MasterJEE
IIT-JEE | Medical | Foundations

Time : 3 hrs. **Answers & Solutions** M.M. : 360

*for*

# JEE (MAIN)-2019

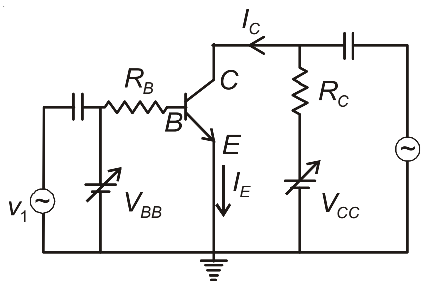
## (Online CBT Mode)

### (Physics, Chemistry and Mathematics)

**Important Instructions :**

1. The test is of **3 hours** duration.
2. The Test consists of **90** questions**.** The maximum marks are **360.**
3. There are ***three*** parts consisting of **Physics**, **Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response.
4. *Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for a question in the answer sheet.*
5. There is only one correct response for each question.

# PHYSICS

1. 


In the figure, given that $V_{BB}$ supply can vary from 0 to 5.0 $V$, $V_{CC}$ = 5$V$, $\beta_{dc}$ = 200, $R_B$ = 100 $K\Omega$, $R_C$ = 1 $K\Omega$ and $V_{BE}$ = 1.0$V$. The minimum base current and the input voltage at which the transistor will go to saturation, will be respectively:

(1) 25 μA and 3.5 V (2) 20 μA and 2.8 V

(3) 25 μA and 2.8 V (4) 20 μA and 3.5 V

**Answer (1)**

**Sol.** $V_{CC} - I_C R_C = 0$

$V_{BB} - I_b R_b = V_{BE}$

$I_C = 200\ I_b$

$I_C = 5$ mA, $I_b = 25$ μA

$V_{BB} - 25 \times 10^{-6} \times 100 \times 10^3 = 1$

$V_{BB} = 3.5$ V

2. A vertical closed cylinder is separated into two parts by a frictionless piston of mass $m$ and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is $l_1$, and that below the piston is $l_2$, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, $m$ will be given by :

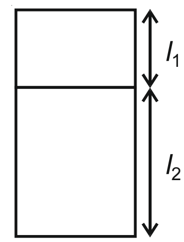
($R$ is universal gas constant and $g$ is the acceleration due to gravity)

(1) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$ (2) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$

(3) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1 l_2}\right]$ (4) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$

**Answer (1)**

**Sol.** $(P_2 - P_1)A = mg$




$$\left[\frac{nRT}{Al_2} - \frac{nRT}{Al_1}\right]A = mg$$

$$\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right] = m$$

3. A galvanometer, whose resistance is 50 $ohm$, has 25 divisions in it. When a current of 4 × $10^{-4}$ $A$ passes through it, its needle(pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 $V$, it should be connected to a resistance of:

(1) 6250 ohm (2) 250 ohm

(3) 200 ohm (4) 6200 $ohm$

**Answer (3)**

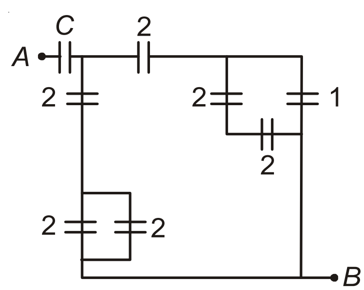
**Sol.** $I_g = 25 \times 4 \times 10^{-4}$

$= 10^{-2}$ A

$V = I_g(R + 50)$

$R = 200\ \Omega$

4. In the circuit shown, find $C$ if the effective capacitance of the whole circuit is to be 0.5 μ$F$. All values in the circuit are in μ$F$.




(1) $\frac{7}{11}$μF

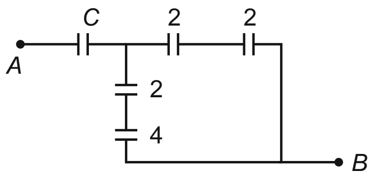
(2) 4μF

(3) $\frac{6}{5}$μF

(4) $\frac{7}{10}$μF

**Answer (1)**

**Sol.**





$$\frac{\frac{7}{3}C}{C+\frac{7}{3}}=\frac{1}{2}$$

$\Rightarrow$ $14C = 3C + 7$

$\Rightarrow$ $C = \frac{7}{11}\mu F$

5. The mean intensity of radiation on the surface of the Sun is about $10^8$ W/m$^2$. The rms value of the corresponding magnetic field is closet to:

(1) $10^2$ T (2) $10^{-4}$ T

(3) 1 T (4) $10^{-2}$ T

**Answer (2)**

**Sol.** $\frac{B_0^2}{2\mu_0}=\frac{10^8}{c}$

$$B_0=\frac{2\times10^8\times4\pi\times10^{-7}}{3\times10^8}$$

$$B_{rms}=\frac{B_0}{\sqrt{2}}$$

out of given option, option (2) is correct.

6. In a radioactive decay chain, the initial nucleus is ${}^{232}_{90}\text{Th}$. At the end there are 6 $\alpha$-particles and particles which are emitted. If the end nucleus is ${}^{A}_{Z}X$, $A$ and $Z$ are given by:

(1) $A$ = 200; $Z$ = 81 (2) $A$ = 202; $Z$ = 80

(3) $A$ = 208; $Z$ = 80 (4) $A$ = 208; $Z$ = 82

**Answer (4)**

**Sol.** $232 - 6 \times 4 = A$

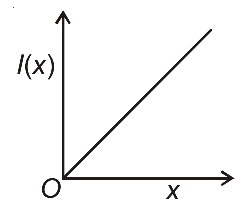
$A = 208$

$Z = 90 - 6 \times 2 + 4 \times 1$

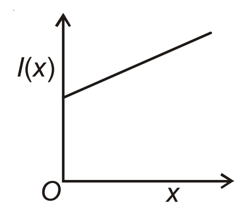
$= 82$

7. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of $x$ from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with $x$ correctly?

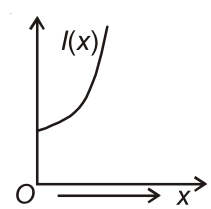
(1)



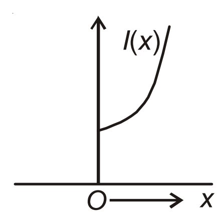

(2)




(3)




(4)




**Answer (3)**

**Sol.** $I(x) = I_0 + mx^2$

Hence option (3) is **correct**.

8. A simple harmonic motion is represented by :

$$y=5(\sin3\pi t+\sqrt{3}\cos3\pi t)\text{cm}$$

The amplitude and time period of the motion are:

(1) $10\,\text{cm}, \frac{2}{3}\text{s}$ (2) $5\,\text{cm}, \frac{2}{3}\text{s}$

(3) $10\,\text{cm}, \frac{3}{2}\text{s}$ (4) $5\,\text{cm}, \frac{3}{2}\text{s}$

**Answer (1)**

$y = 5\ \sin 3\pi t + \sqrt{3}\cos 3\pi t$

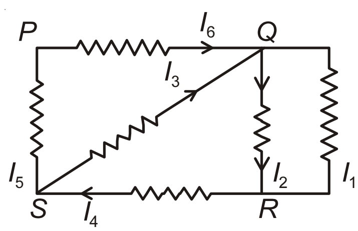
$y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

A = 10 cm

$\frac{2\pi}{T} = 3\pi$

$T = \frac{2}{3}$ s

9. In the given circuit diagram, the currents, $I_1 = -0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents $I_2$, $I_3$ and $I_6$, respectively, are:




(1) 1.1 A, 0.4 A, 0.4 A

(2) 1.1 A, – 0.4 A, 0.4 A

(3) 0.4 A, 1.1 A, 0.4 A

(4) –0.4 A, 0.4 A, 1.1 A

**Answer (1)**

**Sol.** At **Node S**

$I_4 = I_3 + I_5$

$I_4 = I_3 + 0.4$

$0.8 - 0.4 = I_3$, $I_3 = 0.4$ A

At **Node R**

$I_1 + I_2 = I_4$

$-0.3 + I_2 = 0.8$

$I_2 = 1.1$ A

at **Node Q**

$I_3 + I_6 = I_1 + I_2$

$0.4 + I_6 = -0.3 + 1.1$

$I_6 = 0.4$ A

10. When a certain photosensistive surface is illuminated with monochromatic light of frequency $\nu$, the stopping potential for the photo current is $-\frac{V_0}{2}$. When the surface is illuminated by monochromatic light of frequency $\frac{\nu}{2}$, the stopping potential is $-V_0$. The threshold frequency for photoeletric emission is:

(1) $\frac{3\nu}{2}$ (2) $\frac{4}{3}\nu$

(3) $\frac{5\nu}{3}$ (4) 2 $\nu$

**Answer (1)**

**Sol.** $2h\nu = 2\phi + eV_0$

$\frac{h\nu}{2} = \phi + eV_0$

$\frac{3h\nu}{2} = \phi$

$\nu_0 = \frac{3\nu}{2}$

11. A load of mass $M$ kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now th load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8.

The new value of increase in length of the steel wire is:

(1) 4.0 mm (2) zero

(3) 5.0 mm (4) 3.0 mm

**Answer (4)**

**Sol.** Area of wire $A = \pi r^2$

$$\frac{Mg}{\pi r^2} = \frac{\Delta\ell}{\ell_0}Y$$

$$\Rightarrow \frac{Mg}{\pi r^2} = \frac{4\times 10^{-3}}{2}\cdot Y \quad ...(i)$$

$8v_0\rho_0 = \mathrm{M}$

Now when load is immersed in liquid then

$$\frac{8v_0\rho_0 g - 2v_0\rho_0 g}{\pi r^2} = \frac{\Delta\ell'}{\ell_0}Y \quad ...(ii)$$

$$\Rightarrow \quad \frac{v_0 \ \ {}_0 g}{\pi r^2} = \frac{\ell}{\ell_0} Y$$

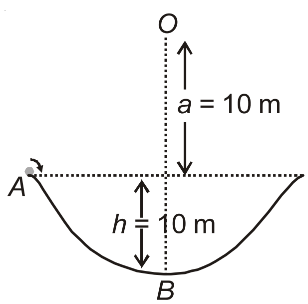
$$\frac{\Delta\ell'}{4\times10^{-3}} = \frac{6v_0\rho_0 g}{8v_0\rho_0 g}$$

$$\Rightarrow \quad \Delta\ell' = \frac{6}{8}\times4\times10^{-3}\text{ m}$$

$$\Rightarrow \quad \Delta\ell' = 3 \times 10^{-3}\text{ m} = 3\text{ mm}$$

12. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point *A*, as shown in the figure. The point *A* is a height *h* from point *B*. The particle slides along the frictionless surface. When the particle reaches point *B*, its angular momentum about *O* will be:

(Take g = 10 m/s$^2$)




(1) 2 kg-m$^2$/s (2) 3 kg-m$^2$/s

(3) 8 kg-m$^2$/s (4) 6 kg-m$^2$/s

**Answer (4)**

**Sol.:** $L = mv_0 r$

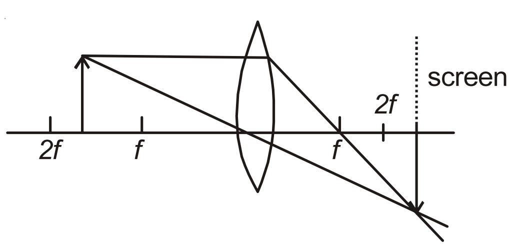
$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_i^2 + mgh$$

$$\Rightarrow \quad v_0^2 = 25 + 2 \times 10 \times 10 = 225$$

$v_0 = 15$ m/s

Now, $L = 20\times10^{-3}\times15\times20 = 6\text{ kg-m}^2\text{s}^{-1}$

13. Formation of real image using a biconvex lens is shown below:




If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?

