



MasterJEE

IIT-JEE | Medical | Foundations

Master JEE CLASSES

Kukatpally, Hyderabad.

JEE-ADVANCE-2013-P1-Model

Max. Marks: 180

PAPER-I

IMPORTANT INSTRUCTIONS:

- 1) This booklet is your Question Paper.
- 2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
- 3) Blank spaces are provided within this booklet for rough work.
- 4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
- 5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

- 6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
- 7) The ORS will be collected by the invigilator at the end of the examination.
- 8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
- 9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

- 10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
- 11) Darken the bubble **COMPLETELY**.
- 12) The correct way of darkening a bubble is as :
- 13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
- 14) Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or

JEE-ADVANCE-2013-P1-Model

Time: 07:30 AM – 10:30 AM

IMPORTANT INSTRUCTIONS

Max Marks: 180

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 10)	Questions with Single Correct Choice	2	0	10	20
Sec – II(Q.N : 11 – 15)	Questions with Multiple Correct Choice	4	-1	5	20
Sec – III(Q.N : 16 – 20)	Questions with Integer Answer Type	4	-1	5	20
Total				20	60

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 30)	Questions with Single Correct Choice	2	0	10	20
Sec – II(Q.N : 31 – 35)	Questions with Multiple Correct Choice	4	-1	5	20
Sec – III(Q.N : 36 – 40)	Questions with Integer Answer Type	4	-1	5	20
Total				20	60

MATHEMATICS:

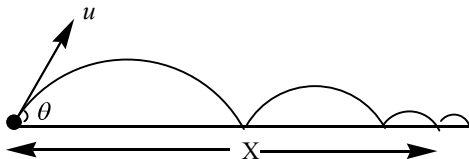
Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 50)	Questions with Single Correct Choice	2	0	10	20
Sec – II(Q.N : 51 – 55)	Questions with Multiple Correct Choice	4	-1	5	20
Sec – III(Q.N : 56 – 60)	Questions with Integer Answer Type	4	-1	5	20
Total				20	60

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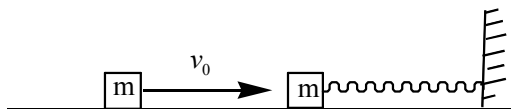
SECTION - I
Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

1. A ball is projected with initial velocity u at an angle θ to the horizontal. Then horizontal displacement covered by ball as it collides third time to the ground would be, if coefficient of restitution is e :



- A) $(1+e)\frac{u^2 \sin 2\theta}{g}$ B) $e\frac{u^2 \sin 2\theta}{g}$ C) $(1-e)\frac{u^2 \sin 2\theta}{g}$ D) $(1+e+e^2)\frac{u^2 \sin 2\theta}{g}$
2. In the figure shown surface is frictionless and spring is in natural condition. If x_1, x_2 and x_3 are the maximum compression in spring for elastic, completely inelastic and inelastic ($e=0.5$) respectively then:

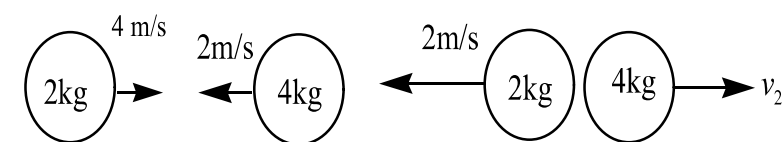


- A) $x_1 > x_2 > x_3$ B) $x_2 > x_3 > x_1$ C) $x_1 > x_3 > x_2$ D) $x_2 > x_1 > x_3$

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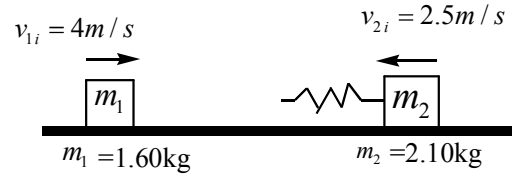
3. A smooth sphere is moving on a horizontal surface with a velocity vector $(2\hat{i} + 2\hat{j}) \text{ m/s}$ immediately before it hit a vertical wall. The wall is parallel to y-z plane and coefficient of restitution between the sphere and the wall is $e = \frac{1}{2}$. The velocity of the sphere after it hits the wall is:
- A) $\hat{i} - \hat{j}$ B) $-\hat{i} + 2\hat{j}$ C) $-\hat{i} - \hat{j}$ D) $2\hat{i} - \hat{j}$
4. Two balls of masses 2 kg and 4 kg are moved towards each other with velocities 4 m/s and 2 m/s respectively on a frictionless surface. After colliding the 2 kg ball returns back with velocity 2 m/s. Then find, coefficient of restitution of collision



- A) 0.5 B) 0.8 C) 1 D) 0.4
5. An elastic collision takes place between two masses m_1 and m_2 , moving on a frictionless surface as shown in fig: The spring constant is $k=600 \text{ N/m}$. What is the magnitude of velocity of block 2 at the instant block 1 is moving to right with a velocity 3 m/s? (i:initial)

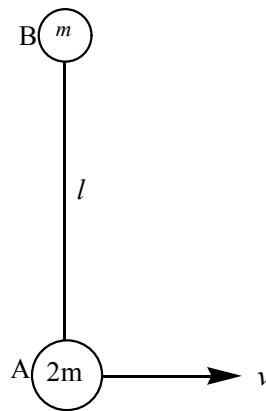
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- A) 1.64 m/s B) 1.74 m/s C) 1.44 m/s D) 1.34 m/s

6. Two masses A and B connected with an inextensible string of length l lie on a smooth horizontal plane. A is given a velocity of $v\text{ m/s}$ along the ground perpendicular to line AB as shown in Fig. Find the tension in string during their subsequent motion.

