) 60^0 2) 30^0

3) 45^0 4) 33^0

43. A ray of light is incident on a prism ABC of refractive index $\sqrt{3}$ as shown in figure.



Find the angle of incidence for which the deviation of light ray by the prism ABC is minimum.

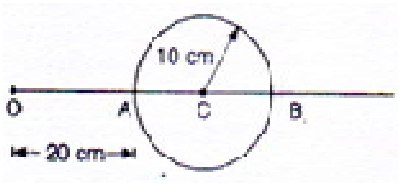
1) 30^0 2) 60^0

3) 90 4) 20^0

44. There are two objects O_1 and O_2 at an identical distances of 20 cm on the two sides of the pole (p) of a spherical concave refracting boundary of radius 60 cm. The indices of refraction of the media on two sides of the boundary are 1 and $\left(\frac{4}{3}\right)$ respectively. Find the location of the object O_1 when seen from O_2

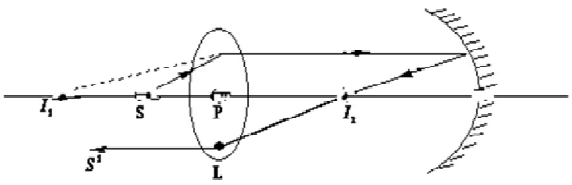
1) 24 cm from p towards c
2) 30 cm from p towards c
3) 36 cm from p towards c
4) 32 cm from p towards c

45. A glass sphere of radius R=10 cm having refractive index $\mu_g = \frac{3}{2}$ is kept inside water. A point object O is placed at 20 cm from A as shown in figure. Find the position and nature of the image when seen from other side of the sphere. [Refractive index of water is $\mu_w = \frac{4}{3}$.]



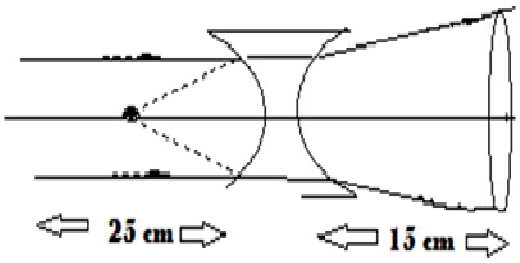
1) 100 cm (towards left)
2) 50 cm (towards left)
3) 100 cm (towards right)
4) 50 cm (towards right)

46. A converging lens of focal length 15cm and a converging mirror of focal length 15 cm and a converging mirror of focal length 20cm are placed with their principal axes coinciding. A point source S is placed on the principal axis at a distance of 12cm from the lens as shown in figure. It is found that final beam comes out parallel to the principal axis. The separation between the mirror and the lens is



1) 50cm
2) 30cm
3) 25cm
4) 40cm

47. A diverging lens with magnitude of focal length 25cm is placed at a distance of 15cm from a converging lens of focal length 20cm. A beam of parallel light falls on the diverging lens. The final image formed is



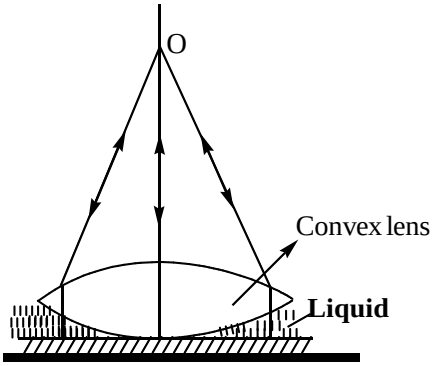
1) Real and at a distance of 40cm from convergent lens

2) Virtual and at a distance of 40cm from convergent lens

3) Real and at a distance of 40cm from divergent

4) Real and at a distance of 6cm from the convergent lens

48. On a horizontal plane mirror a thin equiconvex lens of glass is placed and when the space between the lens and mirror is filled with a liquid and an object held at a distance 30cm vertically above the lens is found to coincide with its own image as shown in the figure. If equiconvex lens of glass has index $\mu = 1.5$ and radius of curvature is 20cm then find Refractive index of the liquid

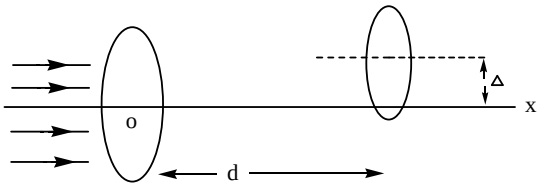


1) $\frac{4}{3}$ 2) $\frac{3}{2}$

3) $\frac{1}{2}$ 4) $\frac{5}{3}$

49. Two thin convex lenses of focal length f_1 and f_2 are separated by a horizontal distance d [where $d < f_1$ and $d < f_2$] and their centers are displaced by a vertical separation Δ as shown. Taking the origin of coordinates O, at the centre of the first lens, the x and y coordinates of the focal point of their lens system, for a parallel

beam of rays coming from the left, are given by



1) $x = \frac{f_1 f_2}{f_1 + f_2}, y = \Delta$

2) $x = \frac{f_1[f_2 + d]}{f_1 + f_2 - d}, y = \frac{\Delta^2}{f_1 + f_2}$

3) $x = \frac{f_1 f_2 + d[f_1 - d]}{f_1 + f_2 - d}, y = \frac{\Delta[f_1 - d]}{f_1 + f_2 - d}$

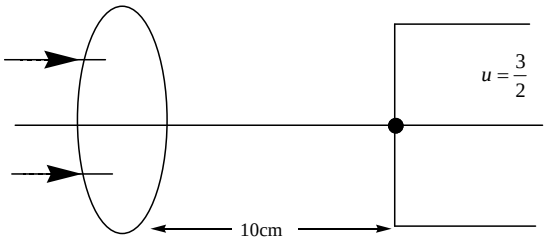
4) $x = \frac{f_1 f_2 + d[f_1 - d]}{f_1 + f_2 - d}, y = O$

50. A plane faces of two identical Plano convex lenses each having focal length of 40cm is pressed against each other to form a usual convex lens. The distance in cm from this lens at which an object must be placed to obtain a real image with magnification unity is

1) 10 2) 20

3) 40 4) 80

51. A plane refracting surface of refractive index $3/2$ is placed at a distance of 10 cm from a thin convex lens of focal length 30cm. The parallel rays incident on lens will converge at a distance of



1) 30cm from the lens

2) 25cm from the lens

3) 20cm from the lens

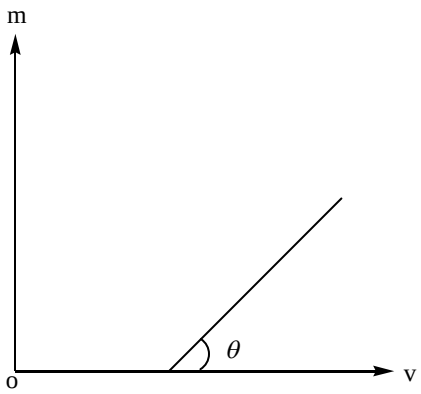
4) 40cm from the lens

52. The diameter of a aperture of Plano convex lens is 6 cm and its maximum thickness is 3mm. If the velocity of light in material of lens is $2\times10^8\ m/s$. Focal length of the lens is

1) 20cm 2) 30cm

3) 15cm 4) 40cm

53. Variation of magnification m produced by a thin convex lens verses distance (V) of image from pole of the lens shown in the graph which of the following statement is not correct?



