23. ALTERNATING CURRENT

1. INTRODUCTION

A majority of electrical power in the world is generated, distributed, and consumed in the form of 50-Hzor60-Hz sinusoidal alternating current (AC) and voltage. It is used for household andindustrial applications.

AC has several advantages over DC. The major advantage of AC is the fact that it can be transformedinto any form, whereas direct current (DC) cannot. A transformer permits voltage to be stepped up or down for the purpose of transmission. Transmission of high voltage (in terms of KV) implies that less current is required to produce the same amount of power. Less current permits thinner wires to be used for transmission.

In this chapter, we will introduce a sinusoidal signal and its basic mathematic equation. We will discuss and analyse circuits where currents i(t) and voltages v(t) vary with time. The phasor analysis techniques will be used to analyse electronic circuits under sinusoidal steady-state operating conditions. The chapter will conclude with single-phase power.

2. SINUSOIDAL WAVEFORMS

AC, unlike DC, flows first in one direction, then in the opposite direction. The most common AC waveform is a sine (or sinusoidal) waveform.

In discussing AC signal, it is necessary to express the current and voltage in termsof maximum or peak values, peak-to-peakvalues, effective values, average values, or instantaneous values. Each of these values has a different meaning and is used to describe a different amount of current or voltage. $V(t)=V_0 \sin \omega t$. Where V_o is the peak voltage, $\omega = 2\pi f$ is the angular frequency expressed in radian per second (rad/s), f is the frequencyexpressed in Hertz (Hz), t is time expressed in second (s).

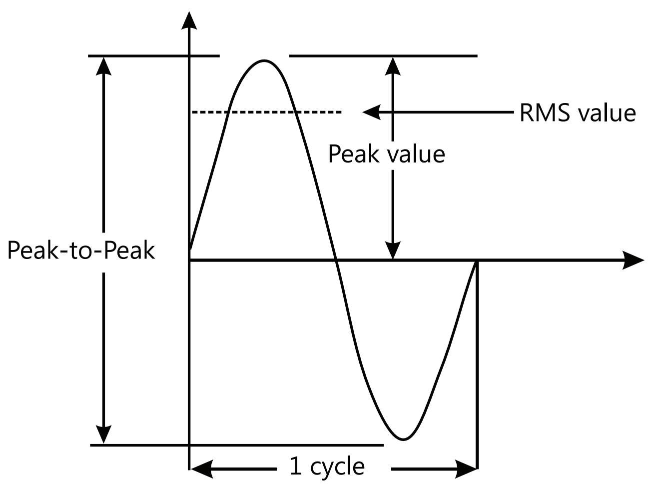


Figure 23.1: Sinusoidal Waveform.

2.1 Instantaneous Value

The instantaneous value of an AC signal is the value of voltage or current at one particular instant. The value may be zero, if the particular instant is the time in the cycle at which the polarity of the voltage is changing. It may also be the same as the peak value, if the selected instant is the time in the cycle at which the voltage or current stops increasing and starts decreasing. There are actually an infinitenumber of instantaneous values between zero and the peak value.

MASTERJEE CONCEPTS

It is always advisable to find symmetries in functions while calculating rms and average value to reduce the period of integration.It helps a lot in avoiding unnecessary calculations when functions are defined part by part.

Yashwanth Sandupatla (JEE 2012, AIR 821)

2.2 Average Value

Average value of a function,from t_1 to t_2, is defined as $\langle f\rangle = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from $t_2 - t_1$.

$I_{avg} = \dfrac{\int_0^t i\,dt}{\int_0^t dt}$,where i is the instantaneous value of the current.

2.2.1 For Sinusoidal Variation of Current and Voltages

Case I: Average value over complete cycle $\dfrac{\int_0^t i_o \sin(\omega\tau + \theta)dt}{\int_0^t dt}$. Similarly $V_{avg} = 0$

Case II: Average value over half cycle $I_{avg} = \dfrac{\int_0^{t/2} i_o \sin(\omega\tau + \theta)dt}{\int_0^{t/2} dt} = \dfrac{2i_o}{\pi}$; Similarly $V_{avg} = \dfrac{2\,i_o}{\pi}$

Illustration 1: An electric heater draws 2.5 A current from a 220-V, 60-Hz power supply. Find **(JEE MAIN)**

(a) The average current

(b) The average of the square of the current

(c) The current amplitude

(d) The supply voltage amplitude

Sol: In AC circuit, the average value of current over a long time interval is zero but I^2 is not zero. The r.m.s. value of current and voltage is given by $I_{rms} = \dfrac{I_{max}}{\sqrt{2}}$ and $V_{rms} = \dfrac{V_{max}}{\sqrt{2}}$.

(a) The average of sinusoidal AC values over any whole number of cycles is zero.

(b) RMS value of current = I_{rms} =2.5 A so, $\left(I^2\right)_{av} = \left(I_{rms}\right)^2 = 6.25\ A^2$

(c) $I_{rms} = \frac{I_m}{\sqrt{2}}$; So, current amplitude $I_m = \sqrt{2}\, I_{rms} = \sqrt{2}\left(2.5A\right) = 3.5\ A$

(d) $V_{rms} = 220V = \frac{V_m}{\sqrt{2}}$, So, supply voltage amplitude $V_m = \sqrt{2}\left(V_{rms}\right) = \sqrt{2}\left(220V\right) = 311\ V.$

2.3 Effective Value (RMS Value)

This is the value of AC signal that will have the same effect on a resistance as a comparable value of direct voltage or current will have on the same resistance. It is possible to compute the effective value of a sine wave of current to a good degree of accuracy by taking equally spacedinstantaneous values of current along the curve and extracting the square root of the average of the sum of the squared values. For this reason, effective value is sometimes called RMS value.

Root mean square value of a function, from t_1 to t_2 is defined as $f_{rms} = \frac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}$

The magnitude of I_{rms} is given by $I_{rms}^{\,2} = \frac{\int_0^T I^2 dt}{\int_0^T dt} = \frac{\int_0^T I_0^2 \sin^2\left(\omega\tau\right) dt}{\int_0^T dt} = \frac{I_0^2}{2}$

$I_{\ eff} = I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$ Where I_0 is the peak value of the current. Similarly V_{eff} or $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ E_0 A$

MASTERJEE CONCEPTS

RMS value is actually more important because in the context of power transmission,the loss in energy due to a resistor plays an important role. And the power is given by i^2R, where R is the resistance.

Yashwanth Sandupatla (JEE 2012, AIR 821)

Illustration 2: Find the RMS value of current I = $I_m \sin\omega t$ from (i) t=0 to t=$\frac{\pi}{\omega}$ (ii)t=$\frac{\pi}{2\omega}$to t=$\frac{3\pi}{2\omega}$ **(JEE MAIN)**

Sol: In AC circuit over time interval $0 \le t \le T$ the RMS value of current is given by

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}} = \sqrt{\frac{\int_0^T I_0^2 \sin^2\left(\omega\tau\right) dt}{\int_0^T dt}} = \frac{I_0}{\sqrt{2}} \quad \text{where } T = \frac{2\pi}{\omega}$$

(i) $I_{rms} = \sqrt{\dfrac{\int_0^{\frac{\pi}{\omega}} I_m^2 \sin^2(\omega t)\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$ (ii) $I_{rms} = \sqrt{\dfrac{\int_{\frac{\pi}{2}}^{\frac{3\pi}{2\pi}} I_m^2 \sin^2(\omega t)\,dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{I_m^2}{2}} = \dfrac{I_m}{\sqrt{2}}$ A

MASTERJEE CONCEPTS

The RMS value of one cycle or half cycle (either a positive or negative cycle) is same.

