Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-MAINS-PAPER-1 **Max.Marks:360**

IMPORTANT INSTRUCTIONS:

1) Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2) The test is of 3 hours duration.
3) The Test Booklet consists of 90 questions. The maximum marks are 360.
4) There are three parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for correct response.
5) Candidates will be awarded marks as stated above in instruction No. 4 for correct response of each question. (1/4) (one fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
6) There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 5 above.

SYLLABUS

PHYSICS:

Kinematics: Motion in 1D, Uniform motion, uniform accelerated motion, motion under gravity and related graphs (70%)

Integration & its application, Complete vectors (Introduction to physical quantities like Position Vectors, displacement Vectors,Velocity Vector ,acceleraction Vector and mathematical relation between them) (30%)

CHEMISTRY:

Thermodynamics: Terms involved in thermodynamics - System & Surrounding, Extensive and Intensive Properties, Path and State function, types of processes, First law of thermodynamics , Reversible and irreversible processes; Work - expressions for reversible, irreversible isothermal and adiabatic processes, Heat and Heat capacity, Thermo chemistry: Enthalpy,(70%)

Gaseous State: Measurable properties of gases; Gas Laws - Ideal gas equation, Dalton's Law of Partial pressure: Graham's Law of diffusion, Kinetic theory of gases; Distribution of molecular velocities; Kinetic energy, Non ideality of gases; Vander Waal's equation of state, Compressibility factor; liquefaction of gases & Critical state (30%)

MATHS:

Measurement of Angles, Trigonometric Ratios & Identities, T- Ratio of Compound angle, Multiple, sub-multiple angles & some special angles (30%); Transformation formulae, Conditional Identities, Graphs of Trigonometric Functions, Periodicity and extreme values (70%)

PHYSICS

1. A train travels from one station to another at a speed of 40km/hr and returns to the first station at the speed of 60km/hr. The average speed and average velocity are respectively

   1) 48 km/hr, 48km/hr

   2)0,48km/hr

   3) 48 km/hr, 0

   4) 48km/hr, 50km/hr

2. A ball is thrown vertically upwards from the top of a tower of height h with velocity v. The ball strikes the ground after time.

   1) $\frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$ 2) $\frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$

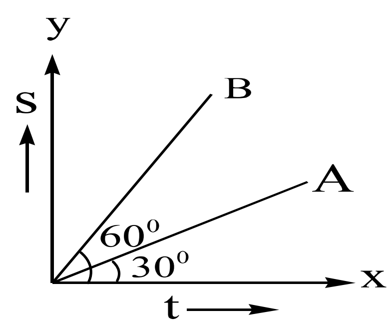
   3) $\frac{v}{g}\left[1+\frac{2gh}{v^2}\right]^{1/2}$ 4) $\frac{v}{g}\left[1-\frac{2gh}{v^2}\right]^{1/2}$

3. A particle is moving up with balloon with constant acceleration (g/8) which starts from rest from ground and at height H particle is dropped from balloon, After this event find time for which particle will be in air

   1) $\sqrt{\frac{H}{g}}$ 2) $2\sqrt{\frac{H}{g}}$

   3) $3\sqrt{\frac{H}{g}}$ 4) $4\sqrt{\frac{H}{g}}$

4. The displacement time graphs of two bodies A and B are shown in figure. the ratio of velocity of A to velocity of B, v_A / v_B is :



   1) $\frac{1}{\sqrt{3}}$ 2) $\sqrt{3}$

   3) $\frac{1}{3}$ 4) 4

5. A body travel 150m in the first two seconds and 210m in the next four seconds. The velocity of the body at the end of 9^{th} sec from the start.

   1) 10m/sec 2) 18m/sec

   3) 15m/sec 4) 20m/sec

6. A stone is thrown upwards. It returns back to earth in 8 sec. find out its location after 5 sec.

(Consider $g = 10m/\sec^2$)

1) 5m from the top

2) 75 m from the bottom

3) Both 1 and 2

4) 5m from the bottom

7. A particle is moving east ward with a velocity of 5m/sec. if in 10sec the velocity changes to 5m/sec north wards. The average acceleration in this time is

1) $0m/\sec^2$ 2) $10m/\sec^2\ N-W$

3) $\frac{1}{\sqrt{2}}m/\sec^2\ N-W$ 4) $\frac{1}{\sqrt{2}}m/\sec^2\ N-E$

8. A point moves along 'x' axis. Its position at time 't' is given by $x^2 = t^2 + 1$. Its acceleration at time 't' is

1) $\frac{1}{x^3}$ 2) $\frac{1}{x}+\frac{t}{x^2}$

3) $\frac{-t}{x^2}$ 4) $\frac{-t^2}{x^3}$

9. A particle starts from rest with uniform acceleration a. its velocity after 'n' second is 'v'. The displacement of the body in the last two second is

1) $\frac{2v(n-1)}{n}$ 2) $\frac{v(n-1)}{n}$

3) $\frac{v(n+1)}{n}$ 4) $\frac{2v(2n+1)}{n}$

10. A car starts from rest and moves with constant acceleration. The ratio of the distance covered in the nth seconds to that covered in n second is:

1) $\frac{2}{n^2}-\frac{1}{n}$ 2) $\frac{2}{n^2}+\frac{1}{n}$

3) $\frac{2}{n}-\frac{1}{n^2}$ 4) $\frac{2}{n}+\frac{1}{n^2}$

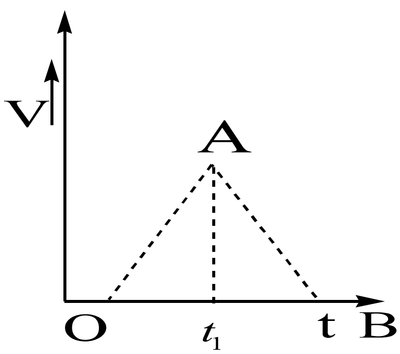
11. A stone is dropped from the top of a tower. When it has fallen by 5m from the top, another stone is dropped from a point 25m below the top. If both stones reach the ground at the same moment, then height of the tower from ground is:

(Take $g = 10m/s^2$)

1) 45 m 2) 50m

3) 60 m 4) 65 m

12. A car accelerates from rest at a constant rate α for some time after which it decelerate at a constant rate β to come to rest. If the total time elaspsed is t sec. the maximum velocity reached by the car is



1) $\frac{\alpha\beta t^2}{\alpha+\beta}$ 2) $\frac{\alpha\beta t}{\alpha+\beta}$

3) $\frac{\alpha+\beta}{\alpha\beta t}$ 4) $\frac{1\alpha\beta}{2(\alpha+\beta)t}$

13. A train starts from rest and moves with uniform acceleration $'\alpha'$ for some time and acquires a velocity v. It then moves with constant velocity for some time and then decelerates at a rate β and finally comes to rest at the next station. If l is the distance between two stations the total time of travel is

1) $\frac{I}{V}+\frac{V}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$ 2) $\frac{I}{V}-\frac{V}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$

3) $\frac{I}{V}+V\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$ 4) $\frac{I}{V}-V\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$

14. Two balls of equal masses are thrown upwards, along the same vertical direction at an interval of 2 seconds, with the same initial velocity of 40 m/s. Then these collide a height of (Take $g=10\,m/s^2$)

1) 120 m 2) 75 m

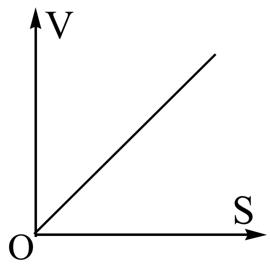
3) 200 m 4) 45 m

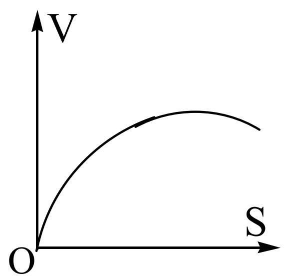
15. A particle is moving with initial velocity $\vec{u}=\hat{i}-\hat{j}+\hat{k}$. What should be its acceleration so that it can remain moving in the same straight line?