(1) Erect real image (2) No change

(3) Image disappears (4) Magnified image

**Answer (3)**

**Sol.:** Initially, $\frac{1}{f} = \left(\frac{3}{2}-1\right)\frac{2}{R}$ $\left[\mu \text{ for glass} = \frac{3}{2}\right]$

$\therefore \quad f = R$

Now for water $\mu_w = 4/3$

$$\therefore \quad \frac{4}{3f'} = \frac{2}{6R} \Rightarrow f' = \frac{6\times4\times R}{3\times2}$$

$\Rightarrow \quad f' = 4R = 4f$

Now object in placed is between focus and lense, so there will not be any real image on screen.

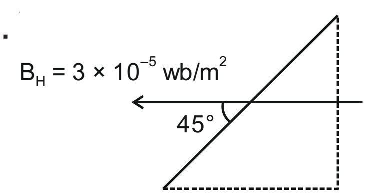
14. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of 5.0 ms$^{-1}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3\times10^{-4}$ Wb/m$^2$. The value of the induced emf in wire is:

(1) $1.1 \times 10^{-3}$ *V* (2) $0.3 \times 10^{-3}$ *V*

(3) $2.5 \times 10^{-3}$ V (4) $1.5 \times 10^{-3}$ V

**Answer (1)**

**Sol.**




$$\varepsilon = vBl = 5\times3\times10^{-5}\times10\times\frac{1}{\sqrt{2}} = 1.06\times10^{-3}\text{ volt}$$

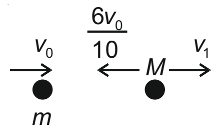
$$\varepsilon \approx 1.1\times10^{-3}\text{ volt}$$

15. An alpha-particle of mass *m* suffers 1-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is:

(1) 1.5 m (2) 3.5 m

(3) 4 m (4) 2 m

**Answer (3)**

**Sol.** 


$$mv_0 = Mv_1 - \frac{6mv_0}{10}$$

$$\Rightarrow \frac{16\,mv_0}{10} = Mv_1$$

Also $v_{App} = v_{sep.}$

$$\therefore\ v_0 = \frac{6\,v_0}{10} + v_1 \ \Rightarrow\ v_1 = \frac{4v_0}{10}$$

So $\frac{16\,mv_0}{10} = \frac{4v_0}{10}M \ \Rightarrow\ M = 4\,m$

16. To double the covering range of a TV transmission tower, its height should be multiplied by:

(1) $\sqrt{2}$ (2) 2

(3) $\frac{1}{\sqrt{2}}$ (4) 4

**Answer (4)**

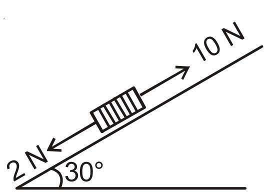
**Sol.** $d = \sqrt{2hR}$

$\therefore$ For $d'$ to be $2d$

$$\frac{2d}{d} = \frac{\sqrt{2h'R}}{\sqrt{2hR}} \Rightarrow 4h = h'$$

17. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is:

[Take g = 10 m/s$^2$]




(1) $\frac{1}{2}$ (2) $\frac{\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{4}$ (4) $\frac{2}{3}$

**Answer (2)**

**Sol.** $2 + mg\frac{1}{2} = \mu mg\frac{\sqrt{3}}{2}$ ...(i)

$\frac{mg}{2} + \mu mg\frac{\sqrt{3}}{2} = 10$ ...(ii)

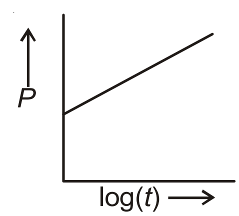
$\Rightarrow\ 2 + mg = 10\ \Rightarrow\ mg = 8$

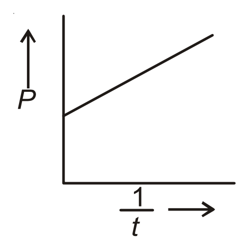
From eq (i), $6 = \mu \times 8 \times \frac{\sqrt{3}}{2}$

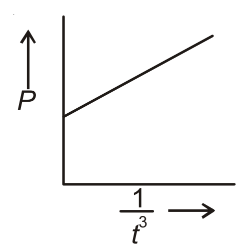
$$\therefore\ \mu = \frac{2\times 6}{8\sqrt{3}}$$

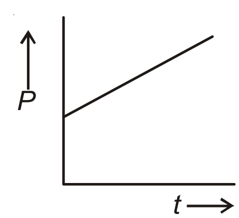
$$\Rightarrow\ \mu = \frac{\sqrt{3}}{2}$$

18. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by:

(1) 


(2) 


(3) 


(4) 


**Answer (No option is correct) [Bonus]**

**Sol.** $P - P_0 = \dfrac{4S}{R}$

$\therefore \quad P = \dfrac{4S}{R} + P_0$

$\therefore \quad P = 4S\left[\dfrac{4\pi}{3v}\right]^{1/3} + P_0$

$P = 4S\left[\dfrac{4\pi}{3kt}\right]^{1/3} + P_0$

Also $v = \dfrac{4}{3}\pi R^3$

$\Rightarrow \quad \left[\dfrac{3v}{4\pi}\right]^{1/3} = R$

Given $v = kt$

Correct form: $P = m\left(\dfrac{1}{t^{1/3}}\right) + c$

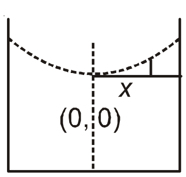
19. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be:

(1) 1.2 (2) 0.1

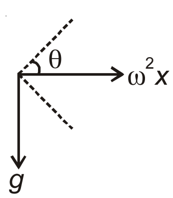
(3) 0.4 (4) 2.0

**Answer (4)**

**Sol.**




$\omega = 4\pi$ rad/sec




$\tan\theta = \dfrac{dy}{dx} = \dfrac{\omega^2 x}{g}$

$\Rightarrow \quad \int_0^h dy = \int_0^x \dfrac{\omega^2 x}{g} \cdot dx$

$\therefore \quad y = \left.\dfrac{\omega^2 x^2}{2g}\right|_0^{5\times10^{-2}}$

$\therefore \quad y = \dfrac{16\pi^2 \times 25 \times 10^{-4}}{2 \times 10} = 1.9 \text{ cm} \approx 2.0 \text{ cm}$

20. A plano-convex lens (focal length $f_2$, refractive index $\mu_2$, radius of curvature R) fits exactly into a plano-concave lens(focal length $f_1$, refractive index $\mu_1$, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be:

(1) $f_1 - f_2$ (2) $\dfrac{R}{\mu_2 - \mu_1}$

(3) $\dfrac{2f_1 f_2}{f_1 + f_2}$ (4) $f_1 + f_2$

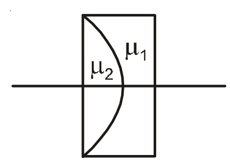
**Answer (2)**

**Sol.** For plano convex lens

$$\frac{1}{f_2} = \frac{\mu_2 - 1}{R} \quad \text{...(i)}$$

For plano – concave lens

$$\frac{1}{f_1} = -\left[\frac{\mu_1 - 1}{R}\right] \quad \text{...(ii)}$$




Now for combination

$$\frac{\mu_2}{v_1} - \frac{1}{\infty} = \frac{\mu_2 - 1}{\infty}$$

$$\frac{\mu_1}{v'_1} - \frac{\mu_2}{v_1} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{f} - \frac{\mu_1}{v'_1} = \frac{1 - \mu_1}{\infty}$$

$$\frac{1}{f} = \frac{(\mu_2 - \mu_1)}{R} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\Rightarrow \quad f = \frac{R}{\mu_2 - \mu_1}$$

21. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

(1) 1700 nm (2) 2020 nm

(3) 250 nm (4) 220 nm

**Answer (3)**

**Sol.** Energy lost by electron = 5.6 – 0.7 = 4.9 eV

$$\frac{hc}{\lambda_{min}} = 4.9$$

$$\Rightarrow \lambda_{min} = \frac{1240}{4.9} = 250 \text{ nm}$$

22. Two satellites, *A* and *B*, have masses *m* and 2 m respectively. *A* is in a circular orbit of radius *R*, and *B* is in a circular orbit of radius 2 *R* around the earth. The ratio of their kinetic energies, $\frac{T_A}{T_B}$ is

(1) 1 (2) $\frac{1}{2}$

(3) 2 (4) $\sqrt{\frac{1}{2}}$

**Answer (1)**

**Sol.** $\frac{V^2}{r} = \frac{GM}{r^2}$

$$\Rightarrow V^2 = \frac{GM}{r}$$

$$\frac{T_A}{T_B} = \frac{\frac{1}{2}mV_A^2}{\frac{1}{2}2mV_B^2} = \frac{1}{2}\left(\frac{V_A}{V_B}\right)^2 = \frac{1}{2}\frac{R_B}{R_A} = 1$$

23. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6 × $10^{-8}$ s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to

(1) 2 × $10^{-7}$ s (2) 3 × $10^{-6}$ s

(3) 0.5 × $10^{-8}$ s (4) 4 × $10^{-8}$ s

**Answer (4)**

**Sol.** $t \propto \frac{V}{\sqrt{T}}$

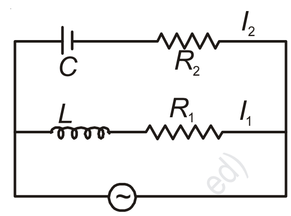
$$V \propto \frac{T}{P}$$

$$t \propto \frac{\sqrt{T}}{P}$$

$$\frac{t_1}{t_2} = \sqrt{\frac{5}{3}} \times \frac{1}{2} \times 6 \times 10^{-8}$$

$= 3.87 \times 10^{-8}$ s

24.




