Master JEE CLASSES
Kukatpally, Hyderabad.

JEE-ADVANCE-2015-P1-Model

Max.Marks:264

2015_PAPER-I

IMPORTANT INSTRUCTIONS:

1) This booklet is your Question Paper.
2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
3) Blank spaces are provided within this booklet for rough work.
4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
7) The ORS will be collected by the invigilator at the end of the examination.
8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS**.** Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
11) Darken the bubble **COMPLETELY**.
12) The correct way of darkening a bubble is as:
13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
14) Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble.

IMPORTANT INSTRUCTIONS

Max Marks: 264

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 8)	Questions with Integer Answer Type	4	0	8	32
Sec – II(Q.N : 9 – 18)	Questions with Multiple Correct Choice	4	-2	10	40
Sec – III(Q.N : 19 – 20)	Matrix Matching (+2/-1 for every match)	8	-4	2	16
Total				**20**	**88**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 28)	Questions with Integer Answer Type	4	0	8	32
Sec – II(Q.N : 29 – 38)	Questions with Multiple Correct Choice	4	-2	10	40
Sec – III(Q.N : 39 – 40)	Matrix Matching (+2/-1 for every match)	8	-4	2	16
Total				**20**	**88**

MATHEMATICS:

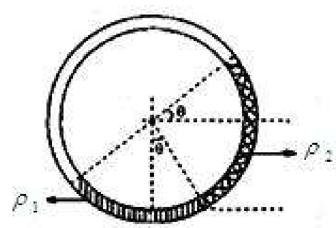
Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 48)	Questions with Integer Answer Type	4	0	8	32
Sec – II(Q.N : 49 – 58)	Questions with Multiple Correct Choice	4	-2	10	40
Sec – III(Q.N : 59 – 60)	Matrix Matching (+2/-1 for every match)	8	-4	2	16
Total				**20**	**88**

PHYSICS **Max Marks: 88**

SECTION – I
(SINGLE INTEGER ANSWER TYPE)

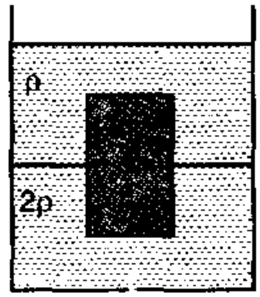
This section contains 8 questions. The answer is a single digit integer ranging from 0 to 9 (both inclusive).
Marking scheme +4 for correct answer, 0 if not attempted and 0 in all other cases.

1. A circular tube of uniform cross-section is filled with two liquids of densities ρ_1 and ρ_2 such that half of each liquid occupies a quarter of volume of the tube. If the line joining the free surface of the liquids makes an angle θ with horizontal, then $\tan\theta = \frac{\rho_1 - k_1\rho_2}{K_2\rho_1 + \rho_2}$, where K_1 and K_2 is integer. Find the numerical value K_1 / K_2?



2. A wooden stick of length L, radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. The minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ is $\pi K R^2 L\rho\left[\sqrt{\frac{\sigma}{\rho}} - 1\right]$. Find the value of K.

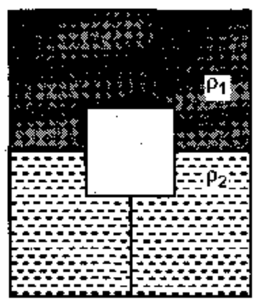
3. A cylinder of uniform cross-section is immersed into two liquids as shown in Fig. Equal volume of cylinder is submerged in both the liquids and density of cylinder is ρ. The magnitude of acceleration of the cylinder at the shown instant is g/K. Where equal volume of cylinder in two liquids. Find the value of K.



4. A liquid is kept in a cylindrical vessel which is rotated about its axis. The liquid rises at the side. If the radius of the vessel is 0.05m and the speed of rotation is 2 rev/sec. Find the difference in the height of the liquid at the centre of the vessel and its sides (in cm). $\left(g = 9.86 m/s^2\right)$

5. A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2, the maximum pressure at any point on the tank is $\sqrt{2}(100 - X)100$ Pascal. Find the value of X. $\left(g = 10 m/s^2\right)$

6. A vessel contains two immiscible liquids of density ρ_1 = 1000kg/m^3 and ρ_2 = 1500 kg/m^3. A solid block of volume V = $10^{-3}m^3$ and density d = 800 kg/m^3 is tied to one end of a string and the other is tied to the bottom of the vessel as shown in fig. The block is immersed with 2/5^{th} of its volume in the liquid of higher density and 3/5^{th} in the liquid of lower density. The entire system is kept in an elevator which is moving upwards with an acceleration of a = g/2. Find the tension (in newtons) in the string. (g = 10 m/s^2)



7. A sphere is just immersed in a liquid. Find the ratio of hydrostatic force acting on bottom half and top half of the sphere.

8. The vertical limbs of a U shaped tube are filled with a liquid of density ρ upto a height h on each side. The horizontal portion of the U tube having length 2h contains a liquid of density 2ρ. The U tube is moved horizontally with an accelerator g/2 parallel to the horizontal arm. The difference in heights in liquid levels in the two vertical limbs, at steady state is $\frac{nh}{7}$ Find n.

SECTION – II
(ONE OR MORE CORRECT ANSWER TYPE)

This section contains 10 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

9. A solid completely immersed in a liquid. The force exerted by the liquid on the solid will

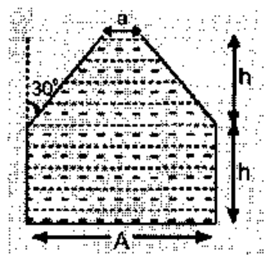
A) Increase if it is pushed deeper inside the liquid

B) Change if its orientation is changed

C) Decrease if it is taken partially out of the liquid

D) Be in the vertically upward direction.

10. The vessel shown in the figure has two sections. The lower part is a rectangular vessel with area of cross-section A and height h. The upper part is a conical vessel of height h with base area 'A' and top area 'a' and the walls of the vessel area inclined at an angle 30^0 with the vertical. A liquid of density ρ fills both the sections upto a height 2h. Neglecting atmospheric pressure (F= force exerted by the liquid on the base of the vessel, w weight of liquid)



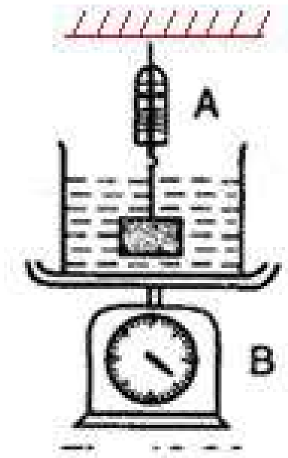
A) the force F exerted by the liquid on the base of the vessel is $2h\rho g\frac{(A+a)}{2}$

B) the pressure P at the base of the vessel is $2h\rho gg\frac{A}{a}$

C) the weight of the liquid W is greater than the force exerted by the liquid on the base

D) the walls of the vessel exert a downward force (F–W) on the liquid

11. The spring balance A reads 2 kg with a block suspended from it. A balance B reads 5 kg when a beaker with liquid is put on the pan of the balance. The two balances are now so arranged that the hanging mass is inside the liquid in the beaker as shown in Fig. In this situation:



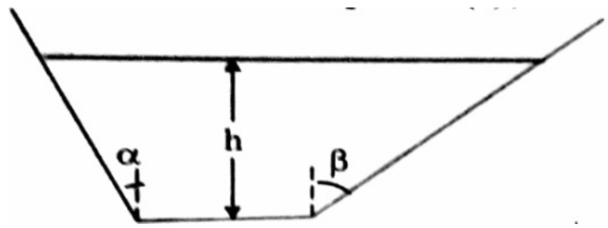
A) the balance A will read more than 2 kg

B) the balance B will read more than 5 kg

C) the balance A will read less than 2 kg and B will read more than 5 kg

D) the balances A and B will read 2 kg and 5 kgrespectivel

12. A liquid is filled up to height h in a vessel, as shown. Find correct (s);



A) If $\alpha = \beta$, horizontal component of forces on left and right side of inclined faces will be equal and opposite.

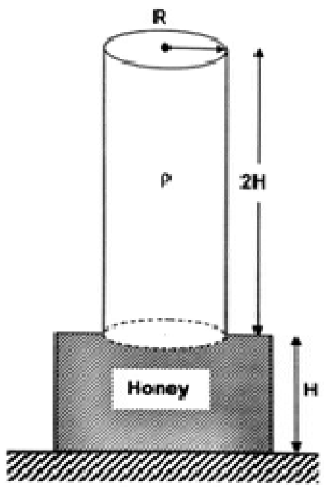
B) If $\alpha \neq \beta$, horizontal component of forces on left and right side of inclined faces will be equal and opposite.

