MasterJEE
IIT-JEE | Medical | Foundations

Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-ADVANCED-PAPER-2012-P2 MODEL
Max.Marks:198

2012_PAPER-II

IMPORTANT INSTRUCTIONS:

1) This booklet is your Question Paper.
2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
3) Blank spaces are provided within this booklet for rough work.
4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
7) The ORS will be collected by the invigilator at the end of the examination.
8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
11) Darken the bubble **COMPLETELY**.
12) The correct way of darkening a bubble is as:
13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
14) Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble.

JEE-ADVANCE-2012-P2-Model

IMPORTANT INSTRUCTIONS **Max Marks: 198**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 8)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : 9 – 14)	Questions with Comprehension Type (3 Comprehensions : 2+2+2 = 6Q)	3	-1	6	18
Sec – III(Q.N : 15 – 20)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 28)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : 29 – 34)	Questions with Comprehension Type (3 Comprehensions : 3+3 = 6Q)	3	-1	6	18
Sec – III(Q.N : 35 – 40)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : (41 – 48)	Questions with Single Correct Choice	3	-1	8	24
Sec – II(Q.N : (49 – 54)	Questions with Comprehension Type (3 Comprehensions : 2+2+2 = 6Q)	3	-1	6	18
Sec – III(Q.N : 55 – 60)	Questions with Multiple Correct Choice	4	0	6	24
Total				**20**	**66**

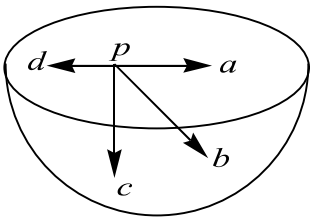
space for rough work

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

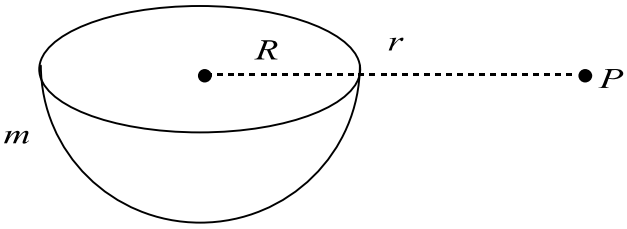
Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

1. What is direction of gravitational field at a point P. P is on a plane containing biggest circle of uniform hollow hemisphere as shown.



A) a B) b C) c D) d

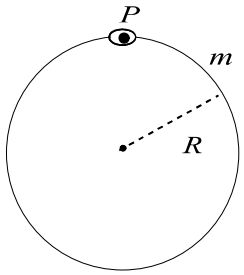
2. Let magnitude of gravitational field at a distance r from centre of uniform hemisphere of mass m at point p shown in figure be E. (R is radius of hemisphere)



A) $E=\frac{Gm}{r^2}$ B) $E=\frac{Gm}{2r^2}$ C) $E>\frac{Gm}{r^2}$ D) $E=\frac{Gm}{(r-R)^2}$

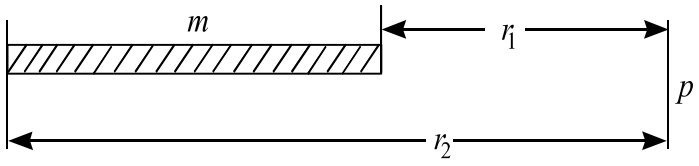
space for rough work

3. Gravitational field at a point p in a small hole made in uniform thin spherical shell of mass m and radius R is approximately



A) $\frac{Gm}{R^2}$ B) $\frac{Gm}{2R^2}$ C) 0 D) $\frac{Gm}{3R^2}$

4. Gravitational field at point P due to rod of mass m is



A) $\frac{Gm}{r_1 r_2}$ B) $\frac{Gm}{\left(\frac{r_1+r_2}{2}\right)^2}$ C) $\frac{Gm}{(r_1+r_2)^2}$ D) $Gm\left(\frac{1}{r_1}+\frac{1}{r_2}\right)^2$

5. A wire of uniform cross sectional area 'a' length l is having density which varies linearly from p_1 at one end to p_2 at other end. Both ends of wire are connected and

wire is transformed into a circular loop. Length of wire doesn't change. The component of gravitational field along the axis of circular loop made of given wire at a distance x from the centre of loop is

A) $\frac{G(p_1+p_2)al}{2x^2}$

B) $\frac{G(p_1+p_2)lax}{\left[\left(\frac{l}{2\pi}\right)^2+x^2\right]^{\frac{3}{2}}}$

C) $\frac{G(p_1+p_2)lax}{2\left[\left(\frac{l}{2\pi}\right)^2+x^2\right]^{\frac{3}{2}}}$

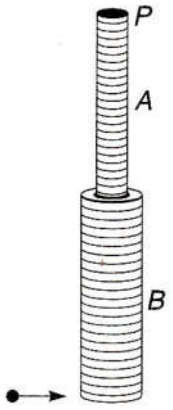
D) $\frac{G\sqrt{p_1p_2}lax}{2\left[\left(\frac{l}{2\pi}\right)^2+x^2\right]^{\frac{3}{2}}}$

6. Gravitational field due to very long hollow cylinder of mass m, length l, and radius R at a point on the axis of cylinder just outside cylinder is $(l >> R)$

A) $\frac{2Gm}{Rl}$ B) $\frac{Gm}{Rl}$ C) $\frac{Gm}{R^2}$ D) $\frac{3Gm}{Rl}$

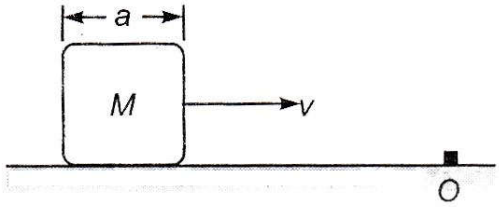
7. Two uniform rods A and B of length 0.6 m each of masses 0.01 kg and 0.02 kg respectively are rigidly joined end to end. The combination is pivoted at the lighter end, P as show in figure. Such that it can freely rotate about point P in a vertical plane. A small object of mass 0.05 kg moving horizontally, hits the lower end of the

combination and sticks to it. What should be the velocity of the object, so that the system could be raised to the horizontal position