In the above circuit, $C = \frac{\sqrt{3}}{2}$ μF, $R_2$ = 20 Ω, $L = \frac{\sqrt{3}}{10}$ H and $R_1$ = 10 Ω. Current in $L$-$R_1$ path is $I_1$ and in $C$–$R_2$ path it is $I_2$. The voltage of A.C source is given by

$V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between $I_1$ and $I_2$ is

(1) 0° (2) 60°

(3) 30° (4) 90°

**Answer (No option is correct) [Bonus]**

**Sol.** $X_C = \frac{1}{\omega C} = \frac{1 \times 2}{100 \times \sqrt{3}} \times 10^6 = \frac{20}{\sqrt{3}}$ kΩ

$$X_C = \omega L = 10\sqrt{3}\ \Omega$$

As $X_C >> R_2$, $I_2$ leads $V$ by 90°.

$I_1$ lags $V$ by 60°

⇒ Phase difference between $I_1$ and $I_2$ 150°.

25. A paramagnetic material has $10^{28}$ atoms/$m^3$. Its magnetic susceptibility at temperature 350 K is 2.8 × $10^{-4}$. Its susceptibility at 300 K is

(1) $3.726 \times 10^{-4}$ (2) $3.672 \times 10^{-4}$

(3) $2.672 \times 10^{-4}$ (4) $3.267 \times 10^{-4}$

**Answer (4)**

**Sol.** $\chi \propto \dfrac{1}{T}$

$\chi_1 T_1 = \chi_2 T_2$

$\chi_2 = \dfrac{2.8 \times 350}{300} \times 10^{-4} = 3.267 \times 10^{-4}$

26. Let $l$, $r$, $c$ and $v$ represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\dfrac{1}{rcv}$ in SI units will be

(1) $[A^{-1}]$ (2) $[LA^{-2}]$

(3) $[LT^2]$ (4) $[LTA]$

**Answer (1)**

**Sol.** $\left[\dfrac{l}{rcv}\right] = \left[\dfrac{l}{TV}\right]$

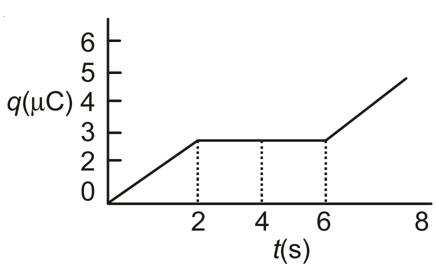
$[ML^2T^{-2}] = [lA^2]$

$\Rightarrow [l] = [ML^2T^{-2}A^{-2}]$

$[V] = \dfrac{ML^2T^{-2}}{AT} = ML^2T^{-3}A^{-1}$

$\Rightarrow \left[\dfrac{l}{rcv}\right] = \dfrac{ML^2T^{-2}A^{-2}}{T\ ML^2T^{-3}A^{-1}} = [A^{-1}]$

27. The charge on a capacitor plate in a circuit, as a function of time, is shown in the figure




What is the value of current at $t$ = 4 s?

(1) 2 μA (2) zero

(3) 3 μA (4) 1.5 μA

**Answer (2)**

**Sol.** $I = \dfrac{dq}{dt}$ = slope of $q - t$ graph.

28. A parallel plate capacitor with plates of area 1 $m^2$ each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is:

$\left(\text{Take } E_0 = 8.85 \times 10^{-12} \dfrac{C^2}{N - m^2}\right)$

(1) $8.85 \times 10^{-10}$ C

(2) $9.85 \times 10^{-10}$ C

(3) $6.85 \times 10^{-10}$ C

(4) $7.85 \times 10^{-10}$ C

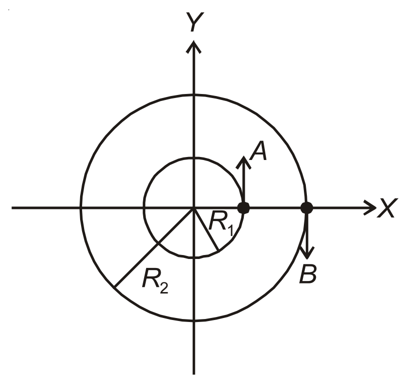
**Answer (1)**

**Sol.** $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{A\epsilon_0}$

$Q = EA\epsilon_0 = 100 \times 1 \times 8.85 \times 10^{-12}$ C

$= 8.85 \times 10^{-10}$ C

29. Two particles $A$, $B$ are moving on two concentric circles of radii $R_1$ and $R_2$ with equal angular speed $\omega$. At $t$ = 0, their positions and direction of motion are shown in the figure




The relative velocity $\vec{v}_A - \vec{v}_B$ at $t = \dfrac{\pi}{2\omega}$ is given by

(1) $\omega(R_2 - R_1)\hat{i}$

(2) $\omega(R_1 - R_2)\hat{i}$

(3) $-\omega(R_1 + R_2)\hat{i}$

(4) $\omega(R_1 + R_2)\hat{i}$

**Answer (1)**

**Sol.** $\omega \quad = \omega \frac{}{2\omega} = \frac{}{2}$

$$\overrightarrow{V_A} = \omega R_1(-\hat{i})$$

$$\overrightarrow{V_B} = \omega R_2\ (-\hat{i})$$

$$\overrightarrow{V_A} - \overrightarrow{V_B} = \omega[R_2 - R_1]\hat{i}$$

30. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to

(1) 322 $ms^{-1}$ (2) 341 $ms^{-1}$

(3) 328 $ms^{-1}$ (4) 335 $ms^{-1}$

**Answer (3)**

**Sol.** $\frac{\lambda_1}{4} = l_0 + 11$

$$\frac{\lambda_2}{4} = l_0 + 27$$

$$\frac{\lambda_2 - \lambda_1}{4} = 16 \text{ cm}$$

$$\Rightarrow V\left[\frac{1}{256} - \frac{1}{512}\right] = 0.64 \text{ m}$$

$\Rightarrow$ $V$ = 512 × 0.64 m/s

= 328 m/s

❑ ❑ ❑

# CHEMISTRY

1. An open vessel at 27°C is heated until two fifth of the air (assumed as an ideal gas) in it has escaped from the vessel. Assuming that the volume of the vessel remains constant, the temperature at which the vessel has been heated is

(1) 750 °C

(2) 750 K

(3) 500 °C

(4) 500 K

**Answer (4)**

**Sol.** Initial number of moles of an ideal gas = $n_1$

Find number of moles of the ideal gas

$$= n_2 = n_1 - \frac{2n_1}{5} = \frac{3n_1}{5}$$

At constant volume and pressure, the number of moles of an ideal gas is inversely proportional to temperature

$$n \propto \frac{1}{T}$$

$$n_1T_1 = n_2T_2$$

$$T_2 = \frac{n_1}{n_2}T_1 = \frac{5}{3} \times 300 = 500\,\text{K}$$

2. Given

(i) C (graphite) + $O_2(g) \rightarrow CO_2(g)$;
$\Delta rH^{\ominus} = x$ kJ mol$^{-1}$

(ii) C (graphite) + $\frac{1}{2}O_2(g) \rightarrow CO_2(g)$;
$\Delta rH^{\ominus} = y$ kJ mol$^{-1}$

(iii) $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$;
$\Delta rH^{\ominus} = z$ kJ mol$^{-1}$

Based on the above thermochemical equations, find out which one of the following algebraic relationships is correct?

(1) $x = y - z$

(2) $x = y + z$

(3) $y = 2z - x$

(4) $z = x + y$

**Answer (2)**

**Sol.** According to Hess's law, the enthalpy change of a reaction does not depend on the number of steps involved in the reaction.

$$\text{C(graphite)} + \frac{1}{2}O_2(g) \rightarrow CO\ (g)\ \Delta H_1^{\circ} = y \text{ kJ mol}^{-1}$$

$$CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)\ \Delta H_2^{\circ} = z \text{ kJ mol}^{-1}$$

---

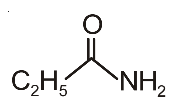
$$\text{C(graphite)} + O_2(g) \rightarrow \Delta^{\circ}H_3 = x \text{ kJ mol}^{-1}$$

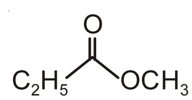
$\because\ \Delta H_3^{\circ} = \Delta H_1^{\circ} + \Delta H_2^{\circ}$

$x = y + z$

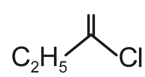
** in reaction ii, Product should be CO (gas) instead of $CO_2$ (gas).

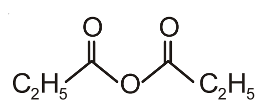
3. The increasing order of the reactivity of the following with $LiAlH_4$ is

(A) 

(B) 

O (C)



(D) 

(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (D) < (C)

(3) (A) < (B) < (D) < (C)

(4) (B) < (A) < (C) < (D)

**Answer (3)**

**Sol.** The reactivity order of carboxylic acid derivatives depends on the leaving tendency of the leaving group. Higher the leaving tendency of the leaving group, higher will be the reactivity of the compound. Therefore, reactivity order towards $LiAlH_4$ is

Acid halide > Acid anhydride > Ester > Amide

4. Among the following, the false statement is

(1) Tyndall effect can be used to distinguish between a colloidal solution and a true solution.

(2) Latex is a colloidal solution of rubber particles which are positively charged

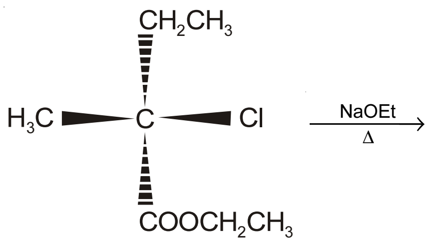
(3) Lyophilic sol can be coagulated by adding an electrolyte.

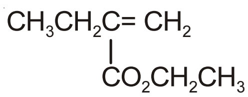
(4) It is possible to cause artificial rain by throwing electrified sand carrying charge opposite to the one on clouds from an aeroplane.