C) If A is the area of the base of the vessel, then force exerted by liquid on walls of the vessel is greater than $\left(P_{atm} + \rho gh\right)A$

D) As above, the force exerted by liquid on walls is equal to $\left(P_{atm} + \rho gh\right)A$

13. A bottle is kept on the ground as shown in the figure. The bottle can be modeled as having two cylindrical zones. The lower zone of the bottle has a cross-sectional radius of $R\sqrt{2}$ and is filled with honey of density 2ρ. The upper zone of the bottle is filled with the water of density ρ and has a cross-sectional radius R. The height of the lower

zone is H while that of the upper zone is $2H$. If now the honey and the water parts are mixed together to form a homogeneous solution: (Assume that total volume does not change)



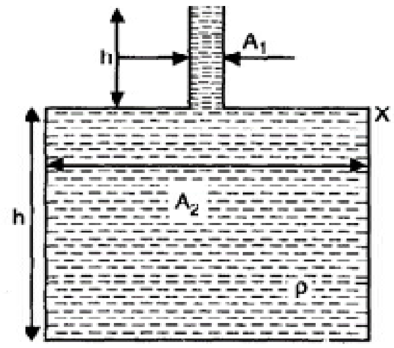
A) The pressure inside the bottle at the base will remain unaltered

B) The normal reaction on the bottle form the ground will remain unaltered

C) The pressure inside the bottle at the base will increase by an amount (1/2) ρgH

D) The pressure inside the bottle at the base will decrease by an amount (1/4) ρgH

14. The vessel shown in the Fig. has two sections of areas of cross-section A_1 and A_2. A liquid of density ρ fills both the sections, up to a height h in each. Neglect atmospheric pressure.



A) the pressure at the base of the vessel is $2h\rho g$

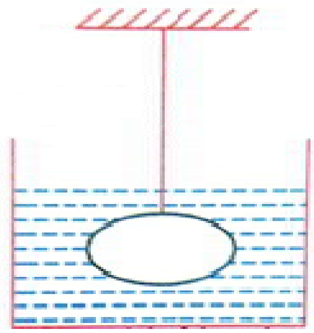
B) the force exerted by the liquid on the base of the vessel is $2h\rho gA_2$

C) the weight of the liquid is $< 2h\rho gA_2$

D) the walls of the vessel at the level X exert a downward force $h\rho g(A_2 - A_1)$on the liquid

15. A solid sphere of mass m, is suspended by means of a string in a liquid as shown. The string has some tension. Magnitudes of net force due to liquid on upper hemisphere

and that on lower hemisphere are F_A and F_B respectively.Which of the following is/are true.



A) Density of material of the sphere is greater than density of liquid

B) Difference of F_B and F_A is dependent of atmospheric pressure

C) F_B-F_A=mg

D) F_B-F_A<mg

16. When a body is weighed in a liquid, the loss in its weight is equal to:

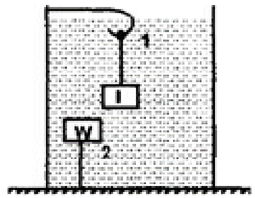
A) weight of liquid displaced by the body

B) weight of water displaced by the body

C) the difference in weights of body in air and liquid

D) the up thrust of liquid on the body

17. An iron block and a wooden block are positioned in a vessel containing water as shown in the figure. The iron block (I) hangs from a massless string with a rigid support from the top while the wooden block floats being tied to the bottom through a massless string. If now the vessel starts accelerating upwards



A) tension in the string 1 will increase.

B) tension in both the strings will increase.

C) tension in both the strings will decrease.

D) tension in the string 2 will remain constant.

18. A piece of ice is floating in a liquid(water soluble liquid) in a container. What happens to the level of liquid when all ice melts?

A) Level remains same if liquid is water

B) Level will fall if liquid is water

C) Level will rise if liquid is denser than water

D) Level will rise if liquid is lighter than water

SECTION - III
(MATRIX MATCHING ANSWER TYPE)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and four statements (P, Q, R and S) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in Q and R, then for the particular question darken the bubbles corresponding to Q and R in the OMR sheet. For each correct matching **will be awarded +2 marks ONLY and 0 if not attempted and -1 in all other cases.**

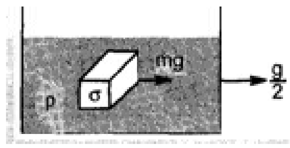
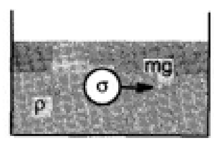
19. Match the following columns

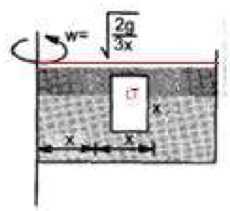
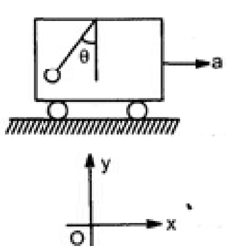
Bucket A contains only water, an identical bucket B contains water, but also contains a solid object in the water. Match the Column-I with Column-II.

(A) The object floats in bucket B, and the buckets have the same water level.	(P) Bucket A has more weight.
(B) The object floats in bucket B and the buckets have the same volume of water.	(Q) Both buckets have the same weight.
(C) The object sinks completely in buckets B, and the buckets have the same water level.	(R) Bucket B has more weight.
(D) The object sinks completely in bucket B, and the buckets have the same volume of water.	(S) The answer cannot be determined from the information given.

space for rough work

20. Match the following columns (liquid is non viscous)

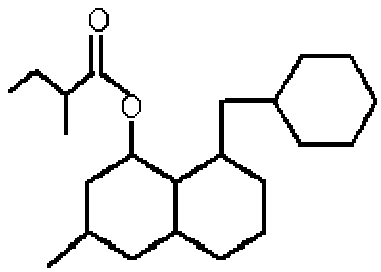
<table>
<tr><td>(A) A cube of mass m and density σ is pulled by a force $\vec{F}_x = mg\hat{i}$ in an accelerating liquid of density ρ. The value of $\frac{a_x}{a_y}$ is
($\sigma > \rho, a_x, a_y$ with respect to ground)

</td><td>(P) $\frac{\rho}{\sigma}$</td></tr>
<tr><td>(B) The sphere of mass m is pulled horizontally by a force F_x = mg in a non accelerating liquid.
The value of $\frac{a_x}{a_y}$ is $(\sigma > \rho)$

</td><td>(Q) 1 : 1</td></tr>
<tr><td>(C) A solid cube of side X is placed in a tube of rotating liquid such that its closer surface is at a distance of X from the</td><td>(R) $\frac{\sigma}{\sigma - \rho}$</td></tr>
</table>

axis. The value of $\frac{a_x}{g}$, where a_x is the horizontal acceleration of the cube away from the axis, is  σ = density of solid cube ρ =density of the liquid	
(D) The string of a pendulum makes on angle θ with vertical when the cart moves with an acceleration a. If the cart is filled with liquid of density ρ = σ (density of the bob), the value of the angle of inclination with vertical becomes θ', then $\frac{\tan\theta}{\tan\theta'}$ is  	(S) $\frac{2\sigma+\rho}{2(\sigma-\rho)}$

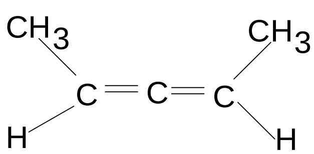
SECTION – I
(SINGLE INTEGER ANSWER TYPE)

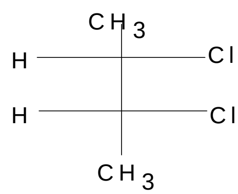
This section contains 8 questions. The answer is a single digit integer ranging from 0 to 9 (both inclusive).
Marking scheme +4 for correct answer, 0 if not attempted and 0 in all other cases.

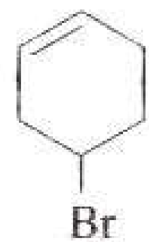
21. How many chiral centres are present in the following compound?