A) 6.3 m/s B) 8.2 m/s C) 4 m/s D) 9 m/s

8. A cubical block of side *a* moving with velocity v on a horizontal smooth plane as shown. It hits a ridge at point o. The angular speed of the block after it hits o is



A) 3v/ 4*a* B) 3v /2*a* C) $\sqrt{3}$ v / $\sqrt{2a}$ D) Zero

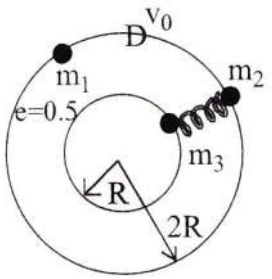
space for rough work

SECTION - II
(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct.**
Marking scheme +3 for correct answer, 0 if not attempted and -1 in all other cases.

Paragraph for Questions 9 and 10

Three particles each of mass m can slide on fixed frictionless plane as shown in figure. Particle m_1 moves with velocity V_0 and hits particle m_2, the coefficient of restitution being $e = \frac{1}{2}$. Assuming that m_2 and m_3 are at rest initially and lie along a radial line before impact and the spring is initially unstretched. Consider only one collision between m_1 and m_2



9. The maximum velocity of m_3 is

A) $\frac{3}{5}V_0$ B) $\frac{3}{10}V_0$ C) $\frac{V_0}{2}$ D) $\frac{V_0}{5}$

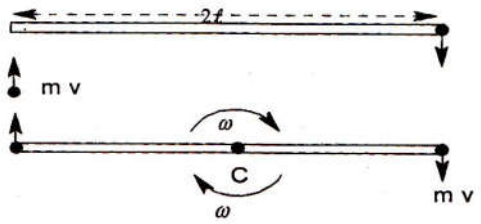
space for rough work

10. The maximum stretch of the spring is

A) $\frac{3}{4}V_0\sqrt{\frac{m}{5k}}$ B) $\frac{V_0}{4}\sqrt{\frac{m}{k}}$ C) $\frac{V_0}{4}\sqrt{\frac{m}{3k}}$ D) $\frac{3V_0}{4}\sqrt{\frac{m}{3k}}$

Paragraph for Questions 11 and 12

Two skaters, each of mass $50kg$, approach each other along parallel paths separated by $3m$. They have equal and opposite velocities of $10ms^{-1}$, The first skater carries a long light pole $3m$ long and the second skater grabs the end of it as he passes on the frictionless surface and the two skaters are connected by the pole



11. The angular velocity of the pole after the skaters are connected is

A) $\frac{22}{3}rads^{-1}$ B) $\frac{20}{3}rads^{-1}$ C) $\frac{17}{3}rads^{-1}$ D) $\frac{14}{3}rads^{-1}$

12. By pulling on the pole, the skaters reduce their distance apart to 1 m. The angular velocity now is

A) $45rads^{-1}$ B) $50rads^{-1}$ C) $55rads^{-1}$ D) $60rads^{-1}$

space for rough work

Paragraph for Questions 13 and 14

A pair of stars rotates about a common center of mass. One of the stars has a mass M and the other has mass m such that M = 2m. The distance between the centres of the stars is d (d being large compared to the size of either star).

13. The period of rotation of the stars about their common centre of mass (in terms of d, m, G.) is

A) $\sqrt{\frac{4\pi^2}{Gm}d^3}$ B) $\sqrt{\frac{8\pi^2}{Gm}d^3}$ C) $\sqrt{\frac{2\pi^2}{3Gm}d^3}$ D) $\sqrt{\frac{4\pi^2}{3Gm}d^3}$

14. The ratio of the angular momentum of the two stars about their common centre of mass (L_m/ L_M) and the ratio of kinetic energies of the two stars (K_m/K_M.) is

A) 1 & 1 B) 2 & 2 C) 2 & 9 D) 9 & 9

space for rough work

SECTION – III
(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

15. Rod inclined at an angle θ with vertical strikes the horizontal frictionless ground. $(\theta \neq 0)$. The angular velocity of rod before striking is zero & linear velocity v is vertically downwards. Which of the following statements is / are correct?

A) Angular momentum of rod is conserved during collision about point of impact on ground.

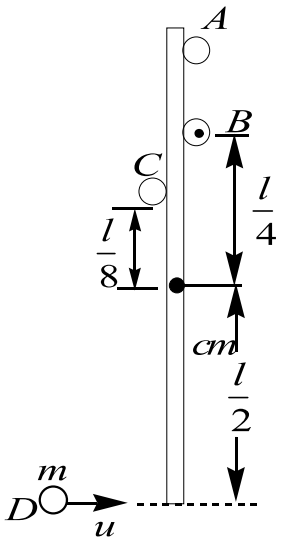
B) For elastic collision the centre of rod rebounds back with velocity v.

C) For elastic collision the centre of mass of rod rebounds back with velocity v.

D) For perfectly inelastic collision the velocity of centre of mass just after collision becomes zero.

16. Mass of all the balls and uniform rod of length l is m and the system of balls A, B, C and rod was initially at rest on horizontal frictionless ground. A, B, C are touching rod but not sticking to it. Ball D collides with one end of rod perpendicularly. Which of the following is correct statement about direction of motion of small balls A, B, C, D & rod shown just after collision of ball D with rod?

space for rough work



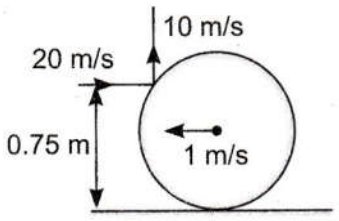
A) *A, B* moves towards right just after collision

B) *C* moves left just after collision

C) *D* moves towards right just after collision

D) Centre of mass of Rod moves towards right just after collision

17. A thin ring of mass 2*kg* and radius 0.5 *m* is rolling without slipping on a horizontal plane with velocity 1 *m/s*. A small ball of mass 0.1 *kg*, moving with velocity 20 *m/s* in the opposite direction, hits the ring at a height if 0.75 *m* and goes vertically up with velocity 10 *m/s*. Immediately after the collision



