



**MasterJEE**  
IIT-JEE | Medical | Foundations

## Master JEE CLASSES

### Kukatpally, Hyderabad.

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IIT-JEE-2013-P2-Model

Max.Marks:180

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## 2013\_PAPER-II

### IMPORTANT INSTRUCTIONS:

- 1) This booklet is your Question Paper.
- 2) Use the Optical Response Sheet (ORS) provided separately for answering the questions
- 3) Blank spaces are provided within this booklet for rough work.
- 4) Write your name, roll number and sign in the space provided on the back cover of this booklet.
- 5) You are allowed to take away the Question Paper at the end of the examination.

### OPTICAL RESPONSE SHEET:

- 6) Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's sheet.
- 7) The ORS will be collected by the invigilator at the end of the examination.
- 8) Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
- 9) Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS.  
Darken the appropriate bubble under each digit of your roll number.

### DARKENING THE BUBBLES ON THE ORS

- 10) Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
- 11) Darken the bubble **COMPLETELY**.
- 12) The correct way of darkening a bubble is as:
- 13) The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
- 14) Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

**IIT-JEE-2013-P2-Model****IMPORTANT INSTRUCTIONS****Max Marks: 180****PHYSICS**

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 1 – 8)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 9 – 16)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 17 – 20)	Matrix Matching Type	3	-1	4	12
<b>Total</b>				<b>20</b>	<b>60</b>

**CHEMISTRY**

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 21 – 28)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 29 – 36)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 37 – 40)	Matrix Matching Type	3	-1	4	12
<b>Total</b>				<b>20</b>	<b>60</b>

**MATHEMATICS**

Section	Question Type	+Ve Marks	- Ve Marks	No.of Qs	Total marks
Sec – I(Q.N : 41 – 48)	Questions with Multiple Correct Choice	3	-1	8	24
Sec – II(Q.N : 49 – 56)	Questions with Comprehension Type (4 Comprehensions – 2 +2+2+2 = 8Q)	3	-1	8	24
Sec – III(Q.N : 57 – 60)	Matrix Matching Type	3	-1	4	12
<b>Total</b>				<b>20</b>	<b>60</b>

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**SECTION – I****(MULTIPLE CORRECT CHOICE TYPE)**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

1. The physical quantities having same units are
  - A) mass and moment of inertia per unit area
  - B) pressure and energy density
  - C) Moment of force and torque
  - D) Linear momentum and angular momentum
2. If U, m L and G denote energy, mass, angular momentum and universal gravitational constant, respectively. The dimensions of  $\frac{m^5 G^2}{UL^2}$  are same as
  - A) co-efficient of restitution
  - B) refractive index
  - C) solid angle
  - D) distance

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3. If  $y = a^3 \sin^{-1}\left(\frac{x}{b} - 1\right)$ , where y is in metre. Then,

A) the dimensions of a is  $\left(M^0 L^{\frac{1}{3}} T^0\right)$

B) the unit of  $\sin^{-1}\left(\frac{x}{b} - 1\right)$  is radian

C) the dimensions of x are same as those of b

D) the unit of a is metre

4. The speeds of two particles are measured to be  $v_1 = (10.20 \pm 0.02)\text{ms}^{-1}$  and  $v_2 = (6.40 \pm 0.01)\text{ms}^{-1}$ . Then, mark the correct option(s).

A) The sum of speeds of particles is  $(16.60 \pm 0.03)\text{ms}^{-1}$

B) The difference of speeds of particles is  $(3.80 \pm 0.03)\text{ms}^{-1}$

C) The difference of speeds of particles is  $(6.40 \pm 0.01)\text{ms}^{-1}$

D) The sum of speeds of particles is  $16.60\text{ms}^{-1}$

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5. If  $y = \sin \theta$ , the error in measurement of  $\theta$  is  $1^\circ$ . The fractional error in  $y$
- A) may be zero
  - B) may be infinity
  - C) will have any value from zero to infinity
  - D) must be zero
6. A student uses a simple pendulum of exactly 1 m length to determine  $g$ , the acceleration due to gravity. He uses a stopwatch with the least count of 1 s for this and records 40 s for 20 oscillations. For the observation, which of the following statements (s) is/are correct?
- A) Error  $\Delta T$  in measuring  $T$ , the time period is 0.05 s
  - B) Error  $\Delta T$  in measuring  $T$ , the time period is 1 s
  - C) Percentage error in the determination of  $g$  is 5%
  - D) Percentage error in the determination of  $g$  is 2.5%

7. Out of the following expressions for couple per unit twist (C), time period (T) of a compound pendulum, time period ( $T^1$ ) of a simple pendulum and frequency (f) of a stretched wire respectively, the dimensionally consistent equations are  
( $\ell$  = length, F = Force, A = area,  $\theta$  = angle, r = radius)

A)  $C = \frac{\pi \eta r^4}{2\ell}$  (here  $\eta = F / A \times \theta$ )

B)  $T = 2\pi \sqrt{\frac{K^2 + \ell}{g}}$  here K = radius of gyration

C)  $T^1 = 2\pi \sqrt{\frac{\ell}{g}}$

D)  $f = \frac{1}{\ell} \sqrt{\frac{T_0}{\mu}}$  here  $\mu$  = linear density,  $T_0$  = tension in string

8. If the dimension of the length are expressed as  $G^x c^y h^z$ , where G, c and h are the universal gravitational constant, speed of light and Planck's constant respectively, then

A)  $x = 1/2, y = 1/2$

B)  $x = 1/2, z = 1/2$

C)  $y = 1/2, z = 3/2$

D)  $y = -3/2, z = 1/2$

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**SECTION - II**  
**(COMPREHENSION TYPE)**

This section contains **4 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.

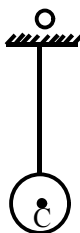
**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

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**Paragraph for Questions 9 and 10**

A small spherical ball suspended from a rigid support using inextensible and light string and free to oscillate without friction is called a simple pendulum. The time

period of small oscillation of simple pendulum is given by  $T = 2\pi\sqrt{\frac{L}{g}}$ .



Here,  $L$  = Distance between suspension point  $O$  and centre  $C$  of the ball,

$g$  = Acceleration due to gravity.

In the determination of value of acceleration due to gravity  $g$  by simple pendulum, the time period is measured by a stopwatch whose least count is 0.5 s and the length of string is measured with a meter scale, having least count of 1 mm. The diameter of the

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bob is measured with the vernier calipers with least count 0.01 cm. The following observations are recorded.

- i. Length of the string = 105.3 cm      ii. Diameter of the bob = 2.45 cm  
iii. Time period = 2.07 s                  iv. Number of oscillations = 10

9. Find percentage error in calculation of acceleration due to gravity.

- A) 3%                  B) 4.9%                  C) 9.8%                  D) 0.28%

10. If we increase the number of oscillations from 10 to 20, find the new percentage error in calculation of g.

- A) 2.6%                  B) 4.9%                  C) 9.8%                  D) 6%

**Paragraph for Question 11 and 12**

The time period of simple pendulum is given by  $T = k\ell^x g^y$  where k is the constant of proportionality,  $\ell$  is the length of pendulum and g, the acceleration due to gravity, x and y are dimensionless constants.