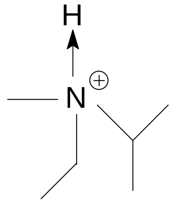
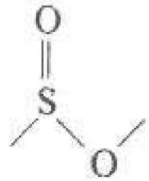
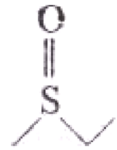


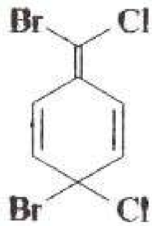
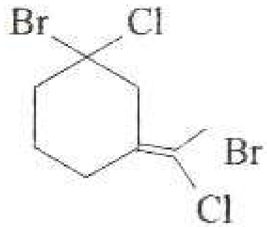
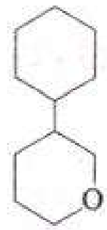
22. Examine the following structural formulae and select those that are chiral.

a) 

b) 

c) 

d)  e)  f) 

g)  h)  i) 

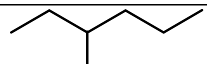
23. A 2:3 racemic mixture (by weight) has an observed specific rotation of 16º. The specific rotation of the optically pure component is $x \times 10$ degrees where x is

24. How many of the following acids are more acidic than acetic acid?

$FCH_2 - COOH$, $DCH_2 - COOH, HOCH_2COOH, C_6H_5COOH, O_2N - CH_2COOH$

25. Total number of stereo isomers $\left(CH_3CH(D)\right)_3 CH$ for the given compound is/are

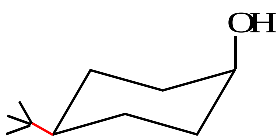
space for rough work

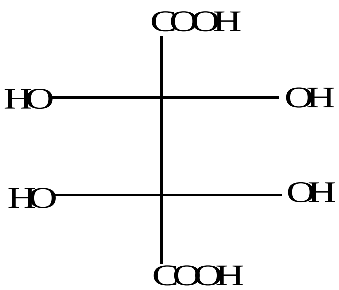
26.  on monochlorination give X number of isomeric products which on fractional distillation give Y number of isomeric products. Find the value of $|X-2Y|$?

27. How many statements are correct?

1. Trans cycloctene is chiral due to presence of chiral (stereo) axis.

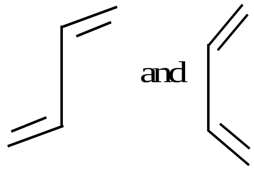
2. Bulkiness order at ortho positions in Biphenyl system follow $-Br > -CH_3 > -Cl > -NO_2$

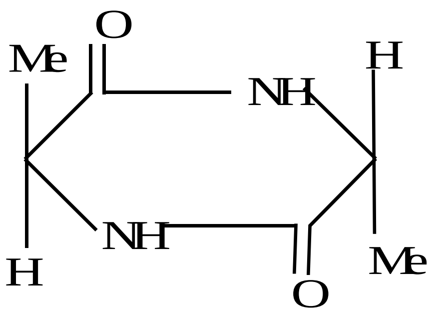
3.  is an optically active compound

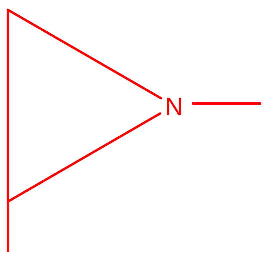
4.  is a meso compound due to plane of symmetry

28. Find the number of incorrect statements

I. Meso tartaric acid has different melting point than (+) Tartaric acid

II.  are conformational diastereomers

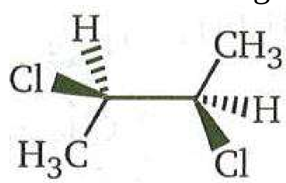
III.  is optically active molecule

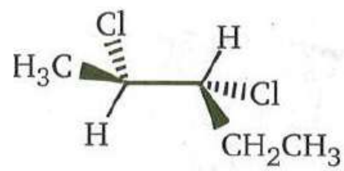
IV.  this molecule has chiral centre and shows optical activity

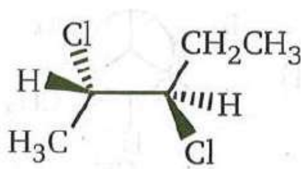
SECTION – II
(ONE OR MORE CORRECT ANSWER TYPE)

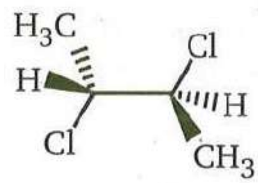
This section contains 10 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

29. Which of the following are not meso compounds?

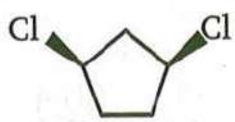
A) 

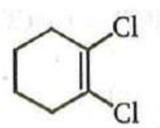
B) 

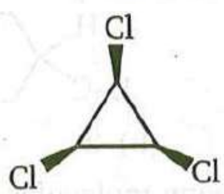
C) 

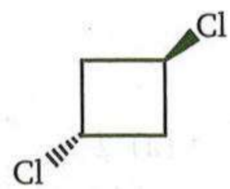
D) 

30. Which of the following compounds has a zero dipole moment ?

A) 

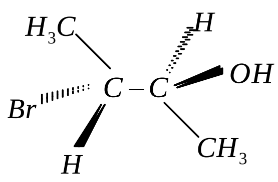
B) 

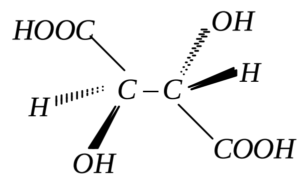
C) 

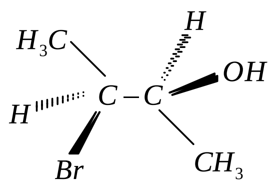
D) 

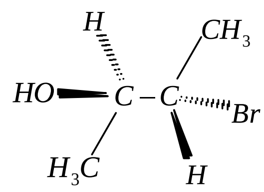
space for rough work

31. In the following structures

(I) 

(II) 

(III) 

(IV) 

find the correct statements.

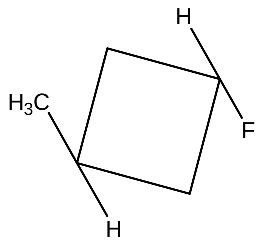
A) Structure II is a meso compound

B) Structures I & III are diastereomers

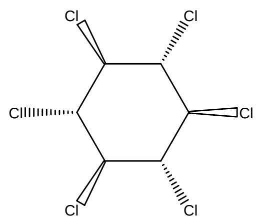
C) Structures III & IV are diastereomers

D) Structures I & IV are enantiomers

32. Plane of symmetry present in

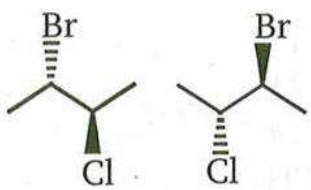
A) 

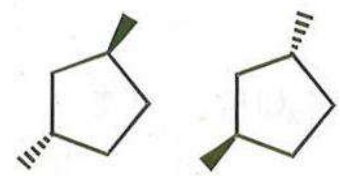
B) 2R, 3S 2,3 dichlorobutane

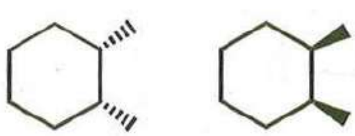
C) 

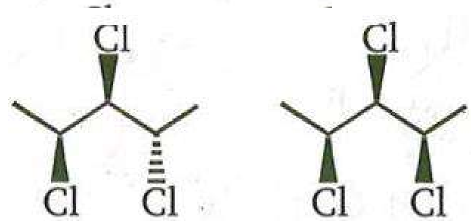
D) $(ph)_3 CH$

33. Which of the following pairs of compounds are not pair of enantiomers?

A) 

B) 

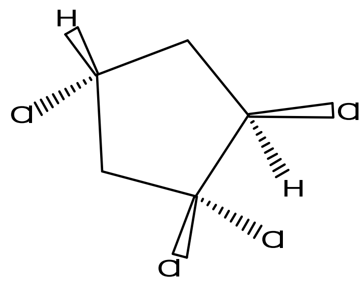
C) 

D) 

34. S-Malic acid at a concentration of $5.0g\ mL^{-1}$ in a 10cm polarimeter cell gave an observed optical rotation of 75° at $25^{\circ}C$. What is the specific rotation of S-Malic acid?