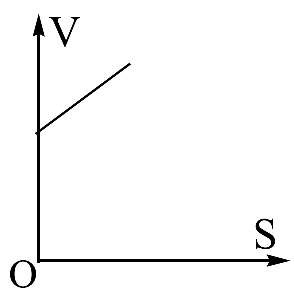
1) $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$ 2) $\vec{a}=-2\hat{i}+2\hat{j}+2\hat{k}$

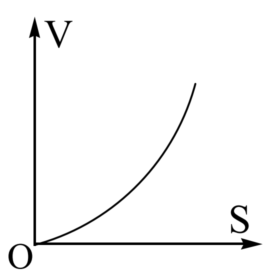
3) $\vec{a}=3\hat{i}+3\hat{j}+3\hat{k}$ 4) $\vec{a}=1\hat{i}-1\hat{j}$

16. A particle starting from rest moves along a straight line with constant acceleration for this velocity displacement graph will have the form-

1) 

2) 

3) 

4) 

17. A particle whose speed is 50 m/s moves along the line from $A(2,1)\, to\, B(9,25)$. Find its velocity in the form of $a\hat{i}+b\hat{j}$

1) $7\hat{i}+24\hat{j}$ 2) $8\hat{i}+24\hat{j}$

3) $14\hat{i}+48\hat{j}$ 4) $i+j$

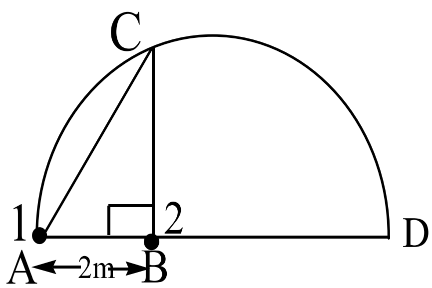
18. If the engine of a train of length $'l'$ moving with constant acceleration crosses an electric pole with a velocity u and the tail of the train crosses the same pole with a velocity v, what is the velocity with which the midpoint of the train crosses the same electric pole?

1) $\frac{v+u}{2}$ 2) $\frac{2vu}{(v+u)}$

3) $\sqrt{\frac{v^2+u^2}{2}}$ 4) $\frac{\sqrt{u^2+u^2}}{2}$

19. A semicircle in the horizontal plane of radius $R=5m$ with diameter AD is shown in figure. Two particles 1 and 2 are at points A and B on shown diameter at $t=0$ and move along segments AC and BC with constant speeds $u_1\, and\, u_2$ respectively.

Then the value of $\frac{u_1}{u_2}$ for both particles to reach point C simultaneously will be:



1) $\frac{5\sqrt{2}}{4}$ 2) $\frac{2\sqrt{2}}{5}$

3) $2\sqrt{2}$ 4) $\sqrt{\frac{5}{4}}$

20. A truck of mass 500 quintal and a car of mass 250 quintal are brought to a hault by application of same breaking force. Find the ratio of distance travelled if they are moving with the same initial velocity.

1) 1: 1 2) 1: 2

3) 2: 1 4) 1: 4

21. A balloon which is ascending at the rate of 12 m/sec is 30.4 m above the ground when a stone is dropped. The tine taken by the stone to reach the ground is (Air resistance is negligible)

1) 3 sec 2) 4 sec

3) 3.5 sec 4) 2 sec

22. A boy sees a bal going up and then back down through a window 2.45 m high. If the total time the ball is in sight is 1 sec, the height above the window the ball rises is

1) 2.45 m 2) 4.9 m

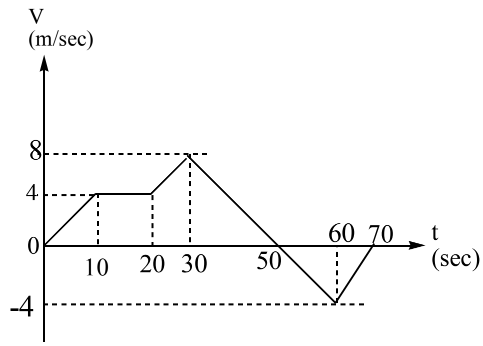
3) 0.3 m 4) 0.49 m

23. A stone is dropped into a well and the sound of impact of stone on the water is heared after 2.056 sec of the release of stone from the top. If velocity of sound in air is 350 m/s. the depth of the well is

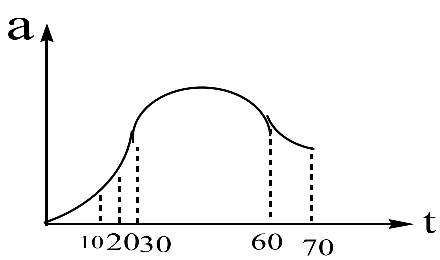
1) 19.6 M 2) 9.8 m

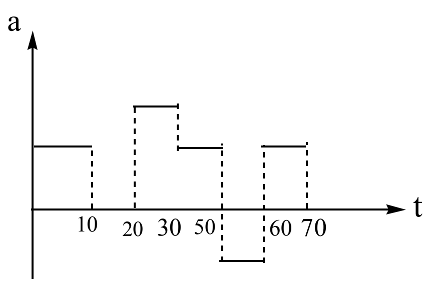
3) 14.7 m 4) 4.9 m

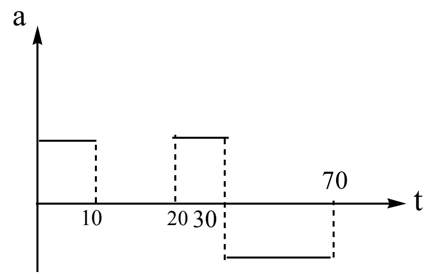
24. A car is moving on a straight road. The velocity of the car varies with time as shown in the figure. initially (at $t=0$), the car was at $x=0$, where, x is the position of the car at any time 't'.

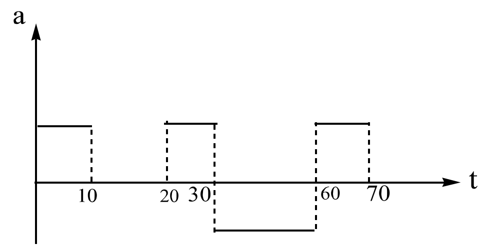


The variation of acceleration (a) with time (t) will be best represented by:

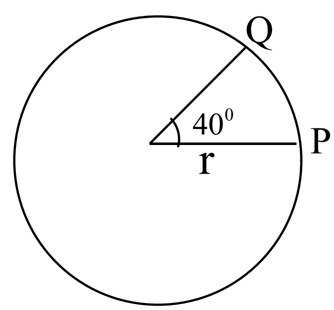
1) 

2) 

3) 

4) 

25. A particle is moving in a circle of radius r with speed v as shown in the figure. The magnitude of change in velocity in moving from P to Q is:



1) $2V\cos 40^0$ 2) $2V\sin 20^0$

3) $2V\cos 20^0$ 4) $V\sin 40^0$

26. The resultant of $\vec{A}$ and $\vec{B}$ makes an angle α with $\vec{A}$ and β with $\vec{B}$, then

1) $\alpha < \beta$ 2) $\alpha < \beta$ if $A < B$

3) $\alpha < \beta$ if $A > B$ 4) $\alpha < \beta$ if $A = B$

27. At the centre of the roof of a cubical room of side 4 m, a bulb is hanging. Taking the origin at one of the corners on the floor, the position vector of the bulb with respect to the origin can be

1) $2\bar{i}+4\bar{j}+2\bar{k}$ 2) $-2\bar{i}+4\bar{j}+2\bar{k}$

3) $2\bar{i}-4\bar{j}+2\bar{k}$ 4) $3\bar{i}+4\bar{j}+2\bar{k}$

28. The ratio of the displacement to distance is always

1) Less than one

2) Greater than one

3) Less than or equal to one

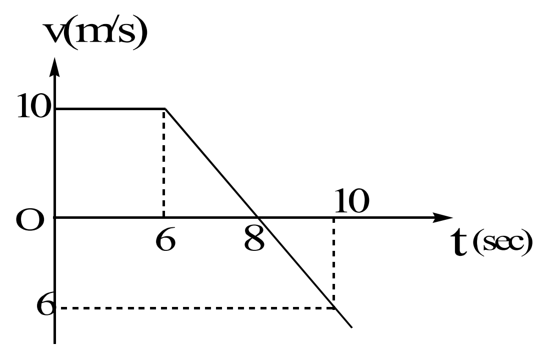
4) Greater than or equal to one

29. A body thrown vertically up from the ground passes the height 10.2m twice at an interval of 10 sec. Its initial velocity was ($g = 10\ m/s^2$)

1) 52 m/s 2) 26 m/s

3) 35 m/s 4) 60 m/s

30. The velocity time graph of a particle is shown in figure. the displacement in 10 sec is



1) 64 m 2) 76 m

3) zero 4) -76 m

CHEMISTRY

31. Consider the following properties.

I) Boiling point

II) Entropy

III) pH

IV) EMF of a cell

V) Volume

VI) Surface tension

Select intensive and extensive properties.

	Intensive	Extensive
1	I, II, III, IV	V, VI
2	I, II, IV	III, V, VI
3	I, III, IV, VI	II, V
4	I, III, IV	II, V, VI

32. The work done (in calories) in adiabatic compression of 2 moles of ideal mono atomic gas against constant external pressure of 2 atm starting from initial pressure of 1 atm and initial temperature of 300K.