GV Abhinav (JEE 2012, AIR 329)

2.4 Difference between Sine and Cosine Representation of AC Signal

The sine and cosine are essentially the same function, but with a 90^0 phase difference. For example, $\sin\omega t = \cos\left(\omega t - 90^0\right)$. Multiples of 360^0 may be added to or subtracted from the argument of any sinusoidal function, without changing the value of the function. To realize this, let us consider

$V_1 = V_{P1}\cos\left(10t + 20^0\right) = V_{P1}\sin\left(10t + 90^0 + 20^0\right)$ ... (i)

$= V_{P1}\sin\left(10t + 110^0\right)$ Leads $V_2 = V_{P2}\sin\left(10t - 40^0\right)$ ... (ii)

by 150^0. It is also correct to say that v_1 lags v_2 by 210^0, since v_1 may be written as

$V_1 = V_{P1}\sin\left(10t - 250^0\right)$ V ... (iii)

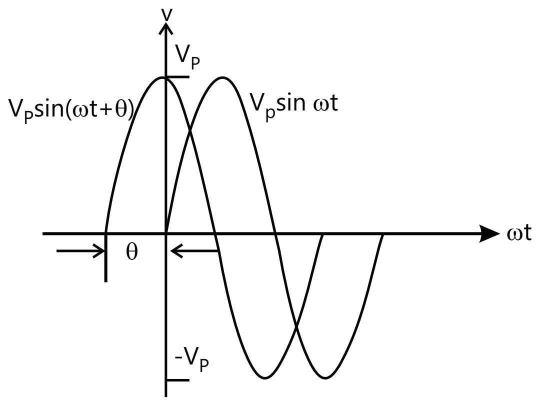


Figure 23.2: Representation of voltage as sine and cosine function

3. POWER IN AC CIRCUITS

Average power in alternating current circuit over time t is defined as $P_{avg}=\frac{\int_0^t vidt}{\int_0^t dt}$, where V andiare the instantaneous values of voltage and current respectively. Let V= $V_0 \sin \omega t;\ i=i_0 \sin(\omega t-\phi)$, Average power over a cycle

$$P_{avg}=\frac{\int_0^T v_o i_o \sin\omega t.\sin(\omega t-\phi)dr}{\int_0^T dt};=\frac{v_o i_o\int_0^T\left(\sin^2\omega t\cos\phi-\frac{1}{2}\sin 2\omega t\sin\phi\right)dt}{T}=\frac{1}{2}V_0 i_0\cos\phi=V_{rms}i_{rms}\cos\phi$$

The term $\cos\phi$ is known as power factor.

If the current leads voltage, it is said to be leading, whereas, if it lags voltage, it is said to be lagging. Thus, a power factor of 0.5 lagging means the current lags voltage by 60° (as $\cos^{-1}0.5 = 60°$). The product of V_{rms} and i_{rms} gives the apparent power, while the true power is obtained by multiplying the apparentpower by the power factor $\cos\phi$

Thus, apparent power = $V_{rms} \times i_{rms}$ and true power=apparent power × power factor

For $\phi = 0°$, the current and voltage are in phase. The power is thus, maximum ($V_{rms} \times i_{rms}$). For $\phi = 90°$ the power is zero. The current is then stated wattless. Such a case will arise when resistance in the circuits is zero. The circuit is purely inductive or capacitive. The case is similar to that of a frictionless pendulum, where the total work done by gravity upon the pendulum cycle is zero.

We shall discuss more about the power and power factor later, shortly after we define impedance and its properties.

Illustration 3: When a voltage $V_s = 200\sqrt{2}$ sin ($\omega t+15°$) is applied to an AC circuit, the current in thecircuit is found to be I=2 sin ($\omega t+\pi/4$) then average power consumed in the circuit is **(JEE MAIN)**

(A) 200 W (B) 400 $\sqrt{2}$ W (C) 100 $\sqrt{2}$ W (D) 200 $\sqrt{2}$ W

Sol: Power in any AC circuit is calculated as $P_{av}=V_{rms}I_{rms}\cos\phi$ where ϕ is phase angle between V and I.

$$P_{av}=V_{rms}\ I_{rms}\cos\phi=\frac{200\sqrt{2}}{\sqrt{2}}.\frac{2}{\sqrt{2}}\cos\left(30^0\right)=100\sqrt{6}\ \ W$$

4. SIMPLE AC CIRCUITS

4.1 Purely Resistive Load

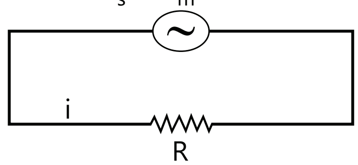


Figure 23.3: AC voltage applied to resistive load

Writing KVL along the circuit (see Fig. 23.3), V_S iR=0

Or $I=\frac{V_s}{R}=\frac{V_m\sin\omega t}{R}=I_m\sin\omega t$.

We see that the phase difference between potential differences across resistance, V_R and i_R is 0.

$$I_m = \frac{V_m}{R} \Rightarrow I_{rms} = \frac{V_{rms}}{R} \quad \langle P \rangle = V_{rms}\, I_{rms} \cos\phi = \frac{V_{rms}{}^2}{R}$$

4.2 Purely Capacitive

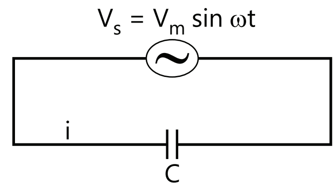


Figure 23.4: AC voltage applied to capacitive load

Writing KVL along the circuit shown in Fig. 23.4

$V_S - \frac{q}{c} = 0$ And current in the circuit is

$$I = \frac{dq}{dt} = \frac{d(cv)}{dt} = \frac{d(cv_m \sin\omega t)}{dt} = cv_m \omega \sin\omega t = \frac{V_m}{1/\omega C}\cos\omega t.$$

$$= \frac{V_m}{X_C}\cos\omega t = I_m \cos\omega t$$

where $X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is Ohm (Ω).

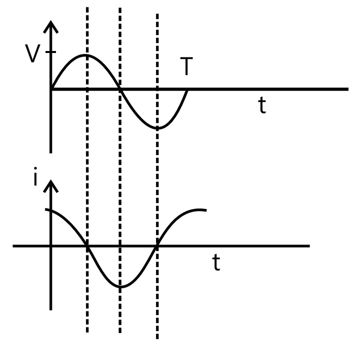


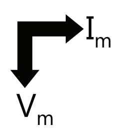
Figure 23.5

From the graph of current versus time and voltage versus time,

it is clear that current attains its peak value at a time $\frac{T}{4}$ before the

time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ phase difference.

$= \omega \Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{\pi}{2}$ i_c leads v_c by $\pi/2$ diagrammatically (phasediagram) represented as



Since $\phi = 90^0$, $\langle P \rangle = V_{rms}\, I_{rms} \cos\phi = 0$

The current leads the voltage by $\pi/2$ in a capacitive circuit

MASTERJEE CONCEPTS

<P>=0 doesn't mean it is zero in any period less than the time period. In actuality, first the capacitor gets charged up, gaining energy during the first half cycle, and loses it for the next half cycle.So overall, power becomes zero.Same goes for the inductor in a different fashion (magnetic field plays a role there).

Yashwanth Sandupatla (JEE 2012, AIR 821)

4.3 Pure Inductive Circuit

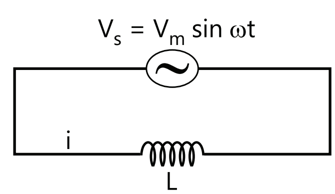


Figure 23.6: AC voltage applied to inductive load

Writing KVL along circuit, $V_s - L\frac{di}{dt} = 0$; $L\frac{di}{dt} = V_m \sin\omega t$; $\int L di = \int V_m \sin\omega t\, dt$;

$i = -\frac{V_m}{\omega L}\cos\omega t + C$; $\langle i \rangle = 0$; C=0;

$\therefore i = -\frac{V_m}{\omega L}\cos\omega t$ $I_m = \frac{Vm}{X_L}$ From the graph of current versus time and voltage versus time, it is clear that voltage attains its peak value at a time $\frac{T}{4}$ before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$, the phase difference $= \omega \Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{T} = \frac{\pi}{2}$

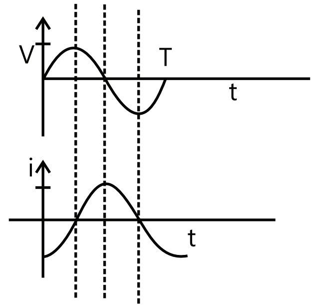
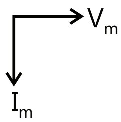


Figure 23.7: Variation of current and voltage with respect to time

Diagrammatically (See Fig. 23.7) it is represented 

as

i_L lags behind V_L by $\pi / 2$ since $\phi = 90^0$, $<P> = V_{rms} I_{rms} \cos\phi = 0$. The current lags voltage by $\pi / 2$ in a purely inductive circuit.