11. In the above equation, dimensions of  $\ell$  and g are respectively

- A)  $[L], [MLT^{-2}]$                   B)  $[L], [L^{-2}T]$   
C)  $[L], [LT^{-2}]$                   D)  $[L^{-1}], [LT^{-2}]$

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12. If the given equation is dimensionally correct then

- A)  $x = 1, y = \frac{1}{2}$                       B)  $x = \frac{1}{2}, y = -\frac{1}{2}$   
C)  $x = -\frac{1}{2}, y = \frac{1}{2}$                       D)  $x = -1, y = \frac{1}{2}$

**Paragraph for Questions 13 and 14**

In a new system of units, let the units of force, energy and velocity be 10 N, 0.1 J and 100 m/s respectively. In this new system,

13. the unit of length is

- A) 0.50 m                      B) 1.00 m                      C)  $10^{-3}$  m                      D)  $10^{-2}$  m

14. the unit of mass is

- A)  $10^{-5}$  kg                      B)  $10^{-3}$  kg                      C)  $10^{-4}$  kg                      D)  $10^{-2}$  kg

**Paragraph for Questions 15 and 16**

The energy 'E' of a particle varies with time t according to the equation

$E = E_0 \sin(\alpha t) e^{\frac{-\alpha t}{\beta x}}$ ; where x is displacement from mean position.  $E_0$  is energy at infinite position and  $\alpha$  &  $\beta$  are constants of appropriate dimensions.

15. Dimensional formula of  $\alpha$  is

- A)  $[M^0 L^0 T^{-1}]$                       B)  $[M^{-1} L^0 T^0]$                       C)  $[M^0 L^{-1} T^0]$                       D)  $[M^0 L^0 T^0]$

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16. Dimensional formula of  $\beta$  is

- A)  $[M^{-1}L^0T^0]$     B)  $[M^0L^{-1}T^0]$     C)  $[M^0L^0T^{-1}]$     D)  $[M^0L^0T^0]$

**SECTION – III**  
**(MATRIX MATCH TYPE)**

This section contains **4 multiple choice questions**. Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

17. Considering Force (F), Velocity (V) and Energy (E) as fundamental quantities, match the correct dimensions of following quantities

	Column I		Column II
I.	Mass	P.	$[F^{-1} V^0 E^1]$
II.	Light year	Q.	$[F^1 V^1 E^{-1}]$
III.	Frequency $\left(\frac{1}{T}\right)$	R.	$[F^3 V^0 E^{-2}]$
IV.	Pressure	S.	$[F^0 V^{-2} E^1]$

Codes

- |    |   |    |     |     |    |     |     |     |     |
|----|---|----|-----|-----|----|-----|-----|-----|-----|
|    | I | II | III | IV  |    | I   | II  | III | IV  |
| A) | S | P  | Q   | S,R | B) | S   | P   | Q   | R   |
| C) | P | Q  | P,Q | P,S | D) | P,Q | R,S | R,S | R,S |

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18. Match the Column I with Column II and mark the correct option from the given below.

	Column I		Column II
I.	If $m, g, \ell$ and $I$ denote mass, acceleration due to gravity, length and moment of inertia, respectively. The formula of $B = \frac{mgl}{I}$	P.	In the dimensional formula of defined quantity, the dimension of mass is 1
II.	If $\ell, F$ and $\mu$ denote length, force and linear mass density, respectively. The formula of $C = \sqrt{\frac{F}{\mu \ell^2}}$	Q.	In the dimensional formula of defined quantity, the dimension of time is (-2)
III.	$G$ = universal gravitational constant. $M_e, R_e$ are mass and	R.	In the dimensional formula of defined quantity, the dimension of

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	radius of earth. The formula of $V = \frac{GM_e}{R_e}$		length is zero
IV.	If U and V represent potential energy and velocity of a particle respectively, The formula of $R = \frac{U}{V^2}$	S.	The dimensional formula of defined quantity is $[ML^0T^0]$

Codes

- |          |     |     |     |    |     |     |         |
|----------|-----|-----|-----|----|-----|-----|---------|
| I        | II  | III | IV  | I  | II  | III | IV      |
| A) Q,R   | R   | Q   | P,S | B) | P,R | Q,R | Q,S P,S |
| C) P,Q,R | P,Q | Q,R | P,S | D) | Q,R | R   | Q,S P,R |

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19. Match the following columns and mark the correct option from the codes given below.

	Column I		Column II
I.	$\frac{1}{3}$	P.	1
II.	$\frac{1.00}{3.00}$	Q.	0.3
III.	$2 - 0.3$	R.	0.33
IV.	$2 - 0.6$	S.	2

Codes

- |    |     |     |     |     |    |   |    |     |    |
|----|-----|-----|-----|-----|----|---|----|-----|----|
|    | I   | II  | III | IV  |    | I | II | III | IV |
| A) | Q   | R   | S   | P   | B) | P | Q  | R   | S  |
| C) | Q,R | Q,R | P,S | P,S | D) | Q | R  | P   | S  |

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20. During determination of acceleration due to gravity by simple pendulum in repeated experiments, following observations are obtained.

S. No. of observations	Acceleration due to gravity (in $\text{ms}^{-2}$ )
1	9.85
2	9.76
3	9.91
4	9.79
5	9.78

Match the following columns and mark the correct option from the codes given below.

	Column I		Column II
I.	Mean value of acceleration due to gravity (in $\text{ms}^{-2}$ )	P	0.05
II.	Mean absolute error (in $\text{ms}^{-2}$ )	Q	9.82
III.	Relative error	R	0.51
IV.	Percentage error	S	0.0051

Codes:

	I	II	III	IV		I	II	III	IV
A)	Q	P	S	R	B)	Q	P	R	S
C)	Q	R	S	P	D)	P	Q	S	R

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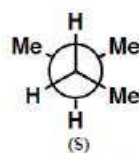
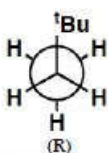
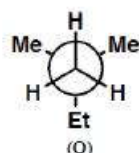
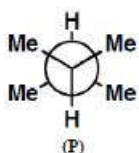
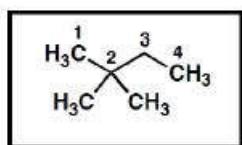
## SECTION – I

## (MULTIPLE CORRECT CHOICE TYPE)

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

21. Identify the structure (s) that represent a conformation of **2,2-dimethylbutane** sighting along any C-C bond



A) P

B) S

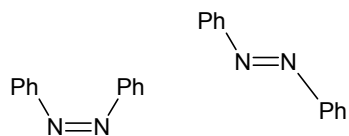
C) Q

D) R

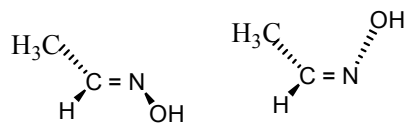
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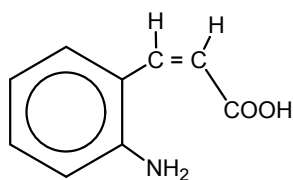
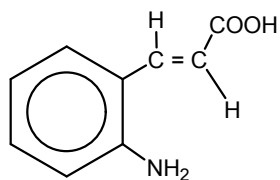
22. For which of the following pairs of compounds are the CORRECT notation given?



