MasterJEE

IIT-JEE | Medical | Foundations

Master JEE CLASSES

Kukatpally, Hyderabad.

JEE-ADVANCE-2014-P1-Model

Max.Marks:180

2014_PAPER-I

IMPORTANT INSTRUCTIONS:

1) This booklet is your Question Paper.
2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
3) Blank spaces are provided within this booklet for rough work.
4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
5) You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET:

6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
7) The ORS will be collected by the invigilator at the end of the examination.
8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work**.
9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
11) Darken the bubble **COMPLETELY**.
12) The correct way of darkening a bubble is as:
13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
14) Darken the bubbles ONLY IF you are sure of the answer. There is NO WAY to erase or "un-darken" a darkened bubble.

JEE-ADVANCE-2014-P1-Model

Time: 07:30 AM to 10:30 AM **IMPORTANT INSTRUCTIONS** **Max Marks: 180**

PHYSICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 10)	Questions with Multiple Correct Choice	3	0	10	30
Sec – II(Q.N : 11 – 20)	Questions with Integer Answer Type	3	0	10	30
Total				**20**	**60**

CHEMISTRY:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 30)	Questions with Multiple Correct Choice	3	0	10	30
Sec – II(Q.N : 31 – 40)	Questions with Integer Answer Type	3	0	10	30
Total				**20**	**60**

MATHEMATICS:

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 50)	Questions with Multiple Correct Choice	3	0	10	30
Sec – II(Q.N : 51 – 60)	Questions with Integer Answer Type	3	0	10	30
Total				**20**	**60**

space for rough work

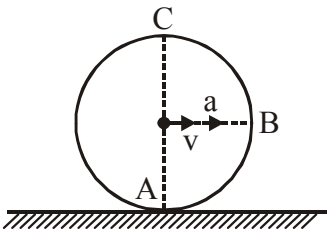
PHYSICS **MaxMarks:60**

SECTION-I

(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B),(C) and (D) out of which ONE or MORE THAN ONE are correct.

1. A wheel is rolling without slipping on a horizontal plane. At a certain instant, it has velocity 'v' and acceleration 'a' of centre of mass as shown in the figure. At that instant, acceleration of



'A' is point of wheel.

A) A is vertically upwards

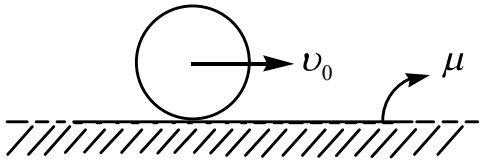
B) B may be vertically downwards

C) C cannot be horizontal

D) some point on the rim may be horizontal in forward direction.

2. A sphere is thrown along a rough surface as shown. Mass of the sphere is M and radius is R. If *t* is total time taken to start pure rolling after thrown, then:

space for rough work



A) speed of the sphere when it starts pure rolling is $\frac{5v_0}{7}$

B) work done by friction is greater in first half i.e. 0 to $\frac{t}{2}$ compare to later half

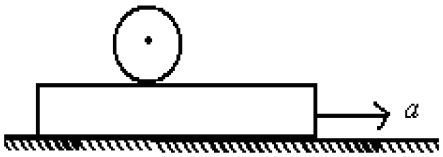
i.e., $\frac{t}{2}$ *to* t

C) work done by friction is zero after time t

D) the net work done on the sphere is $\frac{1}{4}Mv_0^2$

3. An uniform solid cylinder is placed on a horizontal board. The coefficient of friction between them is 0.3. The board is imparted a constant acceleration of $a = 6m/s^2$ in a horizontal direction at right angles of the cylindrical axis. Then (take $g = 10m/s^2$)

space for rough work



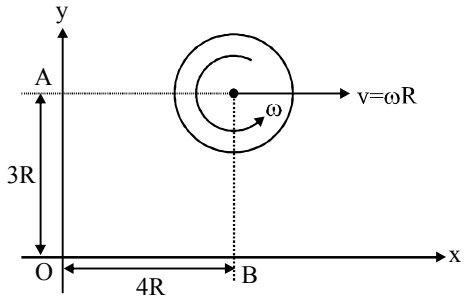
A) The acceleration of the cylinder axis in the absence of slipping will be 1.5 m/s^2.

B) The acceleration of the cylinder axis in the absence of slipping will be $2m/s^2$

C) The limiting value of 'a' for which there will be no slipping is $12m/s^2$

D) The limiting value of 'a' for which there will be no slipping is $9m/s^2$

4. A disc of mass m and radius R moves in the x-y plane as shown in the fig. The angular momentum of the disc at the instant shown is:



A) $\frac{5}{2}mR^2\omega$ about O

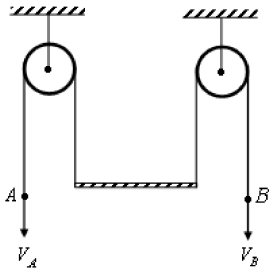
B) $\frac{7}{2}mR^2\omega$ about O

C) $\frac{1}{2}mR^2\omega$ about A

D) $4mR^2\omega$ about A

space for rough work

5. A thin uniform rod of mass 5 kg and length 1 m is held in horizontal position with the help of strings attached to ends of rod. Other ends of strings are held by some external agent. Now end A is pulled down with speed $V_A = 3t$ and end B is pulled down with speed $V_B = t$ where t is time in s. Choose the correct choice (s) at time $t = 0$.



A) Angular acceleration of rod is $2\,rad/s^2$

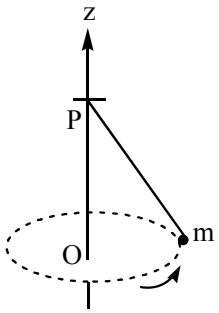
B) Tension in left string is $\frac{185}{6}N$

C) Acceleration of rod is $1\,m/s^2$

D) Tension in right string is $\frac{170}{3}N$

6. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the *x-y* plane with centre at *O*

and constant angular speed ω. If the angular momentum of the system calculated about *O* and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

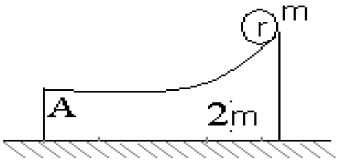
B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

7. A sphere of mass m and radius 'r' is released from rest from the top of a wedge of mass '2m' initially at rest on a smooth horizontal surface as shown. There is sufficient friction between the sphere and the wedge so that the sphere rolls on the wedge without slipping. The sphere leaves the wedge horizontally at A after descending through a vertical displacement h. Let V_1 ,V_2 and ω be the magnitudes of the velocity

of the centre of sphere, velocity of wedge and the angular velocity of the sphere at the instant the sphere leaves the wedge. Choose the correct statements.