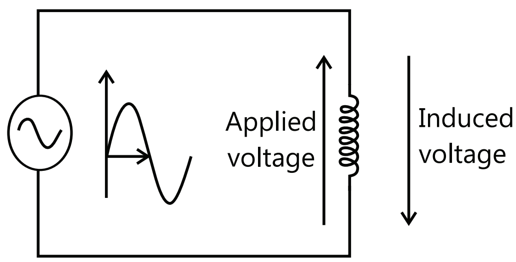


Figure 23.8: AC voltage applied to purely inductive circuit

5. IMPEDANCE

We have already seen that the inductive reactance $X_L = \omega L$ and capacitance reactance $X_C = 1/\omega L$ play the role of an effective resistance in apurely inductive and capacitive circuit respectively. In the series RLC circuit, the effective resistance is the impedance,defined as $Z = \sqrt{R + \left(X_L - X_C\right)^2}$ ...(iv)

The relationship between Z, X_L, and X_C can be represented by the diagram shown in Fig. 23.9.

Following is a diagrammatic representation of the relationship between Z, X_L and X_C.

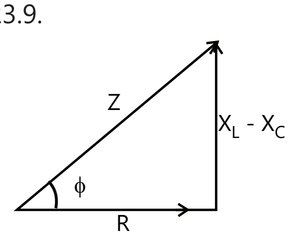


Figure 23.9: Impedance Triangle

The impedance has SI unit of Ω. In terms of Z the current may be rewritten as I(t)

$$= \frac{V_0}{Z}\sin\left(\omega t - \phi\right) \quad \text{...(v)}$$

Notice that the impedance Z also depends on the angular frequency ω ,as do X_L and X_C.

Using the above equations for phase ϕ and Z, we may readily recover the limit for simple circuit (with only one element).

MASTERJEE CONCEPTS

By now, students should get a clear idea of individual behaviour of inductor, capacitor and resistor and be able to visualize phasors. They should never get confused whetherinductor, capacitor is leading, etc.

Chinmay S Purandare (JEE 2012, AIR 698)

The upcoming series of circuits would be easy to understand because they are just a superposition of individual phasor diagrams.

6. MIXED AC CIRCUITS

6.1 LR Circuit

If V_R, V_L and $V_s$are the RMS voltage across are R, L and the AC source respectively. Then,

$V_S = \sqrt{V_R^2 + V_L^2} = I_2\sqrt{R^2 + X_L^2}$ Where I_s is r.m.s value of source current.

The total opposition to the current is called impedance and it is denoted by Z.

$$Z = \frac{V_S}{I_S} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

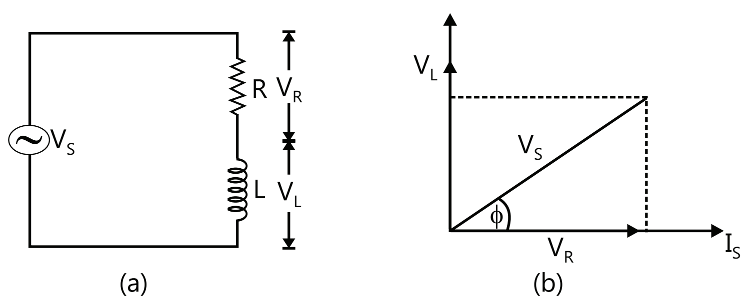


Figure 23.10: (a) AC voltage applied to LR circuit (b) Phasor diagram of voltage drops across R and L

The phase angel ϕ by which the applied voltage leads the current is $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Illustration 4: An alternating voltage of 220V RMS at a frequency of 40 cycles/second is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6Ω in series. Calculate (a) The current, (b) Potential difference across the resistance, (c) Potential different across inductance, (d) The time lag. **(JEE MAIN)**

Sol: Theimpedance of LR circuit is $Z = \sqrt{R^2 + (\omega L)^2}$. The RMS value of the current is $I_{rms} = \frac{V_{rms}}{Z}$. In LR circuit, the current lags the applied voltage by phase angleϕ obtained as $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

The impedence of the L-R series circuit is given by:

$$Z^2 = \left[R^2 + (\omega L)^2\right]^{1/2} = \left[(R)^2 + (2\pi f L)^2\right]^{1/2}$$

$$= \left[62 + \left(2\times3.14\times40\times0.01\right)^2\right]^{1/2} = 6.504\ \Omega$$

(a) RMS value of the current: $I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{6.504} = 33.83$ A

(b) The potential difference across the resistance is given by: $V_R = I_{rms} \times R = 33.83 \times 6 = 202.83$ V

(c) Potential difference across the inductance is given by:

$V_L = I_{rms} \times (\omega L) = 33.83 \times \left(2\times3.14\times0.01\right) = 96.83$ V

(d) Phase angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$; so, $\phi = \tan^{-1}(0.4189 = 22.46)$

Now time lag $= \frac{\phi}{360} = T = \frac{22.46}{360} = 0.0623$ s.

Illustration 5: A $\frac{9}{100\pi}$ H inductor and a 12 Ω resistance are connected in a series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage. **(JEE MAIN)**

Sol: Theimpedance of LR circuit is $Z = \sqrt{R^2 + \left(\omega L\right)^2}$. The RMS value of the current is $I_{rms} = \frac{V_{rms}}{Z}$. In LR circuit, the current lags the applied voltage by phase angleϕ obtained as $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

Here $X_L = \omega L = 2\pi fL = 2\pi\times50\times\frac{9}{100\pi} = 9\Omega$

So, $Z = \sqrt{R^2 + X_L^2} = \sqrt{12^2 + 9^2} = 15\Omega$

(a) $I = \frac{V}{Z} = \frac{225}{15} = 15$A and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1}3/4 = 37^\circ$

i.e., the current will lag the applied voltage by 37^0 in phase.

Illustration 6: A chokecoil is needed to operate an arc lamp at 160 V (RMS) and 50 Hz. The arc lamp has an effective resistance of 5Ω when running of 10 A (RMS). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160V (dc), what additional resistance is required? Compare the power losses in both cases.

(JEE ADVANCED)

Sol: The choke coil is a LR circuit having large inductanceand small resistance. The potential difference across the resistor and inductor is added vectorially:$V^2 = V_R^2 + V_L^2$.

As for the lamp, $V_R = IR = 10\times5 = 50V$, so when it is connected to 160 V ac source though a choke in series,$V^2 = V_R^2 + V_L^2$, $V_L = \sqrt{160^2 - 50^2} = 152V$

And as, $V_L = IX_L = I\omega L = 2\pi fLI$ $L = \frac{V_L}{2\pi fI} = \frac{152}{2\times\pi\times50\times10} = 4.84\times10^{-2}H$

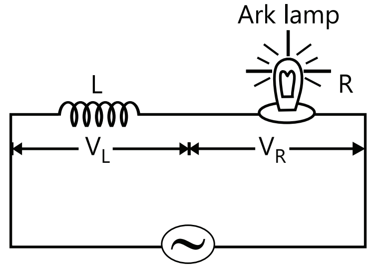


Figure 23.11

Now the lamp is to be operated at 160 V dc; instead of choke, if additional resistance r is I put in a series with it, V = I(R+r), i.e. 160 = 10(5+r) i.e. r = 11Ω In case of AC, as choke has no resistance, power loss in the choke will be zero, while the bulb will consume $P=I^2R=10^2\times 5=500$ W. However, in case of DC,as resistance r is to be used instead of choke, the power loss in the resistance r will be $PL=10^2\times 11=1100$ W

While the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke, more than double the power consumed by the lamp is wasted by the resistance r.

