Solved Examples

JEE Main/Boards

Example 1: The given curves $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and $Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + c = 0$ intersect each other at four concyclic points then prove that $\frac{a-b}{h} = \frac{A-B}{H}$.

Sol: Equation of second degree curve passing through the intersections of the given curves is $S_1 + \lambda S_2 = 0$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c + \lambda(Ax^2 + 2Hxy + By^2 + 2Gx + Fy + C) = 0$ ..... (i)

intersection points of of the two curves are concyclic,

(i) must be a circle for some λ.

$\therefore$ Coefficient of x^2 = coefficient of y^2 and coefficient of xy = 0.

$\therefore a + \lambda A = b + \lambda B$

and $2h + \lambda \cdot 2H = 0$ or $a - b = \lambda(B - A)$

and $h = -\lambda H$

$$\therefore \frac{a-b}{h} = \frac{\lambda(B-A)}{-\lambda H}; \therefore \frac{a-b}{h} = \frac{A-B}{H}.$$

Example 2: Find the equation of a circle which cuts the circle $x^2 + y^2 - 6x + 4y - 3 = 0$ orthogonally and which passes though (3, 0) and touches the y-axis.

Sol: When two circle intersects each other orthogonally then $2(g_1g_2 + f_1f_2) = c_1 + c_2$. Hence by considering centre as (h, k) and using given condition we can solve problem.

Let C(h, k) be the centre of required circle

radius of circle = $\sqrt{(h-3)^2 + k^2} = |h|$

$\therefore \quad (h - 3)^2 + k^2 = h^2$

or $\quad k^2 - 6h + 9 = 0$ .....(i)

Required circle is $(x - h)^2 + (y - k)^2 = h^2$

or $\quad x^2 + y^2 - 2hx - 2ky + k^2 = 0$

It is intersected by $x^2 + y^2 - 6x + 4y - 3 = 0$, orthogonally;

$\therefore \quad 2(-3)(-h) + 2(2)(-k) = k^2 - 3$

or $\quad 6h - 4k + 3 = k^2$ .....(ii)

Solve (i) and (ii) : h = 3, k = 3

Required circle is $x^2 + y^2 - 6x - 6y + 9 = 0$

Example 3: Lines $5x + 12y - 10 = 0$ and $5x - 12y - 40 = 0$ touch a circle C_1 (of diameter 6). If centre of C_1 lies in the first quadrant, find concentric circle C_2 which cuts intercepts of length 8 units on each given line.

Sol: Consider centre of required circle is (h, k) and by using perpendicular distance formula from centre to given tangent we will get value of h and k.

Let centre of circle C_1 be O (h, k), where h > 0 and k > 0

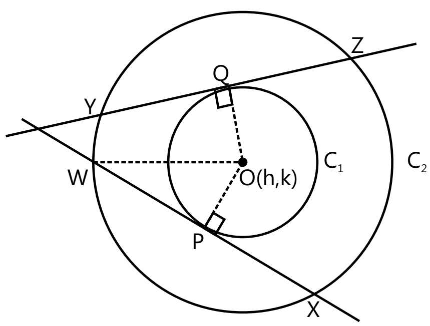
WP = 4 and OP = 3 ... (given)

In $\triangle$OWP,

$$OW = \sqrt{OP^2 + WP^2} = \sqrt{4^2 + 3^2} = 5$$

OW = 5 = radius of C_2.

$$\Rightarrow \frac{|5h + 12k - 10|}{13} = \frac{|5h - 12k - 40|}{13} = 3$$



$$\frac{5h + 12k - 10}{13} = \pm\left(\frac{5h - 12k - 40}{13}\right)$$

$\Rightarrow$ Either $5h + 12k - 10 = 5h - 12k - 40$

$\Rightarrow 24k = -30$

$\Rightarrow k = \frac{-30}{40}$ (Not possible)

Or $5h + 12k - 10 = -5h + 12k + 40$

$\Rightarrow 10h = 50$

$\Rightarrow H = 5$

Substituting h = 5 in

$$\frac{|5h + 12k - 10|}{13} = 3$$

$\Rightarrow k = 2$ (as $k > 0$)

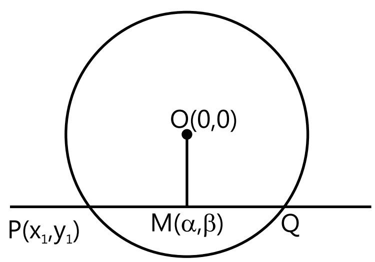
Hence, equation of required circle is

$$(x-5)^2 + (y-2)^2 = 25$$

Example 4: Find the locus of the middle points of the chords of the circle $x^2 + y^2 = a^2$ which pass through a given point (x_1, y_1).

Sol: As line joining centre of given circle to the mid point of chord is perpendicular to the chord and hence product of their slope will be – 1. Therefore by considering mid point of chord as (α, β) and by finding their slope we will get required equation.

Let M (α, β) be the middle point of any chord PQ through the given point (x_1, y_1). The centre of the circle is O (0, 0). Clearly MO is perpendicular to PQ.



Now, slope of PQ = $\frac{\beta - y_1}{\alpha - x_1}$

slope of OM = $\frac{\beta - 0}{\alpha - 0} = \frac{\beta}{\alpha}$

$\therefore \quad \frac{\beta - y_1}{\alpha - x_1} \cdot \frac{\beta}{\alpha} = -1$

or $\quad \alpha(\alpha - x_1) + \beta(\beta - y_1) = 0$

$\therefore$ the equation of the locus of M (α, β) is

$x(x - x_1) + y(y - y_1) = 0$

Alternative

The equation of chord when mid-point is known is $T = S_1$

Let the mid-point be (α, β)

$\therefore x\alpha + y\beta - \cancel{a^2} = \alpha^2 + \beta^2 - \cancel{a^2}$

$\because$ It passes through (x_1, y_1) we get

$x, \alpha + y, \beta = \alpha^2 + \beta^2$

$\Rightarrow \alpha(\alpha - x_1) + \beta(\beta - y_1) = 0$

$\therefore$ Required locus is

$x(x - x_1) + y(y - y_1) = 0$

Example 5: From a point P tangents are drawn to circles $x^2 + y^2 + x - 3 = 0$,

$x^2 + y^2 - \left(\frac{5}{3}\right)x + y = 0$ and $4x^2 + 4y^2 + 8x + 7y + 9 = 0$,

and they are of equal lengths. Find equation of a circle passing through P and touching the line x + y = 5 at A(6, –1).

Sol: By reading the problem we get that P is a radical centre of these circles. Hence by radical axis formula we can obtain co-ordinate of point P, as required circle is passing from these points so we can obtain required equation.

Write third circle as

$x^2 + y^2 + 2x + \left(\frac{7}{4}\right)y + \left(\frac{9}{4}\right) = 0$

By definition, P is radical centre of three circles. Equation of two of the radical axis are

$\left(\frac{8}{3}\right)x - y - 3 = 0$ and $x + \left(\frac{7}{4}\right)y + \left(\frac{21}{4}\right) = 0$

which intersect at P(0, –3). Let required circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

with centre Q(–g, –f)

P(0, –3) lies on it

$\Rightarrow \quad -6f + c + 9 = 0$ ... (i)

A(6, –1) lies on it

$\Rightarrow \quad 12g - 2f + c + 37 = 0$ ... (ii)

Since, PA is perpendicular to x + y = 5

$\therefore \quad \left(\frac{-f+1}{-g-6}\right)(-1) = -1$ ... (iii)

$\Rightarrow \quad f - g = 7$

Solving (i), (ii) and (iii) for f, g and c, we have

$f = \frac{7}{2}, g = -\frac{7}{2}$ and c = 12.

Hence equation of required circle is

$x^2 + y^2 - 7x + 7y + 12 = 0.$

Example 6: Find the equation of a circle which touches the line x + y = 5 at the point P(–2, 7) and cut the circle

$x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally.

Sol: Using the concept of family of circle and the condition for two circles to be orthogonal, we can find the equation of the required circle.

As the circle is touching the line x + y = 5. It (-2, 7).

Consider the equation of circle as

$$(x+2)^2+(y+7)^2+\lambda(x+y-5)=0$$

$$\Rightarrow x^2+y^2+x(4+\lambda)+y(\lambda-14)+53-5\lambda=0 \qquad \text{....(i)}$$

$\therefore$ As the circle given equation (i) is orthogonal to

$x^2+y^2+4x-6y+9=0,$

We have

$$(4+\lambda).2+(\lambda-14)(-3)=53-5\lambda+9$$

$$\Rightarrow 8+2\lambda-3\lambda+42=62-5\lambda$$

$$\Rightarrow 4\lambda=12$$

$$\Rightarrow \lambda=3$$

$\therefore$ Equation of the circle is $x^2+y^2+7x-11y+38=0$.

Example 7: Find the equation of the circle described on the common chord of the circles $x^2 + y^2 - 4x - 5 = 0$ and $x^2 + y^2 + 8y + 7 = 0$ as diameter.

Sol: Use Geometry to find the centre and the radius of the required circle.

For $x^2+y^2-4x-5=0$

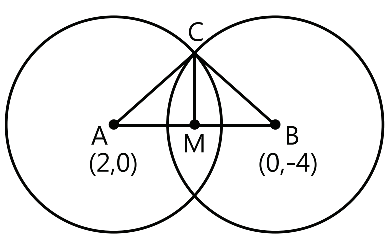
Centre $\equiv (+2, 0)$

Radius = 3

For $x^2+y^2+8y+7=0$

Centre $\equiv (0, -4)$

Radius = 3



The mid point of AB is the centre of the required circle

i.e. $M \equiv (1, -2)$

and Radius $=\sqrt{AC^2-AM^2}$

$=\sqrt{9-5}$

= 2

Equation of circle is $(x-1)^2+(y-2)^2=4$.

Example 8: Prove that, for all $c \in R$, the pole of the line $\frac{x}{a}+\frac{y}{b}$ = 1 with respect to the circle $x^2 + y^2 = c^2$ lies on a fixed line.

Sol: As polar of point (x_1, y_1) with respect to the circle $x^2 + y^2 = c^2$ is same as line $\frac{x}{a}+\frac{y}{b}=1$.

On comparing the two equations, we can prove the given statement.

Let the pole be (x_1, y_1). Then the polar of (x_1,y_1) with respect to the circle $x^2 + y^2 = c^2$ is

$$xx_1 + yy_1 = c^2 \qquad \text{... (i)}$$

Now, the line (i) and $\frac{x}{a}+\frac{y}{b}$ = 1 must be the same line.

$\therefore$ comparing coefficients, $\frac{x_1}{1/a}=\frac{y_1}{1/b}=\frac{c^2}{1}$

or $\quad ax_1=by_1=c^2,$

$\therefore \quad ax_1=by_1$

$\therefore$ (x_1, y_1) always lies on the line ax = by which is a fixed line.

Example 9: Inside the circle $x^2 + y^2 = a^2$ is inscribed an equilateral triangle with the vertex at (a , 0). The equation of the side opposite to this vertex is

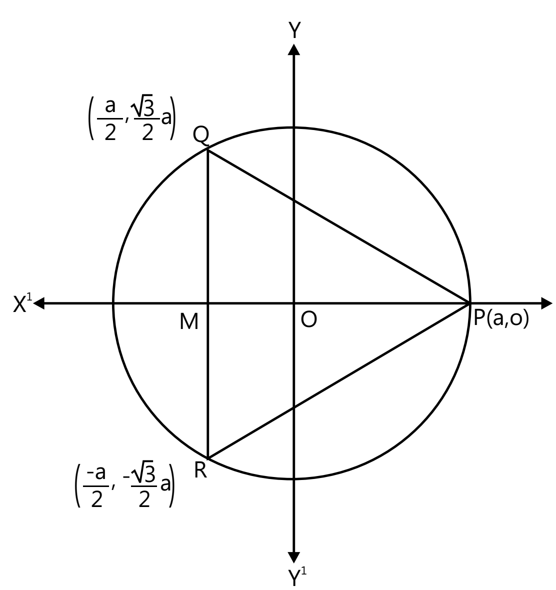
(A) 2x – a = 0 (B) x + a = 0

(C) 2x + a = 0 (D) 3x – 2a = 0

Sol: (C) As P (a , 0) be the vertex of the equilateral triangles PQR inscribed in the circle $x^2 + y^2 = a^2$ Let M be the middle point of the side QR, then MOP is perpendicular to QR and O being the centroid of the triangle OP = 2 (OM).

(Circumcentre and Centroid of an equilateral triangle are same)

So if (h, k) be the coordinates of M, then



$\frac{2h+a}{3}$ = 0 and $\frac{2k+0}{3}$ = 0

$\Rightarrow h = -\left(\frac{a}{2}\right)$ and k = 0

and hence the equation of BC is

$x = -\frac{a}{2}$ or 2x + a = 0.

Example 10 : Find the radical centre of the three circles $x^2 + y^2 = a^2$, $(x - c)^2 + y^2 = a^2$ and $x^2 + (y - b)^2 = a^2$.

Sol: Here by using the formula

$S_1 - S_2 = 0$, $S_2 - S_3 = 0$ and $S_3 - S_1 = 0$

we will get equation of radical axis and by solving them we can obtain requird radical centre.

Radical axis of first & second circle is given by

$(x^2 + y^2) - (x^2 + y^2 - 2cx + c^2) = 0$

or $x = \frac{c}{2}$

Also the radical axis of first and third circle is given by

$(x^2 + y^2) - (x^2 + y^2 - 2by + b^2) = 0$

or $y = \frac{b}{2}$

$\Rightarrow$ The radical centre = $\left(\frac{c}{2}, \frac{b}{2}\right)$.

JEE Advanced/Boards

Example 1: Two distinct chords drawn from the point P(a, b) to the circle $x^2 + y^2 - ax - by = 0$, $(ab \neq 0)$, are bisected by the x-axis. Show that $a^2 > 8b^2$.

Sol: As Circle passes through (0, 0) and P(a, b) .Consider the chord PQ intersect x-axis at A; then, Q is $(\alpha, -b)$. Hence by substituting this point to given equation of circle we can solve above problem.

$\therefore\ \alpha^2 + b^2 - a\alpha + b^2 = 0$ or $\alpha^2 - a\alpha + 2b^2 = 0$

Hence, Discriminant > 0

$\Rightarrow a^2 > 8b^2$

Example 2: Let T_1, T_2 be two tangents drawn from (–2 , 0) to the circle C : $x^2 + y^2 = 1$. Determine circles touching C and having T_1, T_2 as their pair of tangents. Further find the equation of all possible common tangents to these circles, when taken two at time.

Sol: As we know Equation of any tangent to $x^2 + y^2 = 1$, is $y = mx \pm \sqrt{1+m^2}$ and perpendicular distance from centre to tangent is equal to its radius. By using this condition we can solve above problem.

As they are drawn from A(–2, 0), conditions are

$0 = -2m \pm \sqrt{1+m^2}$

$\Rightarrow m = \pm\frac{1}{\sqrt{3}}$

Equations of tangents become

$T_1 : \sqrt{3}y = x + 2$

$T_2 : \sqrt{3}y = -x - 2$

Circles touching C and having T_1 and T_2 as tangents must have their center on x-axis (the angle bisector of T_1 and T_2).

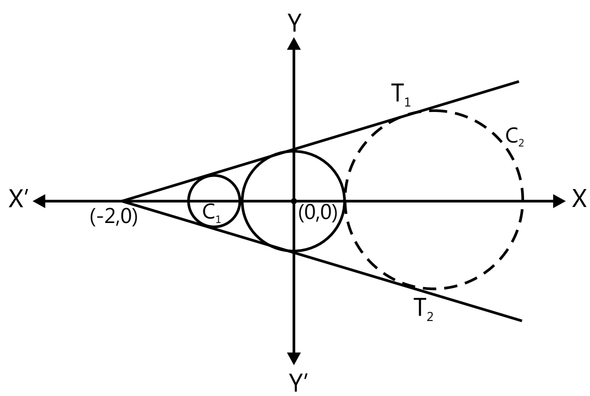
Let C_1 and C_2 be the 2 circles and M $(h_1, 0)$ & L$(h_2, 0)$ be their respective centers where

$h_1 > 0$ and $h_2 < 0$

By tangency of T_1, perpendicular distance from centre M is equal to radius r_1 of the circle C_1

$\therefore\ r_1 = \frac{h_1 + 2}{2}$

As C_1 and C touch each other $r_1 = h_1 - 1$



or $\frac{h_1+2}{2} + 1 = h_1$

or $h_1 = 4$

$\therefore$ For circle C_1 : centre is M(4, 0) and radius = 3.

Similarly for circle C_2, $-h_2 - 1 = \left|\frac{h_2+2}{2}\right|$

$\Rightarrow$ $-2h_2 - 2 = h_2 + 2$

($\because$ $h_2 > -2$; see figure)

$\Rightarrow$ $-3h_2 = 4$

or $h_2 = -\frac{4}{3}$ and radius = $\frac{1}{3}$.

Equations of two circles are $(x-4)^2 + y^2 = 9$ and

$$\left(x+\frac{4}{3}\right)^2 + y^2 = \frac{1}{9}$$

C_1 & C have x = 1 as transverse common tangent and C_2 & C have x = –1 as transverse common tangent.

Example 3 : Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre O. Show that the centroid of the triangle PAB as P moves on the circle is a circle.

Sol: By considering point P(r cos θ , r sinθ) and centroid as point (x_1, y_1) we can obtain required result.

ΔOAB is isosceles with

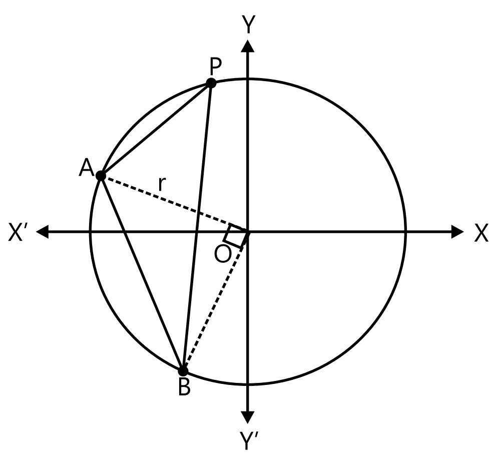
OA = OB = x(say)

We may assume AB is parallel to and below x-axis

$$\therefore \quad x^2 + x^2 = r^2 \quad \Rightarrow \quad x = \frac{r}{\sqrt{2}}$$

$$\therefore \text{ B is } \left(\frac{r}{\sqrt{2}}, -\frac{r}{\sqrt{2}}\right) \text{ and A is } \left(-\frac{r}{\sqrt{2}}, -\frac{r}{\sqrt{2}}\right)$$

Let P be (r cos θ , r sinθ) and centroid of ΔPAB be G (x_1, y_1)



$$\therefore x_1 = \frac{r\cos\theta + \frac{r}{\sqrt{2}} - \frac{r}{\sqrt{2}}}{3},$$

$$y_1 = \frac{r\sin\theta - \frac{r}{\sqrt{2}} - \frac{r}{\sqrt{2}}}{3}$$

$3x_1 = r \cos\theta$; $3y_1 = r(\sin\theta - \sqrt{2})$

Eliminating θ, we get

$$\therefore \quad \left(\frac{3x_1}{r}\right)^2 + \left(\frac{3y_1}{r} + \sqrt{2}\right)^2 = 1$$

or $\quad x_1^2 + \left(y_1 + \frac{\sqrt{2}r}{3}\right)^2 = \frac{r^2}{9}$

∴ Locus of (x_1, y_1) is a circle.

Example 4: Derive the equation of the circle passing through the point P(2, 8) and touches the lines 4x – 3y – 24 = 0 and 4x + 3y – 42 = 0 and coordinates of the centre less than or equal to 8.

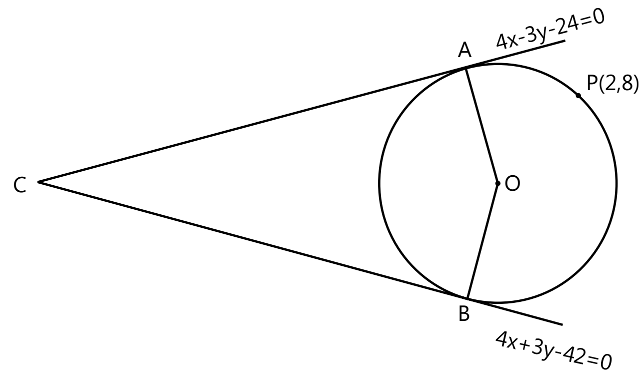
Sol: Here using Equations of bisectors of angle between the lines we will get co-ordinate of centre of circle i.e O. and as OA = OP we can obtain required equation of circle. consider O is the center of circle.