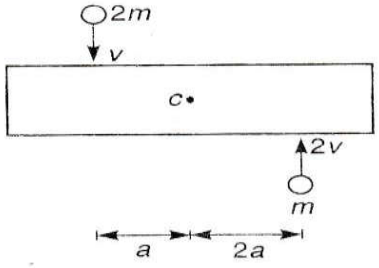
A) The ring has pure rotation about its stationary CM

B) The ring comes to a complete stop

C) Friction between the ring and the ground is to the left

D) There is no friction between the ring and the ground

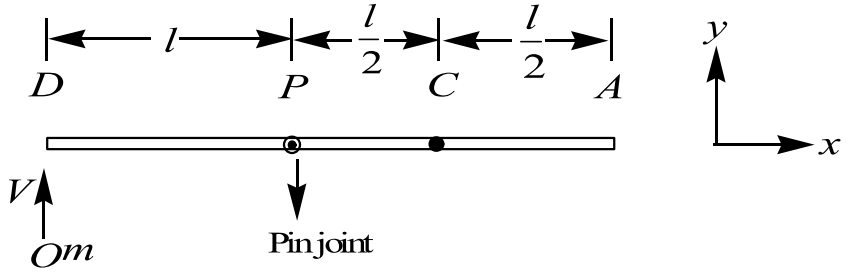
18. A uniform bar of length 6 *a* and mass 8 *m* lies on a smooth horizontal table. Two point masses *m* and 2*m* moving in the same horizontal plane with speed 2v and v respectively, strike the bar [as shown in the figure] and stick to the bar after collision. Denoting angular velocity (about the centre of mass), total energy and centre of mass velocity by ω, E and v_c respectively, we have after collision



A) $v_c = 0$ B) $\omega = \frac{3v}{5a}$ C) $\omega = \frac{v}{5a}$ D) $E = \frac{3}{5}mv^2$

space for rough work

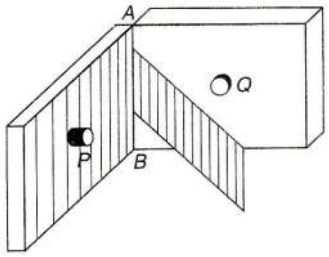
19. Two identical uniform rods connected end to end by a frictionless pin joint are placed along $x-axis$ on a frictionless horizontal surface. These are free to rotate with respect to each other about vertical axis $(z-axis)$ passing through pin joint. A particle of mass m moving with velocity v along $y-axis$ strikes rod at one end as shown. The direction of velocities of different points on rod just after collision will be



A) C along $\hat{j}$ B) C along $-\hat{j}$ C) A along $-\hat{j}$ D) A along $\hat{j}$

20. Two heavy metallic plates are joined together at 90^{o} to each other. A laminar sheet of mass $30kg$ is hinged at the line AB joining the two heavy metallic plates. The hinges are frictionless. The moment of inertia of the laminar sheet about an axis parallel to AB and passing through its centre of mass is $1.2\ kg-m^2$. Two rubber obstacles P and Q are fixed, one on each metallic plate at a distance 0.5 m from the line AB. This

distance is chosen, so that the reaction due to the hinges on the laminar sheet is zero during the impact. Initially the laminar sheet hits one of the obstacles with an angular velocity 1 *rad/s* and turns back. If the impulse on the sheet due to each obstacles is 6 $N-s$. The location of centre of mass of laminar sheet from AB is x & angular velocity of laminar sheet just after impact is ω. Values of x and ω are?



A) ω=0.1 rad/s B) ω=1 rad/s C) x =0.1 m D) x=0.2 m

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

21. In Lassiagne's test, the organic compound is fused with a piece of sodium metal in order to

A) Increase the volatility of the compound

B) Decrease the melting point of the compound

C) Convert the ionic compound into a mixture of covalent compounds

D) Convert the covalent compound into a mixture of ionic compounds

22. During the test for halogens, sodium extract is heated with HNO_3. The reason for this is

A) To decompose NaCN and Na_2S B) To make Na_2S soluble

C) To make NaCN soluble D) to make silver halides insoluble

23. An organic compound was analyzed by Duma's method 0.546g of an organic compound gave 38ml of nitrogen at 350K and 732 mm pressure . Calculate the percentage of nitrogen in the compound (vapour pressure of water at 350K is 32 mm)

A) 6.25% B) 25% C) 12.5% D) 15%

space for rough work

24. Which of the following is the most electronegative carbon?

A) SP hybridised B) SP^2 hybridized

C) SP^3 hybridised D) all have equal electronegative

25. Allyl isocyanide [CH_2CHCH_2NC] has

A) 9 sigma and 4 pi bonds

B) 8 sigma and 5 pi bonds

C) 8 sigma, 3 pi and 4non-bonded electrons

D) 9 sigma, 3 pi and 2non –bonded electrons

26. Lassiagn's test for the detection of nitrogen will fail in case of

A) NH_2CONH_2 B) $H_2NCONHNH_2.HCl$

C) $H_2NNH_2.HCl$ D) $C_6H_5NHNH_2.HCl$

27. The number of tetrahedral bond angles present in methane?

A) 4 B) 2 C) 6 D) 8

28. In the Lassaigne's test for nitrogen in an organic compound, the Prussian blue color is obtained due to the formation of

A) $Na_4\left[Fe(CN)_6\right]$ B) $Fe_4\left[Fe(CN)_6\right]_3$

C) $Fe_2\left[Fe(CN)_6\right]$ D) $Fe_3\left[Fe(CN)_6\right]_4$

SECTION - II
(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to two paragraphs with three questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

Paragraph for Questions 29 and 30:

During the estimation of nitrogen in an organic compound by Kjeldhal's method, the ammonia evolved from 0.5g of the compound, was .absorbed in 50ml of 0.5 M H_2SO_4. The residual acid required 60ml of 0.5M $NaOH$ solution