6.2 RC Circuits

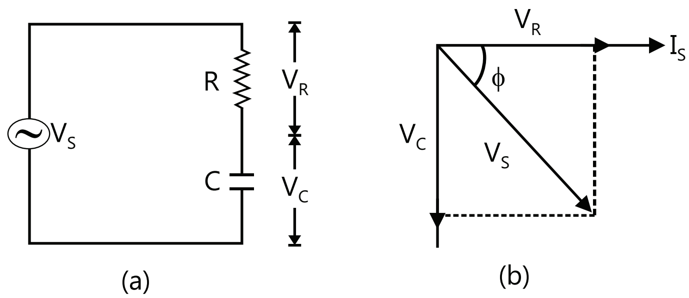


Figure 23.12: (a) AC voltage applied to RC circuit (b) Phasor diagram of voltage drops across R and C

If V_s, V_R and V_C are RMS voltages across a source, resistance and capacitor respectively

$$V_S=\sqrt{V_R^2+V_C^2}=I_S=\sqrt{R^2+X_C^2}$$

Impedance of circuit, $$Z=\frac{V_S}{I_S}=\sqrt{R^2+X_C^2}=\sqrt{R^2+\frac{1}{\omega^2C^2}}$$

V_S leads I_S by $$\phi=\tan^{-1}\left(\frac{X_C}{R}\right)=\tan^{-1}\left(\frac{1}{\omega CR}\right)$$

The current leads the applied voltage by angel ϕ.

Illustration 7: An ac source of angular frequency ω is fed across a resister R and a capacitor C in series. The current registered is I. If now, the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency ω.

(JEE MAIN)

Sol: The impedance of RC circuit is:

$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$. The RMS current is $I_{rms}=\frac{V_{rms}}{Z}$

According to the given problem, $I=\frac{V}{Z}=\frac{V}{\left[R^2+(1/C\omega)^2\right]^{1/2}}$ ... (i)

And for frequency of $\frac{\omega}{3}$, $\frac{I}{2}=\frac{V}{\left[R^2+(3/C\omega)^2\right]^{1/2}}$ ... (ii)

Substituting the value of I from equation (i) in (ii),

$$4\left(R^2+\frac{1}{C^2\omega^2}\right)=R^2+\frac{9}{C^2\omega^2}\quad \text{i.e..}\ \frac{1}{C^2\omega^2}=\frac{3}{5}R^2$$

So that, $\frac{X}{R}=\frac{(1/c\omega)}{R}=\frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R}=\sqrt{\frac{3}{5}}$

Illustration 8: In an RC series circuit, the RMSvoltage of source is 200V, and its frequency is 50 Hz. If R = 100 Ω and $C=\frac{100}{\pi}\mu F$, find

(a) Impedance of the circuit
(b) Power factor angle
(c) Power factor
(d) Current
(e) Maximum current
(f) Voltage across R
(g) Voltage across C
(h) Max voltage across R
(i) Maxvoltage cross C
(j)<P>
(k) $<P_R>$
(l) $<P_C>$

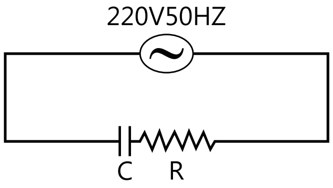


Figure 23.13

(JEE ADVANCED)

Sol: The impedance of RC circuit is

$$Z=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$$

The RMS current is $I_{rms}=\frac{V_{rms}}{Z}$. The phase angle between current and voltage is given by $\tan\phi=\frac{X_C}{R}$. The RMS value of current and voltage is $I_{rms}=\frac{I_0}{\sqrt{2}}$ and $V_{rms}=\frac{V_0}{\sqrt{2}}$. Power developed in circuit is $P=V_{rms}I_{rms}\cos\phi$.

$$X_C=\frac{10^6}{\frac{100}{\pi}(2\pi 50)}=100\ \Omega$$

(a) $Z=\sqrt{R^2+XC^2}=\sqrt{100^2+(100)^2}=100\sqrt{2}\Omega$

(b) $\tan\phi=\frac{X_C}{R}=1 \qquad \therefore \phi=45^0$

(c) Power factor= $\cos\phi=\frac{1}{\sqrt{2}}$

(d) Current $I_{rms}=\frac{V_{rms}}{Z}=\frac{200}{100\sqrt{2}}=\sqrt{2}\,A$

(e) Maximum current =$I_{rms}\sqrt{2}=\sqrt{2}\,A$

(f) Voltage across R=$V_{R,rms}$=I_{rms} R= $\sqrt{2}\times 100$ V

(g) Voltage across C= $V_{Crms}=I_{rms}X_C=\sqrt{2}\times 100$ V

(h) Max voltage across R= $\sqrt{2}\ V_{R,rms}$=200 V

(i) Max voltage across C= $\sqrt{2}\ V_{Crms}$=200 V

(j) $<P> = V_{rms} I_{rms} \cos\phi = 200 \times \sqrt{2} \times \frac{1}{\sqrt{2}}$ W

(k) $<P_R> I_{rms}{}^2 R = 200$ W

(l) $<P_c> = 0$

MASTERJEE CONCEPTS

We observed here that inductor's reactance is directly proportional to the frequency used in the circuit and vice-versa for capacitor. So a combined circuit of them can be used as a frequency filter. High frequencies can be received by noting the voltage across capacitor and low frequencies can be noted using the inductor.

Nitin Chandrol (JEE 2012, AIR 134)

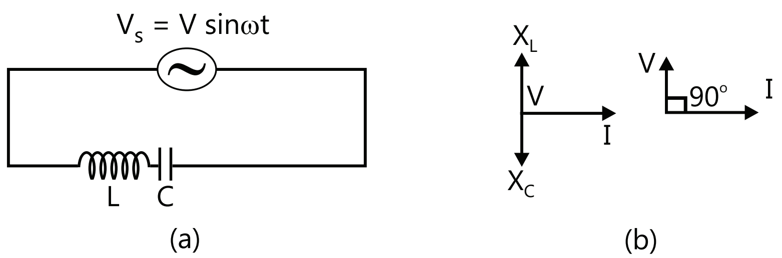


Figure 23.14: (a) AC voltage applied to LC circuit (b) Phasor diagram for voltage drops across L and R

6.3 LC Circuits

From the phasor diagram $V = I\left|\left(X_L - X_C\right)\right| = IZ;\ \phi = 90^0$

6.4 RLC Circuits

For LCR series circuits $V_s = \sqrt{V_R^2 + \left(V_L - V_C\right)^2}$

Impedance of circuits $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}$

V_S leads I_S by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega L - \frac{1}{\omega c}}{R}\right)$

Power in LCR circuit= $V_{rms} I_{rms} \cos f = V_{rms} I_{rms} \frac{R}{Z} = V_R I_{rms}$

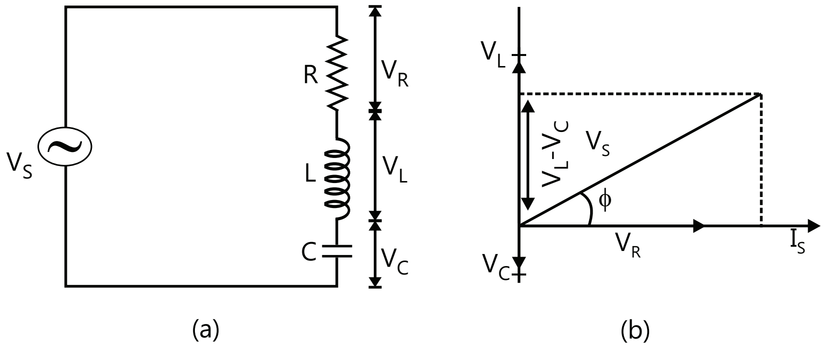


Figure 23.15: (a) AC voltage applied to LCR circuit. (b) Phasor diagram of voltage drops across L, C and R

Where $\cos\phi$ is called the power factor of the LCR circuit.

6.4.1 Resonance in RLC Circuits

At a particular angular frequency ω_o of the source, when $X_L = X_C$ or $\omega_o L = \frac{1}{\omega_0 C}$, the impedance of the circuit becomes minimum and equal to R and therefore, the current will be maximum. The circuit is then said to be in resonance. The resonance angular frequency ω_o andfrequency V_0 given by

$$\omega_0 = \frac{1}{\sqrt{2}}.\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

The variation of RMS current with the frequency of the applied voltage is shown in the Fig. 23.16. If the applied voltage consists of a number of frequency components, the current will be large for the components having frequency V_0. The Q factor of an LCR series circuit is given by Q= $\frac{\omega_0 L}{R}$. A direct current of a flows uniformly throughout the cross-section of theconductor. An alternating current on the other hand, flows mainly along the surface of the conductor. This effect is known as the skin effect. The reason is that when ac flows through aconductor, the flux change in the inner part of the conductor is higher.

