



MasterJEE

IIT-JEE | Medical | Foundations

Master JEE CLASSES Kukatpally, Hyderabad.

IIT-JEE-2012-P1-Model

Max. Marks: 210

PAPER-I

IMPORTANT INSTRUCTIONS:

- 1) This booklet is your Question Paper.
- 2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
- 3) Blank spaces are provided within this booklet for rough work.
- 4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
- 5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

- 6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
- 7) The ORS will be collected by the invigilator at the end of the examination.
- 8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
- 9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

- 10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
- 11) Darken the bubble **COMPLETELY**.
- 12) The correct way of darkening a bubble is as :
- 13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
- 14) Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or

"un-darken" a darkened bubble.

IIT-JEE-2012-P1-Model

IMPORTANT INSTRUCTIONS

Max Marks: 210

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 10)	Questions with Single Correct Choice	3	-1	10	30
Sec – II(Q.N : 11 – 15)	Questions with Multiple Correct Choice	4	0	5	20
Sec – III(Q.N : 16 – 20)	Questions with Integer Answer Type	4	0	5	20
Total				20	70

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 30)	Questions with Single Correct Choice	3	-1	10	30
Sec – II(Q.N : 31 – 35)	Questions with Multiple Correct Choice	4	0	5	20
Sec – III(Q.N : 36 – 40)	Questions with Integer Answer Type	4	0	5	20
Total				20	70

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 50)	Questions with Single Correct Choice	3	-1	10	30
Sec – II(Q.N : 51 – 55)	Questions with Multiple Correct Choice	4	0	5	20
Sec – III(Q.N : 56 – 60)	Questions with Integer Answer Type	4	0	5	20
Total				20	70

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Page 2

SECTION I

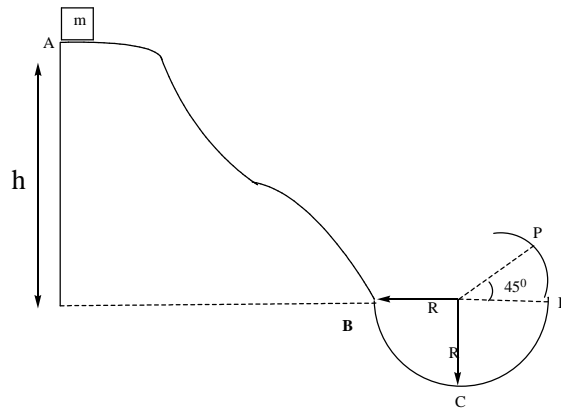
Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

1. A machine delivers power to a body which is proportional to the instantaneous velocity 'v' of the body. If the body starts with a velocity which is almost negligible, then the distance covered by the body is proportional to:

- A) \sqrt{v} B) $\sqrt[3]{\frac{v}{2}}$ C) $v^{\frac{5}{3}}$ D) v^2

2. A particle of mass m slides on a frictionless surface ABCD, starting from rest as shown in the figure. The part BCD is a circular arc. If it loses contact at point P, find the maximum height attained by the particle from point C.



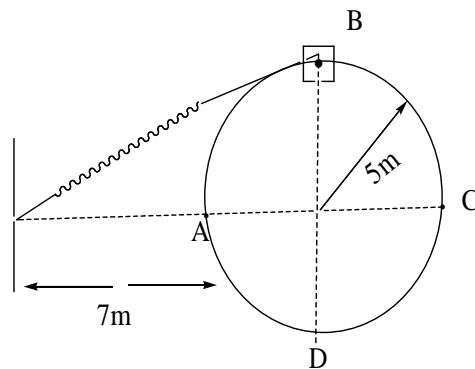
A) $R \left[2 + \frac{1}{2\sqrt{2}} \right] + \frac{h}{2}$

B) $R \left[2 - \frac{1}{2\sqrt{2}} \right] + \frac{h}{2}$

C) $3R + \frac{h}{2}$

D) none of these

3. A collar B of mass 2 kg is constrained to move along a horizontal smooth and fixed circular track of radius 5m. The spring lying in the plane of the circular track and having spring constant 200 N/m, is undeformed when the collar is at A. If the collar starts from rest at B, the normal reaction exerted by the track on the collar when it passes through A is:



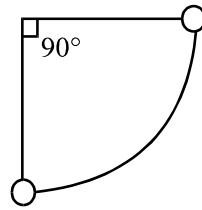
A) 360 N

B) 720 N

C) 1440 N

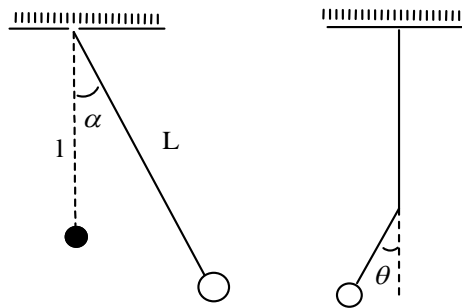
D) 2880 N

4. A simple pendulum is vibrating with an angular amplitude of 90° as shown in fig. For what value of α (angle between string and vertical) during its motion, the total acceleration is directed horizontally?



