MasterJEE

IIT-JEE | Medical | Foundations

Master JEE CLASSES

Kukatpally, Hyderabad.

JEE-ADVANCE-2011-P2-Model

Max.Marks:240

2011-PAPER -II

IMPORTANT INSTRUCTIONS:

1) This booklet is your Question Paper.
2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
3) Blank spaces are provided within this booklet for rough work.
4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
7) The ORS will be collected by the invigilator at the end of the examination.
8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS**.** Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
11) Darken the bubble **COMPLETELY**.
12) The correct way of darkening a bubble is as:
13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
14) Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble.

IMPORTANT INSTRUCTIONS **Max Marks: 240**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 1 – 8)	Questions with Single Correct Choice	3	-1	8	24
Sec – II (Q.N : 9 – 12)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III (Q.N : 13 – 18)	Questions with Integer Answer Type	4	0	6	24
Sec – IV (Q.N : 19 – 20)	Matrix Matching Type	8	0	2	16
Total				**20**	**80**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 21 – 28)	Questions with Single Correct Choice	3	-1	8	24
Sec – II (Q.N : 29 – 32)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III (Q.N : 33 – 38)	Questions with Integer Answer Type	4	0	6	24
Sec – IV (Q.N : 39 – 40)	Matrix Matching Type	8	0	2	16
Total				**20**	**80**

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 41 – 48)	Questions with Single Correct Choice	3	-1	8	24
Sec – II (Q.N : 49 – 52)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III (Q.N : 53 – 58)	Questions with Integer Answer Type	4	0	6	24
Sec – IV (Q.N : 59 – 60)	Matrix Matching Type	8	0	2	16
Total				**20**	**80**

space for rough work

SECTION-I
(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.** For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

1. The aqueous solutions of lithium salts are poor conductor of electricity rather than other alkali metals because of

   A) high ionization energy

   B) high electronegativity

   C) lower ability of Li^+ ions to polarize water molecules

   D) Increase in size due to higher degree of hydration of Li^+ ion

2. Which of the following orders presents the correct sequence of the increasing basic nature of the given oxides?

   A) $Al_2O_3 < MgO < Na_2O < K_2O$

   B) $MgO < K_2O < Al_2O_3 < Na_2O$

   C) $Na_2O < K_2O < MgO < Al_2O_3$

   D) $K_2O < Na_2O < Al_2O_3 < MgO$

3. What happens when sodium metal is heated to a temperature $350^0 C$ in excess of dry air containing carbon dioxide gas?

A) Na_2O_2 is formed

B) Na_2O is formed

C) First Na_2O_2 is formed which then converts into Na_2CO_3

D) First Na_2O_2 is formed which then converts into Na_2CO_3

4. $X + C + Cl_2 \xrightarrow[of\ about\ 1000K]{High\ temparature} Y + CO;\ Y + 2H_2O \rightarrow Z + 2HCl$ Compound Y is found in polymeric chain structure and is an electron deficient Molecule. Y must be

A) BeO B) $BeCl_2$ C) BeH_2 D) $MgCl_2$

5. $BaC_2 + N_2 \xrightarrow{\Delta} (1)$

$CaC_2 + N_2 \xrightarrow{\Delta} (2)$

(1) and (2) are

A) $BaCN_2, CaCN_2$ B) $Ba(CN)_2, Ca(CN)_2$

C) $Ba(CN)_2, CaCN_2$ D) None of these.

6. Which of the following is correct related to change aluminium oxide?

A) $Al_2O_3 \xrightarrow{\text{Dil Hcl}} AlCl_3 \xrightarrow[\text{NaOH}]{\text{Conc}} \left[Al(OH)_6\right]^{3-}$

$\xleftarrow{\quad\Delta\quad}$

B) $Al_2O_3 \xrightarrow{\text{Dil HCl}} AlCl_3 \xrightarrow[\text{NaOH}]{\text{Dil.}} Al(OH)_3$

$\xleftarrow{\quad\Delta\quad}$

C) $Al_2O_3 \xrightarrow{conc.HCl} AlO_2^- \xrightarrow[NaOH]{dil} Al(OH)_3$

$\xleftarrow{\quad\Delta\quad}$

D) $Al_2O_3 \xrightarrow[\Delta]{\text{NaOH}} AlO_2^- \xrightarrow[\text{NaOH}]{\text{Con.}} \left[Al(OH)_6\right]^{3-}$

$\xleftarrow{\quad\Delta\quad}$

7. Borate salts when heated with conc. H_2SO_4 *and* C_2H_5OH produce characteristic green colouration on flame due to the formation of volatile compound:

A) $(C_2H_5)_3B$ B) B_2H_6 C) $B(OC_2H_5)_3$ D) B_2O_3

8. When borax is dissolved in water:

A) $B(OH)_3$ is formed only

B) $\left[B(OH)_4\right]^-$ is formed only

C) both $B(OH)_3$ and $\left[B(OH)_4\right]^-$ are formed

D) $\left[B_3O_3(OH)_4\right]^-$ formed only

SECTION-II
(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

9. Select correct statement(s)

A) Li_2CO_3 is only sparingly soluble in water and no $LiHCO_3$ has been isolated.

B) K_2CO_3 cannot be made by a method similar to the ammonia- soda(solvay) process.

C) Li_2CO_3 and $MgCO_3$ both are thermally stable.

D) Due to alkaline nature of oxides and hydroxides of alkaline earth metals, these are found in the earth crust.

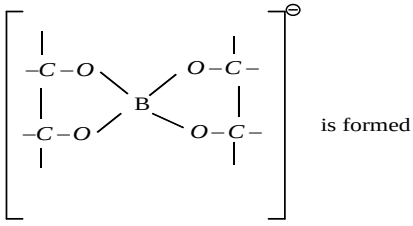
10. Which of the following is/are correct?

A) Ionization energy $= Ca > K$ B) Melting point $= Ca > K$

C) Solubility $= CaSO_4 > K_2SO_4$ D) Hydration energy $= Ca^{2+} > K^+$

11. On addition of cis 1-2 diol in the aqueous solution of boric acid:

A) H^+ ion concentration increases B) H^+ ion concentration decreases

C)  is formed

D) BO_3^{3-} is formed

12. The correct statements among the following

A) $NaBH_4$ is very much less rapidly hydrolysed by water than is $NaAlH_4$

B) The rate of hydrolysis of diborane by water vapour is proportional to $P_{B_2H_6}^{\frac{1}{2}}.P_{H_2O}$

C) 2, 6 – Dimethyl pyridine although stronger base than pyridine do not form adduct with trimethyl boron, however combine with triethyl aluminium

D) A saturated aqueous solution of boric acid do not effect the colour of bromocresol green (pH range 3.8-5.4) but changes its colour on adding excess of boric acid to a solution of KHF_2 due to alkaline.

SECTION-III
[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

13. Out of $Be, Mg, Ca, Sr \text{ and } Ba$, number of elements which do not impart any colour to the flame, is

14. On heating a mixture of 8 moles of Li_2CO_3 and $KHCO_3$ in 3: 1 ratio, the number of moles of CO_2 liberated is

15. Na_2O_2 is a strong oxidizing agent. It oxidized chromic compounds into chromates. How many moles of Na_2O_2 are required to convert two moles of $Cr(OH)_3$ into sodium chromate?

16. In the compound $Na_2\left[B_4O_5(OH)_4\right].8H_2O$, if the

    i)number of B-O-B bonds is'x'

    ii)number of B-B bonds is 'y'

    iii)number of sp^2 hybridized B atoms is'z'

    Calculate the value of x+y+z.

17. 1 mole of $B_2H_6(g)$ on hydrolysis yields……moles of $H_2(g)$.

18. Find the number of B-O bonds in peroxoborate.

SECTION-IV
[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

19. Match the particulars/name of process listed in **column-I** with the formula of the compounds listed in **column II**.