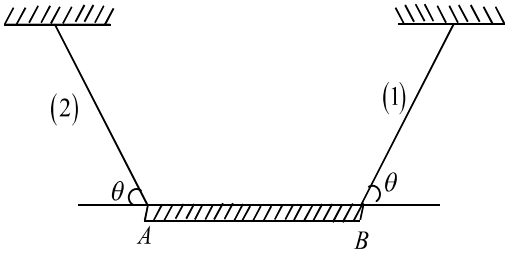
A) $V_1 + V_2 = r\omega$

B) Total mechanical energy of sphere and wedge system is conserved during the process

C) The ratio of total kinetic energy of the sphere and to the wedge at the instant the sphere leaves the wedge is 19:5

D) $V_1 = \sqrt{\frac{5gh}{6}}$

8. A uniform rod AB of length 'L' mass M is suspended by two massless identical threads. Initially the rod AB is in equilibrium in the horizontal position and the system is in vertical plane as shown in fig. Immediately after the thread (1) breaks, (take g is acceleration due to gravity)



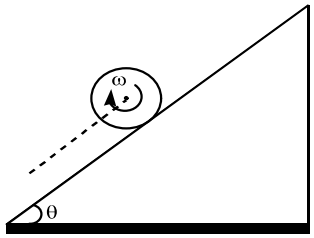
A) Tension in thread (2) is $\frac{Mg\sin\theta}{1+3\sin^2\theta}$

B) Angular acceleration of the rod about it's centre of mass is $\frac{6g\sin^2\theta}{L\left(1+3\sin^2\theta\right)}$

C) The end A of the rod has acceleration component in vertically upward direction

D) Centre of mass of the rod has horizontal acceleration component towards leftward

9. A solid sphere is given an angular velocity ω about a horizontal axis and kept on a rough fixed incline plane. Then choose the correct statement.



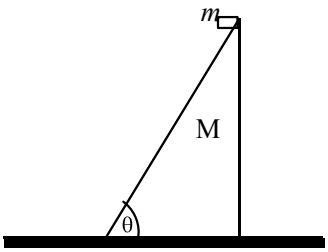
A) If $\mu = \tan\theta$, then the sphere will be in linear equilibrium for some time and after that pure rolling down the plane will start.

B) If $\mu = \tan\theta$, then the sphere will move up the plane and frictional force acting all the time will be $2mg \sin\theta$.

C) If $\mu = \frac{\tan\theta}{2}$, then there will never be pure rolling (consider inclined plane to be long enough.)

D) If incline plane is not fixed and it is on smooth horizontal surface then linear momentum of the system (wedge and sphere) can be conserved in horizontal direction.

10. Friction between the wedge shown and the horizontal floor is sufficient to prevent the wedge from sliding. The mass of the wedge is M and its angle of inclination is θ. A small block of mass *m* is just placed near the top of the wedge and released. Friction coefficient between the wedge and the block is μ.



A) If $\mu > \tan\theta$, the wedge will not topple for any value of m.

B) If $\mu < \tan\theta$, the wedge will topple for $m > \dfrac{M}{3\sin\theta(\sin\theta - \mu\cos\theta)}$.

C) The wedge will topple for any value of m irrespective of μ.

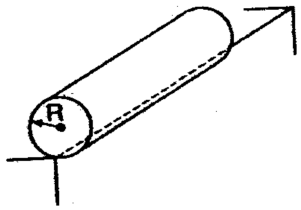
D) If $\mu < \tan\theta$, the wedge will not topple for $m > \dfrac{M}{3\sin\theta(\sin\theta - \mu\cos\theta)}$.

SECTION-II
(Integer Value Correct Type)

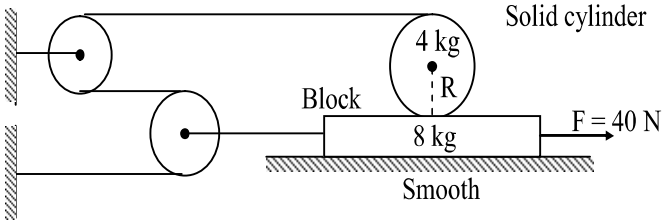
This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

11. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius R is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the horizontal edge are in same vertical plane shown in the figure. There is sufficient friction present at the edge, so that a very small

displacement causes the cylinder to roll off the edge without slipping. Then the ratio of the translational to rotational kinetic energies of the cylinder when its center of mass is in horizontal line with the edge is.....

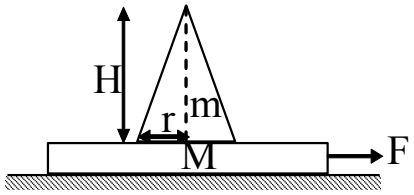


12. An ideal string is wrapped several times on a solid cylinder of mass 4 kg and radius 1 m. The pulleys are ideal and the surface between block and ground is smooth. Cylinder rolls without slipping on the block. If the torque acting on the cylinder is $\frac{10x}{9}$ N-m, then find the value of x.



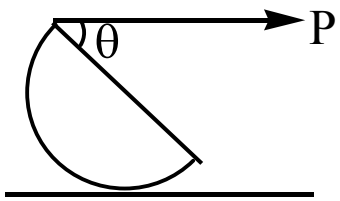
13. A uniform rod of length 2ℓ and mass m is suspended from one end by an inextensible vertical string and other end lies on smooth ground. The angle made by rod with vertical is $\theta = \sin^{-1}\left(1/\sqrt{3}\right)$. If N_1 and N_2 represents the contact force from ground on rod just before and just after cutting the string then find the ratio of N_1 / N_2.

14. A solid cone of mass (m = 1 kg) is placed on a plank of mass M = 1 kg which is placed on a smooth horizontal plane. The coefficient of friction between cone and the plank is 1/4. If a horizontal force F is applied on the plank, then find the maximum value of F for which the cone is in equilibrium with respect to plank.
(Take g = 10 m/s^2 and H/r = 20)

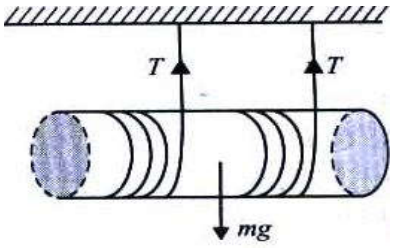


15. The solid semi-cylinder of mass *m* and radius R is rolled slowly through an angle θ by the horizontal force P. If the coefficient of static friction is μ_s, the angle θ at which

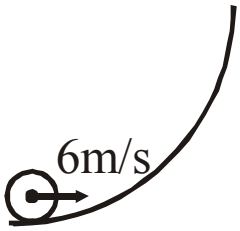
the cylinder begins to slip on the horizontal surface as P gradually increased is $\theta = \sin^{-1}\left(\frac{3\pi\mu_s}{n-3\pi\mu_s}\right)$ .Find 'n'.



16. A solid cylinder of mass m = 4 kg and radius R = 10 cm has two ropes wrapped around it, one near each end. The cylinder is held horizontally by fixing the two free ends of the cords to the hooks on the ceiling and both the cords are exactly vertical. The cylinder is released to fall under gravity. The linear acceleration of the cylinder is $\frac{n}{n+1}g$ then n = ?.

