Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-ADVANCED-2012-P2-Model

Max. Marks: 198

PHYSICS:

Projectile motion on inclined planes,relative velocity (Relative velocity 1D,2D,Rate of approach and rate of separation,condition for collission of two projectiles,relative motion of one projectile with respect to another projectile,rain problems,river boat problems,air craft wind problems etc..)

CHEMISTRY:

Chemical Equilibrium: Equilibrium involving physical processes: solid - liquid; liquid - gas and solid - gas. General characteristics of equilibrium involving physical process, Equilibrium involving chemical processes, characteristics of equilibrium state, Law of mass action, equilibrium constant, characteristics of equilibrium constant, Kp, Kcrelation,Lechatlier's principle - factors effecting equilibrium concentration, pressure, temperature and effect of catalyst, principle applied for synthesis of ammonia and sulphur trioxide and other typical reactions, Thermodynamics of equilibrium constant. Significance of ΔG and ΔG° in chemical equilibrium(60%)

Enthalpy of reaction - different types of enthalpies, Hess's law, Effect of temperature on enthalpy of reaction - Kirchoff's equation, Hess's law and its applications, Determination of enthalpy of hydration of CuSO4, Resonance energy, Limitations of first law of thermodynamics; Second law of Thermodynamics; Carnot cycle, efficiency of thermodynamic system, Entropy - physical significance, expressions in all types of processes ,Gibb's free energy - physical significance, spontneity of processes(30%)

Cumulative Syllabus(10%)

MATHS: Trigonometric Equations (30%); Determinant (60%); Cumulative (10%)

IMPORTANT INSTRUCTIONS **Max Marks: 198**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 8)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : 9 – 14)	Questions with Comprehension Type (3 Comprehensions : 2+2+2 = 6Q)	3	-1	6	18
Sec – III(Q.N : 15 – 20)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 28)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : 29 – 34)	Questions with Comprehension Type (3 Comprehensions : 3+3 = 6Q)	3	-1	6	18
Sec – III(Q.N : 35 – 40)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

MATHEMATICS:

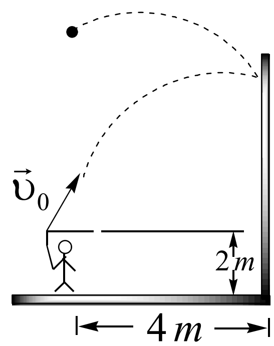
Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : (41 – 48)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : (49 – 54)	Questions with Comprehension Type (3 Comprehensions : 2+2+2 = 6Q)	3	-1	6	18
Sec – III(Q.N : 55 – 60)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

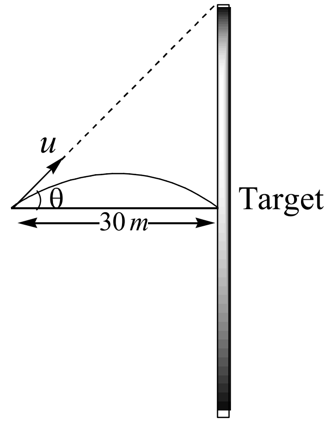
Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

1. A boy throws a ball with velocity $\upsilon_0 = 10\sqrt{2}\ m/s$ at an angle of $45°$ as shown in the figure. After collision with the wall, the vertical component of ball's velocity is unchanged and the horizontal component is reversed in direction. Where does the ball hit the ground from the foot of the wall? $\left[g = 10\,ms^{-2}\right]$



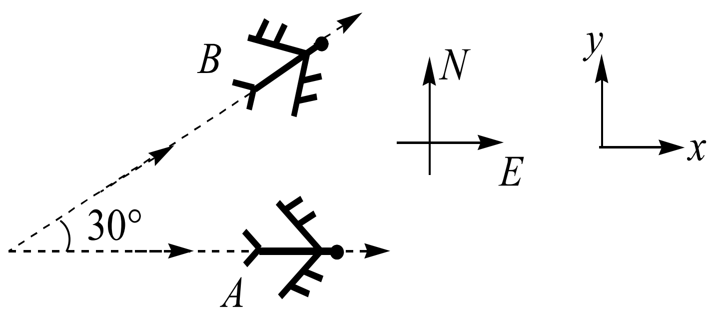
A) $17.8\,m$ B) $18.8m$ C) $19.8m$ D) $16.8m$

2. A bullet with muzzle velocity $100\,m/s$ is to be shot at a target $30\,m$ away in the same horizontal line. How high above the target must the rifle be aimed so that the bullet will hit the target? $\left[g = 10\,ms^{-2}\right]$



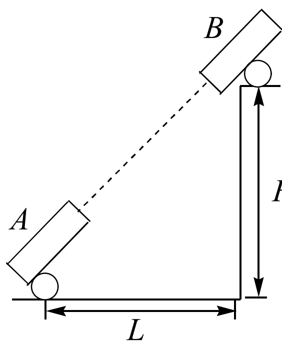
A) $35cm$ B) $45cm$ C) $55cm$ D) $50cm$

3. An aeroplane A is flying horizontally due east at a speed of $400\,km/hr$. Passengers in A, observe that another aeroplane B is moving perpendicular to direction of motion of A. Aeroplane B is actually moving in a direction 30° north of east in the same horizontal plane as shown in the fig. Determine the velocity of B in km/hr.



A) $400\hat{i} + \frac{200}{\sqrt{3}}\hat{j}$ B) $400\hat{i} + 400\hat{j}$ C) $400\hat{i} + \frac{400}{\sqrt{3}}\hat{j}$ D) $400\hat{i} + 200\sqrt{3}\hat{j}$

4. Cannon A is located on a plain at a distance L, from a wall of height H. On top of this wall is an identical cannon (cannon B). Ignore air resistance throughout this problem. Also ignore the size of the cannons relative to L and H. The two groups of gunners aim the cannons directly at each other. They fire at each other simultaneously, with equal muzzle speed υ_0. What is the value of υ_0 for which the two cannon balls collide just as they hit the ground?



A) $\sqrt{\frac{g\left(L^2+H^2\right)}{2H}}$ B) $\sqrt{\frac{2g\left(L^2+H^2\right)}{3H}}$ C) $\sqrt{\frac{2g\left(L^2+H^2\right)}{\sqrt{3}H}}$ D) $\sqrt{\frac{g\left(L^2+H^2\right)}{4H}}$

5. A balloon is ascending vertically with an acceleration of $1ms^{-2}$. Two stones are dropped from it at an interval of $2s$. Find the distance between them $1.5s$ after the second stone is released. ($g = 10\,ms^{-2}$)

A) $45m$ B) $55m$ C) $50m$ D) $65m$

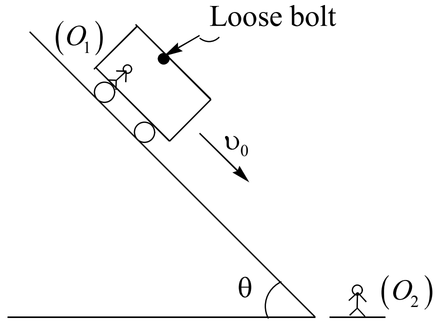
6. Two swimmers leave point A on one bank of the river to reach point B lying straight across on the other bank. The first swimmer reaches point B by swimming at right angles to the stream and then walks the distance that he has been carried away by the stream to get to point B. What was the velocity u of his walking if both the swimmers reached the destination simultaneously? The stream velocity $V_0 = 2\,km/h$ and the velocity V' of each swimmer with respect to water equals $2.5\,km/h^{-1}$.

A) $5\,km.h^{-1}$ B) $4\,km.h^{-1}$ C) $3\,km.h^{-1}$ D) $4.5\,km.h^{-1}$

7. The Telugu Desham Party has to start its procession in an area in Hyderabad where wind is blowing at a speed of $20\,km\,h^{-1}$ along the north – east direction and party flags with the symbols of a cycle, on the vehicles are fluttering. If the procession starts with a speed of $14.14\,km\,h^{-1}$ towards north, find the direction fluttering of flags on the vehicles.

A) $37°$ S of W B) East C) NE D) $53°$ N of E

8. A cart is sliding on a smooth incline. An observer (O_1) is fixed to cart and another observer fixed on ground (O_2) observe, a loose bolt that is released from ceiling. At the instant of release, cart has velocity υ_0 as seen by O_2. Mark the correct option.



A)Trajectory of bolt for O_1 is parabola

B)Trajectory of bolt for O_2 is straight line inclined at an angle θ with vertical

C)Trajectory of bolt for O_2 is a straht line perpendicular to ceiling of cart

D)Trajectory of bolt for O_1 is straight line

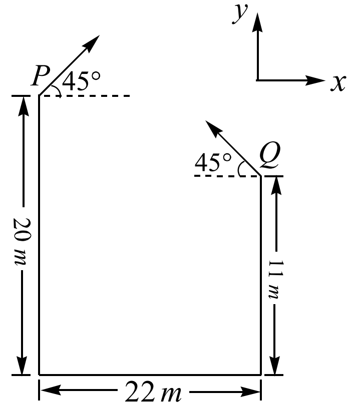
SECTION - II

(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct. Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.**

Paragraph for Questions 9 and 10

Two particles are simultaneously thrown from the roofs of two high buildings as shown in the figure. Their speeds are $V_p = 2\,ms^{-1}$ and $V_Q = 14\,ms^{-1}$.



