MasterJEE

IIT-JEE | Medical | Foundations

Master JEE CLASSES

Kukatpally, Hyderabad.

IIT-JEE-ADVANCED PAPER-2011-MODEL

Max. Marks: 240

2011_PAPER-I

IMPORTANT INSTRUCTIONS:

1) This booklet is your Question Paper.
2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
3) Blank spaces are provided within this booklet for rough work.
4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
7) The ORS will be collected by the invigilator at the end of the examination.
8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
11) Darken the bubble **COMPLETELY**.
12) The correct way of darkening a bubble is as:
13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
14) Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or

"un-darken" a darkened bubble.

JEE-ADVANCE-2011-P1-Model

IMPORTANT INSTRUCTIONS **Max Marks: 240**

CHEMISTRY

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I (Q.N : 1 – 7)	Questions with Single Correct Choice	3	-1	7	21
Sec – II(Q.N : 8 – 11)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III(Q.N : 12 – 16)	Questions with Comprehension Type (2 Comprehensions – 2 + 3 = 5Q)	3	-1	5	15
Sec – IV(Q.N : 17 – 23)	Questions with Integer Answer Type	4	0	7	28
Total				**23**	**80**

PHYSICS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 24 – 30)	Questions with Single Correct Choice	3	-1	7	21
Sec – II(Q.N : 31 – 34)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III(Q.N : 35 – 39)	Questions with Comprehension Type (2 Comprehensions – 2 + 3 = 5Q)	3	-1	5	15
Sec – IV(Q.N : 40 – 46)	Questions with Integer Answer Type	4	0	7	28
Total				**23**	**80**

MATHEMATICS

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 47 – 53)	Questions with Single Correct Choice	3	-1	7	21
Sec – II(Q.N : 54 – 57)	Questions with Multiple Correct Choice	4	0	4	16
Sec – III(Q.N : 58 – 62)	Questions with Comprehension Type (2 Comprehensions – 2 + 3 = 5Q)	3	-1	5	15
Sec – IV(Q.N : 63 – 69)	Questions with Integer Answer Type	4	0	7	28
Total				**23**	**80**

space for rough work

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct

1. Which statement best explains the partial racemization that occurs in SN^1 reactions.

A) The alkyl halide reacts as an intimate ion pair or a solvent-separated ion pair, and the carbocation does not become fully planar but re-attains much of its original configuration.

B) The partial racemization is the result of contribution by a competing E^2 reaction

C) The alkyl halide reacts as an intimate ion pair or a solvent-separated ion pair, and one face of the planar carbocation is partially blocked by the leaving group.

D) The SN^1 reaction yields complete racemization, but afterwards one of the enantiomers is interconverted into the other more stable enantiomer.

2. How does a change to a more polar solvent affect the reaction coordinate diagram for an SN^2 reaction of an alkyl halide with a negatively charged nucleophile?

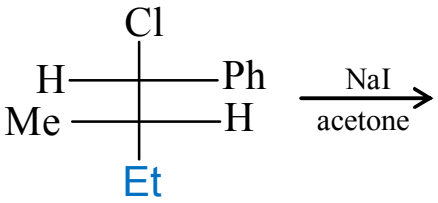
A) The energy of the reactants is increased, and the energy of the transition state remains about the same.

B) The energy of the reactants is decreased slightly, and the energy of transition state is decreased significantly more.

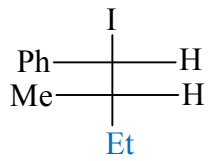
C) The energy of reactants is decreased, and the energy of the tranistion state is decreased significantly less

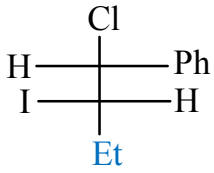
D) The energy of the reactants and the energy of the transition state increase by about the same amount.

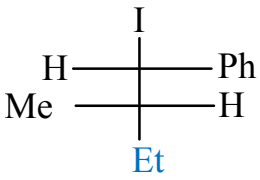
3. In the following reaction,

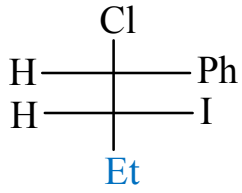


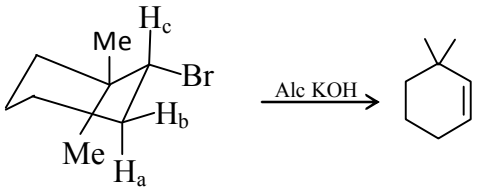
The structure of chief product is

A) 

B) 

C) 

D) 

4. 

+ HBr. Which (H) will get eliminated?

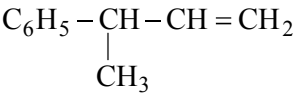
A) H_a B) H_b C) H_c D) none of these

5. Arrange the following compounds in the decreasing order of reactivity with NBS (N–bromosuccinimide).

(I) $C_6H_5 - CH_3$

(II) $C_6H_5 - CH_2 - CH_2 - CH_3$

(III) $C_6H_5 - CH_2 - CH = CH_2$

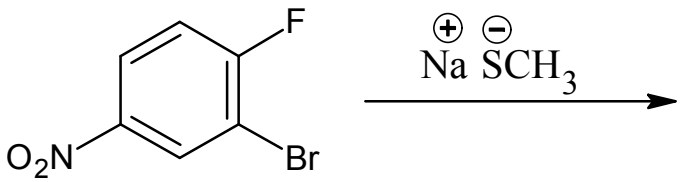
(IV) 

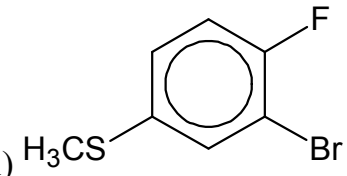
Select the correct answer from the following

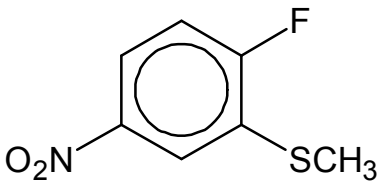
A) (IV) > (III) > (I) > (II) B) (IV) > (III) > (II) > (I)

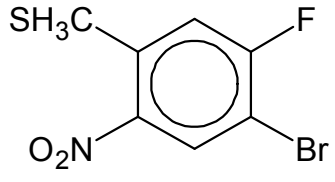
C) (I) > (II) > (III) > (IV) D) (I) > (III) > (II) > (IV)

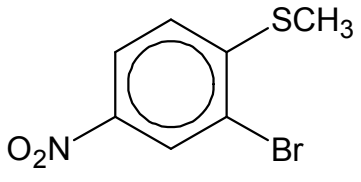
6. Identify the principal organic product of the following reaction.



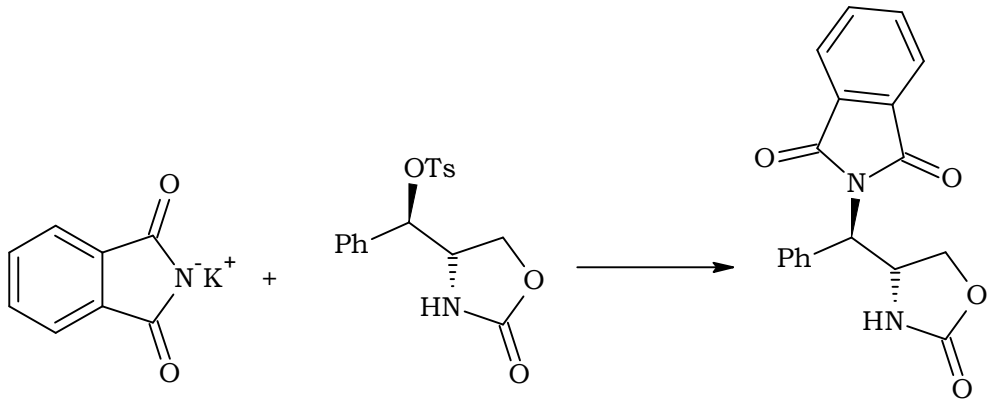
A) 

B) 

C) 

D) 

7. Reason for retention of configuration in the following reaction is



A) OTs is an excellent leaving group.

B) Neighboring group participation by Ph group.

C) Neighboring group participation by NH group.

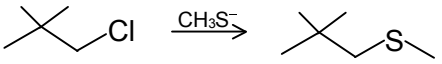
D) The reaction involves S_Ni mechanism.