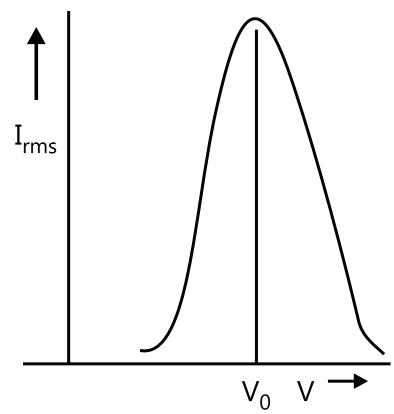


Figure 23.16

MASTERJEE CONCEPTS

The idea of resonance is used in TV channelsfor clarity: a particular frequency is assignedto a channel and when this frequency is received by the receiver,the current corresponding to this frequency becomes maximum. This helps in maximum possible separation of channels, thus increasing their individual clarity.

It is also used by intelligence agencies to intercept the signals of anti-social elements. They generally use frequency of a very high order.

Nivvedan (JEE 2009, AIR 113)

Illustration 9: In the circuit shown in the Fig. 23.17, find **(JEE MAIN)**

(a) The reactance of the circuit

(b) Impedance of the circuit

(c) The current

(d) Reading of the ideal AC voltmeters

(These are hot wire instruments and read RMS values)

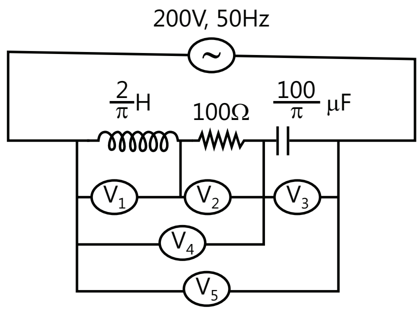


Figure 23.17

Sol: In series LCR circuit, the impedance is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ where X_C and X_L are the capacitive reactance and inductive reactance respectively.

(a) $X_L=2\pi fL=2\pi\times 50\times\frac{2}{\pi}=200\Omega$ $X_C=\dfrac{1}{2\pi 50\dfrac{100}{\pi}\times 10^{-6}}=100\,\Omega$

∴ The reactance of the circuit $X=X_L$-$X_{C=}$200-100=100 Ω

Since $X_L>X_C$, the circuit is called inductive.

(b) Impedance of circuit Z= $\sqrt{R^2+X^2}=\sqrt{100^2+100^2}=100\sqrt{2}\ \Omega$

(c) The current $I_{rms}=\dfrac{V_{rms}}{Z}=\dfrac{200}{100\sqrt{2}}=\sqrt{2}A$

(d) Readings of ideal voltage

$V_1: I_{rms}X_L=200\sqrt{2}\ V$

$V_2: I_{rms}R=100\sqrt{2}\ V$

$V_3: I_{rms}X_c=100\sqrt{2}\ V$

$V_4: I_{rms}\sqrt{R^2+X_L{}^2}=100\sqrt{10}\ \ V$
$V_4: I_{rms}Z=200\ \ V,$, which also happens to be the voltage of source.

Illustration 10: A resistance R, inductance L and a capacitor C all are connected in series with ac supply. The resistance of R is 16 Ω and for a given frequency, the inductive reactance of L is 24 Ω and capacitive reactance of C is 12 Ω. If the current in the circuit is 5 amp, find: **(JEE MAIN)**

(a) The potential difference across R, L and C

(b) The impedance of the circuit

(c) The voltage of ac supply

(d) Phase angle

Sol: In series LCR circuit, the impedance is $Z=\sqrt{R^2+\left(X_C-X_L\right)^2}$ where X_C and X_L are the capacitive reactance and inductive reactance respectively. The phase angle between voltage and current is given by $\phi=\tan^{-1}\left(\dfrac{X_L-X_C}{R}\right)$.

(a) Potential difference across resistance: $V_R = iR = 5 \times 16 = 80$ V

Potential difference across inductance: $V_L = i \times (\omega L) = 5 \times 24 = 120$ V

Potential difference across capacitor: $V_c = i \times (1/\omega C) = 5 \times 12 = 60$ V

(b) $$Z = \sqrt{\left[R^2 + \left(\frac{1}{\omega C} - \omega L\right)^2\right]} = \sqrt{(16)^2 + (12)^2} = 20\Omega$$

(c) The voltage of ac supply is given by: $V = IZ = 5 \times 20 = 100$ V

(d) $$\phi = \tan^{-1}\left(\frac{\omega L - (1/\omega c)}{R}\right) = \tan^{-1}\left(\frac{24-12}{16}\right) = \tan^{-1}(0.75) = 36^0 46''$$

Illustration 11: An oscillating voltage drives an alternating current through a resistor, an inductor, and a capacitor that are all connected in series. Calculate the RMS voltage across each another by multiplying the reactance or resistance of each element by the RMS current. To calculate the RMS current, divide the RMS voltage by the impedance. **(JEE ADVANCED)**

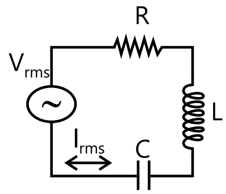


Figure 23.18

Sol: In series LCR circuit, the impedance is $Z = \sqrt{R^2 + (X_C - X_L)^2}$ where X_c and X_L are the capacitive reactance and inductive reactance respectively. The phase angle between voltage and current is given by $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$. Find the current in the series circuit, and multiply the resistance or reactance of each element with the currrent to find the voltage drop across it.

1. Calculate X_C; $X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0\text{Hz})0.15\mu F} = 17.68\ k\Omega$

2. Calculate X_L; $X_L = \omega L = 2\pi(60.0\text{Hz})(25\text{mH}) = 9.42\pi\ \Omega$

3. Calculate the impedance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(9.9k\Omega)^2 + (0.00942k\Omega - 17.68k\Omega)^2} = 20.25\ K\Omega$$

4. Divide the voltage by the impedance: $I_{rms} = \frac{V_{rms}}{Z} = \frac{115\ V}{20.25\ k\Omega} = 5.7$ mA

5. Multiply the current by the resistance: $V_{rms.R} = I_{rms}R = 5.68\ mA(9.9k\Omega) = 56$ V

6. Multiply the current by the inductive reactance: $V_{rms.L} = I_{rms}X_L = 5.68\ mA(9.42k\Omega) = 54$ V

7. Multiply the current by the capacitive reactance:

$$V_{rms.C} = I_{rms}X_C = 5.68\ m\ A(17.68k\Omega) = 100V = 0.10\ KV$$

6.5 Parallel RCL Circuits

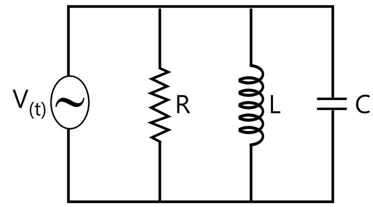


Figure 23.19 Parallel LRC circuit

Consider the parallel RLC circuit illustrated in Fig. 23.19.

The voltage source is $V(t) = V_0 \sin \omega t$.

Unlike the series RLC circuit, the instantaneous voltage acrossall three circuit elements R, L, and C are the same, and each voltage is in phase with the current through the resistor. However, the current through each element will be different.

In analysing this circuit, we make use of the results derived before. The current in the resistor is $I_R(t) = \frac{V(t)}{R} = \frac{V_0}{R} = \sin\omega t = I_{R0}\sin\omega t$ ... (i)

Where $I_{R0} = V_0/R$. The voltage across the inductor is $V_L(t) = V(t) = V_0 \sin\omega t = L\frac{dI_L}{dt}$ ... (ii)

which gives $I_L(t) = \int_0^t \frac{V_0}{L}\sin\omega t'dt' = \frac{V_0}{\omega L}\cos\omega t = \frac{V_0}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right) = I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right)$ ... (iii)

where $I_{L0} = V_0 / X_L$ and $X_L = \omega L$ is the inductive reactance.