Then

9. Find the horizontal distance between P and Q after time t. [in m]

A) $9-12\sqrt{2}\,t$ B) $20-8\sqrt{2}\,t$ C) $18-6\sqrt{2}\,t$ D) $22-8\sqrt{2}\,t$

10. Find the minimum distance between them in the process of their motions. [in$m$]

A) 8 B) 10 C) 6 D) 4

Paragraph for Questions 11 and 12

An elevator whose floor to the ceiling distance is $2.50\,m$, starts ascending with a constant acceleration of $1.25\,ms^{-2}$. One second after the start, a loose bolt begins falling from the ceiling of elevator.

11. The free fall time of the bolt is (in s)

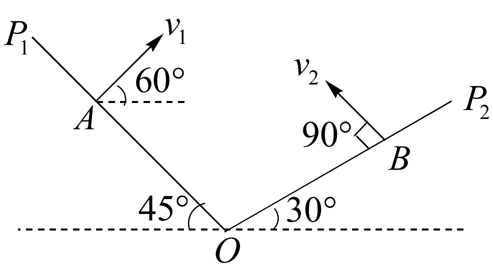
A) $\frac{4}{3}$ B) $\frac{2}{3}$ C) 1 D) $\frac{1}{3}$

12. The distance travelled and magnitude of displacement of the bolt in the ground frame of reference, (take $g = 10\,ms^{-2}$) respectively are (in m)

A) $1.546, 1.39$ B) $1.372, 1.2$ C) $1.214, 0$ D) $2.012, 1.51$

Paragraph for Questions 13 and 14

A particle is projected from an inclined plane OP_1 from A with velocity $\upsilon_1 = 8ms^{-1}$ at an angle $60°$ with horizontal. An another particle is projected at the same instant from B with velocity $16\,ms^{-1}$ and perpendicular to the plane OP_2 as shown in figure. After time $10\sqrt{3}s$ separation between them was minimum and found to be $70m$.



13. Find the distance AB (in m)

A) 300 B) 450 C) 150 D) 250

14. The relative speed of 2 w.r.t 1 is v. Then (as long as both of them are in air)

A) v is constant and is equal to $13.9\,ms^{-1}$

B) v is varying nonlinearly with time

C) v is constant and is equal to $11\,ms^{-1}$

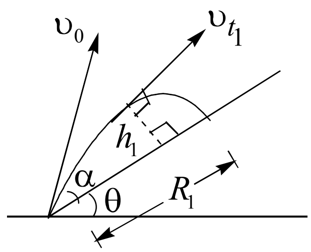
D) v is varying linearly with time

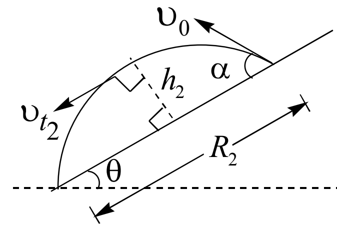
SECTION – III
(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

15. Two balls are thrown from an inclined plane at angle of projection α with the plane, one up of the incline and other down the incline as shown in figure (T stands for total time of flight and α is the angle of projection w.r.t inclined plane).





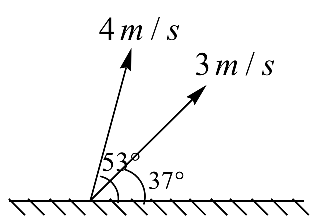
A) $h_1 = h_2 = \dfrac{\upsilon_0^2 \sin^2 \alpha}{2g\cos\theta}$

B) $T_1 = T_2 = \dfrac{2\upsilon_0 \sin\alpha}{g\cos\theta}$

C) $R_2 - R_1 = g(\sin\theta)T_1^2$

D) $\upsilon_{t_2} = \upsilon_{t_1}$

16. Two particles are projected with speed $4\,m/s$ and $3\,m/s$ simultaneously from same point as shown in the figure. Then



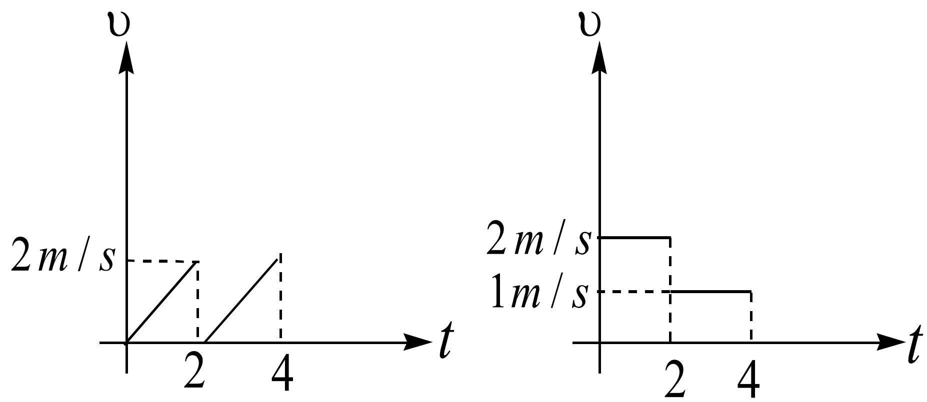
A) their relative velocity is along vertical direction

B) their relative acceleration is non-zero and it is along vertical direction

C) they will hit the surface simultaneously

D) their relative velocity is constant (until no one touches the ground) and has magnitude $1.4\,m/s$

17. The velocity *versus* time of two particles moving along x-axis varies as shown in the following two plots. Then



A)maximum separation between the two particles is $2m$

B)maximum separation between the two particles is $2.5m$

C)maximum separation between them occurs at time $t = 2$s

D)maximum separation between them occurs at time $t = 3$s

18. A particle is moving with uniform acceleration along a straight line AB. Its speeds at A and B are $2 m/s$ and $14 m/s$ respectively. Then

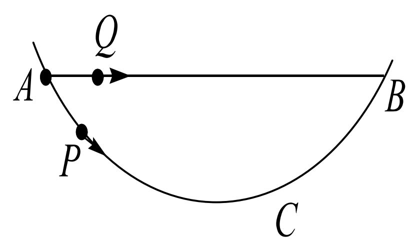
A) its speed at the mid-point of AB is $10 m/s$

B) its speed at a point P such that $AP:PB = 1:5$ is $6 m/s$

C) the time to go from A to the mid-pointof AB is double of that go to from mid-point to B

D) none of the above are correct.

19. A particle P is sliding down a frictionless hemispherical bowl. It passes the point Aat $t=0$. At this instant of time, the horizontal components of its velocity is υ. A bead Q of the same mass as P is ejected from A at $t=0$ along the horizontal string AB, with the speedυ. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then



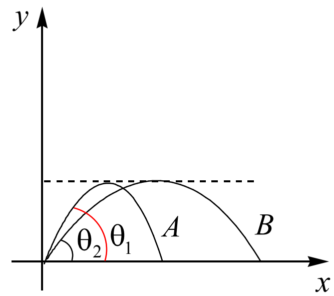
A) $t_P < t_Q$

B) $t_P = t_Q$

C) $t_P > t_Q$

D) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of cord } AB}$

20. Two particle are thrown from the same point in the same vertical plane, as shown in figure simultaneously. Then indicate the correct statements.



A)Time of flight for B is less than that of A

B)Projection speed of B is greater than that of A

C)Horizontal component of velocity for B is greater than that of A

D)The vertical component of velocities of both A and B are always equal throughout the duration for which both the particles in air.

CHEMISTRY: **Max. Marks: 66**

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

21. For the reaction; $X_2O_4(l) \rightarrow 2XO_2(g)$, $\Delta U = 2.1$ kcal, $\Delta S = 20$ cal K^{-1} at 300 K, Hence ΔG is

A) 2.7 kcal B) -2.7 kcal C) 9.3 kcal D) -9.3 kcal

22. For the reaction : $C(s) + H_2O(g) \rightarrow CO(g) + H_2(g)$, $\Delta H^\circ = 133$ kJ/mol and $\Delta S^\circ = 133$ J/Kmol at 298 K. The minimum temperature in °C above which this reaction is spontaneous is--------.