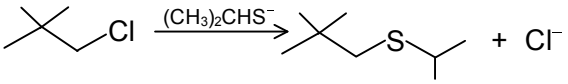
SECTION – II

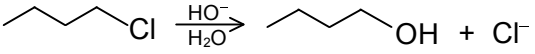
(MULTIPLE CORRECT CHOICE TYPE)

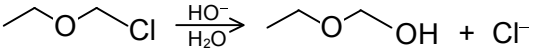
This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/ are correct

8. Which reaction in each of the following pairs, **Reaction-I** will take place more rapidly than **Reaction-II**?

A) Reaction-I 

Reaction-II 

B) Reaction-I 

Reaction-II 

C) Reaction-I $(CH_3)_3C–Br \xrightarrow{H_2O} (CH_3)_3C–OH$

Reaction-II $(CH_3)_2CH–Br \xrightarrow{H_2O} (CH_3)_2CH–OH$

D) Reaction-I $(CH_3)_3C–Br \xrightarrow{H_2O} (CH_3)_3C–OH + HBr$

Reaction-II $(CH_3)_3C–Br \xrightarrow{CH_3CH_2OH} (CH_3)_3C–O–CH_2CH_3 + HBr$

9. The rate law for the substitution reaction of 2–bromobutane and OH^- in 75% ethanol – 25% water at 30°C is

Rate = 3.2×10^{-5} [2–bromobutane] [OH^-] + 1.5×10^{-6} [2–bromobutane]

Which of the following statement(s) is/ are true?

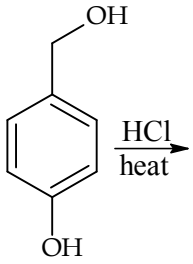
A) The percent of the reaction takes place by the S_N2 mechanism when [OH^-] = 1.0 M is 95.5 aproximately

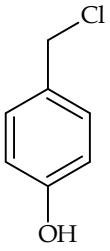
B) The percent of the reaction takes place by the S_N1 mechanism when [OH^-] = 1.0 M is 95.5 aproximately

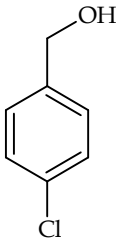
C) The percent of the reaction takes place by the S_N1 mechanism when [OH^-] = 0.001 M is 98 aproximately

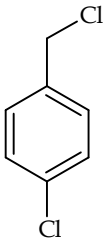
D) The percent of the reaction takes place by the S_N2 mechanism when [OH^-] = 0.001 M is 98 aproximately

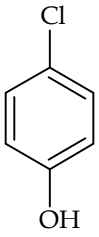
10. Identify correct product of the given reaction:



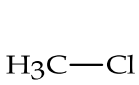
A) 

B) 

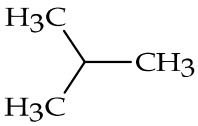
C) 

D) 

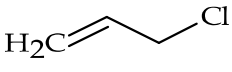
11. KI in acetone , undergoes SN2 reation with each of P,Q,R and S.The rate reaction vary as



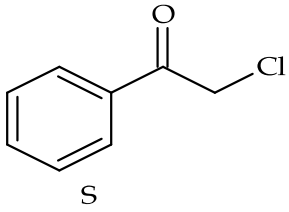
P



Q



R



S

A) P>Q>R>S B) S>P>R>Q C) P>R>Q>S D) R>P>S>Q

SECTION – III

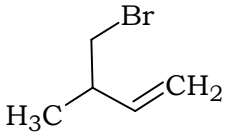
(COMPREHENSION TYPE)

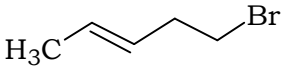
This section contains 2 groups of questions. One group has 3 questions and another group has 2 multiple choice questions based on a paragraph. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

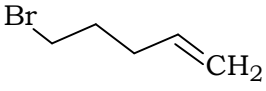
Paragraph for Questions 12 and 14

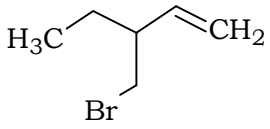
An organic compound **A** has molecular formula C_5H_9-Br. **A** decolourises brown colour of bromine water but does not rotate plane polarized light. **A** on treatment with HBr/$(PhCO)_2O_2$ from $C_5H_{10}Br_2$ which on further treatment with Na/R_2O gives a cyclic compound which on dibromination gives seven isomeric products.

12. The starting compound **A** is most likely to be

A) 

B) 

C) 

D) 

13. Compound **A** on treatment with HBr will produce

A) An achiral dibromide B) A racemic mixture

C) A single pure enantiomer D) A meso dibromide

14. The true statement regarding the monochloro derivative product formed in the reaction below is

$$A \xrightarrow{[H]} Saturated\ Hydrocarbon \xrightarrow[UV]{Cl_2} Monochlorinated\ product$$

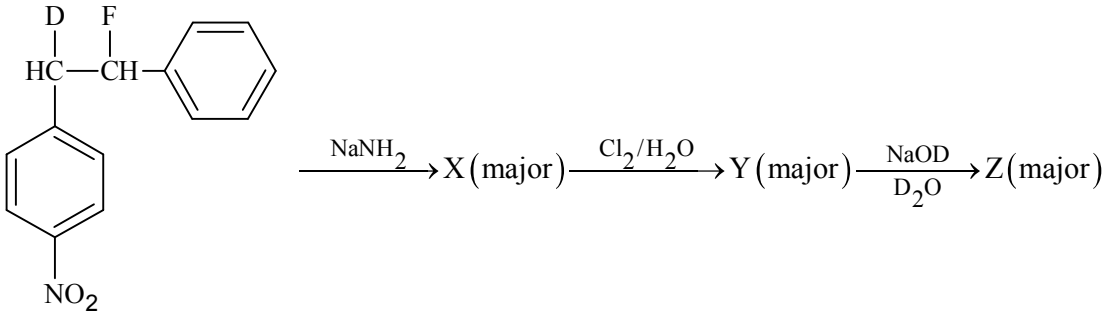
A) Three position isomers are formed

B) Total five isomers are produced

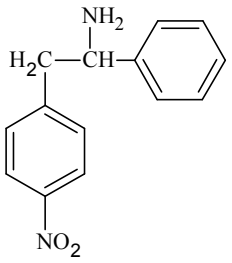
C) Two pair of enantiomers are formed

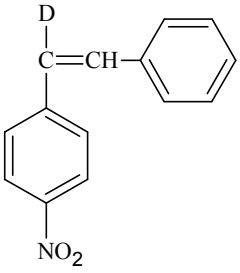
D) A pair of diastereomers are formed

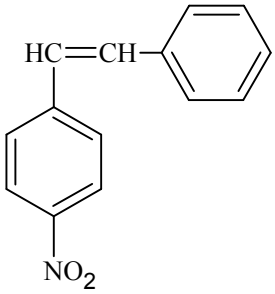
Paragraph for Questions 15 and 16

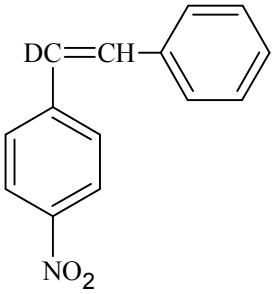


15. The product (X) and the most possible mechanism of its formation are

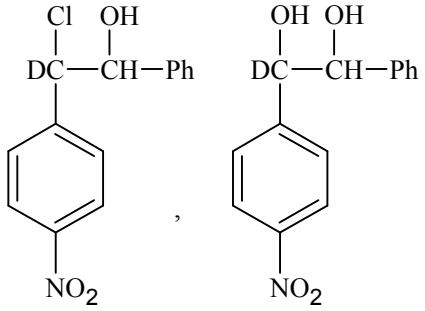
A)  , SN_2

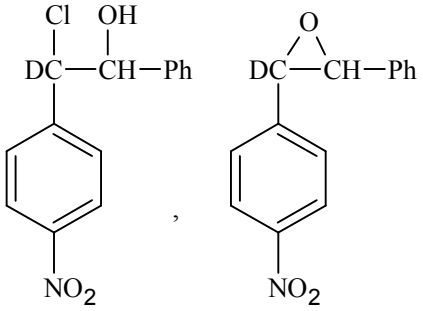
B)  , E_1

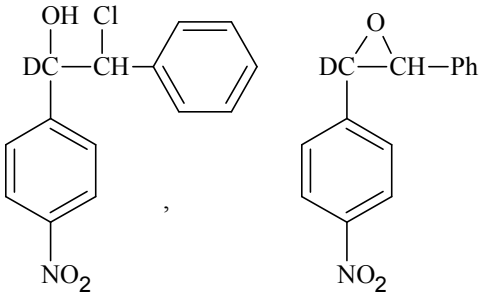
C)  , E_2

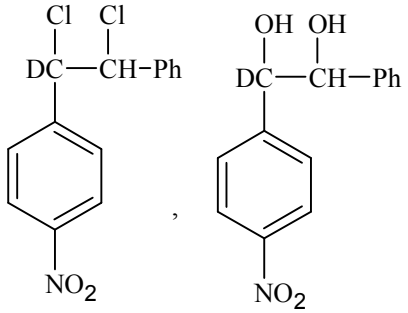
D)  , E_1 cb

16. Product (Y) and (Z) are

A) 

B) 

C) 

D) 

SECTION –IV

(INTEGER ANSWER TYPE)

This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened.