29. What volume of H_2SO_4 is used by NH_3 in the process?

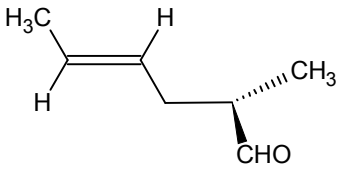
A) 50ml B) 10ml C) 20 ml D) 30 ml

30. The percentage of nitrogen in the compound is

A) 2.8 B) 56 C) 18.6 D) 14

Paragraph for Questions 31 and 32:

Answer questions 31 and 32 based on the following structure.



space for rough work

31. How many pure atomic orbital's of carbon & hydrogen are involved in bond formation in the above molecule?

A) 12 B) 9 C) 15 D) 16

32. How many sp^3 hybridized carbon orbital's are present in above molecule?

A) 4 B) 12 C) 8 D) 16

Paragraph for Questions 33 and 34:

N, S, halogens and phosphorous present in an organic compound are detected by Lassaigne's test, by fusing the compound with *Na* metal. The compounds formed are extracted from fused by boiling it with distilled water. This extract is known as sodium fusion extract or Lassaigne's extract.

33. Violet color in the detection of sulphur with sodium nitroprusside is due to the formation of

A) $Na_3\left[Fe(CN)_5 NOS\right]$ B) $Na_4\left[Fe(CN)_5 NOS\right]$

C) $Na_4\left[Fe(CN)_4 S\right]$ D) $Na_2\left[Fe(CN)_5 S\right]$

34. An organic compound containing *N, S,* and *O* as extra elements is fused with metallic sodium and then extracted with distilled water. Which species is not present in the Lassaigne's extract?

A) NO_3^- B) $C\bar{N}$ C) $\bar{S}CN$ D) S^{2-}

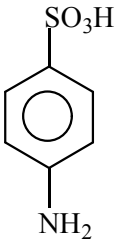
SECTION – III
(MULTIPLE CORRECT CHOICE TYPE)

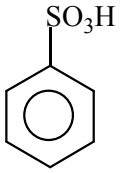
This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

35. Which of the following compounds may give blood red coloration while performing Lassaigne's test for nitrogen?

A) $(NH_2)_2CO$ B) $(NH_2)_2C = S$

C) 

D) 

36. Kjeldahl's method cannot be used for estimation of nitrogen in

A) $C_6H_5NO_2$ B) Pyridine

C) $C_6H_5 - N = N - C_6H_5$ D) $C_6H_5NHCOCH_3$

37. Which of the following compounds will give white precipitate with $AgNO_3$?

A) $C_6H_5\overset{+}{N}H_3I^-$ B) NaCl

C) $NH_2 - \overset{+}{N}H_3Cl^-$ D) $C_6H_5\overset{+}{N}H_3Cl^-$

space for rough work

38. Which has at least one sp^2 hybridized carbon atom?

A) $CH_2 = \bar{C}H$ B) $H - C \equiv \overline{C}$

C) $H_2C = C = CH_2$ D) $O = C = C = C = O$

39. Which of the following is the correct statement regarding ethene?

A) All the atoms are in the same plane

B) π Bond provides the most reactive centre in the molecule.

C) SP^2 Hybridized carbon atoms are present

D) One σ_{C-C} bond is present

40. Which of the following is correct?

A) Oxidation state of Fe in sodium Nitro prusside is 2+

B) AgI is in yellow color due to polarization effect

C) AgBr is a yellow ppt. and is sparingly soluble in excess NH_4OH

D) Yellow ppt is the detection of phosphorus when an organic compound is heated with Na_2O_2 and then boiled with con. HNO_3, followed by the addition of Ammonium molybdate is $(NH_4)_3 PO_4.12MoO_3$

MATHS: **Max. Marks: 66**

SECTION – I
(SINGLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONLY ONE is correct.**

Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

41. If y is a function of x and $ln(x+y)=2xy$, then $y^1(0)=$

A) 1 B) -1 C) 2 D) 0

42. If α be a repeated root of a quadratic equation $f(x)=0$ and $A(x),B(x),C(x)$ be polynomials of degree 3,4,5 respectively, then

$$F(x)=\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A^1(\alpha) & B^1(\alpha) & C^1(\alpha) \end{vmatrix} \text{ is}$$

A) not divisible by $(x-\alpha)$ B) divisible by $(x-\alpha)$, not by $(x-\alpha)^2$

C) divisible by $(x-\alpha)^2$, not by $f(x)$ D) divisible by $f(x)$

43. Let $P(x)=a_0+a_1x+a_2x^2+.....+a_nx^n$. If $|P(x)|\le|e^{x-1}-1|, \forall x\ge 0$, then the maximum value of $|a_1+2a_2+3a_3+.....+na_n|$ is

A) $\frac{1}{n}$ B) 1 C) n D) e

space for rough work

44. Let $g(x) = f^{-1}(x)$ and f is a bijective function such that $f^1(x) = \frac{1}{1+x^4}$, then $g^1(x)$ is equal to

A) $\left(1+g(x)^4\right)^{-1}$ B) $\left(1+\left(f(x)\right)^4\right)^{-1}$ C) $1+\left(f(x)\right)^4$ D) $1+\left(g(x)\right)^4$

45. $f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + ... + \frac{x^2}{2} + x + 1$ then

A) $f^1(1) = f^1(0)$ B) $100 f^1(1) = f^1(0)$

C) $f^1(1) = f^1(-1)$ D) $f^1(1) = 100 f^1(0)$

46. Suppose that f is differentiable function with the property

$f(x+y) = f(x) + f(y) + x^2y^2$ and $\lim_{x\to 0} \frac{f(x)}{x} = 100$, then $f^1(o)$ is equal to

A) 100 B) 20

C) 30 D) none of these

47. If $\sqrt{x+y} + \sqrt{y-x} = c, then \frac{d^2y}{dx^2} is$

A) $\frac{2}{c}$ B) $\frac{-2}{c^2}$ C) $\frac{2}{c^2}$ D) $\frac{-2}{c}$

space for rough work

48. If $\sqrt{x^2+y^2} = a\,e^{Tan^{-1}\left(\frac{y}{x}\right)}; (a>0), (y(0) = ae^{\frac{\pi}{2}})$ then $y^{11}(0) =$

A) $\frac{a}{2}e^{\pi/2}$ B) $ae^{-\pi/2}$ C) $\frac{-2}{a}e^{-\pi/2}$ D) $\frac{a}{2}e^{-\pi/2}$

SECTION - II
(COMPREHENSION TYPE)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE is correct.**
Marking scheme +3 for correct answer , 0 if not attempted and -1 in all other cases.