A) 1000 (B) 325 C) 552 D) 727

23. When $NaNO_3$ is heated in a closed vessel. Oxygen is liberated and $NaNO_2$ is left behind. At equilibrium

A) Addition of $NaNO_2$ favour reverse reaction

B) Addition of $NaNO_3$ favour forward reaction

C) Increasing temperature favour forward reaction

D) Increasing pressure favour forward reaction

24. At 1100K, water vapour decomposes into $H_2(g)$ and $O_2(g)$ only to the extent of $10^{-5}\%$ when pressure is maintained at 1 atm. The value of Kp for the decomposition of one mole of H_2O is $\left(H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2\right)$

A) 5×10^{-22} B) $\sqrt{5}\times10^{-11}$ C) $\sqrt{5}\times10^{-15}$ D) 3.3×10^{-11}

25. 0.5 mole of $H_2(g)$ and 1.0 mole of $HI(g)$ (but no I_2) are added to a 1.0 litre vessel and allowed to reach equilibrium according to the following reaction

$H_2(g)+I_2(g)\rightleftharpoons 2HI(g)$

If x is the equilibrium concentration of $I_2(g)$. Then correct expression for equilibrium constant is:

A) $\frac{(1-2x)^2}{(0.5+x)(x)}$ B) $\frac{(1+2x)^2}{(0.5-x)(x)}$ C) $\frac{x(0.5-x)}{(1+2x)}$ D) $\frac{(0.5+x)x}{(1-2x)^2}$

26. Which of the following statements is correct?

A) If K_1 be the equilibrium constant for $A\rightleftharpoons B$ and K_2 be the equilibrium constant for $C\rightleftharpoons D$ then equilibrium constant for the reaction

$A+D\rightleftharpoons B+C$ is $\frac{K_1}{K_2}$

B) If K_1 be the equilibrium constant for $P\rightleftharpoons Q$ and K_2 be the equilibrium constant for $R\rightleftharpoons S$ then equilibrium constant for the reaction

$P+R \rightleftharpoons Q+S$ is $\dfrac{K_1}{K_2}$

C) If K_1 be the equilibrium constant for $A+B \rightleftharpoons C+D$ the equilibrium constant for

$nA+nB \rightleftharpoons nC+nD$ is $(K_1)^{1/n}$, n can be fraction also

D) The equilibrium constant unit for the gaseous reaction $C_2H_4+H_2 \rightleftharpoons C_2H_6$ is

$\text{mol}^2\text{dm}^{-3}$

27. Calculate the percentage dissociation of $H_2S_{(g)}$ If 0.1 mole of H_2S is kept in 0.5 L vessel at 1000K. The value of K_c for the reaction

$2H_2S_{(g)} \rightleftharpoons 2H_{2(g)}+S_{2(g)}$ is 1.0×10^{-7}

A) 0.1 B) 0.01 C) 1 D) 10

28. For the reaction $CO_{2(g)}+H_{2(g)} \rightleftharpoons CO_{(g)}+H_2O_{(g)}$, K_{eq} is 0.63 at 727^0C and 1.26 at 927^0C. Then calculate the value of K_{eq} at 1227^0C (use $\log_{10}2=0.3$)

A) 8.32 B) 2.52 C) 7.78 D)5.04

SECTION - II
(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to two paragraphs with three questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct.**
Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

Paragraph for Questions 29 and 30:

A flask of 1L having $NH_3(g)$ at 2.0atm and 200K is connected with another flask of volume 800 ml having $HCl(g)$ at 8atm and 200K through a narrow tube of negligible volume. The two gases react to form $NH_4Cl(s)$ with evolution of 43 KJ/mol heat. If heat capacity of $HCl(g)$ at constant volume is 20J/K mol. (neglect heat capacity of flask and NH_4Cl solid, volume of solid NH_4Cl formed and its pressure)

(R = 0.08 $L\ atm\ K^{-1}\ mol^{-1}$).

29. The heat produced is in the reaction is:

A) 5.375KJ B) 4.375KJ C) 6.8 KJ D) 6.375J

30. The final pressure in the flask is

A) 143.9atm B) 1.49atm C) 16.39atm D) 13.78atm

Paragraph for Questions 31 and 32:

Phosphorous pent chloride when heated in a sealed tube at 700 K, it undergoes decomposition as,

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g); \quad K_p = 38 atm$$

vapour density of the mixture is 74.25

31. Percentage dissociation of PCl_5 may be given as:

A) 4.04 B) 40.4 C) 44.0 D) 0.404

32. When inert gas is added to the given reversible process then the equilibrium will:

A) be unaffected B) shift in backward direction

C) shift in forward direction D) cannot be predicted

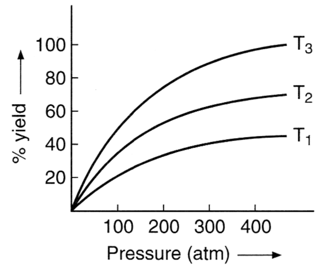
Paragraph for Questions 33 and 34:

In Haber's process the ammonia is manufactured according to the following reaction;

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g); \Delta H^\circ = -(22.4\, kJ)$$

The pressure inside the chamber is maintained at 200 atm and temperature at 500°C. Generally, this reaction is carried out in presence of Fe catalyst.

33. The preparation of ammonia by Haber's process is an exothermic reaction. If the preparation follows the following temperature pressure relationship for its % yield. Then for temperature T_1, T_2 and T_3, the correct option is:



A) $T_3 > T_2 > T_1$

B) $T_1 > T_2 > T_3$

C) $T_1 = T_2 = T_3$

D) nothing could be predicted

34. 500°C is considered optimum temperature for Haber's process because:

A) catalyst has maximum activity at this temperature

B) energy required is easily obtained at this temperature

C) yield is maximum at this temperature

D) rate is fast enough while the yield is also appreciable at this temperature

SECTION – III

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

35. A certain gas is expanded from (1L, 10atm) to (4L, 5atm) against a constant external pressure of 1atm. If the initial temperature of gas is 300K and heat capacity for the process is 50J $^0C^{-1}$ (use 1L atm = 100 J), then

A) $|W|$ = 0.3 kJ   B) $|q|$= 15 kJ

C) $\Delta U = 14.7\ kJ$ D) $\Delta H = 15.7\ kJ$

36. The stander enthalpies of formation of CO_2 gas and $HCOOH_{(l)}$ are – 393.7 kJmol^{-1} and – 409.2 kJmol^{-1} respectively. Which of the following is/are correct ?

A) $C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)} \qquad \Delta H = -393.7\ kJ\ mol^{-1}$

B) $CO_{2(g)} + H_{2(g)} \rightarrow HCOOH_{(l)} \qquad \Delta H = -15.5\ kJ\ mol^{-1}$

C) $H_2O_{(l)} + CO_{(g)} \rightarrow HCOOH_{(l)} \qquad \Delta H = -409.2\ kJ\ mol^{-1}$

D) $C_{(s)} + H_{2(g)} + O_{2(g)} \rightarrow H_2CO_{2(l)} \qquad \Delta H = -409.2\ kJ\ mol^{-1}$

37. Ammonia under a pressure of 20 atm. at 127^0C is heated to 327^0C in a closed vessel. Under these conditions, ammonia is partially decomposed to N_2 and H_2 according to the reaction $2NH_3(g) \rightleftharpoons N_2(g) + 3H_2(g)$

After decomposition at constant volume in the vessel, the pressure increases to 50 atm. which of the following statements is/ are correct?

A) The degree of dissociation of NH_3 in 2/3

B) The Kp of the reaction at 327^0C is 2.7×10^3 atm^2.

C) The pressure of N_2 and NH_3 gas at equilibrium is 10 atm. each

D) The pressure of N_2 at equilibrium is half that of NH_3.

38. Which of the following statements is/are wrong

A) At equilibrium, concentration of reactants and products becomes constant, because the reaction comes to an halt

B) Addition of inert gas at equilibrium for $\Delta n > 0$ at constant volume will favour forward reaction.

C) Addition of catalyst speeds up the forward reaction more than the backward reaction

D) Equilibrium constant of an exothermic reaction increases with increase in temperature.

39. If x and y are arbitrary intensive variables, then:

A) xy is an intensive variable

B) x/y is an intensive variable

C) $(x + y)$ is an extensive property

D) dx/dy is an intensive property

40. Heat of neutralization of oxalic acid ($H_2C_2O_4$) is – 25.4 kcal/mol with sodium hydroxide solution. Identify the correct option(s) among the following:

(ionization in the free acid is negligible)

A) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{-2} \quad \Delta H = 2\ kcal$

B) $NaOH + \frac{1}{2}H_2C_2O_4 \rightarrow \frac{1}{2}Na_2C_2O_4 + H_2O \quad \Delta H = -13.7\ kcal$

C) $NH_4OH + \frac{1}{2}H_2C_2O_4 \rightarrow \frac{1}{2}(NH_4)_2C_2O_4 + H_2O$ energy released is less than 12.7 kcal

D) Heat of neutralization is determined by using calorimeter.