17. In how many of the following pairs the former is more reactive than later with sodium methoxide in methanol at 50^0C?

    (i) Chlorobenzene and o-chloronitrobenzene

    (ii) p-chloronitrobenzene and m-chloronitrobenzene

    (iii) 4-chloro-3-nitroacetophenone and 4-chloro-3-nitrotoluene

    (iv) 2-fluoro-1,3-dinitrobenzene and 2-chloro-1,3-dinitrobenzene

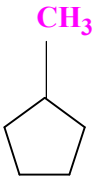
    (v) 4-chloronitrosobenzene and 4-chloroanisole

    (vi)2-iodo-1-(trifluoromethyl)benzene and 2-fluoro-1-(trifluoromethyl)benzene

18. Number of possible monochloro substituted products obtained from the reaction, of $(CH_3)_2CH-CH(CH_3)_2$ is,

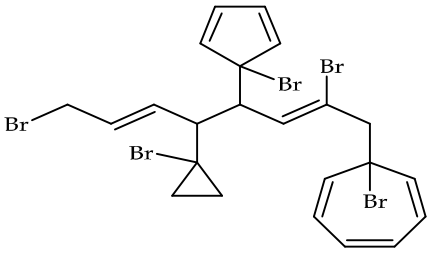
19. A chloroderivative (X) on treatment with Zn-Cu couple in ethanol gives a hydrocarbon (Y). When (X) is dissolved in ether and treated with sodium, 2, 2, 5, 5 - tetramethylhexane is obtained. Identify how many different types of hydrogen atoms are present in Y.

20. When 3,3 dimethyl 2- butanol is treated with HBr the number of 3^0 carbons present in the major product formed is.



21. If mono chlorinated through free radical substitution reaction, the total number of possible isomers (Including stereo isomers) that can be formed for the above compound is (X) which on fractional distillation gives (y) fractions. Find (x-y).

22. How many moles of $AgNO_3$ will react with one mole of given compound?



23. How many of the following are less reactive than CH_3CH_2I in a SN^2 reaction?

(i) MeI (ii) $CH_3CH_2CH_2Br$ (iii) $CH_3CH_2CH_2CH_2Cl$

(iv) Me_2CHI (v) Me_2CHCH_2I (vi) $CH_2 = CH–CH_2I$

(vii) $PhCH_2I$

SECTION – I

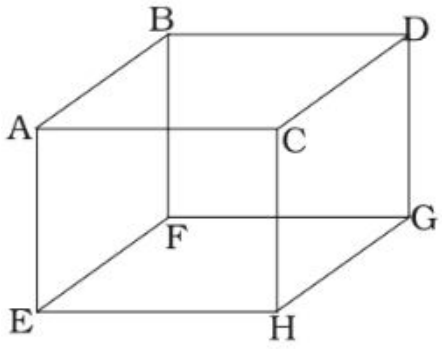
(SINGLE CORRECT CHOICE TYPE)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct

24. 50 calories of heat is requires to raise the temperature of 2 mole of an ideal gas at constant pressure from 30°C to 35°C. The amount of heat required to raise the temperature of the same gas through the same range at constant volume is

A) 30 calories B) 50 calories C) 70 calories D) 90 calories

25. 1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K. One face of the cube (EFGH) is made up of a material which totally adsorbs any gas molecule incident on it at an instant. At this given instant of time,



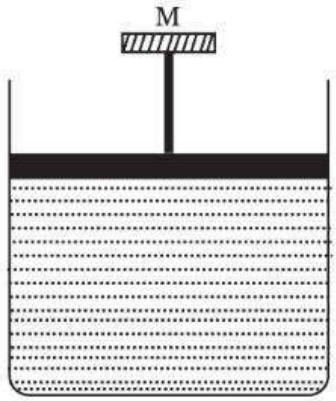
A) The pressure on EFGH would be zero.

B) the pressure on all the faces will the equal.

C) the pressure of EFGH would be double the pressure on ABCD.

D) the pressure on EFGH would be half that on ABCD.

26. A cylinder containing an ideal gas is in vertical position and has a piston of mass M that is able to move up or down without friction. If the temperature is increased slowly



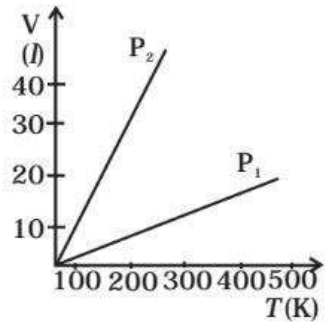
A) both p and V of the gas will change.

B) only p will increase according to Charle's law.

C) V will change but not p.

D) p will change but not V.

27. Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 & P_2 ?



A) $P_1 > P_2$ B) $P_1 = P_2$

C) $P_1 < P_2$ D) data is insufficient.

28. 1 mole of H_2 gas is contained in a box of volume V = 1.00 m^3 at T = 300K. The gas is heated to a temperature of T = 3000K and the gas gets converted to a gas of hydrogen atoms. The final pressure would be (considering all gases to be ideal)

A) same as the pressure initially. B) 2 times the pressure initially.

C) 10 times the pressure initially. D) 20 times the pressure initially.

29. An inflated rubber balloon contains one mole of an ideal gas, has a pressure p, volume V and temperature T. If the temperature rises to 1.1 T, and the volume is increase to 1.05 V, the final pressure will be

A) 1.1 p B) p

C) less than p D) between p and 1.1p

30. Boyle's law is applicable for an

A) adiabatic process. B) isothermal process.

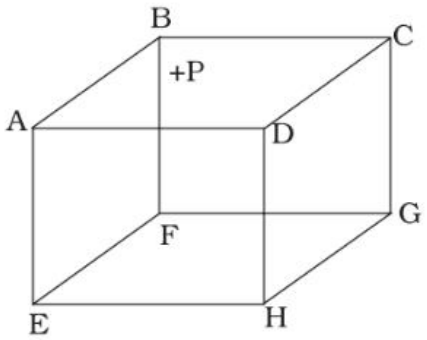
C) isobaric process. D) isochoric process

SECTION – II

(MULTIPLE CORRECT CHOICE TYPE)

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/ are correct

31. In the figure shown, the cube contains ionized hydrogen. The usual kinetic theory expression for pressure



A) will be valid.

B) will not be valid since the ions would experience forces other than due to collisions with the walls.

C) will not be valid since collisions with walls would not be elastic.

D) will not be valid because isotropy is lost.

32. In a diatomic molecule, the rotational energy at a given temperature

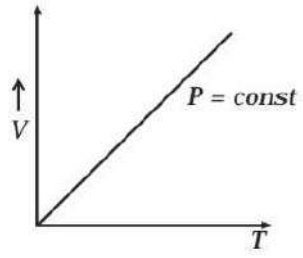
A) obeys Maxwell's distribution.

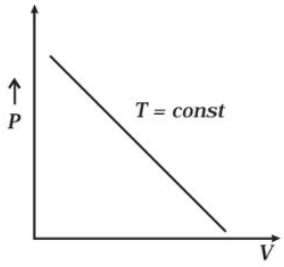
B) have the same value for all molecules.

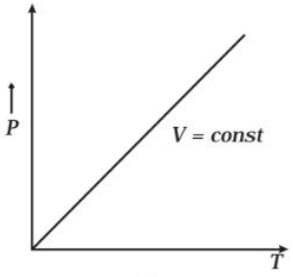
C) equals the translational kinetic energy for each molecule.

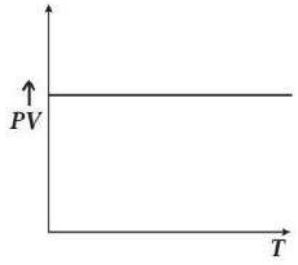
D) is (2/3)rd the translational kinetic energy for each molecule.

33. Which of the following diagrams depicts ideal gas behaviour?

A) 

B) 

C) 

D) 

34. Diatomic molecules like hydrogen have energies due to both translational as well as rotational motion. From the equation in kinetic theory $pV = \frac{2}{3}E$, E is

A) the total energy per unit volume.

B) only the translational part of energy because rotational energy is very small

C) only the translational part of the energy because during collisions with the wall pressure relates to change in linear momentum.

D) the translational part of the energy because rotational energies of molecules can be of either sign and its average over all the molecules is zero.

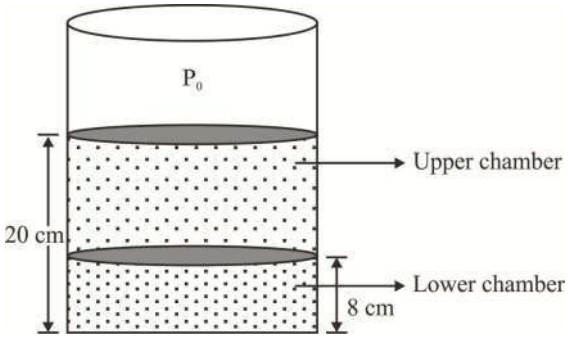
SECTION – III

(PARAGRAPH TYPE)

This section contains 2 paragraphs. Based upon one of the paragraphs 3 multiple choice questions based on the 3 multiple choice questions have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 35 and 36

In a cylindrical container of sufficiently large height, two easily moving pistons enclose certain amount of same ideal gas in two chambers as shown in the figure.