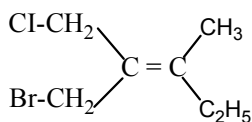
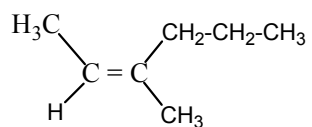
A) Z-azobenzene E-azobenzene



B) E-acetaldoxime Z-acetaldoxime



C) Trans-o-amino cinnamic acid Cis-o-amino cinnamic acid



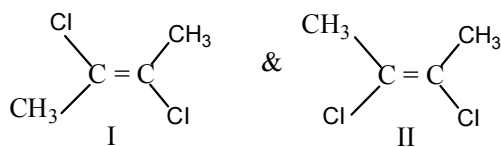
D) Z-isomer E-isomer

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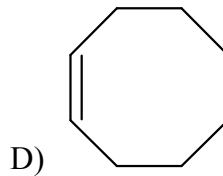
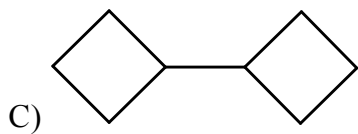
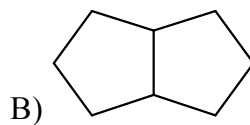
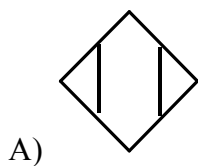


23. Which statement is/are right regarding



- A) They are geometrical isomers
- B) They are diastereoisomers
- C) I is E-isomer and II is Z-isomer
- D) They are conformers

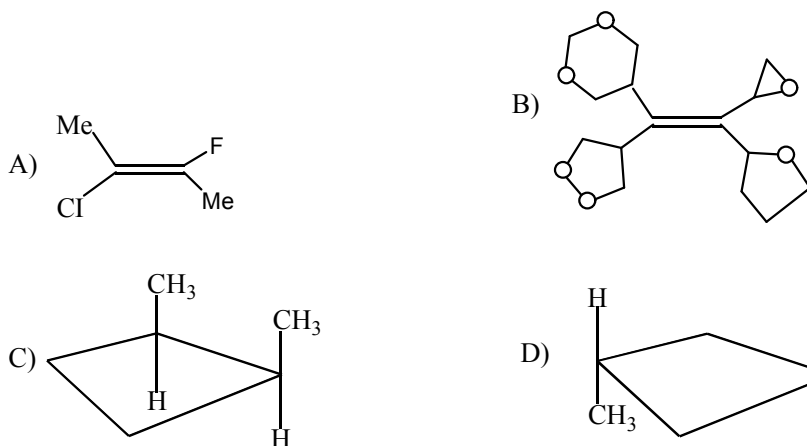
24. Which of the compounds shown below are isomers?



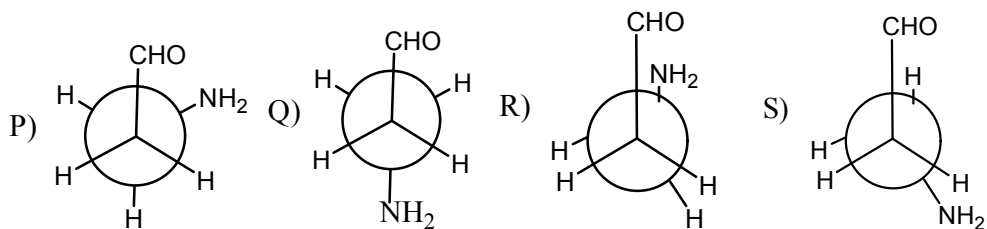
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25. Which of the following molecules are E-isomer?



26. Consider the following conformations of 3-Aminopropanal, amongst the given conformations (P,Q,R,S) one of them is most stable.



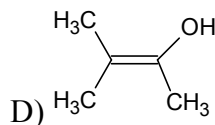
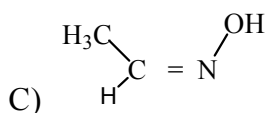
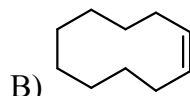
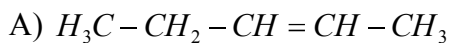
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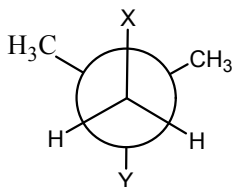
The correct statements for the above are:

- A) H-bonding is present in the most stable conformer
- B) Gauche conformation is the most stable conformer
- C) Anti conformation is the most stable conformer
- D) larger groups being separated by maximum distance in the conformer

27. Which of the following compounds can exhibit geometrical isomerism?



28. In the Newmann projection for 2,2- dimethylbutane



X and Y can respectively be

- A) H and H      B) H and  $C_2H_5$     C)  $C_2H_5$  and H    D)  $CH_3$  and  $CH_3$

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**SECTION - II**  
**(COMPREHENSION TYPE)**

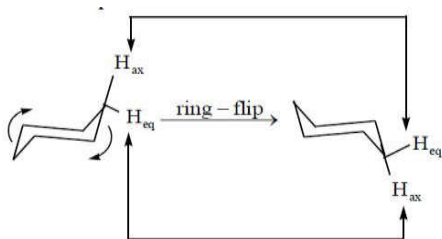
This section contains **4 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

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**Paragraph for Questions 29 and 30**

Cyclohexane exist as two chair conformations in rapid equilibrium at room temperature. Each carbon atom on a cyclohexane ring has one axial and one equatorial hydrogen. Ring flipping converts axial H's to equatorial H's and vice-versa.



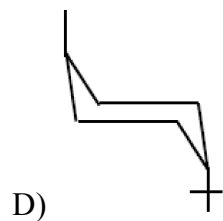
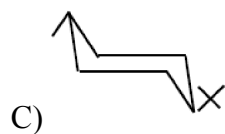
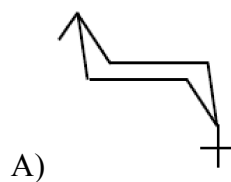
In substituted cyclohexane, groups larger than hydrogen are more stable in the equatorial position. The cis isomers has two groups on the same side of the ring, either both up or both down. The trans isomer has two groups on opposite side of the ring, one up and one down.

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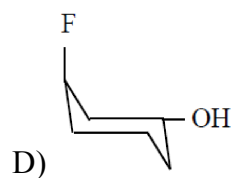
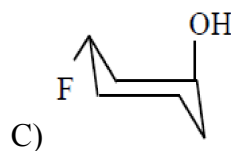
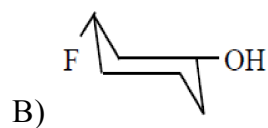
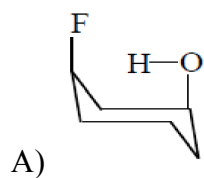
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29. Find out most stable substituted cyclohexane among the following:



30. Find out most stable conformer of cis-3-fluorocyclohexanol



space for rough work

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**Paragraph for Questions 31 and 32****Geometrical Isomerism**

Molecules having same molecular formula but differing in the orientation of atoms in a space due to restricted rotation.

31. Compound which do not exhibit geometrical isomerism

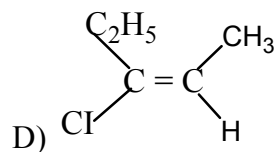
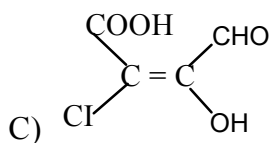
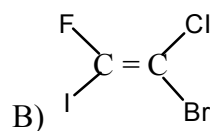
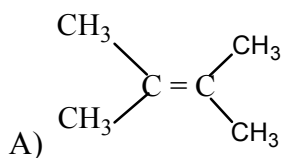
A) diethyl ether

B) 2-butene

C) 3,4- dimethyl-3-hexene

D) 2-chloro-3-methyl-2-pentene

32. Which molecule is in “E” configuration



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**space for rough work**

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### Paragraph for Questions 33 and 34

Stereoisomers, which can be interconverted simply by rotation about sigma bonds, are conformational isomers. While those, which can be converted only by breaking and remaking of bonds and not simply by rotation, are called configurational isomers.