MATHS: **Max. Marks: 66**

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

41. The equation $\sin^4 x+\cos^4 x+\sin 2x+\alpha=0$ is solvable for

A) $-\frac{5}{2}\le\alpha\le\frac{1}{2}$ B) $-3\le\alpha\le 1$ C) $-\frac{3}{2}\le\alpha\le\frac{1}{2}$ D) $-1\le\alpha\le 1$

42. The number of solutions of the Equation $\sin^3 x\cos x+\sin^2 x\cos^2 x+\sin x\cos^3 x=1$, in the interval $\left[0,2\pi\right]$ is __________

A) 4 B) 2 C) 1 D) 0

43. $\tan^6\left(\frac{\pi}{9}\right)-33\tan^4\left(\frac{\pi}{9}\right)+27\tan^2\left(\frac{\pi}{9}\right)$ is equal to

A) 0 B) $\sqrt{3}$ C) 3 D) 9

44. $a \neq p, b \neq q, c \neq r$ and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$ then the value of

$\dfrac{p}{p-a} + \dfrac{q}{q-b} + \dfrac{r}{r-c} =$ ________

A) 1 B) 2 C) 3 D) 4

45. If $\begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$ and α is not root of $ax^2 + bx + c = 0$ then

A) a,b,c are in A.P

B) a,b,c are in G.P

C) a,b,c are in H.P

D) a,b,c are in A.G.P

46. $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ then the value of $5a + 4b + 3c + 2d + e$ is equal to_____

A) 0 B) -16 C) 16 D) -11

47. The maximum value of $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is ________(where $\left(a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3 \in \{-1, 1\}\right)$

A) 0 B) -4 C) 4 D) 6

48. If $a_1, a_2, a_3 5, 4, a_6, a_7, a_8, a_9$ are in HP, then the value of the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ can be expressed in the lowest form as $\frac{p}{q}$, find $(p-2q)$.

A) 5 B) 6 C) 7 D) 8

SECTION - II

(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

Paragraph for Questions 49 and 50

If $S_k = a^k + b^k + c^k$, then $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = \begin{vmatrix} 3 & a+b+c & a^2+b^2+c^2 \\ a+b+c & a^2+b^2+c^2 & a^3+b^3+c^3 \\ a^2+b^2+c^2 & a^3+b^3+c^3 & a^4+b^4+c^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}^2$

$= (a-b)^2 (b-c)^2 (c-a)^2$

If we can express a determinant as a product of two determinants, then it is easier for us. On the basis of above information, answer the following questions:

49. If $a_1, a_2, a_3, b_1, b_2, b_3 \in R$,all distinct, then the value of $\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix}$ is

A) $2(a_1-a_2)(a_1-a_3)(a_3-a_1)$

B) $2(b_1-b_2)(b_2-b_3)(b_3-b_1)$

C) $2(a_1-a_2)(a_2-a_3)(a_3-a_1)$

D) $2(a_1-a_2)(a_2-a_3)(a_3-a_1)(b_1-b_2)(b_2-b_3)(b_3-b_1)$

50. Let α, β be the roots of the equation $ax^2+bx+c=0$. Let $S_n = \alpha^n + \beta^n \ for \ n \geq 1$, then the value of $\begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$ is

A) $\frac{(a+b+c)(b^2-4ac)^2}{a^4}$

B) $\frac{(a+b+c)^2(b^2-4ac)}{a^4}$

C) $\frac{(a+b+c)^2(b^2-4ac)^2}{a^4}$

D) 0

Paragraph for Questions 51and 52

Let $\Delta \neq 0$ and Δ^c denotes the determinant of cofactors, then $\Delta^c = \Delta^{n-1}$ where n (>0) is the order of Δ.

On the basis of above information, answer the following questions:

51. If a,b,c are the roots of the equation $x^3 - 3x^2 + 3x + 7 = 0$, then the value of

$$\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix}$$ is

A) 9 B) 27 C) 81 D) 0

52. If $a^2 + b^2 + c^2 = \lambda^2$, then the value of $\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$ is

A) $8\lambda^6$ B) $27\lambda^9$ C) $8\lambda^9$ D) $27\lambda^6$

Paragraph for Questions 53 and 54

If $\alpha, \beta, \gamma, \delta$ are solutions of the equation $\tan\left(\theta + \frac{\pi}{4}\right) = 3\tan 3\theta$ no two of which have equal tangents. Then

53. $Tan\alpha + Tan\beta + Tan\gamma + Tan\delta =$

A) $\frac{1}{3}$ B) $\frac{5}{3}$ C) $-\frac{8}{3}$ D) 0

54. $Tan\alpha . Tan\beta . Tan\gamma . Tan\delta =$

A) $-\frac{1}{3}$ B) -2 C) 0 D) 1

SECTION – III

(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

55. Let $f(x) = \begin{vmatrix} 1/x & ln\,x & x^2 \\ 1 & -1/2 & 1 \\ 1 & a & a^2 \end{vmatrix}$, then $\frac{d^2}{dx^2} f(x)$ at $x = 1$ is

A) Independent of a B) 3 C) -3 D) zero

56. If $a^2 + b^2 + c^2 = 1$, then

$\begin{vmatrix} a^2 + (b^2 + c^2)\cos\phi & ab(1-\cos\phi) & ac(1-\cos\phi) \\ ba(1-\cos\phi) & b^2 + (c^2 + a^2)\cos\phi & bc(1-\cos\phi) \\ ca(1-\cos\phi) & cb(1-\cos\phi) & c^2 + (a^2 + b^2)\cos\phi \end{vmatrix}$ Is independent of

A) a B) b C) c D) ϕ

57. The value of the determinant

$$\begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{16}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$$, where $i=\sqrt{-1}$, is

A) complex B) real C) irrational D) rational

58. If $0 \le x \le 1$ and $f(x)=\begin{vmatrix} x & 1 & 1 \\ -1 & x & 1 \\ 1 & -1 & x \end{vmatrix}$, then

A) Least value of f(x) is 2

B) Greatest value of f(x) is 4

C) F(x) has a local maximum at x = 2/3

D) f(x) has a local minimum at x = 1/3

59. If $a_i, b_i, c_i \in R (i=1,2,3)$ and $x \in R$ and $\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix}=0$, then

A) x = 1 B) x = -1 C) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$ D) $x=\pm 1$

60. The value of $x \in (-\pi, \pi)$ which satisfy the equation $8^{1+|\cos x|+|\cos x|^2+......\infty}=4^3$ are

A) $\frac{\pi}{3}, \frac{2\pi}{3}$ B) $\frac{\pi}{3}, \frac{-2\pi}{3}$ C) $\frac{-\pi}{3}, \frac{-2\pi}{3}$ D) $\pm\frac{\pi}{3}, \pm\frac{2\pi}{3}$

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KEY SHEET

PHYSICS

1	**A**	2	**B**	3	**C**	4	**D**	5	**B**
6	**C**	7	**B**	8	**D**	9	**D**	10	**C**
11	**B**	12	**A**	13	**D**	14	**A**	15	**ABC**
16	**AD**	17	**BD**	18	**ABC**	19	**A**	20	**BCD**

CHEMISTRY

21	**B**	22	**D**	23	**C**	24	**B**	25	**A**
26	**A**	27	**C**	28	**B**	29	**A**	30	**D**
31	**B**	32	**A**	33	**B**	34	**D**	35	**ABCD**
36	**ABD**	37	**ABC**	38	**ABCD**	39	**ABD**	40	**A,C**

MATHS

41	**C**	42	**D**	43	**C**	44	**B**	45	**B**
46	**D**	47	**C**	48	**D**	49	**D**	50	**B**
51	**D**	52	**C**	53	**D**	54	**A**	55	**AD**
56	**ABC**	57	**A**	58	**AB**	59	**ABCD**	60	**ABCD**

SOLUTIONS

PHYSICS

1. We divide the problem into two parts, i.e., motion from boy to wall and from wall back to ground.

i) Motion from boy to wall

x-components

$\upsilon_0 \cos 45° t = 4m$

$t = \frac{4}{\upsilon_0 \cos 45°}$

$= \frac{4}{10\sqrt{2}.(1/\sqrt{2})}$

$= \frac{2}{5}\sec$

y-components

$y = \upsilon_0 \sin\theta t - \frac{1}{2}gt^2$

$= \frac{10\sqrt{2}}{\sqrt{2}}, \frac{2}{5} - \frac{1}{2} \times 10 \times \left(\frac{2}{5}\right)^2$

$= \frac{16}{5}m$

Ball strikes at a height of $16/5m$ from boy's hand

$\upsilon_x = \upsilon_0 \cos 45° = 10m/s$

$\upsilon_y = \upsilon_0 \sin 45° - gt$

$= \frac{10\sqrt{2}}{\sqrt{2}} - 10 \times \frac{2}{5} = 6\,m/s$

ii) Motion from wall to ground:

We choose point of impact as origin and upward positive

x-components

$x = \upsilon_0 \cos 45° t'$

$= \frac{10\sqrt{2}}{\sqrt{2}} \times 1.78$

$= 17.8\,m$

Hence the ball lands $17.8m$ from the foot of the wall

y-components

$-\frac{26}{5} = 6t' - \frac{1}{2} g\, t'^2$

where t' is time of flight from wall to ground

$-\frac{26}{5} = 6t' - \frac{1}{2} \times 10t'^2$

Or $5t'^2 - 6t' - \frac{26}{5} = 0$

Or $t' = \frac{6 \pm \sqrt{36 + 4(5)(26/5)}}{10}$

$t' = 1.78\sec$ (neglecting $-ve$ value)

2. Horizontal range of bullet is $30\,m$. $\dfrac{u^2 \sin 2\theta}{g} = 30$

Or $\sin 2\theta = \dfrac{30 \times 10}{(100)^2}$

Or $\sin 2\theta = 0.03$

For small θ, $\sin\theta \approx \theta$

i.e., $2\theta = 0.03$

Therefore $\theta = 0.015$

The rifle must be aimed at an angle $\theta = 0.015$ above horizontal.