The upper piston is at a height 20 cm from the bottom and lower piston is at a height 8 cm from the bottom. The mass of each piston is m kg and cross sectional area of each piston is A m^2, where $\frac{mg}{A} = P_0$ and P_0 is the atmospheric pressure $= 1\times10^5\,N/m^2$.

The cylindrical container and pistons are made of conducting material. Initially the temperature of gas is 27^oC and whole system is in equilibrium. Now if the upper

piston is slowly lifted by 16 cm and held in that position with the help of some external force. As a result, the lower piston rises slowly by *l* cm.

35. The value of *l* is:

A) 2 cm B) 4 cm C) 8 cm D) 6 cm

36. Find the ratio of volume of gas in upper chamber to that of in lower chamber in final state:

A) 2: 1 B) 1: 2 C) 4: 1 D) 1: 4

37. Find the pressure of gas in lower chamber in final state:

A) 1.0×10^5 N/m^2 B) 2.0×10^5 N/m^2

C) 3.0×10^5 N/m^2 D) 4.0×10^5 N/m^2

Paragraph for Questions 37 and 39

An ideal gas undergoes a process in which $T = T_0 + aV^3$, where T_0 and "a" are positive constants and V is molar volume.

38 The volume for which pressure will be minimum is:

A) $\left(\frac{T_0}{2a}\right)^{1/3}$ B) $\left(\frac{T_0}{3a}\right)^{1/3}$ C) $\left(\frac{a}{2T_0}\right)^{2/3}$ D) $\left(\frac{a}{3T_0}\right)^{2/3}$

39. In above processes, minimum pressure attainable is:

A) $\frac{3}{4}(a^{5/3}R^{2/3}T_0^{2/3})2^{1/3}$ B) $\frac{3}{2}(a^{2/3}RT_0^{2/3})3^{1/2}$

C) $\frac{3}{2}(a^{1/2}R^{2/3}T_0^{3/4})4^{1/3}$ D) $\frac{3}{2}(a^{1/3}R^{2/3}T_0^{2/3})2^{1/3}$

SECTION –IV

(INTEGER ANSWER TYPE)

This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened.

40. A column of mercury 10 cm long is contained in the middle of a narrow, horizontal 1 m long tube which is closed at both ends. Both the halves of the tube container air at a pressure of 76 cm of mercury. By what distance in centimeter (approx.) will the column of mercury be displaced if the tube is held vertically?

41. A glass tube sealed at one end and containing a quantity of air is immersed in mercury until the sealed end is 10 cm from the surface of mercury in the vessel. At 0^oC the level of mercury in the tube is 5 cm above the level of mercury in the vessel. The length of the tube is 15 cm. Temperature of the air in the tube will be raised by $221\times x$ Kelvin as to fill the tube completely? The atmospheric pressure is 75 cm of Hg. Neglect the expansion of mercury and glass. Value of 'x' is

42. Both limbs of a 'U' tube are of equal length. One of the limbs is sealed and the tube contains a column of 28 cm of air at atmospheric pressure. The air is separated from the atmosphere by mercury. Height of air in the sealed limb, will be approximately $3 \times x$ centimeter if the other limb is now filled to the top with mercury? Atmospheric pressure is 76 cm of mercury. Value of 'x' is

43. The diameter of a gas bubble formed at the bottom of a pond is $d = 4.0\,\mu m$. When the bubble rises to the surface its diameter increases to n = 1.1 times. Depth of pond is 'x' meter. The nearest integer to 'x' is
(The atmospheric pressure is taken 10 m of water column, the gas expansion is assumed to be isothermal).

44. Two glass spheres of equal volume are connected by a small horizontal tube containing a small amount of mercury. The spheres are sealed at 20^oC with exactly 1 litre of air in each side. Cross sectional area of tube is 5×10^{-4} m^2. The mercury will be displaced by 34×10^{-x} centimeter if the temperature of one sphere is raised by 0.1^oC while the other is maintained at 20^oC. Value of 'x' is (neglect the expansion of glass spheres and tube)

45. Two adiabatic cylinders A and B, fitted with insulated pistons, contain equal amounts of an ideal diatomic gas at 300 K. The piston of A is free to move, while that of B is held fixed. The same amount of heat is given slowly to the gas in each cylinder. If the rise in temperature of the gas in cylinder A is 30K then rise in temperature of the gas in cylinder B is 7 × x Kelvin. Value of 'x' is

46. The average degrees of freedom per molecule of a gas are 6. The gas performs 25 J of work in a process when it is expanded at constant pressure. Amount of heat absorbed by the gas is 25 × x. Value of 'x' is

SECTION – I

(SINGLE CORRECT CHOICE TYPE)

This section contains 7 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct

47. The locus of the centre of the circle which touches the y-axis and also touches the circle $(x+1)^2+y^2=1$ externally is

A) $\{(x,y)x^2=4y\}\cup\{(0,y)\mid y\le 0\}$

B) $\{(x,y)\mid y^2=4x\}\cup\{(x,0)\mid x\le 0\}$

C) $\{(x,y)\mid x^2+4y=0\}\cup\{(0,y)\mid y\ge 0\}$

D) $\{(x,y)\mid y^2+4x=0\}\cup\{(x,0)\mid x\ge 0\}$

48. The circle $x^2+y^2=1$ cuts the x-axis at P and Q. Another circle with centre at Q and variable radius intersects to first circle at R above the X-axis and the line segment PQ at S. The maximum area of the triangle QSR is

A) $\frac{2}{9}$ B) $\frac{5\sqrt{2}}{7}$ C) $\frac{4\sqrt{3}}{9}$ D) $\frac{\sqrt{2}}{13}$

49. Equation of a straight line meeting the circle $x^2 + y^2 = 100$ in two points, each point is at a distance of 4 units from the point (8, 6) on the circle, is

A) $4x + 3y - 50 = 0$ B) $4x + 3y - 100 = 0$

C) $4x + 3y - 46 = 0$ D) $4x + 3y - 16 = 0$

50. CD is the common chord of the two circles of equal radii touching a line L at A and B. C is closer to L than D. The ratio of the circum radii of the triangles ACB and ADB is

A) less than 1 B) more than 1

C) equal to 1 D) can't say

51. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$ then $c + d =$

A) 50 B) 25 C) 60 D) 30

52. Tangents drawn from P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touches the circle at the points A and B, respectively. The radius of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circumcircle of the ΔPAB orthogonally is equal to

A) $\frac{\sqrt{73}}{4}$ B) $\frac{\sqrt{71}}{2}$ C) 3 D) 2

53. Consider a family of circles passing through the point of intersection of the lines $\sqrt{3}(y-1)=x-1$ and $y-1=\sqrt{3}(x-1)$ and having its centre on the acute angle bisector of the given lines. Then the common chords of each member of the family and the circle $x^2+y^2+4x-6y+5=0$ are concurrent at

A) $\left(\frac{1}{2},\frac{1}{2}\right)$ B) $\left(\frac{1}{2},\frac{3}{2}\right)$ C) $\left(\frac{3}{2},\frac{3}{2}\right)$ D) $\left(-\frac{1}{2},-\frac{1}{2}\right)$

SECTION – II

(MULTIPLE CORRECT CHOICE TYPE)

This section contains 4 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which ONE OR MORE is/ are correct

54. If two circles $x^2+y^2-6x-12y+1=0$ and $x^2+y^2-4x-2y-11=0$ cut a third circle orthogonally then the radical axis of the two circles passes through

A) $(1,1)$

B) $(0,6)$

C) centre of the third circle

D) mid-point of the line joining the centres of the given circles.

55. Consider the circles $C_1 \equiv x^2 + y^2 - 2x - 4y - 4 = 0$ and $C_2 \equiv x^2 + y^2 + 2x + 4y + 4 = 0$ and the line $L \equiv x + 2y + 2 = 0$, then

A) L is the radical axis of C_1 and C_2

B) L is the common tangent of C_1 and C_2

C) L is the common chord of C_1 and C_2

D) L is perpendicular to the joining centers of C_1 and C_2

56. If C_1, C_2 are two circle touching x-axis and y-axis. They both pass through point P and are orthogonal, then coordinates of P can be

A) $(2, 4 + 2\sqrt{3})$ B) $(3, 6 + 3\sqrt{3})$

C) $(2 - \sqrt{3}, 1$ D) $(1 + \sqrt{2}, 2)$

57. Circle(s) touching the x-axis at a distance of 3 units from the origin and having an intercept of length $2\sqrt{7}$ on the y-axis is (are)

A) $x^2 + y^2 - 6x + 8y + 9 = 0$

B) $x^2 + y^2 - 6x + 7y + 9 = 0$

C) $x^2 + y^2 - 6x - 8y + 9 = 0$

D) $x^2 + y^2 - 6x - 7y + 9 = 0$

SECTION – III

(COMPREHENSION TYPE)

This section contains 2 paragraphs. Based upon one of the paragraphs 3 multiple choice questions based on the 3 multiple choice questions have to be answered. Each of these questions has 4 choices (A), (B), (C) and (D) for its answer, out of which ONLY ONE is correct.