Column-I	**Column-II**
A) Solvay process	P) Na_2O
B) Evolve CO_2 on heating	Q) Na_2O_2
C) Aqueous solution is not neutral towards litmus	R) $NaHCO_3$
D) Used as air purifier in submarine	S) Na_2CO_3

20. Match the Reactions given in **column I** with the Products given in **column II.**

Column-I	**Column-II**
A) $B_2H_6.2NH_3 \xrightarrow{200^0C}$	P) H_2
B) $B_2H_6 \xrightarrow{\text{Red heat}}$	Q) $B_3N_3H_6$
C) $B_2H_6 + 6H_2O \rightarrow$	R) H_3BO_3
D) $B_2H_6 + HCl \rightarrow$	S) B

SECTION-1
(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.** For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

21. Consider four bodies of equal mass - a ring, a cube, a disc and a sphere. Ring, disc and sphere have the same diameter, equal to the length of the cube on each edge. Ring and disc rotate about the axes passing through their centers and perpendicular to their planes. Sphere rotates about its diameter. Cube rotates about an axis passing through centre and parallel to edges. Which one has the largest moment of inertia?

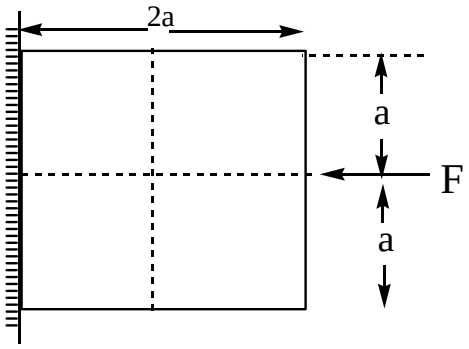
A) Ring B) Cube C) Disc D) Sphere.

22. A circular disc A of radius r is made from an iron plate of thickness t and another circular disc B of radius 4r is made from another iron plate of thickness t/4.The relation between the moments of inertia I_A *and* I_B is(I_A, I_B are the moments of inertia of the objects about normal axes passing through their centers)

A) $I_B = 128 I_A$ B) $I_A = I_B$

C) $I_B = 64 I_A$ D) dependent on the actual values of t and r

23. A block of mass m is held stationary against a rough wall by applying a force F as shown in figure-.Which one of the following statements is incorrect?



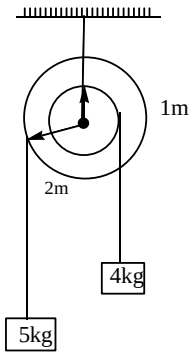
A) frictional force $f = mg$

B) normal reaction N=F

C) F will not produce a torque about centre of the block

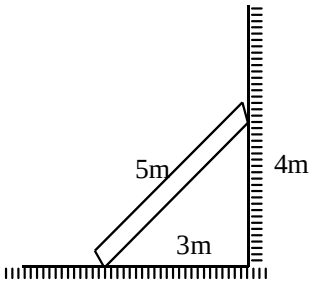
D) N will not produce any torque about centre of the block

24. The moment of inertia of the pulley system as shown in the figure is 4 kgm^2.The radii of bigger and smaller pullies are 2m and 1 m respectively. Angular acceleration of the pulley system is:(take g=10m/s^2)



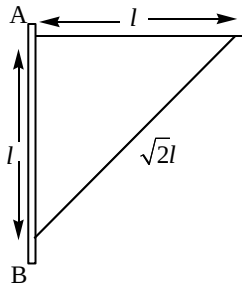
A) $2.1\ rad/s^2$ B) $4.2\ rad/s^2$ C) $1.2\ rad/s^2$ D) $0.6\ rad/s^2$

25. A uniform ladder of length 5m is placed against the wall as shown in the figure. If coefficient of friction μ is the same for both the walls, what is the minimum value of μ for it not to slip?



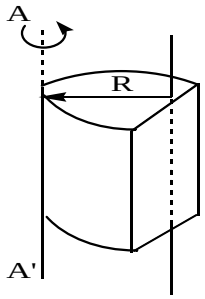
A) $\mu = \frac{1}{2}$ B) $\mu = \frac{1}{4}$ C) $\mu = \frac{1}{3}$ C) $\mu = \frac{1}{5}$

26. Find moment of inertia of the triangular lamina of mass M about the axis of rotation AB shown in figure.



A) $\frac{Ml^2}{6}$ B) $\frac{Ml^2}{3}$ C) $\frac{Ml^2}{12}$ D) $\frac{3}{4}Ml^2$

27. Find moment of inertia of the half cylinder of mass M shown in figure about the axis AA'.



A) $\frac{(9-16\pi)}{6\pi}MR^2$ B) $\left(\frac{9\pi-16}{6\pi}\right)MR^2$ C) $\frac{3}{2}MR^2$ D) $2MR^2$

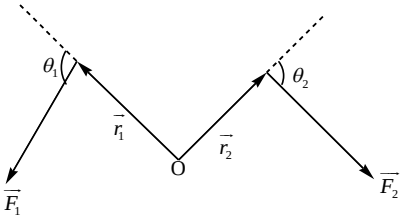
28. Calculate the moment of inertia of a wheel about its axis which is having rim of mass 24M and twenty four spokes each of mass M and length l.

A) $24Ml^2$ B) $32Ml^2$ C) $64Ml^2$ D) $16Ml^2$

SECTION-II
(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

29. In the figure are shown the lines of action and moment arms of two forces about the origin O. Imagining these forces to be acting on a rigid body pivoted at O, all vectors shown being in the plane of the figure, the magnitude and direction of the resultant torque will be:



A) $(F_2 r_2 \sin\theta_2 - F_1 r_1 \sin\theta_1)$ out of the plane of the page.

B) $(F_1 r_1 \sin\theta_1 - F_2 r_2 \sin\theta_2)$ out of the plane of the page.

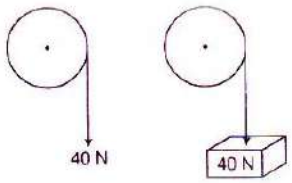
C) $(F_2 r_2 \sin\theta_2 - F_1 r_1 \sin\theta_1)$ into the plane of the page.

D) zero.

30. Two identical semicircular discs of mass 'm' each and radius 'R' are placed in the XY(horizontal) plane and the YZ (vertical) plane, respectively. They are so placed that they have their common diameter along the Y-axis and centre of curvatures are at the origin. Then, the moment of inertia (I_n) of the system about the appropriate axis is given by (I_n refers to moment of inertia about axis n-where n is X,Y,Z)

A) $I_x = \frac{1}{2}mR^2$ B) $I_y = \frac{1}{2}mR^2$ C) $I_z = \frac{3}{4}mR^2$ D) $I_x = I_y = I_z$

31 A cord is wrapped around the rim of a solid cylinder of radius 0.25 m and a constant force of 40 N is exerted on the cord. The cylinder is mounted on frictionless bearings and its moment of inertia is $4kg - m^2$



The angular velocity of the cylinder after 5 m of cord has been unwound, is ω_1. If the 40N force is replaced by a 40 N weight and released from rest at the top, the angular

velocity of the cylinder after 5 m of cord has been unwounded is ω_2. (Given, g = 10 ms^{-2}). Then,

A) $\omega_1 = 10\ rad.s^{-1}$ B) $\omega_2 = \frac{40}{\sqrt{17}}\ rad/s$

C) $\omega_1 = 8.16\ rad.s^{-1}$ D) $\omega_2 = 8\ rad.s^{-1}$

32. A sphere is rotating about a fixed axis along a diameter with constant angular velocity. The which of the following options is/are correct:

A) The particles on the surface of the sphere do not have any linear acceleration.

B) The particles on the diameter mentioned above do not have any linear acceleration

C) Different particles on the surface have different angular speeds

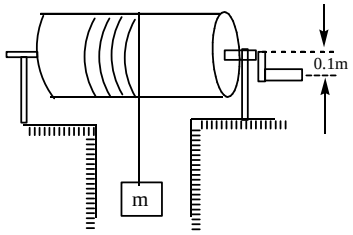
D) All the particles on the surface have the same linear speed

SECTION-III
[INTEGER TYPE]

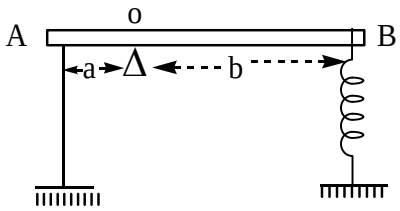
Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section.