- A) 0 B) 90° C) $\cos^{-1}(1/\sqrt{3})$ D) $\sin^{-1}(1/\sqrt{3})$

5. A simple pendulum consisting of a mass M attached to a string of length L is released from rest at an angle α . A pin is located at a distance l below the pivot point. When the pendulum swings down, the string hits the pin as shown in figure. The maximum angle which the string makes with the vertical after hitting the pin is:



A) $\cos^{-1}\left(\frac{L \cos \alpha + l}{L + l}\right)$

B) $\cos^{-1}\left(\frac{L \cos \alpha + l}{L - l}\right)$

C) $\cos^{-1}\left(\frac{L \cos \alpha - l}{L - l}\right)$

D) $\cos^{-1}\left(\frac{L \cos \alpha - l}{L + l}\right)$

6. A particle suspended from a fixed point, by a light inextensible thread of length L is projected horizontally from its lowest position with a velocity $\sqrt{\frac{7gL}{2}}$. The string will slack after swinging through an angle θ , the value of θ is:

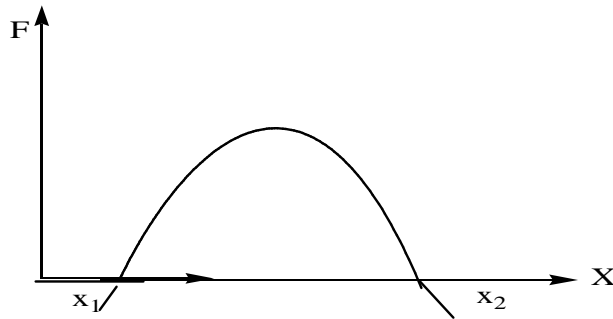
A) 30°

B) 135°

C) 120°

D) 150°

7. The force acting on a body moving along x-axis varies with the position of the particle as shown in figure. The body is in stable equilibrium at:



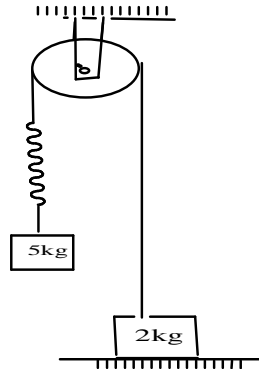
A) $x = x_1$

B) $x = x_2$

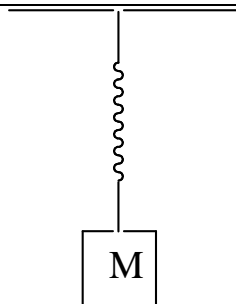
C) both x_1 and x_2

D) neither x_1 nor x_2

8. The system shown in figure is released from rest with spring in natural length and string just taut. The pulley and the springs are massless and friction is absent everywhere. The value of spring constant is $k = 40\text{N/m}$. The speed of 5kg block when 2kg block leaves contact with the ground is:



- A) $\sqrt{2}\text{m/s}$ B) $2\sqrt{2}\text{m/s}$ C) 2 m/s D) None of these
9. Consider the situation shown in figure. Initially the spring is unstretched when the system is released from rest. Assuming no friction in the pulley, find the maximum elongation of the spring?



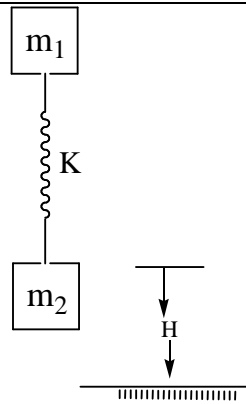
A) $\frac{mg}{k}$

B) $\frac{2mg}{k}$

C) $\frac{mg}{2k}$

D) $\frac{2mg}{3k}$

10. A spring mass system is held at rest at height H from the ground with the spring in relaxed state. Find the minimum value of ' H ' so that the m_2 has a tendency to rebound after hitting the ground during the subsequent motion. Assume that the lower block stops after hitting the ground. (Initial height of m_2 above the ground is H . Neglect the size of m_2)



A) $\frac{m_2 g}{K} \left(\frac{2m_1 + m_2}{2m_1} \right)$

B) $\frac{m_2 g}{2K} \left(\frac{2m_1 + m_2}{2m_1} \right)$

C) $\frac{m_2 g}{K}$

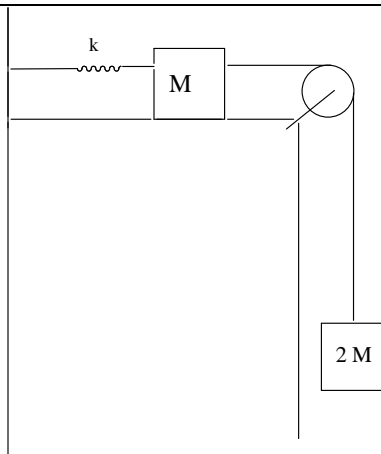
D) $\left(\frac{2m_1 + m_2}{2m_1^2} \right) \frac{g}{k}$

SECTION II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

11. Two blocks of masses M and $2M$ are connected to a light spring of spring constant k that has one end fixed as shown in figure. The horizontal surface and pulley are frictionless. The blocks are released from rest when the spring is in its relaxed state.



A) Maximum extension in the string is $\frac{4Mg}{k}$

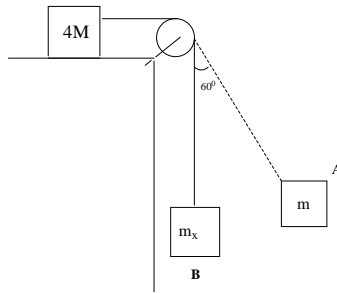
B) Maximum kinetic energy of the system is $\frac{2M^2g^2}{k}$

C) Maximum energy stored in the spring is four times that of maximum kinetic energy of the system

D) When kinetic energy of the system is maximum, energy stored in the spring is

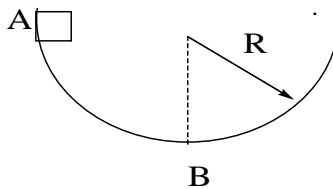
$$\frac{4M^2g^2}{k}$$

12. In the system shown in figure the mass 'm' moves in a circular arc of angular amplitude 60° . Mass $4m$ is stationary, then:



- A) the minimum value of coefficient of friction between the mass $4m$ and the surface of the table is 0.5.
- B) the work done by the gravitational force on the block m is positive when it moves from A to B.
- C) the power delivered by tension when m moves from A to B is zero
- D) the kinetic energy of m in position B equals the work done by gravitational force on the block when it moves position A to B.

13. A small block of mass m is released from rest from position A inside a smooth hemispherical bowl of radius R as shown in figure. Choose the wrong option.