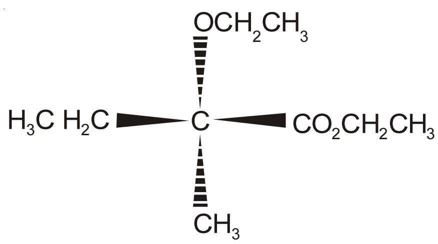
**Answer (2)**

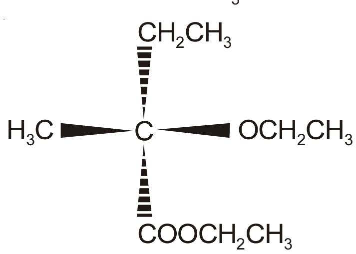
**Sol.** Latex is colloidal solution of rubber particles which are negatively charged.

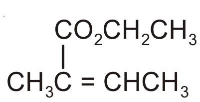
5. The major product of the following reaction is



(1) 

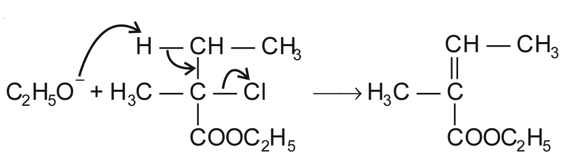
(2) 

(3) 

(4) 

**Answer (4)**

**Sol.** High temperature and strong base favours elimination reaction forming more stable alkene according to Saytzeff rule.



6. The magnetic moment of an octahedral homoleptic Mn(II) complex is 5.9 BM. The suitable ligand for this complex is

(1) CO

(2) Ethylenediamine

(3) $NCS^-$

(4) $CN^-$

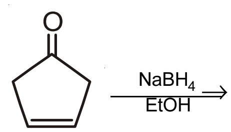
**Answer (3)**

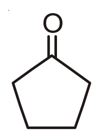
**Sol.** Electronic configuration of $Mn^{2+}$ is

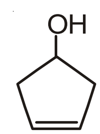
$Mn^{+2}$ : $3d^5$

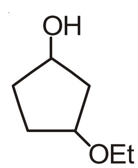
It has 5 unpaired electrons which corresponds to magnetic moment of $\sqrt{35}$ = 5.9 BM. This shows that the homoleptic complex of $Mn^{2+}$ has only weak field ligands and that is $NCS^-$. The remaining three ligands are strong field ligands.

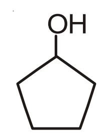
7. The major product of the following reaction is



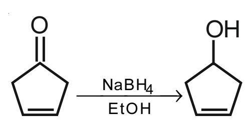
(1) 

(2) 

(3) 

(4) 

**Answer (2)**

**Sol.** 

$NaBH_4$ does not reduces the double bond in β-γ unsaturated aldehydes/ ketones.

Only the keto group will be reduced.

8. If $K_{sp}$ of $Ag_2CO_3$ is $8 \times 10^{-12}$, the molar solubility of $Ag_2CO_3$ in 0.1 M $AgNO_3$ is

(1) $8 \times 10^{-11}$ M

(2) $8 \times 10^{-12}$ M

(3) $8 \times 10^{-13}$ M

(4) $8 \times 10^{-10}$ M

**Answer (4)**

**Sol.** $AgNO_3 \longrightarrow \underset{0.1}{Ag^+} + \underset{0.1}{NO_3^-}$

$$Ag_2CO_3 \rightleftharpoons \underset{\substack{0.1+2x \\ \approx 0.1}}{2Ag^+} + \underset{x}{CO_3^{2-}}$$

$$K_{sp} = [Ag^+]^2[CO_3^{2-}]$$

$$= (0.1)^2\ x = 8 \times 10^{-12}$$

$$0.01\ x = 8 \times 10^{-12}$$

$$x = 8 \times 10^{-10}\ M$$

9. $\overset{\circ}{\wedge}_m$ for NaCl, HCl and NaA are 126.4, 425.9 and 100.5 S $cm^2 mol^{-1}$, respectively. If the conductivity of 0.001 M HA is $5 \times 10^{-5}$ S $cm^{-1}$, degree of dissociation of HA is

(1) 0.25

(2) 0.125

(3) 0.50

(4) 0.75

**Answer (2)**

**Sol.** $\Lambda^\circ_m$ (NaCl) = 126.4 S $cm^2$ $mol^{-1}$

$\Lambda^\circ_m$ (HCl) = 425.9 S $cm^2$ $mol^{-1}$

$\Lambda^\circ_m$ (NaA) = 100.5 S $cm^2$ $mol^{-1}$

$\Lambda^\circ_m$ (HA) = 425.9 – 126.4 + 100.5 = 400 S $cm^2$ $mol^{-1}$

K(HA) = $5 \times 10^{-5}$ S $cm^{-1}$

$$\Lambda^c_m = \frac{K \times 1000}{\text{Molarity}} = \frac{5 \times 10^{-5} \times 1000}{0.001} = 50$$

$$\alpha = \frac{\Lambda^c_m}{\Lambda^\circ_m} = \frac{50}{400} = 0.125$$

10. The major product of the following reaction is

$$\underset{\substack{|\ \ \ \ | \\ Br\ \ Br}}{CH_3CH_2CHCH_2} \xrightarrow[\text{(ii) } NaNH_2 \text{ in liq } NH_3]{\text{(i) KOH alc.}}$$

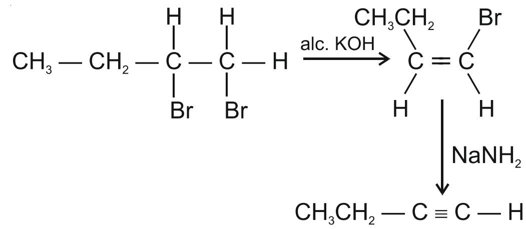
(1) $CH_3CH_2C \equiv CH$

(2) $CH_3CH = CHCH_2NH_2$

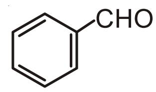
(3) $\underset{\substack{| \\ NH_2}}{CH_3CH_2CH} - \underset{\substack{| \\ NH_2}}{CH_2}$

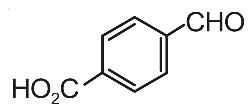
(4) $CH_3CH = C = CH_2$

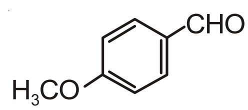
**Answer (1)**

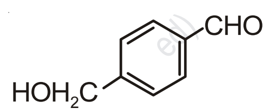
**Sol.** 

11. The aldehydes which will **not** form Grignard product with one equivalent Grignard reagent are

(A) 

(B) 

(C) 

(D) 

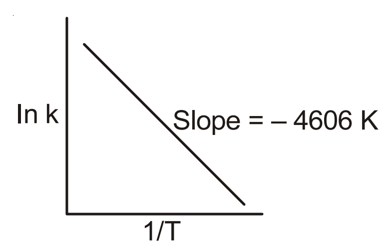
(1) (B), (C)  (2) (B), (D)

(3) (B), (C), (D)  (4) (C), (D)

**Answer (2)**

**Sol.** Grignard reagent will not react with aldehydes if it has a functional group which contains acidic hydrogen. Options (B) and (D) have —COOH and —$CH_2OH$ respectively which contan acidic H-atom.

12. For a reaction, consider the plot of In k versus 1/T given in the figure. If the rate constant of this reaction at 400 K is $10^{-5}$ $s^{-1}$, then the rate constant at 500 K is




(1) $4 \times 10^{-4}$ $s^{-1}$

(2) $10^{-6}$ $s^{-1}$

(3) $2 \times 10^{-4}$ $s^{-1}$

(4) $10^{-4}$ $s^{-1}$

**Answer (4)**

**Sol.** $\ln K = \ln A - \frac{E_a}{RT}$

$$\text{Slope} = \frac{E_a}{R} = 4606\text{ K}$$

$$\ln\left(\frac{K_2}{K_1}\right) = \frac{E_a}{R}\left(\frac{T_2 - T_1}{T_1T_2}\right)$$

$$= \frac{4606(100)}{400 \times 500}$$

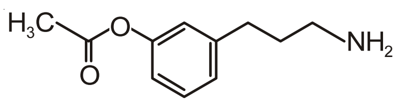
$$= 2.303$$

$$\Rightarrow \quad \log\left(\frac{K_2}{K_1}\right) = 1$$

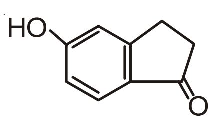
$$\frac{K_2}{K_1} = 10$$

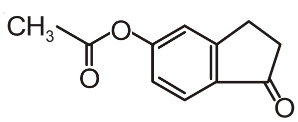
$$\Rightarrow \quad K_2 = 10K_1 = 10^{-5} \times 10 = 10^{-4}\ S^{-1}$$

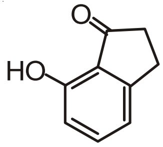
13. The major product of the following reaction is

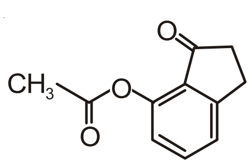



$\xrightarrow[\text{(ii) } CrO_3/H^+ \text{ (iii) } H_2SO_4\text{(conc.)},\Delta]{\text{(i) } NaNO_2/H^+}$

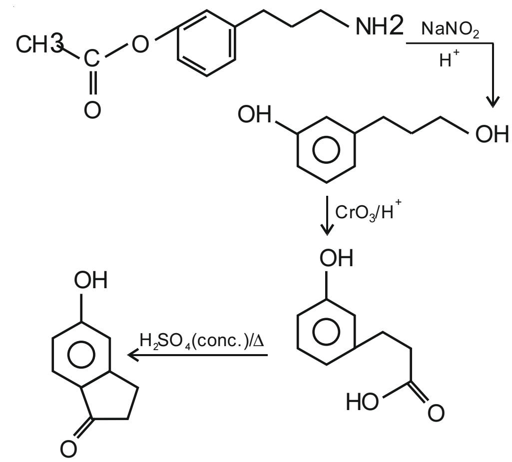
(1) 


(2) 


(3) 


(4) 


**Answer (1)**

**Sol.** 


14. The compound that is NOT a common component of photochemical smog is:

(1) $H_3C - \underset{\underset{O}{||}}{C} - OONO_2$

(2) $CH_2 = CHCHO$

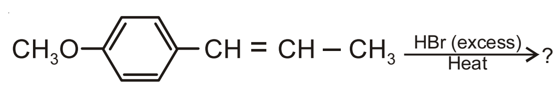
(3) $CF_2Cl_2$

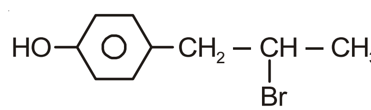
(4) $O_3$

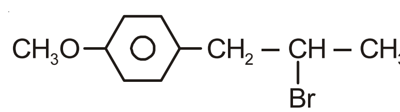
**Answer (3)**

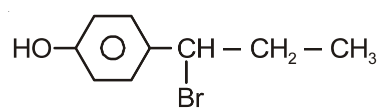
**Sol.** $CF_2Cl_2$ is not a common component of photochemical smog.