A) $> 25^{\circ}$ B) $> 50^{\circ}$ C) $< 25^{\circ}$ D) $< 10^{\circ}$

35. CIP-stereochemical designations used for the following substance



A) 2R B) 3S C) 4R D) 4S

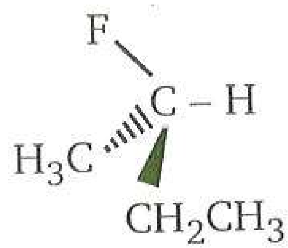
36. Allenes of the type abC=C=Cab (a,b are non stereogenic different ligands) are optically active due to

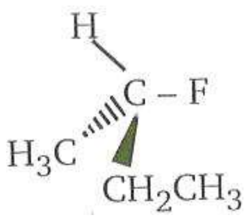
A) chiral centre B) chiral axis

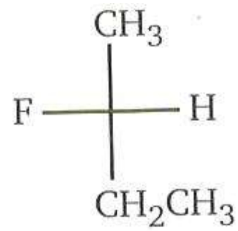
C) chiral plane D) chiral helix

37 The structure of (S) -2- fluorobutane is best represented by

A) 

B) 

C) 

D) 

38 Which of the following are methods of optical resolution ?

A) crystalisation

B) Fractional distillation

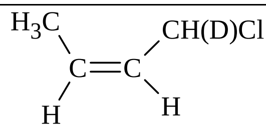
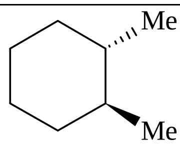
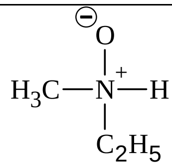
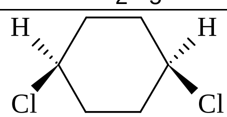
C) Diastereomeric complex formation

D) enantioselective chromatography

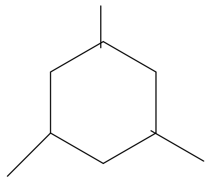
SECTION - III
(MATRIX MATCHING ANSWER TYPE)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and four statements (P, Q, R and S) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in Q and R, then for the particular question darken the bubbles corresponding to Q and R in the OMR sheet. For each correct matching **will be awarded +2 marks ONLY and 0 if not attempted and -1 in all other cases.**

39. Match the Following:-

	Column – I		Column – II
(A)		(P)	Optically active
(B)		(Q)	Optically inactive
(C)		(R)	Plane of symmetry
(D)		(S)	Number of chiral centre = 1
		(T)	Number of of stereocentres = 2

40. Match the Column (I) and (II)

Column – I	**Column – I I(Stereo Isomers)**
A) $H_3C(OH)HC-CH=CH-CH(OH)CH_3$	P) 2
B) 	Q) 4
C) $CH_3-CH=CH-CH=CH-CH_2-CH_3$	R) 6
D) $CH_3-CH=CH-CH=CH-CH-Ph$	S) 8
	T) 12

space for rough work

SECTION – I
(SINGLE INTEGER ANSWER TYPE)

This section contains 8 questions. The answer is a single digit integer ranging from 0 to 9 (both inclusive).
Marking scheme +4 for correct answer, 0 if not attempted and 0 in all other cases.

41. Let $f(x)=2x^3-3(2+p)x^2+12px+\ln\left(16-p^2\right)$. If $f(x)$ has exactly one local maxima and one local minima, then the number of integral values of p is …..

42. A population $P(t)$ of 1000 bacteria introduced into nutrient medium grows according to law $P(t)=1000+\frac{1000t}{100+t^2}$. The maximum size of bacterial population is equal to N then sum of the digits in N is ……

43. The function $f(x)=\sqrt{ax^3+bx^2+cx+d}$ is/has its non zero local minimum and local maxima values at $x=-2$ and $x=2$ respectively. Given 'a' is root of the equation $x^2-x-6=0$.The value of $\left(\frac{a+b+c}{11}\right)$ is equal to …..

44. Let $f(x)=\frac{x^3}{3}+\frac{x^2}{2}+x+2$ and $g'(x)=\left(x^2-9\right)\left(x^2-4x+3\right)\left(x^2-3x+2\right)\left(x^2-2x-3\right)$. If n_1, n_2 and n_3 denote number of points of local minima, number of points of local maxima and number of points of inflection of the function $f(g(x))$ then find the value of $(n_1+n_2+n_3)$.

45. For constant number ‘a’, consider the function $f(x) = ax + \cos 2x + \sin x + \cos x$ on R (the set of real numbers) such that $f(u) < f(v)$ for $u < v$. If the range of ‘a’ for any real numbers u, v is $\left[\frac{m}{n}, \infty\right)$, (m, n or co-prime) then find value of $m - 2n =$ ….

46. Let $A(1,-1), B(4,-2)$ and $C(9,3)$ be the vertices of the triangle ABC. A parallelogram AFDE is drawn with vertices D, E and F on the line segments BC, CA and AB respectively. Find the maximum area of parallelogram AFDE.

47. If $f(x) = \left(\sqrt{4-x^2} - 3\right)^2 + \left(\sqrt{4-x^2} + 1\right)^3$, then the maximum value of $f(x)$ is N then $\frac{N}{7} =$

48. If absolute maximum value of $f(x) = \frac{1}{|x-4|+1} + \frac{1}{|x+8|+1}$ is $\frac{p}{q}$, (p, q are coprime) then $p - q$ is …..

SECTION – II
(ONE OR MORE CORRECT ANSWER TYPE)

This section contains 10 multiple choice questions. Each question has 4 options (A), (B), (C) and (D) for its answer, out of which ONE OR MORE than ONE option can be correct.
Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

49. Let $f(x) = x^2 e^{-x^2}$ then

A) $f(x)$ has local maxima at $x = -1$ and $x = 1$

B) $f(x)$ has local minima at $x = 0$

C) $f(x)$ is strictly decreasing on $x \in R$

D) Range of $f(x)$ is $\left[0, \frac{1}{e}\right]$

50. Let $f(x) = \begin{cases} 2x - 4; & (x \le 2) \\ -x^2 + \dfrac{k^3(k-1)^2}{k^2 - k - 2} + 4; & (x > 2) \end{cases}$, $f(x)$ attains local maximum at $x = 2$, if k lies in

A) $(0,1)$ B) $(3,\infty)$ C) $(-\infty,-1)$ D) $(1,2)$

51. Consider $f(x)=\begin{cases} x^3-3x+3 & ; \quad x\in[-2,0) \\ x+1 & ; \quad x\in[0,5] \\ 1-e^{-x} & ; \quad x>5 \end{cases}$ then

A) $f(x)$ has local maxima at two points

B) $f(x)$ has no local minima

C) Global minima of $f(x)$ does not exist

D) Global maxima value of $f(x)$ is 6

52. The function $f(x)=x^{1/3}(x-1)$

A) has 2 inflection points

B) is strictly increasing for $x>\frac{1}{4}$ and strictly decreasing for $x<\frac{1}{4}$

C) is concave down in $\left(\frac{-1}{2},0\right)$

D) $f''(x)=0$, at $x=\frac{1}{2}$

53. The extremum value of the function $f(x)=\frac{1}{\sin x+4}-\frac{1}{\cos x-4},\ x\in R,$ is

A) $\frac{4}{8-\sqrt{2}}$ B) $\frac{2\sqrt{2}}{8-\sqrt{2}}$ C) $\frac{2\sqrt{2}}{4\sqrt{2}+1}$ D) $\frac{4\sqrt{2}}{8+\sqrt{2}}$

54. Which of the following is true?

A) The least natural number ‘a’ for which $x+ax^{-2}>2,\forall x\in(0,\infty)$ is: 2

B) The least value of the expression $x^2+4y^2+3z^2-2x-2y-6z+14$ is: 2

C) If $\sin\theta+\cos\theta=1$ then the minimum value of $(1+\operatorname{cosec}\theta)(1+\sec\theta)$ is: 9

D) If $a,b\in R$ district numbers satisfying, $|a-1|+|b-1|=|a|+|b|=|a+1|+|b+1|$ then the minimum value of $|a-b|$ is 2.

55. The function $f(x)=2|x|+|x+2|-||x+2|-2|x||$ has a local minimum or a local maximum at x = ……

A) -2 B) $-\frac{2}{3}$ C) 2 D) $\frac{2}{3}$

56. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all 4 – corners. If the total area of removed squares is 100. The resulting box has maximum volume. The lengths of the sides of the rectangular sheet are

A) 24 B) 32 C) 45 D) 60

57. Let $f(x)=(x-1)^4(x-2)^n; n \in N$ then $f(x)$ has

A) a maximum at $x=1$, if n is odd B) a maximum at $x=1$, if n is even

C) a minimum at $x=1$, if n is even D) a minima at $x=2$, if n is even

58. Let S be a square of unit area. Consider any quadrilateral which has one vertex on each side of S, If a, b, c and d denote the lengths of the sides of the quadrilateral.

A) minimum value of $a^2+b^2+c^2+d^2=2$

B) maximum value of $a^2+b^2+c^2+d^2=4$

C) minimum value of $a^2+b^2+c^2+d^2=1$

D) maximum value of $a^2+b^2+c^2+d^2=3$

space for rough work

SECTION - III
(MATRIX MATCHING ANSWER TYPE)

This section contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and four statements (P, Q, R and S) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in Q and R, then for the particular question darken the bubbles corresponding to Q and R in the OMR sheet. For each correct matching **will be awarded +2 marks ONLY and 0 if not attempted and -1 in all other cases.**

59.

	Column – I		Column – II
A)	f(x)= $(x-1)^3(x-2)^5$ has/is	P)	Local maxima
B)	$f(x)=\begin{cases}\sin\left(\frac{\pi x}{4}\right) & ;x\le 2\\ 9-4x & ;x>2\end{cases}$ is/has in $(0,3)$	Q)	Local minima
C)	$f(x)=\{2x\}$ ({.} denotes fractional part of x) is/has in $(0,1)$	R)	Continuous
D)	$f(x)=\begin{cases}\lvert\lvert x-2\rvert-2\rvert & ;x<2\\ [x] & ;x\ge 2\end{cases}$ (where [.] = G.I.F), then $f(x)$ is/has in $(-1,4)$	S)	Non - differentiable
		T)	differentiable

60.