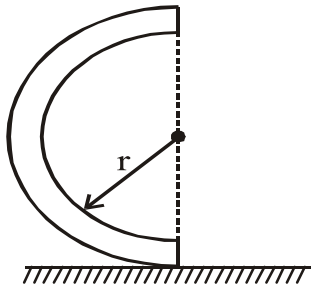


17. A disc of radius 0.1 m rolls without sliding on a horizontal surface with a velocity of 6 m/s. It then ascends a smooth continuous track as shown in figure. The height up to which it will ascend is $\frac{x}{5}$m. Value of x is? ($g = 10 m/s^2$)



18. A constant power is supplied to a rotating disc. Angular velocity (ω) of disc varies with number of rotations (n) made by the disc as $\omega \propto n^{1/x}$. Value of x is

19. A semicircular ring of mass m and radius r is released from rest in the position shown with its lower edge resting on a horizontal surface. The minimum coefficient of static friction μ_s which is necessary to prevent any initial slipping of the ring is $\frac{x}{10}$. Value of x is



20. A binary star consists of two starts A (mass $2.2M_s$) and B (mass $11M_s$), where M_s is the mass of the sun. They are separated by distance r and are rotating about their centre of mass, which is stationary. What is the ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass?

CHEMISTRY **Max Marks: 60**

SECTION-I
(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B) (C) and (D) out of which ONE or MORE are correct.

21. The correct propertie (s) about Boron and Aluminium is/are:

A) Maximum Covalency of Boron is 4 and that of aluminium is 6

B) Aluminium dissolves in dilute alkali and liberate H_2 but Boron does not

C) Oxide of Boron is acidic where as that of aluminium is amphoteric

D) Halides of Boron exist as monomers where as the halides other boron family members exist as dimers.

22. Identify the correct statement(s) regarding hydrogen gas

A) Unlike H_2, D_2 do not show spin isomerism.

B) Orthohydrogen differs from parahydrogen w.r.t their b.p., sp. Heat and thermal conductivity

C) As the temperature is increased from absolute zero some of the para form gradually changes to ortho form.

D) Parahydrogen has lower internal energy than orthohydrogen.

space for rough work

23. The correct statement(s) is/are

A) $I.E_1$ of Pb > $I.E_1$ of Sn

B) CO_2, SiO_2 & GeO_2 are acidic, SnO_2 & PbO_2 are amphoteric

C) SiF_6^{2-} is known but $SiCl_6^{2-}$ is not known

D) $C >> Si > Ge \simeq Sn$ (Catenation)

24. The correct statement(s) is/are:

A) The only solid bicarbonates known are those of the group-1 metals and of NH_4^+

B) Carbon monoxide is quantitatively estimated by I_2O_5

C) $CH_4 > SiH_4 > GeH_4 > PbH_4$ (Thermal stability)

D) Graphite > diamond > fullerenes (Thermal stability)

25. Which of the following methods can be used for the preparation of anhydrous aluminium chloride?

A) Heating $AlCl_3.6H_2O$

B) Heating a mixture of alumina and coke in a current of dry chlorine gas

C) Passing dry HCl gas over heated aluminium powder

D) Passing dry chlorine gas over heated aluminium

26. Select the correct statement about silicates?

A) Cyclic silicate having three silicon atoms contains six $Si-O-Si$ linkages

B) On an average over all 2½ oxygen atoms per tetrahedron unit are shared in double chain silicate

C) $(Si_2O_5)_n^{2n-}$ is the repeating unit or basic unit of double chain silicate

D) SiO_4^{4-} units polymerise to form silicate because Si atom has less tendency to form π-bond with oxygen

27. The correct statement(s) is/are

A) Does not neutralise dilute HNO_3 : SiO_2, PbO_2, CO

B) Reacts with HF : SiO_2, SnO

C) Solid at room temperature : $SiO_2, LiHCO_3$

D) May act as reducing agent : CO, SnO

28. The correct statements among the following are

A) H_2 is more rapidly adsorbed on solid surface than D_2.

B) H-H bond dissociation energy is greater than D-D bond dissociation energy.

C) Deuterated compound can be easily prepared by exchange reaction of deuterium with hydrogen using heavy water rather than D_2.

D) The bond length is shortest in T_2 among the isotopes of hydrogen.

29. $SiO_2 + NaOH \rightarrow A \xrightarrow{H_2F_2} B \xrightarrow{H_2O} C + D$

Hence

A) $A = Na_2SiO_3$

B) $B = SiF_4$

C) $C = H_2SiO_3; D = H_2SiF_6$

D) $C = F_2; D = H_2O_2$

30. The correct statement(s) is/are about carbides:

A) Be_2C and Al_4C_3 are methanides

B) B_4C and SiC are covalent

C) Interstitial carbides retain many of the properties of metals

D) Interstitial carbides are less denser and harder than pure metals

SECTION-II
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

31. Malonic acid $\xrightarrow[150^0C]{P_4O_{10}}$ Foul smell $(A)\xrightarrow{200^0C}(B)$+Carbon

The total number of sigma and Pi bonds present in the compound A are: .....

32. In cyclic silicate ion $Si_6O_{18}^{12-}$ the number of oxygen atoms shared are: .....

33. The number of oxygen atoms not acting as bridges (Si-O-Si) in the mineral kinoite

$Ca_2Cu_2Si_3O_{10}.2H_2O$.

34. How many of the following species have sp^2 hybrid atoms?

Diamond, Graphite, C_{6O},$(SiH_3)_3N$; AlF_3; $AlCl_3(monomer)$; $Al_2(CH_3)_6$; SiC,

H_3BO_3, layered BN, Borazine.

35. How many of the following compounds are amphoteric?

$CO, Al_2O_3; SiO_2 : GeO, SnO, SnO_2, PbO, PbO_2, CO_2$

36. How many of the following reactions produce a paramagnetic gas?

$AlH_3 + H_2O \rightarrow$

$Al_2O_3 + C + N_2 \rightarrow$

$AlN + H_2O \rightarrow$

$HCOOH \xrightarrow[-H_2O]{Conc.H_2SO_4}$

$CH_2(COOH)_2 \xrightarrow[-H_2O]{Conc.H_2SO_4}$

$CO + I_2O_5 \rightarrow$

$Fe_2O_3 + C \rightarrow$

37. In how many of the following reactions hydrogen gas is produced as one of the product.

(i) Electrolysis of acidulated water

(ii) Passing steam over red hot coke.

(iii) Reaction of aluminium with sodium hydroxide

(iv) Reaction of Zinc with dil HCl

(v) Pyrolysis of hydrocarbons

(vi) Reaction of sodium amalgam with water.

(vii) By dissolving hydrolith in water.

(viii) By heating a mixture of steam and hydro carbons

(ix) Electrolysis of brine.

38. How many of them are 3-D silicates?

Feldspars, Ultra marines, Zeolites, Pyroxenes, Amphiboles, Beryl, Zircon, Asbestos

39. X = Oxidation, state of carbon in CO

Y = Oxidation state of carbon in $COCl_2$

They $y - x$ is:

40. How many of the following reactions lead to the formation of CO as one of the products?

$(COOH)_2 \xrightarrow[Conc.H_2SO_4]{\Delta};$ $HCOOH \xrightarrow[Conc.H_2SO_4]{\Delta};$

$Al_2O_3 + C + Cl_2 \rightarrow$ $K_4\left[Fe(CN)_6\right] \xrightarrow[H_2SO_4]{dilute};$

$Ni(CO)_4 \xrightarrow{\Delta}$ $B_2O_3 + C \rightarrow$

SECTION-I
(One or More options Correct Type)

This section contains 10 multiple choice equations. Each question has four choices (A) (B) (C) and (D) out of which ONE or MORE are correct.