- A) Acceleration of block is constant throughout
- B) Acceleration of block is g at A
- C) Acceleration of block is $3g$ at B
- D) Acceleration of block is $2g$ at B.
14. The potential energy of the moving particle along x-axis is given by $U = 20 + 5 \sin(4\pi x)$, Where U is in joule and 'x' is in meter under the action of conservative force.
- A) If mechanical energy is 20J, then at $x=7/8m$; particle is at equilibrium.
- B) If mechanical energy is 20J, then at $x=7/8m$; particle is not at equilibrium.
- C) If mechanical energy is 20J, then at $x=3/8m$; particle is at equilibrium.
- D) If mechanical energy is 20J, then at $x=3/8m$; particle is not at equilibrium.

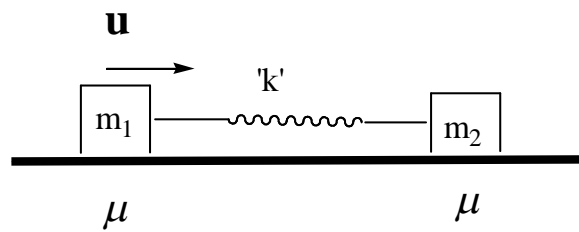
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15. A small sphere of mass 'm' suspended by a thread is taken a side, so that thread forms the right angle with the vertical and then released, then which options are correct (Here ' θ ' \Rightarrow is the angle of with vertical).
- A) The total acceleration of the sphere as a function of ' θ ' is " $g\sqrt{1+3\cos^2\theta}$ "
- B) The total tension in the thread as a function of ' θ ' is ' $3mg \cos\theta$ '
- C) The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be $mg\sqrt{3}$
- D) The thread tension at the moment when the vertical component of the sphere's velocity is maximum will be 'mg'.

SECTION- III

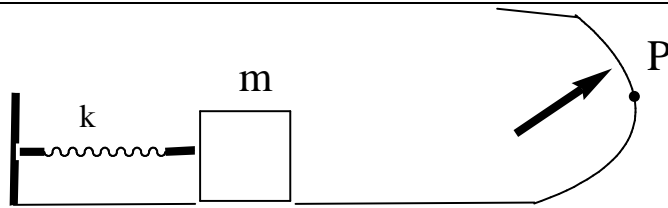
Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

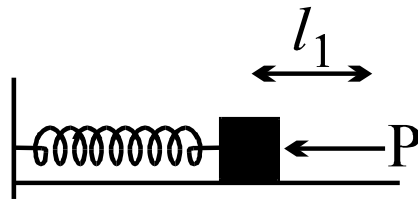
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16. The blocks of masses $m_1 = 1\text{kg}$ and $m_2 = 2\text{kg}$ are connected by a spring and rest on a horizontal surface. The spring is unstretched initially. The spring constant of the spring is $k = 8\text{ N/m}$. The coefficient of friction between blocks and horizontal surface is $\mu = \frac{1}{2}$. Now the left block is imparted a velocity u towards right as shown in figure. Then what is the largest value of u such that the block mass m_2 never moves.



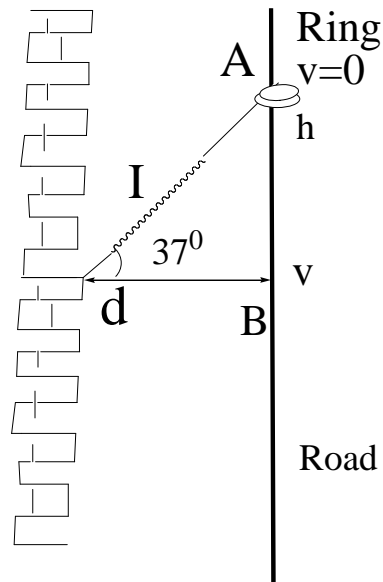
17. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = \left[U_{\text{at infinity}} - U_{\text{at equilibrium}} \right]$, then D is $\frac{b^2}{na}$. Find n ?
18. Figure shows a smooth track, a part of which is a circle of radius 10cm. A block of mass 3kg is pushed against a spring of spring constant 100 N/m fixed at the left end and is then released. If the initial compression of the spring is $10x$ cm, find the value of x so that the block pushes the track with a force of 30 N when it reaches point P, where the radius of the track is horizontal.



19. A block of mass m placed on a smooth horizontal surface is attached to a spring and is held at rest by a force P in equilibrium as shown (at equilibrium compression in spring is l_1). Suddenly the force P changes its direction opposite to the previous one. If the maximum extension of the spring during the subsequent motion is l_2 then find $l_2/l_1 = ?$



20. One end of a light spring of natural length d and spring constant $k=64\text{N/m}$ is fixed on a rigid wall and the other is attached to a smooth ring of mass $m=1\text{ kg}$ which can slide without friction on a vertical rod fixed at a distance $d=3\text{m}$ from the wall. Initially the spring makes an angle 37° with the horizontal as shown in fig. When the system is released from rest find the speed of the ring when the spring becomes horizontal (in m/s) [$\sin 37^\circ = 3/5$]



SECTION I

Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

21. Which of the following solubility order is not correct?
A) $SrCrO_4 > BaCrO_4$ B) $BaC_2O_4 > SrC_2O_4$ C) $KHCO_3 > NaHCO_3$ D) $BaS_2O_3 > SrS_2O_3$
22. The types of bonds present in $CuSO_4 \cdot 5H_2O$ are only
A) electrovalent and covalent
B) electrovalent and co-ordinate covalent
C) electrovalent, covalent and co-ordinate covalent
D) covalent and co-ordinate covalent
23. Carbon suboxide (C_3O_2) has recently been shown as a component of the atmosphere of Venus. Which of the following formulation represents the correct ground state Lewis structure for carbon suboxide?
A) $:O:C::C:C:O:$ B) $:O::C::C:C::O:$
C) $:Ö::C::C::C::Ö:$ D) $:O:C:C:C:O:$

24. Thermodynamic decomposition temperature is maximum for

- A) MgCO_3 B) SrCO_3 C) BaCO_3 D) BeCO_3

25. Which of the following order is not correct?

A) $\text{AlCl}_3 > \text{MgCl}_2 > \text{NaCl}$ [covalent character]

B) $\text{AlF}_3 > \text{MgF}_2 > \text{NaF}$ [melting point]

C) $\text{AlCl}_3 > \text{MgCl}_2 > \text{NaCl}$ [melting point]

D) $\text{AlCl}_3 > \text{AlI}_3 > \text{AlBr}_3$ [melting point]

26. S_1 - size of cation increases due to polarization

S_2 - Al^{+3} has more polarizing power than Ga^{+3}

S_3 - S^{-2} has more polarisability than Cl^-

Which of the following given statements are correct:

- A) S_1, S_2, S_3 B) only S_3 C) S_1, S_3 D) S_2, S_3

27. Among the following, which compound will have the highest lattice energy?

- A) KF B) NaF C) CsF D) RbF.