Let $L_1 \equiv 4x - 3y - 24 = 0$

$L_2 \equiv 4x + 3y - 42 = 0$

and Let A and B denote the respective points of contact

Equations of bisectors of angle between the lines are;



$$\frac{4x-3y-24}{5} = \pm\frac{4x+3y-42}{5}$$

i.e., y = 3 & $x = \frac{33}{4}$

Since O lies on one of these bisectors and x-coordinate of O is less then or equal to 8,

∴ O lies on y = 3.

Let O be (a,3). Then, OA = CP

or $\quad \left(\frac{4a-33}{5}\right)^2 = (a-2)^2 + 25$

or $\quad 16a^2 - 264a + (33)^2 = 25\{a^2 - 4a + 29\}$

or $\quad 9a^2 + 164a - 364 = 0$

or $\quad (a-2)(9a+182) = 0$

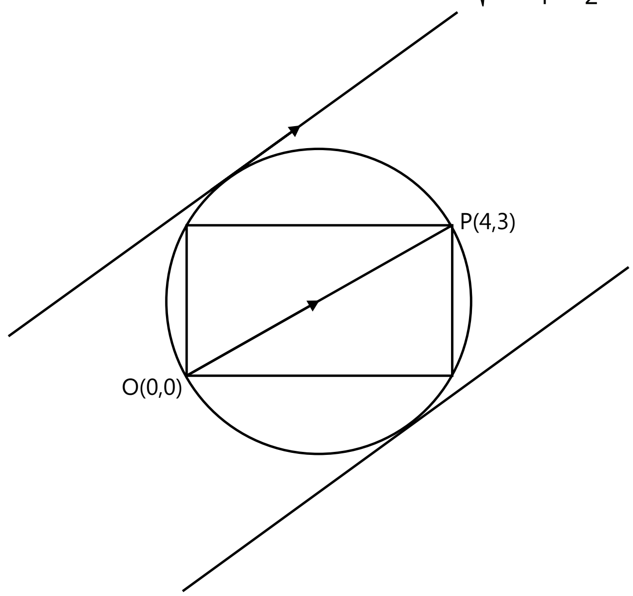
∴ a =2 or $a = -\frac{182}{9}$

and radius = OP.

Example 5: Coordinates of a diagonal of a rectangle are (0, 0) and (4, 3). Find the equations of the tangents to the circumcircle of the rectangle which are parallel to this diagonal.

Sol: Here centre of circle is the mid-point of line OP hence by using slope point form we can get required equation of tangents.

Two extremities are O (0, 0) and P (4, 3). Middle point of the diagonal OP is $M\left(2,\frac{3}{2}\right)$ which is the centre of the circumscribed circle and radius is OM = $\sqrt{4+\frac{9}{4}}=\frac{5}{2}$



A line parallel to OP is $y = \frac{3}{4}x + c$

It is a tangent to the circumscribed circle.

Therefore length of perpendicular from

$M\left(2,\frac{3}{2}\right)$ to it = $\frac{5}{2} \Rightarrow \frac{\left|\frac{3}{4}(2)-\frac{3}{2}+C\right|}{\sqrt{1+\frac{9}{16}}}=\frac{5}{2}$

or $\quad C=\pm\frac{5}{2}\cdot\frac{5}{4}=\pm\frac{25}{8}$

Hence tangents are $y=\frac{3}{4}x\pm\frac{25}{8}$

or $\quad 3x-4y\pm\frac{25}{2} = 0.$

Example 6: The equations two circles are $x^2 + y^2 + \lambda x + c = 0$ and $x^2 + y^2 + \mu x + c = 0$. Prove that one of the circles will be within the other if $\lambda\mu > 0$ and $c > 0$.

Sol: The condition for one circle to be within the other is

$C_1C_2 < |r_1 - r_2|$

Without the loss of generality,

Let $\lambda > \mu$

$\therefore C_1 C_2 < r_1 - r_2 \Rightarrow \left(\frac{\lambda-\mu}{2}\right) < \sqrt{\frac{\lambda^2}{4}-c} - \sqrt{\frac{\mu^2}{4}-c}$

$\Rightarrow \frac{\lambda^2}{4}+\frac{\mu^2}{4}-2\times\frac{\lambda}{2}\times\frac{\mu}{2} < \frac{\lambda^2}{4}-c+\frac{\mu^2}{4}-c$

$-2\sqrt{\left(\frac{\lambda^2}{4}-c\right)\left(\frac{\mu^2}{4}-c\right)}$

$\Rightarrow 2\sqrt{\left(\frac{\lambda^2}{4}-c\right)\left(\frac{\mu^2}{4}-c\right)} < 2.\frac{\lambda\mu}{4}-2c$

$\frac{\lambda^2\mu^2}{16}-c\left(\frac{\lambda^2}{4}+\frac{\mu^2}{4}\right)+c^2 < \frac{\lambda^2\mu^2}{16}+c^2 -$

$2\times c\times\frac{\lambda\mu}{4}$

$c\left(\frac{\lambda^2}{4}+\frac{\mu^2}{4}-2\times\frac{\lambda}{2}\times\frac{\mu}{2}\right) > 0$

$c\left(\frac{\lambda}{2}-\frac{\mu}{2}\right)^2 > 0$

$\Rightarrow C > 0$

Also $\therefore \lambda > \mu$

$\left(\frac{-\mu}{2}, 0\right)$ will be inside

$x^2+y^2+\lambda x+c=0$

$\Rightarrow \frac{\mu^2}{4}+0-\frac{\mu\lambda}{2}+c<0$

$\because \frac{\mu^2}{4}+c>0$

$\therefore \frac{\lambda\mu}{2} > \frac{\mu^2}{4}+c$

$\therefore \frac{\lambda\mu}{2} > 0$

$\Rightarrow \lambda\mu > 0$

Hence, proved.

Example 7: A circle touches the line y = x at a point P such that OP = $4\sqrt{2}$ where O is the origin. The circle contains the point (–10, 2) in its interior and the length of its chord on the line x + y = 0 is $6\sqrt{2}$. Find the equation of the circle.

Sol: In this question, the concept of rotation of axes would be useful.

Let the new co-ordinate axis be rotated by an angle of 45° in the clockwise direction. Then

$X = x\ \cos\ (\theta) + \ y \sin(\theta)$

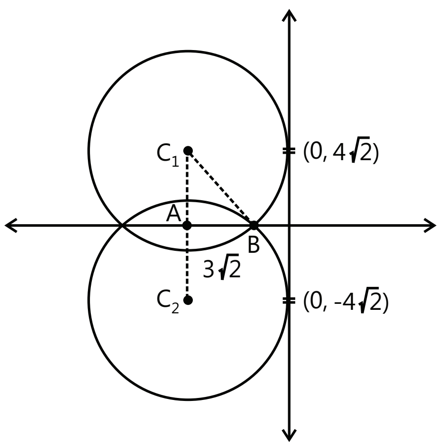
$Y = -x\ \ \sin\ (\theta) + \ y \cos(\theta)$

Where $\theta = 45°$

$\therefore\ X = \frac{x-y}{\sqrt{2}}$

$Y = \frac{x+y}{\sqrt{2}}$

The image after rotation would be



In Δ ABC, AC, $= 4\sqrt{2}$

AB $= 3\sqrt{2}$

$\therefore$ Radius = $\sqrt{\left(4\sqrt{2}\right)^2 + \left(3\sqrt{2}\right)^2} = b\sqrt{2}$

$\therefore$ Equation of the circle is

$$\left(X + 5\sqrt{2}\right)^2 + \left(Y \mp 4\sqrt{2}\right)^2 = \left(5\sqrt{2}\right)^2$$

$$\text{Or, } \left(\frac{x-y}{\sqrt{2}} + 5\sqrt{2}\right)^2 + \left(\frac{x+y}{\sqrt{2}} \mp 4\sqrt{2}\right)^2 = \left(5\sqrt{2}\right)$$

$$\text{or, } (x-y+10)^2 + (x+y\pm 8)^2 = 100$$

But, since (-10, 2) lies inside the circle.

The equation of the circle is

$$(x-y+10)^2 + (x+y+8)^2 = 100$$

$$\text{Or, } x^2 + y^2 + 100 - 2xy - 20y + 20x$$

$$+x^2 + y^2 + 64 + 2xy + 16y + 16x = 100$$

$$\text{Or, } 2x^2 + 2y^2 + 36x - 4y + 64 = 0$$

$$\text{Or, } x^2 + y^2 + 18x - 2y + 32 = 0$$

Example 8: Derive the equation of the circle passing through the centres of the three given circles $x^2 + y^2 - 4y - 5 = 0$,

$x^2 + y^2 + 12x + 4y + 31 = 0$ and

$x^2 + y^2 + 8x + 10y + 32 = 0$.

Sol: Find the relation between the centres of the circle and there use the appropriate form of circle.

Let P, Q and R denote the centres of the given circle

$P \equiv (0, 2)$, $Q = (-6, -2)$ and

$R \equiv\ (-4, -5)$

$$\therefore\ m_{PQ} = \frac{-2-2}{-6-0} = \frac{-4}{-6} = \frac{2}{3}$$

$$m_{QR} = \frac{-5+2}{-4+6} = \frac{-3}{2}$$

$$\therefore\ m_{PQ} . m_{QR} = \frac{2}{3} \times \frac{-3}{2} = -1$$

$\Rightarrow$ PQ is perpendicular to QR

$\therefore$ Using diameter form, we get

(x - 0) (x + 4) + (y - 2) (y + 5) =0

Example 9: Area of Quadrilateral PQRS is 18, side PQ || RS and PQ = 2RS and PS$\perp$ PQ and RS. then radius a circle drawn inside the quadrilateral PQRS touching all the sides is,

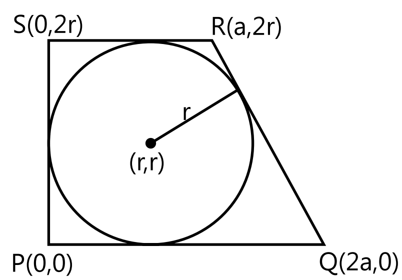
(A) 3 (B) 2 (C) $\frac{3}{2}$ (D) 1

Sol: (B) Let r be the radius of the circle, then PS = 2r. Let P be the origin and PQ and PS as x-axis and y-axis respectively.

$\therefore$ The coordinates of P, Q, R, S are (0, 0), (2a, 0), (a, 2r) and (0, 2r) respectively.

$\therefore$ Area (PQRS) = $\left(\frac{1}{2}\right)$(a + 2a)(2r) = 18

$\Rightarrow$ ar = 6.



$\therefore$ Equation of QR is

$$\left(y - y_1\right) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)\left(x - x_1\right)$$

$\Rightarrow (y-2r) = \left(\dfrac{0-2r}{2a-a}\right)(x-a)$

$\Rightarrow (y-2r) = \dfrac{-2r}{a}(x-a)$

$\Rightarrow ay - 2ar = -2rx + 2ar$

$\Rightarrow 2rx + ay - 4ar = 0$

$\therefore$ QR is a tangent to the circle

$\therefore \left|\dfrac{2r^2 + ar - 4ar}{\sqrt{4r^2+a^2}}\right| = r$

$\Rightarrow \left|\dfrac{r(2r-3a)}{\sqrt{4r^2+a^2}}\right| = r$

$\Rightarrow (2r-3a)^2 = 4r^2 + a^2$

$\Rightarrow 4r^2 + 9a^2 - 12\ ar = 4r^2 + a^2$

$\Rightarrow 8a^2 = 12\ ar$

$\Rightarrow 2a^2 = 3\ ar$

$\Rightarrow 2a^2 = 3 \times 6$

$\Rightarrow a = 3$

$\therefore r = 2\ (\because ar = 6)$

Example 10: A circle having centre at (0, 0) and radius equal to 'a' meets the x - axis at P and Q. A(α) and B(β) are points on this circle such that $\alpha - \beta = 2\gamma$, where γ is a constant. Then locus of the point of intersection of PA and QB is

(A) $x^2 - y^2 - 2ay \tan\gamma = a^2$

(B) $x^2 + y^2 - 2ay \tan\gamma = a^2$

(C) $x^2 + y^2 + 2ay \tan\gamma = a^2$

(D) $x^2 - y^2 + 2ay \tan\gamma = a^2$

Sol: (B) Let the equation of the circle be $x^2 + y^2 = a^2$

$\therefore P \equiv (-a, 0)$ and $Q = (a, 0)$

$\therefore$ Equation of PA is

$(y-0) = \dfrac{a\sin\alpha - 0}{a\cos\alpha + a}(x+a)$

$\Rightarrow y = \dfrac{a\ \sin\alpha}{a(\cos\alpha + 1)}(x+a)$

$\Rightarrow y = \dfrac{a.2\sin\frac{\alpha}{2}.\cos\frac{\alpha}{2}}{a.2\cos^2\frac{\alpha}{2}}(x+a)$

$\Rightarrow y = \tan\dfrac{\alpha}{2}(x+a)$ ...(i)

Similarly, equation of BQ is

$(y-v) = \dfrac{a\sin\beta - 0}{a\cos\beta - a}(x-a)$

$\Rightarrow y = \dfrac{2\ a.\sin\frac{\beta}{2}.\cos\frac{\beta}{2}}{-a \times 2\sin\frac{\beta}{2}}(x-a)$

$\Rightarrow y = -\cot\left(\dfrac{\beta}{2}\right)(x-a)$ ...(ii)

Now, we eliminate α, β using (i) and (ii)

$\because \alpha - \beta = 2r$

$\Rightarrow \dfrac{\alpha}{2} - \dfrac{\beta}{2} = r$

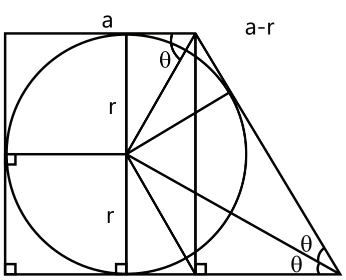
$\Rightarrow \tan\left(\dfrac{\alpha}{2} - \dfrac{\beta}{2}\right) = \tan r$

$\Rightarrow \dfrac{\tan\frac{\alpha}{2} - \tan\frac{\beta}{2}}{1 + \tan\frac{\alpha}{2}.\tan\frac{\beta}{2}} = \tan r$

$\Rightarrow \dfrac{\frac{y}{x+a} - \frac{a-x}{y}}{1 + \frac{y}{a+x} \times \frac{a-x}{y}} = \tan\ \gamma$

$\Rightarrow \dfrac{y^2 - a^2 + x^2}{ay + xy + ay - xy} = \tan\gamma$

$\Rightarrow x^2 + y^2 - 2ay\ \tan\gamma - a^2 = 0$



JEE Main/Boards

Exercise 1

Q.1 Find the equation of the circle whose centre lies on the line $2x - y - 3 = 0$ and which passes through the points (3, –2) and (–2, 0).

Q.2 Show that four points (0, 0), (1, 1), (5, –5) and (6, –4) are concylic.

Q.3 Find the centre, the radius and the equation of the circle drawn on the line joining A(–1, 2) and B(3, –4) as diameter.

Q.4 Find the equation of the tangent and the normal to the circle $x^2 + y^2 = 25$ at the point P(–3, –4).

Q.5 Show that the tangent to $x^2 + y^2 = 5$ at (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$

Q.6 Find the equation of the tangents to the circle $x^2 + y^2 - 2x + 8y = 23$ drawn from an external point (8, –3).

Q.7 Find the equation of the circle whose centre is (–4, 2) and having the line $x - y = 3$ as a tangent

Q.8 Find the equation of the circle through the points of intersections of two given circles

$x^2 + y^2 - 8x - 2y + 7 = 0$ and

$x^2 + y^2 - 4x + 10y + 8 = 0$ and passing through (3, –3).

Q.9 Find the equation of chord of the circle $x^2 + y^2 - 4x = 0$ which is bisected at the point (1, 1).

Q.10 Find the equation of chord of contact of the circle $x^2 + y^2 - 4x = 0$ with respect to the point (6, 0).

Q.11 Find the length of the tangent drawn from the point (3, 2) to the circle $4x^2 + 4y^2 + 4x + 16y + 13 = 0$.

Q.12 Obtain the equations of common tangents of the circles $x^2 + y^2 = 9$ and $x^2 + y^2 - 12x + 27 = 0$.

Q.13 The centres of the circle passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$ are $\left(\frac{1}{2}, \pm\sqrt{2}\right)$.

Q.14 The abscissae of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $x^2 + 2px - q^2 = 0$. Find the equation and the radius of the circle with AB as diameter.

Q.15 Show that the line $x + y = 2$ touches the circles $x^2 + y^2 = 2$ and $x^2 + y^2 + 3x + 3y - 8 = 0$ at the point where the two circles touch each other.

Q.16 One of the diameters of the circle circumscribing the rectangle ABCD is $4y = x + 7$. If A and B are the points (–3, 4) and (5, 4) respectively, find the area of the rectangle.

Q.17 A circle of radius 2 lies in the first quadrant and touches both the axes of co-ordinates, Find the equation of the circle with centre at (6, 5) and touching the above circle externally.

Q.18 If $\left(m_i, \frac{1}{m_i}\right)$; i = 1, 2, 3, 4 are four distinct point on a circle, show that $m_1m_2m_3m_4 = 1$.

Q.19 Show that the circle on the chord $x\cos\alpha + y\sin\alpha - p = 0$ of the circle $x^2 + y^2 = a^2$ as diameter is $x^2 + y^2 - a^2 - 2p(x\cos\alpha + y\sin\alpha - p) = 0$.

Q.20 Find the length of the chord of the circle $x^2 + y^2 = 16$ which bisects the line joining the points (2, 3) and (1, 2) perpendicularly.

Q.21 Find the angle that the chord of circle $x^2 + y^2 - 4y = 0$ along the line $x + y = 1$ subtends at the circumference of the larger segment.

Q.22 Prove that the equation $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$, where λ is a parameter, represents a family of circles passing through two fixed points A and B on the x-axis. Also find the equation of that circle of the family, the tangents to which at A and B meet on the line $x + 2y + 5 = 0$.

Q.23 Find the area of the quadrilateral formed by a pair of tangents from the point (4, 5) to the circle $x^2 + y^2 - 4x - 2y - 11 = 0$ and a pair of its radii.

Q.24 If the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ cut the co-ordinate axes in concyclic points, prove that $a_1a_2 = b_1b_2$.

Q.25 Show that the length of the tangent from any point on the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ to the circle

$x^2 + y^2 + 2gx + 2fy + c_1 = 0$ is $\sqrt{c_1 - c}$.

Q.26 Find the point from which the tangents to the three circles $x^2 + y^2 - 4x + 7 = 0$,

$2x^2 + 2y^2 - 3x + 5y + 9 = 0$

and $x^2 + y^2 + y = 0$ are equal in length. Find also this length.

Q.27 The chord of contact of tangents from a point on the circle $x^2 + y^2 = a^2$ to the circle $x^2 + y^2 = b^2$ touches the circle $x^2 + y^2 = c^2$. Show that a, b, c are in G.P.

Q.28 Obtain the equation of the circle orthogonal to both the circles

$x^2 + y^2 + 3x - 5y + 6 = 0$ and

$4x^2 + 4y^2 - 28x + 29 = 0$ and whose centre lies on the line $3x + 4y + 1 = 0$.

Q.29 From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$, a chord AB is drawn and extended to a point M such that AM = 2AB. Find the equation of the locus of M.

Q.30 From the origin, chords are drawn to the circle $(x - 1)^2 + y^2 = 1$. Find the equation to the locus of the middle points of these chords.

Q.31 Tangent at any point on the circle $x^2 + y^2=a^2$ meets the circle $x^2 + y^2 = b^2$ at P and Q. Find the condition on a and b such that tangents at P and Q meet at right angles.

Q.32 The tangent from a point to the circle $x^2 + y^2 = 1$ is perpendicular to the tangent from the same point to the circle $x^2 + y^2 = 3$. Show that the locus of the point is a circle .

Q.33 A variable circle passes through the point A (a, b) and touches the x-axis. Show that the locus of the other end of the diameter through A is $(x - a)^2 = 4by$.

Q.34 AB is a diameter of a circle. CD is a chord parallel to AB and 2CD = AB. The tangent at B meets the line AC (produced) at E. Prove that AE = 2AB.