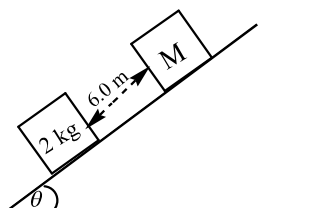


- A) $\frac{2mv^2}{3l}$ B) $\frac{3mv^2}{2l}$ C) $\frac{2mv^2}{l}$ D) $\frac{mv^2}{3l}$

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7. In an elastic head-on collision of bodies of mass m and $(x^2 - 9x + 21)m$, the second body is at rest and the maximum transfer of momentum takes place. Find the value of x
 A) 7 B) 6 C) 5 D) None of these
8. Two blocks of mass 2 kg and $M\text{ kg}$ are at rest on an inclined plane and are separated by a distance of 6.0 m as shown. The coefficient of friction between each block and the inclined plane is 0.25 . The 2 kg block is given a velocity of 10.0 ms^{-1} up the inclined plane. It collides with M , comes back and has velocity of 1.0 ms^{-1} when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Take $\sin\theta \cong \tan\theta = 0.05$ and $g = 10\text{ ms}^{-2}$. Then the approximate value of M is



- A) 2.5 kg B) 5 kg C) 10 kg D) 15 kg
9. A rocket, with an initial mass of 1000 kg , is launched vertically upwards from rest under gravity. The rocket burns fuel at the rate of 10 kg s^{-1} . The burnt matter is ejected vertically downwards with a speed of 2000 ms^{-1} relative to the rocket. If burning ceases

space for rough work

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after one minute, find the maximum velocity of the rocket. (Assume g to be constant and have a value 10ms^{-2}) ($\ln(2.5) = 0.9162$)

- A) 1232.6 m/s B) 1332.6 m/s C) 1032.6 m/s D) 932.6 m/s

10. A rocket set for vertical firing weighs 50 kg and contains 450 kg of fuel. It can have a maximum exhaust velocity of 2 kms^{-1} with respect to rocket. What should be its minimum rate of fuel consumption to give an acceleration of 20 m/s^2

- A) 7.5 kg/s B) 8.5 kg/s C) 6.5 kg/s D) 9.5 kg/s

SECTION - II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

11. A particle of mass m makes a head-on elastic collision with another particle of mass $2m$ initially rest. The velocity of the first particle before and after collision is given to be u_1 and v_1 and the velocity of second particle after the collision is v_2 . Which of the following statements is true in respect of this collision ?

A) For all values of u_1, v_1 will always be less than u_1 in magnitude.

B) The fractional loss in kinetic energy of the first particle is $\frac{8}{9}$

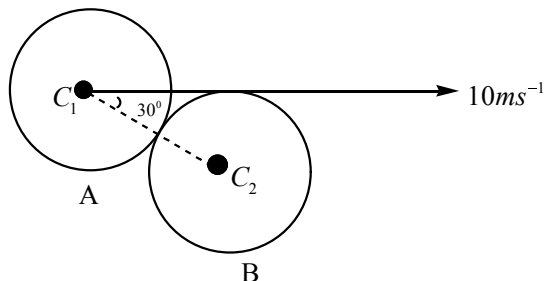
C) The gain in kinetic energy of the second particle is $\left(\frac{8}{9}\right)^{\text{th}}$ of the initial kinetic energy the first particle.

D) There is a net loss in the kinetic energy of the two particle system in the collision.

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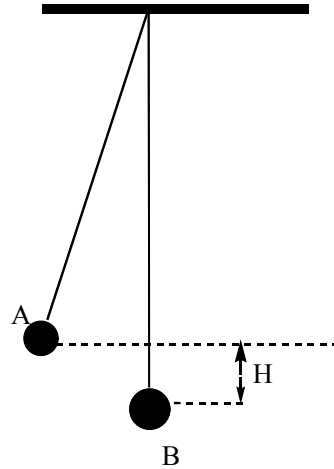
12. A ball A collides elastically with another identical ball B with a velocity of 10 ms^{-1} making an angle with the line joining their centres C_1 and C_2 . select correct statement(s) (there is no friction anywhere)



- A) Velocity of ball A after collision is 5 ms^{-1}
B) Velocity of ball B after collision is $5\sqrt{3} \text{ ms}^{-1}$
C) Kinetic energy will not be conserved here, because collision is not head on.
D) Both the balls move at right angles after collision
13. Two small balls A and B of mass M and $3M$ hang from the ceiling by strings of equal length. The ball A is drawn aside so that it is raised to a height H . Now the ball A is released such that it collides with ball B. select the correct statement (s)

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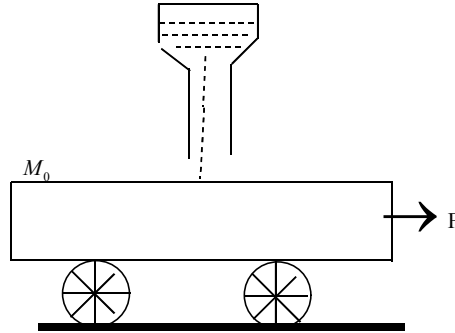


- A) If collision is perfectly elastic, ball B will rise upto a height $\frac{H}{4}$
- B) If the collision is perfectly elastic, ball A will rise upto a height $\frac{H}{4}$
- C) If the collision is perfectly inelastic, the combined mass will rise to a height $\frac{15H}{16}$
- D) If the collision is perfectly inelastic, the combined mass will rise to a height $\frac{H}{16}$
14. The figure shows a long cart moving on a smooth horizontal surface due to an external constant force of magnitude F . Initial mass of the cart is M_0 and its initial velocity is zero when the force F started to act. At $t = 0$, sand starts falling from a stationary hopper onto the cart at constant rate $\mu \text{ kg s}^{-1}$ and sticks to the cart. After time t_0 the sand

space for rough work

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starts leaking from the bottom at the same constant rate $\mu \text{kg s}^{-1}$. Eventually at time $t = 2t_0$, the sand stops falling from the hopper on to the cart and force F also stops acting. Based on above information, which of the following statement(s) is/are correct