Height to be aimed $= 30\tan\theta \approx 30(\theta) \quad = 30 \times 0.015 \quad = 45\,cm$.

3. $\vec{V}_A = 400\hat{i} \qquad ...(i)$

$\vec{V}_B = \upsilon_B \cos 30^\circ \hat{i} + \upsilon_B \sin 30^\circ \hat{j}$

$\vec{V}_B = \dfrac{\upsilon_B \sqrt{3}}{2}\hat{i} + \dfrac{\upsilon_B}{2}\hat{j} \quad ...(ii)$

$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$

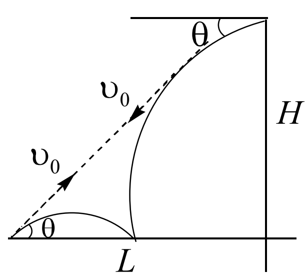
$\vec{V}_{B/A} = \left(\dfrac{\upsilon_B\sqrt{3}}{2}\hat{i} + \dfrac{\upsilon_B}{2}\hat{j}\right) - 400\hat{i}$

$\vec{V}_{B/A} = \left(\dfrac{\upsilon_B\sqrt{3}}{2} - 400\right)\hat{i} + \dfrac{\upsilon_B}{2}\hat{j}$

$\upsilon_B\sqrt{3} - 800 = 0,\ \upsilon_B = \dfrac{800}{\sqrt{3}}$

$\vec{V}_B = 400\hat{i} + \dfrac{400}{\sqrt{3}}\hat{j}$.

4.



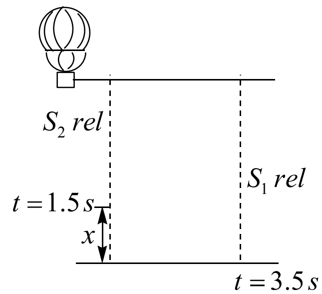
$\upsilon_0 \cos\theta\left(\dfrac{2\upsilon_0 \sin\theta}{g}\right) + \upsilon_0 \cos\theta\left(\dfrac{2\upsilon_0 \sin\theta}{g}\right) = L$

$\sin\theta = \dfrac{H}{\sqrt{H^2 + L^2}}$

$\cos\theta = \dfrac{L}{\sqrt{H^2 + L^2}}$

$\upsilon_0^2 = \dfrac{gL}{4\sin\theta\cos\theta} \qquad \Rightarrow \upsilon_0 = \sqrt{\dfrac{g\left(L^2 + H^2\right)}{4H}}$.

5. Let us solve the problem from the frame of balloon.



The initial velocity of 1st stone wrt balloon is $u_r = 0$

a_s, acceleration of stone $= g$ downwards $= -10\,ms^{-2}$

a_{sb}, acceleration of stone wrt balloon,

$$a_{rel} = a_s - a_b = (-10) - (+1) = -11\,ms^{-2}$$

$$-S_{1_{rel}} = u_r t_1 + \frac{1}{2} a_{rel} t_1^2$$

$$= 0 + \frac{1}{2}(-11)(3.5)^2$$

$$\therefore\ S_{1_{rel}} = 67.375\,m$$

Similarly, $-S_{2_{rel}} = u_r t_2 + \frac{1}{2} a_{rel}\, t_2^2$

$$= 0 + \frac{1}{2}(-11)(1.5)^2$$

$$\therefore\ S_{1_{rel}} = 12.375\,m$$

Required distance is $x = S_{1_{rel}} - S_{2_{rel}} = 67.375 - 12.375 = 55\,m$

Alternatively,

Let at any time $t = 0$, the balloon is at position *A*, where its velocity is *u*. At $t = 2s$, it reaches *B*, where its velocity is *v*, then

$$AB = S = u \times 2 + \frac{1}{2} a \times (2)^2$$

$$= 2u + 2\left(\because a = 1\,ms^{-2}\right)$$

Also, $v = u + at = u + 2$

$$-S_1 = u \times 35 + \frac{1}{2}(-g)(3.5)^2$$

($\because$ for first stone time $= 2 + 1.5 = 3.5\,s$)

And $-S_2 = v\times 1.5+\frac{1}{2}(-g)(1.5)^2$

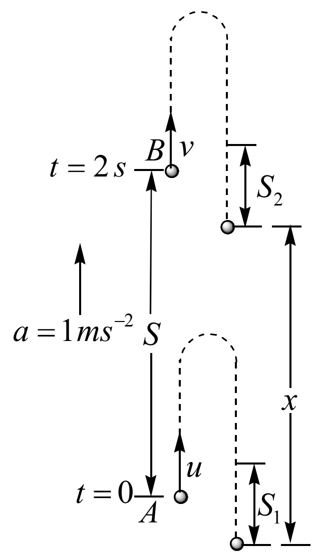
$\therefore\ S_1 = \frac{1}{2}g(3.5)^2 - 3.5u$ and

$S_2 = \frac{1}{2}g(1.5)^2 - 1.5v$

$= \frac{1}{2}g(1.5)^2 - 1.5(u+2)$

$(\because v = u+2)$

So, $S_2 = \frac{1}{2}g(1.5)^2 - 1.5u - 3$



Required distance is

$x = S + S_1 - S_2$

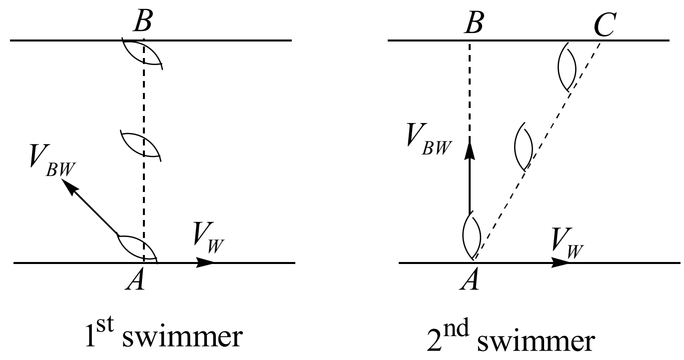
$= (2u+2) + \left[\frac{1}{2}g(3.5)^2 - 3.5u\right] - \left[\frac{1}{2}g(1.5)^2 - 1.5u - 3\right]$

$\therefore\ x = \frac{1}{2}g\left[(3.5)^2 - (1.5)^2\right] + 5$

$= \frac{1}{2}\times 10\times(3.5+1.5)(3.5-1.5)$

$= 5(5)(2) + 5 = 55m$.

6. In the following diagrams, swimmer is substituted by boat.



In the case of 1st swimmer,

$$t_1 = \frac{b}{\sqrt{V_{BW}^2 - V_W^2}} = \frac{b}{\sqrt{(2.5)^2 - 2^2}} = \frac{b}{1.5}$$

In the case of 2nd swimmer,

$$t_2'(= t_{\min}) = \frac{b}{V_{BW}} = \frac{b}{2.5}$$

$$BC = drift$$

$$d = V_W \cdot \frac{b}{V_{BW}} = 2.\frac{b}{2.5} = \frac{4b}{5}$$

$\therefore$ Time taken for walking from C to B is

$$t_2'' = \frac{(4b/5)}{u} = \frac{4b}{5u}$$

Total time taken by the 2nd swimmer to reach point B is

$$t_2 = t'_2 + t''_2 = \frac{b}{2.5} + \frac{4b}{5u} = \frac{2b}{5} + \frac{4b}{5u}$$

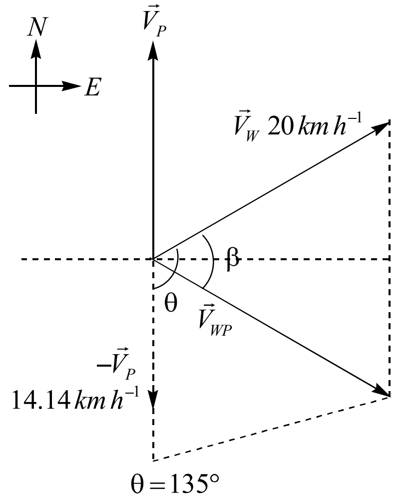
From the problem, $t_1 = t_2$

$$\therefore \frac{b}{1.5} = \frac{2b}{5} + \frac{4b}{5u}; \frac{2b}{3} = \frac{2b}{5} + \frac{4b}{5u}$$

$$\Rightarrow u = 3\,km\,h^{-1}.$$

7. The wind is blowing along north-east direction ($20\,km\,h^{-1}$). The direction of procession is along north direction ($14.14\,km\,h^{-1}$). The direction of flags on the moving vehicles is along $\bar{V}_{WP}$ (velocity of wind wrt procession).