Exercise 2

Single Correct Choice Type

Q.1 Centres of the three circles

$x^2 + y^2 - 4x - 6y - 14 = 0$

$x^2 + y^2 + 2x + 4y - 5 = 0$

and $x^2 + y^2 - 10x - 16y + 7 = 0$

(A) Are the vertices of a right triangle

(B) The vertices of an isosceles triangle which is not regular

(C) Vertices of a regular triangle

(D) Are collinear

Q.2 $2x^2 + 2y^2 + 2\lambda x + \lambda^2 = 0$ represents a circle for :

(A) Each real value of λ

(B) No real value of λ

(C) Positive λ

(D) Negative λ

Q.3 The area of an equilateral triangle inscribed in the circle $x^2 + y^2 - 2x = 0$ is

(A) $\frac{3\sqrt{3}}{4}$ (B) $\frac{3\sqrt{3}}{2}$

(C) $\frac{3\sqrt{3}}{8}$ (D) None of these

Q.4 A circle of radius 5 has its centre on the negative x-axis and passes through the point (2, 3). The intercept made by the circle on the y-axis is

(A)10 (B) $2\sqrt{21}$

(C) $2\sqrt{11}$ (D) imaginary y-intercept

Q.5 The radii of the circle $x^2 + y^2 = 1$, $x^2 + y^2 - 2x - 6y = 6$ and $x^2 + y^2 - 4x - 12y = 9$ are in

(A) A.P. (B) G.P.

(C) H.P. (D) None of these

Q.6 If the equation $x^2 + y^2 + 2\lambda x + 4 = 0$ and $x^2 + y^2 - 4\lambda y + 8 = 0$ represent real circles then the value of λ can be

(A) 5 (B) 2

(C) 3 (D) All of these

Q.7 The equation of the image of the circle $x^2 + y^2 +16x - 24y + 183 = 0$ by the line mirror $4x + 7y + 13 = 0$ is;

(A) $x^2 + y^2 + 32x - 4y + 235 = 0$

(B) $x^2 + y^2 + 32x + 4y - 235 = 0$

(C) $x^2 + y^2 + 32x - 4y - 235 = 0$

(D) $x^2 + y^2 + 32x + 4y + 235 = 0$

Q.8 The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x-axis in points whose abscissae are roots of the equation :

(A) $x^2 + ax + b = 0$ (B) $x^2 - ax + b = 0$

(C) $x^2 + ax - b = 0$ (C) $x^2 - ax - b = 0$

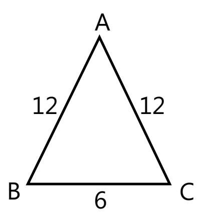
Q.9 A straight line l_1 with equation $x - 2y + 10 = 0$ meets the circle with equation $x^2 + y^2 = 100$ at B in the first quadrant. A line through B, perpendicular to l_1 cuts the y-axis at P (0, t). The value of 't' is

(A) 12 (B) 15 (C) 20 (D) 25

Q.10 If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are four distinct point on a circle of radius 4 units then, abcd is equal to

(A) 4 (B) $\frac{1}{4}$ (C) 1 (D) 16

Q.11 The radius of the circle passing through the vertices of the triangle ABC, is



(A) $\frac{8\sqrt{15}}{5}$ (B) $\frac{3\sqrt{15}}{5}$ (C) $3\sqrt{15}$ (D) $3\sqrt{2}$

Q.12 The points A(a, 0), B(0, b), C(c, 0) and D(0, d) are such that ac = bd and a, b, c, d are all non-zero. Then the points

(A) Form a parallelogram (B) Do not lie on a circle

(C) Form a trapezium (D) Are concyclic

Q.13 Four unit circles pass through the origin and have their centres on the coordinate axes. The area of the quadrilateral whose vertices are the points of intersection (in pairs) of the circle, is

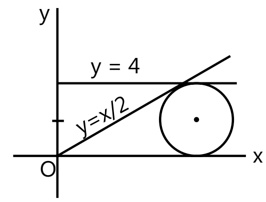
(A) 1 sq. unit

(B) $2\sqrt{2}$ sq. units

(C) 4sq. units

(D) Cannot be uniquely determined, insufficient data

Q.14 The x-coordinate of the center of the circle in the first quadrant (see figure) tangent to the lines $y = \frac{1}{2}x$, y= 4 and the x-axis is



(A) $4 + 2\sqrt{5}$ (B) $4 + \frac{8\sqrt{5}}{5}$

(C) $2 + \frac{6\sqrt{5}}{5}$ (D) $8 + 2\sqrt{5}$

Q.15 From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn and extended to a point M such that AM = 2AB. The equation of the locus of M is,

(A) $x^2 + 8x + y^2 = 0$

(B) $x^2 + 8x + (y - 3)^2 = 0$

(C) $(x - 3)^2 + 8x + y^2 = 0$

(D) $x^2 + 8x + 8y^2 = 0$

Q.16 If L_1 and L_2 are the length of the tangent from (0, 5) to the circles $x^2 + y^2 + 2x - 4 = 0$ and $x^2 + y^2 - y + 1 = 0$ then

(A) $L_1 = 2L_2$ (B) $L_2 = 2L_1$ (C) $L_1 = L_2$ (D) $L_1^2 = L_2$

Q.17 The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the centre of the circles lies on $x-2y = 4$. The radius of the circle is

(A) $3\sqrt{5}$ (B) $5\sqrt{3}$ (C) $2\sqrt{5}$ (D) $5\sqrt{2}$

Q.18 Coordinates of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 1$; $x^2 + y^2 + 2x - 3 = 0$ and $x^2 + y^2 + 2y - 3 = 0$ is

(A) (–1, –1) (B) (3, 3) (C) (2, 2) (D) (–2, –2)

Q.19 The anglebetween the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q.20 In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection of the hypotenuse and the semicircle, then the length AC equals to

(A) $\frac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ (B) $\frac{AB \cdot AD}{AB + AD}$

(C) $\sqrt{AB \cdot AD}$ (D) $\frac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$

Q.21 Locus of all point P (x, y) satisfying $x^3 + y^3 + 3xy = 1$ consists of union of

(A) A line and an isolated point

(B) A line pair and an isolated point

(C) A line and a circle

(D) A circle and an isolated point.

Previous Years' Questions

Q.1 The circle passing through the point(–1, 0) and touching the y-axis at (0, 2) also passes through the point ***(2011)***

(A) $\left(-\frac{3}{2},0\right)$ (B) $\left(-\frac{5}{2},2\right)$ (C) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (D) (–1, –4)

Q.2 Consider the two curves $C_1 : y^2 = 4x$

$C_2 : x^2 + y^2 - 6x + 1 = 0$, then ***(2008)***

(A) C_1 and C_2 touch each other only at one point

(B) C_1 and C_2 touch each other exactly at two points

(C) C_1 and C_2 intersect (but do not touch) at exactly two points

(D) C_1 and C_2 neither intersect nor touch each other

Q.3 If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is ***(2004)***

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 3 (D) 2

Q.4 The centre of circle inscribed in square formed by the lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$, is ***(2003)***

(A) (4, 7) (B) (7, 4) (C) (9, 4) (D) (4, 9)

Q.5 If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is ***(2002)***

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

Q.6 If the circle $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is ***(2000)***

(A) 2 or $-\frac{3}{2}$ (B) –2 or $-\frac{3}{2}$

(C) 2 or $\frac{3}{2}$ (D) –2 or $\frac{3}{2}$

Q.7 The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$. If Q and R have coordinates (3, 4) and (–4, 3) respectively, then $\angle QPR$ is equal to ***(2002)***

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Q.8 The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is ***(1998)***

(A) 0 (B) 1 (C) 3 (D) 4

Q.9 Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$. ***(2007)***

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Statement-I: The tangents are mutually perpendicular.

Statement-II: The locus of the points from which a mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

Q.10 Find the equation of circle touching the line $2x + 3y + 1 = 0$ at the point (1, –1) and is orthogonal to the circle which has the line segment having end points (0, –1) and (–2, 3) as the diameter. ***(2004)***

Q.11 Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C ***(2001)***

Q.12 Consider the family of circles $x^2 + y^2 = r^2$, $2 < r < 5$. If in the first quadrant, the common tangent to a circle of this family and the ellipse $4x^2 + 25y^2 = 100$ meets the coordinate axis at A and B, then find the equation of the locus of the mid points of AB. ***(1999)***

Q.13 C_1 and C_2 are two concentric circle the radius of C_2 being twice that of C_1. From a point P on C_2, tangents PA and PB are drawn to C_1. Prove that the centroid of the triangle PAB lies on C_1. ***(1998)***

Q.14 The length of the diameter of the circle which louches the x-axis at the point $(1,0)$ and passes through the point $(2,3)$ is ***(2012)***

(A) $\frac{10}{3}$ (B) $\frac{3}{5}$ (C) $\frac{6}{5}$ (D) $\frac{5}{3}$

Q.15 The circle through (1, –2) and touching the axis of x at (3,0) also passes through the point ***(2013)***

(A) $(2,-5)$ (B) $(5,-2)$ (C) $(-2,5)$ (D) $(-5,2)$

Q.16 The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$, and having centre at $(0,3)$ is ***(2013)***

(A) $x^2+y^2-6y+7=0$ (B) $x^2+y^2-6y-5=0$
(C) $x^2+y^2-6y+5=0$ (D) $x^2+y^2-6y-7=0$

Q.17 let C be the circle with centre at (1,1) and radius =1. If T is the circle centred at $(0,y)$, passing through origin and touching the circle C externally, then radius of T is equal to ***(2014)***

(A) $\frac{\sqrt{3}}{\sqrt{2}}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Q.18 The number of common tangents to circles $x^2+y^2-4x-6y-12=0$ and $x^2+y^2+6x-18y+26=0$, is ***(2015)***

(A) 2 (B) 3 (C) 4 (D) 1

Q.19 The centres of those circles which touch the circle, $x^2+y^2-8x-8y-4=0$, externally and also touch the x-axis, lie on: ***(2016)***

(A) An ellipse which is not a circle

(B) A hyperbola

(C) A parabola

(D) A circle

JEE Advanced/Boards

Exercise 1

Q.1 Let S : $x^2 + y^2 - 8x - 6y + 24 = 0$ be a circle and O is the origin. Let OAB is the line intersecting the circle at A and B. On the chord AB a point P is taken. The locus of the point P in each of the following cases.

(i) OP is the arithmetic mean of OA and OB

(ii) OP is the geometric mean of OA and OB

(iii) OP is the harmonic mean between OA and OB

Q.2 A circle $x^2 + y^2 + 4x - 2\sqrt{2}y + c = 0$ is the director circle of circle S_1 and S_1 is the director circle of circle S_2 and so on. If the sum of radii of all these circles is 2, then the value of c is equal to $\sqrt{n}$ where $n \in N$. Find the value of n.

Q.3 If the circle $x^2 + y^2 + 4x + 22y + a = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - b = 0$ (where a, b > 0), then find the maximum value of (ab).

Q.4 Real number x, y satisfies $x^2 + y^2 = 1$. If the maximum and minimum value of the expression $z = \frac{4-y}{7-x}$ are M and m respectively, then find the value (2M + 6m).

Q.5 The radical axis of the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ and

$2x^2 + 2y^2 + 3x + 8y + 2c = 0$ touches the circle $x^2 + y^2 + 2x - 2y + 1 = 0$. Show that either $g = \frac{3}{4}$ or f = 2

Q.6 Consider a family of circles passing through two fixed points A (3, 7) & B (6, 5). The chords in which the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ cuts the members of the family are concurrent at a point. Find the coordinates of this point.

Q.7 Find the equation of circle passing through (1, 1) belonging to the system of co-axial circles that are tangent at (2, 2) to the locus of the point of intersection of mutually perpendicular tangent to the circle $x^2 + y^2 = 4$.

Q.8 The circle C : $x^2 + y^2 + kx + (1 + k)y - (k + 1) = 0$ passes through two fixed points for every real number k. Find

(i) the coordinates of these points.

(ii) the minimum value of the radius of a circle C.

Q.9 Find the equation of a circle which is co-axial with circles $2x^2 + 2y^2 - 2x + 6y - 3 = 0$ and

$x^2 + y^2 + 4x + 2y + 1 = 0$. It is given that the centre of the circle to be determined lies on the radical axis of these two circles.

Q.10 Find the equation of the circle passing through the points of intersection of circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 4y - 12 = 0$ and cutting the circle $x^2 + y^2 - 2x - 4 = 0$ orthogonally.

Q.11 The centre of the circles S = 0 lie on line $2x - 2y + 9 = 0$ & S = 0 cuts orthogonally the circle $x^2 + y^2 = 4$. Show that circle S = 0 passes through two fixed points & find their coordinates.

Q.12 Find the equation of a circle passing through the origin if the line pair, $xy - 3x + 2y - 6 = 0$ is orthogonal to it. If this circle is orthogonal to the circle $x^2 + y^2 - kx + 2ky - 8 = 0$ then find the value of k.

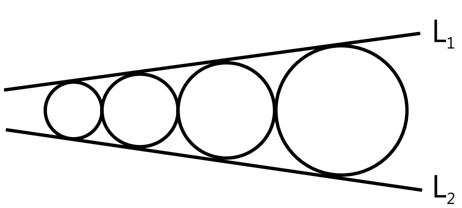
Q.13 Find the equation of the circle which cuts the circle $x^2 + y^2 - 14x - 8y + 64 = 0$ and the coordinate axes orthogonally.

Q.14 Show that the locus of the centres of a circle which cuts two given circles orthogonally is a straight line & hence deduce the locus of the centres of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ & $x^2 + y^2 - 5x + 4y + 2 = 0$ orthogonally. Intercept the locus.

Q.15 Find the equation of a circle which touches the line x + y = 5 at the point (–2, 7) and cuts the circle $x^2 + y^2 + 4x - 6y + 9 = 0$ orthogonally.

Q.16 Find the equation of the circle passing through the point (–6, 0) if the power of the point (1, 1) w.r.t. the circle is 5 and it cuts the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ orthogonally.

Q.17 As shown in the figure, the five circles are tangent to one another consecutively and to the lines L_1 and L_2. If the radius of the largest circle is 18 and that of the smaller one is 8, then find the radius of the middle circle.



Q.18 Find the equation of a circle which touches the line $7x^2 - 18xy + 7y^2 = 0$ and the circle $x^2 + y^2 - 8x - 8y = 0$ and is contained in the given circle.

Q.19 Consider two circle C_1 of radius 'a' and C_2 of radius 'b' (b > a) both lying in the first quadrant and touching the coordinate axes. In each of the conditions listed in column-I, the ratio of b/a is

Column I	Column II
(A) C_1 and C_2 touch each other	(p) $2 + \sqrt{2}$
(B) C_1 and C_2 are orthogonal	(q) 3
(C) C_1 and C_2 intersect so that the common chord is longest	(r) $2 + \sqrt{3}$
(D) C_2 passes through the centre of C_1	(s) $3 + 2\sqrt{2}$
	(t) $3 - 2\sqrt{2}$

Q.20 A circle with centre in the first quadrant is tangent to y = x + 10, y = x – 6, and the y-axis. Let (h, k) be the centre of the circle. If the value of $(h + k) = a + b\sqrt{a}$ where $\sqrt{a}$ is a surd, find the value of a + b.

Q.21 Circles C_1 and C_2 are externally tangent and they are both internally tangent to the circle C_3. The radii of C_1 and C_2 are 4 and 10, respectively and the centres of the three circles are collinear. A chord of C_3 is also a common internal tangent of C_1 and C_2. Given that the length of the chord is $\frac{m\sqrt{n}}{p}$ where m, n and p are positive integers, m and p are relatively prime and n is not divisible by the square of any prime, find the value of (m + n + p).

Q.22 Find the equation of the circle passing through the three points (4, 7), (5, 6) and (1, 8). Also find the coordinates of the point of intersection of the tangents to the circle at the points where it is cut by the straight line 5x + y + 17 = 0.

Q.23 The line 2x – 3y + 1 = 0 is tangent to a circle S = 0 at (1, 1). If the radius of the circle is $\sqrt{13}$. Find the equation of the circle S.

Q.24 Find the equation of the circle which passes through the point (1, 1) & which touches the circle

$x^2 + y^2 + 4x - 6y - 3 = 0$ at the point (2, 3) on it.

Exercise 2

Single Correct Choice Type

Q.1 B and C are fixed points having co-ordinates (3, 0) and (–3, 0) respectively. If the vertical angle BAC is 90°, then the locus of the centroid of the ΔABC has the equation :

(A) $x^2 + y^2 = 1$ (B) $x^2 + y^2 = 2$

(C) $9(x^2 + y^2) = 1$ (D) $9(x^2 + y^2) = 4$

Q.2 Number of points in which the graphs of $|y| = x + 1$ and $(x - 1)^2 + y^2 = 4$ intersect, is

(A) 1 (B) 2 (C) 3 (D) 4

Q.3 $y - 1 = m_1(x - 3)$ and $y - 3 = m_2(x - 1)$ are two family of straight lines, at right angles to each other. The locus of their point of intersection is

(A) $x^2 + y^2 - 2x - 6y + 10 = 0$

(B) $x^2 + y^2 - 4x - 4y + 6 = 0$

(C) $x^2 + y^2 - 2x - 6y + 6 = 0$

(D) $x^2 + y^2 - 4x - 4y - 6 = 0$

Q.4 The points (x_1, y_1), (x_2, y_2), (x_1, y_2) and (x_2, y_1) are always:

(A) Collinear (B) Concyclic

(C) Vertices of a square (D) Vertices of a rhombus

Q.5 Consider 3 non-collinear points A, B, C with coordinates (0, 6), (5, 5) and (–1, 1) respectively. Equation of a line tangent to the circle circumscribing the triangle ABC and passing through the origin is

(A) $2x - 3y = 0$ (B) $3x + 2y = 0$

(C) $3x - 2y = 0$ (D) $2x + 3y = 0$

Q.6 A (1, 0) and B(0, 1) and two fixed points on the circle $x^2 + y^2 = 1$. C is a variable point on this circle. As C moves, the locus of the orthocenter of the triangle ABC is

(A) $x^2 + y^2 - 2x - 2y + 1 = 0$

(B) $x^2 + y^2 - x - y = 0$

(C) $x^2 + y^2 = 4$

(D) $x^2 + y^2 + 2x - 2y + 1 = 0$

Q.7 A straight line with slope 2 and y - intercept 5 touches the circle, $x^2 + y^2 + 16x + 12y + c = 0$ at a point Q. Then the coordinates of Q are

(A) (–6, 11) (B) (–9, –13)

(C) (–10, –15) (D) (–6, –7)

Q.8 A rhombus is inscribed in the region common to the two circles $x^2 + y^2 - 4x - 12 = 0$ and $x^2 + y^2 + 4x - 12 = 0$ with two of its vertices on the line joining the centres of the circles. The area of the rhombus is

(A) $8\sqrt{3}$ sq. units (B) $4\sqrt{3}$ sq. units

(C) $16\sqrt{3}$ sq. units (D) None of these

Q.9 From (3, 4) chords are drawn to the circle $x^2 + y^2 - 4x = 0$. The locus of the mid points of the chords is:

(A) $x^2 + y^2 - 5x - 4y + 6 = 0$

(B) $x^2 + y^2 + 5x - 4y + 6 = 0$

(C) $x^2 + y^2 - 5x + 4y + 6 = 0$

(D) $x^2 + y^2 - 5x - 4y - 6 = 0$

Q.10 The line joining (5, 5) to $(10 \cos \theta, 10 \sin \theta)$ is divided internally in the ratio 2 : 3 at P. If θ varies then the locus of P is :

(A) A pair of straight lines

(B) A circle

(C) A straight line

(D) A second degree curve which is not a circle

Q.11 The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is:

(A) $x^2 + y^2 + 2x - 2y - 13 = 0$

(B) $x^2 + y^2 - 2x - 2y - 11 = 0$

(C) $x^2 + y^2 - 2x + 2y + 12 = 0$

(D) $x^2 + y^2 - 2x - 2y + 14 = 0$

Q.12 The shortest distance from the line $3x + 4y = 25$ to the circle $x^2 + y^2 = 6x - 8y$ is equal to

(A) $\frac{7}{5}$ (B) $\frac{9}{5}$ (C) $\frac{11}{5}$ (D) $\frac{32}{5}$

Q.13 The equation of a line inclined at an angle $\frac{\pi}{4}$ to the axis X, such that the two circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 10x - 14y + 65 = 0$ intercept equal lengths on it, is

(A) $2x - 2y - 3 = 0$ (B) $2x - 2y + 3 = 0$

(C) $x - y + 6 = 0$ (D) $x - y - 6 = 0$

Q.14 The locus of the midpoint of a line segment that is drawn from a given external point P to a given circle with centre O (where O is origin) and radius r, is

(A) A straight line perpendicular to PO

(B) A circle with centre P and radius r

(C) A circle with centre P and radius 2r

(D) A circle with centre at the midpoint PO and radius $\frac{r}{2}$

Multiple Correct Choice Type

Q.15 Locus of the intersection of the two straight lines passing through (1, 0) and (–1, 0) respectively and including an angle of $45°$ can be a circle with

(A) Centre (1, 0) and radius $\sqrt{2}$.