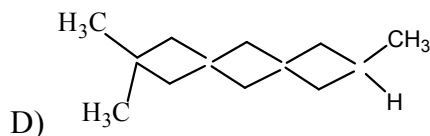
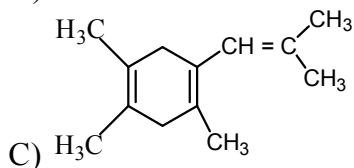
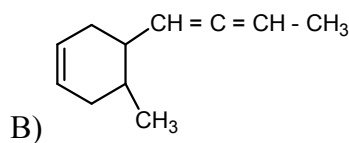
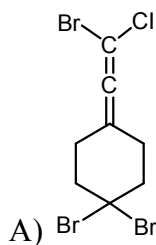
The angle between C – C and C – H bonds on adjacent carbon atoms in any conformation is called dihedral angle. The cyclic compounds most commonly found in nature containing six membered rings can exist in a conformation that is almost completely free of strain. The most stable conformation of cyclohexane is chair form. According to Bayer strain theory, the greater is the deviation from the normal tetrahedral angle, greater is the angle strain or torsional strain and hence lesser is the stability of the cycloalkane.

33. Dihedral angle between two methyl groups of n-butane in the gauche and anti forms respectively are:  
A)  $60^\circ, 0^\circ$       B)  $60^\circ$  and  $180^\circ$       C)  $0^\circ, 60^\circ$       D)  $180^\circ, 60^\circ$
34. Which among the following conformation of cyclohexane is the most stable form?  
A) Chair form      B) Half chair form  
C) Twist boat form      D) Boat form

**Paragraph for Questions 35 and 36**

Those stereoisomers are called configurational isomers if they differ in spatial arrangement of atom or group and are not interconvertible without cleaving a bond. Geometrical isomerism arises due to restricted rotation about a bond or centre. The number of G.I. is  $2^n$  where  $n$  = number of double bonds or rings or both. In some cases the number of G.I. is less than  $2^n$ , where either side nomenclature will be possible.

35. Which of the following compounds can show geometrical isomerism?



36. The number of G.I. of the compound  $C_2H_5 - CH = CH - CH = CH - CH = CH - CH_3$  is

A) 4                      B) 3                      C) 8                      D) 6

**space for rough work**

**Page 24**

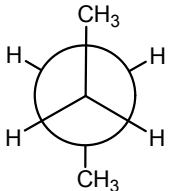
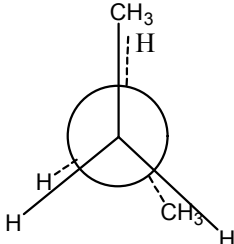


**SECTION – III**  
**(MATRIX MATCH TYPE)**

This section contains **4 multiple choice questions**. Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

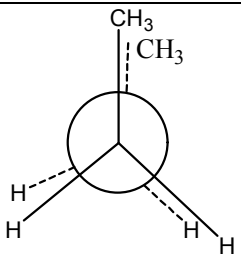
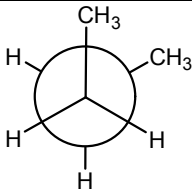
**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

37. Match the conformers of n-Butane along  $C_2 - C_3$  bond in Column I with torsional energies given in Column II

	Column I		Column II
I.		P.	18.4 – 25.5 KJ/mol
II.		Q.	14.2 KJ/mol

space for rough work

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III.		R.	3.3 KJ/mol
IV.		S.	Zero

- |    |   |    |     |    |
|----|---|----|-----|----|
|    | I | II | III | IV |
| A) | P | Q  | S   | R  |
| B) | Q | R  | P   | S  |
| C) | S | Q  | P   | R  |
| D) | P | Q  | R   | S  |

space for rough work

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38. Match the following compounds from Column I with number of Geometrical Isomers from Column II

	Column I		Column II
I.	$CH_3 - CH = C = CH - CH_3$	P.	4
II.	$CH_3 - CH = CH - CH = N - OH$	Q.	8
III.	$CH_3 - CH = C = C = CH - CH_3$	R.	0
IV.	$CH_3 - CH = CH - CH = CH - CH = CH - C_2H_5$	S.	2

- |    |   |    |     |    |
|----|---|----|-----|----|
|    | I | II | III | IV |
| A) | S | P  | R   | Q  |
| B) | R | P  | S   | Q  |
| C) | Q | R  | S   | P  |
| D) | P | Q  | R   | S  |

space for rough work

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39. Match the following compounds in Column I with type of isomerism in Column II

Column I (Molecule)		Column II (Most stable conformer)	
I	Ethane	P	Gauche
II	Ethylene Glycol	Q	Anti
III	3-Fluoropropanal about $C_2-C_3$	R	Staggered
IV	n-Pentane about $C_2-C_3$	S	Fully eclipsed

- I      II      III      IV
- A)    R      S      P      Q
- B)    Q      R      P      S
- C)    R      P      P      Q
- D)    P      Q      R      S

**space for rough work**

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40. Match the following compounds in Column I with their properties from Column II

	Column I		Column II
I.		P.	Torsional strain
II.		Q.	Melting point
III.		R.	Dipole moment
IV.		S.	Boiling point

	I	II	III	IV		I	II	III	IV
A)	P	Q	R	S	B)	Q	P	R	S
C)	S	Q	P	R	D)	R	S	P	Q

space for rough work

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**SECTION – I**  
**(MULTIPLE CORRECT CHOICE TYPE)**

This section contains **8 multiple choice questions**. Each question has 4 choices (A), (B), (C) and (D) for its answer, out of which **ONE OR MORE** is/ are correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

41. Let  $f : (0, \infty) \rightarrow (0, \infty)$  be a derivable function and  $F(x)$  is the primitive of  $f(x)$

such that  $2(F(x) - f(x)) = f^2(x)$  for any real positive  $x$ .

- A)  $f$  is strictly increasing                      B)  $\lim_{x \rightarrow \infty} \frac{f(x)}{x} = 1$   
C)  $f$  is strictly decreasing                      D)  $f$  is non-monotonic

42. Let  $f : R \rightarrow R$  be defined as  $f(x) = \sin x + ax + b$  ( $a, b \in R$ ). Then the equation

$f(x) = 0$  has :

- A) only one real root which is positive if  $a > 1, b < 0$   
B) only one real root which is negative if  $a > 1, b > 0$   
C) only one real root which is negative if  $a < -1, b < 0$   
D) only one real root which is positive if  $a < -1, b < 0$

---

43. Let  $f$  be a real-valued function on  $\mathbb{R}$  defined as  $f(x) = x^4(1-x)^2$ , then which of the following statement (s) is (are) correct ?

A)  $f'(c) = 0$  for some  $c \in (0,1)$

B)  $f''(x)$  vanishes exactly twice in  $\mathbb{R}$ .