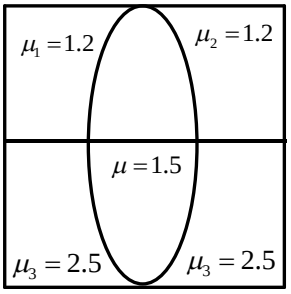
1) Focal length of lens is equal to intercept on V-axis

2) Focal length of the lens is equal to inverse of the slope of the line

3) Magnitude of intercept on m – axis is equal to unity

4) Magnitude of intercept on V –axis is equal to unity

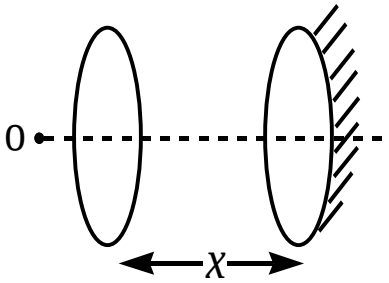
54. A thin lens of refractive index 1.5 and focal length in air 30cm is placed inside, a large container containing two immiscible liquids as shown below. If an object is placed at an infinite distance close to principal axis (Common intersection line of the layers), the distance between the images



1) 25cm 2) 45cm

3) 65cm 4) 85cm

55. The radius of curvature of the curved surface of an equiconvex lens is 32cm and $\mu_L = 1.5$ one of its sides is silvered and placed 14cm away from the object as shown in figure. At what distance x should a convex lens of focal length 24cm be placed so that the image coincides with the object



1) 8cm 2) 10cm

3) 15cm 4) 20cm

56. A thin convex lens of focal length 25cm is cut into two pieces 0.5cm above the principal axis. The top part is placed at (0, 0) and an object is placed at (-50cm, 0).find the coordinates of the image

1) (50cm,-2cm)

2) (50cm,-1cm)

3) (25cm.-1cm)

4) (15cm,-1cm)

57. A hollow double concave lens is made of very thin transparent material. It can be filled with air (or) either of two liquids $L_1(or)L_2$ having refractive indices $\mu_1 and \mu_2$ respectively $[\mu_2 > \mu_1 > 1]$. The lens will diverge a parallel beam of light if it is filled with

1) Air and placed in air

2) Air and immersed in L_1

3) L_1 And immersed in L_2

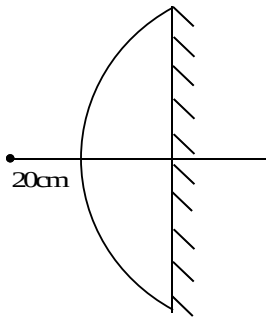
4) L_2 and immersed in L_1

58. The image of an object formed by a Plano convex lens at a distance 8cm behind the lens is real and is one-third the size of the object. Wave length of light inside the lens is $\frac{2}{3}$ times wave length in free space. The radius of the curved surface of the lens is

1) 1m 2) 2m

3) 3m 4) 4m

59. An object is placed 20cm in front of plano convex lens of focal length 60 cm the plane surface of lens is silvered the image will be formed at a distance



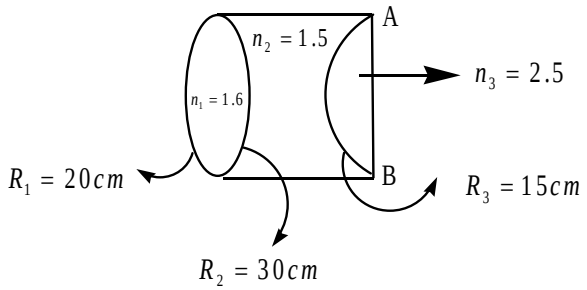
1) 60cm to the left of lens

2) 60cm to right of lens

3) 12cm to left of lens

4) 12cm right of lens

60. A lens is made of three thin different media. Radius of curvature and refractive index of each medium is shown in the figure surface AB is straight. An object is placed at some distance from the lens by which a real image is formed on the screen placed at a distance of 10cm from the lens. Then distance of object from the lens is



1) 5cm
2) Infinite distance
3) 10cm
4) 15cm

MATHS

61. Consider two quadratic expressions $f(x) = ax^2 + bx + c$ and $g(x) = ax^2 + px + q$ $(a, b, c, p, q \in R,\ b \neq p)$ such that their discriminants are equal. If $f(x) = g(x)$ has a root $x = \alpha$ then.
1) α will be A.M of the roots f(x) = 0
2) α will be A.M. of the roots of g(x) = 0
3) α will be A.M. of the roots of f(x) = 0 and g(x) = 0
4) α will be A.M. of the roots of f(x) = 0 and g(-x) = 0

62. If range of the function $f(x) = \dfrac{2\sin^2 x + 2\sin x + 3}{\sin^2 x + \sin x + 1}$ is [a,b] then the value of $3a - 6b + 4$ is
1) 0 2) –3
3) –6 4) –9

63. $If\ x^4 + ax^3 + bx^2 + cx + d$ has four realrootsbetween -2 and 2, $f(2).f(-2) = 256$ Then thevalue of 3a+2b+c+d=
1)0 2)128
3)32 4)64

$x^3 - x - 1 = 0$ then the equation whose roots are $a(b-c)^2, b(a-c)^2, c(a-b)^2$ is

1) $x^3 + x - 1 = 0$

2) $x^3 + 9x^2 + 26x + 23 = 0$

3) $x^3 - 9x^2 + 21x - 27 = 0$

4) $x^3 - x^2 + 10x - 1 = 0$

65. Sequence$\{a_i\}$ (where $a_i > 0$, i = 1, 2, 3, . . , 201) are in G.P. with $a_{101} = 25$ and sum of G.P. is 625. Then value of $\sum_{i=1}^{201}\left(\frac{1}{a_i}\right)$ is equal to

1) $\frac{1}{4}$ 2) $\frac{3}{4}$

3) 1 4) $\frac{7}{4}$

66. *If* $\sum_{k=1}^{n} t_r = \frac{n(n+1)(n+2)}{3}$, *then* $\sum_{r=1}^{n} \frac{1}{t_r} =$

1) $\frac{n}{n+1}$ 2) $\frac{n+1}{n+2}$

3) $\frac{n+2}{n+3}$ 4) None of these

67. The sequence $a_1, a_2, a_3, \ldots$ satisfies $a_1 = 19$, $a_9 = 99$, and, for all $n \geq 3$, a_n is the arithmetic mean of the first n – 1 terms. Find a_2.

1) 29 2) 59

3) 79 4) 179

68. If $S_{(n)} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}, (n \in N)$, then $S_{(1)} + S_{(2)} + \ldots\ldots + S_{(n-1)}$ is equal to

1) $nS_{(n)} - n$ 2) $nS_{(n)} - 1$

3) $(n-1)S_{(n-1)} - n$ 4) $nS_{(n-1)} - n$

69. The sum of three terms of a strictly increasing G.P. is αS and sum of the squares of these terms is $S^2\alpha^2$ lies

1) $\left(\frac{1}{3}, 2\right)$ 2) $(1, 2)$

3) $\left(\frac{1}{3}, 3\right)$ 4) $\left(\frac{1}{3}, 1\right) \cup (1, 3)$

70. The increasing sequence of positive integers $a_1, a_2, a_3, \ldots\ldots$ has the property that $a_{n+2} = a_n + a_{n+1}$ for all $n \geq 1$. Suppose that $a_7 = 120$ What is a_8

the remaining numbers is found to be $35\frac{7}{17}$. The erased number is

1) 6 2) 7

3) 8 4) 9

72 The sequence 1,2,1,2,2,1,2,2,2,1,2,2,2,2,1,2,2,2,2,2,1, 2,..... consists of 1's separated by blocks of 2's with n2's in the n-th block. The sum of the first 1234 terms of this sequence is

1)1996 2)2419

3)2429 4)2439

73. The sum $\sum_{k=1}^{\infty}\frac{6^k}{\left(3^k-2^k\right)\left(3^{k+1}-2^{k+1}\right)}=$

1) 1 2) 2

3) 3 4) 4

74. If $(r)_n$ denotes the numbers r r r …. (n digits) where r=1,2,3……9 and $a=(6)_{100}\ b=(8)_{100}\ c=(4)_{200}$ then

1) $a^2+b+c=0$ 2) $a^2+b-c=0$

3) $a^2+b-2c=0$ 4) $a^2+b-9c=0$

75. a, b, c are in Geometric progression; a, p, q are in Arithmetic progression; $2a, b+p, c+q$ are in Geometric progression, then common difference of A.P has the possible value