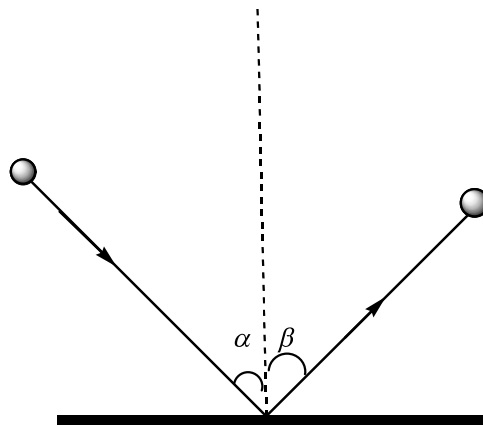


- A) The velocity of the cart at time $t (< t_0)$ is $V = \frac{Ft}{M_0 + \mu t}$
- B) In the same model of above option if the cart is to be moved with constant velocity v , then the power supplied by external agent applying that force is μv^2
- C) In the same model of above option B rate of increase of the kinetic energy of the cart (with sand) is $\frac{1}{2} \mu v^2$
- D) In the same model of above option B the rate of increase of the kinetic energy of the cart (with sand) is $\frac{1}{3} \mu v^2$

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15. A ball strikes the ground at an angle α and rebound at an angle β with the vertical as Shown in the figure. Then (all surfaces are frictionless)



- A) coefficient of restitution is $\frac{\tan \alpha}{\tan \beta}$
- B) if $\alpha < \beta$ the collision is inelastic
- C) if $\alpha = \beta$ the collision is elastic
- D) if $\alpha > \beta$ the collision is inelastic

SECTION - III

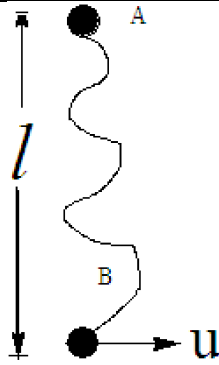
Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

16. A 6000 kg rocket is set for vertical firing. If the exhaust speed is 1000ms^{-1} , the amount of gas that must be ejected per second to supply the thrust needed to overcome the weight of the rocket is $(g = 10\text{ms}^{-2}) 10 \times x \text{ kg}$ the value of x is.... $(g = 10\text{m/s}^2)$
17. A sphere of mass m is moving with a velocity $3\hat{i} + 4\hat{j} + 5\hat{k}$ hits a smooth wall and rebounds with velocity $2\hat{i} + 3\hat{j} + 2\hat{k}$. Then the coefficient of restitution between the sphere and the wall is $n/6$. Find n ?
18. Two particles A and B each of mass m are attached by a light inextensible string of length $2l$. The whole system lies on a smooth horizontal table with B initially at a distance l from A. The particle at end B is projected across the table with speed u perpendicular to AB . Velocity of ball A just after the string is taut is $\frac{u\sqrt{x}}{4}$, the value of x is...

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19. A particle of mass m having collided a stationary particle of mass $M = 5m$ deviate by an angle $\frac{\pi}{2}$ where as the particle M recoiled at angle $\theta = 30^\circ$ to the direction of the initial motion of the particle m . The percentage change in kinetic energy of this system after collision is $8 \times x\%$, the value of x is.....
20. A 1 kg ball, moving at 12ms^{-1} , collides head-on with a 2kg ball moving in the opposite direction at 24ms^{-1} . If the coefficient of restitution is $\frac{2}{3}$, then the energy lost in the collision is $60 \times x$ joule, the value x is...

SECTION - I

Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. Which of the following is false ?
- A) VanderWaals forces are responsible for the formation of molecular crystals.
 - B) Branching lowers the boiling point of isomeric organic compounds due to reduction of Van der Waals forces of attraction
 - C) In graphite, Van der Waals forces act between the hexagonal.
 - D) Diamond has layered structure of carbon atoms with VanderWaals forces between the carbon layers.
22. Which of the following statement(s) is/are true ?
- i) In N_2 , the doubly degenerate π_{2p} orbitals are completely filled .
 - ii) In O_2 , the energy of σ_{2p_z} orbital is lower than the doubly degenerate π_{2p} orbitals .
 - iii) Different molecular species with the same configuration have the same energy.
 - iv) A π_{2p}^* orbital has two nodal planes.
- A) i, ii and iv B) i and ii only
C) i, ii, iii and iv D) ii, iii, iv

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23. Correct bond order for $\overset{NO^-}{(1)}, \overset{NO^+}{(2)}, \overset{NO}{(3)}$
- A) $1 > 2 > 3$ B) $3 > 2 > 1$ C) $2 > 3 > 1$ D) $2 < 1 < 3$
24. s-p orbitals mixing is seen in
- A) F_2 B) C_2 C) O_2^+ D) O_2^-
25. Which of the following species is paramagnetic ?
- A) N_2 B) B_2 C) O_2^{2-} D) C_2
26. Which of the following statement is false ?
- A) The crystal lattice of ice does not form hydrogen bonds.
- B) The density of water increases when heated from 0°C to 4°C due to the change in the structure of the cluster of water molecules.
- C) Above 4°C , the thermal agitation of water molecules increases. Therefore, intermolecular distance increases and water starts expanding.
- D) The density of water increases from 0°C to a maximum at 4°C because the entropy of the system increases.
-

27. Bond order of which of the following is equal to that of O_2 ?
- A) CO B) NO C) CN^+ D) CN^-
28. Metallic lustre is explained by
- A) diffusion of metal ions B) oscillation of loose electrons
C) excitation of free protons D) existence of bcc lattice
29. In the compounds of the type ECl_3 where $E = B, P, As$ (or) Bi the angles $Cl-E-Cl$ for different elements E are in the order
- A) $B > P = As = Bi$ B) $B > P > As > Bi$ C) $B < P = As = Bi$ D) $B < P < As < Bi$
30. Amongst the following statements
- I : The structure of $[BrF_4]^+$ is regular tetrahedral.
II : Bond order of O_2 decreases by the removal of an electron.
III : Br_3^- and Br_3^+ are having same shape and same structure.
IV : The hybrid orbital of phosphorous in PCl_5 is sp^3d_{xy} .
- The incorrect statements is/are
- A) only II B) I, III, IV
C) I, II, III D) I, II, III, IV