	Column – I		Column – II
A)	$f(x)=2x^3-3(a-3)x^2+6ax+a+2$, where $a\in R$ the set of all values of 'a' for which $f(x)$ has no point of extrema is	P)	$(-\infty,-3)\cup\left(3,\frac{29}{7}\right)$
B)	If the function $f(x)=x^3+3(a-7)x^2+3\left(a^2-9\right)x-1$ has a positive point of maximum, then $a\in$	Q)	$(-\infty,-3)\cup(3,\infty)$
C)	The set of values of 'a' for which the function $f(x)=\frac{ax^3}{3}+(a+2)x^2+(a-1)x+2$ possess a negative point of inflection	R)	$a\in[1,9]$
D)	If $f(x)=x^3-(9-a)x^2+3\left(9-a^2\right)x+7$ has points of extrema which are of opposite sign, then parameter 'a' is belongs to	S)	$(-\infty,-2)\cup(0,\infty)$
		T)	$(-\infty,-4)$

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KEY SHEET

PHYSICS

1	**1**	2	**1**	3	**2**	4	**2**	5	**4**
6	**0**	7	**5**	8	**8**	9	**CD**	10	**D**
11	**BC**	12	**ABC**	13	**BC**	14	**ABCD**	15	**AD**
16	**ACD**	17	**AB**	18	**AC**	19	**A-Q B-R C-R D-R**	20	**A-S B-R C-P D-Q**

CHEMISTRY

21	**6**	22	**8**	23	**8**	24	**4**	25	**4**
26	**0**	27	**1**	28	**1**	29	**ABC**	30	**D**
31	**ABCD**	32	**ABC**	33	**ACD**	34	**C**	35	**AC**
36	**B**	37	**C**	38	**ACD**	39	**A-PST B-QRT C-PS D-QRT**	40	**A-R B-P C-R D-S**

MATHS

41	**6**	42	**6**	43	**2**	44	**5**	45	**1**
46	**5**	47	**4**	48	**1**	49	**ABD**		**ACD**
	ACD		**ABC**		**AC**		**ACD**		**AB**
	AC	57	**ACD**	58	**AB**	59	**A-QRT B-PRS**	60	**A-R B-PT C-ST D-PQT**

SOLUTIONS
PHYSICS

1. $P_X = \left(RC\theta + RS\theta\right)\rho_2 g$

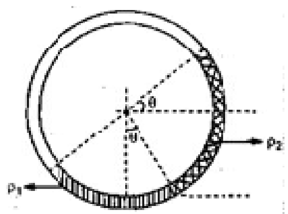
$P_Y = \left(RC\theta - RS\theta\right)\rho_1 g$

$But \quad P_X = P_Y$

$\frac{C\theta + S\theta}{C\theta - S\theta} = \frac{\rho_2}{\rho_1}$

$\tan\theta = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$

$\frac{\theta_1}{\theta_2} = 1$



2. $A = \pi R^2$

$xA\sigma = m + L\rho A$

$\frac{x}{2} = \frac{\left(AL\rho\right)\frac{L}{2}}{m + A\rho L}$

$Solving \; m = \left(\pi R^2\right)L\rho\left(\sqrt{\frac{\sigma}{\rho}} - 1\right)$

3. $F_{Net} = A\left[\rho L - \left(A2\rho + A\rho\right)\frac{L}{2}\right]g$

$\left(A\rho L\right)a = A\rho L\left[1 - \left(\frac{3}{2}\right)\right]g$

$a = \frac{-g}{2} \quad i.e\frac{g}{2}up$

4. $g^h = \frac{1}{2}\left(Rw\right)^2$

Solving h=2 cm

5. $\Delta\rho_{\max} = \rho\left(\sqrt{2}\right)Rg$

=9600 paseal

6. FBD of Block

$$F_{Net} = 0(Vessel \ \ frame)$$

$$T = \left[\frac{3V}{5}\left(10^3\right) + \frac{2V}{5}\left(1.5\times10^3\right) - V\left(0.8\times10^3\right)\right]\left(g+a\right)$$

$$= 6N$$

7. $$F_{4p} - F_{Flat} = \frac{2}{3}\pi R^3 \rho g$$

$$F_{Flat} - F_{down} = \frac{2}{3}\pi R^3 \rho g$$

$$Solving \ \ \frac{F_{up}}{F_{down}} = 5$$

8. Conceptual

9. Conceptual

10. Conceptual

11. Conceptual

12.

13. $$P_q = 2H\rho g + H\left(2\rho\right)g = 4H\rho g$$

Where liquids are mixed

$$\left(\pi R^2\right)2H\rho + \pi\left(2R^2\right)H2\rho = \left[\pi R^2\left(2H\right) + \left(2\pi R^2\right)H\right]\rho^1$$

$$= \ \rho^1 = \frac{3\rho}{2}$$

$$P_f = \left(3H\right)\left(\frac{3\rho}{2}\right)g = \frac{9H\rho g}{2}$$

Bottom

$$\Delta\rho = \frac{H\rho g}{2}$$

14. $$P_{Bot} = 2h\rho g$$

$$F_{Bot} = 2h\rho g A_2$$

$$F_X = \rho_X(A) = h\rho g\left(A_2 - A_1\right)$$

15.

16. Conceptual

17. $T_{IRON}=m(g+a)\left(1-\frac{\sigma}{\rho}\right)$

$T_{wood}=m(g+a)\left(\frac{\sigma}{\rho}-1\right)$

18. Conceptual

19. Conceptual

20. $(A)\leftrightarrow mg=\max=\frac{mg}{2}\left[1+\frac{\rho}{\sigma}\right]$

$\frac{mg}{2}\left(1-\frac{\rho}{\sigma}\right)\max=mg\left(1-\frac{\rho}{\sigma}\right)$

$\frac{a_x}{a_y}=\frac{\sigma+\rho}{2(\sigma-\rho)}$

$(B)\max=\mathrm{mg}$

$\max=\mathrm{mg}\left(1-\frac{\rho}{\sigma}\right)\frac{a_x}{a_y}=\frac{\sigma}{\sigma-p}$

$(c)ma=m\frac{3x}{2}w^2-m\frac{\rho}{\sigma}g=mg\left(1-\frac{\rho}{\sigma}\right)$

$a_x=g\left(1-\frac{\rho}{\sigma}\right)$

$(D)\tan\theta=\frac{ma}{mg}$

$\tan\theta^1=\frac{ma\left(1-\frac{\sigma}{\rho}\right)}{mg\left(1-\frac{\sigma}{\rho}\right)}=\frac{\tan\theta}{\tan\theta^1}=1$

CHEMISTRY

21. Conceptual

22. Conceptual

23. $ee=\frac{1}{5}\times100=20\%$

$\therefore 20=\frac{16}{\alpha}\times100\Rightarrow\alpha=80^0$

$=8\times10\Rightarrow x=8$

24. Except DCH_2-COOH

25. 4 stereo isomers are possible

RRR

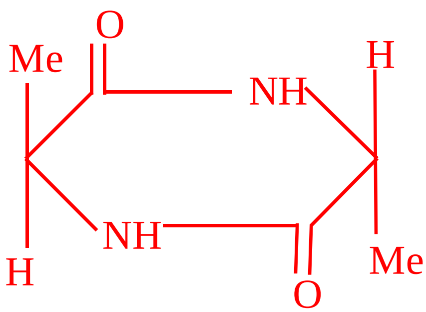
SSS

RRS

SSR

26. X=20, Y=10

27. only option 2 is correct

28. 

is optically active molecule is wrong as it has centre of symmetry

29. Conceptual

30. Conceptual

31. Structure I is $(2S,3R)-3-Bromo-2-$Butanol

Structure II is $(2R,3S)-tartaric\,acid$

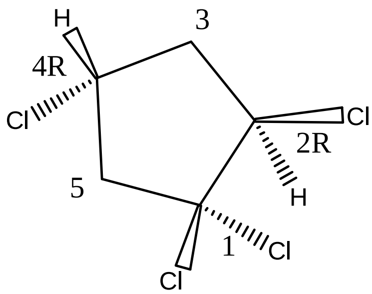
Structure III is $(2S,3S)-3-Bromo-2-$Butanol

Structure IV is $(2R,3S)-3-Bromo-2-$Butanol

32. D has no P.O.S (Propellar shape)

33. Option B is pair of enantiomers

34. $$Specific\,rotation = \frac{Observed\,rotation}{Concentration\left(g\,mL^{-1}\right)\times Length(dm)}$$

35. 