($R = 2cal\,k^{-1}\ mol^{-1}$ and $2^{2/5} = 1.31$)

1) 558 2) 720

3) 800 4) 360

33. Calculate the maximum work done by the gas when pressure on 10 gm of H_2 is reduce from 20 to 2 atm at constant temperature of 300K. The gas behaves ideally:

1) 28.72kJ 2) 38.72J

3) 2.872kJ 4) 2.872J

34. A system is taken from state A to state B along two different paths 1 and 2. The heat absorbed and work done by the system along these paths are Q_1 and Q_2 and W_1 and W_2 respectively. Then

1) $Q_1 = Q_2$

2) $W_1 + Q_1 = Q_2 + W_2$

3) $W_1 = W_2$

4) $Q_1 - W_1 = Q_2 - W_2$

35. The heat capacity ratio 'γ' can be determined by a method based on adiabatic expansion of the gas. For a gas initial temperature, pressure and volume are T_1, P_1 and V_1, while the same in final state are T_2, P_2 and V_2. Which of the following is correct expression for the determination of γ.

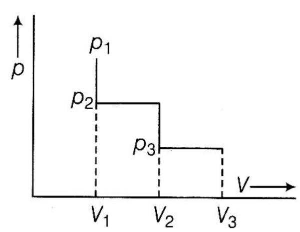
1) $\gamma = \dfrac{\log T_1 - \log T_2}{\log V_2 - \log V_1}$

2) $\gamma = \dfrac{\log V_2 - \log V_1}{\log T_1 - \log T_2}$

3) $\gamma = \dfrac{\log P_1 - \log P_2}{\log V_2 - \log V_1}$

4) $\gamma = 1 + \dfrac{\log P_1 - \log P_2}{\log V_2 - \log V_1}$

36. The following diagram represents the (p-V) changes of a gas



Total work done is

1) $p_2(V_2 - V_1) + p_3(V_3 - V_2)$

2) $p_1(V_2 - V_1) + p_3(V_3 - V_2)$

3) $p_2(V_3 - V_1) + p_3(V_2 - V_1)$

4) $p_2(V_3 - V_2) + p_3(V_2 - V_1)$

37. An ideal gas is expanded isothermally against vaccum till it experiences a change in volume of 50 dm^3. Then , δw and dE respectively are (in J)

1) – 50; Zero 2) Zero; Zero

3) Zero; –50 4) – 50; –100

38. An ideal gas is compressed isothermally until it experiences a change in volume of 20 dm^3 under external pressure of 10atm. Then, Work done (inJ) and change in internal energy respectively are

1) –200; Zero 2) + 20200; Zero

3) –20200; < 0 4) + 20200; > 0

39. 8gm oxygen gas is expanded isothermally at 27°C from 2 dm^3 to 8 dm^3 at a constant external pressure of 4 bar. If the magnitude of work done in this process is used in lifting body of mass 40 kg, determine the mass up to which the body can be lifted? ($g = 10\ ms^{-2}$)

1) 6m 2)4m

3)8m 4) 10m

40. The work done in an open container at 300 K, when 112 g of iron reacts with dil. HCl to form $FeCl_2$ and H_2 gas is- (GMW of Fe=56g)

1)-1.2 k.cal 2) -4 k J

3) -6 k.cal 4) -12 kJ

41. If w_1, w_2, w_3 and w_4 for an ideal gas are work done in isothermal, adiabatic, isobaric and isochoric reversible expansion processes, the correct order will be:

1) $w_1 > w_2 > w_3 > w_4$

2) $w_3 > w_2 > w_1 > w_4$

3) $w_3 > w_2 > w_4 > w_1$

4) $w_3 > w_1 > w_2 > w_4$

42. For an ideal $gas \frac{C_{p.m}}{C_{v.m}} = \gamma$ .The molecular mass of the gas is M, its specific heat capacity at constant volume is:

1) $\frac{R}{M(\gamma - 1)}$ 2) $\frac{M}{R(\gamma - 1)}$

3) $\frac{\gamma RM}{\gamma - 1}$ 4) $\frac{\gamma R}{M(\gamma - 1)}$

43. 10^{23} molecules each having a mass of 10^{-25} kg placed in a 1 lit container, move with a RMS velocity of 10^5 cm/s. Then their total kinetic energy and pressure exerted by them respectively are

1) $10kJ \ \& 3.33 \times 10^6 Pa$

2) $5kJ \ \& 3.33 \times 10^6 Pa$

3) $10kJ \ \& 3.33 \times 10^7 Pa$

4) $5kJ \ \& 3.33 \times 10^7 Pa$

44. Which of the following statements are true or false (T/F) ?

a) In the adiabatic expansion of an ideal gas no cooling effect is observed

b) Rate of diffusion of a gas is directly proportional to its pressure at constant temperature.

c) The order of velocities of a gas at a given temperature is $\overline{C} < Cp < C.$

d) At high temperature the fraction of total no of molecules possessing low velocities is less

1) $a-T, b-F, c-T, d-F$

2) $a-T, b-T, c-F, d-T$

3) $a-T, b-F, c-F, d-T$

4) $a-F, b-F, c-T, d-F$

45. At constant temperature and pressure which one of the following statements is correct for the reaction**?**

$CO\ (g) + 1/2O_2\ (g) \longrightarrow CO_2\ (g)$

1) $\Delta H = \Delta E$
2) $\Delta H > \Delta E$
3) $\Delta H < \Delta E$
4) ΔH is independent physical state of reactant

46. The density of a gaseous substance at 1 atm pressure and 750 K is 0.30 g/lt. If the molecular wt. of the substance is 27 the forces existing among gas molecules is

1) Attractive 2) Repulsive
3) Both 4) None of these

47. Which mixture of gases at room temperature does not obey Dalton's law of partial pressure?

1) $N_2\ \&\ O_2$ 2) $CO\ \&\ CO_2$
3) $NH_3\ \&\ HCl$ 4) $SO_2\ \&\ SO_3$

48. Under identical conditions of temperature, the density of a gas X is three times that of gas Y while molecular mass of gas Y is twice that of X. The ratio of pressures of X and Y will be

1) 6 2) 1/6
3) 2/3 4) 3/2

49. Balloons of 4 litres capacity are to be filled with hydrogen at a pressure of 1 atm and 27°C from an 8 litre cylinder containing hydrogen at 10 atm at the same temperature. The number of balloons that can be filled is

1) 20 2) 18
3) 40 4) 38

50.

Column – I	Column – II
(a) $\left(p+\frac{a}{V^2}\right)(V-b)=RT$	(p) If the force of attraction among the gas molecules be negligible
(b) $pV = RT - \frac{a}{V}$	(q) If the volume of the gas molecules be negligible
(c) $pV = RT + pb$	(r) At normal temperature and pressure
(d) $pV = RT$	(s) At low pressure and at high temperature
	(t) At high pressure

1)A–r;B–q;C–p,t;D–s

2)A–q;B–r;C–p,t;D–s,t

3)A–p,s;B–q;C–r,;D–s

4)A–s;B–r;C–p,t;D–q

51. Air is contained in an open vessel at 27^oC. It is heated to 186^oC when 3 moles of air contained in the vessel have expelled. Assuming there is a 2% increase in the volume of container, the initial number of moles in the vessel are ….

1)9 2)6

3)12 4)18

52. 4 grams of He effused in 10 sec through a pinhole at constant temperature & pressure. Then the amount of CH_4 effused in the same time interval under the same conditions of temperature & pressure would be

1) 1g 2) 8g

3)4g 4)12g

53. Two flasks A and B have equal volumes. A is maintained at 300 K and B at 600 K. while A contains H_2 gas, B has an equal mass of CH_4 gas. Assuming ideal behaviour for the both gases find the ratio of $(u_{av})_A : (u_{av})_B$.