28. Among LiCl, BeCl₂, BCl₃ and CCl₄, the covalent character follows the order?

- A) LiCl < BeCl₂ > BCl₃ > CCl₄ B) LiCl > BeCl₂ < BCl₃ < CCl₄

- C) LiCl < BeCl₂ < BCl₃ < CCl₄ D) LiCl > BeCl₂ > BCl₃ > CCl₄

29. Amongst LiCl, RbCl, BeCl₂ and MgCl₂, the compounds with the greatest and the least ionic character respectively are:

- A) LiCl and RbCl B) RbCl and BeCl₂

- C) RbCl and MgCl₂ D) MgCl₂ and BeCl₂

30. Choose the correct order of lattice energy:-

- A) CsF > CaF₂ B) AlF₃ > Al₂O₃ C) MgI₂ > KI D) MnBr₂ > MnF₂

SECTION II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

31. Which of the following order is incorrect ?
- A) $Cd^{+2} > Hg^{+2}$ (polarizing power)
 - B) $S^{-3} < P^{3-}$ (ionic radius)
 - C) $F > Ne$ (covalent radius)
 - D) $SnF_4 < SnCl_4$ (melting point)
32. Which of the following energy terms are associated with dissolution of $BaCl_2$ in water?
- A) Hydration energy of Ba^{2+}
 - B) lattice energy of $BaCl_2$
 - C) Sublimation energy of Ba
 - D) Electron Affinity of Cl^-
33. The correct order of decreasing electron affinity of B, C, N and O is:
- A) $O > C > N > B$
 - B) $B > N > C > O$
 - C) $O > C > B > N$
 - D) $O > B > C > N$

34. Which of the following orders is/are true?

A) $\text{AgF} > \text{AgCl} > \text{AgBr} > \text{AgI}$ [solubility in water]

B) $\text{ZnCl}_2 < \text{ZnBr}_2 < \text{ZnI}_2$ [covalent character]

C) $\text{LiClO}_4 > \text{NaClO}_4$ [thermal stability]

D) $\text{BeO} > \text{BeF}_2$ [melting Point]

35. Which of the following statements is/are true?

A) low solubility of LiF in water is due to high lattice energy

B) KNO_3 is water soluble due to high entropy change

C) CsI is less soluble in water due to smaller hydration energy

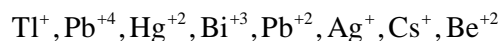
D) KI is more soluble in acetone than KCl

SECTION III

Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

36. How many of the followings cations have non-inert gas configuration



37. What is the value of $|x| + |y|$ where

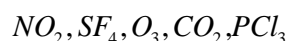
x = lowest Possible formal charge on 'N' atom in Lewis structure(s) of azide ion(N_3^{-1})

y = highest possible formal charge on 'N' atom in Lewis structure(s) of azide ion(N_3^{-1})

38. How many of the following molecule does not follow octet rule?



39. How many of the following molecules contain lone pair(s) on central atom in Lewis dot structures?



40. The number of total electrons shared between nitrogen atoms in N_2 is_____

SECTION I

Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. Let $f : [-1, 3] \rightarrow [-8, 72]$ be defined as $f(x) = 4x^3 - 12x$, then f is

- A) Injective but not surjective
- B) Injective as well as surjective
- C) Neither injective nor surjective
- D) surjective but not injective

42. The function $f : R \rightarrow \{x \in R : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$ is

- A) one-one and into
- B) one-one and onto
- C) many-one and into
- D) many-one and onto

43. For the real number x , let $[x]$ denote the greatest integer less than or equal to x .

let $f : R \rightarrow R$ be defined as $f(x) = 2x + [x] + \sin x \cdot \cos x$ then f is

- A) one-one but not onto
- B) onto but not one-one
- C) both one-one and onto
- D) neither one-one nor onto

44. If the following functions are defined from \mathbb{R} to \mathbb{R} then identify the function which is bijective?

A) $f(x) = \frac{e^x + e^{-x}}{2}$

B) $f(x) = x^4 - 3x^3 + 1$

C) $f(x) = 18x^3 - 21x^2 + 8x - 1$

D) $f(x) = x^3 - 4x^2 + 16x + 17$

45. If the function $f(x)$ and $g(x)$ are defined on $\mathbb{R} \rightarrow \mathbb{R}$ such

that $f(x) = \begin{cases} x+3, & x \in \text{rational} \\ 4x, & x \in \text{irrational} \end{cases}$ and

$g(x) = \begin{cases} x+\sqrt{5}, & x \in \text{irrational} \\ -x, & x \in \text{rational} \end{cases}$ then $(f-g)(x)$ is

A) one-one and onto

B) neither one-one nor onto

C) one-one but not onto

D) onto but not one-one

46. If $f(g(x)) = g(f(x)) = x$ for all real number x , and $f(2) = 5$ and $f(5) = 3$, then the value of $g(3) + g(f(2))$ is

A) 7

B) 5

C) 3

D) 2

47. Let $f(x) = ax + b$, Where \mathbf{a} and \mathbf{b} are integers. If $f(f(0)) = 0$ and $f(f(f(4))) = 9$, then the value of $f(f(f(f(10))))$ is equal to

A) 0

B) 4

C) 9

D) 10

48. $g(x) = \left(4\cos^4(x) - 2\cos(2x) - \frac{1}{2}\cos(4x) - x^7 \right)^{\frac{1}{7}}$, then the value of $g(g(100))$ is equal to

- A)-1 B)0 C)1 D)100

49. Let $f : R \rightarrow R$ be defined as $f(x) = \frac{x^3}{3} + \frac{x^2}{2} + ax + b$. the latest value of 'a' for which $f(x)$ is injective function is.