41. Let $f(x)=g(x)\left(\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\right)$ where $g(x)$ is a continuous function, then $\lim_{x\to 0} f(x)$ exits, if

A) $g(x)$ is any polynomial B) $g(x)=x^2$

C) $g(x)=\sin x$ D) $g(x)=2+3x+4x^3$

42. Let $f:R\to R$ be a function given by

$f(x)=\begin{cases} 5x & if \quad x\in Q \\ x^2+6 & if \quad x\in R-Q \end{cases}$ where Q is set of rational numbers then

A) $f(x)$ is continuous for $x\in R$

B) $f(x)$ is continuous at finite number of values of x

C) $f(x)$ is continuous at $x=2$

D) $f(x)$ is continuous at $x=0$

43. Let $f(x)$ be a non constant polynomial function satisfying the relation $f(x).f(y)=f(x)+f(y)+f(xy)-2$ for all real x and y and $f(0)=1$, suppose $f(4)=65$. Then, which of the following statement(s) is/are true

A) $f'(x)$ is a polynomial of degree 2

B) Roots of $f'(x)=2x+1$ are real

C) $xf'(x)=3[f(x)-1]$

D) $f'(-1)=3$

44. Consider the function $f:R\to R$ defined by $f(x)=[2\sin x]$ where $[\bullet]$ is G.I.F, then which of the following is/are true

A) $f(x)$ is continuous at $x=-\frac{\pi}{2}$

B) $f(x)$ is not differentiable at $x=-\frac{3\pi}{2}$

C) $f(x)$ is continuous at $x=-\pi$

D) $f(x)$ is not differentiable at $x=\frac{\pi}{2}$

45. Let $f:\left[0,\frac{\pi}{2}\right]\to R$ be defined by $f(x)=\max\left\{\sin x,\cos x,\frac{3}{4}\right\}$ then which of the following is not true.

A) $f(x)$ is continuous for all real $x\in[0,\pi/2]$

B) $f(x)$ is not differentiable at $x=\frac{\pi}{4}$

C) Number of points where $f(x)$ is not differentiable is 2

D) Number of points where $f(x)$ is not continuous is 3

46. If $f(x)=\begin{cases}\sin\left(\frac{\pi}{2}(x-[x])\right) & x<5\\ 5(b-1) & x=5\\ ab^2\frac{\left|x^2-11x+24\right|}{x-3} & x>5\end{cases}$ is continuous at $x=5$, $a,b\in R$, then ($[\bullet]$ denotes G.I.F)

A) $a+b=\frac{773}{540}$ B) $a-b=\frac{523}{540}$ C) $a<b$ D) $\frac{a}{b}=\frac{125}{648}$

47. Let $f(x)=\begin{cases} x\left[\frac{1}{x}\right]+x[x] & \text{if } x\neq 0 \\ 0 & \text{if } x=0 \end{cases}$ (where $[x]$ denotes the greatest integer function).

Then the correct statement(s) is/are

A) limit of $f(x)$ exists for $x=-1$

B) $f(x)$ has discontinuity at $x=1$

C) $f(x)$ has discontinuity at $x=2$

D) $f(x)$ is discontinuous at all positive integers

48. The set of points of differentiability of the function $f(x)=\begin{cases} x^n \sin\left(1/x\right) & x\neq 0 (n\geq 2) \\ 0 & x=0 \end{cases}$

contains (Where R is set of real numbers)

A) $(-\infty,0)$ B) $R-\{0\}$ C) $(0,\infty)$ D) R

49. Suppose that $f:R\to R$ is continuous and satisfying the equation $f(x).f(f(x))=1$, for all real x. Let $f(1000)=999$, then which of the following is/are true

A) $f(500)=\frac{1}{500}$ B) $f(199)=\frac{1}{199}$

C) $f(x)=\frac{1}{x}\quad \forall x\in R-\{0\}$ D) $f(1999)=\frac{1}{1999}$

space for rough work

50. Let $f:R\to\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ be a function defined by $f(x)=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then which of the following is/are true

A) $f(x)$ is not differentiable at $x=-1$ and $x=1$

B) $f(x)$ is continuous for all real x

C) $f(x)=2\tan^{-1}x\ \forall x\in R$

D) $f(x)$ is differentiable $\forall x\in R$

SECTION-II
(Integer Value Correct Type)

This section contains 10 questions. The answer to each question is a single digit integer, ranging from 0 to 9 (both inclusive).

51. The number of points where $f(x)=\left[\frac{x}{3}\right]$ in $x\in(0,30)$ is discontinuous is ($[\bullet]$ is G.I.F)

52. Let f be a function defined on $(-1,1)$ as $f(x)=\begin{cases}\frac{2^{\sin^{-1}x}-2^{\tan^{-1}x}}{2^{\tan x}-2^{\sin x}} & \text{if } x\neq 0\\ K & \text{if } x=0\end{cases}$ then the value of K such that $f(x)$ is continuous at $x=0$ is

53. If $f(x)=\begin{cases}\left(\sin\frac{2x^2}{a}+\cos\left(\frac{3x}{b}\right)\right)^{\frac{ab}{x^2}} & x\neq 0\\ e^3 & x=0\end{cases}$ is continuous at $x=0$ where $a,b\in R-\{0\}$ and K is minimum value of 'a' then $|4K|$ is equal to

54. The function $f(x)$ is discontinuous only at $x=0$ such that $f^2(x)=1\ \ \forall x\in R$. The total number of such functions is

55. Number of points where the function $f(x)=\max\left(|\tan x|,\cos|x|\right)$ is non differentiable in the interval $(-\pi,\pi)$ is

56. If $f(x)=\begin{cases}x+2 & x<0\\ -x^2-2 & 0\le x<1\\ x & x\ge 1\end{cases}$ then the number of points of discontinuity of $|f(x)|$ is

57. $f(x)=a|\sin x|+be^{|x|}+c|x|^3$ (a, b, c $\in$ R). If $f(x)$ is differentiable at $x=0$, then the value of $a+b$ is equal to

58. Let $g(x)=\frac{(x-1)^n}{\log_e\cos^m(x-1)};0<x<2$ where $m\neq 0,\text{n}>0$ are integer. Let P be the left hand derivative of $|x-1|$ at $x=1$ and $\lim\limits_{x\to 1^+}g(x)=P$, then $m+n=$

59. Let $f:R\to R$ be a continuous function satisfying $f(0)=1$ and $f(2x)-f(x)=x$ then the value of $f(3)$ is

60. $f(x)=\begin{cases}\dfrac{1-\cos 3x\cos 9x\cos 27x....\cos\left(3^n x\right)}{1-\cos\dfrac{x}{3}\cos\dfrac{x}{9}\cos\dfrac{x}{27}.....\cos\left(\dfrac{x}{3^n}\right)} & if\ x\neq 0\\ 3^{10} & if\ x=0\end{cases}$ If $f(x)$ is continuous at $x=0$, then the value of n is $(n\in N)$

space for rough work

Master JEE CLASSES

Kukatpally, Hyderabad.