1) 4 2)8

3) 16 4) 2

54. A vessel filled the He gas at a pressure of 4 bar had a small thin orifice through which helium escaped into an evacuated space at the rate of 7.07 mmole/ hr. How long would it take for 10m mole of O_2 to leak through a similar orifice if O_2 were confined at same pressure and temperature (Atomic mass of He=4,O =16)

1) 2hr 2) 3hr

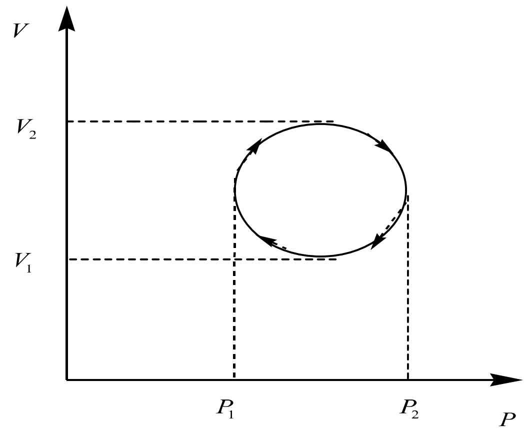
3) 4hr 4) 5 hr

55. A sample of oxygen gas expands its volume from 3L to 5L against a constant pressure of 3atm. If the work done during expansion be used to heat 10mole of water initially present at 290K, its final temperature will be (specific heat of water $= 4.18 JK^{-1}g^{-1}$)

1) 292 K 2) 290.9 K

3) 298 K 4) 293.7K

56. In the cyclic process show in P-V diagram, the magnitude of work done is



1) $\pi\left(\frac{P_2-P_1}{2}\right)^2$ 2) $\pi\left(\frac{V_2-V_1}{2}\right)^2$

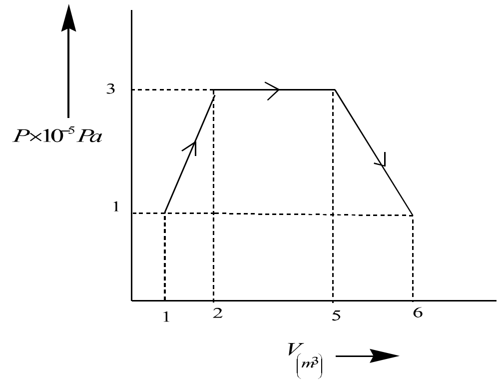
3) $\frac{\pi}{4}(P_2-P_1)(V_2-V_1)$ 4) $\pi(V_2-V_1)^2$

57. Temperature of 1 mole of a gas is increased by 1°C at constant pressure work done is

1) R 2)2R

3) R/2 4)3R

58. The net work done through a series of changes reported in figure for changes ideal gas is



1) $-6\times10^5 J$ 2) $-7\times10^5 J$

3) $-12\times10^5 J$ 4) $+12\times10^5 J$

59. In which cases temperature increases is noticed without providing heat to system?

I) Shaking of liquid in thermos block

II) Adiabatic compression of gas

III) Adiabatic expansion of gas

1) only I,II 2)only I

3) only II 4) I, II, III

60. The slopes of isothermal and adiabatic curve are related as

1) Isothermal curve slope = Adiabatic curve slope

2) Isothermal curve slope = γx Adiabatic curve slope

3) Adiabatic curve slope = γx Isothermal curve slope

4) Adiabatic curve slope = $\frac{1}{2}\times$Isothermal curve slope

MATHS

61. If $\cos x = \tan y, \cot y = \tan z$ *and* $\cot z = \tan x$, *then* $\sin x =$

1) $\frac{\sqrt{5}+1}{4}$ 2) $\frac{\sqrt{5}-1}{4}$

3) $\frac{\sqrt{5}+1}{2}$ 4) $\frac{\sqrt{5}-1}{2}$

62. $\sin x + \sin^2 x + \sin^3 x = 1$ $\Rightarrow \cos^6 x - 4\cos^4 x + 8\cos^2 x =$

1) 4 2) 2

3) 1 4) 0

63. Let $\theta \in \left(0, \frac{\pi}{4}\right)$ *and* $t_1 = (\tan\theta)^{\tan\theta}, t_2 = (\tan\theta)^{\cot\theta}$, $t_3 = (\cot\theta)^{\tan\theta}, t_4 = (\cot\theta)^{\cot\theta}$ then

1) $t_1 > t_2 > t_3 > t_4$ 2) $t_4 > t_3 > t_1 > t_2$

3) $t_3 > t_1 > t_2 > t_4$ 4) $t_2 > t_3 > t_1 > t_4$

64. In a ΔPQR, if 3 sin P+4 cos Q = 6 and 4 sin Q + 3 cos P = 1 then the acute angle 'R' is equal to

1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$

3) $\frac{5\pi}{6}$ 4) $\frac{\pi}{6}$

65. If $\tan\beta = \frac{n\tan\alpha}{1+(1-n)\tan^2\alpha}$ *then* $\tan(\alpha-\beta) =$

1) $(1+n)\tan\alpha$ 2) $(1-n)\tan\alpha$

3) $-(1+n)\tan\alpha$ 4) $-(1-n)\tan\alpha$

66. If $\alpha, \beta, \gamma, \delta$ are the smallest positive angles in ascending order of magnitude which have their sines equal to the positive quantity K the value of $4\sin\left(\frac{\alpha}{2}\right) + 3\sin\left(\frac{\beta}{2}\right) + 2\sin\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\delta}{2}\right) =$

1) $2\sqrt{1-K}$ 2) $2\sqrt{1+K}$

3) $2\sqrt{K}$ 4) $\sqrt{K+1}$

67. If $f_n(x) = \frac{\sin x}{\cos 3x} + \frac{\sin 3x}{\cos 3^2 x} + \frac{\sin 3^2 x}{\cos 3^3 x} + \ldots\ldots + \frac{\sin 3^{n-1} x}{\cos 3^n x}$ then $f_2\left(\pi/4\right) + f_3\left(\pi/4\right) =$

1) 0 2) 1

3) -1 4) 2

68. Let $f_n(\theta) = \tan\frac{\theta}{2}(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)\ldots..(1+\sec 2^n\theta)$, *then* $f_2\left(\frac{\pi}{16}\right) + f_3\left(\frac{\pi}{32}\right) + f_4\left(\frac{\pi}{64}\right) + f_5\left(\frac{\pi}{128}\right) =$

1) 0 2) 2

3) 4 4)8

69. $\cot 16^0.\cot 44^0 + \cot 44^0.\cot 76^0 - \cot 76^0.\cot 16^0 =$

1) 3 2) 0
3) 1 4) 4

70. $\sin\frac{\pi}{14}.\sin\frac{3\pi}{14}.\sin\frac{5\pi}{14}.\sin\frac{7\pi}{14}.\sin\frac{9\pi}{14}.\sin\frac{11\pi}{14}.\sin\frac{13\pi}{14} =$

1) 1/64 2) 3/64
3) 5/64 4) 7/64

71. If $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=\left(a^2-b^2\right)$, and $\frac{ax\sin\theta}{\cos^2\theta}-\frac{by\cos\theta}{\sin^2\theta}=0, then \left(ax\right)^{2/3}+\left(by\right)^{2/3}=$

1) $\left(a^2-b^2\right)^{2/3}$ 2) $\left(a^2+b^2\right)^{2/3}$
3) $\left(a^2+b^2\right)^{3/2}$ 4) $\left(a^2-b^2\right)^{3/2}$

72. If $\left(\tan x-\tan y\right)^2, \left(\tan y-\tan z\right)^2$ *and* $\left(\tan z-\tan x\right)^2$ are in A.P then $\tan x-\tan y, \tan y-\tan z, \tan z-\tan x$ are in

1) A.P 2) G.P
3) H.P 4) A.G.P

73. The value of $2\sin 2^0+4\sin 4^0+6\sin 6^0..........+178\sin 178^0 =$

1) $90.\cos 1^0$ 2) $90.\cot 1^0$
3) $90.\sin 1^0$ 4) 0

74. $0\le a\le 3, 0\le b\le 3$ and the equation $x^2+4+3\cos\left(ax+b\right)=2x$ has at least one solution then the value of (a + b)

1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$
3) $\frac{\pi}{3}$ 4) π

75. If $\frac{\sin^4\theta}{a}+\frac{\cos^4\theta}{b}=\frac{1}{a+b} then \frac{\sin^8\theta}{a^3}+\frac{\cos^8\theta}{b^3}=$

1) $\frac{1}{\left(a+b\right)^3}$ 2) $\frac{1}{\left(a+b\right)^2}$
3) $a+b$ 4) $\frac{1}{a+b}$

76. $3\tan^6 10^0 - 27\tan^4 10^0 + 33\tan^2 10^0 =$

1) 0 2) 1
3) 2 4) 3

77. $\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18} =$

1) $\frac{1}{8}$ 2) $\frac{1}{4}$
3) $\frac{1}{2}$ 4) $\frac{1}{7}$

78. The maximum value of $(\cos\alpha_1)(\cos\alpha_2)......(\cos\alpha_n)$ under the restrictions $0 \le \alpha_1, \alpha_2.......\alpha_n \le \frac{\pi}{2}$ and $\cot\alpha_1.\cot\alpha_2......\cot\alpha_n = 1$ is