SECTION - II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

31. Select the correct statement (s) about the compound $\text{NO}[\text{BF}_4]$:
- A) It has 5σ and 2π bond
 - B) Nitrogen-oxygen bond length in this compound is longer than nitric oxide (NO)
 - C) It is a diamagnetic species
 - D) B – F bond length in this compound is lower than in BF_3
32. Which of the following statement is/are correct ?
- A) O_2 is paramagnetic, N_2 is also paramagnetic
 - B) O_2 is paramagnetic, N_2 is diamagnetic
 - C) O_2 is paramagnetic, O_2^{2-} is also paramagnetic
 - D) In $\text{O}_2 \rightarrow \text{O}_2^+$ and $\text{NO} \rightarrow \text{NO}^+$, bond length decreases.
33. Which of the following species have identical bond order ?
- A) CN^- B) O_2^- C) NO^+ D) Cl_2
34. Which of the following molecular orbital(s) has two nodal planes?
- A) $\Pi_{2\text{Py}}$ B) $\Pi_{2\text{Px}}^*$ C) $\sigma_{2\text{Pz}}^*$ D) $\Pi_{2\text{Py}}^*$

space for rough work

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35. Which of the following is/are correct with respect to hydrogen bonding ?

- A) It creates infinite chains for HF_2^- .
- B) It creates hydrogen bonded chains for molecules like HF, HSO_4^- etc.
- C) It is responsible for creating 3_D network in compounds like $H_2O, H_2PO_4^-$.
- D) It helps in the dimerization of carboxylic acids.

SECTION - III

Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

36. Maximum number of hydrogen bonds a water molecule can form in ice ?

37. Number of hydrogen bonds in $H_9O_4^+$ species is

38. The sum of number of sigma and pi bonds formed between two carbon atoms in CaC_2 are:

39. Number of pi-bonds in B_2 is:

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40. How many of the following statement(s) is/are true ?

- 1) During N_2^+ formation, from N_2 one electron is removed from the bonding molecular orbitals
- 2) During O_2^+ formation, from O_2 one electron is removed from the antibonding molecular orbital.
- 3) During O_2^- formation, from O_2 one electron is added to the bonding molecular orbital.
- 4) During CN^- formation, one electron is added to the bonding molecular orbital.
- 5) σ_{1s} molecular orbital has only one nodal plane.
- 6) σ^*_{1s} molecular orbital has only one nodal plane.

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SECTION - I
Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. Number of values of x for which $\sqrt{1+\cos 2x} = \sqrt{2} \cos^{-1}(\cos x)$ for $x \in \left(\frac{\pi}{2}, \pi\right)$, is
- A) 2 B) 0 C) 3 D) ∞
42. If α, β are the solutions of $\tan^{-1} \sqrt{x(x+1)} + \sin^{-1} \sqrt{x^2+x+1} = \frac{\pi}{2}$ then $\cos^{-1} \alpha + \cos^{-1} \beta =$
- A) $\frac{\pi}{2}$ B) $-\frac{\pi}{2}$ C) $\frac{3\pi}{2}$ D) $-\frac{3\pi}{2}$
43. $\tan^{-1} \frac{2}{4} + \tan^{-1} \frac{2}{9} + \tan^{-1} \frac{2}{16} + \dots + \tan^{-1} \frac{2}{(n+1)^2} + \dots$ to $\alpha =$
- A) $\cot^{-1} 2$ B) $\cot^{-1} 2 + \frac{\pi}{4}$ C) $\frac{\pi}{4} - \cot^{-1} 2$ D) $\frac{\pi}{2} + \cot^{-1} 2$
44. Number of solutions of $2\tan^{-1}(\tan x) = 6 - x$ is
- A) 4 B) 2 C) 3 D) 6
45. The least integral value of k for which $(K-2)x^2 + 8x + k + 4 > \sin^{-1}(\sin 12) + \cos^{-1}(\cos 12) \forall x \in R$, is
- A) -7 B) -5 C) -3 D) 5

46. Let $f(x) = \frac{2}{\pi}(\sin^{-1}[x] + \tan^{-1}[x] + \cot^{-1}[x])$ (where $[.]$ denotes G.I.F), if A and B denote the domain and range of $f(x)$ respectively then the number of integers in $A \cup B$ is
 A) 1 B) 2 C) 3 D) 4
47. If $\cot^{-1}\left(\frac{n^2 - 10n + 26}{2\sqrt{3}}\right) > \frac{\pi}{6}, n \in N$, then the minimum value of n is
 A) 3 B) 5 C) 6 D) 7
48. The number of real solutions of the equation $3\cos^{-1}x - \pi x - \frac{\pi}{2} = 0$ is
 A) 0 B) 1 C) 2 D) infinitely
49. The value of $\sin^{-1}\frac{1}{\sqrt{2}} + \sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right) + \dots + \sin^{-1}\left(\frac{\sqrt{n}-\sqrt{n-1}}{\sqrt{n(n+1)}}\right) + \dots$ to ∞ is equal to ($n \in N$)
 A) π B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\frac{3\pi}{2}$
50. If $(\sin^{-1}a)^2 + (\cos^{-1}b)^2 + (\sec^{-1}c)^2 + (\operatorname{cosec}^{-1}d)^2 = \frac{5\pi^2}{2}$ ($a, b, c, d \in R$) then the value of $(\sin^{-1}a)^2 - (\cos^{-1}b)^2 + (\sec^{-1}c)^2 - (\operatorname{cosec}^{-1}d)^2$ is
 A) $-\pi^2$ B) $\frac{-\pi^2}{2}$ C) 0 D) $\frac{\pi^2}{2}$

SECTION - II

Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

51. If

$$\sin^{-1}(\sin 8) - \tan^{-1}(\tan 10) + \cos^{-1}(\cos 12) - \sec^{-1}(\sec 9) + \cot^{-1}(\cot 6) - \operatorname{cosec}^{-1}(\operatorname{cosec} 7) = a\pi + b$$

Then which of the following is/are correct?

- A) $a - b = 53$ B) $3a + b = -1$ C) $4a + b = 12$ D) $a - b = 27$

52. If $(\sin^{-1} x)^2 + (\sin^{-1} y)^2 + (\sin^{-1} z)^2 = \frac{3\pi^2}{4}$ then the possible value of $x - y + z$ is equal to

- A) 1 B) -1 C) 3 D) -3

53. If $\sin^{-1}\left(a - \frac{a^2}{3} + \frac{a^3}{9} - \dots\right) + \cos^{-1}(1 + b + b^2 + \dots) = \frac{\pi}{2}$ ($|a| < 3$ and $|b| < 1$), then which of the

following is/are correct?