1) $\sqrt{2}$ a 2) $\sqrt{2}(a+b)$

3) $\left(\sqrt{2}+1\right)(a-b)$ 4) $\left(\sqrt{2}-1\right)(b-a)$

76. If the first and $(2n-1)^{th}$ terms of an A.P; a G.P. and H.P. are equal and their n^{th} terms are p,q and s respectively, then

1) $p<q<s$ 2) $p+s=q$

3) $ps=q^2$ 4) $p=q=s$

77. The sum

$$\frac{3}{1!+2!+3!}+\frac{4}{2!+3!+4!}+------+\frac{2012}{2010!+2011!+2012!}=$$

1) $\frac{1}{2012!}$ 2) $1-\frac{1}{2012!}$

3) $\frac{1}{2}-\frac{1}{2012!}$ 4) $\frac{1}{2011!-2012!}$

78. The number a,b,c in that order form a three term A.P and $a+b+c=60$. The number $(a-2),b,(c+3)$ in that order form a three term G.P possible value of $\left(a^2+b^2+c^2\right)$ is

1) 1218 2) 1208

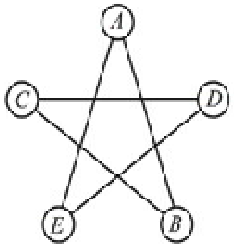
3) 1288 4) 1258

79. The value of $\dfrac{\sum_{r=1}^{n}\frac{1}{r}}{\sum_{k=1}^{n}\frac{k}{(2n-2k+1)(2n-k+1)}}$ is _____

1) 1 2) 2

3)4 4)3

80. In the five-sided star shown, the letters A,B,C,D and E are replaced by the numbers 3,5,6,7 and 9, although not necessarily in that order. The sums of the numbers at the ends of the line segments $\overline{AB},\overline{BC},\overline{CD},\overline{DE}$ and $\overline{EA}$ form an arithmetic sequence, although not necessarily in that order. What is the middle term of this arithmetic sequence.



1)9 2)10

3)11 4)12

81. If a, b, c are in A.P. p, q, r are in H.P. and ap, bq, cr are in G.P. then $\frac{p}{r}+\frac{r}{p}$ is equal to

1) $\frac{a}{b}+\frac{b}{a}$ 2) $\frac{b}{c}+\frac{c}{a}$

3) $\frac{c}{a}+\frac{a}{c}$ 4) $\frac{a}{b}-\frac{b}{a}$

82. $\frac{a-x}{px}=\frac{a-y}{qy}=\frac{a-z}{rz}$ and p, q, r are in A.P. then x, y, z are in.

1)A.P 2)G.P

3)H.P 4)None

83. In an A.P. containing 99 terms, the sum of all the odd numbered terms is 2550. The sum of all the 99 terms of the A.P. is

1) 5010 2) 5050

3) 5100 4) 5049

84. Let a_1,a_2,.....be as sequence with the following properties

(i) a_1=1 and

(ii) $a_{2n}=na_n$ for any positive integer n.

What is the value of $a_{2^{100}}$

1)1 2) 2^{99}

3) 2^{100} 4) 2^{4950}

85. Let a_n = 16, 4, 1, ...be a geometric sequence. Define P_n as the product of the first n terms. The value of $\sum_{n=1}^{\infty} \sqrt[n]{P_n}$

1) 8 2) 16

3) 32 4) 64

86. If sum of the series

$S = 1 + \frac{5}{11} + \frac{12}{(11)^2} + \frac{22}{(11)^3} + \frac{35}{(11)^4} + ...\infty$ can be expressed in the form of a rational number p/q in the lowest form, then the value of (p + q) equals

1) 2573 2) 2753

3) 2375 4) 2537

87. If $x_1, x_2, x_3.....x_{10}$ *are* 10 *distinct* positive numbers in G.P then

$\frac{(x_1 + x_2 + ... + x_{10})(x_1 x_2 x_3 .... x_{10})}{\sum x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8 x_9}$

1) $(x_1 x_2 x_3 .... x_{10})^{1/10}$

2) $(x_1 x_2 .... x_{10})^{1/5}$

3) $(x_1 x_2 .... x_{10})^{2/5}$

4) $x_1 x_2 x_3 .... x_{10}$

88. Numbers of integral values of 'a' for which the quadratic equation $x^2 - (a+1)x + (a-3) = 0$ has both integral roots are

1)2 2)3

3)4 4)5

89 If $\alpha_1, \alpha_2, \alpha_3.....\alpha_{10}$ *are the roots of* $10x^{10} + 9x^9 + 8x^8 + ...2x^2 + x + 1 = 0$ then the value of $\frac{1+\alpha_1}{1-\alpha_1} + \frac{1+\alpha_2}{1-\alpha_2} + ... + \frac{1+\alpha_{10}}{1-\alpha_{10}}$ is equal to

1)4 2) 100

3)7 4)14

90 If $\alpha, \beta, \gamma, \delta$ are the real and positive values of 'x' which satisfies the equation

$(\log_{10} x)^4 - (\log_{10} x)^3 + a(\log_{10} x)^2 + b(\log_{10} x) + 2 = 0$ then $\alpha\beta\gamma\delta$ is equal to

1)1 2)10

3)2 4)100

Space for rough work

Master JEE CLASSES
Kukatpally, Hyderabad.

IIT-JEE-MAINS PAPER-5 Max. Marks: 360

KEY SHEET

CHEMISTRY

1	3	**2**	1	**3**	4	**4**	1	**5**	1	**6**	2
7	3	**8**	1	**9**	4	**10**	1	**11**	1	**12**	3
13	4	**14**	3	**15**	2	**16**	4	**17**	1	**18**	1
19	3	**20**	4	**21**	1	**22**	2	**23**	1	**24**	3
25	3	**26**	3	**27**	2	**28**	3	**29**	1	**30**	1

PHYSICS

31	1	**32**	3	**33**	2	**34**	3	**35**	1	**36**	3
37	2	**38**	2	**39**	3	**40**	2	**41**	1	**42**	1
43	2	**44**	1	**45**	1	**46**	4	**47**	1	**48**	1
49	3	**50**	3	**51**	4	**52**	2	**53**	4	**54**	3
55	1	**56**	2	**57**	4	**58**	3	**59**	2	**60**	2

MATHS

61	3	**62**	4	**63**	1	**64**	2	**65**	3	**66**	1
67	4	**68**	1	**69**	4	**70**	4	**71**	2	**72**	4
73	2	**74**	2	**75**	3	**76**	3	**77**	3	**78**	2
79	2	**80**	4	**81**	3	**82**	3	**83**	4	**84**	4
85	3	**86**	1	**87**	2	**88**	1	**89**	1	**90**	2

1:.3

Hint: sum of the oxidation numbers should be zero

2. 1

3.4

Hint: $FeC_2O_4 \rightarrow Fe^{+3} + 2CO_2$ ($3e^-$ transfer)

$\therefore$ eq.weight = $\frac{M}{3}$

4. 1

Hint: Reductant is Fe_3O_4.