Paragraph for Questions 49 and 50

Let $y = f(x)$ be a continuous function for $-1 \le x \le 6$ such that $f(0) = 0$ and

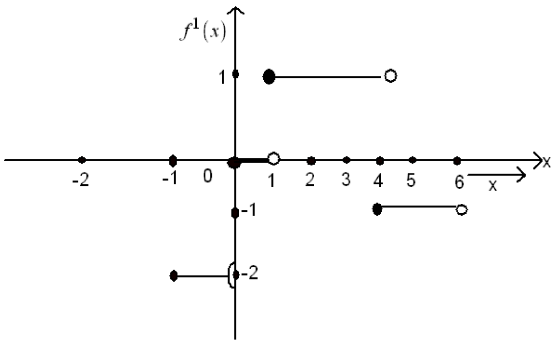
(1) The graph of $f^1(x)$ is made of line segments joined such that left end point is included and right end point is excluded in each sub interval,

($f^1(x)$ takes integral values)

(2) The graph starts at the point $(-1,-2)$.

(3) The derivatives of $f(x)$, where defined, agrees with the step pattern as shown here.

space for rough work



Using the above information answer the following.

49. The range of $f(x)$ is _____

A) $[-1,3]$ B) $(0,3)$ C) $[0,3]$ D) $(0,3]$

50. Number of integral roots of the equation $f(x)=1$ is

A) Exactly 3 B) exactly 4 C) exactly 2 D) none

Paragraph for Questions 51and 52

Let $f:R\to R$ be a continuous and differentiable function such that

$f(x+y)=f(x).f(y)\forall x,y, f(x)\neq 0$ and $f(0)=1$ and $f'(0)=2.$

Let $g(xy)=g(x).g(y)\forall x,y$ and $g'(1)=2; g(1)\neq 0$

51. Identify the correct option

A) $f(2)=e^4;\ g(3)=9$ B) $f(2)=2e^2;\ g(3)=9$

C) $f(1)<4;\ g(3)=3$ D) $f(3)>729;\ g(3)=3$

52. The number of values of x, where f(x)=g(x):

A) 0 B) 1 C) 2 D) 3

Paragraph for Questions 53 and 54

A function of the form x = f(t), y = g(t) where 't' is a parameter, is called as a parametric function, where f, g are differentiable functions of't'.Also it can be used that $\left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)=\left(\frac{dy}{dt}\right)$.Now answer the following questions on the basis of above information.

53. If $x=3\cos\theta-\cos^3\theta, y=3\sin\theta-\sin^3\theta$ then $\frac{dy}{dx}$= ---------- at $\theta=\frac{\pi}{4}$

A) $\frac{-2}{3}$ B) – 1 C) $\frac{1}{2}$ D) 1

54. $x=\sec\theta-\cos\theta\ , y=\sec^6\theta-\cos^6\theta;\theta\neq(2k+1)\pi/2, k\in Z$, then $\left(\frac{dy}{dx}\right)^2\left(\frac{x^2+4}{y^2+4}\right)=$ - - - - -

A) 36 B) 6 C) 30 D) None of these

space for rough work

SECTION – III
(MULTIPLE CORRECT CHOICE TYPE)

This section contains **6 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE is/ are correct.**

Marking scheme +4 for correct answer , 0 if not attempted and 0 in all other cases.

55. A twice differentiable function f(x) is defined for all real numbers and satisfies the following conditions:

f (0)=2; $f'(0) = -5$ and $f''(0) = 3.$

The function g(x) is defined by $g(x) = e^{ax} + f(x) \forall x \in R,$ where 'a' is any constant.

If $g'(0) + g''(0) = 0$ then 'a' can be equal to:

A) 1 B) -1 C) 2 D) -2

56. Let $f(x)$ be a non constant polynomial satisfying the relation

$f(x).f(y) = f(x) + f(y) + f(xy) - 2$ for all real x and y and $f(0) \neq 2$,suppose

$f(4) = 65.$ Then

A) $f^1(x)$ Is a polynomial of degree 2

B) Roots of $f^1(x) = 2x + 1$ are real

C) $xf^1(x) = 3[f(x) - 1]$

D) $f^1(-1) = 3$

space for rough work

57. If $\frac{d}{dx}\{f(x)\} = g(x)$ where $g(x) = e^{-x}.f(x) + e^{x}.f(-x)$ then

$(given\ f(0) = 0)$.

A) $g(x)$ Is an even function

B) $f(x)$ is an odd function

C) $g^{1}(0) = f^{1}(0) = g(0)$

D) $g^{1}(0) \neq f(0)$

58. $y = e^{x\sin(x^3)} + (\tan x)^{x}$ then $\frac{dy}{dx}$ may be equal to:

A) $e^{x\sin(x^3)}\left[3x^3\cos(x^3) + \sin(x^3)\right] + (\tan x)^{x}\left[\ln\tan x + 2x\cos ec2x\right]$

B) $e^{x\sin(x^3)}\left[x^3\cos(x^3) + \sin(x^3)\right] + (\tan x)^{x}\left[\ln\tan x + 2x\cos ec2x\right]$

C) $e^{x\sin(x^3)}\left[\cos(x^3) + x^3\sin(x^3)\right] + (\tan x)^{x}\left[\ln\tan x + 2x\cos ec2x\right]$