Paragraph for Questions 58 and 60

Given equation of two intersecting circle's $S_1 = 0$ & $S_2 = 0$

Equation of family of circles passing through the intersection point's of $S_1 = 0$ & $S_2 = 0$ is $S_1 + \lambda S_2 = 0$, (where $\lambda \neq -1$) Equation of common chord is $S_1 - S_2 = 0$

Equation of chord of contact for circle $x^2 + y^2 + 2gx + 2fy + c = 0$ with respect to external point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

58. The equation of the circle described on the common chord of the circles $x^2 + y^2 + 2x = 0$ and $x^2 + y^2 + 2y = 0$ as diameter is

A) $x^2 + y^2 + x - y = 0$ B) $x^2 + y^2 - x - y = 0$

C) $x^2 + y^2 - x + y = 0$ D) $x^2 + y^2 + x + y = 0$

59. The common chord of the circle $x^2 + y^2 + 6x + 8y - 7 = 0$ and a circle passing through the origin, and touching the line $y = x$, always passes through the point

A) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ B) (1, 1) C) $\left(\frac{1}{2}, \frac{1}{2}\right)$ D) none of these

Paragraph for Questions 61 and 62

A circle C_1 of radius r units rolls outside the circle $C_2: x^2 + y^2 + 2rx = 0$ touching it externally .The line of centers has an inclination 60^0 .Then

60. The point of contact of C_1 & C_2 is

A) $\left(-r, r\sqrt{3}\right)$ B) $\left(-r, -r\sqrt{3}\right)$

C) $\left(\frac{-r}{2}, \frac{-r\sqrt{3}}{2}\right)$ D) $\left(\frac{-r}{2}, \frac{r\sqrt{3}}{2}\right)$

61. The equation of direct common tangents are

A) $\sqrt{3}x - y + r\left(2 \pm \sqrt{3}\right) = 0$

B) $\sqrt{3}x - y + r\left(\sqrt{3} \pm 2\right) = 0$

C) $\sqrt{3}x - y + 2r\left(2 \pm \sqrt{3}\right) = 0$

D) $\sqrt{3}x - y + 2r\left(\sqrt{3} \pm 2\right) = 0$

62. The transverse common tangent is

A) $x + \sqrt{3}y + r = 0$ B) $x + \sqrt{3}y + 2r = 0$

C) $x + \sqrt{3}y - r = 0$ D) $x + \sqrt{3}y - 2r = 0$

SECTION –IV

(INTEGER ANSWER TYPE)

This section contains 7 questions. The answer to each of the questions is a single digit integer, ranging from 0 to 9. The appropriate bubbles below the respective question numbers in the ORS have to be darkened.

63. If the curves $\frac{x^2}{4}+y^2=1$ and $\frac{x^2}{a^2}+y^2=1$ for suitable value of "a" cut on four concyclic points, then find the radius of the smallest circle passing through these 4 points

64. Let $S_1 \equiv x^2 + y^2 - 4x - 8y + 4 = 0$ and S_2 its image in the line y = x. The radius of the circle touching y = x at (1, 1) and orthogonal to $S_2$is $\frac{3}{\sqrt{\lambda}}$, then $\lambda^2+2=$

65. A variable line $ax+by+c=0$, where a,b, c(taken in that order) are in A. P is normal to a circle $(x-\alpha)^2+(y-\beta)^2=\gamma$, which is orthogonal to circle $x^2+y^2-4x-4y=1$. Then the value of $(\alpha+\beta+\gamma)$ is ……..

66. Through a given point P(5, 2) secants are drawn to cut the circle $x^2+y^2=25$ at points $A_1(B_1), A_2(B_2), A_3(B_3), A_4(B_4)$ such that $PA_1+PB_1=5, PA_2+PB_2=5, PA_3+PB_3=7$, $PA_4+PB_4=8$ and $PA_5+PB_5=9$ then the value of $\frac{1}{51}\left[\sum_{i=1}^{5}PA_i^2+\sum_{i=1}^{5}PB_i^2\right]$ is …….

67. Let C, C_1 and C_2 be circles of radii 5, 3 and 2 respectively. C_1 and C_2 touch each other externally and C internally. A circle C_3 touches C_1 and C_2 externally and C internally. If the radius of C_3 is r then the value of [4r] is, (where [.] denotes the greatest integer function)……

68. Point A lies on circle $(x-20)^2+y^2=4$ and points B lies on circle $x^2+y^2=36$. The mid point of AB is M such that all possible positions of M form a region. If area of the region is $2\lambda\pi$. Then λ is equal to ……

69. If eight distinct points can be found on the curve $|x|+|y|=1$ such that from each point two mutually perpendicular tangents can be drawn to the circle $x^2+y^2=a^2$ then the range of $|a|$ is (α,β) then $\left[\frac{1}{\alpha}+\frac{1}{\beta}\right]$, $([.]g.i.f)=$

Master JEE CLASSES
Kukatpally, Hyderabad.

IIT-JEE-ADVANCED PAPER-2011-MODEL

Max.Marks:240

KEY SHEET

CHEMISTRY

1	**C**	2	**C**	3	**A**	4	**B**	5	**B**
6	**D**	7	**C**	8	**ACD**	9	**AC**	10	**A**
11	**B**	12	**C**	13	**B**	14	**A**	15	**D**
16	**B**	17	**4**	18	**3**	19	**1**	20	**2**
21	**4**	22	**2**	23	**4**				

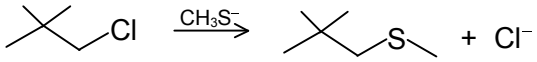
PHYSICS

24	**A**	25	**D**	26	**C**	27	**A**	28	**D**
29	**D**	30	**B**	31	**BD**	32	**AD**	33	**AC**
34	**C**	35	**B**	36	**A**	37	**B**	38	**A**
39	**D**	40	**3**	41	**3**	42	**7**	43	**3**
44	**3**	45	**6**	46	**4**				

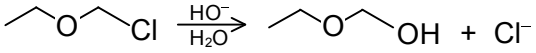
MATHS

47	**D**	48	**C**	49	**C**	50	**C**	51	**A**
52	**A**	53	**B**	54	**AC**	55	**AD**	56	**ABC**
57	**AC**	58	**D**	59	**C**	60	**D**	61	**B**
62	**C**	63	**1**	64	**6**	65	**7**	66	**4**
67	**6**	68	**6**	69	**3**				

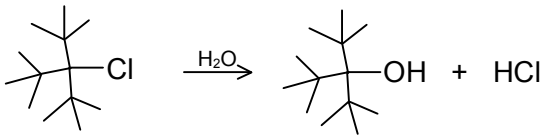
SOLUTIONS
CHEMISTRY

1. CONCEPTUAL
2. CONCEPTUAL
3. CONCEPTUAL
4. Solution (b) Elimination is favoured when departing groups are anti or trans. Here H_b is anti to Br
5. Solution (b) The correct order is decided by the stability of the free radical formed as intermediate.
6. snar
7. conceptual
8. sol: (a) 

The nucleophile CH_3S^- is less sterically hindered than $(CH_3)_2CHS^-$.

(b) 

The electron–withdrawing oxygen increases the electrophilicity of the carbon that the nucleophile attacks.

(c) 

Steric strain is decreased when Cl^- dissociates to form the carbocation in the rate–limiting step since the hybridization of the carbon atom changes from sp^3 to sp^2, allowing the bond angle between the bulky groups to increase from 109.5° to 120°.

(d) $(CH_3)_3C–Br \xrightarrow{H_2O} (CH_3)_3C–OH + HBr$

Because the reactants are neutral, the reaction will be faster in the more polar solvent.