$2Fe_3O_4 \rightarrow 3Fe_2O_3$

total change in O.N. = 2 (+16 to +18)

Therefore, n-factor for Fe_3O_4 is 2/2 = 1

5.1

% C	% H
$\frac{12}{44} \times \frac{12}{4} \times 100$	$100 - \%C$
$= \frac{900}{11}$	$= \frac{200}{11}$

$\therefore$ mole ratio = 3 : 8

$\therefore$ empirical formual = C_3H_8

6.2 Hint: 1 mole of $Mg_3(PO_4)_2$ contains 8 mole of oxygen atoms.

0.25 mole oxygen atom are present in 0.25/8 mole $Mg_3(PO_4)_2$

7.3

Hint:

In SO_2 32 gram of suphur react with 32 gram of oxygen. Similarly for SO_3 fixed mass of sulphur (32 gram) react with 48 gram of oxygen. The ratio of oxygen's mass = 32:48 = 2:3.

Which support law of multiple proportions.

Hence (3) is correct answer.

8.1

9.4

10.1

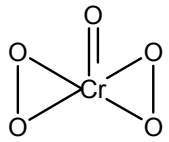
Hint: Let '*x*' be the atomic wt. of metal M, for first oxide

$$\frac{72.4}{x} : \frac{27.6}{16} = 3:4 \Rightarrow M_3O_4$$

X = 56

For second oxide, $\frac{70}{56} : \frac{30}{16} = 2:3 \Rightarrow M_2O_3$

11.1



4 peroxides oxygens and 1 oxide oxygen

12.3

13.4

Hint: Sum of the oxidation numbers is equal to zero.

14.3

Hint: Balance both number of atoms and net charge

15.B

16.4

17.1

18.1

Hint: $\overset{0}{Cl_2} + OH^- \rightarrow Cl^- + H_2O$

$$\overset{+1}{Cu_2}O + 2H^+ \rightarrow \overset{0}{Cu} + \overset{+2}{Cu^{+2}} + H_2O$$

$$2H\overset{+1}{Cu}Cl_2 \rightarrow \overset{0}{Cu} + \overset{+2}{Cu^{+2}} + 4Cl^- + 2H^+$$

19.3

20.4

Hint: $C_8H_{10} + 14\ OH^- \rightarrow C_8H_4O_4^{-2} + 10\ H_2O + 12e^-$

21.1

22.2

Hint: $\psi^2 = \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 e^{-2r/a_0}$

At nucleus $r = 0\ \psi_0^2 = \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 e^0 = \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3$

at $r = a_0\ \psi_n^2 = \frac{1}{\pi}\left(\frac{1}{a_0}\right)^3 e^{-2}$; $\psi_0^2 / \psi_n^2 = e^2$

23.1

24.3

Hint: Only n, l, m

25.3

26.3

27.2

28.3

One m = 0 in each of 3s 3p 3d each containing 2 electrons

$\therefore$ 6 electrons

29.1

Hint: $2\pi r = n\lambda$ $\quad 2\pi.a_0.4^2 = 4\lambda$

30.3

31. A ray of light from the object O undergoes refraction, reflection and then refraction.

Refraction at surface 1:

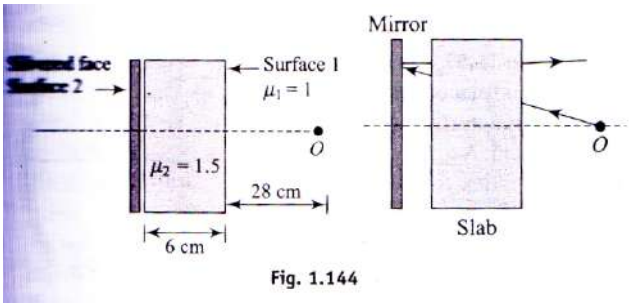


Fig. 1.144

Here $\mu_1 = 1, \mu_2 = 1.5$

$d_1 = 28cm, d_2 = ?$

Since $d_2 = \dfrac{d_1}{n_{relative}} = \dfrac{d_1}{(1/\mu)}; \left[n_{rel} = \dfrac{n_{incident}}{n_{refracted}} = \dfrac{1}{\mu} \right]$

$\Rightarrow d_2 = \mu d_1 = 1.5 \times 28 = 42cm$

Mirror forms images for object distance (42+6) =48cm on behind it.

Again rays form it incidence glass 'slab and final image is formed at a distance

$d_2^I = \dfrac{d_1^I}{\mu_{rel}} = \dfrac{54}{\left(\dfrac{1.5}{1}\right)} = 36cm$ behind the surface 1

So final image distance from silvered surface is (36-6) = 30cm

32. Applying condition of minimum deviation

$$\mu = \frac{\sin\dfrac{(A+\gamma)}{2}}{\sin\dfrac{A}{2}} = \frac{\sin\dfrac{A}{2}\cos\dfrac{\gamma}{2} + \cos\dfrac{A}{2}\sin\dfrac{\gamma}{2}}{\sin\dfrac{A}{2}}$$

$$= \cos\frac{\gamma}{2} + \cot\frac{A}{2}\sin\frac{\gamma}{2}$$

Using $A = 90^0, \mu = \cos\dfrac{\gamma}{2} + \cot 45^0 \sin\dfrac{\gamma}{2}$

$$\Rightarrow \cos\frac{\gamma}{2} + \sin\frac{\gamma}{2} = \mu$$

Squaring, $\cos^2\dfrac{r}{2} + \sin^2\dfrac{\gamma}{2} + \gamma = \mu^2 \Rightarrow \sin\gamma = \mu^2 - 1$

Deviation at grazing incidence,

$\beta = \delta_1 + \delta^2$

$$\beta = \left(\frac{\pi}{2} - c\right) + (e - r_2)$$

$$\Rightarrow \beta = \left(\frac{\pi}{2} - C\right) + \left[e - \left(\frac{\pi}{2} - C\right)\right]$$

$$\Rightarrow \beta = e$$

Or $\sin\beta = \sin e = \mu \sin r_2 = \mu \sin\left(\frac{\pi}{2} - C\right)$

$$\Rightarrow \sin\beta = \mu \cos C$$

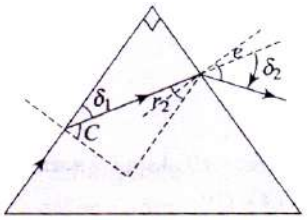


Fig. 1.194

Squaring Eq. (ii)

$$\sin^2\beta = \mu^2 \cos^2 C \Rightarrow \sin^2\beta = \mu^2\left(1 - \sin^2 c\right)$$

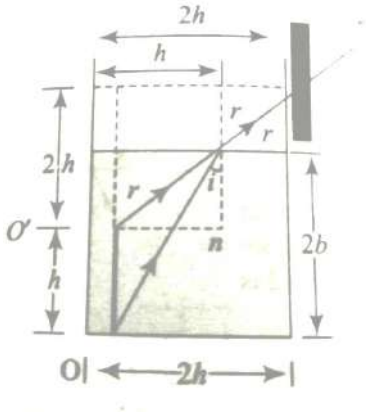
Using $\sin C = \frac{1}{\mu}, \sin^2\beta = \mu^2\left(1 - \frac{1}{\mu^2}\right)$

$$\Rightarrow \sin^2\beta = \mu^2 - 1$$

From Eqs. (i) and (iii),

$$\sin\gamma = \sin^2\beta$$

33. $\frac{\sin^I}{\sin r} = \frac{1}{n}$



$$\tan r = \frac{2h}{2h} = 1$$

$$r = 45^0$$

$$\sin i = \frac{h}{h\sqrt{5}}$$

$$\sin i = \frac{1}{\sqrt{5}}$$

$$\frac{1}{n} = \frac{\frac{1}{\sqrt{5}}}{\frac{1}{\sqrt{2}}} \Rightarrow n = \sqrt{\frac{5}{2}}$$