33. The mechanism shown in figure is used to raise a wooden box of mass m = 50 kg. A string is wrapped around a cylinder that turns on an axle. The cylinder has radius 0.25 m and moment of inertia 1 $kg-m^2$ about the axle. Magnitude of the force F applied

tangentially to the rotating crank handle is required to raise the box with an acceleration of 2.0 m/s^2 is $1500+16a$ N .Here we can neglect the moment of inertia of the axle and the crank. Then $a=$



34. A uniform rod AB of mass 2 kg and length $l=100cm$ is placed on a sharp support O such that $AO=a=40cm$ and $OB=b=60cm$ .A spring of force constant k=600 N/m is attached to end B as shown in figure. To keep the rod horizontal, its end A is tied with a thread such that the spring is elongated by y=1 cm.The reaction of support on the rod immediately after the thread is burnt is 5a N. Then a=

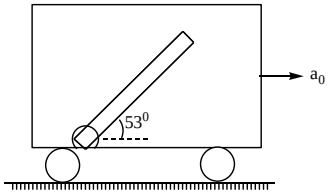


35. Two wooden disc ,one with radius 2 cm and mass 1 kg and the other with radius 4 cm and mass 2 kg, are welded together coaxially and mounted on a frictionless axis through their common centre. A light string is wrapped around the edge of the smaller disc, and a 3 kg block is suspended from the free end of the string. The acceleration of the block after it is released is $\frac{2}{3}a\ m/s^2$ then what is the value of a? Assume that string does not slip on the disc.

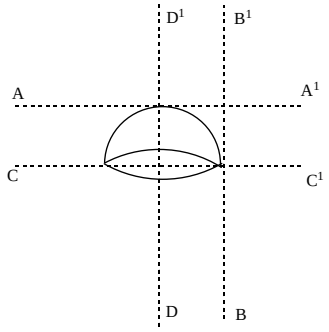
36. A rod of length l is pivoted about an end. Find Moment of inertia of the rod about this axis if the linear mass density of rod varies as $\rho = ax^2 + b$ (x is distance from axis of rotation)is $\frac{al^5}{5(p)} + \frac{bl^3}{q}$ .Then $p \times q =$

37. A uniform rod is pivoted to a trolley as shown in figure. Let α_1 is the angular acceleration of rod with respect to trolley when the trolley is moving with uniform

velocity V_o and α_2 is the angular acceleration of the rod with respect to trolley when the trolley is moving with uniform acceleration a_o so that $\vec{\alpha}_2 = -\vec{\alpha}_1$. Then $\frac{2a_o}{g} = \_\_?$



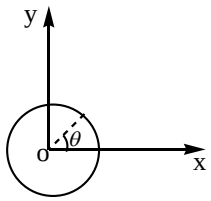
38. Consider a thin uniform shell as shown. Let I_1, I_2, I_3 *and* I_4 are the moments of inertia of the object about axes AA^1, BB^1, CC^1 & DD^1 respectively. $I_1 + I_2 + I_3 + I_4 = \frac{nI_1}{2}$. Then $n - 5 = \_\_\_$



SECTION-IV
[Matrix Matching Type]

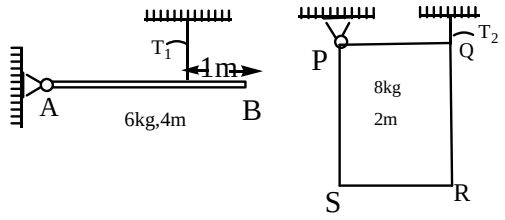
Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

39. A thin circular ring of mass M and radius R is placed at origin in xy plane .Let $I_x, I_y,$ *and* I_z are the moments of inertia of it about x-axis, y-axis and z-axis respectively. If λ is the mass per unit length of ring then



A) If $\lambda = \lambda_o(1+\cos\theta)$, $\lambda_0 = \text{constant}$	P) $I_x = \dfrac{MR^2}{2}$
B) If $\lambda = \lambda_0(1+x), \lambda_0 = \text{constant}$	Q) $I_y = \pi\lambda_o R^3$
C) If $\lambda = \lambda_0(x^2+y^2), \lambda_0 = \text{constant}$	R) $I_x = 2\pi\lambda_o R^3$
D) If $\lambda = \lambda_o = \text{constant}$	S) $I_z = MR^2$

40. A uniform thin rod AB of mass 6 kg, length 4m and a uniform thin square plate PQRS of mass 8 kg, side length 2 m are held horizontally as shown. Let F_1 & F_2 are the forces exerted by the pivots on rod& square plate and F_1^1 & F_2^1 are the forces exerted by the pivots on the objects just after the strings are cut.(take g=10 m/s^2). Then



A) $2F_1$

B) F_2

C) $8F_1^1$

D) $\sqrt{136}F_2^1$

P) 340 N

Q) 40 N

R) 120 N

S) 680 N

SECTION-1
(SINGLE CORRECT CHOICE TYPE)

Section-I (Single Correct Answer Type, Total Marks: 24) contains 8 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.** For each question you will be awarded 3 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks if no bubble is darkened. In all other cases, minus one (-1) mark will be awarded.

41. Let $L=\prod_{n=3}^{\infty}\left(1-\frac{4}{n^2}\right); M=\prod_{n=2}^{\infty}\left(\frac{n^3-1}{n^3+1}\right)$ and $N=\prod_{n=1}^{\infty}\frac{\left(1+n^{-1}\right)^2}{1+2n^{-1}}$; then the value of $L^{-1}+M^{-1}+N^{-1}$ is.

A) 3 B) 6 C) 8 D) 2

42. The value of $\lim_{n\to\infty}\left[\left\{4\sum_{r=0}^{n}(r+1)(r+2)(r+3)\right\}^{\frac{1}{4}}-n\right]$ is

A) 0 B) Does not exist

C) 2/5 D) 5/2

43. If $f(x)$ is a polynomial satisfying $2+f(x)f(y)=f(x)+f(y)+f(xy)$ for all $x,y\in R$ and $f(2)=5$ $(f(1)\neq 1)$, then $\lim_{x\to 5} f(x)=$ ____

A) 35 B) 26 C) 5 D) 126

space for rough work

44. If $\lim_{x\to\infty}\left(a^x+e^x\right)^{1/x}=a;(a>0)$ then

A) $a\in(1,\infty)$ B) $a\in(e,\infty)$ C) $a\in(1,e)$ D) None of these.

45. Let r^{th} term of a series be given by $t_r=\frac{r}{1-3r^2+r^4}$. Then $\lim_{n\to\infty}\sum_{r=1}^{n}t_r$ is

A) 3/2 B) 1/2 C) -1/2 D)-3/2

46. Let $f(x)=\lim_{n\to\infty}\frac{1}{((3/\pi)\tan^{-1}2x)^{2n}+5}$. Then the set of values of x for which $f(x)=0$ is

A) $|2x|>\sqrt{3}$ B) $|2x|<\sqrt{3}$ C) $|2x|\geq\sqrt{3}$ D) $|2x|\leq\sqrt{3}$

47. Let $f(x)$ be defined for all $x\in R$ such that

$\lim_{x\to0}\left[f(x)+\ln\left(1-\frac{1}{e^{f(x)}}\right)-\ln(f(x))\right]=0$ then $\lim_{x\to0}f(x)$ is equal to

A) 0 B) 1 C) e D) 1/e

48. The value of $\lim_{x\to0}\left\{\sin^2\left(\frac{\pi}{2-ax}\right)\right\}^{\sec^2\frac{\pi}{2-bx}}$ is equal to

A) e^{a^2/b^2} B) e^{-a^2/b^2} C) 1 D) 0

space for rough work

SECTION-II
(MORE THAN ONE TYPE)

Section - II (Multiple Correct Answers Type, Total Marks: 16) contains 4 multiple choice questions. Each question has four choices (A), (B), (C) and (D) out of which ONE or MORE may be correct. For each question you will be awarded 4 marks if you darken ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. There are no negative marks in this section.