(B) Centre (1, 0) and radius 2.

(C) Centre (0, 1) and radius $\sqrt{2}$.

(D) Centre (0, –1) and radius $\sqrt{2}$.

Q.16 Consider the circles

$S_1 : x^2 + y^2 + 2x + 4y + 1 = 0$

$S_2 : x^2 + y^2 - 4x + 3 = 0$

$S_3 : x^2 + y^2 + 6y + 5 = 0$

Which of this following statement are correct?

(A) Radical centre of S_1, S_2 and S_3 lies in 1st quadrant.

(B) Radical centre of S_1, S_2 and S_3 lies in 4st quadrant.

(C) Radical centre of S_1, S_2 and S_3 orthogonally is 1.

(D) Circle orthogonal to S_1, S_2 and S_3 has its x and y intercept equal to zero.

Q.17 Consider the circles

$C_1 : x^2 + y^2 - 4x + 6y + 8 = 0$

$C_2 : x^2 + y^2 - 10x - 6y + 14 = 0$

Which of the following statement (s) hold good in respect of C_1 and C_2?

(A) C_1 and C_2 are orthogonal.

(B) C_1 and C_2 touch each other.

(C) Radical axis between C_1 and C_2 is also one of their common tangent.

(D) Middle point of the line joining the centres of C_1 and C_2 lies on their radical axis.

Q.18 A circle passes through the points (–1 , 1), (0, 6) and (5, 5). The point (s) on this circle, the tangent (s) at which is/are parallel to the straight line joining the origin to its centre is/are:

(A) (1, –5) (B) (5, 1) (C) (–5, –1) (D) (–1, 5)

Q.19 The circles $x^2 + y^2 + 2x + 4y - 20 = 0$ and $x^2 + y^2 + 6x - 8y + 10 = 0$

(A) Are such that the number of common tangents on them is 2

(B) Are not orthogonal

(C) Are such that the length of their common tangent is $5\left(\frac{12}{5}\right)^{\frac{1}{4}}$

(D) Are such that the length of their common chord is $5\,\frac{\sqrt{3}}{2}$.

Q.20 Three distinct lines are drawn in a plane. Suppose there exist exactly n circles in the plane tangent to all the three lines, then the possible values of n is/are

(A) 0 (B) 1 (C) 2 (D) 4

Q.21 The equation of a circle C_1 is $x^2+y^2+14x-4y+28=0$. The locus of the point of intersection of orthogonal tangents to C_1 is the curve C_2 and the locus of the point of intersection of perpendicular tangents to C_2 is the curve C_3 then the statement (s) which hold good?

(A) C_3 is a circle

(B) Area enclosed by C_3 is 100π sq. unit

(C) Area of C_2 is $\sqrt{2}$ times the area of C_1.

(D) C_2 and C_3 are concentric circles.

Q.22 The circles $x^2 + y^2 - 2x - 4y + 1 = 0$ and $x^2 + y^2 + 4x + 4y - 1 = 0$

(A) Touch internally

(B) Touch externally

(C) Have $3x + 4y - 1 = 0$ as the common tangent at the point of contact.

(D) have $3x + 4y + 1 = 0$ as the common tangent at the point of contact.

Q.23 Which of the following is/are True? The circles $x^2 + y^2 - 6x - 6y + 9 = 0$ and $x^2 + y^2 + 6x + 6y + 9 = 0$ are such that

(A) They do not intersect.

(B) They touch each other.

(C) Their direct common tangents are parallel.

(D) Their trannsverse common tangents are perpendicular.

Q.24 Two circles $x^2 + y^2 + px + py - 7 = 0$ and $x^2 + y^2 - 10x + 2py + 1 = 0$ intersect each other orthogonally then the value of p is

(A) 1 (B) 2 (C) 3 (D) 5

Q.25 Which of the following statements is/are incorrect?

(A) Two circles always have a unique common normal.

(B) Radical axis is always perpendicular bisector to the line joining the centres of two circles.

(C) Radical axis is nearer to the centre of circle of smaller radius.

(D) Two circles always have a radical axis.

Assertion Reasoning Type

(A) Statement-I is true, statement-II is true and statement-II is correct explanation for statement-I.

(B) Statement-I is true, statement-II is true and statement-II is NOT the correct explanation for statement-I.

(C) Statement-I is true, statement-II is false.

(D) Statement-I is false, statement-II is true.

Q.26 Consider the lines $L : (k + 7)x - (k - 1)y - 4(k - 5)=0$ where k is a parameter and the circle

$C : x^2 + y^2 + 4x + 12y - 60 = 0$

Statement-I: Every member of L intersects the circle 'C' at an angle of $90°$

Statement-II: Every member of L tangent to the circle C.

Q.27 Statement-I: Angle between the tangents drawn from the point P(13, 6) to the circle $S : x^2 + y^2 - 6x + 8y - 75 = 0$ is $90°$.

Statement-II: Point P lies on the director circle of S.

Q.28 Statement-I: From the point (1, 5) as its centre, only one circle can be drawn touching the circle $x^2 + y^2 - 2x = 7$.

Statement-II: Point (1, 5) lies outside the circle $x^2 + y^2 - 2x = 7$.

Q.29 Statement-I: Let C_1 (0, 0) and C_2(2, 2) be centres of two circle and $L : x + y - 2 = 0$ is their common chord. If length of common chord is equal to $\sqrt{2}$, then both circles intersect orthogonally.

Statement-II: Two circles will be orthogonal if their centres are mirror images of each other in their common chord and distance between centres is equal to length of common chord.

Q.30 Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, x_3)$ are the vertices of a triangle ABC.

Statement-I : If angel C is obtuse then the quantity $(x_3 - x_1)(x_3 - x_2) + (y_3 - y_1)(y_3 - y_2)$ is negative.

Statement-II: Diameter of a circle subtends obtuse angle at any point lying inside the semicircle.

Q.31 Let C be a circle with centre 'O' and HK is the chord of contact of pair of the tangents from point A. OA intersects the circle C at P and Q and B is the midpoint of HK, then

Statement-I: AB is the harmonic mean of AP and AQ.

Statement-II: AK is the Geometric mean of AB and AO and OA is the arithmetic mean of AP and AQ.

Comprehension Type

Paragraph for questions 32 to 34

Let A, B, C be three sets of real numbers (x, y) defined as

$A : \{(x, y):\ y \geq 1\}$

$B : \{(x, y): x^2 + y^2 - 4x - 2y - 4 = 0\}$

$C : \{(x, y): x + y = \sqrt{2}\}$

Q.32 Number of elements in the $A \cap B \cap C$ is

(A) 0 (B) 1 (C) 2 (D) infinite

Q.33 $(x + 1)^2 + (y - 1)^2 + (x - 5)^2 + (y - 1)^2$ has the value equal to

(A) 16 (B) 25 (C) 36 (D) 49

Q.34 If the locus of the point of intersection of the pair of perpendicular tangents to the cirlc B is the curve S then the area enclosed between B and S is

(A) 6π (B) 8π (C) 9π (D) 18π

Paragraph for questions 35 to 36

Consider a circle $x^2 + y^2 = 4$ and a point P(4, 2). θ denotes the angle enclosed by the tangents from P on the circle and A, B are the points of contact of the tangents from P on the circle.

Q.35 The value of θ lies in the interval

(A) (0, 15°) (B) (15°, 30°)

(C) (30°, 45°) (D) (45°, 60°)

Q.36 The intercept made by a tangent on the x-axis is

(A) $\frac{9}{4}$ (B) $\frac{10}{4}$ (C) $\frac{11}{4}$ (D) $\frac{12}{4}$

Paragraph for questions 37 to 39

Consider the circle S : $x^2 + y^2 - 4x - 1 = 0$ and the line L : $y = 3x - 1$. If the line L cuts the circle at A and B then

Q.37 Length of the chord AB equal

(A) $2\sqrt{5}$ (B) $\sqrt{5}$ (C) $5\sqrt{2}$ (C) $\sqrt{10}$

Q.38 The angle subtended by the chord AB in the minor arc of S is

(A) $\frac{3\pi}{4}$ (B) $\frac{5\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{\pi}{4}$

Q.39 Acute angel between the line L and the circle S is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Previous Years' Questions

Q.1 Tangents drawn from the point P (1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the point A and B. The equation of the circumcircle of the triangle PAB is ***(2009)***

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$

(B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$

(D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Q.2 Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, its radius is ***(2007)***

(A) 3 (B) 2 (C) $\frac{3}{2}$ (D) 1

Q.3 The locus of the centre of circle which touches $(y - 1)^2 + x^2 = 1$ externally and also touches x axis, is ***(2005)***

(A) $\{x^2 = 4y, y \geq 0\} \cup \{(0, y), y < 0\}$

(B) $x^2 = y$

(C) $y = 4x^2$

(D) $y^2 = 4x \cup (0, y), y \in R$

Q.4 Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals ***(2001)***

(A) $\sqrt{PQ \cdot RS}$ (B) $\frac{PQ + RS}{2}$

(C) $\frac{2PQ \cdot RS}{PQ + RS}$ (D) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

Q.5 Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then the locus of centroid of the triangle PAB as P moves on the circle is ***(2001)***

(A) A parabola (B) A circle

(C) An ellipse (D) A pair of straight lines

Q.6 If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then ***(1999)***

(A) $p^2 = q^2$ (B) $p^2 = 8q^2$

(C) $p^2 < 8q^2$ (D) $p^2 > 8q^2$

Q.7 Consider

$L_1 : 2x + 3y + p - 3 = 0$

$L_2 : 2x + 3y + p + 3 = 0$

where p is a real number and

C : $x^2 + y^2 - 6x + 10y + 30 = 0$ ***(2008)***

Statement-I: If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C. and

Statement-II: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

Paragraph 1: Let ABCD be a square of side length 2 unit. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of square ABCD. L is the line through A. ***(2006)***

Q.8 If P is a point of C_1 and Q is a point on C_2, then $\frac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to

(A) 0.75 (B) 1.25 (C) 1 (D) 0.5

Q.9 A circle touches the line L and the circle C_1 externally such that both the circle are on the same side of the line, then the locus of centre of the circle is

(A) Ellipse (B) Hyperbola

(C) Parabola (D) Parts of straight line

Q.10 A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1. then area of $\Delta T_1T_2T_3$ is

(A) $\frac{1}{2}$ sq unit (B) $\frac{2}{3}$ sq unit

(C) 1 sq unit (D) 2 sq unit

Paragraph 2: A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ. ***(2008)***

Q.11 The equation of circle C is

(A) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(B) $(x - 2\sqrt{3})^2 + \left(y+\frac{1}{2}\right)^2 = 1$

(C) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(D) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

Q.12 Point E and F are given by

(A) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\sqrt{3},0\right)$ (B) $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),\left(\sqrt{3},0\right)$

(C) $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$ (D) $\left(\frac{3}{2},\frac{\sqrt{3}}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

Q.13 Equations of the sides QR, RP are

(A) $y = \frac{2}{\sqrt{3}}x + 1,\ y = -\frac{2}{\sqrt{3}}x - 1$

(B) $y = \frac{1}{\sqrt{3}}x,\ y = 0$

(C) $y = \frac{\sqrt{3}}{2}x + 1,\ y = -\frac{\sqrt{3}}{2}x - 1$

(D) $y = \sqrt{3}x,\ y = 0$

Q.14 Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin O to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. ***(2001)***

Q.15 Let T_1, T_2 and be two tangents drawn from (–2, 0) onto the circle C : $x^2 + y^2 = 1$. Determine the circles touching C and having T_1, T_2 as their pair of tangents. Further, find the equations of all possible common tangents to these circles when taken two at a time. ***(1999)***

Q.16 Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$, where $k > 0$, then the value of $\left[k\right]$ is

[**Note :** $\left[k\right]$ denotes the largest integer less than or equal to k] ***(2010)***

Q.17 The circle passing through the point $(-1,0)$ and touching the $y-$axis at $(0,2)$ also passes through the point ***(2011)***

(A) $\left(-\frac{3}{2},0\right)$ (B) $\left(-\frac{5}{2},2\right)$

(C) $\left(-\frac{3}{2},\frac{5}{2}\right)$ (D) $(-4,0)$

Paragraph 3: A tangent PT is drawn to the circle $x^2+y^2=4$ at the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2+y^2=1$.

Q. 18 A common tangent of the two circles is ***(2012)***

(A) $x=4$ (B) $y=2$

(C) $x+\sqrt{3}y=3$ (D) $x+2\sqrt{2}y=6$

Q.19 A possible equation of L is ***(2012)***

(A) $x-\sqrt{3}y=1$ (B) $x+\sqrt{3}y=1$

(C) $x-\sqrt{3}y=-1$ (D) $x+\sqrt{3}y=5$

Q.20 Let S be the focus of the parabola $y^2=8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is ***(2012)***

Q.21 The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2+y^2=9$ is ***(2012)***

(A) $20(x^2 + y^2) - 36x + 45y = 0$

(B) $20(x^2 + y^2) + 36x$ ñ $45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$

(D) $36(x^2 + y^2) + 20x$ ñ $45y = 0$

Q.22 Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) ***(2013)***

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$

(B) $x^2 + y^2 - 6x + 7y + 9 = 0$

(C) $x^2 + y^2 - 6x - 8y + 9 = 0$

(D) $x^2 + y^2 - 6x - 7y + 9 = 0$

Q.23 The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is ***(2014)***

(A) 3 (B) 6 (C) 9 (D) 15

Q.24 A circle S passes through the point (0, 1) and is orthogonal to the circles $(x-1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then ***(2014)***

(A) Radius of S is 8 (B) Radius of S is 7

(C) Centre of S is (-7, 1) (D) Centre of S is (-8, 1)

Q.25 Let

$$f(x) = \lim_{n\to\infty}\left(\frac{n^n(x+n)\left(x+\frac{n}{2}\right)\ldots\ldots\left(x+\frac{n}{n}\right)}{n!\left(x^2+n^2\right)\left(x^2+\frac{n^2}{4}\right)\ldots\ldots\left(x^2+\frac{n^2}{n^2}\right)}\right)^{\frac{x}{n}},$$

for all x > 0. Then ***(2016)***

(A) $f\left(\frac{1}{2}\right) \ge f(1)$ (B) $f\left(\frac{1}{3}\right) \le f\left(\frac{2}{3}\right)$

(C) $f(2) \le 0$ (D) $\frac{f'(3)}{f(3)} \ge \frac{f'(2)}{f(2)}$

Q.26 The circle $C_1 : x^2 + y^2 = 3$, with centre O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3 respectively. If Q_2 and Q_3 lie on the y-axis, then ***(2016)***

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) Area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) Area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Q.27 Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) ***(2016)***

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$

(B) $\left(\frac{1}{4}, \frac{1}{2}\right)$

(C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$

(D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

MASTERJEE Essential Questions

JEE Main/Boards

Exercise 1

Q.12 Q.18 Q.21
Q.23 Q.29

Exercise 2

Q.3 Q.7 Q.14
Q.15 Q.20

Previous Years' Questions

Q.1 Q.3 Q.5
Q.8 Q.11 Q.13

JEE Advanced/Boards

Exercise 1

Q.5 Q.9 Q.14
Q.17 Q.19 Q.21 Q.24

Exercise 2

Q.2 Q.4 Q.9
Q.13 Q.16 Q.21
Q.22 Q.25 Q.27
Q.22 Q.25 Q.26
Q.29 Q.32

Previous Years' Questions

Q.1 Q.3 Q.6
Q.7 Q.9 Q.13

Answer Key

JEE Main/Boards

Exercise 1

Q.1 $x^2 + y^2 + 3x + 12y + 2 = 0$

Q.3 $(1, -1)$, $\sqrt{13}$, $x^2 + y^2 - 2x + 2y - 11 = 0$

Q.4 $3x - 4y = 7$, $4x + 3y = 0$

Q.6 $13x + 9y = 77$, $3x - y - 27 = 0$

Q.7 $2x^2 + 2y^2 + 16x - 8y - 41 = 0$

Q.8 $23x^2 + 23y^2 - 156x + 38y + 168 = 0$

Q.9 $y = x$

Q.10 $x = 3$

Q.11 $\frac{\sqrt{109}}{2}$

Q.12 $x = 3$ and $y = \pm 3$.

Q.14 $x^2 + y^2 + 2ax + 2py - (b^2 + q^2) = 0$; $\sqrt{a^2 + b^2 + p^2 + q^2}$

Q.15 $x^2 + y^2 + 18x - 2y + 32 = 0$

Q.16 32 sq. units

Q.17 $x^2 + y^2 - 12x - 10y + 52 = 0$

Q.20 $4\sqrt{2}$

Q.21 $\cos^{-1} \frac{1}{2\sqrt{2}}$

Q.22 $x^2 + y^2 - 2x - 6y - 8 = 0$

Q.23 8 sq. units.