34. $\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu}$

$r^I >$ critical angle

$$\sin r^I > \frac{1}{\mu}$$

$$\sin(90 - r) > \frac{1}{\mu}$$

$$\cos r > \frac{1}{\mu}$$

$$\sqrt{1-\sin^2 i} > \frac{1}{\mu}$$

$$\sqrt{1-\frac{\sin^2 i}{\mu^2}} > \frac{1}{\mu}$$

$$\mu > \sqrt{1+\sin^2 i}$$

$$i_{\max} = 90^0, \mu > \sqrt{1+\sin^2 90^0}$$

$$\mu > \sqrt{2}$$

35. The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A will be formed at focus which is 20 cm from the mirror. Now, for the interface between water and air. For observer at air, the image formed by mirror will act as an object for observer with real depth $d = 30 - 20 = 10cm.$

$$d = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5cm$$

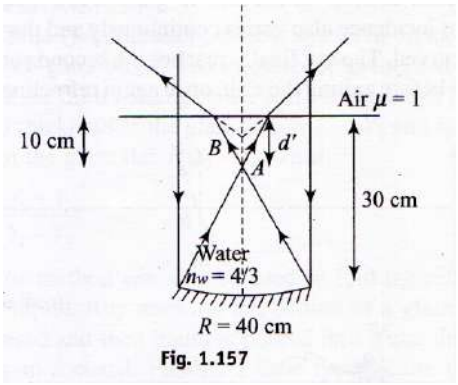


Fig. 1.157

36. $\delta = 30^0, A = 60^0$

Let us test whether the prism is in the position of minimum deviation

$$n = \frac{\sin\left(\frac{30+60}{2}\right)}{\sin\left(\frac{60}{2}\right)} = \frac{\sin 45^0}{\sin 30^0} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

$n = \sqrt{2}$ the ray suffers minimum deviation through prism thus $r_1 = r_2 = r = \frac{A}{2} = 30^0$

Inside the prism, the ray makes an angle of 60^0 with face AB

So it is parallel to base

37. Refraction at first surface (S_1) :

$$\frac{1.5}{V_1}-\frac{1}{\infty}=\frac{(1.5-1)}{+10}$$

$$V_1=30cm$$

First image is the object for refraction at second surface (S_2) : light travel from air to glass

$$\frac{1}{V_2}-\frac{1.5}{(+25)}=\frac{1-1.5}{(+5)}$$

$$V_2=-25cm$$

For refraction at surface (S_3): light travel from glass to air

$$\frac{1.5}{V_3}-\frac{1}{(-35)}=\frac{(1.5-1)}{(-5)}$$

$$V_3=\frac{-35}{3}cm$$

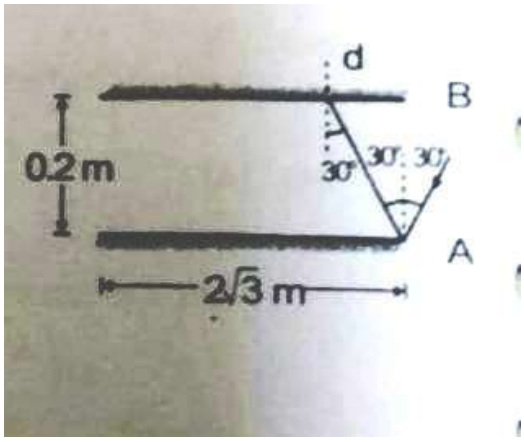
For refraction at surface (S_4): light travel from glass to air

$$\frac{1}{V_4}-\frac{1.5}{\left(\frac{35}{3}+5\right)}=\frac{1-1.5}{-10}$$

$$V_4=-25cm$$

Final image is virtual formed at 25 cm to the left of vertex of surface (S_4):

38. Maximum number of refraction $=\frac{l}{x}=\frac{2\sqrt{3}}{\left(\frac{0.2}{\sqrt{3}}\right)}=30$ (images)

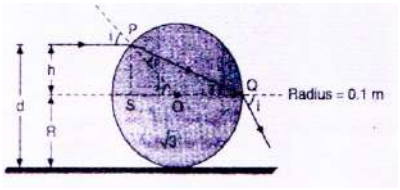


Where $x=0.2\tan 30^0$

$$=\frac{0.2}{\sqrt{3}}$$

39. Since there will be no refraction from P to Q and then from Q to R (all being before identical)Hence ray will now have the same deviation as before the point n, n^1 being same for the ray correct option is (C)

40. Let us first draw the ray diagram for the situation.



(a) Since, $PO = OQ$

$\Rightarrow \angle OPQ = \angle OQP = r$ (Say)

Also, $i = r + r = 2r$

In ΔPOS, we have

$h = OP \sin i = 0.1 \sin i$

$\Rightarrow h = 0.1 \sin 2r$

$\Rightarrow h = 0.2 \sin r \cos r$

Applying Snell's Law at P, we get

$$\sqrt{3} = \frac{\sin i}{\sin r} = \frac{2 \sin r \cos r}{\sin r} = 2 \cos r$$

Substituting in equation (1), we get

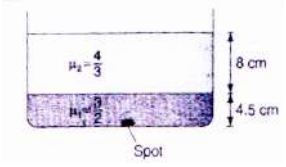
$$h = (0.2)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = 0.086m$$

Hence, height from the mirror is

$d = h + r = 0.1 + 0.086 = 0.186m$

41. The apparent depth is given as

$$d_a = \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} = \frac{4.5}{\left(\frac{3}{2}\right)} + \frac{8}{\left(\frac{4}{3}\right)} = 3 + 6 = 9cm$$

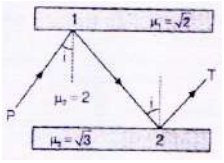


The effective refractive index is given as

$$\mu_{effective} = \frac{\text{Re}al\ \ Depth}{Apparent\ \ Depth}$$

$$\Rightarrow \mu_{effective}\frac{d_r}{d_a}=\frac{d_1+d_2}{d_a}=\frac{4.5+8}{9}=\frac{12.5}{9}=1.39$$

42. Let the critical angles at 1 and 2 be C_1 and C_2 respectively. Then



$$C_1=\sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)=\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=45^0$$

And $C_2=\sin^{-1}\left(\frac{\mu_3}{\mu_2}\right)=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=60^0$

For TIR, $i>C_2$

There for, minimum angle of incidence, for total internal refraction to take place on both slabs must be 60^0

$i_{\min}=60^0$

43. At minimum deviation, we have $r_1=r_2=30^0$

According to Snell's Law, we have

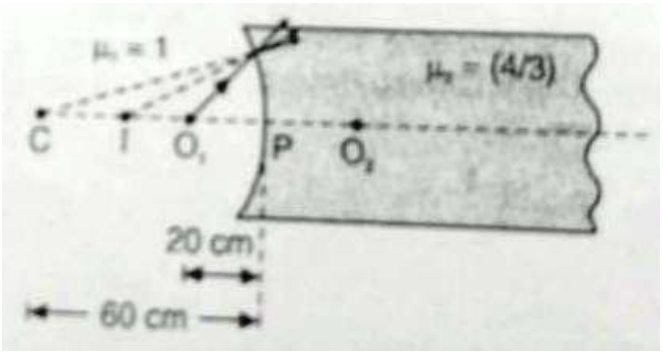
$$\mu=\frac{\sin i_1}{\sin r_1}$$

$$\Rightarrow \sqrt{3}=\frac{\sin i_1}{\sin\left(30^0\right)}$$

$$\Rightarrow \sin i_1=\frac{\sqrt{3}}{2}$$

$$\Rightarrow i_1=60^0$$

44. The formula for refraction from a curved boundary is



$$\frac{\mu_2}{V}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{\frac{4}{3}}{V}-\frac{1}{-20}=\frac{\left(\frac{4}{3}-1\right)}{-60}$$

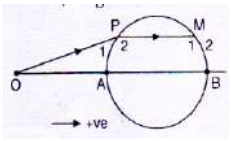
$$V=-24cm$$

The object O_1 will appear at a distance of 24 cm from P towards C

45. A ray of light starting from O gets refracted twice. The ray of light is travelling in a direction from left to right. Hence, the distances measured in this direction are taken positive.