49. If $f(x)=|x-1|-[x]$, where $[x]$ is the greatest integer less than or equal to x, then

A) $f(1^+)=-1, f(1^-)=0$ B) $f(1^+)=f(1^-)=0$

C) $\lim_{x\to 1} f(x)$ exists D) $\lim_{x\to 1} f(x)$ does not exist.

50. If $\lim_{x\to 1}\left(2-x+a[x-1]+b[1+x]\right)$ exists, then a and b can take the values (where $[.]$ denotes the greatest integer function)

A) $a=1/3, b=1$ B) $a=1, b=-1$

C) $a=9, b=-9$ D) $a=2, b=2/3$

51. Given a real valued function f such that $f(x)=\begin{cases}\dfrac{\tan^2\{x\}}{\left(x^2-[x]^2\right)}, & for\ x>0\\ 1 & for\ x=0\\ \sqrt{\{x\}\cot\{x\}}, & for\ x<0\end{cases}$

Where $[x]$ is the integer part and $\{x\}$ is the fractional part of x, then

A) $\lim_{x\to 0^+} f(x)=1$ B) $\lim_{x\to 0^-} f(x)=\sqrt{\cot 1}$

C) $\cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2=1$ D) $\tan^{-1}\left(\lim_{x\to 0^+} f(x)\right)=\frac{\pi}{4}$

52. Which of the following is/are correct.

A) $\lim_{x\to 0}\left(\frac{2^x+2^{2x}+2^{3x}}{3}\right)^{\frac{1}{x}}=4$ B) $\lim_{x\to\infty}\left(\frac{x+6}{x+1}\right)^{x+4}=e^5$

C) $\lim_{x\to 0}\left((\sin x)^{\frac{1}{x}}+\left(\frac{1}{x}\right)^{\sin x}\right)=1,(x>0)$ D) $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{\frac{1}{x}}=e^6$

SECTION-III
[INTEGER TYPE]

Section-III (Integer Answer Type, Total Marks: 24) contains 6 questions. The answer to each of the questions is a single-digit integer, ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS. For each question you will be awarded 4 marks if you darken ONLY the bubble corresponding to the correct answer and zero marks otherwise. There are no negative marks in this section**.**

53. The integer value of n for which $\lim_{x\to 0}\frac{\cos^2 x-\cos x-e^x\cos x+e^x-\frac{x^3}{2}}{x^n}$ is a finite nonzero number is _

space for rough work

54. If $f(x)=\underset{x\to\infty}{lt}\sum_{r=0}^{n}\frac{\tan\left(\frac{x}{2^{r+1}}\right)+\tan^3\left(\frac{x}{2^{r+1}}\right)}{1-\tan^2\left(\frac{x}{2^{r+1}}\right)}$ then $\underset{x\to0}{lt}\frac{f(x)-\sin x}{x^3}=\frac{1}{k}$ Where K is equal to

55. If L= $\lim_{x\to0+}\left\{\lim_{n\to\infty}\left(\frac{\left[1^2(\sin x)^x\right]+\left[2^2(\sin x)^x\right]+.....+\left[n^2(\sin x)^x\right]}{n^3}\right)\right\}$ where[.]denotes the greatest integer function. Then $\frac{3}{L}=$__

56. If $f(n,\theta)=\prod_{r=1}^{n}\left(1-\tan^2\frac{\theta}{2^r}\right)$, then $\lim_{\theta\to0}\left(\lim_{n\to\infty}f(n,\theta)\right)=$__

57. If $\lim_{x\to\infty}f(x)$ exists and is finite and nonzero and if $\lim_{x\to\infty}\left\{f(x)+\frac{3f(x)-1}{f^2(x)}\right\}=3$, then the value of $\lim_{x\to\infty}f(x)$ is__

58. Let $f(x)=\frac{-ax+\sin(x-1)+a}{x+\sin(x-1)-1}$, $g(x)=\frac{1-x}{1-\sqrt{x}}$ and $h(x)=2$.The largest value of the non negative integer for which $\lim_{x\to1}\left(f(x)\right)^{g(x)}=\frac{1}{4}$ is m and $\lim_{x\to1}\left(f(x)\right)^{h(x)}=\frac{1}{4}$ is n then m+n=__

space for rough work

SECTION-IV
[Matrix Matching Type]

Section-IV (Matrix-Match Type, Total Marks: 16) contains 2 questions. Each question has four statements (A, B, C and D) given in Column I and five statements (p, q, r, s and t) in Column II. Any given statement in Column I can have correct matching with ONE or MORE statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS. For each question you will be awarded 2 marks for each row in which you have darkened ALL the bubble(s) corresponding to the correct answer(s) ONLY and zero marks otherwise. Thus, each question in this section carries a maximum of 8 marks. There are no negative marks in this section.

59. Match the following from Column –I to Column-II.

Column-I	**Column-II**
A) $\lim_{x\to 0}\left(\left[200\frac{\tan x}{x}\right]+\left[200\frac{\sin x}{x}\right]\right)$ equals	P)1
B) $\lim_{x\to 0}\left(\left\{100\frac{\tan x}{x}\right\}+\left\{100\frac{\sin x}{x}\right\}\right)$ equals	Q)399
C) $\lim_{x\to 0}\left(100\left[\frac{\sin x}{x}\right]+200\left[\frac{x}{\tan^{-1}x}\right]\right)$ equals	R)199
D) $\lim_{x\to 0}\left(\left\{\frac{\sin^{-1}x}{x}\right\}+\left[200\frac{\tan^{-1}x}{x}\right]\right)$	S) 200

Here $[.]$ and $\{x\}$ are respectively greatest integer function and fractional part functions.

60. Let $\phi(x)=\frac{a_0x^m+a_1x^{m+1}+....a_kx^{m+k}}{b_0x^n+b_1x^{n+1}+.....b_lx^{n+l}}$; where $a_0\neq 0, b_0\neq 0$, then $\lim_{x\to 0}\phi(x)$ is equal to for

(m, n>0 and $k,l\geq 0$)

Column-I	**Column-II**
A) $m>n$	P) ∞
B) m=n	Q) $-\infty$
C) $m<n$ *and* $n-m$ *is even*, $a_0/b_0>0$	R) a_0/b_0
D) $m<n$ *and* $n-m$ *is even*, $a_0/b_0<0$	S) 0

Master JEE CLASSES
Kukatpally, Hyderabad.

JEE-ADVANCE-2011-P2-Model
Max.Marks:240

KEY SHEET

CHEMISTRY

1	**D**	2	**A**	3	**C**	4	**D**	5	**C**
6	**B**	7	**C**	8	**C**	9	**ABD**	10	**ABD**
11	**AC**	12	**ABCD**	13	**2**	14	**7**	15	**3**
16	**7**	17	**6**	18	**8**	19	**A-RS B-R C-PQRS D-Q**	20	**A-PQ B-PS C-PR D-P**

PHYSICS

21	**A**	22	**C**	23	**D**	24	**A**	25	**C**
26	**A**	27	**B**	28	**B**	29	**BC**	30	**BC**
31	**AB**	32	**B**	33	**5**	34	**4**	35	**6**
36	**3**	37	**3**	38	**6**	39	**A-PQS B-PQS C-PS D-PQS**	40	**A-Q B-Q C-R D-S**

MATHS

41	**C**	42	**D**	43	**B**	44	**B**	45	**C**
46	**A**	47	**A**	48	**B**	49	**AD**	50	**BC**
51	**ABCD**	52	**ABC**	53	**4**	54	**2**	55	**9**
56	**1**	57	**1**	58	**2**	59	**A-Q B-P C-S D-R**	60	**A-S B-R C-P D-Q**

SOLUTIONS
CHEMISTRY

1. D.

Due to the larger size of hydrated Li^+ ion conductivity decreases.