- A) $b = \frac{3a-2}{2a}$ B) $b = \frac{2a-3}{3a}$ C) $a = \frac{2}{3-2b}$ D) $a = \frac{3}{2-3b}$

54. If $s_n = \cot^{-1} 3 + \cot^{-1} 7 + \cot^{-1} 13 + \cot^{-1} 21 + \dots$ to n terms then which of the following is/are correct?

- A) $s_{10} = \tan^{-1} \frac{5}{6}$ B) $s_{\infty} = \frac{\pi}{4}$ C) $s_6 = \sin^{-1} \frac{3}{5}$ D) $s_{20} = \cot^{-1}(101)$

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55. If $\sin^{-1}(x^2 + 2x + 2) + \tan^{-1}(x^2 - 3x - k^2) > \frac{\pi}{2}$ ($x \in \mathbb{R}$), then k contains the set

A) (-1,0)

B) (0,1)

C) (1,2)

D) (0,2)

SECTION - III

Integer Answer Type

This section contains **5 questions**. The answer to each question is single digit integer, ranging from 0 to 9 (both inclusive).

56. If the equation $\sin^{-1}(x^2 + x + 1) + \cos^{-1}(\lambda x + 1) = \frac{\pi}{2}$ has exactly two solutions, then the

range of λ is $[a, b)$ where $a + b =$

57. If $r = x + y + z$ ($x, y, z > 0$) and $\tan^{-1}\sqrt{\frac{xr}{yz}} + \tan^{-1}\sqrt{\frac{yr}{xz}} + \tan^{-1}\sqrt{\frac{zr}{xy}} = K\pi$ then $K =$

58. Let $f: [0, 4\pi] \rightarrow [0, \pi]$ be defined as $f(x) = \cos^{-1}(\cos x)$. The number of values of x satisfying the equation $f(x) = \frac{10-x}{10}$ is

59. If $\cot^{-1}\left(2^2 + \frac{1}{2}\right) + \cot^{-1}\left(2^3 + \frac{1}{2^2}\right) + \cot^{-1}\left(2^4 + \frac{1}{2^3}\right) + \dots \text{to } \infty = \cot^{-1}k$, then $K =$

60. If $\sin\left\{2\left[\sin^{-1}\frac{\sqrt{5}}{3} - \cos^{-1}\frac{\sqrt{5}}{3}\right]\right\} = \frac{k\sqrt{5}}{81}$, then the value of k is equal to

space for rough work

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JEE-ADVANCE-2013-P1-Model

Max.Marks:180

KEY SHEET

PHYSICS

1	D	2	C	3	B	4	A	5	B	6	A
7	C	8	D	9	A	10	A	11	ABC	12	ABD
13	ABD	14	ABC	15	ABC	16	6	17	3	18	3
19	5	20	4								

CHEMISTRY

21	D	22	A	23	C	24	B	25	B	26	A
27	C	28	B	29	B	30	D	31	AC	32	BD
33	AC	34	BD	35	BCD	36	4	37	3	38	3
39	1	40	4								

MATHS

41	B	42	C	43	B	44	C	45	D	46	D
47	A	48	B	49	B	50	C	51	ABC	52	ABCD
53	BD	54	ABC	55	ABCD	56	1	57	1	58	3
59	2	60	8								

SOLUTIONS PHYSICS

1. After each collision vertical velocity become e times whereas horizontal velocity remain same. So after each collision time of flight become e times of previous one.

$$\text{So that horizontal displacement} = R + eR + e^2R = (1 + e + e^2) \frac{u^2 \sin 2\theta}{g}$$

2. For elastic collision Velocity is interchanged

$$\therefore \frac{1}{2}mv_0^2 = \frac{1}{2}kx_1^2 \Rightarrow x_1 = \frac{\sqrt{mv_0}}{k}$$

For completely inelastic collision

$$mv_0 = 2mv \Rightarrow v = \frac{v_0}{2}$$

$$\text{Now } \frac{1}{2}kx_2^2 = \frac{1}{2}xmv^2 \Rightarrow x_2 = \sqrt{\frac{mv_0}{2k}}$$

For $e = 0.5$

$$-0.5 = \frac{v_2 - v_1}{0 - v_0}$$

$$v_2 - v_1 = \frac{v_0}{2}$$

$$\text{Also } mv_2 + mv_1 = mv_0$$

$$v_2 = 0.75v_0 \Rightarrow x_3 = \sqrt{\frac{3mv_0}{4k}}$$

3. Along wall momentum is conserved

$$4. \quad e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{1 - (-2)}{4 - (-2)} = \frac{3}{6} = 0.5$$

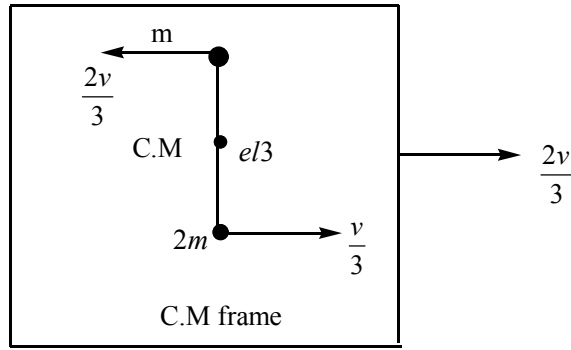
5. From momentum conservation, we obtain $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