JEE-ADVANCE-2014-P1-Model **MAX MARKS : 180**

KEY SHEET
2014-P1

PHYSICS

1	**ABCD**	2	**ABC**	3	**BD**	4	**AC**	5	**AB**
6	**C**	7	**ABCD**	8	**ABD**	9	**AD**	10	**AB**
11	**6**	12	**8**	13	**1**	14	**4**	15	**4**
16	**2**	17	**9**	18	**3**	19	**4**	20	**6**

CHEMISTRY

21	**AC**	22	**BCD**	23	**ABCD**	24	**ABCD**	25	**BCD**
26	**BCD**	27	**ABD**	28	**AC**	29	**ABC**	30	**ABC**
31	**8**	32	**6**	33	**8**	34	**7**	35	**5**
36	**0**	37	**9**	38	**3**	39	**2**	40	**5**

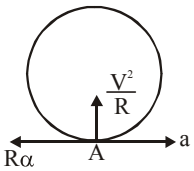
MATHS

41	**BC**	42	**BC**	43	**ABCD**	44	**ABD**	45	**BD**
46	**ACD**	47	**ABCD**	48	**ABCD**	49	**AB**	50	**AB**
51	**9**	52	**1**	53	**1**	54	**6**	55	**4**
56	**1**	57	**0**	58	**4**	59	**4**	60	**4**

SOLUTIONS

PHYSICS

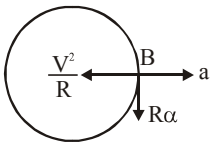
1. At A :



for a rolling wheel, a = Rα

∴ (A) is correct.

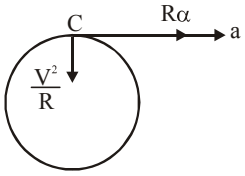
At B :



If $\frac{V^2}{R} = a$ then a_B may be vertically downwards

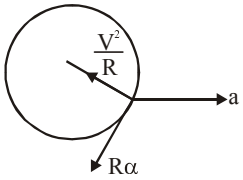
∴ (B) is correct.

At C :



∴ (C) is correct.

Consider this



∴ (D) is correct.

2. $\mu mgR = \frac{2}{5}mR^2\alpha$

$\alpha = \frac{5}{2}\frac{\mu g}{R}$

$\omega = \alpha t$; $v = v_0 - \mu g t$

For pure rolling, $V = R\omega$

$V_0 - \mu g t = R\alpha t$

$t = \frac{2}{7}\frac{V_0}{\mu g}$

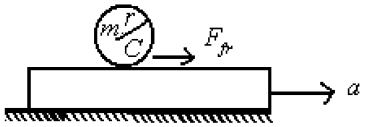
$\therefore\ V = V_0 - \frac{2}{7}V_0 = \frac{5}{7}V_0$

3. $ma_c = F_{fr}$

$I \propto = rF_{fr} = rma_c$

$a - a_c = \alpha r = \frac{rma_c}{I}r$

or, $a - a_c = \frac{mr^2 a_c}{\frac{mr^2}{2}} = 2a_c$



or, $3a_c = a,\ a_c = \frac{a}{3} = \frac{6}{3} = 2m/s^2$

So, $f_{FR} = \frac{ma}{3}$ so maximum value of this force is equal μmg

Hence $\mu mg = \frac{ma_{lim}}{3}$ or, $a_{\lim} = 3\mu g = 3 \times 0.3 \times 10 = 9m/s^2$

4. (a) m (R ω) (3R) – $\frac{MR^2}{2}\omega$

$= \frac{5}{2}mR^2\ \omega$

(c) Conceptual

5. $a_C + \frac{L}{2}\alpha = a_1$

$a_C - \frac{L}{2}\alpha = a_2$

$\Rightarrow\ \alpha = \frac{(a_1 - a_2)}{L} = 2\,rad/s^2$

$\alpha = \frac{(a_1 + a_2)}{2} = 2m/s^2$

$T_1 + T_2 - mg = ma_c$

$$(T_1 - T_2)\frac{L}{2} - \frac{mL^2}{12}\alpha$$

$$\Rightarrow T_1 = 185/6\ N \text{ and } T_2 = 175/5\ N$$

6. $\vec{L} = m\left(\vec{r} - \vec{r}_O\right) \times \vec{v}$

$$\vec{L}_O = m\left(\vec{r} - \vec{r}_O\right) \times \vec{v} = mr\vec{e}_r \times mr\vec{e}_t = m\omega r^2 \hat{z}$$

$$\vec{L} = m\left(\vec{r} - \vec{r}_P\right) \times \vec{v} = m\left(r\vec{e}_r - h\hat{z}\right) \times mr\vec{e}_t = mh\omega r\vec{e}_r + m\omega r^2 \hat{z}$$

7. $V_1 = 2V_2 \quad i \quad rw - V_1 = V_2$ and by conservation of energy

$$mgh = \frac{1}{2}mV_1^2 + \frac{1}{2}2mV_2^2 + \frac{1}{2}\frac{2}{5}mr^2w^2$$

$$\Rightarrow V_1 = \sqrt{\frac{5gh}{6}}$$

Non-conducing of mass M and radius R

8. $T\cos\theta = ma_{cm}x$

$$Mg - T\sin\theta = ma_{cm}y$$

$$T\sin\theta L/2 = I_{cm}\alpha$$

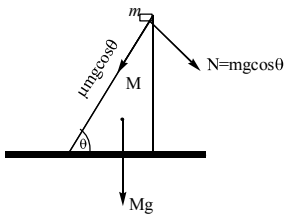
And apply thread constrained along length of the thread acceleration is zero.

$$\tau = \frac{mg\sin\theta}{1 + 3\sin^2\theta}$$

$$\alpha = \frac{6g\sin^2\theta}{L\left(1 + 3\sin^2\theta\right)}$$

9. Due to torque of friction about CM, ω eventually, decreases to zero, initially there is no translation. Friction is sufficient for pure rolling therefore after sometime pure rolling begin. There is no external force in x direction therefore momentum is conserved along x direction.

10.



$$N\sin\theta h > Mg\frac{b}{3} + \mu mg\cos\theta h\cos\theta$$

$$mg\cos\theta.\sin\theta h > Mg\frac{b}{3} + \mu mgh\cos^2\theta$$

$$3m\sin\theta\left(\sin\theta - \mu\cos\theta\right) > M$$

$$m > \frac{M}{3\sin\theta\left(\sin\theta - \mu\cos\theta\right)}$$

11. from C.O.M.E, $\frac{\upsilon^2}{R} = \frac{4}{3}g(1-\cos\theta)$

From dynamics of circular motion $mg\ \cos\theta = \frac{m\upsilon^2}{R}$

On solving above eqs. $\theta = \cos^{-1}\frac{4}{7}$

$$K_R = \frac{mgR}{7}$$

When the cylinder loses its constant i.e., the frictional force vanishes, also torque due to gravitational force is zero. Hence its angular velocity is constant and rotational kinetic energy becomes constant, while its translational kinetic energy increases.