Similarly, the voltage across the capacitor is $V_C(t) = V_0 \sin\omega t = Q(t)/c$, which implies

$$I_C(t) = \frac{dQ}{dt} = \omega C V_0 \cos\omega t = \frac{V_0}{X_C}\sin\left(\omega t + \frac{\pi}{2}\right) = I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{... (iv)}$$

where $I_{C0} = V_0 / X_C$ and $X_C = 1/\omega L$ is the capacitive reactance.

Using Kirchhoff's junction rule, the total current is simply the sum of all three currents.

$$I(t) = I_R(t) + I_L(t) + I_c(t) = I_{R0}\sin\omega t + I_{L0}\sin\left(\omega t - \frac{\pi}{2}\right) + I_{C0}\sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{... (v)}$$

The current can be represented with the phasor diagram shown in Fig. 23.20

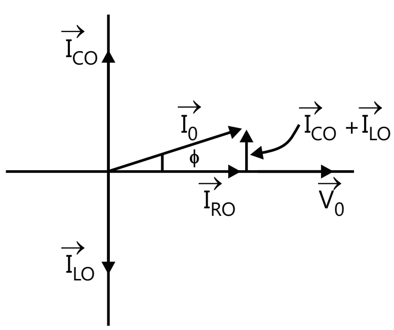


Figure 23.20: Phase difference between current and voltage

From the phasor diagram, we see that. $\vec{I}_0 = \vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0}$ ... (vi)

And the maximum amplitude of the total current, I_0, can be obtained as

$$\vec{I}_0 = \left|\vec{I}_0\right| = \left|\vec{I}_{R0} + \vec{I}_{L0} + \vec{I}_{C0}\right| = \sqrt{I^2_{R0} + (I_{c0} - I_{L0})^2} = V_0\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = V_0\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{... (vii)}$$

Note however, since $I_R(t)$, $I_L(t)$ and $I_C(t)$ are not in phase with one another, I_0 is not equal to the sum of the maximum amplitudes of the three currents: $I_0 \neq I_{R0} + I_{L0} + I_{C0}$ ... (viii)

With $I_0 = V_0/Z$, the (inverse) impedance of the circuit is given by:

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_C} - \frac{1}{X_L}\right)^2} \quad \text{... (ix)}$$

The relationship between Z, R, X_L and X_C is shown in Fig. 23.21 which shows a relationship between Z, R, X_L and X_C in a parallel RLC circuit.

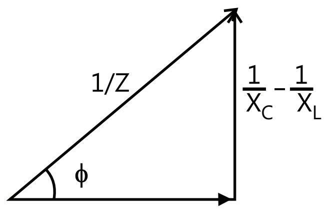


Figure 23.21: Impedance triangle

From the phasor diagram, we see that the phase can be obtained as: $\tan\phi = \left(\frac{I_{C0} - I_{L0}}{I_{R0}}\right) = \frac{\frac{V_0}{X_C} - \frac{V_0}{X_L}}{\frac{V_0}{R}} = R\left(\frac{V_0}{X_C} - \frac{V_0}{X_L}\right)$

$$= R\left(\omega t - \frac{\pi}{2}\right) \quad \text{... (x)}$$

The resonance condition for the parallel RLC circuit is given by $\phi = 0$, which implies:

$$\frac{1}{X_C} = \frac{1}{X_L} \quad \text{... (xi)}$$

The resonant frequency is: $\omega_0 = \frac{1}{\sqrt{LC}}$ ... (xii)

which is the same as for the series RLC circuit. From Eq. (xii), we readily see that 1/Z is minimum (or Z is maximum) at resonance. The current in the inductor exactly cancels out the current in the capacitor, so that the total current in the circuit reaches minimum, and is equal to the current in the resistor: $I_0 = \frac{V_0}{R}$ ... (xiii)

As in the series RLC circuit, power is dissipated only through the resistor. The average power is

$$\langle P(t)\rangle = \langle I_R(t)V(t)\rangle = \langle I_R^2(t)R\rangle = \frac{V_0^2}{R}\langle \sin^2\omega t\rangle = \frac{V_0^2}{2R} = \frac{V_0^2}{2Z} = \left(\frac{Z}{R}\right) \quad \text{... (xiv)}$$

Thus, the power factor in this case is

$$\text{Power factor} = \frac{\langle P(t)\rangle}{V_0^2/2Z} = \frac{Z}{R} = \frac{1}{\sqrt{1 + \left(R\omega C - \frac{R}{\omega L}\right)^2}} = \cos\phi \quad \text{... (xv)}$$

Illustration 12: The image shows an inductor (L=0.22 mH) in series with a 15Ω resistor. These elements are in parallel with a second 15Ω resistor. An AC generator powers the circuit with an RMS voltage of 65V.

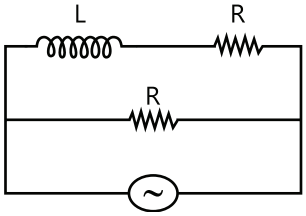


Figure 23.22

In the limit of high frequency, the inductor behaves like a very large resistor. In such a case, nearly all of the current flows through the branch with the lone resistor. Calculate the current by dividing the RMS voltage by the single resistor.

In the limit of low frequency, the reactance of the inductor approaches zero. In such a case, the current flows through each resistor equally. Calculate the equivalent resistor and divide the voltage by the equivalentresistance to determine the current. **(JEE ADVANCED)**

Sol: For very high source frequency, the reactance of the inductor becomes practically infinite so that the current doesn't flow through the inductor. Thus, the inductor acts as an open circuit. For very low source frequency, the reactance of the inductor becomes practically zero, and theinductor behaves as a short circuit.

1. Calculate the current at high frequency: $I_{rms}=\frac{V_{rms}}{R}=\frac{65\text{ V}}{15\ \Omega}=4.3\text{ A}$

2. Calculate the equivalent resistance at low frequency: $R_{eq}=\left(\frac{1}{R}+\frac{1}{R}\right)^{-1}=\frac{R}{2}=\frac{15\Omega}{2}=7.5\Omega$ 3.

Divide the voltage by the equivalent resistance: $I_{rms}=\frac{V_{rms}}{R_{eq}}=\frac{65V}{7.5\Omega}=8.7\text{ A}$

Illustration 13: For the circuit shown in Fig. 23.23, current in inductance is 0.8 A while its capacitance is 0.6A. What is the current drawn from the source? **(JEE ADVANCED)**

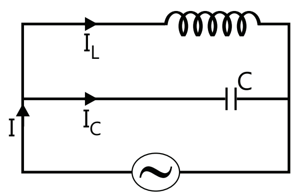


Figure 23.23

Sol: For LC circuit, total current in the circuit is $I=I_0\sin(\omega t+\phi)=I_L+I_C$. The current in the inductor lags the applied voltage by phase difference of $\frac{\pi}{2}$ while in capacitor, the current leads applied voltage by $\frac{\pi}{2}$ In parallel ac circuit, $V=V_0\sin\omega t$ is applied across both the inductorand capacitor, current in inductor lags the applied voltage while current in capacitor leads the applied voltage.

So, $I_L=\frac{V}{X_C}\sin\left(\omega t-\frac{\pi}{2}\right)=-0.8\cos\omega t$; $I_C=\frac{V}{X_C}\sin\left(\omega t+\frac{\pi}{2}\right)=0.6\cos\omega t$

So, the current drawn from the source, $I=I_L+I_C=-0.2\cos\omega t$, i.e. $|I_0|=0.2\text{ A}$

7. MORE ON POWER FACTOR

(a) The factor $\cos\phi$ present in the relation for average power of an ac circuit is called power factor.

So, $\cos\phi=\frac{P_{ac}}{E_{rms}I_{rms}}=\frac{P_{avg}}{p_V}$. Thus, ratio of average power and virtual power in the circuit is equal to power factor.

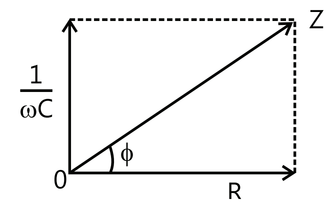


Figure 23.24

(b) Power factor is also equal to the ratio of the resistance and the impedance of the ac circuit.