Q.26 $(2, -1)$; 2

Q.28 $4(x^2 + y^2) + 2y - 29 = 0$

Q.29 $x^2 + y^2 + 8x - 6y + 9 = 0$ **Q.30** $x^2 + y^2 - x = 0$ **Q.31** $2a^2 = b^2$

Exercise 2

Single Correct Choice Type

Q.1 D **Q.2** B **Q.3** A **Q.4** B **Q.5** A **Q.6** D

Q.7 D **Q.8** B **Q.9** C **Q.10** C **Q.11** A **Q.12** D

Q.13 C **Q.14** A **Q.15** B **Q. 16** C **Q.17** A **Q.18** D

Q.19 C **Q.20** D **Q.21** A

Previous Years' Questions

Q.1 D **Q.2** B **Q.3** C **Q.4** A **Q.5** C **Q.6** A

Q.7 C **Q.8** B **Q.10** $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **Q.11** (a,b) and (0,0)

Q.12 $4x^2+25y^2=4x^2y^2$ **Q.14** A **Q.15** B **Q.16** D **Q.17** D

Q.18 B **Q.19** C

JEE Advanced/Boards

Exercise 1

Q.1 (i) $x^2 + y^2 - 4x - 3y = 0$, (ii) $x^2 + y^2 = 24$, (iii) $4x + 3y = 24$ **Q.2** 32

Q.3 625 **Q.4** 4 **Q.6** $\left(2, \frac{23}{3}\right)$

Q.7 $x^2 + y^2 - 3x - 3y + 4 = 0$ **Q.8** (1, 0)& $\left(\frac{1}{2}, \frac{1}{2}\right)$; $r = \frac{1}{2\sqrt{2}}$ **Q.9** $4x^2 + 4y^2 + 6x + 10y - 1 = 0$

Q.10 $x^2 + y^2 + 16x + 14y - 12 = 0$ **Q.11** (–4, 4); $\left(-\frac{1}{2}, \frac{1}{2}\right)$ **Q.12** $x^2 + y^2 + 4x - 6y = 0$; k= 1;

Q.13 $x^2 + y^2 = 64$ **Q.14** $9x - 10y + 7 = 0$; radical axis **Q.15** $x^2 + y^2 + 7x - 11y + 38 = 0$

Q.16 $x^2 + y^2 + 6x - 3y = 0$ **Q.17** 12 **Q.18** $x^2 + y^2 - 12x - 12y + 64 = 0$

Q.19 (A) S; (B) R; (C) Q ; (D) P **Q.20** 10 **Q.21** 19

Q.22 (–4, 2), $x^2 + y^2 - 2x - 6y - 15 = 0$

Q.23 $x^2 + y^2 - 6x + 4y = 0$ OR $x^2 + y^2 + 2x - 8y + 4 = 0$ **Q.24** $x^2 + y^2 + x - 6y + 3 = 0$

Exercise 2

Single Correct Choice Type

Q.1 A	**Q.2** C	**Q.3** B	**Q.4** B	**Q.5** D	**Q.6** A
Q.7 D	**Q.8** A	**Q.9** A	**Q.10** B	**Q.11** B	**Q.12** A
Q.13 A	**Q.14** D				

Multiple Correct Choice Type

Q.15 C, D	**Q.16** B, C, D	**Q.17** B, C	**Q.18** B, D	**Q.19** A, C, D	**Q.20** A, C, D
Q.21 A, B, D	**Q.22** B, C	**Q.23** A, C, D	**Q.24** B, C	**Q.25** A, B, D	

Assertion Reaosing Type

Q.26 C	**Q.27** A	**Q.28** D	**Q.29** A	**Q.30** A	**Q.31** A

Comprehension Type

Paragraph 1:	**Q.32** B	**Q.33** C	**Q.34** C
Paragraph 2:	**Q.35** D	**Q.36** B	
Paragraph 3:	**Q.37** D	**Q.38** A	**Q.39** C

Previous Years' Questions

Q.1 B	**Q.2** B	**Q.3** A	**Q.4** A	**Q.5** B	**Q.6** D
Q.7 C	**Q.8** A	**Q.9** C	**Q.10** C	**Q.11** D	**Q.12** A
Q.13 D	**Q.14** $3(3+\sqrt{10})$	**Q.15** $\left(X+\frac{4}{3}\right)^2 + y = \left(\frac{1}{3}\right)^2$; $y = \pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right)$			**Q.16** 3
Q.17 D	**Q.18** D	**Q.19** A	**Q.20** 4	**Q.21** A	**Q.22** C
Q.23 D	**Q.24** B C	**Q.25** A C D	**Q.26** C	**Q.27** A C	

Solutions

JEE Main/Boards

Exercise 1

Sol 1: $\therefore$ Centre lies on $2x - y - 3 = 0$

$\therefore$ Let the centre be $C \equiv (h, 2h - 3)$

It also passes through $A \equiv (3, -2)$ and $B \equiv (-2, 0)$

$\therefore$ AC = BC

$\Rightarrow (h-3)^2 + (2h-1)^2 = (h+2)^2 + (2h-3)^2$

$\Rightarrow -6h + 9 - 4h + 1 = 4h + 4 - 12h + 9 - 2h = 3$

$\therefore h = -\frac{3}{2}$ $\quad\therefore C = \left(\frac{-3}{2}, -6\right)$

$\therefore$ Equation of the circle is

$(x-h)^2 + (y-k)^2 = R^2$

$\Rightarrow \left(x + \frac{3}{2}\right)^2 + (y+6)^2 = \left(\frac{-3}{2}, -3\right)^2 + \left(-6+2\right)^2$

$\Rightarrow x^2 + 3x + \frac{9}{4} + y^2 + 12y + 36 = \frac{81}{4} + 16$

$x^2 + y^2 + 3x + 12y + 2 = 0$

Sol 2: We can see that (0, 0), (1, 1) & (6, –4) form a right angled triangle with (0, 0) & (6, –4) as diameter

Equation of circle is $(x-0)(x-6) + y(y+4) = 0$

$\Rightarrow C = x(x-6) + y(y+4) = 0$

We can see that (5, –5) satisfies this equation

$\therefore$ 4 points are concyclic

Sol3: $A \equiv (-1, 2)$, and $B \equiv (3, -4)$

Equation of the circle is $(x+1)(x-3) + (y-2)(y+4)$

$\Rightarrow x^2 + y^2 - 2x + 2y - 11 = 0$

$\therefore C = \left(-g, -f\right) = \left(1, -1\right)$

Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{1+1+11} = \sqrt{13}$

Sol 4: Given equation of circle is $x^2 + y^2 = 25$ $P \equiv \left(-3, -4\right)$

$\therefore$ Slope of normal $OP = \frac{0+4}{0+3} = \frac{4}{3}$

$\therefore$ Equation of normal is $\left(y+4\right) = \frac{4}{3}\left(x+3\right)$

$\therefore$ Slope of tangent at P is $\frac{-1}{4/3} = \frac{-3}{4}$

$\therefore$ Equation of tangent is $\left(y+4\right) = \frac{-3}{4}\left(x+3\right)$

$\Rightarrow 4y + 16 = -3x - 9$

$\Rightarrow 3x + 4y + 25 = 0$

Sol 5: Equation of tangent at (1, –2) is

$x x1 + y y1 = a^2$

$x - 2y = 5$

$\therefore y = \frac{1}{2}x - \frac{5}{2}$

Equation of C_2 is $(x-4)^2 + (x+3)^2 = (\sqrt{5})^2$

Now the tangent will touch C_2 If $c^2 = r^2(1 + m^2)$

$c^2 = \left(\frac{5}{2}\right)^2$

$r^2(1 + m^2) = 5 \times \left(1 + \frac{1}{4}\right) = \left(\frac{5}{2}\right)^2$

$\therefore$ The given line is tangent to C_2

Sol 6: Equation of circle is

$C \equiv (x-1)^2 + (y+4)^2 = (2\sqrt{10})^2$

Shifting origin to (1, –4)

$\therefore C' \equiv X^2 + Y^2 = (2\sqrt{10})^2$ & $P = (7, 1)$

$\therefore Y - 1 = m(X - 7)$

$\therefore Y = mX + (1 - 7m)$

$\therefore c^2 = a^2(1 + m^2)$

$\left(1-7m\right)^2 = 40\left(1+m^2\right)$

$\Rightarrow 9m^2 - 14m - 39 = 0$

$\Rightarrow 9m^2 - 27m + 13m - 39 = 0$

$m = 3$ or $m = \frac{-13}{9}$

Since slope remains same in both system

$\therefore$ Equation of lines in old co-ordinates are

$(y+3) = 3(x-8)$ $\quad$ & $(y+3) = \frac{-13}{9}(x-8)$

Or $3x - y - 27 = 0$ and $13x + 9y = 77$

Sol 7: Centre = (– 4, 2)

Tangent is x – y = 3

$\therefore$ Radius = $\left|\frac{-4-2-3}{\sqrt{2}}\right| = \left(\frac{9}{\sqrt{2}}\right)$

$\therefore C \equiv (x + 4)^2 + (y - 2)^2 = \frac{81}{2}$

Sol 8: Using the concept of family of circles, let the equation of circle be

$\left(x^2+y^2-8x-2y+7\right)+\lambda\left(x^2+y^2-4x+10y+8\right)=0$

As (3, -3) lies on it

$\therefore \left(9+9-24+6+7\right)+\lambda\left(9+9-12-30+8\right)=0$

$\Rightarrow 7-16\lambda=0$

$\Rightarrow \lambda=\frac{7}{16}$

$\therefore$ Equation of the circle is

$\left(x^2+y^2-8x-2y+7\right)+\frac{7}{16}\left(x^2+y^2-4x+10y+8\right) = 0$

or $23x^2+23y^2-156x+38y+168=0$

Sol 9: C $\equiv x^2 + y^2 - 4x = 0$

Centre = (2, 0)

Slope of line perpendicular to chord $=\frac{1-0}{1-2}=-1$

$\therefore$ Slope of chord = 1

$\Rightarrow$ y – 1 = 1 (x – 1)

$\therefore$ y = x is the equation of chord

Alternative

Equation of a chord bisected at a given point is $T=S_1$

$\therefore xx_1+yy_1-2\left(x+x_1\right)=x_1^2+y_1^2-4x_1$

Or, $x+y-2x-2=1+1-4$

Or, x – y = 0

Sol 10: Equation of chord of contact

$xx_1 + yy_1 + g(x + x_1) + f(g+g_1) + c = 0$

$\Rightarrow$ 6x – 2(x + 6) = 0 ; x = 3

Sol 11: Length of tangent from a point

$=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+C}$

$=\sqrt{\frac{4(3)^2+4(2)^2+4\times3+16\times2+13}{4}}=\sqrt{\frac{109}{4}}=\frac{\sqrt{109}}{2}$

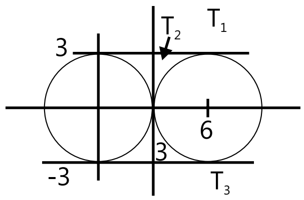
Sol 12: $C_1 \equiv x^2 + y^2 = 9$

Centre = (0, 0 & R_1 = 3)

$C_2 \equiv x^2 + y^2 - 12x + 27 = 0$

Centre = (6, 0 & R_2 = 3)

$\therefore$ The circles touch each other externally



$\therefore$ The equation of tangents are

y = 3, x = 3 & y = –3 (from figure itself)

Sol 13: Family of circles passing through two points is

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + \lambda L = 0$

$\therefore x(x - 1) + y^2 + \lambda y = 0$

$\therefore x^2 + y^2 - x + \lambda y = 0$

Centre = $\left(\frac{1}{2},\frac{-\lambda}{2}\right)$

Now since the circle touches internally [$\because$ (0, 0), & (1, 0) lie inside the circle]

$\therefore r_1 - r_2$ = distance between their centres

$\therefore 3-\sqrt{\frac{1}{4}+\frac{\lambda^2}{4}}=\sqrt{\frac{1}{4}+\frac{\lambda^2}{4}}$

$\therefore 9 = 4\left(\frac{1+\lambda^2}{4}\right)$

$\therefore \lambda = \pm 2\sqrt{2}$ $\quad\therefore$ Centre = $\left(\frac{1}{2},\pm\sqrt{2}\right)$

Sol 14: Let the coordinates of diameter be (h_1, k_1) & (h_2, k_2)

$\therefore$ Equation of circle is

$(x - h_1)(x - h_2)+(y - k_1)(y - k_2) = 0$

$\Rightarrow x^2 - y^2 - (h_1 + h_2)x - (k_1 - k_2)y + (h_1h_2+k_1k_2)=0$

$\Rightarrow x^2 + y^2 - (-2a)x - (-2p)y + (-b^2 - q^2) = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - (b^2 + q^2) = 0$

$\therefore R = \sqrt{a^2+p^2+b^2+q^2}$

Sol 15: Given equation of line is x + y = 2 ...(i)

On solving (i), with $x^2+y^2=2$, we get

$x^2+(2-x)^2=2$

$\Rightarrow x^2+4-4x+x^2=2$

$\Rightarrow 2x^2-4x+2=0$

$\Rightarrow (x-1)^2=0 \Rightarrow x=1$ This means the line represented by (i) and the circle intersects only at (1, 1)

Similarly, on solving x + y = 2 and

$x^2+y^2+3x+3y-8=0$, we get

$2x^2-4x+4+3(2)-8=0$, we get

$\Rightarrow 2x^2-4x+2=0$

$\Rightarrow 2(x-1)^2=0$

$\Rightarrow x=1$

Hence, the line intersects only at one point (1, 1)

Hence, proved.

Sol 16: Let $C\equiv(h, k)$ be the center of the circle

$\therefore 4k = h + 7$

$\therefore$ AC = BC

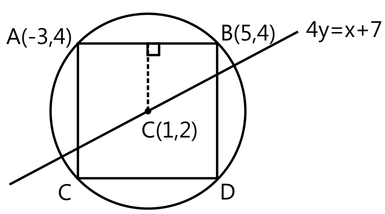
$\Rightarrow (4k-7+3)^2+(k-4)^2=(4k-12)^2+(k-4)^2$

$\Rightarrow (4k-4)^2=(4k-12^2)$

$\Rightarrow K=2$

$\therefore C\equiv(1, 2)$

Now



Equation of chord AB is y – 4 = 0

$\therefore$ Perpendicular distance of center from chord AB is

$\left|\frac{2-4}{1}\right| = 2$

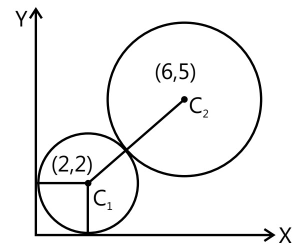
$\therefore$ AB = 8 and BC = 2PQ = 4

$\therefore$ Area of rectangle = 8 × 4 = 32

Sol 17: Radius of C_1 = 2

$\therefore$ Centre = (2, 2)

As circle with center (6,5) touches it externally



$\therefore C_1C_2 = r_1 + r_2$

$\Rightarrow \sqrt{(6-2)^2+(5-2)^2} = 2 + r$

$\Rightarrow r^2 + 4r + 4 = 16 + 9$

$\Rightarrow r^2 + 4r - 21 = 0$

$\Rightarrow r^2 + 7r - 3r - 21 = 0$

$\therefore r = 3$ ($\because$ r cannot be negative)

$\therefore$ Equation of C_2 is $(x-6)^2 + (y-5)^2 = 9$

Sol 18: Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Let $\left(m, \frac{1}{m}\right)$ be point on the circle.

On substitution we get

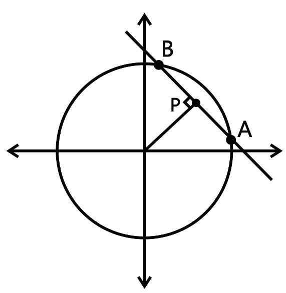
If $m^4 + 2gm^3 + 2fm + cm^2 + 1 = 0$

m_1, m_2, m_3, m_4 are roots of this equation

then, $m_1m_2m_3m_4 = 1$

Sol 19: Equation of line

$\Rightarrow$ x cos x + y sin x – p = 0



Now family of circle passing through the intersection of the circle & line is

$x^2 + y^2 - a^2 + \lambda$ (x cos x + y sin x – p) = 0

$\therefore$ Radius of circle = $AM=\sqrt{a^2-p^2}$

$\Rightarrow \sqrt{\left(\frac{\lambda\cos x}{2}\right)^2+\left(\frac{\lambda\sin x}{2}\right)^2+a^2+\lambda p} = \sqrt{a^2-p^2}$

$\Rightarrow \frac{\lambda^2}{4} + \lambda p + p^2 = 0$

$(\lambda + 2p)^2 = 0 \Rightarrow \lambda = -2p$

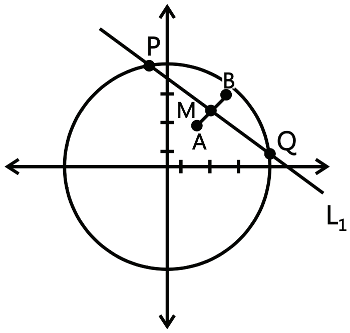
$\therefore S \equiv x^2 + y^2 - 2px\cos x$

$\quad - 2py\sin x + 2P^2 - a^2 = 0$

Sol 20: Slope of AB = 1

$\therefore$ Slope of $L_1 \times$ slope of AB = –1

$\Rightarrow$ Slope of $L_1 = -1$



And mid-point of AB, $M \equiv \left(\frac{3}{2}, \frac{5}{2}\right)$

$\therefore$ Equation of line L_1 is

$$\left(y - \frac{5}{2}\right) = -1\left(x - \frac{3}{2}\right)$$

Or, (2y - 5) = - (2x - 3)

Or 2x + 2y – 8 = 0

Or, x + y – 4 = 0

$\therefore$ Length of perpendicular from (0, 0) on L_1 is

$$\left|\frac{0+0-4}{\sqrt{2}}\right| = 2\sqrt{2}$$

$\therefore$ Length of the chord $= 2\sqrt{(a)^2 - \left(2\sqrt{2}\right)^2}$

$$= 2\sqrt{16 - \left(2\sqrt{2}\right)^2}$$

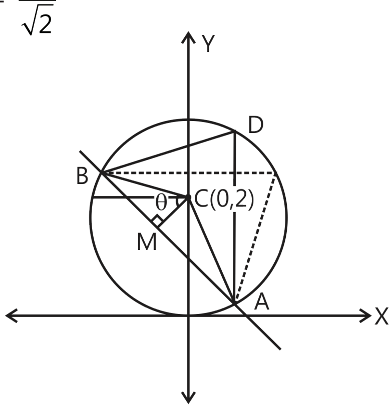
$$= 4\sqrt{2}$$

Sol 21: Equation of circle is $x^2 + y^2 - 4y = 0$

$\therefore$ Centre = (0, 2) & radius = 2

Perpendicular distance of center from the line x + y = 1 is

$$\frac{0+2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$



Let θ be the angle subtended at the circumference

$\therefore$ Angle subtended at circumference

$= \frac{1}{2}$ (Angle subtended at centre)

$\therefore \cos\theta = \frac{1}{2\sqrt{2}} \Rightarrow \theta = \cos^{-1}\frac{1}{2\sqrt{2}}$

Sol 22: Given $x^2 + y^2 - 2x - 2\lambda y - 8 = 0$

$\Rightarrow \left(x^2 + y^2 - 2x - 8\right) - 2\lambda\left(y\right) = 0$ ....(i)

Let $S \equiv x^2 + y^2 - 2x - 8 = 0$ and ...(iii)

$L \equiv y = 0$ ...(iii)

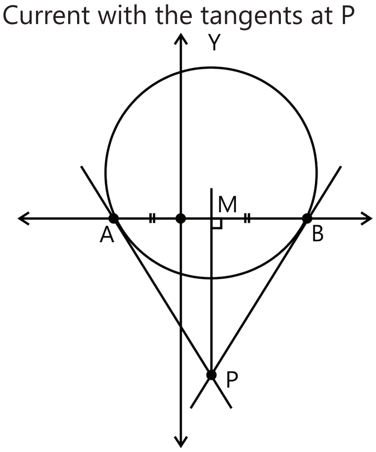
$\therefore$ The equation is represents a family of circles passing through the intersection of S = 0 & L = 0.

$\therefore$ On solving (ii) and (iii), we get

$x^2 - 2x - 8 = 0$

$\Rightarrow x = \frac{2 \pm \sqrt{4+32}}{2} = 4$ or -2

$\therefore$ The fixed point are A (4, 0) and B (-2, 0) from the diagram, the perpendicular bisector of AB is con. Current with the tangents at P



$\therefore M \equiv (1, 0)$

And Equation of line MP is x = 1 .....(iv)

$\therefore$ On solving (iv) with x + 2y + 5 = 0

We get 1 + 2y + 5 = 0

$\Rightarrow$ 2y + 6 = 0 $\Rightarrow y = \frac{-6}{2} = -3$

$\therefore P \equiv (1, -3)$

Centre of circle (i) is $C \equiv (1, \lambda)$

If P is the point of intersection of tangents then CB is perpendicular to BP

$$\therefore \left(\frac{\lambda - 0}{1-4}\right) \times \left(\frac{0+3}{4-1}\right) = -1$$

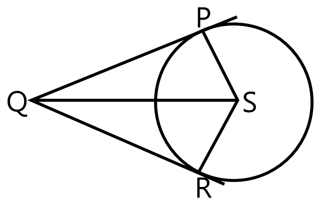
$\therefore \frac{\lambda}{-3} \times \frac{3}{3} = -1 \Rightarrow \lambda = 3$

$\therefore$ Equation of the required circle is

$x^2 + y^2 - 2x - 6y - 8 = 0$

Sol 23: Length of tangent = $\sqrt{S_{11}}$

$\therefore$ QP = $\sqrt{4^2 + 5^2 - 4^2 - 10 - 11}$ = 2



Area of PQRS = 2ΔPQS = 2 × $\frac{1}{2}$ × PS x QPPS

= Radius of circle = $\sqrt{2^2 + 1^2 + 11}$ = 4

$\therefore$ Area of PQRS = 4 × 2 = 8

Sol 24: The equation of any curve passing through the intersection of

$L_1 \equiv a_1x + b_1y + c_1 = 0$

$L_2 \equiv a_2x + b_2y + c_2 = 0$

$L_3 \equiv y = 0$ & $L_4 \equiv x = 0$ is $L_1 L_2 + \lambda L_3 L_4$

$\Rightarrow (a_1x + b_1y + c_1)(a_2x + b_2y + c_2) + \lambda xy = 0$

where λ is a parameter

This curve represents a circle if coeff. of x^2 = coeff. of y^2

$\therefore a_1a_2 = b_1b_2$

Sol 25: Let any point on c_2 be (h, k)

Length of tangent from any point to circle

$= \sqrt{S_1}$

$\therefore l = \sqrt{h^2 + k^2 + 2gh + 2fk + c_1}$

Now since (h, k) satisfies circle 1

$\therefore h^2 + k^2 + 2gh + 2fk = -c$

$\therefore l = \sqrt{c_1 - c}$

Sol 26: The tangents to the these circle are equal in length

$\therefore$The point is radical centre

The equation of radical axes are $S_1 - S_2 = 0$

$\therefore S_1 - S_2 = \left(\frac{3}{2} - 4\right)x - \frac{5}{2}y + \frac{7-9}{2} = 0$

$\Rightarrow 5x + 5y - 5 = 0 \Rightarrow x + y - 1 = 0$

and $S_1 - S_3 = 0 \Rightarrow -4x - y + 7 = 0$

$4x + y - 7 = 0$

$\therefore$The radical centre is (2, −1)

Length of tangent = $\sqrt{S_1} = \sqrt{2^2 + 1^2 - 8 + 7} = 2$

Sol 27: Let (h, k) be the point on circle $x^2 + y^2 = a^2$

$\Rightarrow \therefore h^2 + k^2 = a^2$ ...(i)

Equation of chord of contact for $x^2 + y^2 = b^2$ is $hx + ky = b^2$ ...(ii)

As (ii) touches the circle $x^2 + y^2 = c^2$

$\therefore \left|\frac{-b^2}{\sqrt{h^2 + k^2}}\right| = c$

$\Rightarrow b^2 = ac$

$\therefore$ a, b and c are in G.P.