Applying conservation of energy at position (1) and position (3)

Decrease in gravitational P.E. = Gain in rotational K.E. + translational K.E.

$$K_T = \left(mgR\right) - \frac{mgR}{7} = \frac{6}{7}mgR$$

From (3) and (4), we have

$$\frac{K_T}{K_R} = \frac{\frac{6}{7}mgR}{\frac{mgR}{7}} \text{ or } \frac{K_T}{K_R} = 6$$

$$mgR + 0 = mgR\cos\theta + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

But $\omega = \frac{v}{R}$

And $I = \frac{1}{2}mR^2$

Therefore, $mgR = mgR\cos\theta + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(v^2/R^2\right) + \frac{1}{2}mv^2$

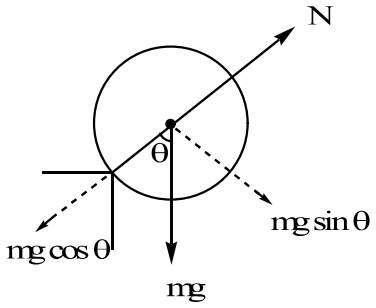
Or $\frac{v^2}{R} = \frac{4}{3} g(1-\cos\theta)$ ..... (1)

When the cylinder leaves the constant, normal reaction N=0 and $\theta = \theta_c$. Hence

$$mg\cos\theta_c = \frac{mv^2}{R}$$

Or $\frac{v^2}{R} = g\cos\theta_c$ ..... (2)

From eqns. (1) and (2),



$$\frac{4}{3} g(1-\cos\theta_c) = g\cos\theta_c$$

Or $\cos\theta_c = \frac{4}{7}$

Or $\theta_c = \cos^{-1}\left(\frac{4}{7}\right)$

All the time it leaves the contact,

$\cos\theta = \cos\theta_c = 4/7$,

On substituting it in eqn. (1), we get

$$\therefore \qquad v = \sqrt{\frac{v}{2} gR\left(1-\frac{4}{7}\right)} = \sqrt{\frac{4}{7} gR}$$

Therefore, speed of CM of cylinder just before it leaves the constant is $\sqrt{\frac{4}{7} gR}$.

(c) At the moment, when the cylinder leaves the constant,

$$v = \sqrt{\frac{4}{7} gR}$$

Therefore, rotational kinetic energy,

$$K_R = \frac{1}{2} I\omega^2$$

Or $K_R = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right) = \frac{1}{4}mv^2$

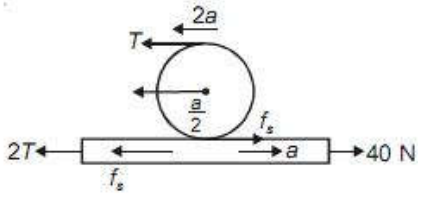
12. FBD of cylinder and block are as shown by Newton's laws

$40 - 2T - f_s = 8a$ (1)

$T - f_s = 4a/2$ (2)

Subtracting equation (2) from equation (1)

$40 - 3T = 6a$



$T = (40 - 6a)/3$

Also, by $\tau = I\alpha$, we get

$T \times R + f_s \times R = I\frac{3a}{2R}$

$\Rightarrow \quad T + f_s = 3Ia/2R^2$

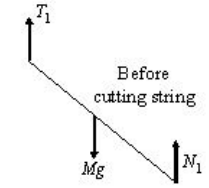
$\Rightarrow (40 - 6a)/3 + (40 - 12a)/3 = 3Ia/2R^2 \Rightarrow a = 80/(18 + 9I/2R^2)$

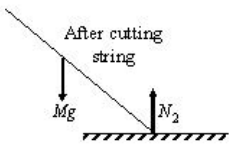
As $\tau = I\alpha = I \times (3a/2R)$

$\therefore \quad \tau = I\alpha = I \times (3a/2R)$

$\therefore \quad \tau = \frac{3I}{2R}\frac{80}{\left(18 + \frac{9I}{2R^2}\right)} \Rightarrow \tau = \frac{3 \times 80}{18 + 9} = \frac{80}{9}\text{N} - \text{m}$

13. Before cutting string





$N_1 = T = Mg/2$

After cutting string

$Mg - N_2 = Ma \qquad (i)$

$N_2 \ell \sin\theta = Ma(2\ell)^2/12 \qquad (ii)$

By constraint that lower end will have acceleration only in horizontal direction.

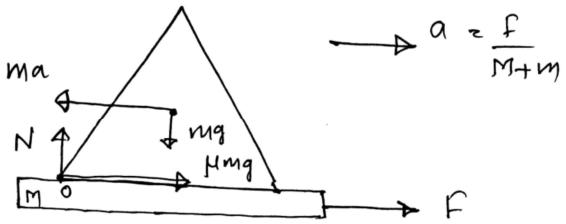
$\ell\,\alpha \sin\theta = a \qquad (iii)$

from (i), (ii) and (iii)

$\Rightarrow N_2 = Mg/(3\sin^2\theta + 1)$

$\Rightarrow N_1/N_2 = (3\sin^2\theta + 1)/2 = 1$

14.



τ about O=0

$ma\dfrac{H}{4} = mgr$

$\dfrac{F}{M+m}\cdot\dfrac{H}{4} = gr$

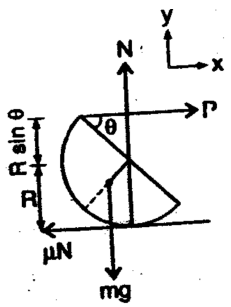
$F = 4N$

15. $P = \mu_s N$

$N = mg$

$\sum \tau = P(\mathrm{R} + \mathrm{R}\sin\theta) - \mathrm{mg}\dfrac{4R}{3\pi}\sin\theta = 0$

Also $\sin\theta = \dfrac{3\pi\mu_s}{4 - 3\pi\mu_s}$



16. $mg - 2T = Ma$

$2TR = IX = \frac{mr^2}{2}\left[\frac{a}{R}\right]$

$T = \frac{Ma}{4}$ And $a = \frac{2a}{3}$

17. Let m be the mass of the disc. Then translational energy of the disc is

$K_T = \frac{1}{2}mv^2$ When it ascends on a smooth track its rotational kinetic energy will remain same while translational kinetic energy will go on decreasing. At highest point.

$K_T = mgh$ or $\frac{1}{2}mv^2 = mgh$ or $h = \frac{v^2}{2g} = \frac{(6)^2}{2\times 19} = 1.8\,m$.

18. Comparing this with linear motion,

Power P = F.v

We have $P = \tau.\omega$ or $I\cdot\left(\omega\frac{d\omega}{d\theta}\right)\cdot\omega = P$

or $\omega^2 d\omega = \frac{P}{\alpha}d\theta$ on integration, we find that $\omega \propto \theta^{1/3}$

or $\omega \propto (n)^{1/3}$.

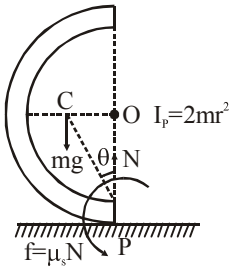
19. Centre of mass of semicircular ring lies at a distance of $\frac{2r}{\pi}$ from the center.

$OC = \frac{2r}{\pi}$, $OP = r$

Hence, $CP = \sqrt{(OC)^2 + (OP)^2}$

or CP = 1.185r.

If there is no slipping, then the instantaneous axis of rotation passes through P.