2. A

As metallic character of element attached to oxygen atom increases, the difference between the electronegativity values of element and oxygen increases and thus basic character of oxides increases and vice versa, Hence, the increasing correct order of basic nature is $Al_2O_3 < MgO < Na_2O < K_2O$

3. C

$2Na + O_2 \xrightarrow{350^0 C} Na_2O_2; 2Na_2O_2 + 2CO_2 \rightarrow 2Na_2CO_3 + O_2$

4. D

$BeO + C + Cl_2 \rightarrow BeCl_2 + CO$

(y)

$BeCl_2 + 2H_2O \rightarrow Be(OH)_2 + 2HCl$

(z)

5. C

$BaC_2 + N_2 \xrightarrow{\Delta} Ba(CN)_2$

$CaC_2 + N_2 \xrightarrow{\Delta} CaCN_2$

6. A)

$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$

$AlCl_3 + 6NaOH \rightarrow Na_3[Al(OH)_3] \xrightarrow{\Delta} Na_3AlO_3 + 3H_2O$

B)

$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$

$AlCl_3 + 3NaOH \rightarrow Al(OH)_3 \xrightarrow{\Delta} Al_2O_3 + 3H_2O$

C)

$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O(\text{Not } AlO_2^-)$

$AlCl_3 + 3NaOH \rightarrow Al(OH)_3$

D)

$$Al_2O_3 + NaOH \rightarrow AlO_2^- \xrightarrow[NaOH]{Conc} Na_3\left[Al(OH)_6\right] \xrightarrow{\Delta} Na_3AlO_3 + 3H_2O$$

7. C

$$H_3BO_3 + 3C_2H_5OH \rightarrow (C_2H_5)BO_3 + 3H_2O$$

Methyl borate

(green flame)

8. C

$$\left[B_4O_5(OH)_4\right]^{2-} + 5H_2O \rightleftharpoons 2B(OH)_3 + 2\left[B(OH)_4\right]^-$$

or $\left[B_4O_7\right]^{2-} + 7H_2O \rightarrow 2B(OH)_3 + 2\left[B(OH)_4\right]^-$

9. ABD

The solubility of the alkali metal carbonates increases down the group from Li to Cs. This is because of the fact that down the group with increasing size of cation the lattice energy as well as hydration energy also decrease but the change in lattice energy is more as compare to that of hydration energy. lithium hydrogen carbonate is not obtained in the solid form while all other elements form solid hydrogen carbonates.

b) K_2CO_3 cannot be prepared by solvary process because intermediate formed, $KHCO_3$ is appreciable soluble in water.

c) Li_2CO_3 and $MgCO_3$ both are not thermally stable.

d) Fact ref. NCERT

10. ABD

$$K_2SO_4 > CaSO_4$$

Water soluble Insoluble

11. AC

Conceptual

12. ABCD

A) Boron do not contain vacant d-orbitals will A*l* contain vacant d-orbitals.

B) $B_2H_6 \underset{}{\overset{fast}{\rightleftharpoons}} 2BH_3$

$BH_3 + H_2O \xrightarrow{slow} products$

C) Due to steric hindrance in 2,6-dimethyl pyridine.

D) $B(OH)_3 + 2HF_2^- \rightarrow BF_4^- + 2H_2O + OH^-$ or $H_3BO_3 + 2KHF_2 \rightarrow KBF_4 + 2H_2O + 2KOH$

13. 2

Out of s-black elements Be and Mg do not impart any colour to flame.

14. 7

8 moles of a mixture of Li_2CO_3 and $KHCO_3$ contain 3:1 ratio means it contain 6 moles of Li_2CO_3 and 2 moles $KHCO_3$.

On heating, $Li_2CO_3 \rightarrow Li_2O + CO_2$

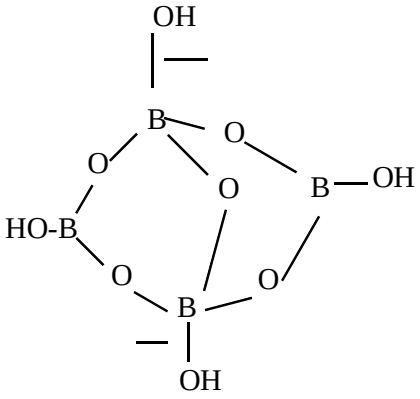
$2KHCO_3 \rightarrow K_2CO_3 + H_2O + CO_2$

6 moles of CO_2 will be evolved on heating 6 moles of Li_2CO_3 and 1 mol. of CO_2 is evolved from 2 moles of $KHCO_3$

15. 3

$2Cr(OH)_3 + 3Na_2O_2 \rightarrow 2Na_2CrO_4 + 2NaOH + 2H_2O$

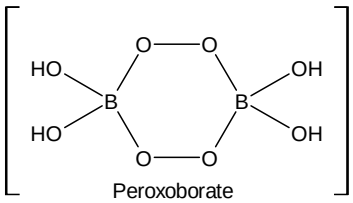
16. 7



17. 6

$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3 + 6H_2$

18. 8



19. $(a \rightarrow r,s);(b \rightarrow r);(c \rightarrow pqrs);(d \rightarrow q)$

a)solvay process is used for the manufacture of $NaHCO_3$ and Na_2CO_3;the raw material is NaCl

b) $2NaHCO_3 \xrightarrow{\Delta} Na_2CO_3 + CO_2 + H_2O$

c) $Na_2O + 2H_2O \rightarrow 2NaOH$;NaOH being basic turns red litmus blue.

$Na_2O_2 + 2H_2O \rightarrow 2NaOH + H_2O_2$;NaOH being basic turns red litmus blue but H_2O_2 being agent bleaches coloured litmus.

$NaHCO_3 + H_2O \rightleftharpoons NaOH + H_2CO_3$ solution is alkaline and turns red litmus blue.

d) $Na_2O_2 + CO_2 \rightarrow Na_2CO_3; 2Na_2O_2 + 2CO \rightarrow 2Na_2CO_3 + O_2$,Hence, it absorbs CO and CO_2 and liberates oxygen.

20. $(a \rightarrow pq);(b \rightarrow p,s);(c \rightarrow p,r);(d \rightarrow p)$

Reactions are $\rightarrow$

$B_2H_6 \xrightarrow[heat]{\text{Red}} 2B + 3H_2$

$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3(aq) + 6H_2$

$B_2H_6 + HCl \xrightarrow{AlCl_3} B_2H_5Cl + H_2$

$B_2H_6 + NH_3 \xrightarrow{200^0C} B_3N_3H_6$

(Borazole)

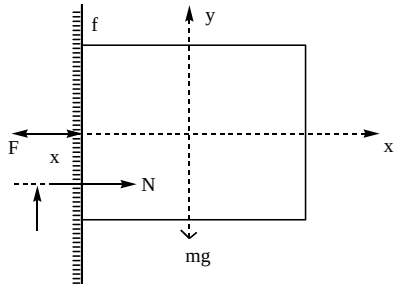
(Inorganic benzene borazine)

PHYSICS

21. Compare the standard moment of inertias of the bodies

22. $I_A = \dfrac{(8\pi r^2 t)r^2}{2}, I_B = \left(8\pi 16r^2 \dfrac{t}{4}\right)\dfrac{16r^2/e}{2}, I_B = 64I_A$

23. The different forces on the block are shown in figure.



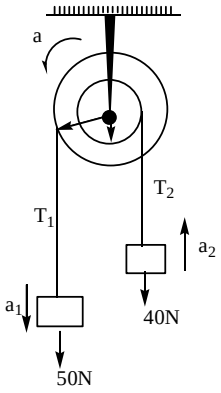
For equilibrium, $f = mg$

And normal reaction ,N=F

Torque due to $F = 0$ (F passes through centre)

As the body is in equilibrium and hence, torque due to friction about centre of mass is equal to torque due to normal reaction about centre of mass.