- A) $\frac{1}{4}$ B) $\frac{1}{8}$ C) $\frac{1}{2}$ D)1

50. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in R$. then the set of all x satisfying $(f \circ g \circ f \circ g)(x) = (g \circ g \circ f \circ f)(x)$, where $(f \circ g)(x) = f(g(x))$, is

- A) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$ B) $\pm\sqrt{n\pi}, n \in \{1, 2, \dots\}$
C) $\frac{\pi}{2} + 2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$ D) $2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

SECTION - II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

51. The maximum value of the function defined by $f(x) = \min(e^x, 2 + e^x, 7)$ is α then integral value of x satisfying the inequality $\frac{x(x - [\alpha])}{x^2 - [\alpha]x + 12} < 0$ is

- A)1 B) 3 C) 5 D) 6

52. Let $f : R \rightarrow R$ defined by $f(x) = \min(|x|, 1 - |x|)$ Then which of the following hold(s) good?

- A) Rang of f is $(-\infty, 1]$ B) f is even function
C) f Is function odd D) f is neither injective nor surjective.

53. Let: $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions and $g \circ f : A \rightarrow C$ is defined. Then which of the following statement(s) is (are) incorrect?

- A) If $g \circ f$ is onto then f must be onto
B) If f is into and g is onto then $g \circ f$ must be onto function.
C) If $g \circ f$ is one-one then g is necessarily one-one.
D) If f is injective and g is surjective then $g \circ f$ must be bijective mapping.

54. Let f , g & h be three function defined as follows:

$$f(x) = \frac{32}{4 + x^2 + x^4}, g(x) = 9 + x^2 \text{ and } h(x) = -x^2 - 3x + k$$
 Identify which of the following

statement(s) is (are) correct?

- A) Number of integers in the range of $f(x)$ is 8.
B) Number of integral value of k for which $h(f(x)) > 0$ and $h(g(x)) < 0 \forall x \in R$ is 20
C) Number on integral value of k for which $h(f(x)) > 0$ and $h(g(x)) < 0 \forall x \in R$ is 19.
D) Maximum value of $g(f(x))$ is 73.

55. Let $f(x)$ be a function defined on $[0, 1]$ such that

$$f(x) = \begin{cases} x & ; x \in \text{rational} \\ 1-x & ; x \in \text{irrational} \end{cases}$$

If $f \circ f(x) = p + qx, \forall x \in [0, 1]$ Where $p, q \in I$, then the value of $(p^2 + 2q)$ is greater than

- A) 0 B) 1 C) 2 D) 3

SECTION III

Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

56. Let $f(x) = \left[\frac{1}{\cos\{x\}} \right]$ where $[y]$ and $\{y\}$ denote greatest integer and fractional part

function respectively and $g(x) = 2x^2 - 3x(k+1) + k(3k+1)$. if $g(f(x)) < 0 \forall x \in R$ then find the number of integral value of k .

57. If $h(x) = Ax^5 + B \sin x + C \ln\left(\frac{1+x}{1-x}\right) + 7$, Where A, B, C are non-zero real constants and

$$h\left(\frac{-1}{2}\right) = 6, \text{ then find the value of } h\left(\frac{\text{sgn}(e^{-x})}{2}\right).$$

58. Let $f : \mathbb{R} \rightarrow [0, \infty)$ be defined as $f(x) = \log_e(\sqrt{9x^2 - 12x + \lambda} + 1)$ be an onto function where λ is a real parameter belong to $(0, 10)$ Find the greatest possible value of λ .

59. Let a function f defined from $\mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = \begin{cases} x+p^2, & \text{for } x \leq 2 \\ 2px+5, & \text{for } x > 2 \end{cases}$ If the function is surjective, then the sum of all possible integral value of p in $[-100, 100]$ is

$$\sum_{r=1}^{100} r - k \text{ then } (k-2) =$$

60. Let the equation $(a-1)x^2 = x(2b+3)$ be satisfied by three distinct value of x , where $a, b \in \mathbb{R}$. if $f(x) = (a-1)x^3 + (2b+3)x^2 + 2x + 1$, and $f(g(x)) = 6x - 7$ where $g(x)$ is a linear function then find the value of $g(2)$ is equal to

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KEY SHEET

PHYSICS

1	D	2	A	3	C	4	C	5	C
6	C	7	B	8	B	9	B	10	A
11	ABC	12	ABCD	13	AC	14	AC	15	ABC
16	5	17	4	18	3	19	3	20	9

CHEMISTRY

21	D	22	C	23	C	24	C	25	C
26	A	27	B	28	C	29	B	30	C
31	ABD	32	AB	33	C	34	ABD	35	ABCD
36	3	37	3	38	6	39	3	40	6

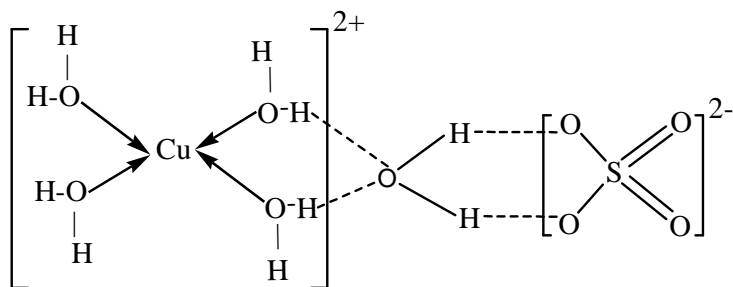
MATHS

41	D	42	B	43	A	44	D	45	B
46	A	47	D	48	D	49	A	50	A
51	ACD	52	BD	53	ABCD	54	ACD	55	AB
56	1	57	8	58	4	59	8	60	2

SOLUTIONS

CHEMISTRY

22. The bond between Cu^{2+} and SO_4^{2-} ion is ionic ; between S and O in SO_4^{2-} ions and between H and O atoms in H_2O are covalent ; those between Cu^{2+} and H_2O molecules are coordinate.