1) $\frac{1}{2^{n/2}}$ 2) $\frac{1}{2^n}$

3) $\frac{1}{2n}$ 4) 1

79. If $\begin{matrix} \cos x + \cos y + \cos z = 0 \\ = \sin x + \sin y + \sin z \end{matrix}$ then the possible value of $\cos\left(\frac{x-y}{2}\right) =$

1) $\frac{1}{2}$ 2) $\frac{1}{4}$

3) 1 4) -1

80. Let $a^2 + b^2 = \alpha^2 + \beta^2 = 2$ then the maximum value of $S = (1-a)(1-b) + (1-\alpha)(1-\beta)$ *is*

1) 4 2) 8

3) 2 4) 6

81. In ΔABC, $\operatorname{cosec}\frac{A}{2} + \operatorname{cosec}\frac{B}{2} + \operatorname{cosec}\frac{C}{2}$ *is*

1) $\ge 3\sqrt{3}$ 2) $\ge \frac{3}{2}$

3) ≥ 6 4) $\le \frac{1}{8}$

82. If $\cos(\theta-\alpha), \cos\theta, \cos(\theta+\alpha)$ are in H.P. then $\cos\theta.\sec\frac{\alpha}{2} =$

1) $-\sqrt{2}$ 2) $\sqrt{2}$

3) $\pm\sqrt{2}$ 4) 1

83. If $\alpha = \frac{2\pi}{7}$, *then*

$\tan\alpha \ \tan 2\alpha + \tan 2\alpha \ \tan 4\alpha + \tan 4\alpha \ \tan\alpha =$

1) -1 2) -3

3) -5 4) -7

84. If $a\sin x + b\cos(x+\theta) + b\cos(x-\theta) = d$ then the minimum value of $|\cos\theta|$ is equal to

1) $\frac{1}{2|a|}\sqrt{d^2-a^2}$ 2) $\frac{1}{2|d|}\sqrt{d^2-a^2}$

3) $\frac{1}{2|b|}\sqrt{d^2-a^2}$ 4) $\sqrt{d^2-a^2}$

85. The maximum value of the expression $\left|\sqrt{\sin^2 x + 2a^2} - \sqrt{2a^2 - 1 - \cos^2 x}\right|$, where a and x are real numbers is

1) $\sqrt{3}$ 2) $\sqrt{2}$

3) 1 4) $\sqrt{5}$

86. If

$U = \sqrt{a^2 \cos^2\theta + b^2 \sin^2\theta} + \sqrt{a^2 \sin^2\theta + b^2 \cos^2\theta}$, then the difference between the maximum and minimum values of u^2 is given by

1) $2(a^2+b^2)$ 2) $2\sqrt{a^2+b^2}$

3) $(a+b)^2$ 4) $(a-b)^2$

87. Maximum value of the expression

$\frac{1}{\sin^2\theta + 3\sin\theta.\cos\theta + 5\cos^2\theta} is$

1) 2 2) 3

3) 1 4) ∞

88. If $f(x) = \frac{4\sin\frac{\pi x}{2} + 3\cos\frac{\pi x}{3} - 7}{4 + 2\sin^2\frac{\pi}{14}x + 3\cos\frac{\pi x}{7}}$ then the period of $f(x)$ is

1) 84 2) 56

3) 12 4) none

89. The period of

$f(x) = \tan(x+3x+5x+.....10\,terms)$ is

1) $\frac{\pi}{10}$ 2) $\frac{\pi}{50}$

3) $\frac{\pi}{100}$ 4) π

90. If the period of $f(x) = \frac{\sin(nx)}{\cos\left(\frac{x}{n}\right)} is\ 4\pi$ then the positive integer ‘n’ is

1) 1 2) 2

3) 3 4) 4

Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-MAINS-PAPER-1 Max. Marks: 360

KEY SHEET

PHYSICS

1	**3**	2	**1**	3	**2**	4	**3**	5	**3**	6	**3**
7	**3**	8	**1**	9	**1**	10	**3**	11	**1**	12	**2**
13	**1**	14	**2**	15	**1**	16	**2**	17	**3**	18	**3**
19	**4**	20	**3**	21	**2**	22	**3**	23	**1**	24	**4**
25	**2**	26	**3**	27	**1**	28	**3**	29	**1**	30	**1**

CHEMISTRY

31	**3**	32	**1**	33	**1**	34	**4**	35	**3**	36	**1**
37	**2**	38	**2**	39	**1**	40	**1**	41	**4**	42	**1**
43	**2**	44	**2**	45	**3**	46	**2**	47	**3**	48	**1**
49	**2**	50	**1**	51	**1**	52	**2**	53	**4**	54	**3**
55	**2**	56	**3**	57	**1**	58	**2**	59	**1**	60	**3**

MATHS

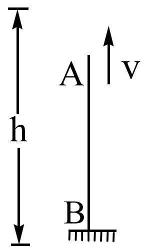
61	**4**	62	**1**	63	**2**	64	**4**	5	**2**	6	**2**
67	**3**	68	**3**	69	**1**	70	**1**	71	**1**	72	**3**
73	**2**	74	**4**	75	**1**	76	**2**	77	**1**	78	**1**
79	**1**	80	**2**	81	**3**	82	**3**	83	**4**	84	**3**
85	**2**	86	**4**	87	**1**	88	**1**	89	**3**	90	**2**

SOLUTIONS
PHYSICS

1. Average speed $= \frac{2v_1v_2}{v_1+v_2} = \frac{2\times40\times60}{100} = 48km/hr$

Average velocity $\frac{Total\ \ displacemnet}{total\,time} = \frac{0}{7} = 0$

For accelerated motion both acceleration & velocity must have same sign

2. 

For AB AB

$s = ut + \frac{1}{2}at^2 \quad -h = vt - \frac{g}{2}t^2 \quad \frac{g}{2}t^2 - vt - h = 0$

$t = \frac{v \pm \sqrt{v^2 + 4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}} \qquad t = \frac{v\pm\sqrt{v^2+2gh}}{g}$

$t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right], t = \frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$ [as time cannot be negative so we neglect it

$\therefore$ we neglect] $\qquad \therefore t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$

3. $u_0 = \sqrt{2(g/8)H} = \sqrt{gH/2} \quad -H = u_0T - \frac{1}{2}gT^2$

4. For A, $\frac{ds}{dt} = V_A = \frac{1}{\sqrt{3}}$ For B, $\frac{ds}{dt} = V_B = \sqrt{3} \quad \frac{V_A}{V_B} = \frac{1}{3}$

5. Let u be the initial velocity of the body consider $S = ut + \frac{1}{2}at^2$ given t = 2sec, and S = 150 m

$\Rightarrow 150 = 2u + 4a\, or\, u + a = 75$ _______ (1)

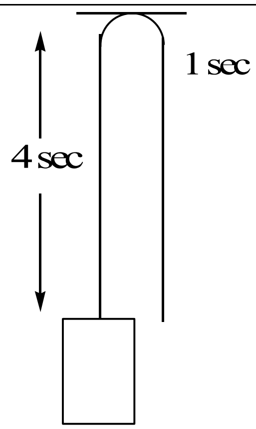
If $t = 4\sec, S = 150 + 210 = 360m$

$\Rightarrow 360 = 6u + 18a\, or\, 4 + 3a = 60$ _________ (2)

$(2)-(1) \Rightarrow 2a = -15\, or\, a = -7.5m/\sec^2$

Put value of a in (1) u = 82.5 m/sec.

$\therefore V = u + at = 82.5 - 7.5\times9 = 15m/\sec$

6. 