Sol 28: Let the required circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ... (i)

The given circles are

$x^2 + y^2 + 3x - 5y + 6 = 0$ ... (ii)

and $x^2 + y^2 - 7x + \frac{29}{4} = 0$ ... (iii)

Now 1, 2 & 1, 3 are orthogonal

$\therefore 2g\frac{3}{2} + 2f\frac{-5}{2} = c + 6$

$3g - 5f = c + 6$ & $2g \times \frac{-7}{2} + 2f \times 0 = c + \frac{29}{4}$

$\Rightarrow -7g = c + \frac{29}{4}$

$\therefore 10g - 5f = \frac{-5}{4}$

$\therefore 8g - 4f = -1$

Equation of circle is

$x^2 + y^2 + 2gx + \frac{(8g+1)}{2}y + c = 0$

The centre lies on the line

$3x + 4y + 1 = 0$

$\Rightarrow 3(-g) - 4\frac{(8g+1)}{4} + 1 = 0$

$\Rightarrow -11g = 0$

$\Rightarrow g = 0,\ f = \frac{1}{4}$ and $c = -\frac{29}{4}$

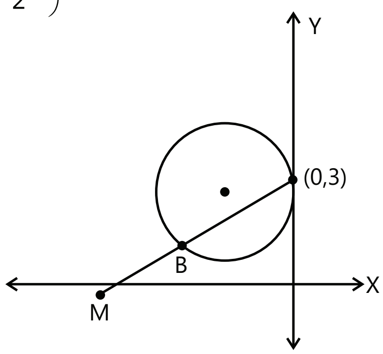
$\therefore$ Equation of circle is

$4x^2 + 4y^2 + 2y - 29 = 0$

Sol 29: Given equation of circle is $x^2 + 4x + (y-3)^2 = 0$

Let $M \equiv (h, k)$ $\quad \therefore B \equiv \left(\frac{0+h}{2}, \frac{3+k}{2}\right)$

$\therefore B \equiv \left(\frac{h}{2}, \frac{3+k}{2}\right)$



As point B lies on the circle

$\therefore \frac{h^2}{4} + 4 \times \frac{h}{2} + \left(\frac{3+k}{2} - 3\right)^2 = 0$

$\Rightarrow \frac{h^2}{4} + 2h + \frac{k^2}{4} + \frac{9}{4} - 2\frac{k}{2} \times \frac{3}{2} = 0$

$\Rightarrow h^2 + k^2 + 8h - 6k + 9 = 0$

$\therefore$ The value of point B is

$x^2 + y^2 + 8x - 6y + 9 = 0$

Sol 30: Let (h, k) be middle points

Equation of chord through (h, k) is

$xh - (x + h) + yk = h^2 - 2h + k^2$ ... (i)

As the chord given by equation (i) passes through (0, 0)

$\therefore$ On substituting, x = 0 and y = 0, we get

$-h = h^2 - 2h + k^2$

$\therefore$ Locus of midpoint is $x^2 - x + y^2 = 0$

Sol 31: Given, OM = a and OP = b

From the diagram,

$\angle PRQ = 90°$

And PR = QR

$\angle QPR = \angle PQR = 45°$

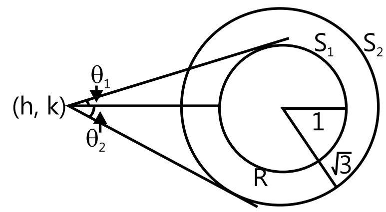
$\angle OPR = 90° - \angle QPR = 45°$

$\therefore$ In ΔOMP, $\sin 45° = \frac{OM}{OP}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a}{b}$

$\Rightarrow b = \sqrt{2}a$.

Sol 32: According to condition



$\theta_1 + \theta_2 = 90°$

$\therefore \tan\theta_1 \tan\theta_2 = 1$

$\tan\theta_1 = \frac{r_1}{\text{Length of tangent}} = \frac{1}{\sqrt{h^2 + k^2 - 1}}$

$\tan\theta_2 = \frac{\sqrt{3}}{\sqrt{h^2 + k^2 - 3}}$

According to condition –

$\therefore 3 = (h^2 + k^2 - 1)(h^2 + k^2 - 3)$

$3 = (h^2 + k^2)^2 - 4(h^2 + k^2) + 3$

$\therefore h^2 + k^2 = 0$

or $h^2 + k^2 = 4$

Now $h^2 + k^2 \neq 0$ as no tangent will be possible.

$\therefore$ The locus of point is a circle

Sol 33: Let the other end of diameter be (h, k)

$\therefore$ Equation of circle is

$(x - a)(x - h) + (y - b)(y - k) = 0$

$\therefore$ Center $\equiv \left(\frac{a+h}{2}, \frac{b+k}{2}\right)$

Since the circle touches the x-axis

$\therefore$ | y-coordinate| = radius

$\Rightarrow \left|\frac{b+k}{2}\right| = \sqrt{\left(\frac{a+h}{2}\right)^2 + \left(\frac{b+k}{2}\right)^2 - (ah + bk)}$

$\therefore \left(\frac{a+h}{2}\right)^2 = (ah + bk)$

$\therefore$ Locus of point is

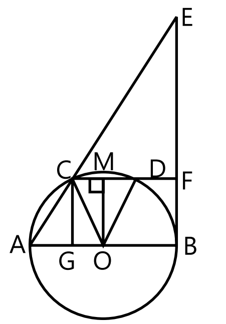
$x^2 + 2ax + a^2 = 4ax + 4by$

$(x - a)^2 = 4by$

Sol 34: Let G be perpendicular from C on AB

And M be midpoint of CD

Let radius = R



$\therefore MO^2 + MD^2 = OD^2$ (O is centre)

$MO^2 = R^2 - \frac{R^2}{4}$

$MO = \frac{\sqrt{3}R}{4}$

$\Rightarrow CG = \frac{\sqrt{3}R}{4}$

$AG = AO - GO = AO - CM = R - \frac{R}{2} = \frac{R}{2}$

$AC^2 = AG^2 + GC^2 = \frac{3R^2}{4} + \frac{R^2}{4}$

$\therefore AC = R$

$\therefore \frac{AE}{AC} = \frac{AB}{AG}$ (As Δ AEB ~ Δ ACG)

$\Rightarrow \frac{AE}{R} = \frac{AB}{\frac{R}{2}} \Rightarrow AE = 2AB$

Exercise 2

Single Correct Choice Type

Sol 1: (D) The centers are A = (2, 3) ; B = (–1, –2) ; C = (5, 8)

$\therefore$ Slope of AB = $\frac{3-(-2)}{2-(-1)} = \frac{5}{3}$

and slope AC = $\frac{8-3}{5-2} = \frac{5}{3}$

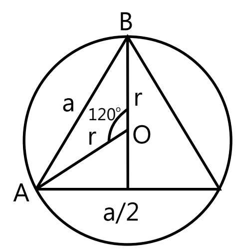
$\therefore$ The three points are collinear

Sol 2: (B) $S \equiv x^2 + y^2 + \lambda x + \frac{\lambda^2}{2} = 0$

Radius = $\sqrt{g^2 + f^2 - c} = \sqrt{\left(\frac{\lambda}{2}\right)^2 - \frac{\lambda^2}{2}} = \sqrt{-\frac{\lambda^2}{4}}$

$\therefore$ Radius is not defined for any real value of λ

Sol 3: (A) For an equilateral triangle inscribed in circle of radius r, in Δ OAB using cosine rule, we get



$\cos 120° = \frac{r^2 + r^2 - a^2}{2r^2}$

$\Rightarrow -r^2 = 2r^2 - a^2$

$\Rightarrow a = \sqrt{3}\, r$

Area of equilateral triangle

$= \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (\sqrt{3}\, r)^2 = \frac{3\sqrt{3}}{4} r^2$

Radius of given circle = $\sqrt{g^2 + f^2 - c} = 1$

$\therefore A = \frac{3\sqrt{3}}{4}$

Sol 4: (B) Let the centre of circle be (– h, 0)

where h > 0

Radius = 5

$\therefore$ Equation of circle is $(x + h)^2 + y^2 = 25$

It passes through the point (2, 3)

$\therefore (h + 2)^2 = (4)^2 \Rightarrow h = 2$ or $h = -6$

But $h > 0 \Rightarrow h = 2 \Rightarrow (2+h)^2 + 9 = 25 \Rightarrow h = 2$ or -6

$\therefore$ Equation of the circle is $x^2 + y^2 + 2x - 21 = 0$

$\therefore$ Intercept made on y-axis = $2\sqrt{f^2 - c} = 2\sqrt{21}$

Sol 5: (A) $S_1 : x^2 + y^2 = 1$

$S_2 : x^2 + y^2 - 2x - 6y = 6$

$S_3 : x^2 + y^2 - 4x - 12y = 9$

$r_1 = 1$; $r_2 = \sqrt{1^2 + 3^2 + 6} = 4$; $r_3 = \sqrt{2^2 + 6^2 + 9} = 7$

$\therefore r_1, r_2, r_3$ are in A.P.

Sol 6: (D) $S_1: x^2 + y^2 + 2\lambda x + 4 = 0$

$S_2: x^2 + y^2 - 4\lambda x + 8 = 0$

Since both represent real circles

$\therefore r_1 \geq 0 \;\&\; r_2 \geq 0$

$\therefore \lambda^2 - 4 \geq 0 \therefore \lambda \leq -2$ or $\lambda \geq 2$ ... (i)

$\therefore 4\lambda^2 - 8 \geq 0 \therefore \lambda \leq -\sqrt{2}$ or $\lambda \geq \sqrt{2}$ ... (ii)

From 1, 2 λ $(-\infty, -2] \cup [2, \infty)$

All of these lie within the range

Sol 7: (D) $s = x^2 + y^2 + 16x - 24y + 183 = 0$

Centre ≡ (–8, 12) Radius = 5

Let (x_1, y_1) be the image of (–8, 12) w.r.t. to the line

$4x + 7y + 13 = 0$

$$\therefore \frac{x_1-(-8)}{4} = \frac{y_1-12}{7}$$

$$= \frac{-2\{4\times(-8)+7\times 12+13\}}{4^2+7^2}$$

$$\frac{x_1+8}{4} = \frac{y_1-12}{7} = -2$$

$x_1 = -16, y_1 = -2$

Equation of required circle is

$(x + 16)^2 + (y + 2)^2 = 5^2$

$x^2 + y^2 + 32x + 4y + 235 = 0$

Sol 8: (B) Equation of circle is

$(x - 0)(x - a) + (y - 1)(y - b) = 0$ ...(i)

Let the circle given by eq. (i) cut the x-axis at (h, 0)

$h(h - a) + b = 0$

$h^2 - ah + b = 0$

The abscissa are roots of equation $x^2 - ax + b = 0$

Sol 9: (C) $x = 2y - 10 \;\&\; x^2 + y^2 = 100$

$\Rightarrow 4y^2 - 40y + y^2 = 0$

$\Rightarrow 5y(y - 8) 0$

$\therefore$ y = 8 (as point lies in 1st quadrant & x = + 6)

The line perpendicular to $x - 2y + 10 = 0$ passing through (6, 8) is $(y - 8) = -2(x - 6)$

$2x + y = 20$

It cuts the y-axis at (0, 20)

Sol 10: (C) Let equation of circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$

Let $\left(x, \frac{1}{x}\right)$ be a point on the circle.

$\therefore x^4 + 2gx^3 + cx^2 + 2fx + 1 = 0$

$\Rightarrow abcd = \frac{1}{1} = 1$

Sol 11: (A) Circumradius $R = \frac{abc}{4\Delta}$, where Δ is the area of a triangle

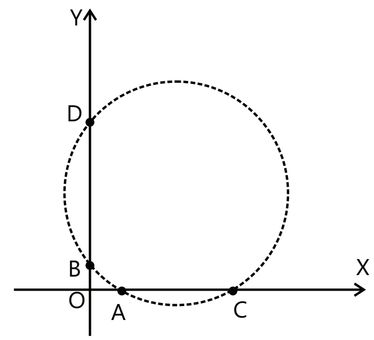
$$\Rightarrow R = \frac{12\times 12\times 6}{4\times\left(\frac{1}{2}\times 6\times \text{height}\right)}$$

Height = $\sqrt{12^2-3^2} = 3\sqrt{15}$

$$\therefore R = \frac{12\times 6}{3\sqrt{15}} = \frac{8\sqrt{15}}{5}$$

Sol 12: (D) Given, ac = bd

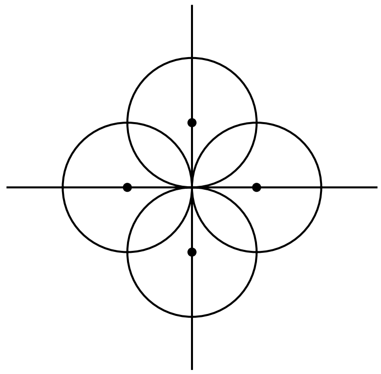
$\Rightarrow AO \times OC = OB \times OD$



This is true in case of circle and two secants

$\therefore$A, B, C and D lie on a circle.

Sol 13: (C)



Since the centres lie on co-ordinate axes

The centre are (1, 0), (–1, 0), (0, 1) and (0, –1)

Consider two circles with centre (1, 0) & (0, 1)

Their point of intersection will lie on the line y = x

Putting y = x in $(x-1)^2 + y^2 = 1$

$\Rightarrow 2x^2 - 2x = 0$

$\Rightarrow x = 1 \;\&\; y = 1$ (ignoring x = y = 0)

(1, 1) is the point

By symmetry the other 3 points are

(1, –1) (–1, 1) (–1, –1).

It is a square of side 2 units

Area = 4 sq. units

Sol 14: (A) The y co-ordinate = 2, centre = (h, 2) & radius = 2

On using the condition of tangency on $y = \frac{x}{2}$,

we get $\frac{2 \times 2 - h}{\sqrt{5}} = \pm 2$

$\Rightarrow h = 4 \pm 2\sqrt{5}$

But h > 0

x coordinate is $4 + 2\sqrt{5}$.

Sol 15: (B) Let the midpoint of chord be (h, k)

$\therefore$ Equation of chord is $T = S_1$

$\Rightarrow xh + 2(x + h) + yk - 3(y + k) + 9$

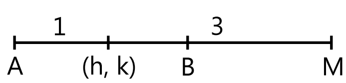
$= h^2 + 4h + k^2 - 6k + 9$

Since (0, 3) lies on this chord

$2h + 3k - 3(3+k) = h^2 + 4h + k^2 - 6k$

Locus of midpoint is

$h^2 + 2h + k^2 - 6k + 9 = 0$



$\therefore$ Let M be (x , y)

$(h, k) = \left(\frac{3 \times 0 + x}{4}, \frac{9 + y}{4}\right)$

Substituting in 1 we get locus of M.

$\therefore \left(\frac{x}{4}\right)^2 + \frac{2(x)}{4} + \left(\frac{y+9}{4}\right)^2 - \frac{6 \times (y+9)}{4} + 9 = 0$

$\Rightarrow x^2 + y^2 + 8x - 6y + 81 - 216 + 144 = 0$

$\Rightarrow x^2 + 8x - (y-3)^2 = 0$

Alternate:

Since, $\frac{AM}{AB} = 2 \Rightarrow \frac{AB}{BM} = 1$

Let M be (h, k)

Then, $B \equiv \left(\frac{h}{2}, \frac{k+3}{2}\right)$

Which lies on Circle.

Substitute to get the required Locus.

Sol 16: (C) P = (0, 5)

$S_1 = x^2 + y^2 + 2x - 4 = 0$

$S_2 = x^2 + y^2 - y + 1 = 0$

$L_1 = \sqrt{25 - 4} = \sqrt{21}$

$L_2 = \sqrt{21}$

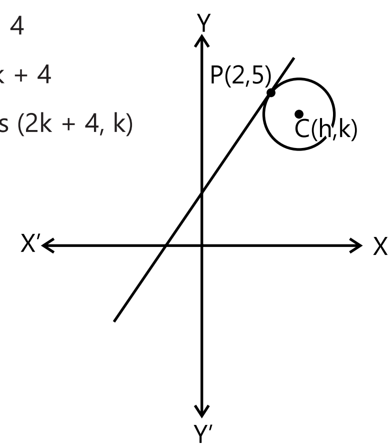
$\therefore L_1 = L_2$

Sol 17: (A) Let centre of circle be (h, k)

$\therefore h - 2k = 4$

$\Rightarrow h = 2k + 4$

$\therefore$ Centre is (2k + 4, k)



Now CP $\perp$ tangent

$\therefore \frac{5-k}{2-2k-4} \times 2 = -1$

$\therefore 5 - k = \frac{(2k+2)}{2}$

$\therefore 5 - k = k + 1$

$\therefore k = 2$

Center is (8, 2)

Radius = $\sqrt{(8-2)^2 + (2-5)^2} = 3\sqrt{5}$

Sol 18: (D) Let circle be $S \equiv x^2+y^2+2gx+2fy + c = 0$

$S_1 \equiv x^2 + y^2 = 1$

$S_2 \equiv x^2 + y^2 + 2x - 3 = 0$

$S_3 \equiv x^2 + y^2 + 2y - 3 = 0$

$\Rightarrow S - S_1 = 0$ is the equation of chord of contact & it passes through centre of S_1

$\Rightarrow 2gx + 2fy + c + 1 = 0$

Satisfying (0, 0) $\Rightarrow$ c = – 1,

Similarly $S - S_2 = 0$

$\Rightarrow (2g - 2)x + 2fy + 2 = 0$

Satisfying (-1, 0), we get $2 - 2g + 2 = 0$

$\Rightarrow g = 2$

Similarly, $S - S_3 = 0$

$\Rightarrow (2gx + (2f - 2)y + 2 = 0$

(Satisfying (0, –1), we get $\Rightarrow f = 2$

$\therefore$ Centre is (–2, –2)

Sol 19: (C) Let tangent from origin be y = mx

Using the condition of tangency, we get

$$\Rightarrow \frac{7m+1}{\sqrt{m^2+1}} = 5$$

$(7m + 1)^2 = 25(m^2 + 1)$

$\Rightarrow 24m^2 + 14m - 24 = 0$

$\Rightarrow 12m^2 + 7m - 12 = 0$

$\Rightarrow 12m^2 + 16m - 9m - 12 = 0$

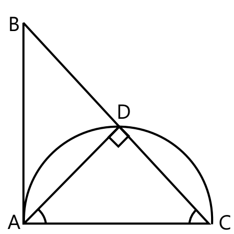
$(4m - 3)(3m + 4)$

$\therefore m = \frac{3}{4}$ and $m = -\frac{4}{3}$

The angle between tangents = $\frac{\pi}{2}$

Sol 20: (D) Since A, D, C lies on the circle with AC as the diameter

AD $\perp$ DC



$\therefore \Delta ADC \sim \Delta ABC$

$$\Rightarrow \frac{AC}{BC} = \frac{AD}{AB} \quad \text{...(i)}$$

Also, $BC^2 = \sqrt{AB^2 + AC^2}$ ...(ii)

$$AC^2 = (AB^2 + AC^2)\frac{AD^2}{AB^2}$$

[From (i) and (ii)]

$$AC^2 = \frac{AB^2AD^2}{AB^2 - AD^2}$$

$$AC = \frac{AB.AD}{\sqrt{AB^2 - AD^2}}$$

Sol 21: (A) $x^3 + y^3 + 3xy - 1 = 0$

$\Rightarrow (x + y)^3 - 3xy(x + y) + 3xy - 1 = 0$

$\Rightarrow (x + y)^3 - 3xy(x + y - 1) - 1^3 = 0$

$\Rightarrow (x + y)^3 - 1^3 = 3xy(x + y - 1)$

$\Rightarrow (x + y - 1)\{(x + y)^2 + (x + y) + 1\} - 3xy(x + y - 1) = 0$

We get,

$\therefore (x + y - 1)\{(x + y)^2 + (x + y) + 1 - 3xy\} = 0$

$(x + y - 1)(x^2 + y^2 - xy + x + y + 1) = 0$

For the curve $x^2 + y^2 - xy + x + y + 1 = 0$

$ab - h^2 = 1 - \frac{1}{4} > 0$

and

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & -1/2 & 1/2 \\ -1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix}$$

$$= 1 \times \left(1 - \frac{1}{4}\right) + \frac{1}{2}\left(-\frac{1}{2} - \frac{1}{4}\right) + \frac{1}{2}\left(-\frac{1}{4} - \frac{1}{2}\right)$$

$$= \frac{3}{4} - \frac{3}{4} = 0$$

$\therefore$ It is a point

Previous Years' Questions

Sol 1: (D) Equation of circle passing through a point (x_1, y_1) and touching the straight line L, is given by