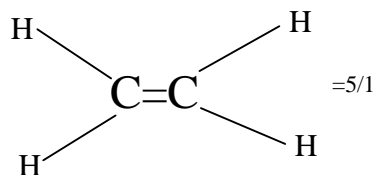
23. In (c), each C and O atom has octet of electrons.
24. A σ -bond is stronger than a π -bond hence option (a) is not correct.
- .
- .
27. For compounds containing ions of same charge, lattice energy increases as the size the ions decrease. Thus, NaF has highest lattice energy.
28. As we move from $\text{Li} \rightarrow \text{Be} \rightarrow \text{B} \rightarrow \text{C}$, the electronegativity (EN) increases and hence the EN difference between the element and Cl decreases and accordingly the covalent character increases.
- Thus option (c) i.e. $\text{LiCl} < \text{BeCl}_2 < \text{BCl}_3 < \text{CCl}_4$ is correct.
29. Electronegativity difference (EN) is highest in RbCl ($3 - 0.8 = 2.2$) and least in BeCl_2 ($3 - 1.5 = 1.5$) and hence option (b) is correct.

30. In $\text{H}-\overset{\text{O}}{\parallel}{\text{C}}-\text{C}-\text{O}-\text{H}$ the asterisked carbon has a valency of 5 and hence this formula is not correct.

31. Because of the triple bond, the carbon-carbon bond distance in ethyne is shortest.
- .

33. Conceptual.

36.



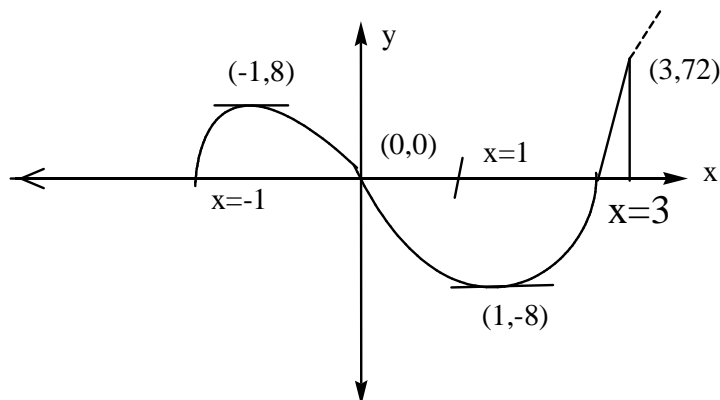
38. BeCl_2 ; BF_3 ; NO ; ClO_2 ; PF_5 ; SF_6

39. Conceptual.

40. Each N atom in N_2 molecule shares three electrons, i.e., $:\text{N}::\text{N}:$

MATHS

41. $f(x) = 4x^3 - 12x$
 $f'(x) = 12(x^2 - 1) = 12(x+1)(x-1)$



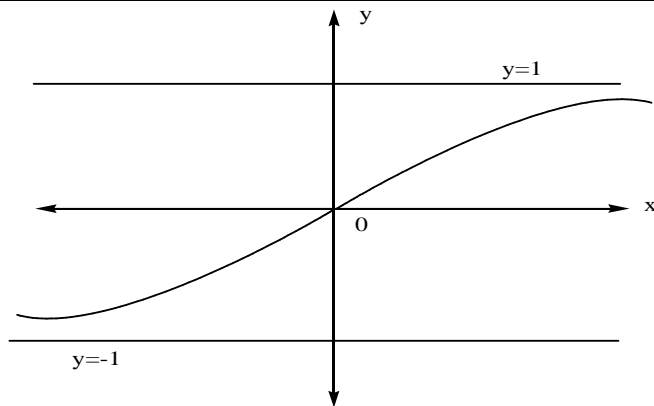
42. $f(x) = \begin{cases} \frac{x}{1-x}, & x < 0 \\ 0, & x = 0 \\ \frac{x}{1+x}, & x > 0 \end{cases}$

$\therefore f(0) = 0$. Now, $y = \frac{x}{1-x}, x < 0$

$\Rightarrow x = \frac{y}{1+y} < 0 \Rightarrow y \in (-1, 0)$

Also, $y = \frac{x}{1+x}, x > 0 \Rightarrow x = \frac{y}{1-y} > 0 \Rightarrow y \in (0, 1)$

So, range of function = $(-1, 1)$



43. If $x=a$, where 'a' is an integer then $f(a) = 2a + a + \frac{1}{2} \sin 2a$

But $\lim_{h \rightarrow 0} f(a-h) = 2a + (a-1) + \frac{1}{2} \sin 2a$

\therefore Value between $\lim_{h \rightarrow 0} f(a-h)$ and $f(a)$ are never achieved.

Also, $f'(x) = 2 + \cos 2x > 0$, i.e., $f(x)$ is strictly increasing.

44. (A) $f(x) = \frac{e^x + e^{-x}}{2}$ is even function.

(B) $f(x) = x^4 - 3x^3 + 1$ is an even degree polynomial function.

(C) $f(x) = 18x^3 - 21x^2 + 8x - 1$

$f(x)$ is an odd degree polynomial function hence its range is \mathbb{R} .

$f'(x) = 3x^2 - 8x + 16 \Rightarrow D < 0$, hence $f(x)$ is one-one as well as onto.