Applying

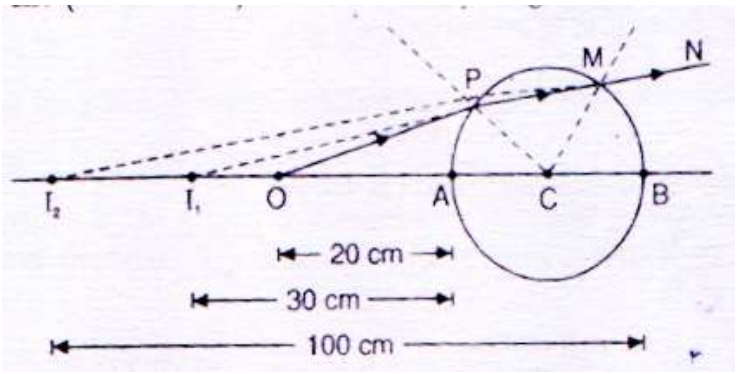
$\frac{\mu_1}{-u}+\frac{\mu_2}{-v}=\frac{\mu_2-\mu_1}{R}$, twice with appropriate sings at the two refracting surfaces, we get



$$\frac{\left(\frac{4}{3}\right)}{-(-20)}+\frac{\frac{3}{2}}{AI_1}=\frac{\left(\frac{3}{2}-\frac{4}{3}\right)}{10}$$

$$\Rightarrow AI_1=-30cm$$

Now, the first image I_1, acts as an object for the second surface, so, we have



$$BI_1=u=-(30+20)=-50cm$$

Again applying $\frac{\mu_2}{-u}+\frac{\mu_1}{v}=\frac{\mu_2-\mu_1}{R}$, we get

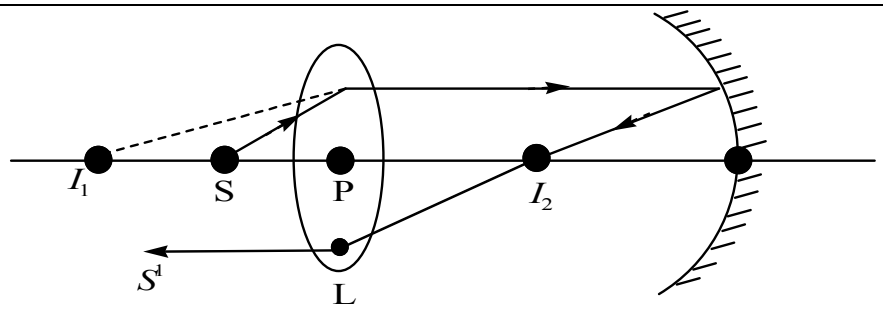
$$\frac{\left(\frac{3}{2}\right)}{-(-50)}+\frac{\frac{4}{3}}{BI_2}=\frac{\frac{4}{3}-\frac{3}{2}}{-10}$$

$$\Rightarrow BI_2=-100cm$$

i.e., the final image I_2 is virtual and is formed at a distance 100 cm (towards left) from B.

The ray diagram is as shown below.

46.



Let us first locate image of S formed by lens L

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{15}-\frac{1}{12}$$

V= 60

Image I_1 acts as source for the various. The minor forms an image I_2 of source I_1. This image I_2 then acts source for the lens and the final beam comes out parallel to the principal axis. Clearly I_2 must be at the focus of the lens

$$I_1, I_2 = P_1P + PI_2 = 60 + 15 = 75cm$$

Suppose distance of minor from I_2 is x

For reaction from the minor

$$u = MI_1 = -\left[75 + x\right]$$

V = -x

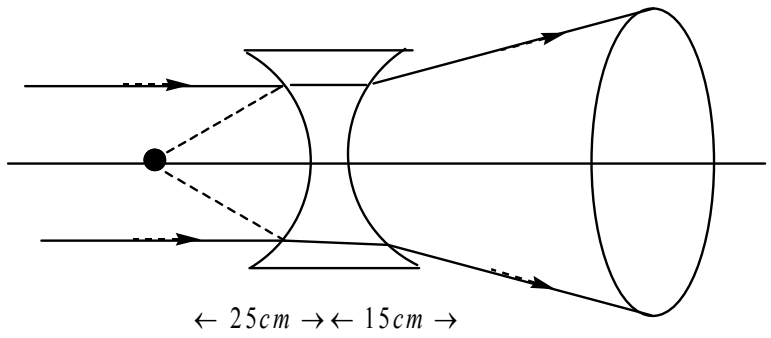
F = -20 cm

$$\frac{1}{x}+\frac{1}{75+x}=\frac{1}{20}$$

$$x = 25(or) - 60$$

Taking the $x = 25cm$ eparation between lens and mirror is 15 + 25 = 40 cm

47.



For lens L_1, (diverging lens)

$$\frac{1}{v}-\frac{1}{(-\infty)}=\frac{1}{(-25)}$$

V = -25 cm

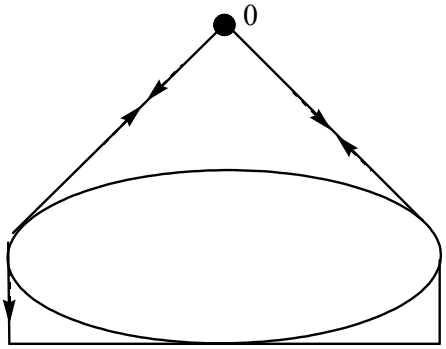
For lens L_2,(converging lens)

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{(-110)}=\frac{1}{(+20)}$$

$v=+40cm$ final image is form at a distance 40 cm from converging lens and also image is real

48.



If an object has to centroid with its image then the ray have to retrace its path hence it is conducted that object is at the focus of combination

$$\frac{1}{f}=\frac{1}{f_g}+\frac{1}{f_1}$$

$$\frac{1}{f_g}=(1.5-1)\frac{2}{R}=\frac{1}{R}$$

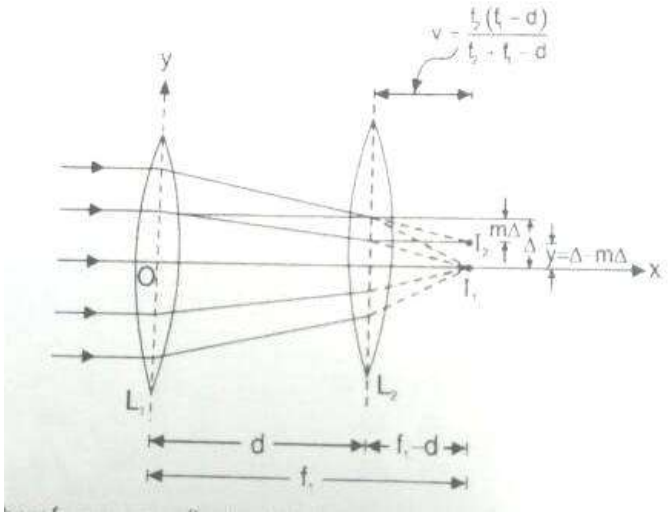
$$\frac{1}{f_l}=(u-1)\left[-\frac{1}{R}-\frac{1}{\alpha}\right]=\frac{(u-1)}{R}$$

$$\frac{1}{f}=\frac{1}{R}-\frac{(u-1)}{R}=\frac{(2-u)}{R}$$

$$F=\frac{R}{2-u}\Rightarrow u=2-\frac{R}{F}$$

$$u=\frac{2}{1}-\frac{2d}{3u}=\frac{6-2}{3}=\frac{4}{3}$$

49.