36. Allenes have axial chirality.

37. Conceptual

38. Optical resolution methods.

39. Conceptual.

40. Conceptual.

MATHS

41. We have, $f(x) = 2x^3 - 3(2+p)x^2 + 12px + \ln(16-p^2)$

Now, $f'(x) = 6\left(x^2 - (2+p)x + 2p\right)$

$\therefore f'(x) = 0 \Rightarrow x = 2, p$

As, $f(x)$ has exactly one local maxima and one local minima

So, $p \neq 2$

Also, $16 - p^2 > 0$

$\Rightarrow p^2 - 16 < 0$

$\Rightarrow (p+4)(p-4) < 0$

$\Rightarrow -4 < p < 4$

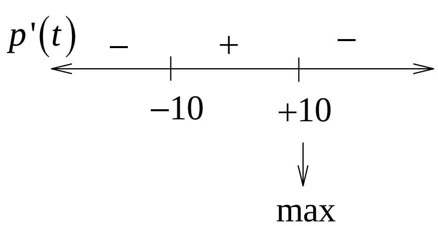
$\therefore$ The possible integral values of p are $-3, -2, -1, 0, 1, 3$

42. We have, $P(t) = 1000 + \dfrac{1000t}{100+t^2}$

$$\Rightarrow P'(t) = \frac{1000\left(100 - t^2\right)}{\left(100+t^2\right)^2}$$

$\therefore Put \;\; P'(t) = 0$

$\Rightarrow t = \pm 10$



So, $P_{\max}(t = 10) = 1050 = N$

sum of the digits of N = 6

43. Since minimum occurs before maximum, so $a<0$

Also, a is root of $x^2-x-6=0$

$\Rightarrow a=-2$

Let $g(x)=ax^2+bx^2+cx+d$

$\Rightarrow g(x)=-2x^3+bx^2+cx+d$

So, $g'(x)=-6x^2+2bx+c=-6(x+2)(x-2)$

$\Rightarrow b=0, c=24$

$\Rightarrow a+b+c=-2+0+24=22$

(Clearly, the equation $ax^2+bx+c=0$ is $-2x^2+0\,.\,x+24=0 \Rightarrow x=\pm 2\sqrt{3}$, so roots of above equation are opposite in sign)

44. $f'(x)=x^2+x+1>0 \forall x \in R$

$g'(x)=(x-3)^3(x+3)(x-1)^2(x-2)(x+1)$

$$\begin{array}{ccccccccccc} + & & - & & + & & + & & - & & + \\ & -3 & & -1 & & 1 & & 2 & & 3 & \end{array}$$

$n(x)=f(g(x))$

$n'(x)=f'\underbrace{(g(x))}_{+ve}g'(x)$

$\therefore n_1=2, n_2=2, n_3=1$

$\Rightarrow n_1+n_2+n_3=5$

45. We have $f(x)=ax+\cos 2x+\sin x+\cos \Rightarrow f'(x)=a-2\sin 2x+\cos x-\sin x$

As $f'(x)\geq 0$ for any real number $x \Rightarrow a\geq 2\sin 2x+\sin x-\cos x$

Let $t=\sin x-\cos x, 1-\sin 2x=t^2 \Rightarrow \sin 2x=1-t^2$

$\therefore\ a\geq -2t^2+t+2$, where $-\sqrt{2}\leq t\leq\sqrt{2}$

$a\geq -2t^2+t+2\Big]_{\max}$, but $2-2t^2+t\Big]_{\max}=\frac{17}{8}$, when $t=\frac{1}{4}$

$a\geq\frac{17}{8}\Rightarrow a\in\left[\frac{17}{8},\infty\right)$

Hence, $(m-2n)=1$

46. Let $AF = x = DE$ and $AE = y = DF$

As ΔCAB is ΔCED

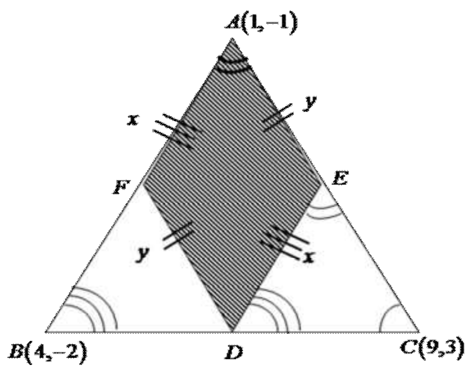
So, $\frac{CE}{CA}=\frac{DE}{AB} \Rightarrow \frac{b-y}{b}=\frac{x}{c} \Rightarrow y=b\left(1-\frac{x}{c}\right)$

(Here $BC=a, AC=b$ and $AB=C$)

Now, area of parallelogram AFDE

$= S = (AF)(EM) = xy\sin A \Rightarrow S = x.b\left(1-\frac{x}{c}\right)\sin A$ [Note: sin A is fixed]

Now, differentiating both sides of equation (i) with respect to x, we get



$\frac{dS}{dx}=\frac{b}{c}(c-2x)\sin A = 0 \Rightarrow x=\frac{c}{2}$

Also, $\left.\frac{d^2S}{dx^2}\right]_{x=\frac{c}{2}} = \frac{-2b}{c} < 0$

So, S is maximum when $x=\frac{c}{2}$

Now, $S_{\max}=\frac{1}{4}bc\sin A$

$=\frac{1}{2}\left(\frac{1}{2}bc\sin A\right)=\frac{1}{2}\left[area(\Delta ABC)\right]=\frac{1}{4}\left|\begin{vmatrix} 1 & -1 & 1 \\ 4 & -2 & 1 \\ 9 & 3 & 1 \end{vmatrix}\right| = 5(sq)$

47. $f(x)=\left(\sqrt{4-x^2}-3\right)^2+\left(\sqrt{4-x^2}+1\right)^3$

$$\left(\begin{array}{ll} & Let\ \sqrt{4-x^2}=a \\ \Rightarrow & a\in[0,2]\ for x\in[-2,2]\end{array}\right)$$

$$f(a)=(a-3)^2+(a+1)^3$$

$$\Rightarrow \quad f'(a)=3a^2+8a-3$$

$$\Rightarrow \quad f'(a)=0$$

$a=\frac{1}{3},-3$ (to be rejected)

$$f_{\max}=\max\left\{f(0),f(2),f\left(\frac{1}{3}\right)\right\}$$

$$=\max\left\{10,28,\frac{256}{27}\right\}=28$$

48. $g(x)=\frac{1}{|x-4|+1}, g(x+12)=\frac{1}{|x+8|+1}$

When $x<-8$, both $g(x)$ and $g(x+12)$ are increasing, hence maximum value cannot occur in this interval. ‘

Similarly, for $x\in(4,\infty)$ Both $g(x)$ and $g(x+12)$ are decreasing.

Hence maximum value cannot occur in this interval.

now, for all values of $x\in(-8,4)$

$$f(x)=\frac{1}{x+9}+\frac{1}{5-x}$$

$$\Rightarrow f'(x)=\frac{14(2x+4)}{(x+9)^2(5-x)^2}$$

$-\quad +$

-2

minimum

$\therefore$ minimum at $x=-2$, and maximum occurs at $x=4$ or $x=-8$

Here $f(4)=f(-8)=\frac{14}{13}=\frac{p}{q}$

$p-q=1$

49. $f(x)=x^2e^{-x^2}$

$f'(x)=2xe^{-x^2}\left(1-x^2\right)$

$$\begin{array}{ccccccccc} & + & & - & & + & & - & \\ -\infty & & -1 & & 0 & & 1 & & \infty \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & \text{max} & & \text{min} & & \text{max} & & \end{array}$$

$f(x)$ has local maximum at $x=-1\,\&\,1$ and local minimum at $x=0$

$f(0)=0$, $f(1)=\dfrac{1}{e}$, As $x\to\infty\Rightarrow f(x)\to 0$

So, Range of $f(x)$ is $\left[0,\dfrac{1}{e}\right]$

50. When $f(x)$ is continuous at $x=2$

$f\left(2^-\right)=f\left(2^+\right)\Rightarrow 0=\dfrac{k^3(k-1)^2}{k^2-k-2}\Rightarrow k=0,1$

$$\begin{array}{ccccc} f'(x) & + & & - & \\ & & 2 & & \\ & & \downarrow & & \\ & & \text{max} & & \end{array}$$

When $f(x)$ is discontinuous at $x=2$

$f'(x)=\begin{cases} 2; & (x\le 2) \\ -2x; & (x>2) \end{cases}$

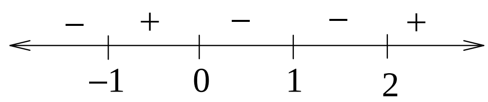
$f'(x)$ does not exist at $x=2$, and $f'(x)$ changes from + to $-$