24. From figure shown



$50 - T_1 = 5(2\alpha)$

$T_2 - 40 = 4(\alpha)$

$T_1(2) - T_2(1) = 4\alpha$

From (1),(2) and (3),

$2(50 - 10\alpha) - (40 + 4\alpha) = 4\alpha$

$\Rightarrow 100 - 20\alpha - 40 - 4\alpha = 4\alpha$

$\Rightarrow 28\alpha = 60$

$\Rightarrow \alpha = \frac{60}{28} = 2.1 m/s^2$

25. For equilibrium of rod considering limiting friction we use

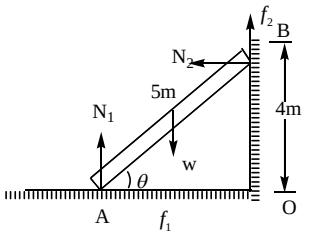
$f_1 = N_2$

$\mu N_1 = N_2$

$-w + N_1 + f_2 = 0$

$N_1 + \mu N_2 = w$

$N_1 + \mu^2 N_1 = W$



Balancing all torque about point A, we use

$$-w\left(\frac{3}{2}\right)+f_2(3)+N_2(4)=0$$

`

$$\Rightarrow \frac{-3W}{2}+3f_2+4N_2=0$$

$$\Rightarrow 3\mu N_2+4N_2=\frac{3}{2}(w)$$

$$\Rightarrow \mu N_1\left(3\mu^2+4\mu\right)=\frac{3}{2}N_1\left(1+\mu^2\right)$$

$$2\left(3\mu^2+4\mu\right)=3+3\mu^2$$

$$6\mu^2+8\mu=3+3\mu^2$$

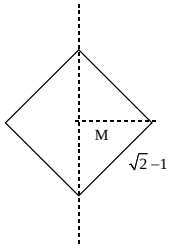
$$3\mu^2+8\mu-3=0$$

$$\Rightarrow 3\mu^2+9\mu-\mu-3=0$$

$$3\mu(\mu+3)-1(\mu+3)=0$$

$$\mu=\frac{1}{3}\ (As\quad \mu\neq -3)$$

26. As



$$I=\frac{1}{4}I_{\text{full square}}$$

$$=\frac{1}{4}M_{full}l^2{}_{full}/12$$

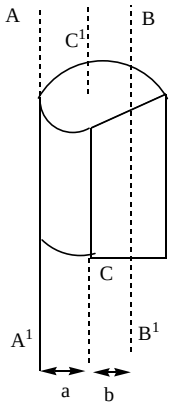
$$I=\frac{1}{4}(4m)\frac{\left(\sqrt{2}l\right)^2}{12}$$

$$I = \frac{Ml^2}{6}$$

27. Using parallel axis theorem

$$I_{CC^1} = I_B - Mb^2$$

$$I_{AA^1} = I_{CC^1} + Ma^2$$



28. $I = (24M)R^2 + 24\left(\frac{Ml^2}{3}\right)$

$= 24Ml^2 + 8Ml^2$

$I = 32Ml^2$

29. Applying net torque= $\sum \vec{r_i} \times \vec{F_i}$

30. B,C

From figure we have

$$I_Y = \frac{1}{2}mR^2 \;\&\; I_X = I_Z = \frac{3}{4}mR^2$$

31. <u>Case 1</u>:

$$FR = I\alpha$$

$$40 \times 0.25 = 4\alpha$$

$$\alpha = \frac{5}{2} rad/s^2$$

$$\omega_1 = \sqrt{2\alpha\theta}$$

$$= \sqrt{2\alpha\frac{l}{r}}$$

$$= \sqrt{2 \times \frac{5}{2} \times \frac{5}{0.25}}$$

= 10 rad/s

Case 2:

Mass of the block is $m = \frac{40}{10} = 4kg$

For conservation of energy ,

[Loss in GPE of 4 kg mss] = [Gain in KE of 4kg mass] + {Gain in RKE of Cylinder]

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

As $v = R\omega$,

$$mgh = \frac{1}{2}m(R\omega)^2 + \frac{1}{2}I\omega^2$$

$$(4)(10)(5) = \frac{1}{2}(4)\left(\frac{1}{4}\right)^2\omega^2 + \frac{1}{2}(4)\omega^2$$

$$\Rightarrow \quad \omega_2 = \frac{40}{\sqrt{17}}\ rad/s$$

32. As the sphere is rotating uniformly the tangential acceleration of any particle on sphere will be zero and all particles will only have normal acceleration.

33. $N2L = block$

$T - 50\times10 = 50\times2 \Rightarrow T = 600N$

$\&1\times\frac{2}{0.25} = F\times0.10 - 0.25\times600$

$\Rightarrow 0.10F = 8+150$

34. Just after the thread is burnt

Spring force$= = 600\times0.01 = 6N$ remains same

Torque equation$= I\alpha = 6\times0.60 + 20\times0.10(c.w)$

$\Rightarrow 2\left(\frac{12}{12}+0.01\right)\alpha = 5.6$

$\Rightarrow \alpha = \frac{5.6\times12}{2\times1.12}$

Applying $= N2L = 6+20-N = 2\times0.10\times\frac{5.6\times12}{2\times1.12}$

$\Rightarrow N = 20$

35. Torque equation (discus)

$(I_1 + I_2)\alpha = r_1 T$

$N\,2L = block$

$mg - T = mr_1\alpha$

36. A rod of length l is pivoted about an end. Find the moment of inertia of the rod about this axis if the linear mass density of rod varies as $\rho = ax^2 + b\ kg/m$

As mass of the rod varies with its length, here we cannot use the expression $\frac{Ml^2}{3}$.which is only used for uniformly distributed mass along the length of rod pivoted at an end. In this case we consider an elemental length dx from the axis at a distance x.Let its mass be dm ,where

$dm = \rho dx$

$= (ax^2 + b)dx$

During rotation of rod this dm revolves in a circle of radius x, hence its moment of inertia dl is given as

$dl = dmx^2$

$= (ax^2 + b)x^2 dx$

The moment of inertia of the whole rod is given by integrating the above expression within limits from O to l as $I = \int_0^1 (ax^2 + b)x^2 dx$

Or $= \left[\frac{ax^5}{5} + \frac{bx^3}{3}\right]_0^l$

Or $= \frac{al^5}{5} + \frac{bl^3}{3}$

37. i) $\frac{Ml^2}{3}\alpha_1 = mg\frac{l}{2}\cos 53^0$

ii) $-\frac{Ml^2}{3}\alpha_1 = mg\frac{l}{2}\cos 53^0 - ma_o\frac{l}{2}\sin 53^0$

$\Rightarrow a_0 \sin 53^0 = 2g\cos 53^0$

$a_0 = 2g\frac{3}{4} = \frac{3}{2}g$

38. $I_3 = I_1 = I_4 = \frac{2}{3}MR^2$

$I_2 = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$

$\Rightarrow I_1 + I_2 + I_3 + I_4 = 3\frac{2}{3}MR^2 + \frac{5}{3}MR^2$

$= \frac{11}{3}MR^2 = \frac{11}{2}I_1$

39. $dm = \lambda R d\theta$

$dI_z = R^2 \sin^2\theta dm \Rightarrow I_x = \int R^2 \sin^2\theta dm$

$dI_y = R^2 \cos^2\theta dm \Rightarrow I_y = \int R^2 \cos^2\theta dm$

$dI_z = R^2 dm \Rightarrow I_z = \int R^2 dm$

40. Applying rotational equation and then applying torque equation about the respective pivots

MATHS

41. Given $L = \left(1-\frac{4}{3^2}\right)\left(1-\frac{4}{4^2}\right)\left(1-\frac{4}{5^2}\right).....$

$= \left[\left(1-\frac{2}{3}\right)\left(1-\frac{2}{4}\right)\left(1-\frac{2}{5}\right)\left(1-\frac{2}{6}\right).....\right] \times \left[\left(1+\frac{2}{3}\right)\left(1+\frac{2}{4}\right)\left(1+\frac{2}{5}\right)....\right]$

$= \left[\frac{1}{3}\times\frac{2}{4}\times\frac{3}{5}\times\frac{4}{6}\times.....\right] \times \left[\left(\frac{5}{3}\right)\left(\frac{6}{4}\right)\left(\frac{7}{5}\right).....\right]$

$= \frac{1}{3}\times\frac{2}{4} = \left(\frac{1}{6}\right)$

(Remembering terms in two braces are reciprocal of each other)