Thus, $\cos\phi = \frac{R}{Z}$

(c) Power factor depends upon the nature of the components used in the circuit.(d) If a pure resistor is connected in the ac circuit then,

$$\phi = 0, \cos\phi = 1; \qquad p_{av} = \frac{E_0 I_0}{2} = \frac{E_0^2}{2R} = \text{Erms } I_{rms}$$

Thus, the power loss is maximum and electrical energy is converted in the form of heat.

(e) If a pure inductor or capacitor are connected in the ac circuit, then

$\phi \neq 90^o$, $\cos\phi = 0$ $\therefore$ P_{av} =0 (minimum)

Thus is no loss of power.

(f) If a resistor and an inductor or a capacitor are connected in an ac circuit, then $\phi \neq 0$ or $\phi \neq 90^o$. Thus ϕ is in between 0 & 90^o.

(g) If the components L, C and R are connected in series in a circuit, then

$$\tan\phi = \frac{X}{R} = \frac{(\omega L - 1/\omega C)}{R} \text{and } \cos\phi = \frac{R}{Z} = \frac{R}{\left[R^2 (\omega L - 1/\omega C)^2\right]^{1/2}}; \text{ Power factor } \cos\phi = \frac{R}{Z}$$

(h) Power factor is a unit less quantity.

(i) If there is only an inductance coil in the circuit, there will be no loss of power, and energy will be stored in the magnetic field.

(j) If a capacitor is only connected in the circuit, there will also be no loss of power, and energy will be stored in the electrostatic field.

(k) In reality, an inductor and capacitor do have some resistance. So, there is always some loss of power.

(l) In the state of resonance, the power factor is one.

8. WATTLESS CURRENT

(a) The component of current whose contribution to the average power is nil, is called wattless current.

(b) The average wattle of power iszero because the average of the second component of instantaneous power for a full cycle will be

$$E_0 \sin\omega t (I_0 \sin\phi) \sin(\omega t - \pi/2) = 0$$

(c) The component of current associated with this part is called Wattless current. Thus the current

$(I_0 \sin\phi) \sin\ (\omega t - \pi/2)$ is a wattless current whose amplitude is $I_0 \sin\phi$.

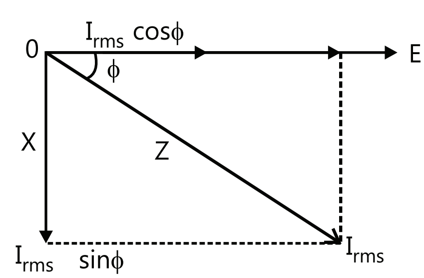


Figure 23.25

(c) If RMS value of current in the circuit is I_{rms}, then the RMS value of a wattless current will be I_{rms}, $\sin\phi$. A wattless current lags or leads the e.m.f. by an angle $\pi/2$. RMS value of wattless current:

$I_{rms}\sin\phi = \frac{I_0}{\sqrt{2}}\sin\phi\,; = \frac{I_0}{\sqrt{2}}\frac{X}{Z}$. Since $\sin\phi = \frac{X}{Z}$, where X is the resultant reactance of the circuit.

9. TRANSFORMERS

A transformer is a device used to convert low alternating voltage at higher current into high alternating voltage at lower current, and vice-versa. In other words, a transformer is an electrical device used to increase or decrease alternating voltage.

9.1 Types of Transformers

(a) Step-up transformers: The transformerwhich converts low alternating voltage at higher current into a high alternating voltage at lower current is called a step-up transformer.

(b) Step-down transformers: The transformer which converts high alternating voltage at lower current into a low alternating voltage athigher current is called a step-down transformer.

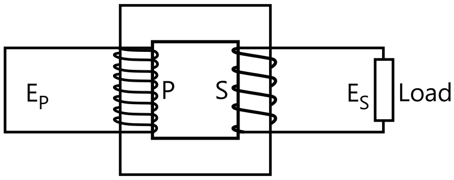


Figure 23.26

Principle: A transformer is based on the principle of mutual induction. An e.m.f. is induced in a coil, when a changing current flows through its nearby coil.

Construction: Itconsists of two separate coils of insulated wires wound on the same iron core. One of the coil connected to a.c. input is called primary (p) and the other winding giving output is called secondary (S) winding or coil.

Theory: When an alternating source of e.m.f. E_p is connected to the primary coil, an alternating current flows through it. Due to the flow of alternating current in the primary coil, an alternating magnetic flux induces an alternating e.m.f. in the secondary coil (E_s). Let N_p and N_s be the number of turns in the primary and secondary coil respectively. The iron core is capable of coupling the whole of the magnetic flux ϕ produced by the turns of the primary coil with the secondary coil.

According to Faraday's law of electromagnetic induction, the induced e.m.f in the primary coil,

$$E_P = -N_P\frac{d\phi}{dt} \quad \text{... (i)}$$

The induced e.m.f in the secondary coil. $E_S = -N_S\frac{d\phi}{dt}$ ... (ii)

Dividing (ii) by (i), we get $\frac{E_S}{E_p} = \frac{N_S}{N_P}$; Where $\frac{N_S}{N_P}$ =K the transformation ratio or ratio.

Then, $\frac{E_S}{E_p} = \frac{N_S}{N_P}$ =K

K< 1 for step down transformer. In this case, $N_S < N_P$ and $E_S < E_p$ i.e. E_p, and output alternating voltage <input alternating voltage.

K>1 for step up transformer. In this case, $N_S > N_P$ and $E_S > E_p$ i.e., output alternating voltage is greater than the input alternating voltage.

For an ideal transformer (in which there in no energy losses), output power= input power .... (iii)

Let I_p and I_s be the current in the primary and secondary coil respectively.

Then output power= $E_s I_S$; input power= $E_p I_p$; from equation (iii) $E_p = E_s$ or $\frac{E_s}{E_p} = \frac{I_P}{I_S}$; In general, $E \propto \frac{1}{I}$. For same power transfer, voltage increases with the decrease in current and vice-versa. Thus, whatever is gained in voltage ratio is lost in the current ratio and viceversa. So, astep-up transformer increases the alternating voltage by decreasing the alternating current, and a step- down transformer decreases the alternating voltage by increasing the alternating current.

For a transformer, efficiency, n= $\frac{\text{ouputpower}}{\text{inputpower}} = \frac{E_s I_S}{E_P I_P}$ For an ideal transformer, efficiency, n is 100%. But in a real transformer, the efficiency varies from 90-99%. This indicates that there are some energy losses in the transformer.

10. CHOKING COIL

Let us consider a choke coil of large inductance L and low resistance R. Then, the power factor of the given circuit will be given by $\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{R}{\omega L}$ (as $R << \omega L$)

Now, as we know that $R<<\omega L$, the power factor is small and hence the power absorbed will be very small. And also, on account of its large impedance (large inductance), current passing through the coil is very small. Hence, such a coil is preferred in electrical circuits for the purpose of adjusting the current to any desired value without having a significant energy waste.

Illustration 14: An ac circuit consists of a $220\,\Omega$ resistance and a 0.7 H choke. Find the power absorbed from a 220V and 50 Hz source connected in this circuit if the resistance and choke are joined, (a) in series (b) in parallel

(JEE ADVANCED)

Sol: For a seriesLR circuit, impedance is $Z = \sqrt{R^2 + \omega^2 L^2}$ and average power dissipated in circuit is calculated as $P = V_{rms} I_{rms} \cos\phi$.

In parallel LR circuit $\frac{1}{Z^2} = \frac{1}{R^2} + \frac{1}{\omega^2 L^2}$. But for a choke, L is very large, so $\frac{1}{\omega^2 L^2} \approx 0$.

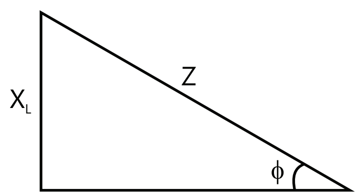


Figure 23.27

(a) in series the impedance of the circuit is:

$$Z = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2} = \sqrt{(220)^2 + (2 \times 3.14 \times 50 \times 0.7)^2} = 311\Omega$$

$$\therefore I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{311} = 0.707\text{A}, \ \cos\phi = \frac{R}{Z} = \frac{220}{311} = 0.707$$

and the power absorbed in the circuit, $P = V_{rms} i_{rms} \cos\phi = (220)(0.707)(0.707) = 110.08$ W

(b) When the resistance and choke are in parallel, the entire power is absorbed in resistance, as the choke (having zero resistance) absorbs no power. $\therefore P = \frac{V^2_{rms}}{R} = \frac{(220)^2}{220}$ =220W

PROBLEM-SOLVING TACTICS

(a) In this chapter, we have seen how a phasor provides a powerful tool for analysing the AC circuits.