9. sol: (a) Rate = [2–bromobutane] $(3.2 \times 10^{-5} + 1.5 \times 10^{-6})$

$= 33.5 \times 10^{-6}$ [2–bromobutane]

% of S_N1 reaction = $\frac{1.5 \times 10^{-6}[2-\text{bromobutane}]}{33.5 \times 10^{-6}\ [2-\text{bromobutane}]} \times 100 = 4.5\%$

% of S_N2 reaction = $\frac{3.2 \times 10^{-5}\ [2-\text{bromobutane}]}{33.5 \times 10^{-6}\ [2-\text{bromobutane}]} \times 100 = 95.5\%$

(b) Rate = [2–bromobutane] $(3.2 \times 10^{-8} + 1.5 \times 10^{-6})$

$= 1.532\ 10^{-6}$ [2–bromobutane]

$$\% \text{ of } S_N1 \text{ reaction} = \frac{1.5\times10^{-6}[2-\text{bromobutane}]}{1.532\times10^{-6}\ [2-\text{bromobutane}]}\times100 \approx 98\%$$

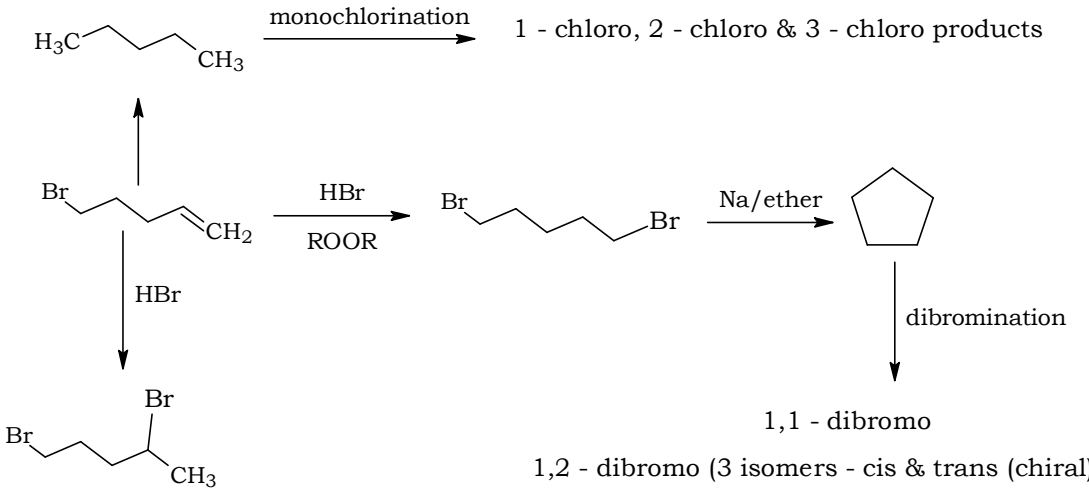
$$\% \text{ of } S_N2 \text{ reaction} = \frac{0.032\times10^{-6}\ [2-\text{bromobutane}]}{1.532\times10^{-6}\ [2-\text{bromobutane}]}\times100 \approx 2\%$$

10. CONCEPTUAL

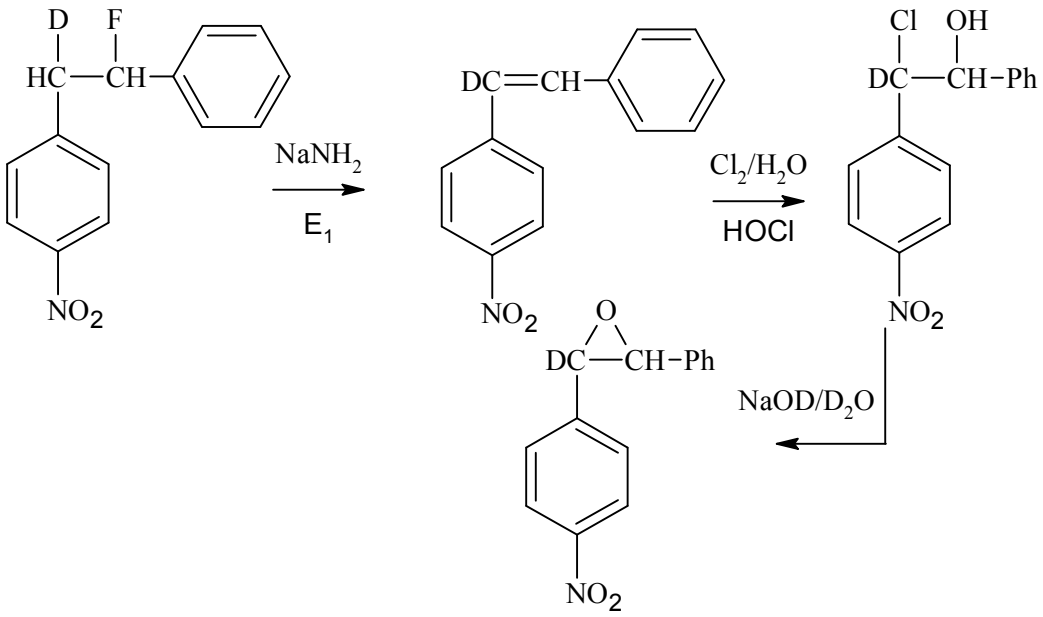
11. CONCEPTUAL

11. sol:- All the four are correct choices. The first three reactions involve benzyne intermediate

12, 13, 14 Sol:



15, 16, SOL:

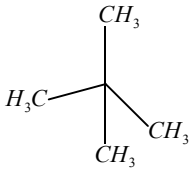


17. SOL: In (ii), (iii), (iv) and (v) first compound is more reactive than the second compound.

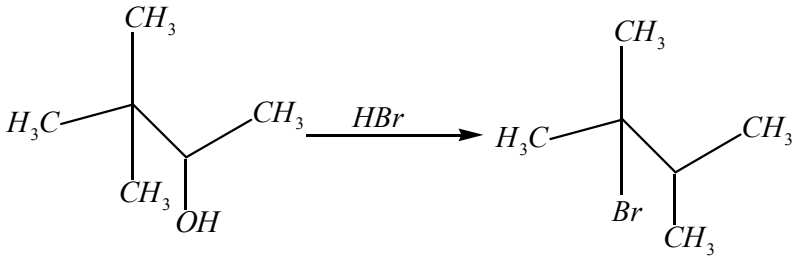
18. sol: 

(racemic mixture)

19. y is



20. ans: 2



Sol:

21. sol: (x) = 10 , (y) = 4

22. CONCEPTUAL

23. sol: (ii),(iii),(iv),(v) are less reactive than CH_3CH_2I in a SN^2 reaction

PHYSICS

24. $\Delta Q = \Delta U + W$

25. Comment for discussion: In the ideal case that we normally consider, each collision transfers twice the magnitude of its normal momentum. On the face EFGH, it transfers only half of that.

26. This is a constant pressure (p = Mg/ A) arrangement.

28. Comment for discussion: The usual statement for the perfect gas law somehow emphasizes molecules. If a gas exists in atomic form (perfectly possible) or a combination of atomic and molecular form, the law is not clearly stated.

29. Comment for discussion: In this chapter, one has discussed constant pressure and constant volume situations but in real life there are many situations where both change. If the surfaces were rigid, p would rise to 1.1 p. However, as the pressure rises, V also rises such that pv finally is 1.1 RT with $p_{final} > p$ and $V_{final} > V$.

32. Comment : The equation <K.E. of translation> = (3/2) RT , <Rotational energy> = RT is taught. The fact that the distribution of the two is independent of each other is not emphasized. They are independently Maxwellian.

35-37. Let P_1 and P_2 be the initial pressure in lower chamber of gas and upper chamber of gas.

$$P_2 + P_0 + \frac{mg}{A} = 2P_0, V_2 = A \times 12 \times 10^{-2}\,m^3$$

If P_2' and P_2' are final pressure and volume in upper chamber

$$V_2' = A \times (28 - l) \times 10^{-2}\,m^3$$

$$P_2V_2 = P_2'V_2' \quad \Rightarrow P_2' = \frac{P_2V_2}{P_2'} = \frac{24P_0}{28 - l}$$

Now consider lower chamber

$$P_1 = P_0 + \frac{2mg}{A} = 3P_0$$

and $V_1 = A \times 8 \times 10^{-2}\,m^3$

$$P_1' = P_2' + \frac{mg}{A} = P_0\left[\frac{52 - l}{28 - l}\right]$$

and $V_1' = A \times (8 \times l) \times 10^{-2}\,m^3$

$$P_1V_1 = P_1'V_1'$$

$$3P_0A \times 8 \times 10^{-2} = P_0\left[\frac{52 - l}{28 - l}\right] \times A \times (8 + l) \times 10^{-2}$$

$$24 = \left[\frac{52 - l}{28 - l}\right] \times (8 + l)$$

Solving we get $l = 4$ cm

$$\Rightarrow P_1' = 2P_0 = 2 \times 10^5\,A/m^2$$

$$\Rightarrow P_2' = \frac{24P_0}{28 - 1} = P_0 = 1 \times 10^5\,N/m^2$$

$$\frac{V_2'}{V_1'} = \frac{28 - l}{8 + l} = \frac{24}{12} = 2$$

38. From ideal gas equation,

$$pV = nRT$$

$$p = \frac{nRT}{V} = \frac{nR(T_0 + aV^3)}{V}$$

$$p = \frac{nRT_0}{V} + nRaV^2$$

$$\frac{dp}{dV} = 0$$

$$\frac{-nRT_0}{V^2} + 2anRV = 0$$

$$\frac{T_0}{V^2} = 2aV$$

$$V^3 = \frac{T_0}{2a}$$

$$V = \left(\frac{T_0}{2a}\right)^{1/3}$$

39. From ideal gas equation,

$$p = \frac{nRT}{V}$$

$$p = \frac{nR(T_0 + aV^3)}{V}$$

$$o = \frac{nRT_0}{V} + anRV^2$$

$$p = \frac{nRT_0}{T_0^{1/3}}(2a)^{1/3} + anR.\frac{T_0^{2/3}}{(2a)^{2/3}}$$

for n = 1

$$\Rightarrow\ p = RT_0^{2/3}(2)^{1/3}(a)^{1/3}$$

$$+\frac{a^{1/3}.RT_0^{2/3}}{2^{2/3}}$$

$$\Rightarrow\quad p = \frac{3RT_0^{2/3}a^{1/3}}{2^{2/3}} \qquad \Rightarrow\ p = \frac{3}{2}(a^{1/3}RT_0^{2/3})(2)^{1/3}$$