Also, $M = \frac{2^3-1}{2^3+1}, \frac{3^3-1}{3^3+1}, \frac{4^3-1}{4^3+1}....$

$\left(Now\ \frac{x^3-1}{x^3+1} = \frac{(x-1)(x^2+x+1)}{(x+1)(x^2-x+1)}\right)$

$M = \left(\frac{2-1}{2+1}, \frac{3-1}{3+1}, \frac{4-1}{4+1}, \frac{5-1}{5+1}....\right) \times$

$\left(\frac{2^2+2+1}{2^2-2+1}\right)\left(\frac{3^2+3+1}{3^2-3+1}\right)\left(\frac{4^2+4+1}{4^2-4+1}\right)$

$$M=\lim_{n\to\infty}\frac{1\times 2}{(n+1)(n)}\frac{n^2+n+1}{3}=\frac{2}{3};$$

$$N=\prod_{n=1}^{\infty}\frac{\left(1+\frac{1}{n}\right)^2}{\left(1+\frac{2}{n}\right)}=\underset{n\to\infty}{Lt}\prod\frac{(n+1)2/n^2}{(n+2)/n}$$

$$=\underset{n\to\infty}{Lt}\prod_{r=1}^{n}\left(\frac{(r+1)^2}{r(r+2)}\right)$$

$$=\underset{n\to\infty}{Lt}\left(\prod_{r=1}^{n}\frac{r+1}{r}.\frac{r+1}{r+2}\right)$$

$$=\underset{n\to\infty}{Lt}\left(\frac{2}{1}\times\frac{3}{2}\times\frac{4}{3}\times....\times\frac{n+1}{n}\right)\times\left(\frac{2}{3}\times\frac{3}{4}\times...\times\frac{n+1}{n+2}\right)$$

$$=\underset{n\to\infty}{Lt}\left(\frac{2(n+1)}{n+2}\right)=2$$

$$L^{-1}+M^{-1}+N^{-1}=6+\frac{3}{2}+\frac{1}{2}=8$$

42. $$\lim_{n\to\infty}\left[\left\{4\sum_{r=0}^{n}(r+1)(r+2)(r+3)\right\}^{\frac{1}{4}}-n\right]$$

$$\lim_{n\to\infty}\left[\left\{\frac{4(n+1)(n+2)(n+3)(n+4)}{4}\right\}^{\frac{1}{4}}-n\right]$$

$$\lim_{n\to\infty}\left[\left\{(n+1)(n+2)(n+3)(n+4)\right\}^{\frac{1}{4}}-n\right]$$

$$\lim_{n\to\infty}\left[n\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)\right\}^{\frac{1}{4}}-n\right]$$

$$\lim_{n\to\infty}\left[\frac{\left\{\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}\right)\left(1+\frac{4}{n}\right)\right\}^{1/4}-1}{\frac{1}{n}}\right]$$

$$\lim_{x\to 0}\left[\frac{\left\{(1+x)(1+2x)(1+3x)(1+4x)\right\}^{1/4}-1}{x}\right]$$

$$\lim_{x\to 0}\left[\frac{\left(1+10x+35x^2+50x^3+24x^4\right)^{1/4}-1}{x}\right]=\frac{10}{4}=\frac{5}{2}\left(By\ L.H.Rule\right)$$

43. $2+f(x)f(y)=f(x)+f(y)+f(xy);y\to x$

$2+\left[f(x)\right]^2=2f(x)+f\left(x^2\right)$

put $x=1;$

$\Rightarrow 2+f(1)^2=3f(1)$

$\Rightarrow\left(f(1)\right)^2-3f(1)+2=0$

$\Rightarrow\left(f(1)-1\right)\left(f(1)-2\right)=0$

$\Rightarrow f(1)=1\ or\ f(1)=2,\text{Re}\ placing\ y\ by\ \frac{1}{x}$

$\Rightarrow 2+f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)+f(1)$

$\Rightarrow 2+f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)+f(1)$

If $f(1)=2$

$\Rightarrow f(x)+f\left(\frac{1}{x}\right)=f(x).f\left(\frac{1}{x}\right)$

$\Rightarrow f(x)=1\pm x^2\ but\ f(2)=5$

$\Rightarrow 1\pm(2)^n$

$\Rightarrow n=2\ and\ f(x)=1+x^2$

If $f(1)=1,Then\ 2+f(x)(1)=f(x)+1+f(x)$

$\Rightarrow f(x)=1\forall x\in R$

But $f(2)=5$

$\Rightarrow f(1)\neq 1$

$f(x)=1+x^2$ which is a continuous function

$\lim_{x\to 5}f(5)=1+5^2=26$

44. $\lim_{x\to\infty}\left(a^x+e^x\right)^{\frac{1}{x}}=a,a>0$

$$a=\lim_{x\to\infty}\left(a^x+e^x\right)^{1/x}$$

$$\Rightarrow \ln a=\lim_{x\to\infty}\frac{\ln\left(a^x+e^x\right)}{x}$$

This gives as ∞/∞ format.

Hence L.H rule, we get $\lim_{x\to\infty} Lna=\left[\frac{a^x\ln a+e^x}{a^x+e^x}\right]$

$$\lim_{x\to\infty} Lna=\left[\frac{a^x\ln a+e^x}{a^x+e^x}\right]$$

$$\Rightarrow a\in(e,\infty)$$

45. $$\sum_{r=1}^{n} t_r=\sum_{r=1}^{n}\frac{r}{1-3r^2+r^4}=\sum_{r=1}^{n}\frac{r}{\left[\left(r^2-1\right)^2-r^2\right]}$$

$$\sum_{r=1}^{n}\frac{1}{2}\frac{\left(r^2+r-1\right)-\left(r^2-r-1\right)}{\left(r^2-r-1\right)\left(r^2+r-1\right)}$$

$$=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{r^2-r-1}-\frac{1}{r^2+r-1}\right]=\frac{1}{2}\left[-1-\frac{1}{\left(n^2+n-1\right)}\right]$$

$$=\frac{1}{2}\left[\frac{-n^2-n+1-1}{n^2+n-1}\right]=\frac{1}{2}\left[\frac{-n^2-n}{n^2+n-1}\right];\lim_{n\to\infty}\sum_{r=1}^{n} t_r=\frac{-1}{2}$$

46. Here $\left|\frac{3}{\pi}\tan^{-1}2x\right|>1$

$$\Rightarrow\left|\tan^{-1}(2x)\right|>\frac{\pi}{3}\Rightarrow|2x|>\sqrt{3}$$

For $|2x|>\sqrt{3}, \left|\left(\frac{3}{\pi}\tan^{-1}2x\right)\right|>1$

$$\Rightarrow\left(\frac{3}{\pi}\tan^{-1}2x\right)^{2\pi}\to\infty\ as\ n\to\infty$$

$$\Rightarrow\lim_{n\to\infty}\frac{1}{\left(\frac{3}{\pi}\tan^{-1}(2x)\right)^{2\pi}+5}=0$$

$$\Rightarrow f(x)=0\quad for\ |2x|>\sqrt{3}$$

47. $$\lim_{x\to0}\left[f(x)+\ln\left(1-\frac{1}{e^{f(x)}}\right)-\ln\left(f(x)\right)\right]$$

$$\lim_{x\to 0}\left[f(x)+\ln\left(e^{f(x)}-1\right)-\ln e^{f(x)}-\ln\left(f(x)\right)\right]$$

$$\lim_{x\to 0}\left[\ln\left(e^{f(x)}-1\right)-\ln f(x)\right]=\lim_{x\to 0}\left[\ln\left(\frac{e^{f(x)}-1}{f(x)}\right)\right]$$

$= 0$ *if* $f(x)\to 0$ *as* $x\to 0$

As $f(x)$ is defined $\forall x\in R$

$f(0)=0$

48. $\lim_{x\to 0}\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)^{\sec^2\frac{\pi}{2-bx}}$

Let $k=\lim_{x\to 0}\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)^{\sec^2\frac{\pi}{2-bx}}$