Construct the parallelogram with sides $\bar{V}_W$ and $-\bar{V}_P$.



The diagonal gives $\vec{V}_{WP}$

In the figure, $\theta = 135°$

$$\tan\beta = \frac{14.14\sin 135°}{20 + 14.14\cos 135°}$$

$$= \frac{14.14\left(\frac{1}{\sqrt{2}}\right)}{20 + 14.14\left(-\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{10}{20-10} = 1 \Rightarrow \beta = 45°$$

$\therefore$ Flags flutter along east direction.

8. For observer on cart

$\vec{\upsilon}_{rel} = 0$

$\vec{S}_{rel} = \frac{1}{2}\vec{a}_{rel}t^2$

Trajectory is straight line along $\vec{a}_{rel}$ for observer on ground trajectory is parabola because $\vec{\upsilon}_0$ and $\vec{g}$ are at an angle θ initially.

9. Horizontal distance between P and Q after time t is $x = 22 - \left|\left(U_{QP}\right)_x\right| t = \left(22 - 8\sqrt{2}t\right)m$.

10. $S_{\min} = \left[200\left(\frac{23}{10\sqrt{2}}\right)^2 - 460\sqrt{2}\left(\frac{23}{10\sqrt{2}}\right) + 565\right]^{\frac{1}{2}} = 6\,m$.

11. After 1s i.e., at the beginning of the free fall, (initial) velocity of bolt is

$= u + at = 0 + 1.25 \times 1 = 1.25\, ms^{-1}$

Acceleration of lift $= 1.25\, ms^{-2}$ (upwards) = $+1.25\, ms^{-2}$

And acceleration of bolt $10\, ms^{-2}$ (downwards) $= -10\, ms^{-2}$

Relative acceleration (of both wrt lift)

$= (-10) - (1.25) = -11.25\, ms^{-2}$

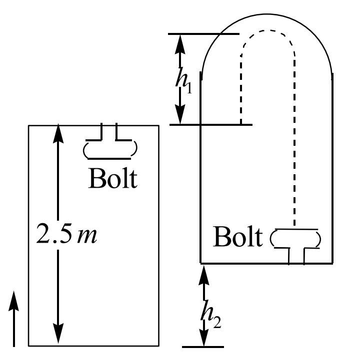
$S_r = -2.5\, m; u_r = 0$ ($\because$ At the start of falling velocities of bolt and lift are equal).

$$S_r = u_r t + \frac{1}{2} a_r t^2$$

$$\Rightarrow -2.5 = 0 + \frac{1}{2}(-11.25)t^2$$

$$\Rightarrow t = \frac{2}{3} s$$

12. Maximum height attained by the bolt from the starting point is



$v\alpha = 1.25 ms^{-1}$ (at the starting of fall of bolt)

$$h_= \frac{u^2}{2g} = \frac{(1.25)^2}{2 \times 10} = 0.078\, m$$

Distance travelled by elevator when the bolt falls on its floor is

$$h_2 = (1.25)\frac{2}{3} + \frac{1}{2} \times 1.25 \times \left(\frac{2}{3}\right)^2 = 1.11\, m$$

$\therefore$ From ground frame,

Distance travelled by bolt

$= 2h_1 + (2.5 - h_2) = 2 \times 0.078 + (2.5 - 1.11)$

$= 1.546\, m$

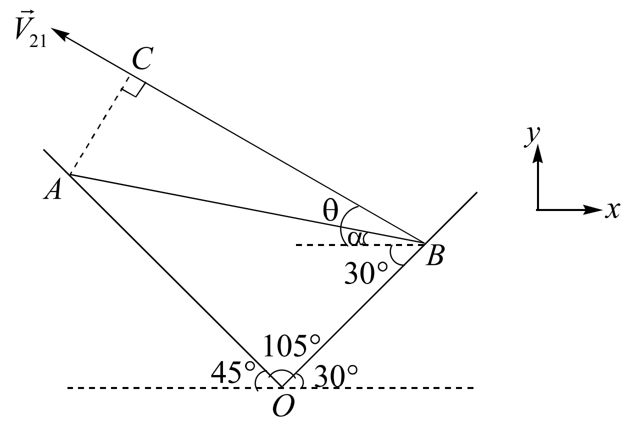
Displacement of the bolt

$= 2.5 - h_2 = 2.5 - 1.11 = 1.39\, m$.

13 & 14. $\left|(\upsilon_{21})_x\right| = (\upsilon_1 + \upsilon_2)\cos 60° = 12\, ms^{-1}$

$\left|(\upsilon_{21})_y\right| = (\upsilon_1 + \upsilon_2)\sin 60° = 4\sqrt{3}\, ms^{-1}$

$\Rightarrow \upsilon_{21} = \sqrt{(12)^2 + \left(4\sqrt{3}\right)^2} = 13.9\, ms^{-1}$



And $\theta = \tan^{-1}\left(\frac{4\sqrt{3}}{12}\right) = 30°,\ BC = (\upsilon_{21})t = 240m\ \ AC = 70\, m$

a) Hence using Pythagora's Theorem, we have

$AB = \sqrt{(240)^2 + (70)^2} = 250\, m$

b) $\angle\alpha = \angle\theta - \angle ABC = 30° - \sin^{-1}\left(\frac{AC}{AB}\right)$

$\Rightarrow \angle\alpha = 30° - \sin^{-1}\left(\frac{70}{250}\right) = 13.7°$

Applying sine law in $\text{A}ABO$, we get

$\frac{AB}{\sin(105°)} = \frac{AO}{\sin(\alpha + 30°)}$

$= \frac{BO}{\sin\left(180° - (\alpha + 30° + 105°)\right)}$

Solving this we get,

$AO = 181.2\, m$ and $BO = 134.6\, m$

So, $h_A = AO\sin 45° = 128\, m$ and $h_B = BO\sin 30° = 67.3\, m$.

15. $h_{1\max} = \dfrac{(\upsilon_0 \sin\alpha)^2}{2g\cos\theta} = h_{2\max} \Rightarrow (a)$ is correct

$T_1 = \dfrac{2\upsilon_0 \sin\theta}{g\cos\theta} = T_2 \Rightarrow (b)$ is correct

$R_1 = (\upsilon_0 \cos\theta)T_1 - \dfrac{1}{2} g \sin\theta T_1^2$

$R_2 = (\upsilon_0 \cos\theta)T_2 + \dfrac{1}{2} f \sin\theta T_2^2$

$\Rightarrow (R_2 - R_1) = g \sin\theta T_1^2$

$\Rightarrow (c)$ is correct

υ_{t_1} & υ_{t_2} are the velocities of the particles at their maximum heights. Let the particles reach their maximum heights at time t_1 and t_2 respectively. Hence

$0 = (\upsilon_0 \sin\alpha) - (g\cos\theta)t_1$

$\Rightarrow t_1 = \dfrac{\upsilon_0 \sin\alpha}{g\cos\theta}$

Similarly $t_2 = \dfrac{\upsilon_0 \sin\alpha}{g\cos\theta}$. Hence $t_2 = t_1$

Hence $\upsilon_{t_1} = \upsilon_0 \cos\alpha + (g\sin\theta)t_1$

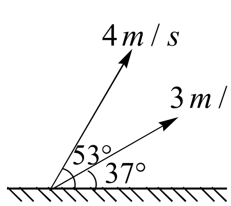
$\upsilon_{t_2} = \upsilon_0 \cos\alpha + (g\sin\theta)t_2$

$\Rightarrow \upsilon_{t_1} \neq \upsilon_{t_2}$.

16. So, velocity of first particle

$= 3\cos 30°\hat{i} + 3\sin 30°\hat{j}$

$= \dfrac{12}{5}\hat{i} + \dfrac{9}{5}\hat{j}$



Velocity of second particle

$= 4\cos 53°\hat{i} + 4\sin 53°\hat{j}$

$= \dfrac{12}{5}\hat{i} + \dfrac{16}{5}\hat{j}$

So, relative horizontal velocity is zero. So their relative velocity is vertical only. Since both particles are moving under gravity, so their relative acceleration is zero.

Their relative velocity $=\frac{16}{5}-\frac{9}{5}=\frac{7}{5}=1.4\,m/s$.

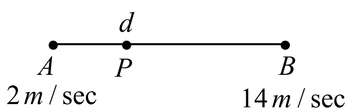
17. Separation between them will be maximum when both particles have same velocity. This situation come at $t=2\sec$, but just after it, first particle comes to rest and second $1\,m/s$. So first particle will again gain this velocity in next one second. So, maximum separation will occur after 3 seconds.