40. by 3 cm downwards

41. 663°C

42. 21.77 cm

43. 3.3 m

44. 0.034cm

45. 42 K

46. 100 J

47. Key. D

Sol. Let $P(x_1, y_1)$ be the centre of the touching $(x+1)^2 + y^2 = 1$ externally and touching y-axis $\therefore 1 - x_1 = (x_1 + 1)^2 + y_1^2 \Rightarrow y_1^2 + 4x_1 = 0$ Also every circle with centre on positive x-axis and touching y-axis at origin satisfy the condition.

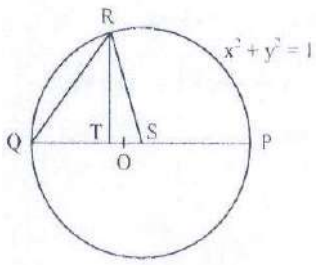
48. Key: C

Sol: Q is (-1, 0)

The circle with centre at Q and variable radius r has the equation

$(x+1)^2 + y^2 = r^2$ This circle meets the line segment QP at S where QS = r

It meets the circle $x^2 + y^2 = 1$ at $R\left(\frac{r^2-2}{2}, \frac{r}{2}\sqrt{4-r^2}\right)$ found by solving the equations of the two circles simultaneously. A = area of the triangle QSR



$= \frac{1}{2} QS \times RT$

$= \frac{1}{2} r\left(\frac{r}{2}\sqrt{4-r^2}\right)$ since RT is the y coordinate of R

$$\frac{dA}{dr} = \frac{1}{4}\left\{2r\sqrt{4-r^2} + \frac{r^2(-r)}{\sqrt{4-r^2}}\right\} = \frac{\left\{2r\left(4-r^2\right) - 4^3\right\}}{4\sqrt{4-r^2}} = \frac{8r-3r^3}{4\sqrt{4-r^2}}$$

$\frac{dA}{dr} = 0$ when $r\left(8-3r^2\right) = 0$ giving $r = \sqrt{\frac{8}{3}}$

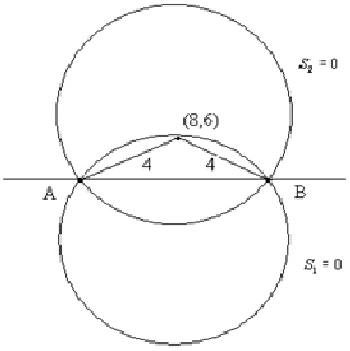
$$\frac{d^2A}{dr^2} = \frac{4\sqrt{4-r^2}\left(8-9r^2\right) - \left(8r-3r^3\right)\frac{(-r)^4}{\sqrt{4-r^2}}}{16\left(4-r^2\right)}, \text{ where, } r = \sqrt{\frac{8}{3}}, \frac{d^2A}{dr^2} < 0$$

Hence A is maximum when r = $\sqrt{\frac{8}{3}}$ and the maximum area =

$$\frac{8}{4\times 3}\sqrt{4-\frac{8}{3}} = \frac{16}{12\sqrt{3}} = \frac{4}{3\sqrt{3}} = \frac{4\sqrt{3}}{9}$$

49. Key. C

SOL. $S_1 = x^2 + y^2 = 100$



equation ofcircle centred at (8,6) & radius 4 units is $(x-8)^2 + (y-6)^2 = 16$

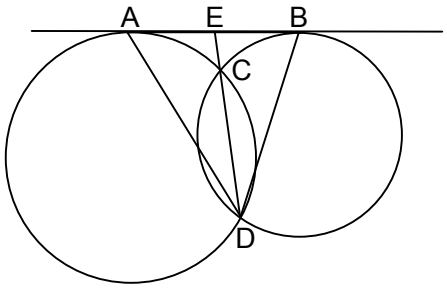
required line AB is the common chord of $S_1 = 0$ & $S_2 = 0$, is $S_1 - S_2 = 0$

$4x + 3y - 46 = 0$

50. C

Let R_1 be the circum radii of Δ ACB

$\Rightarrow R_1 = \frac{AB}{2\sin\angle ADB}$



Now AE = EB

(common chord bisects the common tangent)

Also $\angle EAC = \angle ADE = \theta$

$\Rightarrow \angle ACE = 90 - \theta \Rightarrow \angle ACB = 180 - \angle ADB$

$\Rightarrow R_1 = R_2$

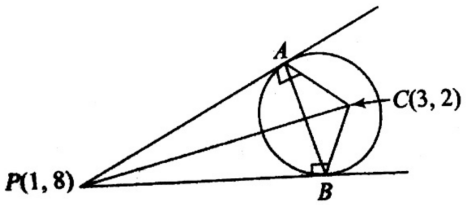
51. A

The common chord of the two circles passes through (1, –4)

$\Rightarrow 6(1) + 14(-4) + c + d = 0 \Rightarrow c + d = 50$

52. A

Equations of circle circumscribing ΔPAB is



$(x-1)(x-3)+(y-8)(y-2)=0$

$\Rightarrow x^2+y^2-4x-10y+19=0$

Equation of circle passing throughpoints of intersection of circles

$x^2+y^2-2x-6y+6=0$ and $x^2+y^2+2x-6y+6=0$ is given by

$\left(x^2+y^2-2x-6y+6\right)+\lambda\left(x^2+y^2+2x-6y+6\right)=0$

$\Rightarrow x^2+y^2+\dfrac{(2\lambda-2)}{\lambda+1}x-6y+6=0$

As circle (ii) is orthogonal to circle(i), we have

$-2\left(\dfrac{2\lambda-2}{\lambda+1}\right)-5(-6)=19+6$

$\Rightarrow 4\lambda-4=5\lambda+5$

$\Rightarrow \lambda=-9$

Hence, required equation of circle is

$x^2+y^2+\dfrac{5}{2}x-6y+6=0$

$\therefore$ Radius of circle $=\sqrt{\dfrac{25}{16}+9-6}=\dfrac{\sqrt{73}}{4}$

53. B

The given lines are $\sqrt{3}(y-1)=x-1$

$y-1=\sqrt{3}(x-1)$

These lines intersect at the point (1, 1)

Also lines make an angle 30^0 and 60^0 with the axes so acute angle bisector makes an angle 45^0 with the axes There for eacute angle bisector is x=y

Since centre lies on this line left it be (a,a)

$\therefore$ Equation of the circle is

$(x-a)^2+\left(y-a^2\right)=(1-a)^2+(1-a)^2$

Or $x^2+y^2-2ax-2ay+4a-2=0$

The common chord of this variable circle with the given circle

$x^2+y^2+4x-6y+5=0$ is

$(4+2a)x+(2a-6)y+(7-4a)=0$

i.e $(4x-6y+7)+2a(x+y-2)=0$

Which represents a family of straightlines passingthroughthe pointof intersection of the lines $4x-6y+7=0$ And $x+y-2=0$ Which is $\left(\frac{1}{2},\frac{3}{2}\right)$

54. Key.A,C

Sol. Radical axis of the given circles is $x+5y-6=0$ which passes through $(1,1)$

Let the given circles intersect the circle

$x^2+y^2+2gx+2fy+c=0$ Orthogonally then $2g(-3)+2f(-6)=c+1$

$2g(-2)+2f(-1)=c-11 \quad \Rightarrow 2g(-1)+2f(-5)=12 \Rightarrow -g-5f-6=0$

$\Rightarrow$the radical axis passes through the centre $(-g,-f)$ of the third circle.

55. Key. A, C, D

Sol. $C_1 \equiv x^2+y^2-2x-4y-4=0$ .....(i)

And $C_2=x^2+y^2+2x+4y+4=0$ .....(ii)

$\therefore$ Radical axis is $C_1-C_2=0$

$\Rightarrow -4x-8y-8=0$

Or $x+2y+2=0$ which is $L=0$

(a) option is correct

centre and radius of $C_1=0$ are $(1,2)$ and 3.