$$(1.60)(4.00) + (2.10)(-2.50) = (1.60)(3.00) + (2.10)v_{2f}$$

$$v_{2f} = -1.74 \text{ m/s}$$

Negative sign implies that block 2 is still continuing in the same direction

6. Solve the problem in CM frame. In CM frame particles A and B moving along circular path with same ω



Velocity of $2m$ mass $= v - \frac{2v}{3} = \frac{v}{3}$

Angular velocity of particle A

$$\omega = \frac{v/3}{l/3} = \frac{v}{l}$$

$$T = M\omega^2 r = 2m \cdot \frac{v^2}{l^2} \cdot \frac{l}{3}$$

$$\text{or } T = \frac{2mv^2}{3l}$$

7 For maximum momentum transfer $m_1 = m_2$

8. Conceptual

9. Since, we know that

$$v = (u - gt) + v_r \ln\left(\frac{m_0}{m}\right)$$

According to the problem we have $u = 0, t = 60s, g = 10ms^{-2}, v_r = 2000ms^{-1}, m_0 = 1000kg$

and $m = 1000 - 10 \times 60 = 400kg$

$$\Rightarrow v = 0 - 600 + 2000 \ln\left(\frac{1000}{400}\right)$$

$$\Rightarrow v = 2000 \ln(2.5) - 600$$

The maximum velocity of the rocket is

$$v = 200(10 \ln 2.5 - 3) = 1232.6ms^{-1}$$

10. Net acceleration $a = 20ms^{-2}$

Since we have $ma = F_1 - mg$

$$\Rightarrow a = \frac{F_1}{m} - g$$

$$\Rightarrow a = \frac{v_r}{m} \left(-\frac{dm}{dt} \right) - g$$

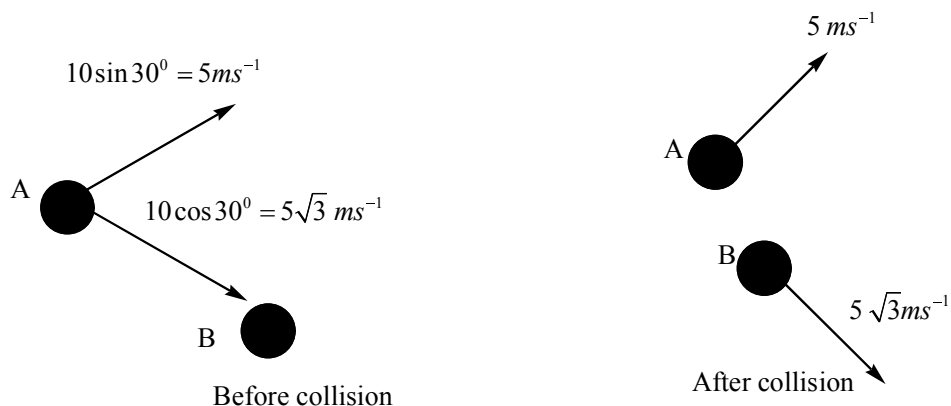
$$\Rightarrow \left(-\frac{dm}{dt} \right) = \frac{m(g + a)}{v_r}$$

$$\Rightarrow \left(-\frac{dm}{dt} \right) = \frac{(450 + 50)(10 + 20)}{2 \times 10^3} = 7.5kgs^{-1}$$

11. Impulse $j = \frac{m(v+4)}{t}$

Coefficient of restitution $e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v}{u}$

12.



Velocity components in common tangent direction will remain unchanged while velocity components in common normal direction are interchanged in case of an elastic

collision Hence both A and B move at right angles after collision with $v_A = 5 \text{ ms}^{-1}$ and $v_B = 5\sqrt{3} \text{ ms}^{-1}$. Kinetic energy is conserved in an elastic collision. Whether it a head on or an oblique collision

13. Before collision $v_A = \sqrt{2gH}$ and $v_B = 0$

$$v_A = \left(\frac{m_A - m_B}{m_A + m_B} \right) v_A$$

$$\Rightarrow v_A = \left(\frac{M - 3M}{M + 3M} \right) \sqrt{2gH} = -\frac{\sqrt{2gH}}{2}$$

$$\text{and } v_B = \left(\frac{2m_A}{m_A + m_B} \right) v_A$$

$$\Rightarrow v_B = \left(\frac{2M}{M + 3M} \right) \sqrt{2gH} = \frac{\sqrt{2gH}}{2}$$

$$\text{So, } h_A = \frac{v_A^2}{2g} = \frac{H}{4} \text{ and } h_B = \frac{v_B^2}{2g} = \frac{H}{4}$$

$$v_A = \frac{m_A v_A}{m_A + m_B} = \frac{M \sqrt{2gH}}{M + 3M} = \frac{\sqrt{2gH}}{4}$$

The combined mass will rise to height

$$h = \frac{v^2}{2g} = \frac{H}{16}$$

14. By Newton's Law

$$F - \mu v = (M + \mu t) \frac{dv}{dt}$$

$$\Rightarrow \int_0^v \frac{dv}{F - \mu v} = \int_0^t \frac{dt}{M + \mu t}$$

$$\Rightarrow v = \frac{Ft}{M_0 + \mu t}$$

For constant velocity $\frac{dv}{dt} = 0$

So, from (1) we get $F = \mu v^2$

$$K = \frac{1}{2} M v^2$$

For constant velocity $v = \text{constant}$

$$\Rightarrow \frac{dk}{dt} = \frac{v^2}{2} \frac{dn}{dt}$$

Since $M = M_0 + \mu t$

$$\Rightarrow \frac{dM}{dt} = \mu$$

$$\Rightarrow \frac{dk}{dt} = \frac{1}{2} \mu v^2$$

15. Let velocity of ball before collision and after collision be u and v respectively, then

$$e = \frac{v \cos \beta}{u \cos \alpha}$$

And by law of conservation of momentum along horizontal direction we get

$$u \sin \alpha = v \sin \beta$$

From (1) and (2) we get

$$e = \frac{\tan \alpha}{\tan \beta}$$

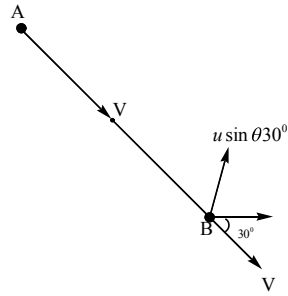
16. $M_0 g = \text{Thrust} = v \frac{dM}{dt}$

$$\Rightarrow \frac{dM}{dt} = \frac{M_0 g}{v}$$

$$\Rightarrow \frac{dM}{dt} = \frac{60000}{1000} = 60 \text{ kg s}^{-1}$$

17. $\frac{\vec{V} \cdot (\vec{V} - \vec{U})}{\vec{U} \cdot (\vec{V} - \vec{U})} = e$