C)  $f(x)$  is an even function

D) Monotonic increasing in  $\left(0, \frac{2}{3}\right) \cup (1, \infty)$

44. For which of the following functions Rolle's theorem is applicable ?

A)  $f(x) = \frac{x}{2} + \frac{2}{x}, x \in [1, 4]$

B)  $f(x) = x + 1 - x^{3/2}, x \in [0, 1]$

C)  $f(x) = |x+1|^3, x \in [-2, 0]$

D)  $f(x) = \operatorname{sgn}(x) + \operatorname{sgn}(-x), x \in \left[-\frac{5}{2}, \frac{1}{2}\right]$  ( $\operatorname{sgn}(x)$  denotes signum functions) of  $x$ )

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**space for rough work**

**Page 31**

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45. Let  $f(x)$  be a twice differentiable function such that  $f''(x) > 0$  in  $[0, 2]$ . Then :

A)  $f(0) + f(2) = 2f(c)$  for atleast one  $c \in (0, 2)$

B)  $f(0) + f(2) < 2f(1)$

C)  $f(0) + f(2) > 2f(1)$

D)  $2f(0) + f(2) > 3f\left(\frac{2}{3}\right)$

46. Let  $h(x) = f(x) - (f(x))^2 + (f(x))^3$  for every real number  $x$ . Then

A)  $h$  is increasing whenever  $f$  is increasing

B)  $h$  is increasing whenever  $f$  is decreasing

C)  $h$  is decreasing whenever  $f$  is decreasing

D)  $h$  is decreasing whenever  $f$  is increasing

47. In  $[0, \pi]$  Rolle's theorem is applicable to

A)  $f(x) = \sin x$

B)  $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

C)  $f(x) = \cos 2x$

D)  $f(x) = \sin^2 x + \sin x$

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**space for rough work**

**Page 32**



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48. if  $\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n+1} + \dots + \frac{a_{n-1}}{2} + a_n = 0$  then  $a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$  has

A) no solution in  $(0,1)$                       B) at least one solution in  $(0,1)$

C) exactly one solution in  $(0,1)$                       D) at least one solution in  $(2,3)$

**SECTION - II**

**(COMPREHENSION TYPE)**

This section contains **4 groups of questions**. Each group has 2 multiple choice questions based on a paragraph. Each question has 4 choices A), B), C) and D) for its answer, out of which **ONLY ONE** is correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

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**Paragraph for Questions 49 and 50**

Consider,  $f(x) = (\lambda^4 - 2\lambda^2)x + 3\cos x, \lambda \in R$ .

49. The least positive integral value of  $\lambda$  for which  $f(x)$  is monotonically increasing for all  $x \in R$ .

A) 2                      B) 3                      C) 4                      D) 5

50. For  $\lambda = -2$ , if  $f(\pi x - x^2) > f(\sin x)$ , then the sum of all integral values of  $x$ , is:

A) 3                      B) 5                      C) 6                      D) 10

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**space for rough work**

**Page 33**

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**Paragraph for Questions 51 and 52**

Consider a cubic,  $f(x) = ax^3 + bx^2 + cx + 4, a, b, c \in R$  and  $f''\left(\frac{-2}{3}\right) = 0$  and tangent

drawn to the graph of the function  $y = f(x)$  at  $x = \frac{-2}{3}$  is  $y = \frac{5x}{3} + \frac{100}{27}$

51. The value of  $(a + b + c)$  is equal to:

- A) 4                      B) 6                      C) 7                      D) 10

52. If  $g$  is the inverse of  $f$ , then  $\frac{d}{dx}(g(x) \cdot f(g(x)))$  at  $x = 4$  is equal to :

- A)  $\frac{1}{4}$                       B)  $\frac{4}{7}$                       C)  $\frac{3}{4}$                       D)  $\frac{4}{3}$

**Paragraph for Questions 53 and 54**

Consider,  $f(x) = \cos 2x + 2x\lambda^2 + (2\lambda + 1)(\lambda - 1)x^2, \lambda \in R$ .

53. For  $\lambda = 1$ , if  $f(3x^2 - 2x + 1) < f(x^2 - 2x + 9)$ , then number of integral values of  $x$  in  $[-10, 10]$ :

- A) 3                      B) 5                      C) 16                      D) 18

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**space for rough work**

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54. If  $f(x)$  is increasing for all  $x \in R$ , then number of values of  $\lambda$

- A) 1                      B) 2                      C) 3                      D)  $\infty$

**Paragraph for Questions 55 and 56**

Let  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$  and  $g(x) = f(\sin x) + f(\cos x)$

55. which of the following is true in  $\left(0, \frac{\pi}{2}\right)$ ?

- A)  $g'$  is increasing                      B)  $g'$  is decreasing  
C)  $g'$  is non-increasing                      D)  $g'$  is neither increasing nor decreasing

56. which of the following is true ?

- A)  $g(x)$  is decreasing in  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$   
B)  $g(x)$  increasing in  $\left(0, \frac{\pi}{4}\right)$   
C)  $g(x)$  is monotonically increasing  
D) None of these

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**space for rough work**

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**SECTION – III**  
**(MATRIX MATCH TYPE)**

This section contains **4 multiple choice questions**. Each question has matching lists. The codes for the lists have choices (A), (B), (C), and (D) out of which **ONLY ONE** is correct.

**Marking scheme: +3 for correct answer, 0 if not attempted and -1 in all other cases.**

57. Match the column

For the function  $f(x) = ax^2 - b|x|$

Column – I

Column – II

(A)  $f(x)$  has local max. at  $x = 0$

(p) When  $a > 0, b > 0$

(B)  $f(x)$  has local min at  $x = 0$

(q) When  $a > 0, b < 0$

(C)  $f(x)$  has local extremum at  $x = \frac{b}{2a}$

(r) When  $a < 0, b < 0$

(D)  $f(x)$  is not diff. at  $x = 0$

(s) When  $a < 0, b > 0$

A

B

C

D

A) Q,R    S,R    P,S    P,Q,R,S

B) P,S    Q,R    P,R    P,Q,R,S

C) P,R,S    Q,R    P,Q    P,Q,R,S

D) P,S    P,Q    Q,R    P,Q,R,S

**space for rough work**

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58. Observe the following list

	Column I		Column II
A.	$f(x) = x^2 - 2x + 5$ is increasing in	P.	$\phi$
B.	$f(x) = e^{-x}$ is increasing in	Q.	$(-\infty, 1) \cup (2, \infty)$
C.	$f(x) = \log_e x$ increasing in	R.	$(1, \infty)$
D.	$f(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + 5$ is increasing in	S.	$(0, \infty)$
		T.	R

- |    |          |          |          |          |
|----|----------|----------|----------|----------|
|    | <b>A</b> | <b>B</b> | <b>C</b> | <b>D</b> |
| A) | R        | P        | S        | Q        |
| B) | P        | R        | Q        | S        |
| C) | Q        | R        | S        | P        |
| D) | S        | Q        | P        | R        |

space for rough work

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59. Observe the following lists

	Column I		Column II
A.	$f(x) = x + \frac{1}{x}$ is decreasing in	P.	R
B.	$f(x) = 1 - x^3$ is decreasing in	Q.	$\left(0, \frac{1}{e}\right)$
C.	$f(x) = xe^{-x}$ is decreasing in	R.	$(-1, 0) \cup (0, 1)$
D.	$f(x) = x^x$ is decreasing in	S.	$(1, \infty)$

The correct matching for list -I from list -II

- |    |   |   |   |   |
|----|---|---|---|---|
|    | A | B | C | D |
| A) | R | S | Q | P |
| B) | Q | R | P | S |
| C) | R | P | S | Q |
| D) | Q | R | S | P |

space for rough work

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60. Let  $f(x) = (2^x - 1)(2^x - 2)$  and  $g(x) = 2 \sin x + \cos 2x$  in  $[0, \pi]$

	Column I		Column II
A.	f increases on	P.	$(\log_2(3/2), \infty)$
B.	f decreases on	Q.	$(-\infty, \log_2(3/2))$
C.	g decreases on	R.	$\left(0, \frac{\pi}{6}\right)$
D.	g increases on	S.	$\left(\frac{5\pi}{6}, \pi\right)$