$\because$ length of $\perp$ from $(1,2)$ on $L=0$

is $\frac{|1+4+2|}{\sqrt{1+4}}=\frac{7}{\sqrt{5}} \neq$ radius

$\therefore$ (b) option is wrong

L is also the common chord of C_1 and C_2

$\therefore$ (c) option is correct.

$\because$ centres of $C_1=0$ and $C_2=0$ are $(1,2)$ and $(-1,-2)$

$\therefore$ slope of line joining centres of circles $C_1=0 \& C_2=0$ is

$\frac{-2-2}{-1-1}=\frac{4}{2}=2=m_1$ (say)

And slope of $L=0$ is $-\frac{1}{2}=m_2$ (say)

$\therefore m_1m_2=-1$

Hence, L is perpendicular to the line joining centres of C_1 and C_2

$\therefore$ (d) option is correct

56. A, B, C

In the given situation $a^2 - 4ab + b^2 = 0$ where P is (a, b).

57. A,C

The figure shows circlestouchingthe x-axisat A(3, 0) and havingintercept BD

$=BD=2\sqrt{7}$ on the y-axis from thefigure, theabscissa of center is 3

Also BC = Radius of the circle $=\sqrt{CM^2+BM^2}=\sqrt{9+7}=4$

Therefore , AC = 4

So, the centerof the circles are

C(3, 4) and $C'(3,-4)$

Hence, theequations of circles are

$(x-3)^2+(y\pm4)^2=16$

Or $x^2+y^2-6x\pm8y+9=0$

58. Key. D

59. Key. C

58. Equation of common chord is $2x - 2y = 0$

Equation of family of circle is

$x^2 + y^2 + 2x + \lambda(2x - 2y) = 0$

Centre of circle is $(-\lambda-1, +\lambda)$

Centre lies on $y = x$

$\lambda = -\lambda - 1$

$2\lambda = -1$

$\lambda=-\frac{1}{2}$

Equation is $x^2 + y^2 + x + y = 0$

59. Let the second circle $x^2 + y^2 + 2gx + 2fy = 0$

Hence, $x^2 + y^2 + 2gx + 2fy = 0$ lies equal roots f + g= 0

Equation of common chord is

$2(g-3)x + 2(-g-4)y + 7 = 0$

$(-6x - 8y + 7) + g(2x - 2y) = 0$

Passes through the intersection point of

$-6x - 8y + 7 = 0$ and $2x - 2y = 0$

$\Rightarrow \quad \left(\frac{1}{2}, \frac{1}{2}\right)$

$\cos 60° = \frac{\sqrt{(h+1)^2 + (k-1)^2}}{2}$

60. Key.D

61. Key. B

62. Key.C

Sol. 60,61,62

The point of contact is at a distance of r units from (-r,0) on a line of inclination 60^0

$\therefore$ Point of contact $= \left(-r + r\cos 60^0, 0 + r\sin 60^0\right)$

$= \left(-\frac{r}{2}, \frac{r\sqrt{3}}{2}\right)$ C_1 centre $= \left(-r + 2r\cos 60^0, 0 + 2r\sin 60^0\right) = \left(0, r\sqrt{3}\right)$

DCT's are parallel to line of centres.

Equation of DTCs are of the form : $x\sqrt{3} - y + K = 0$ to find K use CP=r.

Transverse common tangent is $\perp r$ to line of centers and passes through point of contact.

63. Key: 1

Hint: $\left(\frac{x^2}{4} + y^2 - 1\right) + \lambda\left(\frac{x^2}{a^2} + y^2 - 1\right) = 0$

$x^2\left(\frac{a^2 + 4\lambda}{4a^2(1+\lambda)}\right) + y^2 = 1$

Clearly radius is 1 unit

64. Key. 6

Sol. Centre of circle $S_1 = (2, 4)$

Centre of circle $S_2 = (4, 2)$

Radius of circle S_1 = radius of circle $S_2 = 4$

$\therefore$ equation of circle S_2

$(x - 4)^2 + (y - 2)^2 = 16$

$\Rightarrow \quad x^2 + y^2 - 8x - 4y + 4 = 0$ . .. (i)

Equation of circle touching y = x at (1, 1) can be taken as

$(x - 1)^2 + (y - 1)^2 + \lambda(x - y) = 0$

or, $x^2 + y^2 + x(\lambda - 2) + y(-\lambda - 2) + 2 = 0$ . . . (ii)

As this is orthogonal to S_2

$\Rightarrow 2\left(\frac{\lambda - 2}{2}\right) \cdot (-4) + 2\left(\frac{-\lambda - 2}{2}\right) \cdot (-2) = 4 + 2$

$\Rightarrow -4\lambda + 8 + 2\lambda + 4 = 6$

$\therefore$ required equation of circle is

$x^2 + y^2 + x - 5y + 2 = 0.$

Radius = $\sqrt{\frac{1}{4} + \frac{25}{4} - 2} = \sqrt{\frac{26 - 8}{4}} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$.

65. $ax + by + c = 0$

Also $a - 2b + c = 0$

Comparing we get x=1, y= -2

Thus variable lines are concurrent at $(1, -2)$

These lines are normal to the circle

Thus (1, –2) is the centre of circle

So, $\alpha = 1, \beta = -2$

Now $(x-1)^2 + (y+2)^2 = \gamma$

$\Rightarrow x^2 + y^2 - 2x + 4y + (5 - \gamma) = 0$

And $x^2 + y^2 - 4x - 4y - 1 = 0$

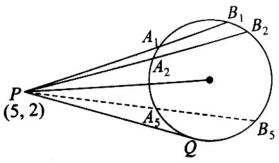
Above two circles interest orthogonally

$\Rightarrow 2(-1)(-2) + 2(2)(5 - \gamma) + (-1)$

$\Rightarrow \gamma = 8$

So, $\alpha + \beta + \gamma = 1 + (-2) + 8 = 7$

66. 8



$$\sum_{i=1}^{5} PA_i^2 + \sum_{i=1}^{5} PB_i^2$$

$$= \sum_{i=1}^{5} \left(PA_i + PB_i \right)^2 - 2\sum_{i=1}^{5} PA_i + PB_i$$

$$= 5^2 + 5^2 + 7^2 + 8^2 + 9^2 - \sum_{i=1}^{5} PQ^2$$

(Where PQ= length of tangent from P to the circle)

$$= 5^2 + 5^2 + 7^2 + 8^2 + 9^2 - 5(4)$$

$$= 50 + 49 + 64 + 81 - 20$$

$$= 244 - 20 = 224$$

67. 6

Let O, O_1, O_2, O_3 be the centres and r be the radius of C_3

C and C_1 touches internally

$\therefore OO_1 = 5 - 3 = 2$

C_1 and C_2 touches internally

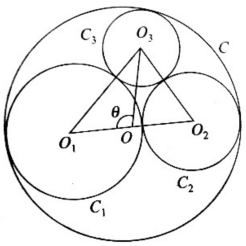
$\therefore OO_2 = 5 - 2 = 3$

C_3 touches C_1 and C_2 externally

$\therefore O_1O_3 = r + 3$ and $O_2O_3 = r + 2$

C_3 touches C internally

$\therefore OO_3 = 5 - r$



In ΔOO_1O_3 by cosine rule at vertex ' O' we get

$$O_1O_3^2 = OO_1^2 + OO_3^2 - 2OO_1OO_3 \cos \angle O_1OO_3$$

$\therefore (r+3)^2 = 4 + (5-r)^2 - 2.2(5-r)\cos\theta$

In ΔOO_2O_3 by cosine rule at vertex 'O' we get $O_2O_3^2 = OO_2^2 + OO_3^2 - 2OO_2OO\cos\angle O_2OO_3$

$(r+2)^2 = 9 + (5-r)^2 + 2.3.(5-r)\cos\theta$

By eliminating $\cos\theta$, we get $r = \frac{30}{19}$

68. 6

Parametric points on given circles are A(20+2 cosα, 2sinα) and B(6 cos β, 6sin β)

Midpoint of AB is (h,k) $(h,k) \equiv (10 + \cos\alpha + 3\cos\beta, \sin\alpha + 3\sin\beta)$

$\therefore h - 10 = \cos\alpha + 3\cos\beta$

And $k = \sin\alpha + 3\sin\beta$

Squaring and adding, we get $(h-10)^2 + k^2 = 10 + 6\cos(\alpha - \beta)$

Or we have region bounded by $(x-10)^2 + y^2 = 16$ (where $\cos(\alpha - \beta) = 1$)

And $(x-10)^2 + y^2 = 4$ (when $\cos(\alpha - \beta) = -1$)

69. $\frac{1}{2} < a < \frac{1}{\sqrt{2}}$ $\alpha + \beta = 2 + \sqrt{2}[\alpha + \beta] = 3$