$\Rightarrow f(x)$ attains maximum at $x=2$

$\Rightarrow \underbrace{\lim_{x\to 2^-} f(x)=f(2)}_{0}>\lim_{x\to 2^+} f(x)$

$\Rightarrow 0>\dfrac{k^3(k-1)^2}{(k-2)(k+1)}$

$\Rightarrow k^3(k-1)^2(k-2)(k+1)<0$ and $k \neq 2,-1$



$k \in (-\infty,-1) \cup [0,2)$

51. $f(x)$ is continuous $[-2,0)$

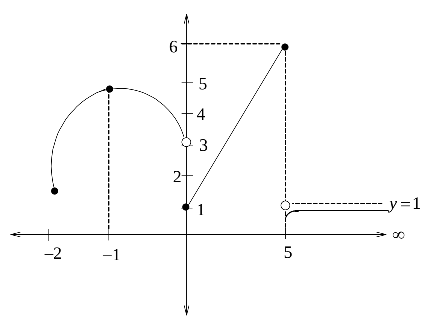
$\Rightarrow f(x)$ has local maximum, at $x=-1$, and discontinuous at $x=5$

But $f(x)$ has local maximum at $x=5$

$\left(f(5) > f\left(5^+\right)\right)$

$\rightarrow$ global minima of $f(x)$ does not exist

$\rightarrow$ global maxima = 6

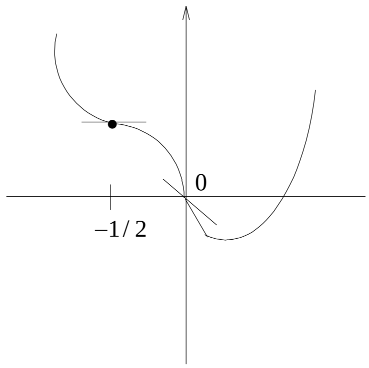


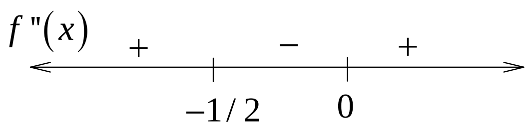
52. $y = x^{1/3}(x-1) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{3x^{2/3}}(4x-1)$, hence f is increasing for $x > \dfrac{1}{4}$ and f is decreasing for $x < \dfrac{1}{4}$ then $f(x)$ has local minimum at $x = \dfrac{1}{4}$

$f'(x) = \dfrac{4x-1}{3x^{2/3}}$ [non existence at x = 0; Vertical Tangent]

$f''(x) = \dfrac{2}{9x^{2/3}}\left(\dfrac{2x+1}{x}\right)$

$f''(x) = 0 \Rightarrow$ at $x = \dfrac{-1}{2}$ (inflection point)





53. $f(x)=\dfrac{1}{\sin x+4}-\dfrac{1}{\cos x-4}\Rightarrow f'(x)=\dfrac{-\cos x}{(\sin x+4)^2}-\dfrac{\sin x}{(\cos x-4)^2}$

$f'(x)=0\Rightarrow(\sin x+\cos x)\left[1-\sin x\cos x+16+8(\sin x-\cos x)\right]=0$

$\Rightarrow \sin x+\cos x=0$

$\Rightarrow \tan x=-1$

$\Rightarrow x=\dfrac{3\pi}{4}$ or $\dfrac{7\pi}{4}$

Global minimum : $x=2n\pi+\left(\dfrac{3\pi}{4}\right)$

Global maximum : $x=2n\pi+\left(\dfrac{7\pi}{4}\right)$

$M=\dfrac{4}{8-\sqrt{2}},m=\dfrac{4}{8+\sqrt{2}}=\dfrac{2\sqrt{2}}{4\sqrt{2}+1}$

54. a) $x+\dfrac{a}{x^2}>2,\forall x\in(0,\infty)$

$\Rightarrow \dfrac{x}{2}+\dfrac{x}{2}+\dfrac{a}{x^2}>2$

$\therefore AM\geq GM$

$\Rightarrow \dfrac{\frac{x}{2}+\frac{x}{2}+\frac{a}{x^2}}{3}\geq\left(\dfrac{x}{2}.\dfrac{x}{2}.\dfrac{a}{x^2}\right)^{1/3}$

$$\Rightarrow x+ax^{-2}\geq 3\frac{a^{1/3}}{4^{1/3}}>2$$

$$\Rightarrow \frac{3^3 a}{4}>8\Rightarrow a>\frac{32}{27}$$

b) $f(x,y,z)=x^2-2x+1+4y^2-12y+9+3z^2-6z+3+1$

$=(x-1)^2+(2y-3)^2+3(z-1)^2+1$

Hence least value of $f(x,y,z)=1$, when $x=1, y=\frac{3}{2}, z=1$

c) $A.M\geq G.M$

$$\Rightarrow \frac{\sin x+\cos x}{2}\geq\sqrt{\sin\theta\cos\theta}\Rightarrow \sin\theta\cos\theta\leq\frac{1}{4}\Rightarrow xy\leq\frac{1}{4}$$

Let $\sin\theta = x, \cos\theta = y$ and $(1+\cos ec\theta)(1+\sec\theta)\geq p$

$$\Rightarrow\left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\geq p$$

$\Rightarrow (x+1)(1+y)\geq pxy$

$\Rightarrow 1+x+y+xy\geq pxy$

$\Rightarrow 2\geq (p-1)xy$

$\Rightarrow xy\leq\frac{2}{p-1}$ (But $xy\leq\frac{1}{4}$)

$\Rightarrow \frac{2}{p-1}\leq\frac{1}{4} \Rightarrow p-1\leq 8 \Rightarrow p\leq 9$

d) Let $a<b$ and $f(x)=|x-a|+|x-b|, \forall x\in R$

So, $f(x)$ is decreasing in $(-\infty,a]$, constant in $[a,b]$ and increasing in $[b,\infty)$ We have, $f(0)=f(1)=f(-1)$

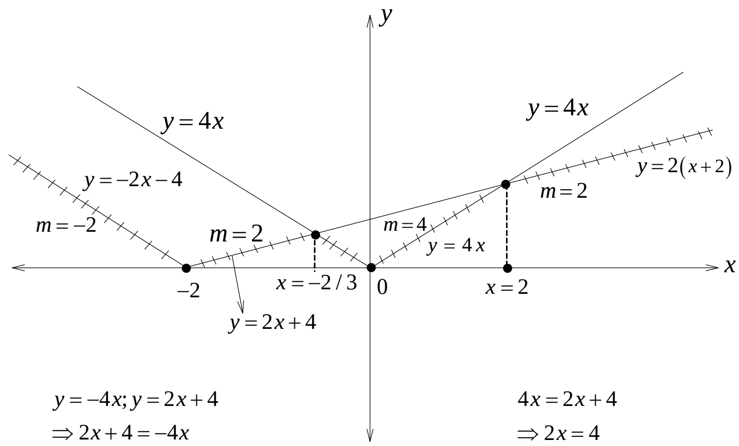
$\Rightarrow \{-1,0,1\}\in[a,b] \therefore |a-b|_{\min}=2$

55. $$\frac{f+g-|f-g|}{2}=\begin{cases}\dfrac{f+g-(f-g)}{2}=g(x)=\min(f,g) & (if\ \ f>g)\\ \dfrac{f+g+f-g}{2}=f(x)=\min(f,g) & (if\ \ f<g)\end{cases}$$

$$\left[\frac{f+g-|f-g|}{2}=\min\left(f(x),g(x)\right)\right]$$

$$\frac{2|x|+|x+2|-\left\|x+2|-2|x\right\|}{2}=\min\left(|x+2|,2|x|\right)$$

$$\Rightarrow 2|x|+|x+2|-\left\|x+2|-2|x\right\|=\min\left(2|x+2|,4|x|\right)$$



$y=-4x; y=2x+4$
$\Rightarrow 2x+4=-4x$
$\Rightarrow 6x=-4$
$\Rightarrow x=-2/3$

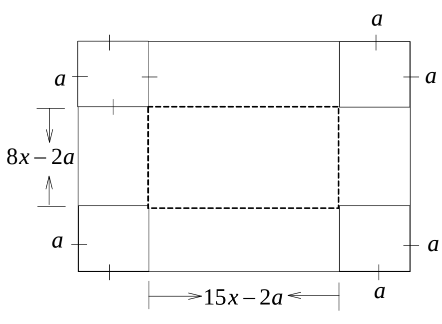
$4x=2x+4$
$\Rightarrow 2x=4$
$\Rightarrow x=2$

no of points of local maximum/minimum = 3

(by observation of slopes ‘m’)

$$\begin{array}{ccccccccc} - & & + & & - & & + & & + \\ & -2 & & -2/3 & & 0 & & 2 & \\ & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ & \text{min} & & \text{max} & & \text{min} & & \text{no change} & \end{array}$$