Time of flight $= T = \frac{2u}{g} = 8 \Rightarrow u = 4gm/\sec = 40m/\sec$

Maximum height $= \frac{u^2}{2g} = \frac{40\times40}{2\times10} = 80m$

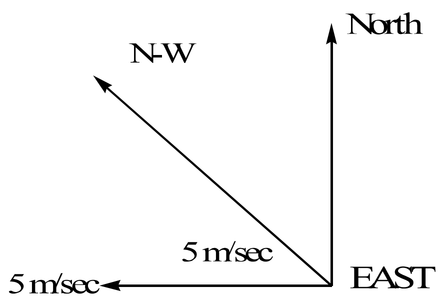
Time of ascent = 4sec Distance travelled in the first second $= \frac{1}{2}gt^2 = \frac{1}{2}\times10\times1 = 5m$

$\therefore$ The body at a distance of 5m from the top or 75m from the bottom

7. Change in velocity $= \sqrt{5^2+5^2} = 5\sqrt{2}$

Acceleration $= \frac{Change\,in\,velocity}{time}$

$= \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}} m/\sec^2 = \frac{1}{\sqrt{2}} m/\sec^2\ N-W$



8. $x = \sqrt{t^2+1}$; $\frac{dx}{dt} = \frac{1}{2\sqrt{t^2+1}}(2t) = \frac{t}{\sqrt{t^2+1}}$

$$a = \frac{d^2x}{xt^2} = \frac{\left(\sqrt{t^2+1}\right) - \frac{t}{2\sqrt{t^2+1}}(2t)}{\left(\sqrt{t^2+1}\right)^2} = \frac{1}{\left(\sqrt{t^2+1}\right)^3} = \frac{1}{x^3}$$

9. distance travelled in last 2 seconds $S_2 = ut + \frac{1}{2}at^2$

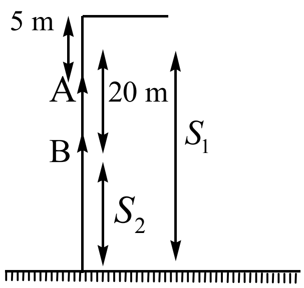
$= 2a(n-2) + 2a = 2a(n-1)$ Also $\frac{v}{n} = a \Rightarrow s = 2\times\frac{v}{n}(n-1)$

10. distance covered nth second is $\frac{a}{2}(2n-1)$

Distance covered in n seconds $=\frac{1}{2}a\,n^2$

This implies ratio $=\frac{2}{n}-\frac{1}{n^2}$

11.



Vel. Of 1st stone when passing at A $\rightarrow$

$V^2=0+2.10.5$ $\quad$ $V=10\,m/s$ $\quad$ $S_1-S_2=20m$.

$$\Rightarrow\left(10.t+\frac{1}{2}10.t^2\right)-\left(\frac{1}{2}10.t^2\right)=20 \qquad t=2s$$

$S_2=\frac{1}{2}.10.4=20m$ $\quad$ $Ht=25+20=45m$.

12. Initial velocity = 0, Let v be the maximum velocity. Hence $v=0+t_1\alpha\Rightarrow t_1=\frac{v}{\alpha}$ ____(1)

And finally it comes to rest hence to rest hence $0=v+(t-t_1)(-\beta)$

$$\Rightarrow t-t_1=\frac{V}{\beta}\,or\,t-\frac{v}{\alpha}=\frac{v}{\beta}\,or\;t=V\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\Rightarrow t=V\left(\frac{\alpha+\beta}{\alpha\,\beta}\right)or\,V=\frac{\alpha\beta\,t}{\alpha+\beta}$$

$t_1=\frac{v}{\alpha};t_3=\frac{v}{\beta}$

13. t_2 = time during which the train moves with constant velocity. Consider the equation

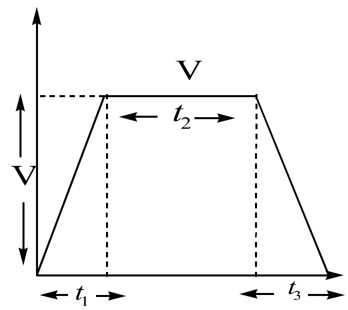
$v^2-u^2=2as;I_1=\frac{v^2}{2\alpha}.I_3=\frac{v^2}{2\beta}$

Consider the equation $s=vt$ then $I_2=vt_2$

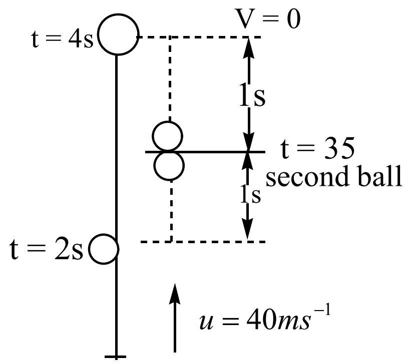
Total distance $=I=I_1+I_2+I_3$

$$=\frac{v^2}{2\alpha}+vt_2+\frac{v^2}{2\beta}or\frac{I}{v}=\frac{v}{2\alpha}+\frac{v}{2\beta}+t_2 \quad \therefore t_2=\frac{I}{v}-\left[\frac{v}{2\alpha}+\frac{v}{2\beta}\right]$$

Total time of travel $t_1=t_1+t_2+t_3$ $\quad =\frac{v}{\alpha}+\frac{v}{\beta}+\frac{I}{v}-\frac{v}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$ $\quad t=\frac{I}{v}+\frac{v}{2}\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$



14.



$u = 40ms,\ g = 10m/s^2$

Let be time taken by the first ball to reach the highest point

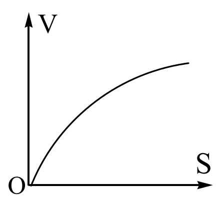
$V = u - gt \quad O = 40 - 10t \quad t = 4s$

After reaching the first ball at the highest point now both will collide after 1 sec as both the balls cover equal distances in opposite directions during 1 sec.

There fore, the height of collision point = height gained by the second ball in 3 sec.

15. To move in a straight line $\vec{a} \| \vec{u}$

16.



17. Position vector of point $A = 2\hat{i} + \hat{j}$

Position vector of point $B = 9\hat{i} + 25\hat{j}$ $\quad \therefore \overrightarrow{AB} = (9\hat{i} + 25\hat{j}) - (2\hat{i} + \hat{j}) = 7\hat{i} + 24\hat{j}$

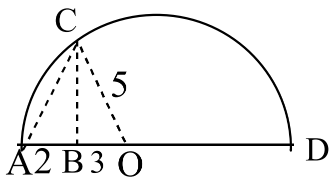
Unit vector in the direction of $\overrightarrow{AB}$ $\quad \widehat{AB} = \frac{\overrightarrow{AB}}{|AB|} = \frac{7\hat{i} + 24\hat{j}}{25}$ $\quad \therefore \vec{V} = 50\,\hat{AB} = 14\hat{i} + 48\hat{j}$.

18. Let a be the acceleration of the train $\therefore v^2 - u^2 = 2as\ (or)\ a = \frac{v^2 - u^2}{2I}$

v_1 = velocity with which mid point of the train crosses the same pole

$V_1^2 = u^2 + 2a.\frac{I}{2}$ $\quad = u^3 + 2.\left(\frac{v^2 - u^2}{2l}\right).\frac{l}{2} \Rightarrow v_1 = \sqrt{\frac{u^2 + v^2}{2}}$

19.



From triangle BCO $\Rightarrow BC = 4$

From triangle BCA $\Rightarrow AC = \sqrt{2^2 + 4^2} = 2\sqrt{5}$

$AC = u_1 t,\ BC = u_2 t$ $\quad \therefore \frac{u_1}{u_2} = \frac{AC}{BC} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{\sqrt{4}}$

20. since they comes to rest, final velocity = v = 0 $a = \left(\frac{-F}{m}\right)$ [as it is retardation]

If S is the distance travelled by the body before coming to rest consider $v^2 + u^2 = 2as$

$0 = u^2 - 2\left(\frac{-F}{m}\right)S$ $\quad \Rightarrow S = \frac{mu^2}{2F} \Rightarrow \frac{S_T}{S_c} = \frac{m_r}{m_c} = \frac{500}{250} = \frac{2}{1}$

[since F and u are same] $\therefore$ car comes to rest in shorter distance.

21. Stone dropped from rising balloon is similar to body projected vertically upwards from the top of a tower.

So consider the equation $h = \frac{1}{2}gt^2 - ut$

i.e., $30.4 = \frac{1}{2} \times 9.8 \times t^2 - 12 \times t$ $\quad 4.9t^2 - 12 \times t - 30.4 = 0$

$49t^2 - 12t - 304 = 0$ $\quad (49t + 76)(t - 4) = 0$ $\quad \Rightarrow t = \frac{-76}{49}\ or\ t = -4\sec$

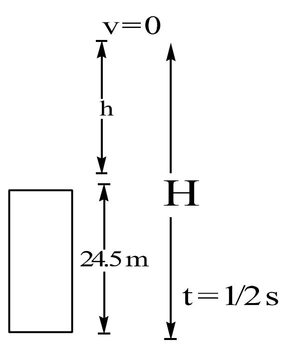
Since time is always positive $t = 4\sec$

22. Given that total time the ball in sight is $1\sec$ $\therefore$ it takes 1/2 sec to go up across the window and equal tine to come down across it. Let h be the height rises by the ball above the window.