D) $e^{x\sin(x^3)}\left[3x^3\cos(x^3) + \sin(x^3)\right] + (\tan x)^{x}\left[\ln\tan x + \frac{x\sec^2 x}{\tan x}\right]$

59. Let $g(x)=\ln f(x)$ where $f(x)$ is twice differentiable positive function on $(0,\infty)$ such that $f(x+1)=x\,f(x)$ then for $N=1,2,3,.....;\ g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=.....$

A) $-4\left(1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N-1)^2}\right)$

B) $4\left(1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N-1)^2}\right)$

C) $-4\left(1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N+1)^2}\right)$

D) $4\left(1+\frac{1}{9}+\frac{1}{25}+.....+\frac{1}{(2N+1)^2}\right)$

60. If $f(x-y), f(x)f(y)$ and $f(x+y)$ are in A.P $\forall\ x, y\in R$; $f(0)\neq 0$ then

A) $f(4)=f(-4)$

B) $f(2)+f(-2)=0$

C) $f'(4)+f'(-4)=0$

D) $f'(2)=f'(-2)$

space for rough work

Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-ADVANCED-PAPER-2012-P2 MODEL
MAX MARKS : 198

KEY SHEET
2012-P2

PHYSICS

1	**C**	2	**C**	3	**B**	4	**A**	5	**C**
6	**B**	7	**A**	8	**A**	9	**B**	10	**A**
11	**B**	12	**D**	13	**D**	14	**D**	15	**A**
16	**CD**	17	**AC**	18	**ACD**	19	**BD**	20	**BC**

CHEMISTRY

21	**D**	22	**A**	23	**A**	24	**A**	25	**D**
26	**C**	27	**C**	28	**B**	29	**C**	30	**B**
31	**C**	32	**D**	33	**B**	34	**A**	35	**BC**
36	**ABC**	37	**BCD**	38	**AC**	39	**ABCD**	40	**ABCD**

MATHS

41	**A**	42	**D**	43	**B**	44	**D**	45	**D**
46	**A**	47	**C**	48	**C**	49	**C**	50	**C**
51	**A**	52	**B**	53	**B**	54	**A**	55	**AD**
56	**ABCD**	57	**AB**	58	**AD**	59	**A**	60	**AC**

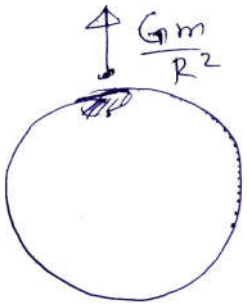
SOLUTIONS

PHYSICS

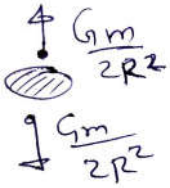
1. Conceptual

2. Conceptual

3. Net Gravitational field



Field due to local surface



Field at hole = (Net Gravitational field) – (Field due to local surface)

$$= \left(\frac{Gm}{2R^2} \right)$$

4. 

$$E = \int \frac{Gdm}{r^2} = \int \frac{G\lambda dr}{r^2}$$

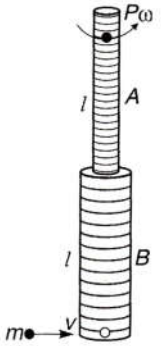
$$= G\lambda \left(\frac{1}{r} - \frac{1}{r_2} \right) = \frac{G\lambda (r_2 - r_1)}{r_1 r_2} \quad E = \frac{Gm}{r_1 r_2}$$

5. $$m = \frac{(p_1 + p_2)al}{2} \; R = \frac{l}{2\Pi}$$

$$E = \frac{Gm}{\left(R^2 + x^2\right)^{\frac{3}{2}}}$$

6. Conceptual

7. System is free to rotate but not free to translate. During collision, net torque on the system (rod A+ rod B+ mass m) about point P is zero. Therefore, angular momentum of system before collision = angular momentum of system just after collision (about P). Let ω be the angular velocity of system just after collision, then

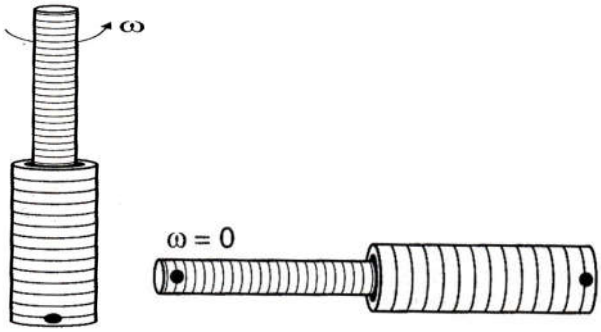


...(i)

$L_i = L_f \Rightarrow mv(2l) = I\omega$ Here. I= moment of incrtia of system about p

$= m(2l)^2 + m_A\left(l^2/3\right) + m_B\left[\frac{l^2}{I2} + \left(\frac{l}{2}+l\right)^2\right]$ Given, $l = 0.6m, m = 0.05kg, m_A$

$= 0.01kg\ and\ m_B = 0.02kg$. Substituting the values, we get $I = 0.09kg - m^2$



Therefore, from Eq. (i)

$$\omega = \frac{2mvl}{I} = \frac{(2)(0.05)(v)(0.6)}{0.09}$$

$$\omega = 0.67v$$

Now, after collision, mechanical energy will be conserved. Therefore, decrease in rotational KE = increase in gravitational PE

or $\frac{1}{2}I\omega^2 = mg(2l) + m_A g\left(\frac{l}{2}\right) + m_B g\left(l + \frac{1}{2}\right)$

or $\omega^2 = \frac{gl(4m + m_A + 3m_B)}{I}$

$$= \frac{(9.8)(0.6)(4\times 0.05 + 0.01 + 3\times 0.02)}{0.09}$$

$$= 17.64(rad/s)^2$$

$$\therefore \omega = 4.2 rad/s$$

Equation Eqs. (ii) and (iii), we get

$$v = \frac{4.2}{0.67} m/s \text{ or } v = 6.3 m/s$$

8. Conservation of angular momentum

$$\left(\frac{ma^2}{6} + \frac{ma^2}{2}\right)\omega = \left(\frac{mva}{2}\right)$$