Δ Let IDE and ΔO^1AE are similar

$$\frac{y}{\Delta}=\frac{f_1-x}{f_1-d}-----(1)$$

$\Delta ICB \text{ and } \Delta O^1B$

$$\frac{\Delta-y}{\Delta}=\frac{(f_2+d)-x}{f_2}$$

$$\frac{-y}{\Delta}=\frac{d}{f_2}-\frac{x}{f_2}$$

$$\frac{y}{\Delta}=\frac{x}{f_2}-\frac{d}{f_2}------(2)$$

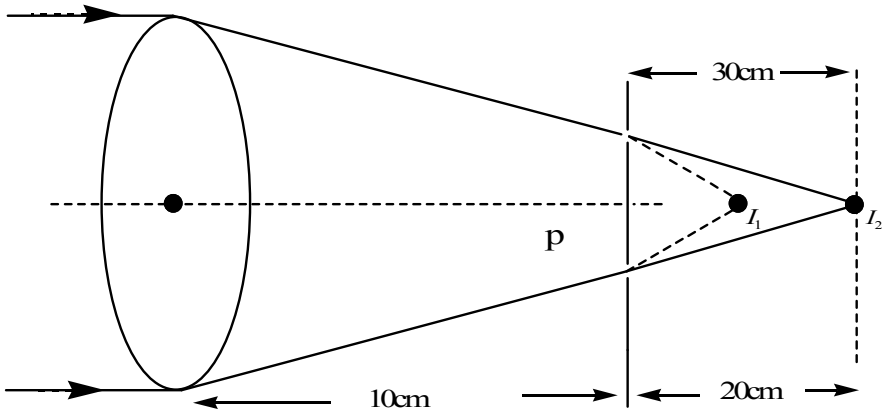
From 1 & 2, $x=\frac{f_1f_2+d(f_1-d)}{f_1+f_2-d}$

Linear magnification for lens L_2 is given by $m=\frac{v}{u}=\left(\frac{f_2[f_1-d]}{f_2+f_1-d}\right)\frac{1}{(f_1-d)}=\left(\frac{f_2}{f_2+f_1-d}\right)$

Second image will be formed at a distance $m\Delta$ or $\left(\frac{f_2}{f_1+f_2-d}\right)\Delta$ below its optic axis

Y co ordinate of the focus of system is given by $y=\Delta-\left(\frac{f_2\Delta}{f_2+f_1-d}\right)\ \ y=\left(\frac{f_1-d}{f_2+f_1-d}\right)\Delta$

50. Two Plano convex lenses of focal length f on combining give convex lens of focal length f/2 = 40/2 =20 cm obtain real image the object must be place at a $2F=2\times20=40cm$



51.

The lens will converge the rays as its focus

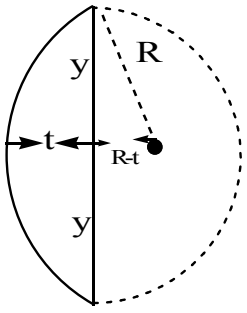
i.e. 30 cm from refracting surface now

$PI_2 = \mu\left(PI_1\right)$

$= \frac{3}{2} \times 20$

$= 30cm$

Hence rays will converge at a distance 40 cm from the lens.



52.

$R^2 = y^2 + \left(R - t\right)^2$

$R^2 = y^2 + R^2 + t^2 - 2Rt$

$t << R$, t^2 is neglected

$y^2 = 2Rt$

$R = \frac{y^2}{2R} = \frac{3^2}{2 \times 0.3} = 15cm$

$$\mu = \frac{C}{V} = \frac{3\times10^8}{2\times10^8} = (1.5-1)\left[\frac{1}{15} - \frac{1}{\alpha}\right]$$

$$f = 30cm$$

53. for lens $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$

$$m = \frac{-V}{u} = \frac{V}{f} - 1$$

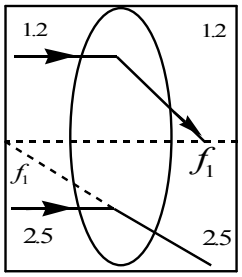
Graph between m and V will be straight line with -1 intercept on m-axis

And slope $\tan\theta = \frac{1}{f}$

Putting $m = O$ it gives $V = f$

a, b, c Options are correct.

54.



for upper portion, $f_1 = \frac{\mu_2 - 1}{\left(\frac{\mu_2}{\mu_1} - 1\right)}(f_a) = \frac{1.5-1}{\left(\frac{1.5}{1.2} - 1\right)} \times 20 = 40cm$

For lower portion, $f_1 = \frac{\mu_2 - 1}{\left(\frac{\mu_2}{\mu_3} - 1\right)} f_a = \frac{1.5-1}{\left(\frac{1.5}{2.5} - 1\right)} = -25cm$

Object is at infinite then images are formed at focus points.

Distance between images $= 40 + 25 = 65cm$

55. Equivalent focal length of silvered

$$\frac{1}{F} = \frac{1}{f_L} + \frac{1}{f_m} + \frac{1}{f_L} = \frac{2}{f_L} + \frac{1}{f_m}$$

$$= 2\left[\frac{3}{2} - 1\right]\left[\frac{1}{32} - \frac{1}{(-32)}\right] + \frac{1}{\left(\frac{32}{2}\right)}$$

$$F = 8cm$$

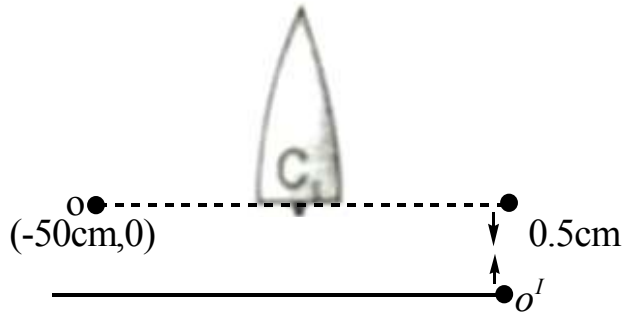
The image by lens should be formed at centre of curve of mirror

$$= \frac{1}{-(16-x)} - \frac{1}{-(14-x)} = \frac{1}{24}$$

$$x^2 - 30x + 176 = 0$$

$$x = 8cm(or)22cm$$

$$\therefore x = 8cm$$



56.

To part of cutted lens is placed at (0.0) and object placed at (-50cm, 0). There is no effect on the focal length or lens

By applying lens formula $\frac{1}{V} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{V} = \frac{1}{u} + \frac{1}{f} = \frac{-1}{50} + \frac{1}{25} = \frac{1}{50}$$

$$V = 50cm$$

Magnification $m = \frac{v}{u} = \frac{-50}{50} = -1$

So image is found at 50cm from pole and 0.5cm below the principal axis

With respect X axis passes through the edge of cut lens. Coordinates of image are (50cm,-1cm)

57. $$\frac{1}{f} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For divergence $\mu_2 > \mu_1$

Here $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is negative

There force (d) is the correct option

58. $$\mu = \frac{\lambda_0}{\lambda_m} = \frac{3}{2}$$

$$\frac{1}{f} = \frac{(\mu - 1)}{R} = \frac{1}{2R}$$

$$\frac{1}{f}=\frac{1}{V}-\frac{1}{u}$$

$$\frac{1}{8}-\left(-\frac{1}{24}\right)=\frac{1}{2R}$$

$$\Rightarrow \frac{3+1}{24}=\frac{1}{2R}$$

$$R=3m$$

59. Since $p=2p_l+p_m$

$$-\frac{1}{F}=\frac{2}{f_l}-\frac{1}{f_m}$$

$$-\frac{1}{F}=\frac{2}{60}-\frac{1}{\infty}$$

$F=-30\,\text{cm}$ the problem is reduced to simple case where a point object is placed in front of a concave mirror of focal length 30cm

Using mirror formula

$$\frac{1}{V}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{V}+\frac{1}{-20}=\frac{1}{-30}$$

$V=60cm$ the image is virtual and erect

60. $$\frac{1}{f_1}=(1.6-1)\left[\frac{1}{20}-\frac{1}{-30}\right]=\frac{1}{20}$$

$$\frac{1}{f_2}=(1.5-1)\left[\frac{4}{-30}-\frac{1}{15}\right]=\frac{1}{20}$$

$$\frac{1}{f_3}=(2.5-1)\left[\frac{1}{15}-\frac{1}{\infty}\right]=\frac{1}{10}$$

$$\frac{1}{f_{eq}}=\frac{1}{20}-\frac{1}{20}+\frac{1}{10}=\frac{1}{10}$$

$$f_{eq}=10cm$$

Since the image is found on the screen which is 10cm from the lens, so object is at infinite.