	A	B	C	D
A)	R	P	Q	S
B)	Q	S	R	P
C)	P	Q	S	R
D)	S	R	Q	P

space for rough work

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# Master JEE CLASSES

## Kukatpally, Hyderabad.

**IIT-JEE-2013-P2-Model**

**Max.Marks:180**

### KEY SHEET

#### PHYSICS

1	<b>ABC</b>	2	<b>ABC</b>	3	<b>ABC</b>	4	<b>AB</b>	5	<b>ABC</b>
6	<b>AC</b>	7	<b>ACD</b>	8	<b>BD</b>	9	<b>B</b>	10	<b>A</b>
11	<b>C</b>	12	<b>B</b>	13	<b>D</b>	14	<b>A</b>	15	<b>A</b>
16	<b>B</b>	17	<b>B</b>	18	<b>A</b>	19	<b>A</b>	20	<b>A</b>

#### CHEMISTRY

21	<b>CD</b>	22	<b>ABC</b>	23	<b>ABC</b>	24	<b>BCD</b>	25	<b>ABD</b>
26	<b>AB</b>	27	<b>ABC</b>	28	<b>BD</b>	29	<b>C</b>	30	<b>A</b>
31	<b>A</b>	32	<b>D</b>	33	<b>B</b>	34	<b>A</b>	35	<b>B</b>
36	<b>C</b>	37	<b>C</b>	38	<b>B</b>	39	<b>C</b>	40	<b>D</b>

#### MATHS

41	<b>AB</b>	42	<b>ABC</b>	43	<b>AD</b>	44	<b>ABCD</b>	45	<b>BD</b>
46	<b>AC</b>	47	<b>ACD</b>	48	<b>B</b>	49	<b>A</b>	50	<b>C</b>
51	<b>B</b>	52	<b>D</b>	53	<b>A</b>	54	<b>A</b>	55	<b>A</b>
56	<b>D</b>	57	<b>B</b>	58	<b>A</b>	59	<b>C</b>	60	<b>C</b>



## SOLUTIONS PHYSICS

1. a) MASS  $\rightarrow kg = I \Rightarrow mr^2 = kgm^2 \quad I / A = \frac{kgm^2}{m^2} = kg$
- b) Pressure =  $F / A (N / m^2)$ ,  $\frac{energy}{volume} = \frac{kgm^2 / sec^2}{m^3} = kg / m sec^2$
- $$= \frac{kgm / sec^2}{m^2} = \frac{kg}{m sec^2}$$
- c) moment of force = force x distance = time
- d)  $p = kgm / sec \quad L = kg (m / sec)m = kgm^2 / sec$

2.  $e = \frac{V_{rel}}{u_{rel}}, \mu = \frac{\sin i}{\sin r}, \Omega = A / r^2$

$$\frac{m^5 G^2}{UL^2} = \frac{kg^5 \left[ \frac{Nm^2}{kg^2} \right]^2}{N - m \times [(kg)(m/sec)(m)]^2}$$

$$= \frac{kg^5 N^2 m^4 \times sec^2}{kg^4 \times N - m \times kg^2 \times m^2 \times m^2}$$

$$= \frac{kg \times kg \times m / sec^2}{kg^2 m^2} = \text{dimensionless}$$

3. a) Here,  $\sin^{-1} \left( \frac{x}{b} - 1 \right)$  is dimensionless

b) Let  $\sin^{-1} \left( \frac{x}{b} - 1 \right) = \theta$

$$\therefore \sin \theta = \left( \frac{x}{b} - 1 \right)$$

Here,  $\theta$  should be in radian.

So, the unit of  $\sin^{-1} \left( \frac{x}{b} - 1 \right)$  is radian

c) Dimensions of  $\frac{x}{b}$  = Dimensions of 1 = dimensionless

$\therefore$  Dimension of x = Dimension of b

4. a)  $Sum = v_1 + v_2 = (10.20 \pm 0.02) + (6.40 \pm 0.01)$

$$= (16.60 \pm 0.03) ms^{-1}$$

b) Difference  $= v_1 - v_2 = (10.20 \pm 0.02) - (6.40 \pm 0.01)$

$$= 3.80 \pm (0.02 + 0.01)$$

$$= (3.80 \pm 0.03) ms^{-1}$$

5.  $y = \sin \theta$  or  $\frac{dy}{d\theta} = \cos \theta$

Or  $dy = \cos \theta d\theta$

Or  $\Delta y = \cos \theta \Delta \theta$

Or  $\frac{\Delta y}{y} = \frac{\cos \theta \Delta \theta}{y} = \frac{\cos \theta \Delta \theta}{\sin \theta}$

Fractional error in  $y = \frac{\Delta y}{y} = \frac{\Delta \theta}{\tan \theta}$

But  $\tan \theta$  has any value from zero to infinity. Hence, fractional error will have any value from infinity to zero. Hence, options (a), (b), (c) are correct.

6.  $T = \frac{40s}{20} = 2s$

Further,  $t = nT = 20T$  or  $\Delta t = 20\Delta T$

$$\therefore \frac{\Delta t}{t} = \frac{\Delta T}{T}$$

Or  $\Delta T = \frac{T}{t} \Delta t = \left(\frac{2}{40}\right)(1) = 0.05s$

Further,  $T = 2\pi \sqrt{\frac{l}{g}}$  or  $T \propto g^{-1/2}$

$$\therefore \frac{\Delta T}{T} \times 100 = -\frac{1}{2} \times \frac{\Delta g}{g} \times 100$$

Percentage error in determination of  $g$  is

$$\frac{\Delta}{g} \times 100 = -200 \times \frac{\Delta T}{T} = \frac{200 \times 0.05}{2} = -5\%$$

$\therefore$  Correct options are (a) and (c).

7. a) is consistent, C is torque per unit twist,  $\eta$  is modulus of rigidity, r is radius,  $\ell$  is length  
 b) is inconsistent, T is time period, k is the radius of gyration, g is acceleration due to gravity  
 c) is consistent  
 d) is consistent, f is frequency, T is tension  $\mu$  is mass per unit length

8.  $L = (M^{-1}L^3T^{-2})^x (LT^{-1})^y (ML^2T^{-1})^z$   
 $-x + z = 0, 3x + y + 2z = 1, -2x - y - z = 0$   
 $\Rightarrow x = 1/2, y = -3/2, z = 1/2$

9.  $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2}$   
 $= 105.3 + \frac{2.45}{2} = 106.5 \text{ 25cm}$

$$T = 2.07s$$

$$\therefore g = 4\pi^2 \frac{L}{T^2}$$

$$\therefore \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2 \frac{\Delta T}{T} = \frac{0.1 + 0.005}{106.525} + \frac{2 \times 0.5}{2.07 \times 10}$$

$$= 0.00098 + 0.048 = 0.04898 = 0.049$$

$$\therefore \text{Percentage error in } g = \frac{\Delta g}{g} \times 100 = 0.049 \times 100 = 4.9\%$$

10.  $\frac{\Delta g}{g} = 0.00098 + \frac{2 \times 0.5}{2 \times 20.07} = 0.026$

$$\therefore \text{percentage error in calculation of } g \text{ is } \frac{\Delta g}{g} \times 100 = 0.026 \times 100 = 2.6\%$$

11.  $\ell$  and g are the length pendulum and the acceleration due to gravity. Hence, their respective dimensions are L and  $LT^{-2}$