Angular acceleration of ring about P is

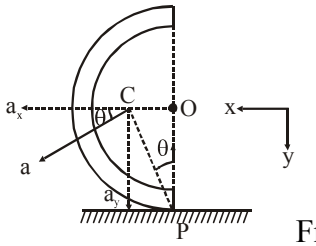
$\alpha = \frac{\tau_p}{I_p} = \frac{(mg)(CO)}{(2mr^2)}$

$$= \frac{(mg)\left(\frac{2r}{\pi}\right)}{(2mr^2)} = \frac{g}{\pi r} \quad \ldots(i)$$

Acceleration of center of mass,

$a = (CP)(\alpha)$

$$= (1.185r)\left(\frac{g}{\pi r}\right) = \left(\frac{1.185}{\pi}\right)g \quad \ldots(ii)$$



From Newton's law,

$\Sigma F_y = ma_y$

or $mg - N = ma_y$

or $mg - N = ma \sin\theta$

or $$mg - N = m\left(\frac{1.185}{\pi}\right)g\left(\frac{2r}{\pi}\right)\left(\frac{1}{1.185r}\right)$$

$$= \frac{2}{\pi^2}mg = 0.2\,mg$$

or $N = mg - 0.2mg$

or $N = 0.8\ mg \quad \ldots(iii)$

$\Sigma F_x = ma_x$

or $\mu_s N = (m)(a\cos\theta)$

or $$\mu_s(0.8mg) = (m)\left(\frac{1.185g}{\pi}\right)\left(\frac{r}{1.185r}\right)$$

or $\mu_s = 0.398$

Therefore, minimum value of static friction is 0.398.

20. $$I_{system} = m_A r_A^2 + m_B r_B^2 = \left[m_A m_B / (m_A + m_B)\right] r^2$$

$$L_{system} = I_{system}\omega = \left[m_A m_B / (m_A + m_B)\right]\omega r^2$$

$$I_B = m_B r_B^2 \left[m_B m_A^2 / (m_A + m_B)^2\right] r^2$$

$$L_B = I_B\omega = \left[m_B m_A^2 / (m_A + m_B)^2\right]\omega r^2$$ SS

21. (b) Boron liberates hydrogen with alkali.

22. Both H_2 and D_2 show spin isomerism. Ortho-H_2 has more m.pt and b.pt than Para-H_2 both increase in temperature para convertes into Ortho. Due to opposite spins of protons Para H_2 has lesser internal energy than Ortho-H_2

23. (a) Lanthanide contraction.
    (c) Due to large size of chlorine.

24. (b) $5CO + I_2O_5 \rightarrow 5CO_2 + I_2$

25. Salts of aluminium suffer hydrolysis on heating.

26. Concept of silicates.
    (d) Silicon has less tendency to form multiple bonds with oxygen.

27. (a) SiO_2 is acidic, PbO_2 is amphoteric and CO is neutral.
    (c) $LiHCO_3$ cannot exist in solid form.

28. Due to more rate of diffusion. H_2 will be adsorbed rapidly than D_2 on solid surface. Since O-D bond is polar while D-D bond is non-polar. So, exchange reactions with D_2O is eaiser. Atomic sizes of H_2, D_2 and T_2 are same.

29. $SiO_2 + NaOH \rightarrow Na_2SiO_3 \xrightarrow{H_2F_2} SiF_4 \xrightarrow{H_2O} H_2SiO_3 + H_2SiF_6$

30. Interstitial carbides are more denser and harder than pure metals.

31. A is C_3O_2 $O = C = C = C = O$

32. In cyclic silicate ion $Si_6O_{18}^{12-}$ the number of oxygen atoms shared is 6.

33. The number of monovalent oxygen atoms are present in the mineral kinoite $Ca_2Cu_2Si_3O_{10}.2H_2O$ is 8.

34. Graphite, $C_{6O}, (SiH_3)_3N; AlCl_3(monomer), H_3BO_3,$ layered BN, $B_3N_3H_6$ (Borazine) contain sp^2 hybrid atoms.

35. $Al_2O_3, SnO, SnO_2, PbO, PbO_2$

36. $AlH_3 + 3H_2O \rightarrow Al(OH)_3 + 3H_2$

    $Al_2O_3 + 3C + N_2 \rightarrow 2AlN + 3CO$

    $AlN + 3H_2O \rightarrow Al(OH)_3 + NH_3$

$HCOOH \rightarrow CO$

$3CH_2(COOH)_2 + P_4O_{10} \rightarrow 3C_3O_2 + 4H_3PO_4$

$5CO + I_2O_5 \rightarrow I_2 + 5CO_2$

$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO$

$CH_2(COOH)_2 \xrightarrow[-H_2O]{P_4O_{10}}$

37. All give Hydrogen gas.

38. Feldspars, Ultra marines, Zeolites

39. X = +2, Y = +4.

40. $(COOH)_2 \xrightarrow[-H_2O]{Conc.\ H_2SO_4} CO + CO_2$

$HCOOH \xrightarrow[-H_2O]{Conc.\ H_2SO_4} CO$

$Al_2O_3 + 3C + N_2 \rightarrow 2AlN + 3CO$

$K_4\left[Fe(CN)_6\right] + 3H_2SO_4(dilute) \rightarrow 2K_2SO_4 + FeSO + 6HCN$

$Ni(CO)_4 \xrightarrow{180^oC} Ni + 4CO$

$2B_2O_3 + 7C \rightarrow B_4C + 6CO$

MATHS

41. $\lim\limits_{x\to 0^+} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} = 1$ and $\lim\limits_{x\to 0^-} \dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} = -1$, $\lim\limits_{x\to 0} f(x)$ exists, if $\lim\limits_{x\to 0} g(x) = 0$

42. $f(x)$ is continuous at 'x' satisfying $5x = x^2 + 6$ i ,e $x = 2, 3$

43. Given $f(x).f(y) = f(x) + f(y) + f(xy) - 2 \forall x, y \in R$

Put $x = y = 1$ then $f(1)^2 - 3f(1) + 2 = 0$

$\Rightarrow f(1) = 1 (or)\ f(1) = 2$

If $f(1) = 1$ then $f(x) = 1 \forall x \in R$.

A contradiction ($\because$ degree of $f(x)$ is positive)

$\therefore f(1) \neq 1$, hence $f(1) = 2$

Replace 'y' with $\dfrac{1}{x}$ then $f(x).f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right) (\because f(1) = 2)$

$\therefore f(x)$ must be in the form $x^n+1(or)x^n-1$.