Maximum separation = Displacement of seconds particle – Displacement of first particle in first 3 seconds.

$$(2\times2+1\times1)-\left(\frac{1}{2}\times2\times2+\frac{1}{2}\times1\times1\right)$$

$$=5-2.5=2.5\,m.$$

18. $14^2=2^2+2\times a\times d$



At mid-point let velocity is υ

$$\Rightarrow \upsilon^2=2^2+2\times a\times\frac{d}{2}$$

$$\Rightarrow \upsilon^2=4+\frac{14^2-2^2}{2}$$

$$\upsilon^2=4+\frac{192}{2}=100$$

$$\upsilon=10\,m/\sec \rightarrow \text{if } \frac{AP}{PB}=\frac{1}{5}AP=\frac{d}{6}$$

Let velocity at P is υ_1

$$\upsilon_1^2=2^2+2\times a\times\frac{d}{6}=4+\frac{14^2-2^2}{6}$$

$$\Rightarrow \upsilon_1=6\,m/\sec$$

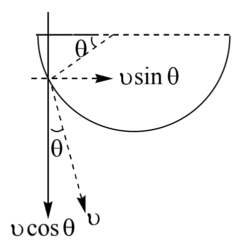
Let time taken to reach mid-point from A is t_1, and t_2 be time taken to reach B from mid-point.

$$6=2+at_1 \quad ...(i)$$

$14 = 6 + at_2 \quad ...(ii)$

$\frac{t_1}{t_2} = \frac{4}{8} = \frac{1}{2} \Rightarrow t_2 = 2t_1$.

19. Since Q moves along a smooth horizontal rod its velocity remains constant. But as P moves downwards its speed increases. Therefore its horizontal component of velocity $\upsilon \sin\theta$ increases and becomes maximum at lowest point. Afterwards it decreases gradually & becomes minimum at B; but at B, the horizontal component of velocity is equal to that at A. Hence horizontal component of velocity of P, is never less than velocity of Q. Since horizontal displacements of both are same, therefore, P takes less time or $t_P < t_Q$. Hence (a) is correct.



20. Both A and B have same $h_{\max}$. Hence

$$\frac{(u_A \sin\theta_A)^2}{2g} = \frac{(u_B \sin\theta_B)^2}{2g}$$

$$\Rightarrow u_A \sin\theta_A = u_B \sin\theta_B \rightarrow (u_y)_A = (u_y)_B$$

Hence option ‘d’ is correct.

Again time of flight $\frac{2(\text{vertical velocity of projection})}{g}$.

Hence time of flight of A is equal to that of B. Hence ‘a’ is wrong.

Since range of A is less than that of B and time of flight of A and B are equal, therefore $(u_x)_A < (u_x)_B$.

Hence ‘c’ is correct.

Speed of projection $= \sqrt{(u_x)^2 + (u_y)^2}$

Since u_y is same for both and $(u_x)_A < (u_x)_B$, therefore, speed of projection of A is less than that of B. Hence ‘b’ is correct.

CHEMISTRY :

21. $\Delta H = \Delta U + \Delta nRT$, $\Delta G = \Delta H - T\Delta S$

22. $\Delta G° = \Delta H° - T\Delta S°$

23. $NaNo_3(s) \rightleftharpoons NaNo_2(s) + \frac{1}{2}O_2(g)$

$NaNo_3$ and $NaNo_2$ are in solid state, changing their amount has no effect on equilibrium. Increases temperature will favour forward reaction due to endothermic nature of reaction. Also, increase pressure will favour backward reaction in which some $O_2(g)$ will combine with.

$NaNo_2(s)$ forming $NaNo_3(s)$.

24. $H_2O(g) \rightleftharpoons H_2(g) + \frac{1}{2}O_2(g)$

At equilibrium : $1-10^{-7}$ $\quad 10^{-7}$ $\quad \frac{10^{-7}}{2}$

Total moles = 1, Total pressure = 1 atm

$$\text{Kp} = \frac{{}^{P}H_2\sqrt{{}^{P}O_2}}{{}^{P}H_2O}$$

$$\therefore \text{Kp} = \frac{10^{-7} \times \sqrt{\frac{10^{-7}}{2}}}{1} \qquad = 10^{-7} \times \sqrt{5} \times 10^{-4} = \sqrt{5} \times 10^{-11}$$

25. $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

Initial con 0.5 mol/lit 0 mol/lit 1.0mol/lit

Equilibrium conc $(0.5+\text{x})$mol/lit x mol/lit $(1-2\text{x})$mol/lit

$$K_c = \frac{(1-2x)^2}{(0.5+x)(x)}$$

26. i.e. $A + D \rightleftharpoons B + C$ is $\frac{K_1}{K_2}$

27. $2H_2S \rightleftharpoons 2H_{2(g)} + S_{2(g)}$

Let x be the degree of dissociation $K_c = \frac{\left(\frac{0.1x}{v}\right)^2\left(\frac{0.1x}{2v}\right)}{\left(\frac{0.1-0.1x}{v}\right)^2} = 10^{-7}$

$\frac{x^3}{2v} = 10^{-6}$ $\qquad$ x = 0.01

Degree of dissociation (α)

$= \frac{0.1x}{0.1} = x$

$\therefore 1\%$ dissociation of H_2S

28. Using the vant' Hoff equation

$\Delta H = 8.32 kcal/mol$

Now, K_2 be the equilibrium const at $T_2 = 1500K$

$T_1 = 1000K$ and then $K_1 = 0.63$

$$\log_{10}\left(\frac{K_2}{0.63}\right) = \frac{8.32\times 10^3}{2.303(2)}\left(\frac{1500-1000}{1500\times 1000}\right)$$

$$\log_{10}\frac{K_2}{0.63} \approx 0.6 = \log_{10} 4$$

$\therefore K_2 = 0.63\times 4 = 2.52$

29. $NH_{3(g)} \quad + \quad HCl_{(g)} \quad \rightarrow \quad NH_4Cl_{(s)} \quad \Delta H = -43.0 KJ$

	$NH_{3(g)}$	$HCl_{(g)}$	$NH_4Cl_{(s)}$
Initial mole	$\frac{1\times 2}{0.08\times 200} = 0.125$	$\frac{8\times 0.8}{0.08\times 200} = 0.4$	0
Final moles	0	0.275	0.125

$\therefore$ heat produced $= 0.125\times 43 = 5.375 KJ$

The heat produced is used to increase the temperature of HCl left in flask since heat capacity of flask and $NH_4Cl = 0$

$\therefore Q = nC_V \Delta T$

$\therefore 5.375\times 10^3 = 0.275\times 20\times \Delta T$

$\therefore \Delta T = 977.27K$

$\therefore$ Final temperature = 200 + 977.27 = 1177.27K

Also final pressure $= \frac{nRT}{V} = \frac{0.275\times 0.08\times 1127.27}{1.8} = 13.78 atm$

30. (A)

31. (B)

32. (A)

33. (B)

34. (D) On increasing temperature, equilibrium will shift in backward direction.

35. $q = C\Delta T,\ w = -p_{ext}\left(V_2 - V_1\right),\ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

36. (A, B, D)

37. (A,B,C)

$$2NH_3 \rightleftharpoons N_2 + 3H_2$$

At $400k$	$20\,\text{atm}$	0	0
At $600k$	$30\,\text{atm}$	0	0

At constant value let $2x$ atm. NH_3 disappear

$30-2x \quad x \quad 3x$

${}^{P}Total = 30 + 2x = 50 \quad 2x = 20 \quad x = 10\,\text{atm}$

$\alpha = \frac{2x}{30} = \frac{20}{30} = \frac{2}{3}$

$\therefore Kp = \frac{10 \times 30^3}{10^2} = \frac{27 \times 10^4}{10^2} = 2.7 \times 10^3$

MATHS :

41. $\sin^2 2x - 2\sin x - 2 - 2\alpha$

$\Rightarrow \sin 2x = 1 \pm \sqrt{3+2\alpha}$

$\Rightarrow -1 \le 1 - \sqrt{3+2\alpha} \le 1$

$\Rightarrow \frac{-3}{2} \le \alpha \le \frac{1}{2}$

42. $\sin x \cos x\left(\sin^2 x + \cos^2 x\right) + \sin^2 x \cos^2 x$

$\Rightarrow \sin^2 x \cos^2 x + \sin x \cos x - 1 = 0$

$\Delta < O$

No solution

43. $\tan\left(3\left(20^0\right)\right) = \sqrt{3}$

$\frac{3\tan 20^0 - \tan^3\left(20^0\right)}{1 - 3\tan^2\left(20^0\right)} = \sqrt{3}$

Squaring on both sides

44. $\begin{vmatrix} p & b & c \\ a-p & q-b & o \\ o & b-q & r-c \end{vmatrix} = 0$

$p\left[(q-b)(r-c)\right] - b\left[(a-p)(r-c)\right] + c\left[(a-p)(b-q)\right] = 0$

Dividing with $a-p, b-q, c-r$

45. $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ 0 & 0 & a\alpha^2+2b\alpha+c \end{vmatrix} = 0$