56.



$$V=lbh=(15x-2a)(8x-2a)(a)$$

$$V=120ax^2-46a^2x+4a^3$$

$$\frac{dv}{da}=120x^2-2.46ax+12a^2$$

$$= 2\left[6a^2 - 46ax + 60x^2\right]$$

$$\frac{d^2v}{da^2} = 12a - 46x$$

$$\frac{dv}{da} = 0 \Rightarrow 6a^2 - 46ax + 60x^2 = 0$$

$$\Rightarrow 6x(x-3) - 5(x-3) = 0$$

$$\Rightarrow x = 3, \frac{5}{6}$$

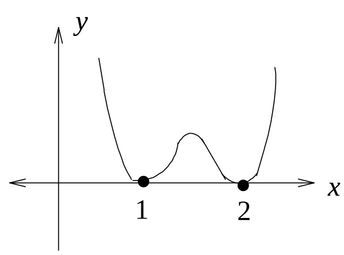
At $x = 3$, $\frac{d^2v}{da^2} = 60 - 46\times 3 < 0 \Rightarrow V$ is maximum, when $x = 3$

At $x = \frac{5}{6}, \frac{d^2v}{da^2} = 60 - 46\times\frac{5}{6} > 0 \Rightarrow V$ is minimum, when $x = \frac{5}{6}$

$\therefore$ Thus sides are $8x = 24$ and $15x = 45$

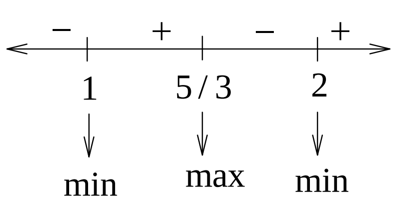
57. Graph of $f(x)$,

When n is even :

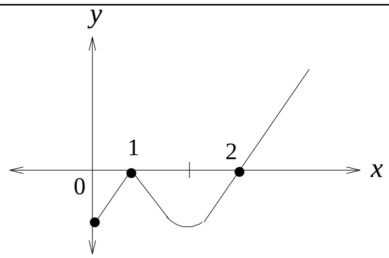


$$f(x) = (x-1)^4 (x-2)^2$$

$$f'(x) = 2(x-1)^3 (x-2)[3x-5]$$



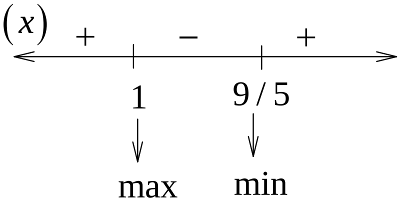
When n is odd:



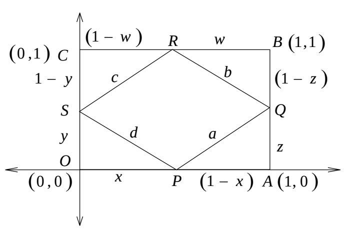
$f(x)=(x-1)^4(x-2)$

$f'(x)=(x-1)^3(5x-9)$

$f'(x)$



58.



$$a^2+b^2+c^2+d^2=\left\{x^2+(1-x)^2\right\}+\left\{y^2+(1-y)^2\right\}+\left\{x^2+(1-x)^2\right\}+\left\{w^2+(1-w)^2\right\}$$

Where $0\le x,y,z,w\le 1$

Let us consider a for

$f(x)=x^2+(1-x)^2 .0\le x\le 1$

$f'(x)=2x-2(1-x)$

$f'(x)=0$, for max/min, when $x=\dfrac{1}{2}$

$\therefore f(x)$ is minimum at $x=\dfrac{1}{2}$, and maximum at $x=1$

$2\le a^2+b^2+c^2+d^2\le 4$

59. a) $f(x)=(x-1)^3(x-2)^5$

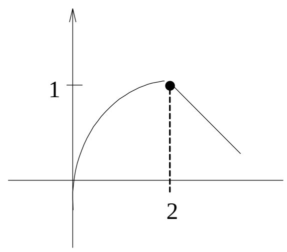
$f'(x)=(x-1)^2(x-2)^4(8x-11)$

at $x=\frac{11}{8}$ minimum

$f|x|$ is continuous and differentiable

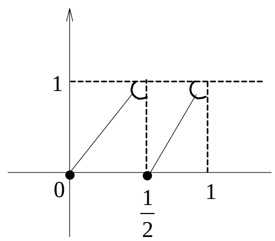
B) $f(x)$ has local maximum at $x=2$ and continuous on $(0,2)$

$f(x)$ is non different at $x=2$ $(B\to p,r,s)$



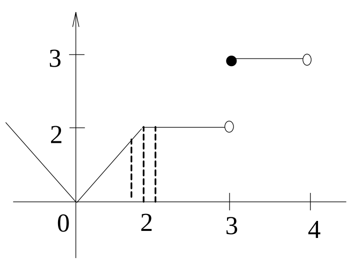
C) $f(x)$ is non differentiable at $x=\frac{1}{2}$ and $f(x)$ has local minimum at $x=\frac{1}{2}$

$(C\to q,s)$



$f\left(\frac{1}{2}\right)<f\left(\frac{1}{2}^{-}\right)$ and $f\left(\frac{1}{2}\right)<f\left(\frac{1}{2}^{+}\right)$

D) $D\to(q,s)$



$f(x)$ is non differentiable at $x=0,2$, 3 and $f(x)$ has minimum at $x=0$

60. A) $f(x)$ has no point of extrema

$\Leftrightarrow$ $f(x)$ is increasing on R

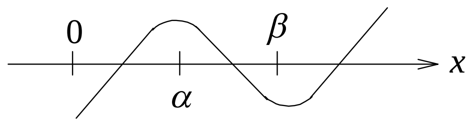
$\Leftrightarrow f'(x)\geq 0, \forall x\in R$

$$\Leftrightarrow x^2-(a-3)x+a\ge 0,\forall x\in R$$

$$\Leftrightarrow \Delta\le 0,\Rightarrow (a-3)^2-4a\le 0$$

$$\Rightarrow (a-9)(a-1)\le 0$$

$$\Rightarrow 1\le a\le 9$$

B) 

$$f(x)=x^3+3(a-7)x^2+3\left(a^2-9\right)x-1$$

$$f'(x)=3x^2+6(a-7)x+3\left(a^2-9\right)$$

$\Rightarrow \Delta>0, f'(0)>0$ and sum of the roots > 0

Condition (1) : $36(a-7)^2-4\times 3\times 3\left(a^2-9\right)>0$

$$\Rightarrow -14a+58>0\Rightarrow a<\frac{29}{7}$$

Condition (2) : $f'(0)>0$

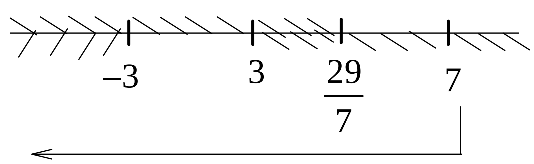
$$\Rightarrow a^2-9>0$$

$$\Rightarrow a\in(-\infty,-3)\cup(3,\infty)$$

Condition (3) : $S_1>0\Rightarrow \dfrac{-6(a-7)}{3}>0$

$$\Rightarrow a-7<0$$

$$\Rightarrow a<7$$



$$a\in(-\infty,-3)\cup\left(3,\frac{29}{7}\right)$$

C) $f(x)=\dfrac{ax^3}{3}+(a+2)x^2+(a-1)x+2$

$f'(x) = ax^2 + 2(a+2)x + (a-1)$

$f''(x) = 2ax + 2(a+2) = 0 \Rightarrow x = \frac{-(a+2)}{a}$

($\because$ since negative point of inflection $\frac{d^2y}{dx^2} = 0$, we get x and put $x < 0$)

$\Rightarrow \frac{-(a+2)}{a} < 0$

$\Rightarrow a \in (-\infty, -2) \cup (0, \infty)$

(or)

For negative point of inflection $\Rightarrow \frac{-b}{2a} < 0$

D) $f(x) = x^3 - (9-a)x^2 + 3\left(9-a^2\right)x + 7$

$f'(x) = 3x^2 - 2(9-a)x + 3\left(9-a^2\right)$

Since x_1 and x_2 are opposite signs

$\Rightarrow x_1x_2 < 0$

$\Rightarrow \frac{3\left(9-a^2\right)}{3} < 0$

$\Rightarrow a^2 - 9 > 0$

$\Rightarrow a \in (-\infty, -3) \cup (3, \infty)$