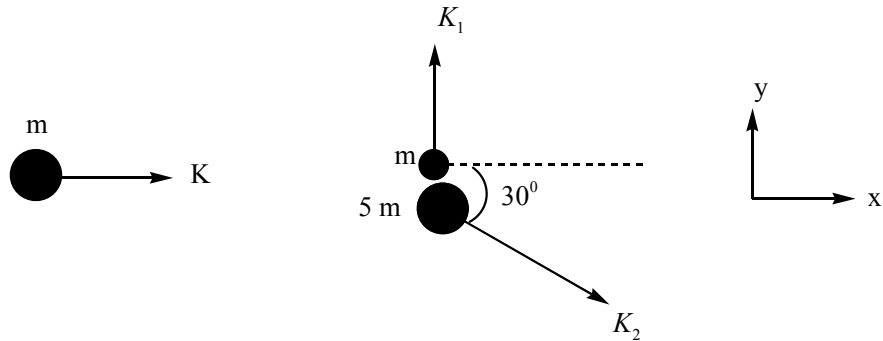
18. when the string becomes taut both particle begin to move with velocity components v in the direction AB. Applying Law of conservation of linear momentum in the direction AB we get $mu \cos 30^\circ = mv + mv$



$$\Rightarrow v = \frac{u\sqrt{3}}{4}$$

Hence the velocity of ball A just after the string becomes taut is $\frac{u\sqrt{3}}{4}$

19. Let K be the initial kinetic energy of mass m and K_1 and K_2 the kinetic energy after collision in the direction shown in figure initial momentum of the particle is $\sqrt{2mK}$
So applying conservation of linear momentum in vector form, we get



$$\sqrt{2mK} \hat{i} = \sqrt{2mK_1} \hat{j} + \sqrt{2mK_2} (\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$$

$$\Rightarrow \sqrt{2mK} = \sqrt{10mK_2} \frac{\sqrt{3}}{2}$$

$$\sqrt{2mK_1} = \sqrt{10mK_2} \frac{1}{2}$$

Solving we get $K_2 = \frac{4}{15}K$ and $K_1 = \frac{K}{3}$

$$\Rightarrow K_1 + K_2 = \frac{9}{15}K = 0.6K$$

So, Loss in $KE = K - 0.6K = 0.4K$

Hence, KE will be decreased by 40%

20. $(1)(12) + (2)(-24) = 1v_1 + 2v_2$

$$\Rightarrow v_1 + 2v_2 = -36$$

Further

$$e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right)$$

$$\Rightarrow \frac{2}{3} = -\left(\frac{v_2 - v_1}{-24 - 12}\right)$$

$$\Rightarrow \frac{2}{3} = \frac{v_2 - v_1}{36}$$

$$\Rightarrow v_2 - v_1 = 24$$

Add(1) and (2)

$$3v_2 = -12$$

$$\Rightarrow v_2 = -4ms^{-1}$$

$$\Rightarrow v_1 = -28ms^{-1}$$

$$\Rightarrow \text{Loss} = \text{Total initial} - \text{Total final}$$

Since

$$\text{Total initial} = \frac{1}{2}(1)(144) + \frac{1}{2}(2)(16)$$

$$\Rightarrow E_i = 72 + 576 = 648J$$

Similarly

$$E_f = \frac{1}{2}(1)(784) + \frac{1}{2}(2)(16)$$

$$\Rightarrow E_f = 392 + 16$$

$$\Rightarrow E_f = 408J$$

$$\Rightarrow \text{Loss} = 648 - 408$$

$$\Rightarrow \text{Loss} = 240J$$

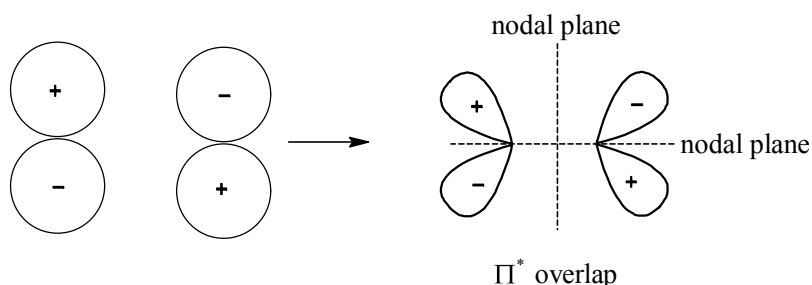
CHEMISTRY

21. Diamond is a crystalline covalent solid.
22. The σ_{2p_z} orbital in O_2 is higher than the doubly degenerate π_{2p} orbitals and more than π_{2p} orbitals in N_2 from MOT.
23. $B.O \Rightarrow NO = 2.5$
 $NO^+ = 3.0; NO^- = 2.0$
 $NO^+ > NO > NO^-$
24. $s-p$ mixing is observed in B_2, C_2, N_2 or in species with total electrons ≤ 14
25. B_2 has two unpaired electrons
26. Ice also has hydrogen bonds, a water molecule in ice is surrounded by 4 more water molecules through H-bonds
27. Bond order of $CN^+ = \frac{Nb - Na}{2}$