$\because f(4)=65, f(x)=x^3+1 \Rightarrow f^1(x)=3x^2$

44. Plotting the graph we get points of discontinuity at $x=\frac{-3\pi}{2},-\pi,0,\frac{\pi}{2}$ and is continuous at $x=\frac{-\pi}{2}$, $f(x)$ is not differentiable at $x=\frac{-3\pi}{2},\frac{\pi}{2}$

45. $f(x)=\begin{cases}\cos x & 0\le x\le \cos^{-1}(3/4)\\ 3/4 & \cos^{-1}(3/4)<x<\sin^{-1}(3/4)\\ \sin x & \sin^{-1}(3/4)\le x\le \pi/2\end{cases}$

$f(x)$ is continuous for all x and is not differentiable at 2 points

46. $\lim\limits_{x\to 5^-} f(x)=\lim\limits_{x\to 5^+} f(x)=f(5)$

$1=5(b-1)=3ab^2 \Rightarrow a=\frac{25}{108}, b=\frac{6}{5}$

47. $f(1^+)=\lim\limits_{x\to 1^+}\left(x\left[\frac{1}{x}\right]+x[x]\right)$

$=\lim\limits_{x\to 1^-}(x(0)+x(1))$

$=1$

$f(1^-)=\lim\limits_{x\to 1^-}\left(x\left[\frac{1}{x}\right]+x[x]\right)$

$=\lim\limits_{x\to 1^-}(x(1)+x(0))$

$=1$

$f(2^+)=\lim\limits_{x\to 2^+}\left(x\left[\frac{1}{x}\right]+x[x]\right)$

$=\lim\limits_{x\to 2^+}(x(0)+x(2))$

$=4$

$f(2^-)=\lim\limits_{x\to 2^-}\left(x\left[\frac{1}{x}\right]+x[x]\right)$

$=\lim\limits_{x\to 2^-}(x(0)+x(1))$

$=2$

48. Clearly for $x\neq 0$ $f(x)$ is differentiable.

At the point $x=0$,

$$f^1(0)=\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$

$$=\lim_{x\to 0}\frac{x^n\sin(1/x)}{x}$$

$$=\lim_{x\to 0}x^{n-1}\sin(1/x)=0$$

f is differentiable at $x=0$ also, $\therefore f$ is differentiable $\forall x$

49. $f(1000)f(f(1000))=1$

$\Rightarrow f(1000)f(999)=1$

$\Rightarrow 999f(999)=1$

$\therefore f(999)=\frac{1}{999}$

The numbers 999 and $\frac{1}{999}$ are in the range of f.

Hence, by intermediate value property of continuous function, function takes all values between 999 and $\frac{1}{999}$, then there exists

$\alpha\in\left(\frac{1}{999},999\right)$ such that $f(\alpha)=500$

Then $f(\alpha)f(f(\alpha))=1\Rightarrow f(500)=\frac{1}{500}$

Similarly, $199\in\left(\frac{1}{999},999\right)$, thus $f(199)=\frac{1}{999}$

But there is nothing to show that 1999 lies in the range of f. Thus (D) is not correct and (C) is also incorrect.

50. $\sin^{-1}\left(\frac{2x}{1+x^2}\right)=\begin{cases}-\pi-2\tan^{-1}x & if \quad x<-1\\ 2\tan^{-1}x & if \quad -1\le x\le 1\\ \pi-2\tan^{-1}x & if \quad x>1\end{cases}$

$f(x)$ is continuous for all x, and is not differentiable $x=-1,1$

51. $f(x)=\left[\frac{x}{3}\right]$ is not continuous at $x=3,6,9,12,15,18,21,24,27$

52. $K=\lim_{x\to 0}f(x)=\lim_{x\to 0}\frac{2^{\sin^{-1}x}-2^{\tan^{-1}x}}{2^{\tan x}-2^{\sin x}}$

$$= \lim_{x\to 0} \frac{2^{\sin^{-1}x-\tan^{-1}x}-1}{2^{\tan x-\sin x}-1} \times \frac{2^{\tan^{-1}x}}{2^{\sin x}}$$

$$= \lim_{x\to 0} \frac{2^{\sin^{-1}x-\tan^{-1}x}-1}{\sin^{-1}x-\tan^{-1}x} \times \frac{\sin^{-1}x-\tan^{-1}x}{\frac{2^{\tan x-\sin x}-1}{\tan x-\sin x}\times(\tan x-\sin x)} \times \frac{2^{\tan^{-1}x}}{2^{\sin x}} = 1$$

53. $\lim_{x\to 0} f(x) = f(0)$

$$\lim_{x\to 0} \frac{ab}{x^2}\left(\sin\frac{2x^2}{a}+\cos\frac{3x}{b}-1\right)=3$$

$$ab\left(\frac{2}{a}-\frac{9}{2b^2}\right)=3$$

$$4b^2-6b-9a=0$$

$\because$ b is real, $D\geq 0 \Rightarrow a \geq -1/4$

54. (i) $f(x)=1 \quad x\geq 0$

$=-1 \quad x<0$

(ii) $f(x)=-1 \quad x\geq 0$

$=1 \quad x<0$

(iii) $f(x)=1 \quad x>0$

$=-1 \quad x\leq 0$

(iv) $f(x)=-1 \quad x>0$

$=1 \quad x\leq 0$

(v) $f(x)=1 \quad \forall x\neq 0$

$=-1 \quad x=0$

(vi) $f(x)=-1 \quad \forall x>0$

$=1 \quad x=0$

55. Draw the graph, we get the function is not differentiable and continuous at two points between $x=-\pi/2$ and $x=\pi/2$ also function is not continuous at $x=\pi/2$ and $x=-\pi/2$

56. $$|f(x)| = \begin{cases} |x+2| & x<0 \\ |-x^2-2| & 0\leq x<1 \\ |x| & x\geq 1 \end{cases}$$

$$|f(x)| = \begin{cases} -x-2 & if \quad x \le -2 \\ x+2 & if \quad -2<x<0 \\ x^2+2 & if \quad 0 \le x<1 \\ x & if \quad x \ge 1 \end{cases}$$

Clearly $|f(x)|$ is discontinuous at $x=1$

57. $f(x) = a\sin x + be^x + cx^3 \ if \ x \ge 0$

$= -a\sin x + be^{-x} - cx^3 \ if \ x < 0$

Clearly $f(x)$ is continuous at $x=0$

$$f^1(x) = \begin{cases} a\cos x + be^x + 3cx^2 & if \ x \ge 0 \\ -a\cos x - be^{-x} - 3cx^2 & if \quad x < 0 \end{cases}$$

L.H.D=R.H.D

$\Rightarrow -a-b- = a+b \Rightarrow a+b=0$

58. $\lim\limits_{x\to 1^+} \dfrac{(x-1)^n}{\log_e \cos^m(x-1)} = -1$

$\lim\limits_{x\to 1^+} \dfrac{n(x-1)^{n-1}}{m.\dfrac{-\sin(x-1)}{\cos(x-1)}} = -1$

$\Rightarrow \lim\limits_{x\to 1^+} \dfrac{n}{m} \dfrac{(x-1)^{n-1}}{\tan(x-1)} = 1$

$\Rightarrow \dfrac{n}{m} = 1, n = 2, m+n = 4$

59. $f(2x) - f(x) = x$

$f(x) - f\left(\dfrac{x}{2}\right) = \dfrac{x}{2}$

$f\left(\dfrac{x}{2}\right) - f\left(\dfrac{x}{4}\right) = \dfrac{x}{4}$

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$f\left(\dfrac{x}{2^{n-1}}\right) - f\left(\dfrac{x}{2^n}\right) = \dfrac{x}{2^n}$

Adding $f(2x)-f\left(\frac{x}{2^n}\right)=x+\frac{x}{2}+\frac{x}{4}+....+\frac{x}{2^n}$

$\lim_{n\to\infty}\left[f(2x)-f\left(\frac{x}{2^n}\right)\right]=\lim_{n\to\infty}x+\frac{x}{2}+\frac{x}{4}+....+\frac{x}{2^n}$

$f(2x)-1=\frac{x}{1-\frac{1}{2}}$

$f(2x)=2x+1$

$f(x)=x+1$

60. Conceptual