45. We have $(f \circ g)(x) = (f(x) \circ g(x)) = (f - g)(x) = (f(x) - g(x)) = \begin{cases} 2x+3, & x \in \text{rational} \\ 4x, & x \in \text{irrational} \end{cases}$

As, $f\left(\frac{-3}{2}\right) = 0 = f\left(\frac{\sqrt{5}}{3}\right)$ and so on.

$\Rightarrow f(x)$ is many one function.

Also, $-\sqrt{5}$ does not belong to the range, because if $3x - \sqrt{5} = -\sqrt{5}$

$\therefore x = 0 \notin \mathbb{Q}$

$\Rightarrow f(x)$ is into function

46. Since g and f are inverses, $g(3)=5$ and so $g(3)+g(f(2))=5+2=7$.

47. $f(x) = ax + b$

$$f(0) = b$$

$$f(f(0)) = ab + b = 0 \Rightarrow (a+1)b = 0 \Rightarrow a = -1 \text{ or } b = 0$$

$$f(f(f(4))) = 9$$

$$ff(f(4a+b)+b) = 9$$

$$\Rightarrow f(a(4a+b)+b) = 9$$

$$\Rightarrow f(4a^2 + ab + b) = 9$$

$$\Rightarrow a(4a^2 + ab + b) + b = 9$$

$$\Rightarrow 4a^3 + a^2b + ab + b = 9$$

$$\text{when } b = 0, 4a^3 = 9 \Rightarrow a^3 = \frac{9}{4} \Rightarrow a \text{ is not an integer}$$

$$\text{When } a = -1, -4 + b - b + b = 9 \Rightarrow b = 13$$

$$\therefore f(x) = -x + 13$$

$$f(f(x)) = -(-x + 13) + 13 = x$$

$$\therefore f(f(f(f(10)))) = f(f(10)) = 10$$

48. We have

$$4 \cos^4 x - 2 \cos 2x - \frac{1}{2} \cos 4x - x^7$$

$$= 4 \cos^4 x - 2(2 \cos^2 x - 1) - \frac{1}{2}(2 \cos^2 2x - 1) - x^7$$

$$\therefore \text{We get } g(x) = \left(\frac{3}{2} - x^7 \right)^{\frac{1}{7}}$$

$$\Rightarrow g(g(x)) = \left(\frac{3}{2} - (g(x))^7 \right)^{\frac{1}{7}} = \left(\frac{3}{2} - \left(\frac{3}{2} - x^7 \right) \right)^{\frac{1}{7}} = x$$

49. If $f(x)$ is one-one then $f(x)$ must be monotonic.

$$\text{Now, } f'(x) = x^2 + x + a \geq 0 \forall x \in \mathbb{R}$$

$$\Rightarrow D \leq 0 \text{ i.e., } 1 - 4a \leq 0 \Rightarrow a \geq \frac{1}{4}$$

$$\text{So, } a_{\min} = \frac{1}{4}$$

50.

$$f(g(g(f(x)))) = g(g(f(x))) \Rightarrow \sin^2(\sin x^2) = \sin(\sin x^2)$$

$$\Rightarrow \sin(\sin x^2) = 0, 1$$

$$\Rightarrow \sin x^2 = m\pi, (4k+1)\frac{\pi}{2}, \text{ where } m, k \in I.$$

$$\text{But } -1 \leq \sin x^2 \leq 1$$

$$\Rightarrow \sin x^2 = 0$$

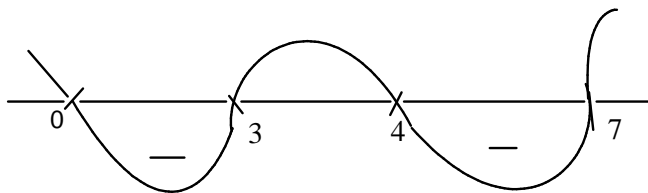
$$\Rightarrow x^2 = n\pi, n = 0, 1, 2, \dots$$

$$\Rightarrow x = \pm\sqrt{n\pi}, n \in \{0, 1, 2, \dots\}$$

51. $f(x) = \min(e^x, 2 + e^2, 8)$ From the graph it is clear that maximum value of $f(x)$ is, $\alpha = e^2$

$$\therefore [\alpha] = [e^2] = 7$$

$$\frac{x(x-7)}{x^2-7x+12} < 0 \Rightarrow \frac{x(x-7)}{(x-3)(x-4)} < 0$$



52. $f(x) = \text{Min}(|x|, 1 - |x|)$

$$D_f = R$$

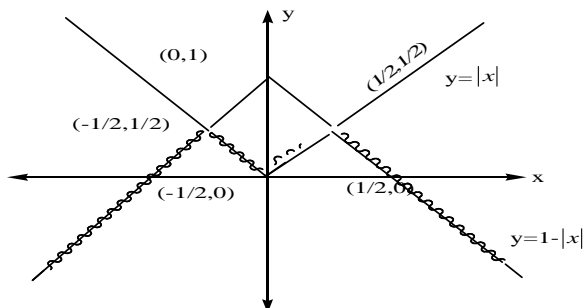
$$f(-x) = f(x)$$

$\Rightarrow f$ is even function.