$b^2 - ac = 0 \Rightarrow a, b, c \quad in \quad G.P$

46. Expand and compare

47. Maximum possible value is 4

48. We have, $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Since, $\begin{vmatrix} a_1 & a_2 & a_3 \\ 5 & 4 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

Therefore, $D = \begin{vmatrix} 20 & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix} = \frac{50}{21} = \frac{p}{q}$

$= 50 - 42 = 8$

Passage -1

49. $\begin{vmatrix} (a_1-b_1)^2 & (a_1-b_2)^2 & (a_1-b_3)^2 \\ (a_2-b_1)^2 & (a_2-b_2)^2 & (a_2-b_3)^2 \\ (a_3-b_1)^2 & (a_3-b_2)^2 & (a_3-b_3)^2 \end{vmatrix}$

$= \begin{vmatrix} a_1^2 & -2a_1 & 1 \\ a_2^2 & -2a_2 & 1 \\ a_3^2 & -2a_3 & 1 \end{vmatrix} \times \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_2 & a_2^2 \\ 1 & a_3 & a_3^2 \end{vmatrix} \times \begin{vmatrix} 1 & b_1 & b_1^2 \\ 1 & b_2 & b_2^2 \\ 1 & b_3 & b_3^2 \end{vmatrix}$

50. $\because \begin{vmatrix} 3 & 1+S_1 & 1+S_2 \\ 1+S_1 & 1+S_2 & 1+S_3 \\ 1+S_2 & 1+S_3 & 1+S_4 \end{vmatrix}$

$$=\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$=\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\times\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$$

$$=(\alpha-1)^2(\beta-1)^2(\alpha-\beta)^2$$

$$=\{\alpha\beta-(\alpha+\beta)1\}^2\{(\alpha+\beta)^2-4\alpha\beta\}$$

$$=\left(\frac{c}{a}+\frac{b}{a}+1\right)^2\left(\frac{b^2}{a^2}-\frac{4c}{a}\right)$$

$$=\frac{(a+b+c)^2(b^2-4ac)}{a^4}$$

Passage -2

51. $\because x^3-3x^2+3x+7=0$

$$\Rightarrow (x-1)^3+8=0$$

$$\Rightarrow (x-1)^3=(-2)^3$$

$$\Rightarrow \left(\frac{x-1}{-2}\right)=1$$

$$\Rightarrow \frac{x-1}{-2}=(1)^{1/3}=1,\omega,\omega^2$$

$$\Rightarrow x-1=-2,-2\omega,-2\omega^2$$
$$or\ x=-1,1-2\omega,1-2\omega^2$$

$$\therefore a=-1, b=1-2\omega, c=1-2\omega^2$$

$$\therefore \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$ (row by row)

$$=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \times \begin{vmatrix} -a & c & b \\ -b & a & c \\ -c & b & a \end{vmatrix}$$

$$=\begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & c^2 \end{vmatrix}$$

$$=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2$$

$$=\left(a^3+b^3+c^3-3abc\right)^2$$

$$=\left\{\left(a+b+c\right)\left(a^2+b^2+c^2-ab-bc-ca\right)\right\}^2$$

$$=\frac{1}{4}\left(a+b+c\right)^2\left\{\left(a-b\right)^2+\left(b-c\right)^2+\left(c-a\right)^2\right\}^2$$

$$=\frac{9}{4}\left\{-12\left(1+\omega+\omega^2\right)\right\}=0$$

52. Let $\Delta=\begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix}$

$$\Delta^c=\begin{vmatrix} a^2+\lambda^2 & ab+c\lambda & ca-b\lambda \\ ab-c\lambda & b^2+\lambda^2 & bc+a\lambda \\ ac+b\lambda & bc-a\lambda & c^2+\lambda^2 \end{vmatrix} \times \begin{vmatrix} \lambda & c & -b \\ -c & \lambda & a \\ b & -a & \lambda \end{vmatrix} = \Delta^2 \times \Delta = \Delta^3$$

Passage – 3

53 & 54

$$\frac{1+Tan\theta}{1-Tan\theta}=3\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$$

Put $\tan\theta=t$

$$\frac{1+t}{1-t}=\frac{3\left(3t-t^3\right)}{\left(1-3t^2\right)}$$

$$\Rightarrow 3t^4-6t^2+8t-1=0$$

Its roots are $\tan\alpha, \tan\beta, \tan\gamma, \tan\delta$

$\therefore$ (1) $\sum \tan\alpha = 0$

(2) $\pi \tan\alpha = \frac{-1}{3}$

(3) $\sum \frac{1}{Tan\alpha} = 8$

55. $\frac{d^2}{dx^2} at\ x = 1\ is \begin{vmatrix} 2 & -1 & 2 \\ 1 & -1/2 & 1 \\ 1 & a & a^2 \end{vmatrix} = 0$

56. Taking a,b,c common from $R_1, R_2\ and\ R_3$ respectively and then multiplying a,b,c in C_1, C_2, C_3 respectively, then

$$\begin{vmatrix} a^2+\left(b^2+c^2\right)\cos\phi & b^2\left(1-\cos\phi\right) & c^2\left(1-\cos\phi\right) \\ a^2\left(1-\cos\phi\right) & b^2+\left(c^2+a^2\right)\cos\phi & c^2\left(1-\cos\phi\right) \\ a^2\left(1-\cos\phi\right) & b^2\left(1-\cos\phi\right) & c^2+\left(a^2+b^2\right)\cos\phi \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, then

$$\begin{vmatrix} 1 & b^2\left(1-\cos\phi\right) & c^2\left(1-\cos\phi\right) \\ 1 & b^2+\left(c^2+a^2\right)\cos\phi & c^2\left(1-\cos\phi\right) \\ 1 & b^2\left(1-\cos\phi\right) & c^2+\left(a^2+b^2\right)\cos\phi \end{vmatrix}$$

$\left(\therefore a^2+b^2+c^2=1\right)$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$m then

$$\begin{vmatrix} 1 & b^2\left(1-\cos\phi\right) & c^2\left(1-\cos\phi\right) \\ 0 & \cos\phi & 0 \\ 0 & 0 & \cos\phi \end{vmatrix}$$

$\left(\therefore a^2+b^2+c^2=1\right)$

$= \cos^2\phi$, which is independent of a,b, and c.

57. $Let\,\Delta = \begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{16}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$

Applying $R_2 \to R_2 - \sqrt{2}R_1$

And $R_3 \to R_3 - \sqrt{3}R_1$

$$\therefore \Delta = \begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ 0 & \sqrt{3} & i\sqrt{6}-\sqrt{12} \\ 0 & \sqrt{2} & 2i-\sqrt{18} \end{vmatrix} = \sqrt{6}\left(-\sqrt{6}\right)$$

= -6 (real and rational)

58. Applying $R_2 \to 2R_1 \ and \ R_3 \to R_3 - R_1$

Then, $A = \begin{bmatrix} 1 & 4 & 5 \\ \lambda-2 & 0 & 8\lambda-16 \\ \lambda^2 & 8\lambda & 2\lambda+16 \end{bmatrix}$

For $\lambda = 2$,

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 0 & 0 \\ 4 & 16 & 20 \end{bmatrix} R_3 \to R_3 - 4R_1 \begin{bmatrix} 1 & 4 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \quad p(A) = 1$

For $A = \begin{bmatrix} 1 & 4 & 5 \\ -3 & 0 & -24 \\ 1 & -8 & 14 \end{bmatrix}$

$$R_3 \to R_3 - R_1 \begin{bmatrix} 1 & 4 & 5 \\ 0 & 12 & -9 \\ 0 & -12 & 9 \end{bmatrix}$$

$$R_3 \to R_3 + R_2 \begin{bmatrix} 1 & 4 & 5 \\ 0 & 12 & -9 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 2$

For $\lambda \neq -1, 2, \rho(A) = 3$

59. Applying $C_1 \to C_1 - xC_2$, then

$$\begin{vmatrix} a_1\left(1-x^2\right) & a_1x+b_1 & c_1 \\ a_2\left(1-x^2\right) & a_2x+b_2 & c_2 \\ a_3\left(1-x^2\right) & a_3x+b_3 & c_3 \end{vmatrix} = 0$$

$$\Rightarrow \left(1-x^2\right)\begin{vmatrix} a_1 & a_1x+b_1 & c_1 \\ a_2 & a_2x+b_2 & c_2 \\ a_3 & a_3x+b_3 & c_3 \end{vmatrix}=0$$

Now, applying $C_2 \to C_2 - xC_1$, then

$$\left(1-x^2\right)\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

$$\therefore x=\pm 1 \ and \ \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$

60. $8^{\frac{1}{1-|\cos x|}}=8^2$

$$\Rightarrow |\cos x|=\frac{1}{2} \Rightarrow \cos x=\pm\frac{1}{2}$$

$$x=\pm\frac{\pi}{3},\pm\frac{2\pi}{3}$$