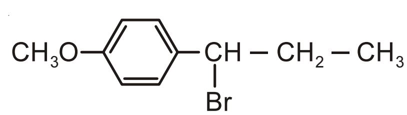
15. The major product in the following conversion is



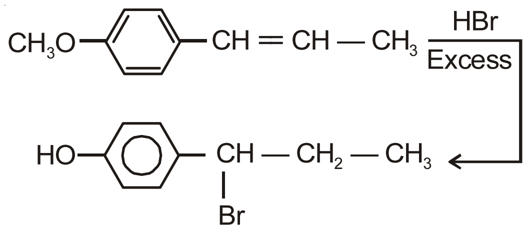

(1) 


(2) 

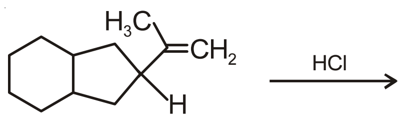

(3) 

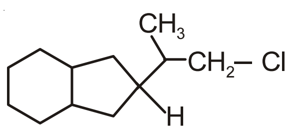

(4) 

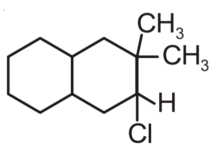

**Answer (3)**

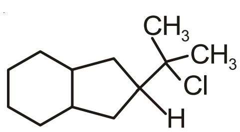
**Sol.** 

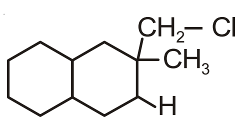
16. The major product of the following reaction is



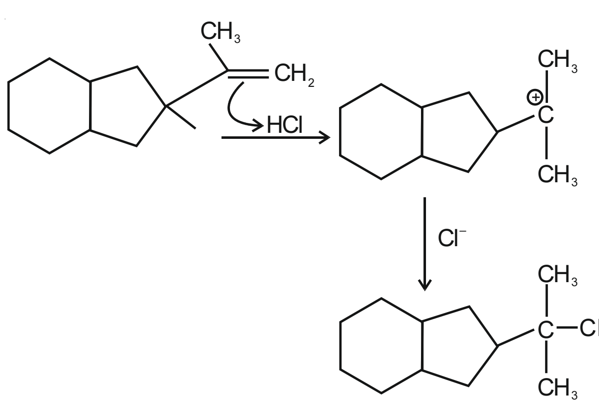
(1) 

(2) 

(3) 

(4) 

**Answer (3)**

**Sol.** 

17. Molecules of benzoic acid ($C_6H_5COOH$) dimerise in benzene. 'w' g of the acid dissolved in 30 g of benzene shows a depression in freezing point equal to 2 K. If the percentage association of the acid to form dimer in the solution is 80, then w is

(Given that $K_f$ = 5 K kg mol$^{-1}$, Molar mass of benzoic acid = 122 g mol$^{-1}$)

(1) 1.5 g

(2) 2.4 g

(3) 1.8 g

(4) 1.0 g

**Answer (2)**

**Sol.** $2C_6H_5COOH \longrightarrow (C_6H_5COOH)_2$

| | | |
|---|---|---|
| t = 0 | 1 | 0 |
| t | $1 - 2\alpha$ | $\alpha$ |

Moles at equilibrium = $1 - 2\alpha + \alpha = 1 - \alpha$

$2\alpha = 0.8$, $\alpha = 0.4$

Moles at equilibrium = 0.6

i = 0.6

$$\Delta T_f = ik_f m \Rightarrow \quad 2 = 0.6 \times 5 \times \left(\frac{\frac{w}{122}}{30}\right) \times 1000$$

w = 2.4 g

18. Chlorine on reaction with hot and concentrated sodium hydroxide gives

(1) $Cl^-$ and $ClO^-$

(2) $Cl^-$ and $ClO_2^-$

(3) $ClO_3^-$ and $ClO_2^-$

(4) $Cl^-$ and $ClO_3^-$

**Answer (4)**

**Sol.** $3Cl_2 + 6NaOH \longrightarrow 5NaCl + NaClO_3 + 3H_2O$

19. The correct statement(s) among I to III with respect to potassium ions that are abundant within the cell fluids is/are

I. They activate many enzymes

II. They participate in the oxidation of glucose to produce ATP

III. Along with sodium ions, they are responsible for the transmission of nerve signals

(1) I and III only

(2) I, II and III

(3) III only

(4) I and II only

**Answer (2)**

**Sol.** $K^+$ ions act as carriers for nerve signals, are activators for many enzymes and participate in the oxidation of glucose to form ATP.

20. If the de Broglie wavelength of the electron in $n^{th}$ Bohr orbit in a hydrogenic atom is equal to $1.5\ \pi a_0$ ($a_0$ is Bohr radius), then the value of n/z is

(1) 0.40 (2) 1.50

(3) 0.75 (4) 1.0

**Answer (3)**

**Sol.** $n\lambda = 2\pi r$

$$r = a_0 \frac{n^2}{z}$$

$$n\lambda = \frac{2\pi a_0 n^2}{z}$$

$$\lambda = \frac{2\pi a_0 n^2}{z}$$

$$1.5\pi a^0 = 2\pi a_0 \frac{n}{2}$$

$$\frac{n}{z} = \frac{3}{4} = 0.75$$

21. The volume strength of 1M $H_2O_2$ is

(Molar mass of $H_2O_2$ = 34 g $mol^{-1}$)

(1) 11.35

(2) 22.4

(3) 5.6

(4) 16.8

**Answer (1)**

**Sol.** Volume strength ≈ 11.2 × M

≈ 11.2

22. The correct order of atomic radii is

(1) Ce > Eu > Ho > N (2) N > Ce > Eu > Ho

(3) Eu > Ce > Ho > N (4) Ho > N > Eu > Ce

**Answer (3)**

**Sol.** Atomic radii follows the order

| Eu | > | Ce | > | Ho | > | N |
|---|---|---|---|---|---|---|
| 199 pm | | 183 pm | | 176 pm | | 70 pm |

23. The element that does NOT show catenation is

(1) Sn

(2) Ge

(3) Pb

(4) Si

**Answer (3)**

**Sol.** Lead Pb

24. The two monomers for the synthesis of nylon 6, 6 are

(1) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_4NH_2$

(2) $HOOC(CH_2)_6COOH$, $H_2N(CH_2)_6NH$

(3) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_6NH_2$

(4) $HOOC(CH_2)_4COOH$, $H_2N(CH_2)_4NH_2$

**Answer (3)**

**Sol.** Monomer of Nylon–6, 6 are adipic acid and hexamethylene diammine.

25. The pair that does NOT require calcination is

(1) $Fe_2O_3$ and $CaCO_3 \cdot MgCO_3$

(2) $ZnCO_3$ and CaO

(3) ZnO and MgO

(4) ZnO and $Fe_2O_3 \cdot xH_2O$

**Answer (3)**

**Sol.** ZnO and MgO

They are oxides while other are carbonates or hydrated oxides which require calcination.

26. The upper stratosphere consisting of the ozone layer protects us from the sun's radiation that falls in the wavelength region of

(1) 200 – 315 nm (2) 600 – 750 nm

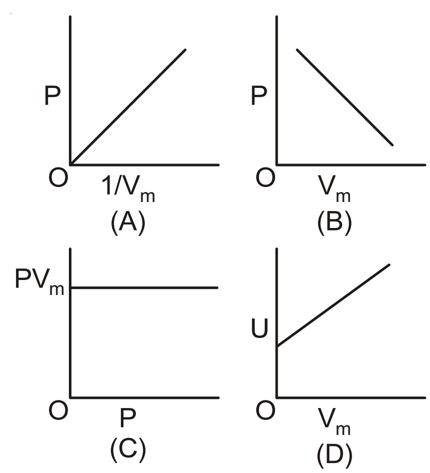
(3) 400 – 550 nm (4) 0.8 – 1.5 nm

**Answer (1)**

**Sol.** Ozone layer protects from ultra violet radiation.

∴ Wavelength range lies in 200 – 315 nm

27. The combination of plots which does not represent isothermal expansion of an ideal gas is




(1) (A) and (C) (2) (A) and (D)

(3) (B) and (C) (4) (B) and (D)

**Answer (4)**

**Sol.** (B) and (D) are not correct representation for isothermal expansion of ideal gas.

28. 8 g of NaOH is dissolved in 18 g of $H_2O$. Mole fraction of NaOH in solution and molality (in mol $kg^{-1}$) of the solution respectively are

    (1) 0.2, 22.20

    (2) 0.167, 22.20

    (3) 0.167, 11.11

    (4) 0.2, 11.11

**Answer (3)**

**Sol.** Mole faction $= \frac{n_2}{n_2 + n_1} = \frac{\frac{1}{5}}{\frac{1}{5}+1} = 0.167$

$$n_2 = \frac{8}{40} \quad n_1 = \frac{18}{18}$$

Molality $= \frac{8}{40} \times \frac{1000}{18} = 11.11\,m$

29. The element that shows greater ability of form $p\pi - p\pi$ multiple bonds, is

    (1) Sn

    (2) Si

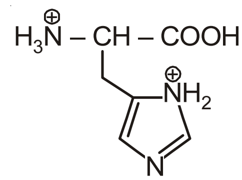
    (3) Ge

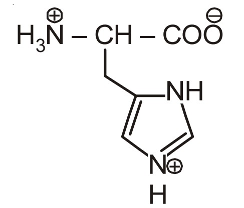
    (4) C

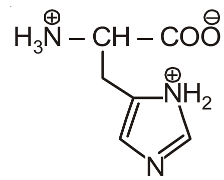
**Answer (4)**

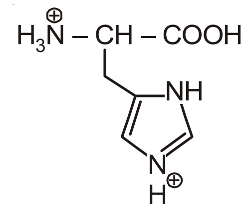
**Sol.** Carbon has small size so effective, lateral overlapping between 2p and 2p.