$$a\left(\frac{1}{3} + 1\right)\omega = v$$

$$\omega = \frac{3v}{4a}$$

9. Conservation of angular momentum during collision of m_1 & m_2

$$m_1 V_0 (2R) = (m_1 V_1)2R + (m_2 V_2)2R$$

$$V_0 = V_1 + V_2 \text{ -----} (1)$$

$$V_2 - V_1 = \frac{V_0}{2} \text{-----} (2)$$

$$2V_2 = \frac{3V_0}{2} \qquad V_2 = \frac{3V_0}{4}$$

By conservation of angular momentum when the masses will have same ω

$$m\left(\frac{3V_0}{2}\right)R = 5mR^2\omega$$

$$\frac{3V_0}{10} = R\omega$$

$$= (\max imum\ speed\ of\ m_3)$$

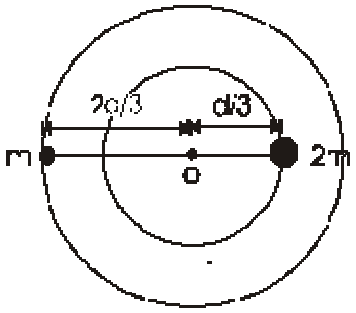
10. $$\frac{1}{2}kx^2 + \frac{1}{2}(5mR^2)\omega^2 = \frac{1}{2}m\left(\frac{3V_0}{4}\right)^2$$

$$x = \sqrt{\frac{mV_0^2}{80k}}$$

11. Conceptual

12. Conceptual

13. Let the angular speed of revolution of both stars be w about the common centre , that is, centre of mass of system.



The centripetal force on star of mass m is

$m\omega^2 \frac{2d}{3} = \frac{Gm(2m)}{d^2}$. Solving we get $T = \sqrt{\frac{4\pi^2}{3Gm} d^3}$

14. The ratio of angular momentum is simply the ratio of moment of inertia about center of mass of system.

$$\frac{L_m}{L_M} = \frac{I_m}{I_M \omega} = \frac{m\left(\frac{2d}{3}\right)^2}{2m\left(\frac{d}{3}\right)^2} = 2$$

Similarly, The ratio of kinetic energy is simply the ratio of moment of inertia about center of mass of system.

$$\frac{K_m}{K_M} = \frac{\frac{1}{2} I_m \omega^2}{\frac{1}{2} I_M \omega^2} = \frac{m\left(\frac{2d}{3}\right)^2}{2m\left(\frac{d}{3}\right)^2} = 2$$

15. Conceptual

16. Conceptual

17. The data is incomplete. Let us assume that friction form ground on ring is not impulsive during impact. From linear momentum conservation in horizontal direction. We have

$(-2 \times 1) + (0.1 \times 20) = (0.1 \times 0) + (2 \times v)$ $\overset{-ve \quad +ve}{\longleftrightarrow}$

Here, v is the velocity of CM of ring after impact. Solving the above equation, we have v = 0 Thus CM becomes stationary.

$\therefore$ Correct option is (a).

Linear impulse during impact

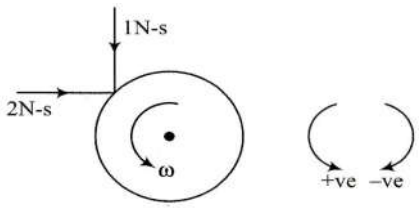
(i) In horizontal direction

$J_1 = \Delta p = 0.1 \times 20 = 2N - s$

(ii) In vertical direction

$J_2 = \Delta p = 0.1 \times 10 = 1N - s$

Writing the equation (about CM)



Angular impulse = Change in angular momentum We have,

$$1 \times \left(\frac{\sqrt{3}}{2} \times \frac{1}{2} \right) - 2 \times 0.5 \times \frac{1}{2} = 2 \times (0.5)^2 \left[\omega - \frac{1}{0.5} \right]$$

Solving this equation ω comes out to be positive or ω anti-clockwise. So just after collision rightwards slipping is taking place. Hence, friction is leftwards. Therefore, option (c) is also correct. $\therefore$ Correct options are (a) and (c)

18. $P_i = 0$

$\therefore P_f = 0$ or $v_C = 0$

$L_i = L_f$ or $(2mv)a + (2mv)(2a) = I\omega .....(i)$

Here, $I = \frac{(8m)(6a)^2}{12} + m(2a)^2 + (2m)(a^2) = 30ma^2$

Substituting in Eq. (i), we get $\omega = \frac{v}{5a}$

Further, $E = \frac{1}{2} I\omega^2 = \frac{1}{2} \times (30ma^2) \left(\frac{v}{5a} \right)^2 = \frac{3mv^2}{5}$

$\therefore$ Correct options are (a), (c) and (d).

19. Conceptual

20. Let r be the perpendicular distance of CM from the line AB and ω the angular velocity of the sheet just after colliding with rubber obstacle for the first time.

Obviously the linear velocity of CM before and after collision will be

$v_i = (r)(1 rad/s) = r$ and $v_f = r\omega$. v_i and v_f will be in opposite directions.

Now, linear impulse on CM

= change in linear momentum of CM

or $6 = m(v_f + v_i) = 30(r + r\omega)$

or $r(1+\omega) = \frac{1}{5}$ .....(i)

Similarly, angular impulse about AB = change in angular momentum about AB

Angular impulse = Linear impulse

× Perpendicular distance of impulse from AB

Hence, $6(0.5m) = I_{AB}(\omega + 1)$

(Initial angular velocity = 1 rad/s)

or $3 = \left[I_{CM} + Mr^2\right](1+\omega)$

or $3 = \left[1.2 + 30r^2\right](1+\omega)$ .....(ii)

Solving Eqs. (i) and (ii) for r, we get

r = 0.4 m and r = 0.1 m

But at r = 0.4 m, ω comes out to be negative (-0.5 rad/s)

Which is not acceptable. Therefore,

(a) r = distance of CM from AB = 0.1m

(b) Substituting r = 0.1m in Eq. (i), we get ω = 1 rad/s ie, the angular velocity with which sheet comes back after the first impact is 1 rad/s.