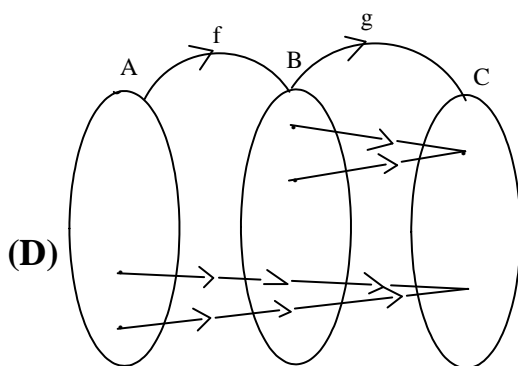
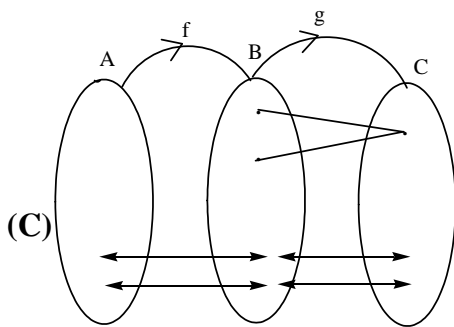
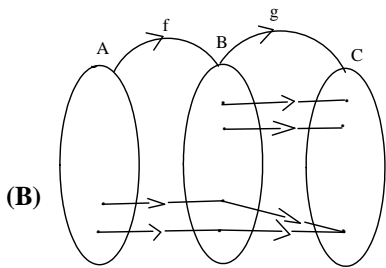
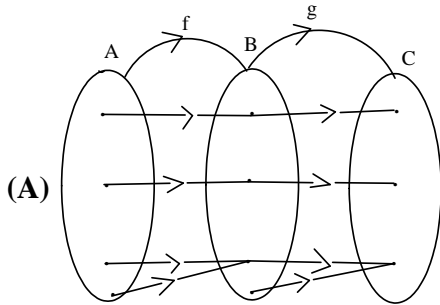
Put

$$|x| = 1 - |x|$$

$$\therefore x = \pm \frac{1}{2}$$



53. (A) We have $f : A \rightarrow B$
 $g : B \rightarrow C$ and $g \circ f : A \rightarrow C$



54. $f(x) = \frac{32}{4+x^2+x^4}$; $g(x) = 9+x^2$; $h(x) = -x^2-3x+k$

(A) Range of f is $[0,8]$

(B) $h(f(x)) > 0$ and $h(g(x)) < 0$

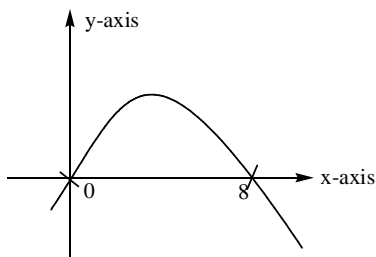
$\Rightarrow h(0) \geq 0 \Rightarrow k \geq 0$

$$h(8) > 0 \Rightarrow -64 - 24 + k > 0 \Rightarrow k > 88$$

$$\text{Also, } h(9) < 0 \Rightarrow -81 - 27 + k < 0 \Rightarrow k < 108$$

\Rightarrow Number of integral value of k is 19.

(D) Maximum value of $g(f(x))$ is $g(8) = 64 + 9 = 73$.



55. Given, $f(x) = \begin{cases} x & ; x \in Q \\ 1-x & ; x \notin Q \end{cases}$

$$fof(x) = \begin{cases} x & ; x \in Q \\ 1-x & ; x \notin Q \end{cases}$$

$$\text{So, } fof(x) = \begin{cases} x & ; x \in Q \\ 1-x & ; x \notin Q \end{cases}$$

$$\therefore f(f(x)) = x, \forall x \in [0, 1]$$

$$\Rightarrow p = 0 \text{ and } q = 1$$

$$\text{hence, } (p^2 + 2q) = 2$$

56. $f(x) = 1 \forall x \in R$

$$\Rightarrow g(1) < 0 \forall x \in$$

$$\Rightarrow 2 - 3(k+1) + 3k^2 + k < 0$$

$$\Rightarrow 3k^2 - 2k - 1 < 0$$

$$\Rightarrow (3k+1)(k-1) < 0$$

$$\Rightarrow k \in \left(-\frac{1}{3}, 1 \right)$$

57. Given $h\left(\frac{-1}{2}\right) = 6$

$$h(x) = Ax^5 + B \sin x + c \ln\left(\frac{1-x}{1+x}\right) + 7$$

$$\Rightarrow h(-x) = -Ax^5 - B \sin x + C \ln\left(\frac{1-x}{1+x}\right) + 7$$

$$\therefore h(x) + h(-x) = 14$$

$$\Rightarrow h\left(\frac{1}{2}\right) + h\left(\frac{-1}{2}\right) = 14$$

$$\therefore h\left(\frac{1}{2}\right) + 6 = 14$$

$$\Rightarrow h\left(\frac{1}{2}\right) = 8$$

58. Given $f(x) = \log(\sqrt{9x^2 - 12x + \lambda} + 1)$

$D_f = R$ & $R_f = [0, \infty]$ So, it is possible when $9x^2 - 12x + \lambda = 0$ hence equal roots

$$\Rightarrow \text{Disc.} = 0$$

$$\text{So, } 144 = 36\lambda$$

$$\Rightarrow \lambda = 4$$

59. For f to be surjective range = Co-domain $f(x) = \begin{cases} x+p^2, & \text{for } x \leq 2 \\ 2px+5, & \text{for } x > 2 \end{cases}$

For $x \leq 2$, range of f is $(-\infty, 2+p^2]$ and

For $x > 2$, range of f is $(4p+5, \infty)$. [Note that $p > 0$]

For range of f to be R , $4p+5 \leq p^2+2$

$$p^2 - 4p - 3 \geq 0 \Rightarrow (p-3)(p+1) \geq 0 \Rightarrow p \in (-\infty, -1) \cup [3, \infty) \text{ but } p > 0.$$

Hence, $p \in [3, \infty)$

$$\Rightarrow s = 3 + 4 + \dots + 100 = 5050 - 3 = 5047$$

60. $(a-1)x^2 = x(2b+3)$

The above equation is satisfied by three distinct values of x therefore it is an identity.

$$\therefore 2-2a=0 \Rightarrow a=1 \text{ and } 2b+3=0 \Rightarrow b = \frac{-3}{2}$$

Now, $f(x) = 2x+1$

$$\text{Let } g(x) = px+q \Rightarrow g'(x) = p$$

$$f(g(x)) = 6x-7 \Rightarrow 2(px+q)+1 = 6x-7$$

$$\Rightarrow 2px+2q+1 = 6x-7$$

$$\Rightarrow 2p=6 \Rightarrow p=3 \text{ and } q=-4$$

$$\therefore g'(2012) = 3$$