30. The correct structure of histidine in a strongly acidic solution (pH = 2) is

(1) 

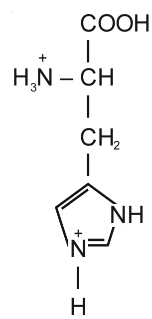
(2) 

(3) 

(4) 

**Answer (4)**

**Sol.** Histidine (in strongly acidic solution)



❑ ❑ ❑

# MATHEMATICS

1. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither of NCC nor for NSS is

(1) $\frac{5}{6}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{2}{3}$

**Answer (3)**

**Sol.** $A$ = Set of students who opted for NCC

$B$ = Set of Students who opted for NSS

$n(A) = 40$, $n(B) = 30$, $n(A \cap B) = 20$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 40 + 30 - 20$

$= 50$

$\therefore$ Required probability $= 1 - \frac{50}{60} = \frac{1}{6}$

2. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors, out of which vectors $\vec{b}$ and $\vec{c}$ are non-parallel. If $\alpha$ and $\beta$ are the angles which vector $\vec{a}$ makes with vectors $\vec{b}$ and $\vec{c}$ respectively and $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{1}{2}\vec{b}$, the $|\alpha - \beta|$ is equal to

(1) 90° (2) 45°

(3) 30° (4) 60°

**Answer (3)**

**Sol.** $\because$ $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$

Now $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{1}{2}\bar{b}$

$(\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c} = \frac{1}{2}\bar{b}$

$\therefore$ $\bar{a} \cdot \bar{c} = \frac{1}{2}$ and $\bar{a} \cdot \bar{b} = 0$

$|\bar{a}||\bar{c}|\cos\beta = \frac{1}{2}$ and $\alpha = 90°$

$\beta = 60°$

$\therefore$ $|\alpha - \beta| = |90° - 60°| = 30°$

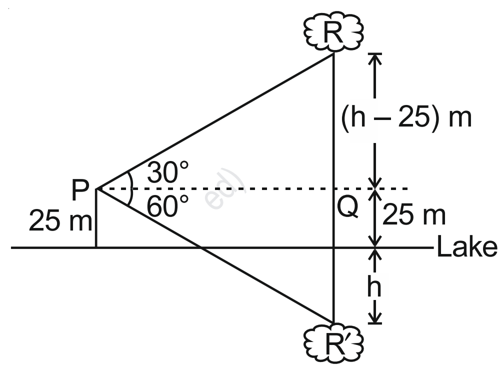
3. If the angle of elevation of a cloud from a point $P$ which is 25 m above a lake be 30° and the angle of depression of reflection of the cloud in the lake from $P$ be 60°, then the height of the cloud (in meters) from the surface of the lake is

(1) 45 (2) 50

(3) 42 (4) 60

**Answer (2)**

**Sol.** Let height of the cloud from the surface of the lane is $h$ meters.




$\therefore$ In $\Delta PQR$ :

$\tan 30° = \frac{h - 25}{PQ}$

$\therefore$ $PQ = (h - 25)\sqrt{3}$ ...(1)

and in $\Delta PQR'$ : $\tan 60° = \frac{h + 25}{PQ}$

$PQ = \frac{h + 25}{\sqrt{3}}$ ...(2)

From Eq. (1) and (2),

$(h - 25)\sqrt{3} = \frac{h + 25}{\sqrt{3}}$

$\therefore$ $h = 50$ m

4. The tangent to the curve $y = x^2 - 5x + 5$, parallel to the line $2y = 4x + 1$, also passes through the point

(1) $\left(\frac{1}{4}, \frac{7}{2}\right)$ (2) $\left(\frac{1}{8}, -7\right)$

(3) $\left(\frac{7}{2}, \frac{1}{4}\right)$ (4) $\left(-\frac{1}{8}, 7\right)$

**Answer (2)**

**Sol.** $\because$ Tangent is parallel to line $2y = 4x + 1$

Let equation of tangent be $y = 2x + c$ ...(1)

Now line (1) and curve $y = x^2 - 5x + 5$ has only one point of intersection.

$\therefore$ $2x + c = x^2 - 5x + 5$

$x^2 - 7x + (5 - c) = 0$

$\therefore$ $D = 49 - 4(5 - c) = 0$

$\therefore$ $c = -\dfrac{29}{4}$

$\therefore$ Equation of tangent: $y = 2x - \dfrac{29}{4}$

5. If $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$; then for all

$\theta \in \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right)$, det $(A)$ lies in the interval:

(1) $\left(1, \dfrac{5}{2}\right]$ (2) $\left(0, \dfrac{3}{2}\right]$

(3) $\left[\dfrac{5}{2}, 4\right)$ (4) $\left(\dfrac{3}{2}, 3\right]$

**Answer (4)**

**Sol.** $|A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 2 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix} R_1 \to R_1 + R_3$

$= 2(\sin^2\theta + 1)$

as $\theta \in \left(\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right)$

$\Rightarrow$ $\sin^2\theta \in \left(0, \dfrac{1}{2}\right)$

$\therefore$ $\det(A) \in [2, 3)$

$[2, 3) \subset \left(\dfrac{3}{2}, 3\right]$

6. In a game, a man wins Rs. 100 if he gets 5 or 6 on a throw of a fair die and loses Rs. 50 for getting any other number on the die. If he decides to throw the die either till he gets a five or a six or to a maximum of three throws, then his expected gain/loss (in rupees) is

(1) $\dfrac{400}{3}$ gain (2) 0

(3) $\dfrac{400}{9}$ loss (4) $\dfrac{400}{3}$ loss

**Answer (2)**

**Sol.** Probability of getting 5 or 6 = $P(E) = \dfrac{2}{6} = \dfrac{1}{3}$

Probability of not getting 5 or 6 = $P(E) = 1 - \dfrac{1}{3} = \dfrac{2}{3}$

$E$ will consider as success.

| Event | Success in I[st] attempt | Success in II[nd] attempt | Success in 3[rd] attempt | No success in 3 attempt |
|---|---|---|---|---|
| Probability | $\frac{1}{3}$ | $\frac{2}{3}\times\frac{1}{3}$ | $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{3}$ | $\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}$ |
| Gain/loss | 100 | 50 | 0 | –150 |

His expected gain/loss

$= \dfrac{1}{3}\times 100 + \dfrac{2}{9}\times 50 + \dfrac{8}{27}\times(-150)$

$= \dfrac{900 + 300 - 1200}{27}$

$= 0$

7. If a curve passes through the point (1, –2) and has slope of the tangent at any point $(x, y)$ on it as $\dfrac{x^2 - 2y}{x}$, then the curve also passes through the point

(1) (–1, 2) (2) $(\sqrt{3}, 0)$

(3) (3, 0) (4) $(-\sqrt{2}, 1)$

**Answer (2)**

**Sol.**

$\because$ $\dfrac{dy}{dx} = \dfrac{x^2 - 2y}{x}$

$\dfrac{dy}{dx} + \dfrac{2}{x}y = x$

I.F. $= e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Solution of equation

$$y \cdot x^2 = \int x \cdot x^2 dx$$

$$x^2 y = \frac{x^4}{4} + C$$

This curve passes through point (1, –2)

$\therefore \quad C = \frac{-9}{4}$

$\therefore$ equation of curve : $y = \frac{x^2}{4} - \frac{9}{4x^2}$

clearly it passes through $(\sqrt{3}, 0)$

8. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha, \beta \in [0, \pi]$, then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to

(1) $\sqrt{2}$ (2) $-\sqrt{2}$

(3) –1 (4) 0

**Answer (2)**

**Sol.**

$\because \sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}$

$\sin\alpha \cdot \cos\beta, \alpha, \beta \in [0, \pi]$

By A.M., G.M. ineqality;

$$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \left(\sin^4\alpha \cdot 4\cos^4\beta \cdot 1 \cdot 1\right)^{\frac{1}{4}}$$

$$\sin^4\alpha + 4\cos^4\beta + 1 + 1 \geq 4\sqrt{2}\sin\alpha \cdot |\cos\beta|$$

When $\cos\beta < 0$ then inequality still holds but L.H.S. is positive than $\cos\beta > 0$

Here, L.H.S. = R.H.S

$\therefore \quad \sin^4\alpha = 1$ and $\cos^4\beta = \frac{1}{4}$

$\therefore \quad \alpha = \frac{\pi}{2}$ and $\beta = \frac{\pi}{4}$

$\therefore \quad \cos(\alpha + \beta) - \cos(\alpha - \beta)$

$$= \cos\left(\frac{\pi}{2} + \beta\right) - \cos\left(\frac{\pi}{2} - \beta\right)$$

$$= -\sin\beta - \sin\beta$$

$$= -2\sin\frac{\pi}{4} = -\sqrt{2}$$

9. The integral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\} \log_e x\, dx$ is equal to

(1) $\frac{3}{2} - e - \frac{1}{2e^2}$ (2) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$

(3) $\frac{1}{2} - e - \frac{1}{e^2}$ (4) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

**Answer (1)**

**Sol.** $I = \int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\} \log_e x dx$

Let $\left(\frac{x}{e}\right)^x = t$

$\Rightarrow \quad x\ln\left(\frac{x}{e}\right) = \ln t$

$\Rightarrow \quad x(\ln x - 1) = \ln t$

On differentiating both sides w.r.t $x$ we get.

$$\ln x \cdot dx = \frac{dt}{t}$$

$$I = \int_{\frac{1}{e}}^{1}\left(t^2 - \frac{1}{t}\right) \cdot \frac{dt}{t} = \int_{\frac{1}{e}}^{1}\left(t - \frac{1}{t^2}\right)dt$$

$$= \left(\frac{t^2}{2} + \frac{1}{t}\right)_{\frac{1}{e}}^{1}$$

$$= \left(\frac{1}{2} + 1\right) - \left(\frac{1}{2e^2} + e\right)$$

$$= \frac{3}{2} - e - \frac{1}{2e^2}$$

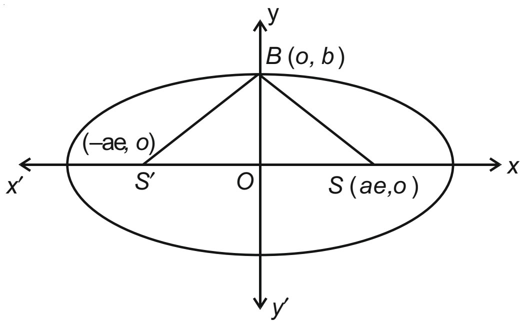
10. Let $S$ and $S'$ be the foci of an ellipse and $B$ be any one of the extremities of its minor axis. If $\Delta S'BS$ is a right angled triangle with right angle at $B$ and area ($\Delta S'BS$) = 8 sq. units, then the length of a latus rectum of the ellipse is