12. We have,

$$T = k l^x g^y \quad \dots (I)$$

Writing dimensions on both sides of eq. (I), we get

$$T = L^x (LT^{-2})^y$$

$$\text{Or } M^0 L^0 T = M^0 L^{x+y} T^{-2y}$$

Equating exponents of M, L and T on both sides, we get

$$x + y = 0$$

$$\text{And } -2y = 1$$

$$\text{Solving, } y = -\frac{1}{2} \text{ and } x = \frac{1}{2}.$$

13. force = 10N energy = 0.1J velocity = 100m/sec

Limit of surface

$$L = F^x E^y v^z = [MLT^{-2}]^x [ML^2T^{-2}]^y [LT^{-1}]^z$$

$$x + y = 0 \quad x + 2y + z = 1 \quad -2x - 2y - z = 0$$

$$x = -y \Rightarrow -2(-y) - 2y = z \quad 2y - 2y = z \Rightarrow$$

$$-y + 2y = 0 = 1 \quad L = 10^{-1} \times 0.1^1 \times 100^0 = 10^{-2}$$

$$14. \quad M = [MLT^{-2}]^x [MLT^{-2}]^y [LT^{-1}]^z$$

$$x + y = 1 \quad x + 2y + z = 0 \quad -2x - 2y - z = 0 \quad -x = 0$$

$$M = 10^0 \times (0.1)^1 \times 100^{-2}$$

$$15. \quad LT \text{ should be dimensionless } [M^0 L^0 T^{-1}][T] = M^0 L^0 T^0$$

$$16. \quad \beta x \text{ should be dimensionless } [M^0 L^0 T^{-1}][L] = M^0 L^0 T^0$$

17. Conceptual

$$18. \quad (i) [B] = \left[ \frac{mgl}{l} \right] = \frac{[MLT^{-2}] \times [L]}{[ML^2]} = [T^{-2}]$$

Thus, (i)  $\rightarrow (q, r)$

$$(ii) [C] = \sqrt{\frac{F}{\mu l^2}} = \sqrt{\frac{[MLT^{-2}]}{[ML^{-1}][L^2]}} = [T^{-1}]$$

Thus, (ii)  $\rightarrow (r)$

$$(iii) [V] = \frac{[M^{-1} L^3 T^{-2}]}{L} = M^{-1} L^2 T^{-2}, \text{ Thus, (iii) } \rightarrow (Q)$$

$$(iv) [R] = \frac{[ML^2T^{-2}]}{(LT^{-1})^2} = ML^0T^0$$

Thus, (iv)  $\rightarrow (p, s)$

$$19. a) (i) \frac{1}{3} = 0.333 = 0.3$$

Thus, (i)  $\rightarrow (q)$

$$(ii) \frac{1.00}{3.00} = 0.333 = 0.3$$

Thus, (ii)  $\rightarrow (r)$

$$(iii) 2 - 0.3 = 2 - 0 = 2$$

Thus, (iii)  $\rightarrow (s)$

$$(iv) 2 - 0.6 = 2 - 1 = 1$$

Thus, (iv)  $\rightarrow (p)$

$$20. (i) \frac{g}{g} = \frac{g_1 + g_2 + g_3 + g_4 + g_5}{5}$$

$$= \frac{9.85 + 9.76 + 9.91 + 9.79 + 9.78}{5} = 9.82ms^{-2}$$

Thus, (i)  $\rightarrow (q)$

$$(ii) \Delta g_1 = |g_1 - \bar{g}| = 0.032ms^{-2}$$

$$\Delta g_2 = |g_2 - \bar{g}| = 0.060ms^{-2}$$

$$\Delta g_3 = |g_3 - \bar{g}| = 0.090ms^{-2}$$

$$\Delta g_4 = |g_4 - \bar{g}| = 0.030ms^{-2}$$

$$\Delta g_5 = |g_5 - \bar{g}| = 0.040ms^{-2}$$

$$\therefore \Delta \bar{g} = \frac{\Delta g_1 + \Delta g_2 + \Delta g_3 + \Delta g_4 + \Delta g_5}{5} = 0.05ms^{-2}$$

Thus, (ii)  $\rightarrow (p)$

$$(iii) \text{Relative error} = \frac{\Delta \bar{g}}{g} = \frac{0.05}{9.82} = 0.0051$$

Thus, (iii)  $\rightarrow (s)$

$$(iv) \text{Percentage error} = \frac{\Delta \bar{g}}{g} \times 100 = 0.51$$

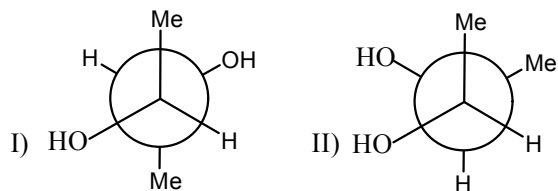
Thus, (iv)  $\rightarrow (r)$

## CHEMISTRY

21. CONCEPTUAL

22. CONCEPTUAL

23. Geometrical isomers are diastereomers



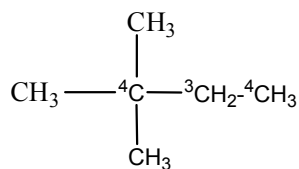
24.

25. As per CIP same priority groups on opposite gives E-isomer and same side Z-isomers

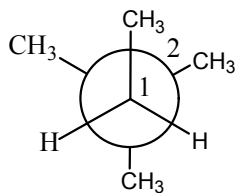
26. 1) Stable due to H-bonding 2) P is most stable due to H-bonding in it's gauche conformations.

27. CONCEPTUAL

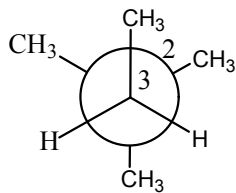
28. Structural formula 2,2-dimethylbutane is



(I) Newman projection using  $C_1 - C_2$  bond



(II) Newman projection using  $C_3 - C_2$  bond



29. In stable conformation, larger group occupies equatorial position

30. Due to H-bonding, structure (A) is most stable

31. To exhibit geometric isomerism, compound must have unsaturation or compound must be cyclic with at least two different groups

32. Same priority groups are on opposite side

33.  $60^\circ$  and  $180^\circ$
34. Chair form is the most stable form
35. Allenes with even number of carbons are not planar & do not show G.I.
36.  $2^n = 2^3 = 8$
37. CONCEPTUAL
38. No of geometrical isomers =  $2^n$
39. CONCEPTUAL
40. trans - compound has higher melting point than cis – compound. cis - compound is more polar and hence has higher boiling point

### MATHS

41.  $f(n) > 0$   $2(f(x) - f(x)) = (f(x))^2$   $F(x) = \frac{(f(x))^2}{2} + f(x)$

$$\Rightarrow F'(x) = f(x)f'(x) + f'(x)$$

But  $f'(x) = f(x)$

$$\therefore f'(x) = \frac{f(x)}{1+f(x)} = 1 - \frac{1}{1+f(x)} > 0$$

$\therefore f$  is strictly increasing

$$\lim_{x \rightarrow \infty} \frac{f(x)}{x} = \lim_{x \rightarrow \infty} \frac{f'(x)}{1} = \lim_{x \rightarrow \infty} \frac{f(x)}{1+f(x)} = 1$$

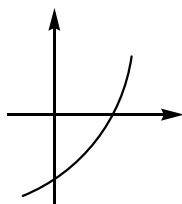
As  $\lim_{x \rightarrow \infty} f(x) \rightarrow \infty$