Let u be initial velocity at the bottom of the window consider $S = ut - \frac{1}{2}gt^2$

$2.45 = u \times \frac{1}{2} - \frac{1}{2} \times 9.8 \times \frac{1}{4}$ $\quad \Rightarrow 4.9 = u - 2.45\ or\ u = 7.35\,m/\sec$ consider $H = \frac{u^2}{2g}$

$\Rightarrow h + 2.45 = \frac{7.35 \times 7.35}{2 \times 9.5} \Rightarrow h \cong 0.3m$



23. If the depth if well is h and time taken by stone to reach the bottom t_1, then

$h = \frac{1}{2}gt_1^2$ ______(1)

And time taken by sound to come to the surface (velocity of sound is constant)

$t_2 = \frac{h}{350} or\, t_2 = \frac{g_1^2}{700}$ ____ (2)

But according to given problem $t_1 + t_2 = 2.056$ ____ $(3) \Rightarrow t_1 + \frac{gt_1^2}{700} = 2.056$

$9.8t_1^2 + 700t_1 = 1439.2\, or\, 14t_1^2 + 1000t_1 - 2056 = 0$ or $(14t_1 + 1028)(t_1 - 2) = 0$

So $t_1 = 2\, or\ t_1 = \left(\frac{1028}{14}\right)$

Physically not acceptable so $t_1 = 2s$ and so the depth of well h $= \frac{1}{2} \times 9.82 \times 2^2 = 19.6m$

24. a = slope of v-t curve

25. velocity at $Q = -v \sin 40^0 \hat{i} + v \cos 40^0 \hat{j}$

Magnitude of change in velocity $2v \sin 20^0$

26. $\text{Tan}\ \alpha = \frac{B \sin\theta}{A + B\cos\theta} = \frac{1}{\frac{A}{B}\text{cosec}\theta + \cot\theta}$; $\text{Tan}\ \beta = \frac{A\sin\theta}{B + A\cos\theta} = \frac{1}{\frac{B}{A}\text{cosec}\theta + \cot\theta}$

When A < B, $\text{Tan}\ \alpha > \text{Tan}\ \beta \Rightarrow \alpha > \beta$ $\Rightarrow$ C is the answer

27. coordinates of bulb B are (2, 4,2)

Position vector of B with respect 0, $\vec{r}_{BO} = (2 - 0)\ \bar{i} + (4 - 0)\ \bar{j} + (2 - 0)\ \bar{k} = 2\bar{i} + 4\bar{j} + 2\bar{k}$

28. $s / d \le 1$

29. Displacement is same in both cases

$s = ut + 1/2\ at^2$ $\quad 10.2 = ut - \frac{1}{2}(10)t^2$ $\quad \Rightarrow t = \frac{u \pm \sqrt{u^2 - 204}}{10}$

$t_1 = \frac{u - \sqrt{u^2 - 204}}{10}$ and $t_2 = \frac{u + \sqrt{u^2 - 204}}{10}$ $\Rightarrow \Delta t = t_2 - t_1 = 10$ sec

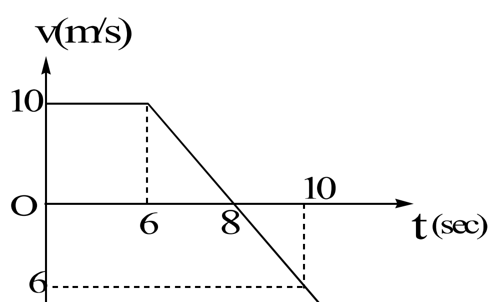
$\sqrt{u^2 - 204} = 50$ $\quad \Rightarrow u^2 = 2500 + 204$ $\quad \Rightarrow u = 52$ m/s

30. Displacement t = Area of OABC –Area of CED = Area of OABD + Area if FBC-Area of CED

$= \left[10 \times 6 + \frac{1}{2} \times 10 \times (2)\right] - \left[\frac{1}{2} \times 2 \times 6\right]$

$= [60 + 10] - [6]$

$= 70 - 6 = 64m$



CHEMISTRY

31. Conceptual

32. Sol: $w = \frac{nR(T_2 - T_1)}{r-1}$

$P_1 T_1 \frac{r}{1-r} = P_2 T_2 \frac{r}{1-r}$

$\left(\frac{300}{T_2}\right)\frac{r}{T-r} = 2 \quad \left(\frac{T_2}{300}\right) = 2^{2/5}$

$T_2 = 300x2^{2/5}$

=393

$w = \frac{2x2x93}{\frac{5}{3}-1} = 558$ cal.

33. Sol:W = 2.303 x $\frac{10}{2}$ x 8.314x 300 log $\frac{20}{2}$

= 28.72 Kj

34. change internal energy is same

35. sol: $PV^{\gamma} = \text{constant}$

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$

$\log \frac{P_1}{P_2} = \gamma \log \frac{V_2}{V_1}$

$\gamma = \frac{\log P_1 - \log P_2}{\log V_2 - \log V_1} \qquad \gamma - 1 = \frac{\log T_1 - \log T_2}{\log V_2 - \log V_1}$

36. Conceptual

37. Conceptual

38. Conceptual

39. W= mgh

40. $W = -\Delta nRT$

41. Conceptual

42. $\frac{C_P}{C_V} = r, C_p - C_v = R$

$C_v = \frac{R}{(r-1)M}$

43. $P = \frac{1}{3}\frac{mnc^2}{v}, K.E = \frac{1}{2}mnc^2$

44.

45. $\Delta n = -ve$

46. $Z = \frac{PV}{nRT}$ $\qquad$ $d = \frac{PM}{RT}$

$= \frac{PM}{dRT} = \frac{1 \times 27}{0.3 \times .082 \times 750} = 1.46$

$\therefore$ $\quad Z > 1$ forces are repulsive

47. Conceptual

48. Conceptual

49. Conceptual

50. Conceptual

51. $\frac{n_1T_1}{V_1} = \frac{n_2T_2}{V_2}$

$= \frac{n_1}{n_2}$

52.

$\frac{4}{w_2} = \sqrt{\frac{4}{16}}$

$w_2 = 8g$

53. Since $(u_{av})_A = \sqrt{\frac{8RT_A}{\pi M_A}} = \sqrt{\frac{8R(300K)}{\pi(0.002kg\ mol^{-1})}}$

$(u_{av})_B = \sqrt{\frac{8RT_B}{\pi M_B}} = \sqrt{\frac{8R\ (600K)}{\pi(0.016\ kg\ mol^{-1})}}$

Therefore, $\frac{(u_{av})_A}{(u_{av})_B} = \sqrt{\frac{300}{2}} \times \sqrt{\frac{16}{600}} = 2$

$\frac{r_{He}}{r_{O_2}} = \frac{P_{He}}{P_{O_2}}\sqrt{\frac{M_{O_2}}{M_{He}}}$

54.

$\frac{n_{He} / t_{He}}{n_{O_2} / t_{O_2}} = \frac{P_{He}}{P_{O_2}}\sqrt{\frac{M_{O_2}}{M_{He}}}$

$\frac{7.07xt_{O_2}}{1x10} = \sqrt{\frac{32}{4}} = \sqrt{8} = 2\sqrt{2}$

$t_{O_2} = \frac{2x14.14}{7.07} = 4hr$

55. $W = P\Delta V = 6; \therefore P\Delta V = mS\Delta T$

$W = 607.8J \quad 607.8 = 180 \times 4.184 \times \Delta T$

$\Delta T = 0.81K$

$\therefore T_{final} = 290.9K$

56. Conceptual

57. Conceptua

58. Net work done = area covered under PV curve

$$= \left[\frac{1}{2}\left(3\times10^5 - 1\times10^5\right)\times(3-1) + 3\times10^5(5-3) + \frac{1}{2}\left(1\times10^5 - 3\times10^5\right)\times(6-5)\right] = -7\times10^5 J$$

59. Conceptual

60. PV=constant for isothermal

$\therefore \log P + \log V = \log C$

$\therefore slope\ of\ \log\ PVs\ \log V\ curves = -1$

$\therefore PV^{\gamma} = const.\ for\ adiabatic$

$\log P + \gamma \log V = \log C^1$

Slope of $\log PV_s\ \log V.Curves = -\gamma$

$\gamma \times$ Slope of isothermal curve = slope of adiabatic curve

MATHS

61. $$\left.\begin{array}{l}\cos x=\tan y\\ \tan z=\cot y\\ \cot z=\tan x\end{array}\right\} multyplying\ weget\ \cos x=\tan x$$

$\Rightarrow \quad \cos^2 x=\sin x$

$\Rightarrow \quad \sin^2 x=\sin x-1=0$

$$\sin x=\frac{-1\pm\sqrt{5}}{2}$$

62. $Sinx\left(1+\sin^2 x\right)=\cos^2 x$

Squaring both side weget

$(1-\cos x)\left(2-\cos^2 x\right)^2=\cos^4 x$

Simplify further

63. $In\left(0,\frac{\pi}{4}\right),\tan\theta<1\ and\ \cot\theta>1$

$(\cot\theta)^{\cot\theta}>(\cot\theta)^{\tan\theta}>(\tan\theta)^{\tan\theta}>(\tan\theta)^{\cot\theta}$