(1) $4\sqrt{2}$ (2) 4

(3) $2\sqrt{2}$ (4) 2

**Answer (2)**

**Sol.** (Slope of BS) × (Slope of BS′) = –1




$$\frac{b}{-ae}\times\frac{b}{ae}=-1$$

$b^2 = a^2e^2$ …(i)

∵ Area g ΔSBS′ = 8

$$\Rightarrow \frac{1}{2}\cdot 2ae\cdot b=8$$

$b^2 = 8$ …(ii)

∵ $a^2e^2 = 8$

$$\because \quad e^2=1-\frac{b^2}{a^2}$$

$a^2e^2 = a^2 - b^2$

$8 = a^2 - 8$

$a^2 = 16$

$$\therefore \quad \text{Length of latus rectum} =\frac{2b^2}{a}=\frac{2.8}{4}=4\,\text{units}$$

11. Let $f$ be a differentiable function such that $f(1) = 2$ and $f'(x) = f(x)$ for all $x \in R$. If $h(x) = f(f(x))$, then $h'(1)$ is equal to

(1) $2e$ (2) $2e^2$

(3) $4e$ (4) $4e^2$

**Answer (3)**

**Sol.** $f'(x) = f(x)$

$$\frac{f'(x)}{f(x)}=1$$

$$\Rightarrow \frac{f'(x)\,dx}{f(x)}=dx$$

$\Rightarrow \ln|f(x)| = x + c$

$f(x) = \pm e^{x+c}$

∵ $f(1) = 2$

$\Rightarrow f(x) = e^{x+c} = e^c e^x$

$\Rightarrow 2 = e^{1+c} = e\cdot e^c$

$$\Rightarrow f(x)=\frac{2}{e}e^x$$

$$\Rightarrow f'(x)=\frac{2}{e}e^x$$

$h(x) = f(f(x))$

$h'(x) = f'(f(x))\cdot f'(x)$

$$h'(1)=f'(2)\cdot f'(1)=\frac{2}{e}e^2\cdot\frac{2}{e}\cdot e=4e$$

⇒ Option (3) is correct.

12. If the function $f$ given by $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, for some $a \in R$ is increasing in (0, 1] and decreasing in [1, 5), then a root of the equation,

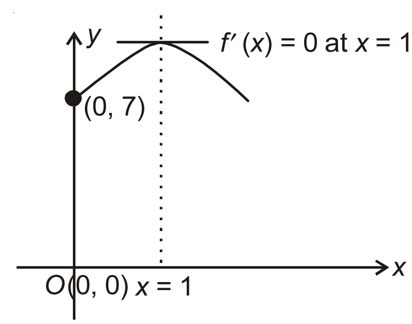
$$\frac{f(x)-14}{(x-1)^2}=0\ (x\neq 1)\ \text{is:}$$

(1) –7 (2) 5

(3) 6 (4) 7

**Answer (4)**

**Sol.** $f(x) = x^3 - 3(a - 2)x^2 + 3ax + 7$, $f(0) = 7$




$\Rightarrow f'(x) = 3x^2 - 6(a - 2)x + 3a$

$f'(1) = 0$

$\Rightarrow 1 - 2a + 4 + a = 0$

$\Rightarrow a = 5$

$$\Rightarrow \frac{f(x)-14}{(x-1)^2}=0$$

$$\Rightarrow \frac{x^3-9x^2+15x+7-14}{(x-1)^2}=0$$

$$\Rightarrow \frac{(x-1)^2(x-7)}{(x-1)^2}=0$$

$\Rightarrow x = 7$

13. There are $m$ men and two women participating in a chess tournament. Each participant plays two games with every other participant. If the number of games played by the men between themselves exceeds the number of games played between the men and the women by 84, then the value of $m$ is

(1) 9 (2) 7

(3) 11 (4) 12

**Answer (4)**

**Sol.** ${}^{m}C_2 \times 2 = {}^{m}C_1 \cdot {}^{2}C_1 \times 2 + 84$

$m(m-1) = 4m + 84$

$m^2 - 5m - 84 = 0$

$m^2 - 12m - 7m - 84 = 0$

$m(m - 12) + 7(m - 12) = 0$

$m = 12, \quad m = -7$

$\because \; m > 0$

$m = 12$

14. Let $Z$ be the set of integers.

If $A = \{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\}$ and

$B = \{x \in Z : -3 < 2x - 1 < 9\}$, then the number of subsets of the set $A \times B$, is

(1) $2^{15}$ (2) $2^{12}$

(3) $2^{18}$ (4) $2^{10}$

**Answer (1)**

**Sol.** $2^{(x+2)(x^2-5x+6)} = 1$

$\Rightarrow (x+2)(x-2)(x-3) = 0$

$x = -2, 2, 3$

$A = \{-2, 2, 3\}$

$\Rightarrow n(A) = 3$

$B : -3 < 2x - 1 < 9$

$-1 < x < 5$ and $x \in Z$

$\therefore B = \{0, 1, 2, 3, 4\}$

$n(B) = 5$

$n(A \times B) = 3 \times 5 = 15$

$\therefore$ Number of subsets of $A \times B = 2^{19}$

15. The expression $\sim(\sim p \to q)$ is logically equivalent to

(1) $p \wedge q$ (2) $p \wedge \sim q$

(3) $\sim p \wedge \sim q$ (4) $\sim p \wedge q$

**Answer (3)**

**Sol.** $\sim (\sim p \to Q) \equiv \sim (p \vee q) \equiv \sim p \wedge \sim q$

16. The mean and the variance of five observations are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4; then the absolute value of the difference of the other two observations, is

(1) 5 (2) 7

(3) 3 (4) 1

**Answer (2)**

**Sol.** Let two observations are $x_1$ and $x_2$

$$\frac{x_1 + x_2 + 3 + 4 + 4}{5} = 4$$

$\Rightarrow x_1 + x_2 = 9 \qquad ...(i)$

$$\text{Variance} = \frac{\sum x_i^2}{N} - (\bar{x})^2$$

$$5 \cdot 20 = \frac{9 + 16 + 16 + x_1^2 + x_2^2}{5} - 16$$

$(21 \cdot 20)5 = 41 + x_1^2 + x_2^2$

$x_1^2 + x_2^2 = 65 \qquad ...(ii)$

From (i) and (ii);

$x_1 = 8, x_2 = 1$

$|x_1 - x_2| = 7$

17. Let $S$ be the set of all real values of $\lambda$ such that a plane passing through the points $(-\lambda^2, 1, 1)$, $(1, -\lambda^2, 1)$ and $(1, 1, -\lambda^2)$ also passes through the point $(-1, -1, 1)$. Then $S$ is equal to

(1) $\{1, -1\}$ (2) $\{\sqrt{3}\}$

(3) $\{\sqrt{3}, -\sqrt{3}\}$ (4) $\{3, -3\}$

**Answer (3)**

**Sol.** $P(-\lambda^2, 1, 1)$, $Q(1, -\lambda^2, 1)$, $R(1, 1, -\lambda^2)$, $S(-1, -1, 1)$ lie on same plane

$$\therefore \begin{vmatrix} 1-\lambda^2 & 2 & 0 \\ 2 & 1-\lambda^2 & 0 \\ 2 & 2 & -\lambda^2 - 1 \end{vmatrix} = 0$$

$\Rightarrow (\lambda^2 + 1)((1 - \lambda^2)^2 - 4) = 0$

$\Rightarrow (3 - \lambda^2)(\lambda^2 + 1) = 0$

$\lambda^2 = 3$

$\lambda = \pm\sqrt{3}$

$S = \{-\sqrt{3}, \sqrt{3}\}$

18. If an angle between the line, $\frac{x+1}{2}=\frac{y-2}{1}=\frac{z-3}{-2}$ and the plane, $x-2y-kz=3$ is $\cos^{-1}\left(\frac{2\sqrt{2}}{3}\right)$, then a value of $k$ is

(1) $\sqrt{\frac{3}{5}}$ (2) $\sqrt{\frac{5}{3}}$

(3) $-\frac{5}{3}$ (4) $-\frac{3}{5}$

**Answer (2)**

**Sol.** Let angle between line and plane is $\theta$

$$\sin\theta = \left|\frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\cdot|\vec{n}|}\right|$$

$$= \left|\frac{(2\hat{i}+\hat{j}-2\hat{k})\cdot(\hat{i}-2\hat{j}-K\hat{k})}{\sqrt{9}\cdot\sqrt{1+4+K^2}}\right|$$

$$= \left|\frac{2-2+2K}{3\sqrt{5+K^2}}\right|$$

$$= \frac{2|K|}{3\sqrt{5+K^2}}$$

But $\cos\theta = \frac{2\sqrt{2}}{3} \Rightarrow \sin\theta = \frac{1}{3}$

$$\frac{2|K|}{3\sqrt{5+K^2}} = \frac{1}{3}$$

$4K^2 = 5 + K^2$

$3K^2 = 5$

$K = \pm\sqrt{\frac{5}{3}}$

19. Let $z_1$ and $z_2$ be two complex numbers satisfying $|z_1| = 9$ and $|z_2 - 3 - 4i| = 4$. Then the minimum value of $|z_1 - z_2|$ is

(1) 0 (2) $\sqrt{2}$

(3) 1 (4) 2

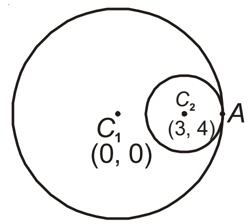
**Answer (1)**

**Sol.** $|z_1| = 9$, $|z_2 - 3 - 4i| = 4$

$z_1$ lies on a circle with centre $C_1(0, 0)$ and radius $r_1 = 9$

$z_2$ lies on a circle with centre $C_2(3, 4)$ and radius $r_2 = 4$

Minimum value of $|z_1 - z_2|$ is zero at point of contact (i.e. $A$)




20. The number of integral values of $m$ for which the quadratic expression, $(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, $x \in R$, is always positive, is:

(1) 8

(2) 3

(3) 6

(4) 7

**Answer (4)**

**Sol.** Given quadratic expression

$(1+2m)x^2 - 2(1+3m)x + 4(1+m)$, is positive for all $x \in R$, then

$1 + 2m > 0$ ...(i)