$$= \frac{8-4}{2} = 2$$

Bond order of $O_2 = 2$

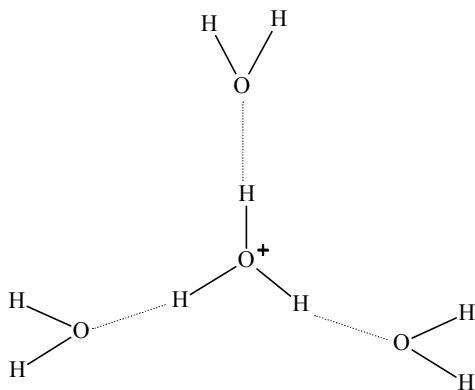
28. Metallic lustre is due to free valency electrons.
29. With decrease in electronegativity of central atom, bond angle decreases. BCl_3 has trigonal planar structure with 120° angles
30. I : BrF_4^+ : Trigonal bipyramidal structure and see-saw shape
 II : Bond order of O_2 increases by the removal of e^- , thus bond order increases.
 III : Br_3^- and Br_3^+ are of different shape and structure.
 IV : In PCl_5 , hybrid orbital of P is $sp^3d_{z^2}$
31. Bond order of NO^+ is = 3, one sigma bond and two pie bonds.
 No of pie bonds = 2
 No of sigma bonds = 5
 B.O. of $NO^+ = 3$
 B.O. of $NO = 2.5$
 NO^+ is the diamagnetic cation and BF_4^- is also a diamagnetic anion.
 B – F bonds are longer in BF_4^- than in BF_3 due to absence of $P\pi - d\pi$ back bonding in $[BF_4]^-$.
32. O_2 is paramagnetic due to 2 unpaired electrons while N_2 & O_2^{2-} are diamagnetic. In conversion of $O_2 \rightarrow O_2^+$ and $NO \rightarrow NO^+$, e^- from antibonding orbital ($CN^-, N_2, NO^+, C_2^{2-}$) is removed, thus bond order increases & bond length decreases.
33. Isoelectronic species has same bond order
- 34.



35. $\text{HF}_2^{(-)}$ exists in discrete form as salt, KHF_2 .

36. A water molecule can form 4 hydrogen bonds in ice.

37.



38. C_2^{2-} has one sigma and two pi bonds

39. B_2 has two pi electrons in bonding MO's, so it translates into one π -bond

40. During O_2^- formation, one electron is added to the antibonding molecular orbital.
 $\sigma^* 1s$ molecular orbital has only one nodal plane.

MATHEMATICS

41. $\sqrt{1 + \cos 2x} = \sqrt{2} \cos^{-1} |\cos x| \Rightarrow \cos x = x$ for $x \in \left(\frac{\pi}{2}, \pi\right)$

42. $x(x+1) \geq 0$ & $x^2 + x + 1 \leq 1 \Rightarrow x(x+1) = 0 \Rightarrow x = -1, 0$

43. $s_n = \sum \tan^{-1} \left(\frac{2}{n^2 + 2n + 1} \right) = \tan^{-1}(n+1) + \tan^{-1}(n+2) - \tan^{-1}1 - \tan^{-1}2$

$$s_\infty = \frac{\pi}{4} + \cot^{-1} 2$$

44. $\tan^{-1} \tan(x) = 3 - \frac{x}{2}$

No. of solutions is equal to no. of points of integer of $y = \tan^{-1}(\tan x)$ and $y = 3 - \frac{x}{2}$

45. $\cos^{-1} \cos 12 + \sin^{-1} \sin 12 = 0$

$$\therefore (k-2)x^2 + 8x + k + 4 > 0 \forall x \in R$$

$$\Rightarrow K - 2 > 0 \text{ \& } 64 - 4(K-2)(K+4) < 0$$

$$\Rightarrow K > 4$$

46. $A = [-1, 2), B = \{0, 1, 2\}$

47. $\frac{n^2 - 10n + 26}{2\sqrt{3}} < \sqrt{3}$

48. $\cos^{-1} x = \frac{\pi}{3}x + \frac{\pi}{6}$ Draw the graph

49. $s_n = \sin^{-1} \left[\frac{\sqrt{n} - \sqrt{n-1}}{\sqrt{n(n+1)}} \right] = \sum \tan^{-1} \frac{\sqrt{n} - \sqrt{n-1}}{1 + \sqrt{n}\sqrt{n-1}} = \sum (\tan^{-1} \sqrt{n} - \tan^{-1} \sqrt{n-1})$

$$s_{\infty} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

50. $(\sin^{-1} a)^2 = \frac{\pi}{4}, (\cos^{-1} b)^2 = \pi^2, (\sec^{-1} c)^2 = \pi^2, (\cos e^{-1} d)^2 = \frac{\pi^2}{4}$

51. $\sin^{-1} \sin 8 = 3\pi - 8, \tan^{-1}(\tan 10) = 10 - 3\pi, \cos^{-1} \cos 12 = 4\pi - 12$

$$\cot^{-1} \cot^6 = 6 - \pi, \sec^{-1} \sec 9 = 9 - 2\pi, \cos e c^{-1} \cos e c 7 = 7 - 2\pi$$

52. $\sin^{-1} x = \pm \frac{\pi}{2}, \sin^{-1} y = \pm \frac{\pi}{2}, \sin^{-1} z = \pm \frac{\pi}{2}$

53. $a - \frac{a^2}{3} + \frac{a^3}{9} \dots = 1 + b + b^2 + \dots$

54. $s_n = \sum_{r=1}^n (\tan^{-1}(r+1) - \tan^{-1} r)$

55. Domain = $\{-1\}$

$$\therefore \frac{\pi}{2} + \tan^{-1}(4 - K^2) > \frac{\pi}{2}$$

$$4 - K^2 > 0$$

56. $x^2 + x + 1 = \lambda x + 1 \Rightarrow x = 0, \lambda - 1$

$$-1 \leq \lambda x + 1 \leq 1 \Rightarrow \lambda \in [0, 1)$$

57. Take $x = y = z$

58. Draw the graph of $y = \cos^{-1} \cos x$ and $y = \frac{10-x}{10}$

59. $\sum_{r=1}^{\infty} \tan^{-1} \left(2^{2^{r+1}} + \frac{1}{2^2} \right) = \sum_{r=1}^{\infty} (\tan^{-1}(2^{r+1}) - \tan^{-1}(2^2)) = \cot^{-1} 2$

60. $2 \left\{ \left[\sin^{-1} \frac{\sqrt{5}}{3} - \cos^{-1} \frac{\sqrt{5}}{3} \right] \right\}$

$$= \sin \left\{ 2 \left(\frac{\pi}{2} - 2 \cos^{-1} \frac{\sqrt{5}}{3} \right) \right\}$$

$$= \sin \left(4 \cos^{-1} \left(\frac{\sqrt{5}}{3} \right) \right)$$

$$= \frac{8\sqrt{5}}{81}$$