Taking $\ln$ both sides, $\ln k$

$$=\sec^2\frac{\pi}{2-bx}\ln\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)$$

$$=\ln k=\lim_{x\to 0}\left[\frac{\ln\left(\sin^2\left(\frac{\pi}{2-ax}\right)\right)}{\cos^2\left(\frac{\pi}{1-bx}\right)}\right]$$

By L.H rule, we get $\ln k=-\frac{a^2}{b^2}$

$\Rightarrow k=e^{-\frac{a^2}{b^2}}$

49. a,d

$$f(1+0)=\lim_{h\to 0}\{|1+h-1|\}-\{1+h\}=\lim_{h\to 0}\{h-1\}=-1$$

$$f(1-0)=\lim_{h\to 0}\{|1-h-1|\}-\{1-h\}=\lim_{h\to 0}\{h-0\}=0$$

50. b.c

Since the greatest integer function is discontinuous (sensitive) at integral values of x, for a given limit to exist both left0and right-hand must be equal.

$$L.H.L=\lim_{x\to 1^-}\left(2-x+a[x-1]+b[1+x]\right)$$

$$=2-1+a(-1)+b(1)=1-a+b$$

$R.H.L = \lim_{x\to 1^+}\left(2-x+a\left[x-1\right]+b\left[1+x\right]\right)$

$= 2-1+a(0)+b(2) = 1+2b$

On comparing, we have –a=b

51. a,b,c,d

We have $\lim_{x\to 0^+} f(x) = \lim_{h\to 0^+}\dfrac{\tan^2\{x\}}{\left(x^2-[x]^2\right)}$

$= \lim_{x\to 0^+}\dfrac{\tan^2 x}{x^2} = 1 \qquad \ldots\ldots 1$

$Also\ \lim_{k\to 0^+} f(x) = \lim_{x\to 0}\sqrt{\{x\}\cot\{x\}} = \sqrt{\cot 1} \qquad \ldots\ldots 2$

$\left(\therefore x\to 0, [x] = -1 \Rightarrow \{x\} = x+1 \Rightarrow \{x\}\to 1\right)$

$Also, \cot^{-1}\left(\lim_{x\to 0^-} f(x)\right)^2 = \cot^{-1}(\cot 1) = 1$

52. a,b,c

53. Given that $\lim_{x\to 0}\dfrac{\cos^2 x-\cos x = e^x\cos x+e^x-\dfrac{x^3}{2}}{x^n}$

$= \lim_{x\to 0}\dfrac{(\cos x-1)\left(\cos x-e^x\right)-\dfrac{x^3}{2}}{x^n}$

$= \lim_{x\to 0}\dfrac{\left(1-\dfrac{x^5}{2!}+\dfrac{x^4}{4!}-\dfrac{x^6}{6!}+\ldots..-1\right)\left(1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\ldots...\right)-\left(1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}=....\right)-\dfrac{x^3}{2}}{x^n}$

= n=4

54. Conceptual

55. $\lim_{x\to 0+}\left[\dfrac{\lim_{n\to\infty}\sum_{k=1}^{n}\left[k^2(\sin x)^x\right]}{n^3}\right] = \dfrac{1}{3}\lim_{x\to 0+}(\sin x)^x$

L=1/3

56. $f(n,\theta) = \dfrac{2^n\tan\dfrac{\theta}{2^n}}{\tan\theta}$

$\therefore \lim_{n\to\infty} f(n,\theta) = \dfrac{\theta}{\tan\theta}$

57. $\lim_{x\to\infty}\left(f(x)+\frac{3f(x)-1}{f^2(x)}\right)=3$

$or\ \left(\lim_{x\to\infty}f(x)+\frac{3\lim_{x\to\infty}f(x)-1}{\left(\lim_{x\to\infty}f(x)^2\right)}\right)=3$

$or\left(y+\frac{3y-1}{y^2}\right)=3$

$or\ y^2-3y^2+3y-1=0$

$or(y-1)^3=0$

$or\ y=1$

58. m=0,n=2

59. p) $\frac{\tan x}{x}>1 and\ \frac{\tan x}{x}\to 1\ for\ x\to 0$

$\Rightarrow\frac{\tan x}{x}=1+h;h\to 0$

$\Rightarrow 200\frac{\tan x}{x}=200+200h;h\to 0$

$\Rightarrow\left[200\frac{\tan}{x}\right]=200$

Also $\frac{\sin x}{x}<1\ and\ \frac{\sin x}{x}\to 1\ for\ x\to 0$

$\Rightarrow\frac{\sin x}{x}=1-h;h\to 0^+$

$\Rightarrow 200\left(\frac{\sin x}{x}\right)=200-200h;h\to 0$

$\Rightarrow\left[200\frac{\sin x}{x}\right]=199$

$\lim_{x\to 0}\left(\left[200\frac{\tan x}{x}\right]+\left[200\frac{\sin x}{x}\right]\right)=399$

q) $100\frac{\tan x}{x}=100(1+h)=100+100h;h\to 0$

$\left\{100\frac{\tan x}{x}\right\}=100\ h\to 0\ as\ h\to 0$

Also $\left\{100\frac{\sin x}{x}\right\}=\{100(1-h)\}=\{100-100h\}$

$\{99+(1-100h)\}=1-100h\to 1\ as\ h\to 0$

$\lim\limits_{x\to 0}\left\{100\frac{\tan x}{x}\right\}+\left\{100\frac{\sin x}{x}\right\}=0+1=1$

r) $\frac{\sin x}{x}=1-h;h\to 0^+\ as\ x\to 0^+$

$\Rightarrow\left[\frac{\sin x}{x}\right]=[1-h]=0\ \ and\ \frac{x}{\tan^{-1}x}>1\ \ for\ x\to 0$

$\Rightarrow\frac{x}{\tan^{-1}x}=1+h;h\to 0^+$

$\Rightarrow\left[\frac{x}{\tan^{-1}x}\right]=1$

$\lim\limits_{x\to 0}\left(100\left[\frac{\sin x}{x}\right]+200\left[\frac{x}{\tan^{-1}x}\right]\right)=0+200=200$

s)As $x\to 0,\frac{\sin^{-}x}{x}>1 and\ \frac{\sin^{-1}x}{x}\to 1\ \ as\ x\to 0$

$\Rightarrow\frac{\sin^{-1}x}{x}=1+h;h\to 0^+$

$\left\{\frac{\sin^{-1}x}{x}\right\}=h\to 0\,as\ h\to 0^+$

Also $\frac{\tan^{-1}x}{x}<1\ and\ \frac{\tan^{-1}x}{x}\to 1\ as\ x\to 0$

$\Rightarrow 200\frac{\tan^{-1}x}{x}=200(1-h);h\to 0^+$

$\Rightarrow\left[200\frac{\tan^{-1}x}{x}\right]=[200-200h]=199$

60. $(p)\to d;(q)\to c;(r)\to a;(s)\to b$

$\phi(x)=\frac{a_ox^m+a_1x^{m+1}+....a_kx^{m+k}}{b_0x^n+....+b_lx^{l+e}}$

$\phi(x)=\left(\frac{a_0+a_1x+.....a_ka^k}{b_o+b_1x+.....b_lx^l}\right).x^{(m-n)}$

p) $m>n$

$\Rightarrow \lim_{x\to 0}\left(\phi(x)\right)$

$\Rightarrow \frac{a_0}{b_0}(0)^{m-n} = 0(m > n)$

$(p) \to (d)$

q) $m = n \Rightarrow \lim_{x\to 0}\phi(x) = \frac{a_o}{b_o}$

$(q) \to c$

r) $m < n$

$\Rightarrow \lim_{x\to 0}\phi(x) = \frac{a_0}{b_o}\left(\frac{1}{(0)^{n-m}}\right) = \infty if \quad \frac{a_0}{b_0} > 0, n-m = even$

$(r) \to a$

s)By the similar logic $\lim_{x\to 0}\phi(x) = -\infty, \left(\frac{a_0}{b_0} < 0\right)$ and n-m is even

$(s) \to b$