$D < 0$

$\Rightarrow 4(1+3m)^2 - 4(1+2m)4(1+m) < 0$

$\Rightarrow 1 + 9m^2 + 6m - 4[1 + 2m^2 + 3m] < 0$

$\Rightarrow m^2 - 6m - 3 < 0$

$m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

$\therefore m > -\frac{1}{2}$

So $m \in (3 - 2\sqrt{3}, 3 + 2\sqrt{3})$

So integral values of $m$ = {0, 1, 2, 3, 4, 5, 6}

Number of integral values of $m$ = 7

21. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in $A.P.$, then $n$ can be:

(1) 12

(2) 9

(3) 14

(4) 11

**Answer (3)**

**Sol.** $2\,{}^nC_5 = {}^nC_4 + {}^nC_6$

$$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$$

$$2 = \frac{5}{n-4} + \frac{n-5}{6}$$

$\Rightarrow$ $12(n-4) = 30 + n^2 - 9n + 20$

$\Rightarrow$ $n^2 - 21n + 98 = 0$

$(n-7)(n-14) = 0$

$(n-7)(n-14) = 0$

$n = 7,\ n = 14$

22. If a circle of radius $R$ pases through the origin $O$ and intersects the coordinate axes at $A$ and $B$, then the locus of the foot of perpendicular from $O$ on $AB$ is:

(1) $(x^2 + y^2)^2 = 4Rx^2y^2$

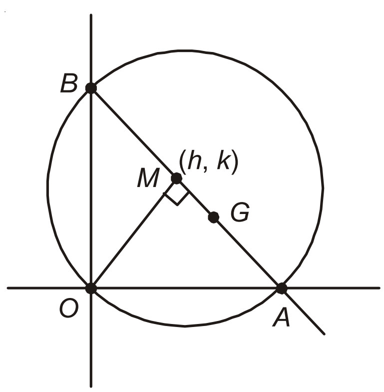
(2) $(x^2 + y^2)^2 = 4R^2x^2y^2$

(3) $(x^2 + y^2)^3 = 4R^2x^2y^2$

(4) $(x^2 + y^2)(x + y) = R^2xy$

**Answer (3)**

**Sol.** As $\angle AOB = 90°$




$AB \rightarrow$ Diameter

$M(h, k)$ is foot of perpendicular

$$M_{AB} = \frac{-h}{k}$$

Equation of $AB$ $(y-k) = \frac{-h}{k}(x-h)$

$\Rightarrow$ $hx + ky = h^2 + k^2$

$$A\left(\frac{h^2+k^2}{h}, 0\right)$$

$$B\left(0, \frac{h^2+k^2}{k}\right)$$

$AB = 2R$

$\Rightarrow$ $AB^2 = 4R^2$

$$\Rightarrow \left(\frac{h^2+k^2}{h}\right)^2 + \left(\frac{h^2+k^2}{k}\right)^2 = 4R^2$$

$\Rightarrow$ Locus is $(x^2 + y^2)^3 = 4R^2x^2y^2$

23. $\lim\limits_{n\to\infty}\left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \cdots + \frac{1}{5n}\right)$ is equal to:

(1) $\pi/4$ (2) $\tan^{-1}(3)$

(3) $\tan^{-1}(2)$ (4) $\pi/2$

**Answer (3)**

**Sol.** $I = \lim\limits_{n\to\infty}\sum\limits_{r=1}^{2n}\frac{n}{n^2+r^2}$

$$= \int_0^2 \frac{dx}{1+x^2} \qquad \frac{r}{n} \rightarrow x, \frac{1}{r} \rightarrow dx$$

$$= \left[\tan^{-1}x\right]_0^2$$

$= \tan^{-1}2$

24. The integral $\int \frac{3x^{13}+2x^{11}}{(2x^4+3x^2+1)^4}dx$ is equal to (where C is a constant of integration)

(1) $\frac{x^4}{6(2x^4+3x^2+1)^3} + C$

(2) $\frac{x^{12}}{(2x^4+3x^2+1)^3} + C$

(3) $\frac{x^{12}}{6(2x^4+3x^2+1)^3} + C$

(4) $\frac{x^4}{(2x^4+3x^2+1)^3} + C$

**Answer (3)**

**Sol.** $I = \int \frac{3x^{13}+2x^{11}}{x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$

$$I = \int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

Let $2+\frac{3}{x^2}+\frac{1}{x^4}=t$, $-2\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx = dt$

$$I = \int \frac{-\frac{dt}{2}}{t^4} = -\frac{1}{2}\frac{t^{-4+1}}{-4+1}+C$$

$$I = \frac{-1}{2}\times\frac{1}{(-3)}\frac{1}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3}+C$$

$$I = \frac{1}{6}\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$$

25. The equation of a tangent to the parabola, $x^2 = 8y$, which makes an angle $\theta$ with the positive direction of $x$-axis, is:

(1) $x = y\cot\theta - 2\tan\theta$

(2) $y = x\tan\theta + 2\cot\theta$

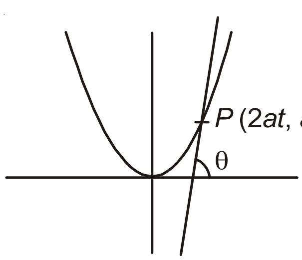
(3) $x = y\cot\theta + 2\tan\theta$

(4) $y = x\tan\theta - 2\cot\theta$

**Answer (3)**

**Sol.** $x^2 = 8y$

Equation of tangent at $P$




$tx = y + at^2$

$y = tx - at^2$

$t = \tan\theta$

$y = \tan\theta x - 2\tan^2\theta$

$\Rightarrow \cot\theta y = x - 2\tan\theta$

$$x = y\cot\theta + 2\tan\theta$$

26. If a straight line passing through the point $P(-3, 4)$ is such that its intercepted portion between the coordinate axes is bisected at $P$, then its equation is:

(1) $3x - 4y + 25 = 0$

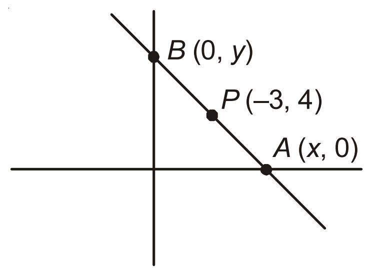
(2) $4x - 3y + 24 = 0$

(3) $x - y + 7 = 0$

(4) $4x + 3y = 0$

**Answer (2)**

**Sol.** $P$ is mid point of $AB$




So $x = -3 \times 2$

$x = -6$

and $y + 0 = 2 \times 4$

$y = 8$

Now equation $AB$ is

$$\frac{x}{-6}+\frac{y}{8}=1$$

$\Rightarrow \quad 4x - 3y + 24 = 0$

27. $\lim\limits_{x\to 1}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to:

(1) $\sqrt{\frac{2}{\pi}}$

(2) $\sqrt{\frac{\pi}{2}}$

(3) $\sqrt{\pi}$

(4) $\frac{1}{\sqrt{2\pi}}$

**Answer (1)**

**Sol.** $\lim\limits_{x\to 1^-}\dfrac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$

$$=\lim_{h\to 0} f(1-h)$$

$$=\lim_{h\to 0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{1-(1-h)}}$$

$$=\lim_{h\to 0}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}(1-h)}}{\sqrt{h}}$$

$$=\lim_{h\to 0}\frac{-\dfrac{1}{2\sqrt{2\sin^{-1}(1-h)}}\times 2\times\dfrac{1}{\sqrt{1-(1-h)^2}}(-1)}{\dfrac{1}{2\sqrt{h}}}$$

$$=\lim_{h\to 0}\frac{\dfrac{1}{\sqrt{2\sin^{-1}(1-h)}}\dfrac{1}{\sqrt{h(2-h)}}}{\dfrac{1}{2\sqrt{h}}}$$

$$=2\times\frac{1}{\sqrt{\pi}}\times\frac{1}{\sqrt{2}}=\sqrt{\frac{2}{\pi}}$$

28. if the sum of the first 15 terms of the series $\left(\frac{3}{4}\right)^3+\left(1\frac{1}{2}\right)^3+\left(2\frac{1}{4}\right)^3+3^3+\left(3\frac{3}{4}\right)^3+\ldots$ is equal to 225 $k$, then $k$ is equal to:

(1) 108 (2) 27

(3) 9 (4) 54

**Answer (2)**

**Sol.** $S=\left(\frac{3}{4}\right)^3+\left(\frac{3}{2}\right)^3+\left(\frac{9}{4}\right)^3+(3)^3+\ldots$

$$S=\left(\frac{3}{4}\right)^3+\left(\frac{6}{4}\right)^3+\left(\frac{9}{4}\right)^3+\left(\frac{12}{4}\right)^3+\ldots$$

$$T_r=\left(\frac{3r}{4}\right)^3$$

$$225K=\sum_{r=1}^{15}T_r=\left(\frac{3}{4}\right)^3\sum_{r=1}^{15}r^3$$

$$225K=\frac{27}{64}\times\left(\frac{15\times 16}{2}\right)^2$$

$K = 27$

29. The total number of irrational terms in the binomial expansion of $\left(7^{1/5}-3^{1/10}\right)^{60}$ is:

(1) 48 (2) 49

(3) 54 (4) 55

**Answer (3)**

**Sol.** $T_{r+1}={}^{60}C_r\left(7^{\frac{1}{5}}\right)^{60-r}\left(-3^{\frac{1}{10}}\right)^r$

$$={}^{60}C_r.(7)^{12-\frac{r}{5}}(-1)^r.(3)^{\frac{r}{10}}$$

So for getting rational terms, $r$ should be multiple of L.C.M. of (5, 10)

So $r$ can be 0, 10, 20, 30, 40, 50, 60.

Now total number of terms = 61

Total irrational terms = 61 – 7 = 54

30. The set of all values of $\lambda$ for which the system of linear equations

$x-2y-2z=\lambda x$

$x+2y+z=\lambda y$

$-x-y=\lambda z$

(has a non-trivial solution)

(1) Contains exactly two elements

(2) Contains more than two elements

(3) Is a singleton

(4) Is an empty set

**Answer (3)**

**Sol.** $x(1-\lambda)-2y-2z=0$

$x+(2-\lambda)y+z=0$

$-x-y-\lambda z=0$

for getting a non-trivial solution

$\Delta = 0$

$$\begin{vmatrix}1-\lambda & -2 & -2\\ 1 & 2-\lambda & 1\\ -1 & -1 & -\lambda\end{vmatrix}=0$$

$\Rightarrow (1-\lambda)^3=0$

$\lambda = 1$

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