(c) Since, the sheet returns with same angular velocity of 1 rad/s, the sheet will never come to rest.

CHEMISTRY

21. To convert the covalent compound into a mixture of ionic compounds

22. NaCN and Na_2S decomposes into HCN and H_2S

23. $V_{N_2} = 38ml$

$P = 732 - 32 = 700$

$V_{N_2}\ at\ STP = \frac{700 \times 38}{350} \times \frac{273}{760} ml$

$22400ml---28gm\,N_2$

$\frac{2\times38\times273}{760}----=\frac{28}{22400}\times\frac{2\times38\times273}{760}gm$

% Nitrogen = $\frac{28\times2\times38\times273\times100}{22400\times760\times0.546}=6.25\%$

24. $SP>SP^2>SP^3$

25. $CH_2=CH-CH_2-N\equiv C:$

26. $NH_2NH_2.HCL$

27. 6

28. $Fe_4\left[Fe(CN)_6\right]_3$

29. / 30. The residual acid requires, 60×0.5 meq of $NaOH$ = $30meq\,NaOH$

The no.of meq of H_2SO_4 that reacts with NH_3 = $(50\times1-30)=20meq$

no.of meq of NH_3 is 20 meq

Weight of NH_3 = $20\times10^{-3}\times17g$

= $0.34g$

Weight of Nitrogen in $0.34g\,NH_3$ is 0.28g

% by weight of $N=\frac{0.28}{0.5}\times100=56\%$

31. Conceptual

32. Conceptual

33. $Na_2\left[Fe(CN)_5NO\right]+Na_2S\rightarrow Na_4\left[Fe(CN)_5NOS\right]$

34. CN^-,CNS^-,S^{2-}

35. $Fe^{3+}+SCN^-\rightarrow\left[Fe(SCN)\right]^{2+}$

36. Nitro compounds, Pyridine and *azo* compounds

37. $AgCl$

38. Conceptual

39. NCERT - 327

40. NCERT

41. $\frac{1}{x+y}\left(1+y^1\right)=2\left(xy^1+y\right)$ Put $y=0$

42. $F^1(\alpha)=0\Rightarrow F(x)$ is divisible $f(x)$

43. $\left|P^1(x)\right|=\left|a_1+2a_2x+3a_3x^2+....+na_nx^{n-1}\right|\le\left|e^{x-1}\right|$

Put $x=1\Rightarrow\left|a_1+2a_2+...+na_n\right|\le 1$

44. $f\left(g(x)\right)=x\Rightarrow f^1\left(g(x)\right)g^1(x)=1$

$\therefore g^1(x)=\frac{1}{f^1\left(g(x)\right)}=1+\left(g(x)\right)^4$

45. Conceptual

46. Observe $f(0)=0$ and $\underset{x\to 0}{Lt}\frac{f'(x)}{1}=100\Rightarrow f'(0)=100$

Now using first principle

$f'(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}=\underset{h\to 0}{Lt}\frac{f(x)+f(h)+x^2h^2-f(x)}{h}$

$=\underset{h\to 0}{Lt}\frac{f(h)}{h}+0$

$f'(0)=100$

47. Conceptual

48. Take 'log' and differentiate twice and use $y'(0)=-1$

49. $f(x)=-2x+a:-1\le x<0$

$=b:0\le x<1$

$=x+c:1\le x<4$

$=-x+d:4\le x<6$

50. $a=b=1+x,4+c=d-4$

$f(0)=0\Rightarrow a=0=b,c=-1,d=7$

$\therefore f(x)=-2x:-1<x<0$

$=0:0\le x<1$

$=x-1:1\le x<4$

$=7-x:4\le x\le 6$

51. & 52.

$f(x)=e^{2x}$

$g(x)=x^2$

53 $\dfrac{\dfrac{dy}{d\theta}}{\dfrac{dx}{d\theta}}=\dfrac{dy}{dx}=-\cot^2\theta$

54. $\dfrac{dx}{d\theta}=Tan\theta\left(\sec\theta+\cos\theta\right)=Tan\theta\sqrt{x^2+4}$

$\dfrac{dy}{dx}=6Tan\theta\left(\sec^6\theta+\cos^6\theta\right)=6Tan\theta\sqrt{y^2+4}$

$\dfrac{dy}{dx}=\dfrac{6\sqrt{y^2+4}}{\sqrt{x^2+4}}\Rightarrow\left(\dfrac{dy}{dx}\right)^2\dfrac{\left(x^2+4\right)}{\left(y^2+4\right)}=36$

55. $g'(x)=ae^{ax}+f'(x)\Rightarrow g'(0)=a-5$

$g''(x)=a^2e^{ax}+f''(x)\Rightarrow g''(x)=a^2+3$

$a=-2,1$

56. Put $y=\dfrac{1}{x}$ to get $f(x)=1+x^3$

57. $g(-x)=g(x)$ and $f'(x)=g(x)$

$\therefore g(x)$ is even and f is odd

58. $e^{x\sin\left(x^3\right)}\left[3x^3\cos\left(x^3\right)+\sin\left(x^3\right)\right]+\left(\tan x\right)^x\left[\ln\tan x+2x\cos ec2x\right]$

$e^{x\sin\left(x^3\right)}\left[3x^3\cos\left(x^3\right)+\sin\left(x^3\right)\right]+\left(\tan x\right)^x\left[\ln\tan x+\dfrac{x\sec^2x}{\tan x}\right]$

59. $g(x+1)-g(x)=\ln x$ differentiable twice and put $x=\frac{1}{2},\frac{3}{2},\frac{5}{2},\ldots.\dfrac{2N-1}{2}$

60. $2f(x)f(y)=f(x-y)+f(x+y)$

$\therefore\ f(x)=\cos x$ is suitable function

Since $f(0)\neq 0$