42.  $f: R \rightarrow R$  and  $f(x) = \sin x + ax + b$   $f'(x) = \cos x + a$

if  $a > 1$ , then  $f$  is increasing

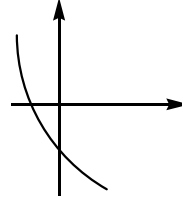
If  $a < -1$  then  $f$  is decreasing

If  $a > b, f(0) = b$  and  $b < 0 \Rightarrow f(0) < 0$



y shown in the diagram

$\therefore f(x) = 0$  has only the +ve root hence a is correct



If  $f(0) > 0$  again  $f$  has only one negative root

If  $a < -1, b < 0$  then  $f$  is decreasing

43.  $f(x), x(1-x)^2$

$$f'(x) = 2x^3(x-1)(3x-2)$$

$$f'(x) = 0 \Rightarrow x = 0, 2/3, 1$$

$$f''(x) = 2x^2(15x^2 - 20x + b)$$

$$f''(0) = 0 \text{ and } 15ax^2 - 20x + b = 0 \text{ has two roots}$$

44. CONCEPTUAL

45.  $X \in [0, 2]$  and  $f''(x) > 0$

$$\Rightarrow f'(x) \text{ is increasing in } [0, 2]$$

$$\Rightarrow \text{LMVT is applicable let } c_1 \in (0, 1) \text{ and } c_2 \in (1, 2)$$

$$\text{By LMVT, } f'(c_1) = \frac{f(1) - f(0)}{1 - 0}$$

$$f'(c_2) = \frac{f(2) - f(1)}{2 - 1}$$

$$f'(x) \text{ is increasing and } c_1 < c_2$$

$$\Rightarrow f'(c_1) < f'(c_2)$$

$$\Rightarrow f(1) - f(0) < f(2) - f(1)$$

$$f(0) + f(2) > 2f(1)$$

$$\text{Let } c_1 \in (0, 2/3) \text{ and } c_2 \in (2/3, 2)$$

$$f'(1) < f'(2)$$

$$\Rightarrow \frac{f(2/3) - f(0)}{\frac{2}{3} - 0} < \frac{f(2) - f(2/3)}{2 - 2/3}$$

$$\Rightarrow 2f(0) + f(2) > 3f(2/3)$$



46.  $h(x) = f(x) - (f(x))^2 + (f(x))^3$

$$h'(x) = (3(f(x))^2 - 2f(x) + 1)f'(x)$$

$$3(f(x))^2 - 2f(x) + 1 > 0, \forall x \in R$$

$$h'(x) > 0 \text{ if } f'(x) > 0$$

$\therefore h$  is increasing whenever  $f$  is increasing

$$h'(x) < 0 \text{ if } f'(x) < 0$$

$\therefore h$  is decreasing whenever  $f$  is decreasing

47. CONCEPTUAL

48. CONCEPTUAL

PASSAGE I  $f(x) = (\lambda^4 - 2\lambda^2)x + 3$  is  $x, \lambda \in R$

49.  $f'(x) = \lambda^4 - 2\lambda^2 - 3\sin x \geq 0$

$$-1 \leq \sin x \leq 1 \Rightarrow \lambda^4 - 2\lambda^2 - 3 \geq 0$$

$$\Rightarrow x \leq \sqrt{3} \text{ or } \lambda \geq \sqrt{3}$$

Least positive integer  $\lambda = 2$

$$\Rightarrow f'(x) = 16 - 8 - 3\sin x = 8 - 3\sin x > 0$$

$\Rightarrow f$  is increasing

50.  $f(\pi x - x^2) > f(\sin x)$

$$\Rightarrow \pi x - x^2 > \sin x$$

$$\Rightarrow x = 1, 2, 3 \text{ respectively}$$

$$\sin x = 1 + 2 + 3 = 6$$

PASSAGE II

$$f(x) = ax^3 + bx^2 + cx + 4$$

$$f'(-2/3) = 5/3, f''\left(-\frac{2}{3}\right) = 0$$

Degree of  $f'(x)$  is two  $f''(x)$  is one

$$\Rightarrow f'(x) = k(x + 2/3)^2 + c$$

$$\Rightarrow f'(-2/3) = c = 5/3$$

$$f'(x) = \frac{k}{2} \left( x + \frac{2}{3} \right)^2 + 5/3$$

$$f(x) = \frac{k}{2} \left( x + \frac{2}{3} \right)^3 + \frac{5x}{3} + b$$

$$\text{But } f\left(\frac{-2}{3}\right) = 0 - \frac{10}{9} + b$$

$$\text{And } y\left(\frac{-2}{3}\right) = \frac{5}{3}(-2/3) + \frac{100}{27}$$

$$\Rightarrow b = \frac{100}{27}$$

$$\text{But coefficient } x^3 = 1$$

$$\therefore f(x) = \left(x + \frac{2}{3}\right)^3 + \frac{5}{3}x + \frac{100}{27}$$

$$= x^3 + 2x^2 + 3x + 4$$

$$a = 1, b = 2, c = 3$$

$$51. \quad a + b + c = 6$$

$$52. \quad g = f^{-1} \text{ and } f(g(x)) = x$$

$$\frac{d}{dx}(g(x) \times (g(x))) = \frac{d}{dx}(g(x) \cdot x)$$

$$= x \cdot g'(x) + g(x)$$

$$\text{at } x = 4, \text{ the derivative}$$

$$= 4 \cdot g'(x) + g(x)$$

$$= 4 \cdot \frac{1}{3} + 0$$

$$\text{Hence } f^{-1}(4) = 0 = g(x) \text{ and } g'(x) = \frac{1}{f'(0)}$$

PASSAGE III

$$f(x) = \cos 2x + 2x\lambda^2 + (2\lambda + 1)(\lambda - 1)x^2, \lambda \in R$$

$$53. \quad \lambda = 1 \Rightarrow f(x) = \cos 2x + 2x$$

$$f'(x) \geq 0 \Rightarrow f \text{ is increasing}$$

$$\Rightarrow 3x^2 - 2x + 1 \leq x^2 - 2x + 9$$

$$\Rightarrow -2 < x < 2$$

$$\Rightarrow x = -1, 0, 1$$

54.  $f$  is increasing  $\forall x \in R \Rightarrow f'(x) = -2\sin 2x + 2\lambda^2 + (2\lambda + 1)(\lambda - 1)/2x > 0$

at  $\lambda = \frac{-1}{2}, f'(x) < 0$

$\therefore$  this is valid only at  $\lambda = 1$

55.  $f'(\sin x) < 0$  and  $f''(\sin x) > 0 \forall x \in [0, \pi/2]$

$$g(x) = f(\sin x) + f(\cos x)$$

$$g'(x) = f'(\sin x)\cos x + f'(\cos x)(-\sin x)$$

$$g''(x) = f''(\sin x) \cdot \cos^2 x - \sin x + (\sin x)$$

+       +       +       -

$$g''(x) > 0 \Rightarrow g''(x) \text{ is increasing}$$

56.  $g'\left(\frac{\pi}{4}\right) = 0 \Rightarrow g'(x) > 0, \text{ for } x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

and  $g'(x) < 0 \text{ for } x \in \left(0, \frac{\pi}{4}\right)$

$g$  is increasing in  $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$g$  is decreasing in  $\left(0, \frac{\pi}{4}\right)$

57. CONCEPTUAL

58. CONCEPTUAL

59. CONCEPTUAL

60. CONCEPTUAL