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PROBLEM-SOLVING TACTICS

(a) In this chapter, we have seen how a phasor provides a powerful tool for analysing the AC circuits.

Below are some important tips:

1. Keep in mind the phase relationship for simple circuits.

(i) For a resistor, the voltage and phase are always in phase.

(ii) For an inductor, the current lags the voltage by90°.

(iii) For a capacitor, the current leads the voltage by 90°.

- (b) When circuit elements are connected in series, the instantaneous current is the same for allelements, and instantaneous voltages across the elements are out of phase. On the otherhand, when circuit elements are connected in parallel, the instantaneous voltage is the same for all elements, and the instantaneous currents across the elements are out of phase.
- (c) For a series connection, draw a phasor diagram for the voltage. The amplitude of the voltage drop across all the circuit elements involved should be represented with phasors. In Fig. 23.28, the phasor diagram for a series RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_L < X_C$. Below is a phasor diagram for the series RLC circuit for (a) $X_L > X_C$ (b) $X_L < X_C$.



Figure 23.28: Phase angle between applied voltage and current (a) in RC circuit, (b) in LC circuit

From Fig. 23.28(a), we see that $V_{L0} > V_{C0}$ in the inductive case and \overline{V}_0 leads \overline{I}_0 by a phase ϕ . On the other hand, in the capacitive case shown in Fig. 23.28(b), $V_{C0} > V_{L0}$ and \overline{I}_0 leads \overline{V}_0 by a phase ϕ .

- (d) Students should directly learn the formula for reactance, impedance, etc.to solve any problem easily.
- (e) For parallel connection, draw a phasor diagram for the currents. The amplitudes of the current across all the circuit elements involved should be represented with phasors. In the following Fig. 23.29, the phasor diagram for a parallel RLC circuit is shown for both the inductive case $X_L > X_C$ and the capacitive case $X_I < X_C$.



Figure 23.29

(f) Phasor diagram for the parallel RLC circuit for (a) $X_L > X_C$ And (b) $X_L < X_C$: From Fig. 23.29(a), we see that $I_{L0} > I_{C0}$ in the inductive case and $\overline{V_0}$ lead $\overline{I_0}$ by a phase ϕ . On the other hand, in the capacitive case shown in Fig. 23.29 (b), $I_{C0} > I_{L0}$ and $\overline{I_0}$ leads $\overline{V_0}$ by a phase ϕ .

FORMULAE SHEET

- (a) In an AC circuit, sinusoidal voltage source of amplitude V₀ is represented as:V(t) = V₀ sinwt. The current in the circuit has amplitude I₀ and lags the applied voltage by phase angle ϕ . Current is represented as: I(t) = I₀ sin ($\omega t - \phi$)
- (b) For a single-element circuit (a resistor, a capacitor or an inductor) connected to the AC voltage source, we summarise the results in the below table:

Circuit elements	Resistance/Reactance	Current Amplitude	Phase angel ϕ
← ^R →	R	$I_{R_0} = \frac{V_0}{R}$	0
←L	Inductive Reactance $X_L = \omega L$	$I_{L_0} = \frac{V_0}{X_L}$	$(\pi / 2)$ i.e.,current lags voltage by 90°
← ^C →	Capacitive Reactance $X_{C} = \frac{1}{\omega C}$	$I_{C_0} = \frac{V_0}{X_C}$	(- π / 2) i.e. current leads voltage by 90°

(c) For a circuit having more than one circuit element connected ina series, we summarise the results in the below table:

Circuit elements	Impedance Z	Current amplitude	Phase angle ϕ
•	$\sqrt{R^2 + X_L^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_L^2}}$	$0 < \phi < \left(\frac{\pi}{2}\right)$
	$\sqrt{R^2 + X_C^2}$	$I_0 = \frac{V_0}{\sqrt{R^2 + X_C^2}}$	$\left(-\frac{\pi}{2}\right) < \phi < 0$
←∰ ^L w_ ^C →	$\sqrt{R^2 + \left(X_L - X_C\right)^2}$	$I_{0} = \frac{V_{0}}{\sqrt{R^{2} + (X_{L} - X_{C})^{2}}}$	

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- (d) For series LCR circuit,
 - (i) the impedance is $Z = \sqrt{R^2 + (X_L X_C)^2}$
 - (ii) the current lags the voltage by phase angle $\phi = \tan^{-1} \frac{(X_L X_C)}{R}$
 - (iii) the resonant frequency is $\omega_0 = \sqrt{\frac{1}{LC}}$.

At resonance, the current in the series LCR circuit is maximum, while that in parallel LCR circuit is minimum.

(e) Impedance for parallel LCR circuit, is given by

$$\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{\omega L} - \omega C\right)^2} = \sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

The phase angle by which the current lags the voltage is

$$\phi = \tan^{-1} R\left(\frac{1}{X_L} - \frac{1}{X_C}\right) = \tan^{-1} R\left(\frac{1}{\omega L} - \omega C\right)$$

(f) The RMS (root mean square) value of voltage and current in an AC circuit are given as

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$
, and $I_{rms} = \frac{I_0}{\sqrt{2}}$

- (g) Average power of an AC circuit is $\langle P(t) \rangle = I_{rms} V_{rms} \cos \phi$ where $\cos \phi = \frac{R}{Z}$ is the power factor of the circuit.
- (h) Quality factor Q of LCR circuit is $Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$
- (i) For a transformer, the ratio of secondary coil voltage to that of primary coil voltage is $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ where N₁ is number of turns in primary coil, and N₂ is number of turns in secondary coil. For the step-up transformer, N₂ > N₁; for step down transformer, N₂ < N₁.