Below are some important tips:

1. Keep in mind the phase relationship for simple circuits.

(i) For a resistor, the voltage and phase are always in phase.

(ii) For an inductor, the current lags the voltage by90^0.

(iii) For a capacitor, the current leads the voltage by 90^0.

(b) When circuit elements are connected in series, the instantaneous current is the same for allelements, and instantaneous voltages across the elements are out of phase. On the otherhand, when circuit elements are connected in parallel, the instantaneous voltage is the same for all elements, and the instantaneous currents across the elements are out of phase.

(c) For a series connection, draw a phasor diagram for the voltage. The amplitude of the voltage drop across all the circuit elements involved should be represented with phasors. In Fig. 23.28, the phasor diagram for a series RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$. Below is a phasor diagram for the series RLC circuit for (a) $X_L > X_C$ (b) $X_L < X_C$.

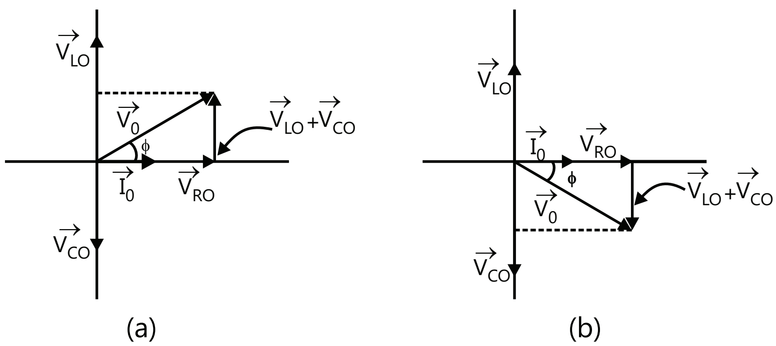


Figure 23.28: Phase angle between applied voltage and current (a) in RC circuit, (b) in LC circuit

From Fig. 23.28(a), we see that $V_{L0} > V_{C0}$ in the inductive case and $\overline{V}_0$ leads $\bar{I}_0$ by a phase ϕ. On the other hand, in the capacitive case shown in Fig. 23.28(b), $V_{C0} > V_{L0}$ and $\bar{I}_0$ leads $\overline{V}_0$ by a phase ϕ.

(d) Students should directly learn the formula for reactance, impedance, etc.to solve any problem easily.

(e) For parallel connection, draw a phasor diagram for the currents. The amplitudes of the current across all the circuit elements involved should be represented with phasors. In the following Fig. 23.29, the phasor diagram for a parallel RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$.

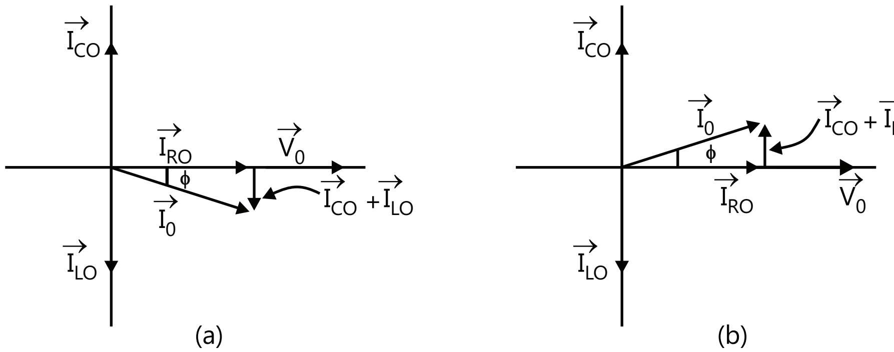


Figure 23.29

(f) Phasor diagram for the parallel RLC circuit for (a) $X_L > X_C$ And (b) $X_L < X_C$: From Fig. 23.29(a), we see that $I_{L0} > I_{C0}$ in the inductive case and $\overline{V_0}$ lead $\overline{I_0}$ by a phase ϕ.On the other hand, in the capacitive case shown in Fig. 23.29 (b), $I_{C0} > I_{L0}$ and $\overline{I_0}$ leads $\overline{V_0}$ by a phase ϕ.

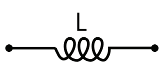
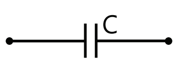
FORMULAE SHEET

(a) In an AC circuit, sinusoidal voltage source of amplitude V_0 is represented as:$V(t) = V_0 \sin wt$.

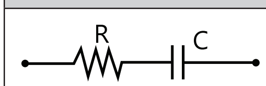
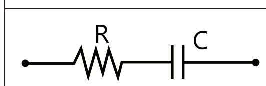
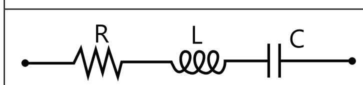
The current in the circuit has amplitude I_0 and lags the applied voltage by phase angle ϕ.

Current is represented as: $I(t) = I_0 \sin(\omega t - \phi)$

(b) For a single-element circuit (a resistor, a capacitor or an inductor) connected to the AC voltage source, we summarise the results in the below table:

Circuit elements	Resistance/Reactance	Current Amplitude	Phase angel ϕ
	R	$I_{R_0} = \frac{V_0}{R}$	0
	Inductive Reactance $X_L = \omega L$	$I_{L_0} = \frac{V_0}{X_L}$	$(\pi/2)$ i.e.,current lags voltage by 90^0
	Capacitive Reactance $X_C = \frac{1}{\omega C}$	$I_{C_0} = \frac{V_0}{X_C}$	$(-\pi/2)$ i.e. current leads voltage by 90^0

(c) For a circuit having more than one circuit element connected ina series,we summarise the results in the below table:

Circuit elements	Impedance Z	Current amplitude	Phase angle ϕ
	$\sqrt{R^2 + X_L^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$	$0 < \phi < \left(\frac{\pi}{2}\right)$
	$\sqrt{R^2 + X_C^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$	$\left(-\frac{\pi}{2}\right) < \phi < 0$
	$\sqrt{R^2 + (X_L - X_C)^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + (X_L - X_C)^2}}$	$\phi > 0$ if $X_L > X_C$ $\phi < 0$ if $X_L < X_C$

(d) For series LCR circuit,

(i) the impedance is $Z=\sqrt{R^2+(X_L-X_C)^2}$

(ii) the current lags the voltage by phase angle $\phi=\tan^{-1}\frac{(X_L-X_C)}{R}$

(iii) the resonant frequency is $\omega_0=\sqrt{\frac{1}{LC}}$.

At resonance, the current in the series LCR circuit is maximum, while that in parallel LCR circuit is minimum.

(e) Impedance for parallel LCR circuit, is given by

$$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\left(\frac{1}{\omega L}-\omega C\right)^2}=\sqrt{\frac{1}{R^2}+\left(\frac{1}{X_L}-\frac{1}{X_C}\right)^2}$$

The phase angle by which the current lags the voltage is

$$\phi=\tan^{-1}R\left(\frac{1}{X_L}-\frac{1}{X_C}\right)=\tan^{-1}R\left(\frac{1}{\omega L}-\omega C\right)$$

(f) The RMS (root mean square) value of voltage and current in an AC circuit are given as

$V_{rms}=\frac{V_0}{\sqrt{2}}$, and $I_{rms}=\frac{I_0}{\sqrt{2}}$

(g) Average power of an AC circuit is $\langle P(t)\rangle=I_{rms}V_{rms}\cos\phi$ where $\cos\phi=\frac{R}{Z}$ is the power factor of the circuit.

(h) Quality factor Q of LCR circuit is $Q=\frac{\omega_0 L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$

(i) For a transformer, the ratio of secondary coil voltage to that of primary coil voltage is $\frac{V_2}{V_1}=\frac{N_2}{N_1}$

where N_1 is number of turns in primary coil, and N_2 is number of turns in secondary coil.

For the step-up transformer, $N_2>N_1$; for step down transformer, $N_2<N_1$.