61. $a\alpha^2+b\alpha+c=a\alpha^2+P\alpha+q\Rightarrow\alpha=\dfrac{q-c}{b-p}$

$b^2-4ac=p^2-4aq$

$b^2-p^2=4a(c-q)$

$b+p=\dfrac{4a(c-q)}{b-p}$

$b+p=-4a\alpha$

$\alpha=\dfrac{-(b+p)}{4a}$ which is A.M. of the roots of f(x) = 0 and g(x) = 0

62. $\dfrac{2\sin^2 x+2\sin x+3}{\sin^2 x+\sin x+1}=2+\dfrac{1}{\left(\sin x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}}$

$\therefore a=\dfrac{7}{3},b=\dfrac{10}{3}$

63. $f(2).f(-2)=(4-\alpha^2)(4-\beta^2)(4-\gamma^2)(4-\delta^2)=256$ $\alpha,\beta,\gamma,\delta$ *are real* $\alpha=\beta=\gamma=\delta=0$

65. Let first term of G.P. is a and common ratio is r

$\dfrac{a\left(1-r^{201}\right)}{1-r}=625$ . . . (i)

$\sum_{i=1}^{201}\dfrac{1}{a_i}=\dfrac{1}{a_1}+\dfrac{1}{a_2}+\ldots+\dfrac{1}{a_{201}}$

$=\dfrac{1}{a}+\dfrac{1}{ar}+..\dfrac{1}{ar^{200}}$

$=\dfrac{\dfrac{1}{a}\left(\left(\dfrac{1}{r}\right)^{201}-1\right)}{\left(\dfrac{1}{r}-1\right)}=\dfrac{1}{a}\left(\dfrac{1-r^{201}}{1-r}\right).\dfrac{1}{r^{200}}$

$=\dfrac{1}{a}\times\dfrac{625}{a}\times\dfrac{1}{r^{200}}$ (from (i))

$=\dfrac{625}{\left(ar^{100}\right)^2}=\dfrac{625}{\left(a_{101}\right)^2}=\dfrac{625}{625}=1.$

66. $S_n=\dfrac{n(n+1)(n+2)}{3}$

$\therefore t_r=S_r-S_{r=1}$

$$= \frac{r(r+1)(r+2)}{3} - \frac{(r-1)r(r+1)}{3} \qquad = r(r+1)$$

$$\therefore \frac{1}{t_r} = \frac{1}{r(r+1)} = \frac{1}{r} - \frac{1}{r+1}$$

$$\therefore \sum_{r=1}^{n} \frac{1}{t_r} = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

67. (E) For $n \geq 3, a_n = \dfrac{a_1 + a_2 + ... + a_{n-1}}{n-1}$

Thus $(n-1)a_n = a_1 + a_2 + ... + a_{n-1}$. It follows that

$$a_{n+1} = \frac{a_1 + a_2 + ... + a_{n-1} + a_n}{n} = \frac{(n-1)\cdot a_n + a_n}{n} = a_n.$$

for $n \geq 3$. Since $a_9 = 99$ and $a_1 = 19$, it follows that $99 = a_3 = \dfrac{19 + a_2}{2}$.

and hence that $a_2 = 179$. (The sequence is 19, 179,99,99.....)

73. $$\frac{6^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})} = \frac{3^k - 2^k}{(3^k - 2^k)(3^{k+1} - 2^{k+1})}$$

$$= \frac{3^k}{3^k - 2^k} - \frac{3^{k+1}}{3^{k+1} - 2^{k+1}}$$

$$\sum_{k=1}^{\infty} \frac{3^k}{3^k - 2^k} - \frac{3^{k+1}}{3^{k+1} - 2^{k+1}} = \frac{3}{3-2} - \underset{n\to\infty}{Lt} \frac{3^2}{3^n - 2^n}$$

$$= 3 - 1 = 2$$

74. a=666……..n digits = $6x1+6x10+6\times10^2.....6\times10^{n-1} = \dfrac{6}{9}(10^n - 1) = \dfrac{2}{3}(10^n - 1)$

$b = \dfrac{8}{9}(10^n - 1) \quad c = \dfrac{4}{9}(10^{2n} - 1)$

79. $A = \sum_{k=1}^{n} \dfrac{1}{2n-2k+1} - \sum_{k=1}^{n} \dfrac{1}{2n-k+1}, B = \sum_{r=1}^{n} \dfrac{1}{r}$

$$A = \left(\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{2n-1}\right) - \left(\frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2m}\right)$$

$$B - A = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} .... + \frac{1}{2m}\right) - \left(\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + ... + \frac{1}{2n-1}\right)$$

$$B - A = \frac{1}{2}B \Rightarrow \frac{B}{2} = A$$

$$\frac{A}{B} = \frac{1}{2}$$

80. Each number appears in two sums, so the sum of the sequence is 2(3+5+6+7+9)=60. The middle term of a five term arithmetic sequence is the mean of its terms, so 12 is the middle term.

81. $b=\frac{a+c}{2}; q=\frac{2qr}{p+r} and\ apcr=b^2q^2 \quad apcr=\left(\frac{a+c}{2}\right)^2.\left(\frac{2pr}{p+r}\right)^2=\frac{(a+c)^2-p^2r^2}{(p+r)^2}$

$=\frac{(p+r)^2}{pr}=\frac{(a+c)^2}{ac} \qquad =\frac{p}{r}+\frac{r}{p}=\frac{a}{c}+\frac{c}{a}$

82. $\frac{\frac{a}{x}-1}{p}=\frac{\frac{a}{y}-1}{q}=\frac{\frac{a}{z}-1}{r} \quad \therefore \frac{\frac{a}{x}-\frac{a}{y}}{p-q}=\frac{\frac{a}{y}-\frac{a}{z}}{q-r}$

Since p, q, r are in A.P Þ p – q = q – r

$\frac{a}{X}-\frac{a}{Y}=\frac{a}{y}-\frac{a}{z}\Rightarrow\frac{1}{x},\frac{1}{y},\frac{1}{z} are\ in\ A.P$ .x, y, z in H.P.]

88. $(x-a)(x-1)=3$

x-1=1 or -1 or 3 or -3 simultaneously solve for $x-a=3,-3,1,-1$ respectively

89. $10x^{10}+9x^9+.....+2x^2+x+1=(x-\alpha_1)(x-\alpha_2).....(x-\alpha_{10})$

apply log on both sides ,differentiate and put x=1

$\frac{10^2+9^2+....+2^2+1^2}{10+9+8+....+1}=\frac{1}{1-\alpha_1}+....\frac{1}{1-\alpha_{10}}$

$\frac{1}{2}\left(\frac{1+\alpha_1+1-\alpha_1}{1-\alpha_1}+\frac{1+\alpha_2+1-\alpha_2}{1-\alpha_2}+....\frac{1+\alpha_{10}+1-\alpha_{10}}{1-\alpha_{10}}\right)=7$

90. $\log_{10}x=t$ Sums roots of give $\alpha\beta\gamma\delta$