$\therefore t_4>t_3>t_1>t_2$

64. Squaring both the equations and adding

65. $$Tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\ \tan\beta}$$

$$=\frac{\tan\alpha-\dfrac{n\tan\alpha}{1+(1-n)\tan^2\alpha}}{\tan\alpha-\dfrac{n\tan\alpha}{1+(1-n)\tan^2\alpha}}$$

$$=\frac{\tan\alpha(1-n)\tan^3\alpha-n\tan\alpha}{1+(1-n)\tan^2\alpha+n\tan^2\alpha}=(1-n)\tan\alpha$$

66. $\beta=180^0-\alpha,\gamma=360^0+\alpha,\delta=540^0-\alpha$

Then $4\sin\frac{\alpha}{2}+3\sin\frac{\beta}{2}+2\sin\frac{1}{2}+\sin\frac{\delta}{2}$

$=4\sin\frac{\alpha}{2}+3\cos\frac{\alpha}{2}-2\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}$

$=2\sqrt{1+\sin\alpha}=2\sqrt{1+K}$

67. $f_2(x)+f_3(x)$

$=\frac{1}{2}(\tan 9x-\tan x)+\frac{1}{2}(\tan 27x-\tan x)$

$f_2\left(\frac{\pi}{4}\right)+f_3\left(\frac{\pi}{4}\right)=\frac{1}{2}(-1-1)+\frac{1}{2}(-1-1)=-1$

68. $f_n(\theta)=\tan 2^n\theta$

69. Write $\cot\theta=\frac{\cos\theta}{\sin\theta}$ and use $\sin(A+B+C)$

Formula $,A=76,B=16,C=44$

70. Write $\left(\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\right)=\left(\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\right)^2$ and use the formula

$\cos\theta\,\cos 2\theta\,\cos 4\theta......\cos 2^{(n-1)}\theta = \dfrac{2\sin 2^{n-1}}{2^n \sin\theta}$

71. Conceptual

72. Let $a=\tan x-\tan y,\ b=\tan y-\tan z,\ c=\tan z-\tan x$ then $a+b+c=0$

$b=-(a+c);\ b^2=a^2+c^2+2ac$

Given $2b^2=a^2+c^2, b^2=-2ac$

$-b=\dfrac{2ac}{b}=\dfrac{2ac}{-(a+c)}, b=\dfrac{2ac}{a+c}$

$\tan x-\tan y, \tan y-\tan z, \tan z-\tan x$ *are in H.P*

73. Conceptual

74. Conceptual

75. $\dfrac{\sin^4\theta}{a}+\dfrac{\left(1-\sin^2\theta\right)}{b}=\dfrac{1}{a+b}$

$(a+b)^2\sin^4\theta-2a(a+b)\sin^2\theta+a^2=0$

$(a+b)\sin^2\theta=a$

$\therefore Sin^2\theta=\dfrac{a}{a+b}, \cos^2\theta=\dfrac{b}{a+b}$

$Now\dfrac{Sin^8\theta}{a^3}+\dfrac{\cos^8\theta}{b^3}=\dfrac{1}{a^3}\left(\dfrac{a}{a+b}\right)^4+\dfrac{1}{b^3}\left(\dfrac{b}{a+b}\right)^4=\dfrac{a+b}{(a+b)^4}=\dfrac{1}{(a+b)^3}$

76. Let $t=\tan 10^0, \tan 30^0=\dfrac{1}{\sqrt{3}}$

$\dfrac{3t-t^3}{1-3t^2}=\dfrac{1}{\sqrt{3}}$

Squaring $3t^6-27t^4+33t^2=1$

77. $Sin\dfrac{\pi}{7}\sin\dfrac{2\pi}{7}\sin\dfrac{4\pi}{7}$

$=\dfrac{1}{2}\left(\cos\dfrac{\pi}{7}-\cos\dfrac{3\pi}{7}\right)\sin\dfrac{4\pi}{7}$

$=\dfrac{1}{4}\left[\sin\dfrac{5\pi}{7}+\sin\dfrac{3\pi}{7}-\left(\sin\pi+\sin\dfrac{\pi}{7}\right)\right]$

$=\dfrac{1}{4}\left[\sin\dfrac{2\pi}{7}+\sin\dfrac{4\pi}{7}+\sin\dfrac{8\pi}{7}\right]=\dfrac{\sqrt{7}}{8}$

78. $Cot\alpha_1, Cot\alpha_2.......Cot\alpha_n=1$

$Cos\alpha_1.Cos\alpha_2.......Cos\alpha_n=sin\alpha_1, \sin\alpha_2.....\sin\alpha_n$

Multiplying both sides by $\cos\alpha, \cos\alpha_2......\cos\alpha_n$

$\left(\cos\alpha_1.\cos\alpha_2......\cos\alpha_n\right)^2$

$=\dfrac{1}{2^n}\sin 2\alpha_1, \sin 2\alpha_2.\sin 2\alpha_n$

$\le\dfrac{1}{2^n}\Rightarrow \cos\alpha_1.\cos\alpha_2.........\cos\alpha_n\le\dfrac{1}{2^{n/2}}$

79. $\cos x+\cos y=-\cos z.................(1)$

$\sin x+\sin y=-\sin z.................(2)$

Squaring and adding

$$\cos^2 x+\cos^2 y+2\cos x\cos y=\cos^2 z$$
$$\sin^2 x+\sin^2 y+2\sin x\ \sin y=\sin^2 z$$
$$1+1+2\cos(x-y)=1;2(1+\cos(x-y))=1$$
$$2\times 210s^2\left(\frac{x-y}{2}\right)=1,\cos\left(\frac{x-y}{2}\right)=\pm\frac{1}{2}$$

80. $a=\sqrt{2}\ \cos\theta, b=\sqrt{2}\sin\theta$
$$\alpha=\sqrt{2}\cos\phi,\beta=\sqrt{2}\sin\phi$$
$$\Rightarrow s=2-2\left[\sin\left(\theta+\frac{\pi}{4}\right)+\sin\left(\theta+\frac{\pi}{4}\right)\right]+2\left[\cos(\theta-\iota)\right]$$
$$=2+4+2=8$$

81. $Cot70^0+4\cos 70^0=\dfrac{\cos 70^0+4\sin 70^0\cos 70^0}{\sin 70^0}$
$$=\frac{\cos 70^0+2\sin 140^0}{\sin 70^0}$$
$$=\frac{\cos 70^0+2\sin\left(180^0-40^0\right)}{\sin 70^0}$$
$$=\frac{\sin 20^0+\sin 40^0+\sin 40^0}{\sin 70^0}$$
$$=\frac{2\sin 30^0+\cos 10^0+\sin 40^0}{\sin 70^0}=\sqrt{3}$$

82. Apply H.P condition

83. $\sum\tan\alpha.\tan=\alpha=1-\dfrac{1}{\cos\alpha.\cos 2\alpha.\cos 4\alpha}$
$$=1-\frac{(8).\sin\alpha}{\sin 8\alpha}=-7$$

84. $a\ \sin x+2b\ \cos x\ \cos\theta=d$
$$d=a\sin x+2b\cos x\cos\theta$$
$$|d|\le\sqrt{a^2+4b^2\cos^2\theta}$$
$$d^2\le a^2+b^2\cos^2\theta$$
$$\frac{d^2-a^2}{4b^2}\le\cos^2\theta$$
$$|\cos\theta|\ge\frac{\sqrt{d^2-a^2}}{2|b|}$$

85. The expression is max when $2a^2-1-\cos^2 x=0$
$$\cos^2 x=2a^2-1$$
$$\sin^2 x=1-\cos^2 x$$
$$=1-2a^2+1$$
$$=2\left(1-a^2\right)$$
$$2a^2+\sin^2 x=2$$
$$\max\ value=\sqrt{2}-0$$
$$=\sqrt{2}$$

86. Conceptual

87. Maximum value of $a\cos\theta+b\sin\theta+c\ is\ c+\sqrt{a^2+b^2}$ and minimum value is $c-\sqrt{a^2+b^2}$

88. Period $= LCM\{4,6,14,7\} = 84$

89. $f(x) = \tan(x + 3x + 5x + ... + 19x)$

$= \tan(100x)$

Period $= \dfrac{\pi}{100}$

90. $L.CM\left\{\dfrac{2\pi}{n}, 2n\pi\right\} = 4\pi$

$2n\pi = 4\pi$

$= n = 2$