$(x - x_1)^2 + (y - y_1)^2 + \lambda L = 0$

$\therefore$ Equation of circle passing through (0, 2) and touching x=0

$\Rightarrow (x - 0)^2 + (y - 2)^2 + \lambda x = 0$ .....(i)

Also, it passes through (-1, 0)

$\Rightarrow 1 + 4 - \lambda = 0 \;\; \lambda \Rightarrow 5$

$\therefore$ Eq. (i) becomes,

$x^2 + y^2 - 4y + 4 + 5x = 0$

$\Rightarrow x^2 + y^2 + 5x - 4y + 4 = 0,$

For x-intercept put y = 0

$\Rightarrow x^2 + 5x + 4 = 0$

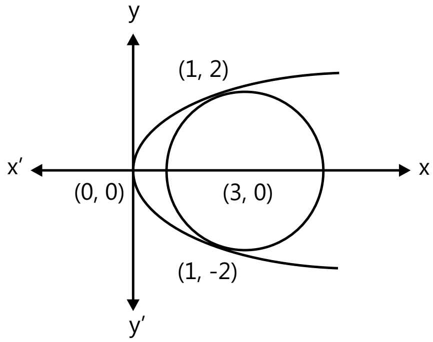
$(x+1)(x+4) = 0$

$\therefore x = -1, -4$

Sol 2: (B) For the point of intersection of the two given curves

$C_1 : y^2 = 4x$

and $C_2 : x^2 + y^2 - 6x + 1 = 0$



We have, $x^2 + 4x - 6x + 1 = 0$

$\Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0$

$\Rightarrow x = 1$ (equal real roots)

$\Rightarrow y = 2, -2$

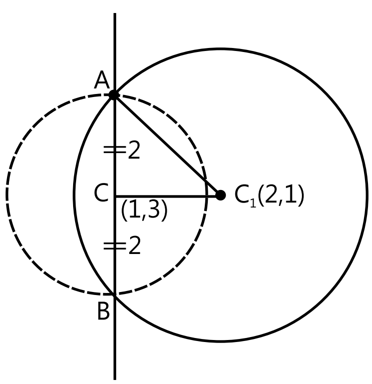
Thus, the given curves touch each other at exactly two point (1, 2) and (1, -2).

Sol 3: (C) Here radius of smaller circle, AC– $\sqrt{1^2 + 3^2 - 6} = 2$ Clearly, from the figure the radius of bigger circle

$r^2 = 2^2 + [(2-1)^2 + (1-3)^2]$

$r^2 = 9$

$\Rightarrow r = 3$

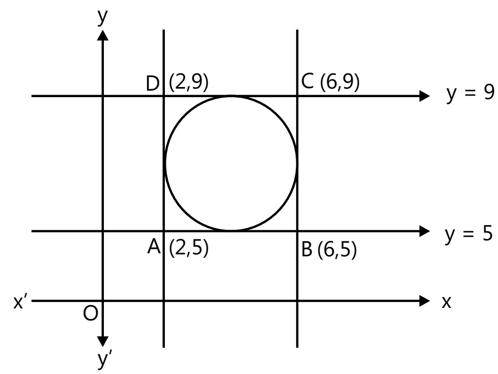


Sol 4: (A) Given, circle is inscribed in square formed by the lines

$x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$

$\Rightarrow x = 6$ and $x = 2, y = 5$ and $y = 9$

Which could be plotted as



Where ABCD clearly forms a square

$\therefore$ Centre of inscribed circle

= Point of intersection of diagonals

= Mid point of AC or BD

$= \left(\frac{2+6}{2}\right), \left(\frac{5+9}{2}\right) = (4, 7)$

$\Rightarrow$ Centre of inscribed circle is (4, 7)

Sol 5: (C) The line 5x-2y+6=0 meets

The y-axis at the point (0, 3) and therefore the tangent has to pass through the point (0, 3) and required length

$= \sqrt{x_1^2 + y_1^2 + 6x_1 + 6y_1 - 2}$

$= \sqrt{0^2 + 3^2 + 6(0) + 6(3) - 2} = \sqrt{25} = 5$

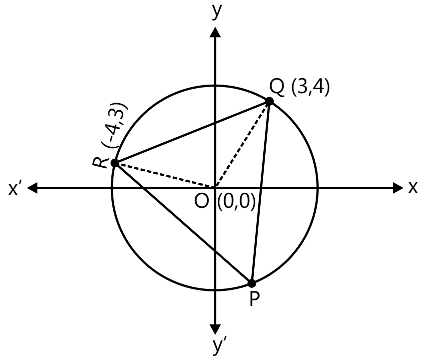
Sol 6: (A) Since, the given circles intersect orthogonally.

$2\left(g_1\ g_2 + f_1\ f_2\right) = G + C_2$

$\therefore \quad 2(-1)(0) + 2(-k)(-k) = 6 + k$

$\Rightarrow \quad 2k^2 - k - 6 = 0 \;\Rightarrow\; k = -\frac{3}{2}, 2$

Sol 7: (C) Let O is the point at centre and P is the point at circumference. Therefore, angle QOR is double the angle QPR. So it is sufficient to find the angle QOR.



Now, slope of OQ, m1 = 4/3, slope of OR, m_2 = −3/4

Here, $m_1m_2 = -1$

Threfore, $\angle QOR = \pi/2$

Which implies that $\angle QPR = \pi/4$

Sol 8: (B) Given, $x^2 + y^2 = 4$

Centre $\equiv C_1 \equiv (0,0)$ and $R_1 = 2$

Again, $x^2 + y^2 - 6x - 8y - 24 = 0$, then $C_2 \equiv (3,4)$

and $R_2 = 7$ again, $C_1C_2 = 5 = R_2 - R_1$

Since, the given circles touch internally therefore, they can have just one common tangent at the point of contact.

Sol 9: Since, the tangents are perpendicular.

So, locus of perpendicular tangents to circle

$x^2 + y^2 = 169$ is a director circle having equation

$x^2 + y^2 = 338$

Sol 10: The equation of circle having tangent 2x+3y+1=0 at (1, -1)

$\Rightarrow (x-1)^2 + (y+1)^2 + \lambda(2x+3y+1) = 0$

$x^2 + y^2 + 2x(\lambda - 1) + y(3\lambda + 2) + (\lambda + 2) = 0$ ....(i)

Which is orthogonal to the circle having end point of diameter (0, -1) and (-2, 3)

$\Rightarrow x(x+2) + (y+1)(y-3) = 0$

or $x^2 + y^2 + 2x - 2y - 3 = 0$ ...(ii)

$\therefore \frac{2(2\lambda - 2)}{2}\cdot 1 + \frac{2(3\lambda + 2)}{2}(-1) = \lambda + 2 - 3$

$\Rightarrow 2\lambda - 2 - 3\lambda - 2 = \lambda - 1$

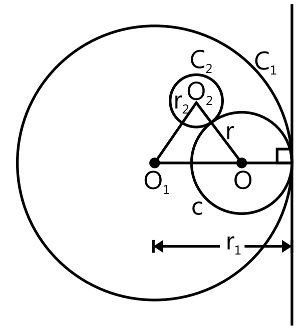
$\Rightarrow 2\lambda = -3 \Rightarrow \lambda = -3/2$

$\therefore$ From Eq. (i) equation of circle,

$2x^2 + 2y^2 - 10x - 5y + 1 = 0$

Sol 11: Let the given circles C_1 and C_2 have centres O_1 and O_2 and radii r_1 and r_2 respectively.

Let the variable circle C touching C_1 internally, C_2 externally have a radius r and centre at O



Now, $OO_2 = r + r_2$ and $OO_1 = r_1 - r$

$\Rightarrow OO_1 + OO_2 = r_1 + r_2$

Which is greater than

O_1O_2 as $O_1O_2 < r_1 + r_2$ ($\because C_2$ lies inside C_1)

$\Rightarrow$ Locus of O is an ellipse with foci O_1 and O_2

Alternate Solution

Let equations of C_1 be $x^2 + y^2 = r_1^2$ and of C_2 be $(x-a)^2 + (y-b)^2 = r_2^2$

Let centre C be (h, k) and radius r, then by the given condition

$\sqrt{(h-a)^2 + (k-b)^2} = r + r_2$ and $\sqrt{h^2 + k^2} = r_1 - r$

$\Rightarrow \sqrt{(h-a)^2 + (k-b)^2} + \sqrt{h^2 + k^2} = r_1 + r_2$

Required locus is

$\sqrt{(x-a)^2 + (y-b)^2} + \sqrt{x^2 + y^2} = r_1 + r_2$

Which represents an ellipse whose foci are at (a, b) and (0, 0).

Sol 12: Equation of any tangent to circle $x^2 + y^2 = r^2$ is

$x\cos\theta + y\sin\theta = r$ ....(i)

Suppose Eq. (i) is tangent to $4x^2 + 25y^2 = 100$

Or $\frac{x^2}{25} + \frac{y^2}{4} = 1$ at (x_1, y_1)

Then, Eq. (i) and $\frac{xx_1}{25} + \frac{yy_1}{4} = 1$ are identical

$\therefore \frac{x_1/25}{\cos\theta} = \frac{\frac{y_1}{4}}{\sin\theta} = \frac{1}{r}$

$\Rightarrow x_1 = \frac{25\cos\theta}{r}, y_1 = \frac{4\sin\theta}{r}$

The line (i) meet the coordinates axes in $A(r\sec\theta, 0)$ and $\beta(0, r\text{cosec}\theta)$. Let (h, k) be mid point of AB.

Then, $h = \frac{r\sec\theta}{2}$ and $k = \frac{r\text{cosec}\theta}{2}$

Therefore, $2h = \frac{r}{\cos\theta}$ and $2k = \frac{r}{\sin\theta}$

$\therefore \quad x_1 = \frac{25}{2h}$ and $y_1 = \frac{4}{2k}$

As (x_1, y_1) lies on the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$

We get $\frac{1}{25}\left(\frac{625}{4h^2}\right) + \frac{1}{4}\left(\frac{4}{k^2}\right) = 1$

$\Rightarrow \frac{25}{4h^2} + \frac{1}{k^2} = 1$

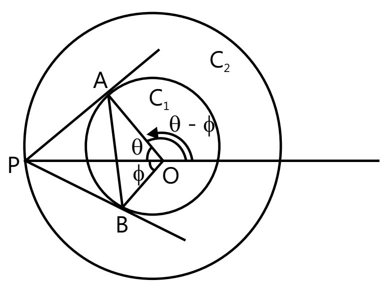
or $\quad 25k^2 + 4h^2 = 4h^2k^2$

Therefore, required locus is $4x^2 + 25y^2 = 4x^2y^2$

Sol 13: Let the coordinate of point P be $(2r\cos\theta, 2r\sin\theta)$

We have, OA = r, OP = 2r

Since, ΔOAP is a right angled triangle.



$\cos\phi = 1/2$

$\Rightarrow \quad \phi = \pi/3$

$\therefore$ Coordinates of A and B are

$\{r\cos(\theta - \pi/3), r\sin(\theta - \pi/3)\}$ and

$\left\{r\cos\left(\theta + \frac{\pi}{3}\right)\right\}, r\sin\left(\theta + \frac{\pi}{3}\right)$

If p,q is the centroid of ΔPAB, then

$p = \frac{1}{3}[r\cos(\theta - \pi/3) + r\cos(\theta + \pi/3) + 2r\cos\theta]$

$= \frac{1}{3}[r\{\cos(\theta - \pi/3) + \cos(\theta + \pi/3)\} + 2r\cos\theta]$

$= \frac{1}{3}\left[r\left(2\cos\frac{\theta - \frac{\pi}{3} + \theta + \frac{\pi}{3}}{2} \cdot \cos\frac{\theta - \frac{\pi}{3} - \theta - \frac{\pi}{3}}{2}\right) + 2r\cos\theta\right]$

$= \frac{1}{3}\left[r\left(2\sin\frac{\theta - \frac{\pi}{3} + \theta + \frac{\pi}{3}}{2} \cdot \cos\frac{\theta - \frac{\pi}{3} - \theta - \frac{\pi}{3}}{2}\right) + 2r\sin\theta\right]$

$= \frac{1}{3}[r\{2\cos\theta\cos\pi/3\} + 2r\cos\theta]$

$= \frac{1}{3}[r \cdot \cos\theta + 2r\cos\theta] = r\cos\theta$

and $q = \frac{1}{3}[r\sin\left(\theta - \frac{\pi}{3}\right) + r\sin\left(\theta + \frac{\pi}{3}\right) + 2r\sin\theta]$

$= \frac{1}{3}[r\{\sin\left(\theta - \frac{\pi}{3}\right) + \sin\left(\theta + \frac{\pi}{3}\right)\} + 2r\sin\theta]$

$= \frac{1}{3}[r(2\sin\theta\cos\pi/3) + 2r\sin\theta]$

$= \frac{1}{3}[r(\sin\theta) + 2r\sin\theta]$

$= r\sin\theta$

Now, $(p, q) = (r\cos\theta, r\sin\theta)$ lies on $x^2 + y^2$ $= r^2$, which is C_1

Sol 14: (A) Eq. of circle touching $x - a \times y$ at $(1,0)$ u

$(x-1)^2 + (y-k)^2 = k^2$

Circle passes through $(2,3)$, then

$(x-1)^2 + (3-k)^2 = k^2$

$1 + 9 - 6k + k^2 = k^2$

$\Rightarrow 6k = 10$

$\Rightarrow 2k = \frac{10}{3}$

Sol 15: (B) The eq. of circle touching the

$a - a \times \hat{u}$ at $(3,0)$ is

$(1-3)^2 + (-2,-k)^2 = k^2$

$\Rightarrow 4 + 4 + 4k + k^2 = k^2$

$\Rightarrow 4k = -8$

$\Rightarrow k = -2$

Circle: $(x-3)^2 (-2-k)^2 = k^2$

Point $(5,-2)$

$(5,-3)^2 + (-2+2)^2 = 2 + 2 = 4$

Only $(5,-2)$ lies on circle.

Sol 16: (D) $\frac{x^2}{10}+\frac{y^2}{9}=1$

to is $=\left(\pm\sqrt{7},0\right)$

Circle having cente as $\left(0,3\right)$

$x^2+\left(y-3\right)^2=\gamma^2$ passes through focus , then

$\left(\pm\sqrt{7}\right)^2+\left(0-3\right)^2=\gamma^2$

$7+9=\gamma^2$

$\Rightarrow \gamma^2=16$

$\Rightarrow x^2+\left(y-3\right)^2=16$

$\Rightarrow x^2+y^2-6y-7=0$

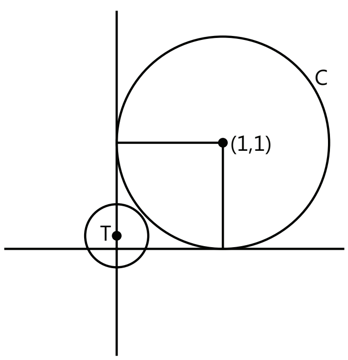
Sol 17: (D) If circles C and T touch each other externally then

$1+y=\sqrt{\left(1-0\right)^2+\left(1-y\right)^2}$

$\Rightarrow \left(1+y\right)^2=1+\left(1-y^2\right)$

$\Rightarrow 1^2+y^2+2y=1+1+y^2-2y$

$\Rightarrow y=\frac{1}{4}$



Sol 18: (B) $\left(x-2\right)^2+\left(y-3\right)^2=25$

$\left(x+3\right)^2+\left(y+9\right)^2=64$

$C_1C_2=\sqrt{\left(2+3\right)^2+\left(3+9\right)^2}$

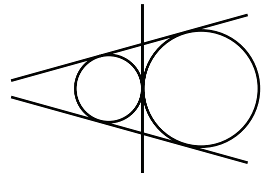
$=\sqrt{25+144}$

$=\sqrt{169}=13$

$r_1+r_2=3+5=13$

$\Rightarrow r_1+r_2=c_1\ c_2$

Circles touch each other externally therefrom, three tangents are possible



Sol 19: (C) $x^2+y^2-8x-8y-4=0$

$\left(x-4\right)^2+\left(y-4\right)^2=36$

Circles touch each other exotically

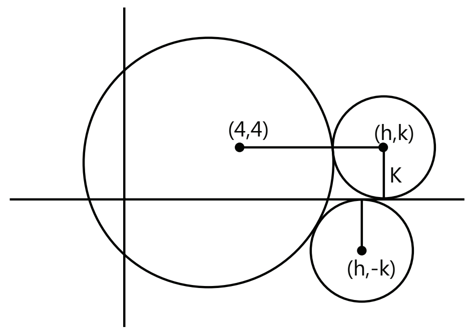
$k+6=\sqrt{\left(n-4\right)^2+\left(k-4\right)^2}$

$k^2+36+12k=h^2+16-3h+k^2-3k+16$

$\Rightarrow h^2-3h-9=20k$

$\Rightarrow x^2-3x-20y-4=0$

If $y<0$



$\left(-k+6\right)^2=\left(n-4\right)^2+\left(k-4\right)^2$

$\Rightarrow h^2-8h+4k-4=0$

$\Rightarrow x^2-8x+4y-4=0$

Locus is Parabola

JEE Advanced/Boards

Exercise 1

Sol 1: The equation of line through origin is y = mx

Let point on circle be (h_1, mh_1) and (h_2, mh_2)

$S = x^2 + y^2 - 8x - 6y + 24 = 0$

O = origin

(i) The equation of chord of S whose mid-point is (h, k) is

$hx + ky - 4(x + h) - 3(y + k) + 24$

$= h^2 + k^2 - 8h - 6k + 24$

Since it passes through origin

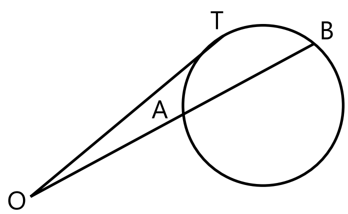
$\therefore -4h - 3k = h^2 + k^2 - 8h - 6k$

$\therefore$ Locus of point is

$x^2 + y^2 - 4x - 3y = 0$

(ii) OP = $\sqrt{OA \times OB}$

It is a known property that



$OA \times OB = OT^2 = OP^2$

$\therefore OP = OT$ = constant k

$OT = \sqrt{S_{(0,0)}} = \sqrt{24}$

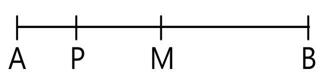
$\therefore$ The locus of P is the circle of radius $\sqrt{24}$ and centre = origin

$\Rightarrow x^2 + y^2 = 24$ is the locus of P

(iii) $OP = \dfrac{2OA.OB}{OA + OB} = \dfrac{OA.\,OB}{OM}$

$\therefore OP \times OM = OA \times OB$

$\therefore$ A and M are harmonic conjugates of P & B



$\therefore \dfrac{AM}{PM} = \dfrac{AB}{MB} \Rightarrow \dfrac{AM}{PM} = 2$

$\therefore$ P is mid-point of A & M

$\therefore$ Locus of P:

$x^2 + y^2 - 8x - 6y + 24 - (x^2 + y^2 - 4x - 3y) = 0$

$\therefore 4x + 3y = 24$ is locus of P

Sol 2: Radius of given circle= $\sqrt{4 + 2 - C} = \sqrt{6 - C}$

$r = \sqrt{2}\, r_1$ and $r_1 = \sqrt{2}\, r_2$, $r_2 = \sqrt{2}\, r_3$

Sum of radii of all circles

$= r + \dfrac{r}{\sqrt{2}} + \dfrac{r}{2} + \ldots\ldots\ldots\ldots = \dfrac{r}{1 - \dfrac{1}{\sqrt{2}}} \Rightarrow \dfrac{r}{1 - \dfrac{1}{\sqrt{2}}} = 2$

$\therefore r = 2 - \sqrt{2}$

$\Rightarrow \sqrt{6 - C} = 2 - \sqrt{2} \Rightarrow 6 - C = 4 + 2 - 4\sqrt{2}$

$\therefore C = 4\sqrt{2} = \sqrt{32} \Rightarrow n = 32$

Sol 3: Equation of common chord

$= x^2 + y^2 + 4x + 22y + a - (x^2 + y^2 - 2x + 8y - b)$

$= 6x + 14y + (a + b) = 0$

Now centre of second circle lies on this

$\therefore 6 \times 1 + 14 \times (-4) + (a + b) = 0$

$\therefore (a + b) = 50$

Now a , b > 0

$\therefore$ AM > GM

$\Rightarrow \dfrac{a + b}{2} > \sqrt{ab}$

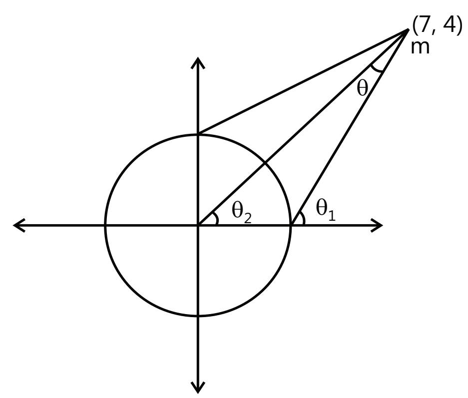
$\therefore 25 > \sqrt{ab}$

$ab < 625$

Sol 4: $x^2 + y^2 = 1$

$Z = \dfrac{y - 4}{x - 7}$

In this the slope from the point (7, 4); $\tan\theta_2 = \dfrac{4}{7}$



$\tan\theta = \tan(\theta_1 - \theta_2) = \dfrac{m - \tan\theta_2}{1 + m\tan\theta_2}$

$\left|\dfrac{r}{\ell_{tangent}}\right| = \dfrac{m - 4/7}{1 + m \times \dfrac{4}{7}}$

$\Rightarrow \pm\dfrac{1}{8} = \dfrac{m - 4/7}{1 + \dfrac{4m}{7}} = \dfrac{7m - 4}{7 + 4m}$

$\therefore M = \dfrac{3}{4}$ and $m = \dfrac{5}{12}$

$\therefore 2M + 6m = \dfrac{2 \times 3}{4} + \dfrac{6 \times 5}{12} = 4$

Sol 5: The radical axis of 2 circles is

$\left(2g-\frac{3}{2}\right)x + (2f-4)y = 0$

Centre of the given circle = (–1, 1)

and radius = 1

Since it is a tangent to the circle

$\Rightarrow \frac{(2f-4)-\left(2g-\frac{3}{2}\right)}{\sqrt{\left(2g-\frac{3}{2}\right)^2+(2f-4)^2}} = 1$

$\Rightarrow (2f-4)^2 + \left(2g-\frac{3}{2}\right)^2 + 2(2f-4)\left(2g-\frac{3}{2}\right)$

$= \left(2g-\frac{3}{2}\right)^2 + (2t-4)^2$

$\therefore (2f-4)\left(2g-\frac{3}{2}\right) = 0$

$\therefore$ Either f = 2 or $g = \frac{3}{4}$

Sol 6: The line passing through points A(3, 7) and B(6, 5) is 2x + 3y – 27 = 0

The family of circles passing through these points is

$(x-3)(x-6) + (y-7)(y-5) + \lambda(2x+3y-27) = 0$

$\Rightarrow x^2 - 9x + 18 + y^2 - 12y + 35 + \lambda(2x+3y-27) = 0$

$\therefore$Chord of contact = $s_1 - s_2$

$\Rightarrow -5x - 6y + 50 + \lambda(2x+3y-27) = 0$

$\Rightarrow L_1 + \lambda L_2 = 0$

The point which passes through intersection of L_1 and L_2 is the point of intersection of all λ

5x + 6y = 50

2x + 3y = 27

$\therefore x = 2$ & $y = \frac{23}{3}$ $\therefore$ $P = \left(2,\frac{23}{3}\right)$

Sol 7: The locus of point of intersection of mutually perpendicular tangent is the director circle

$\therefore$Locus of point = $x^2 + y^2 = 8$

The equation of family of circle touch a given circles & at (x_1, y_1) is S + λ(L) where L = tangent

$x^2 + y^2 - 8 + \lambda(x \times 2 + y \times 2 - 8) = 0$

Now this passes through (1, 1)

$-6 + \lambda(-4) = 0$

$\lambda = \frac{-3}{2}$

Equation of circle is

$x^2 + y^2 - 8 - 3x - 3y + 12 = 0$

$\Rightarrow x^2 + y^2 - 3x - 3y + 4 = 0$

Sol 8: C: $x^2+y^2+y-1+k(x+y-1)=0$

It is the family of circle passing through points of intersection of a circle & L.

Putting x = 1 – y in C_1

We get $y^2 - 2y + 1 + fy^2 + y - 1 = 0$

$\Rightarrow 2y^2 - y = 0 \Rightarrow y = 0, \frac{1}{2}$ & $x = 1$ or $\frac{1}{2}$

$\therefore$ The point of intersection are A(1, 0) and $B\left(\frac{1}{2},\frac{1}{2}\right)$

The minimum value of radius is when point act as diameter

$\therefore r_{min} = \frac{1}{2}\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \frac{1}{2}\times\frac{1}{2}\sqrt{2} = \frac{1}{2\sqrt{2}}$

Sol 9: The equation of circle co-axial with 2 circle is $S_1 + \lambda S_2 = 0$

$2x^2 + 2y^2 - 2x + 6y - 3 +$

$\lambda(x^2 + y^2 + 4x + 2y + 1) = 0$

$= (2+\lambda)x^2 + (\lambda+2)y^2 + (4\lambda-2)x$

$+ (2\lambda+6)y + \lambda - 3 = 0$

$x_{centre} = \frac{2-4\lambda}{2(2+\lambda)} = \frac{1-2\lambda}{\lambda+2}$

$y_{centre} = \frac{-(\lambda+3)}{\lambda+2}$

Radical axis of the two circle is $s_1 - s_2 \equiv 5x - y + \frac{5}{2} = 0$

Centre lies on radical axis

$\therefore 5 \times \frac{(1-2\lambda)}{\lambda+2} + \frac{\lambda+3}{\lambda+2} + \frac{5}{2} = 0$

$\Rightarrow 10 - 20\lambda + 2\lambda + 6 + 5\lambda + 10 = 0$

$\Rightarrow 13\lambda = 26 \therefore \lambda = 2$

$\therefore$Equation of circle is $4x^2 + 4y^2 + 6x + 10y - 1 = 0$

Sol 10: $s_1 \equiv x^2 + y^2 - 4x - 6y - 12 = 0$

$s_2 \equiv x^2 + y^2 + 6x + 4y - 12 = 0$

$s_3 \equiv x^2 + y^2 - 2x - 4 = 0$

The circle passing through point of intersection of s_1 and s_2 is $s \equiv s_1 + \lambda s_2 = 0$

$\Rightarrow x^2 + y^2 - 4x - 6y - 12 + \lambda$

$(x^2 + y^2 + 6x + 4y - 12) = 0$

$\Rightarrow (\lambda + 1)x^2 + (\lambda + 1)y^2 + (6\lambda - 4)x$

$+ (4\lambda - 6)y - 12(\lambda + 1) = 0$

Since it is orthogonal to s_3

$\therefore 2gg_1 + 2ff_1 = c + c_1$

$\Rightarrow \frac{(6\lambda - 4)}{\lambda + 1} x - 1 + 0 = \frac{-12(\lambda + 1)}{(\lambda + 1)} - 4$

$\therefore 4 - 6\lambda = -16(\lambda + 1)$

$10\lambda = -20$

$\lambda = -2$

$s \equiv -x^2 - y^2 - 16x - 4y + 12 = 0$

$\therefore x^2 + y^2 + 16x + 14y - 12 = 0$

Sol 11: Let $s = x^2 + y^2 + 2gx + 2fy + c = 0$

Now $(-g, -f)$ lies on $2x - 2y + 9 = 0$

$\Rightarrow -2g + 2f + 9 = 0$

and it is orthogonal to $x^2 + y^2 - 4 = 0$

$2g \times 0 + 2f \times 0 = c - 4$

$C = 4$

and $f = g - \frac{9}{2}$

$s = x^2 + y^2 + 2gx + 2\left(g - \frac{9}{2}\right)y + 4 = 0$

$s \equiv x^2 + y^2 - 9y + 4 + 2g(x + y)$

$\therefore$ It passes through point of intersection of S and L

Putting $x = -y$ in s

$2y^2 - 9y + 4 = 0 \Rightarrow 2y^2 - 8y - y + 4 = 0$

$\therefore y = \frac{1}{2}$ or $y = 4$ & $x = -\frac{1}{2}, -4$

$\therefore$ The points are $\left(\frac{-1}{2}, \frac{1}{2}\right)$ & $(-4, 4)$

Sol 12: Let the equation of circle be

$x^2 + y^2 + 2gx + 2fy = 0$

(it passes through origin)

The line pair is

$xy - 3x + 2y - 6 = 0$

$x(y - 3) + 2(y - 3) = 0$

$(x + 2)(y - 3) = 0$

The centre is point of intersection of these two lines

$c \equiv (-2, 3)$

$g = 2$ and $f = -3$

$s \equiv x^2 + y^2 + 4x - 6y = 0$

$s_1 = x^2 + y^2 - kx + 2ky - 8 = 0$

Since s & s_1 are orthogonal

$\therefore 2gg_1 + 2ff_1 = 0 - 8$

$\Rightarrow 2(-k) + (-3) \times 2k = 0 - 8$

$\therefore k = 1$

Sol 13: Since the circle cuts co-ordinate axis orthogonally

$\therefore C \equiv (0, 0)$

$\therefore s \equiv x^2 + y^2 - a^2 = 0$

$s \equiv x^2 + y^2 - 14x - 8y + 64 = 0$

Since s & s_1 are orthogonal

$\therefore 2 \times 0 \times -7 + 2 \times 0 \times -4 = -a^2 + 64$

$\therefore a^2 = 64$

$\therefore s \equiv x^2 + y^2 - 64 = 0$

Sol 14: Let the given circles

$S_1 \equiv x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$

$S_2 \equiv x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$

Let the circle orthogonal to the two circles be

$x^2 + y^2 + 2gx + 2fy + c = 0$

$\therefore 2gg_1 + 2ff_1 = c_1 + c$

and $2gg_2 + 2ff_2 = c + c_2$

$\Rightarrow 2g(g_1 - g_2) + 2f(f_1 - f_2) = c_1 - c_2$

Now the centre is $(-g, -f)$

$\therefore x = -g$ & $y = -f$ substituting instead of g & f

We get $2x(g_1 - g_2) + 2y(f_1 + f_2) = (c_1 - c_2)$

Which is the radical axis & (straight line)

The locus of centres of given s_1, s_2 is $s_1 - s_2 = 0$

$4x + 5x - 6y - 4y + 7 = 0$

$9x - 10y + 7 = 0$

Sol 15: Consider a point circle at $(-2, 7)$

$(x + 2)^2 + (y - 7)^2 = 0$

Now the equation a circle touching a circle at point is $s + \lambda L$

Where L is tangent to L

$\therefore s \equiv (x+2)^2 + (y-7)^2 + \lambda(x + y - 5) = 0$

$\Rightarrow x^2 + y^2 + (\lambda + 4)x + (\lambda - 14)y + (53 - 5\lambda) = 0$

$\therefore s_2 \equiv x^2 + y^2 + 4x - 6y + 9 = 0$

Since s & s_2 are orthogonal

$\therefore (\lambda + 4) + (\lambda - 14)\, x - 3 = 53 - 5\lambda + 9$

$\Rightarrow 4\lambda = 12 \Rightarrow \lambda = 3$

$\therefore$ Equation of circle

$s \equiv x^2 + y^2 + 7x - 11y + 38 = 0$

Sol 16: Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$

(–6, 0) lies on the circle

$\therefore 36 - 12g + c = 0$ ... (i)

The power of (i, i) is 5

$\Rightarrow 1 + 1 + 2g + 2f + c = 5$

$\Rightarrow 2g + 2f + c = 3$ ... (ii)

S is orthogonal to

$x^2 + y^2 - 4x - 6y - 3 = 0$

$\Rightarrow 2g(-2) + 2f(-3) = c - 3$

$\therefore 4g + 6f + c - 3 = 0$ ... (iii)

From (ii) and (iii)

$2g + 2c = 6 \Rightarrow g + c = 3$ ... (iv)

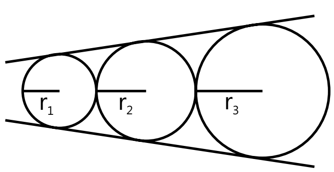
From i and iv g = 3

$\therefore c = 0 \quad \& \; f = \frac{-3}{2}$

$s \equiv x^2 + y^2 + 6x - 3y = 0$

Sol 17: Radius of largest circle = 18

Radius of smallest circle = 8



When 3 circle touching each other have direct common tangent

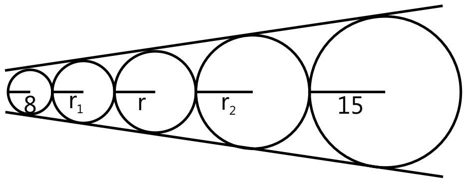
The radius of the middle circle is GM of radius of other 2 circles

$\therefore r_2^2 = r_1 r_3$

In the given problem

Let radius of middle circle be r,

2nd smallest circle be r_1 & 2nd largest circle be r_2



$\therefore r_1^2 = 8r \;\&\; r_2^2 = 18\,r$

$r^2 = r_1 r_2$

$\therefore r^4 = 8r \times 18\,r$

$r = \sqrt{8 \times 18} = 12$

Sol 18: The pair of lines is

$7x^2 - 18\,xy + 7y^2 = 0$

Since co-eff of x = coeff of y.

angle bisectors are

$(x - y) = 0 \;\&\; x + y = 0$

Since the given circle lies in the 1st quadrant

$\therefore$ Our circle should also lie in the 1st quadrant

$\therefore$ Its centre should lie on y = x

Centre $\equiv$ (h , h)

Now $(x - h)^2 + (y - h)^2 = k^2$

Let y = mx be equation of tangent

$\frac{h - mh}{\sqrt{1 + m^2}} = R$

$\therefore R^2(m^2 + 1) = h^2(m^2 - 2m + 1)$

$\therefore (R^2 - h^2)m^2 + 2h^2 m + R^2 - h^2 = 0$

Comparing to pair of lines

We get $\frac{2h^2}{R^2 - h^2} = \frac{-18}{7}$

$14h^2 = -18R^2 + 18h^2$

$\therefore 4h^2 = 18R^2$

$\therefore h = \frac{3}{\sqrt{2}} R$ h is in 1st quadrant

Since C touches C_1

$= (R_1 - R)$ = distance between centres

$\therefore (4\sqrt{2} - R)^2 = \left(4 - \frac{3R}{\sqrt{2}}\right)^2 + \left(4 - \frac{3R}{\sqrt{2}}\right)^2$

$\Rightarrow 4\sqrt{2} - R = \pm \sqrt{2}\left(4 - \frac{3R}{\sqrt{2}}\right)$

$\therefore 4\sqrt{2} - R = -\sqrt{2}\left(4 - \frac{3R}{\sqrt{2}}\right)$

$8\sqrt{2} = \frac{3\sqrt{2}R}{\sqrt{2}} + R$

$8\sqrt{2} = 4R$

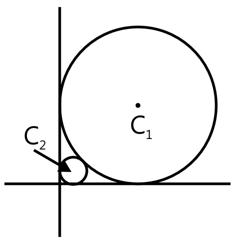
$R = 2\sqrt{2}$

$h = \frac{3\times 2\sqrt{2}}{\sqrt{2}} = 6$

Equation is $(x-6)^2+(y-6)^2 = (2\sqrt{2})^2$

Sol 19: (r, p, q) (A) Centre of $C_1 \equiv (a, a)$ & radius = a for C_2 centre ≡ (b, b) & radius = b

C_1 & C_2 cannot touch other internally



$\sqrt{(b-a)^2 + (b-a)^2} = (b + a)$

$\therefore \sqrt{2}(b-a) = (b+a)$

$\therefore (\sqrt{2} - 1)b = (\sqrt{2} + 1)a$

$\therefore \frac{b}{a} = \frac{\sqrt{2}+1}{\sqrt{2}-1} = 3 + 2\sqrt{2}$

(B) Equation of

$C_1 \equiv x^2 + y^2 - 2ax - 2ay + a^2 = 0$

$C_2 \equiv x^2 + y^2 - 2bx - 2by + b^2 = 0$

C_1 & C_2 are orthogonal

$2(-a_x - b) + 2(-a_x - b) = a^2 + b^2$

$4ab = a^2 + b^2$

$a^2 - 4ab + b^2 = 0$

$\frac{b}{a} = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm \sqrt{12}}{2}$

$= 2 \pm \sqrt{3}$

But b > a $\therefore \frac{b}{a} = 2 + \sqrt{3}$

(C) C_1 and C_2 intersect such that common chord is longest

$\therefore C_2$ bisects C_1

Equation of common chord is

$2(b-a)x + 2(b-a)y = b^2 - a^2$

$\therefore 2x + 2y = a + b$

It passes through (a, a)

$4a = a + b$

$b = 3a$

$\Rightarrow \frac{b}{a} = 3$

(D) C_2 passes through centre of C_1

$\therefore a^2 + a^2 - 2ab - 2ab + b^2 = 0$

$\Rightarrow b^2 - 4ab + 2a^2 = 0$

$\Rightarrow \left(\frac{b}{a}\right)^2 - 4\left(\frac{b}{a}\right) + 2 = 0$

$\therefore \frac{b}{a} = \frac{4 \pm \sqrt{16-8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$

But b > a

$\therefore \frac{b}{a} = 2 + \sqrt{2}$

Sol 20: y = x + 10 & y = x – 6 are tangents

The centre of circle passes through

$y = x + \frac{(10-6)}{2} = y = x + 2$

Also radius, = $\frac{1}{2}$ ⊥ distance between lines

$= \frac{1}{2} \times \frac{c_1 - c_2}{\sqrt{a^2+b^2}} = \frac{1}{2} \times \frac{16}{\sqrt{2}} = 4\sqrt{2}$

$\therefore$ Circle is $(x-h)^2 + (y-(h+2))^2 = (4\sqrt{2})^2$

$h + k = 2h + 2 = a + b\sqrt{a}$

Since y – axis is tangent

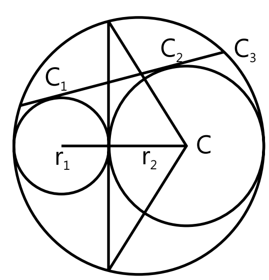
$\therefore$ h = Radius

$\therefore h = 4\sqrt{2}$

and $h + k = 2h + 2 = 8\sqrt{2} + 2$

$\therefore a + b = 10$

Sol 21:



Since, centres of the Circle are collinear.

$\therefore$ Radius of bigger circle = $\frac{2r_1 + 2r_2}{2}$ = 14

Now distance of point of intersection from centre = R – $(2r_1)$ = 14 – 2 × 4 = 6 = d

Length of chord

= $2\sqrt{R^2 - d^2}$ = $2(14)^2 - (6)^2$ = $4\sqrt{40}$ = $8\sqrt{10}$

m + n + p = 1 + 8 + 10 = 19

Sol 22: Equation of a circle passing through two points

(x – 1) (x – 4) + (y – 7) (y – 8) + λ(L) =0

L passing through (4, 7) & (1, 8)

is $y - 8 = \frac{-1}{3}(x - 1)$

3y + x – 25 = 0

$\therefore$ (x – 1) (x – 4) + (y – 7) (y – 8) + λ(3y + x – 25) =0

(5, 6) satisfies this equation

$\lambda = +\frac{(4+2)}{+2} = 3$

Equation of circle is $x^2 + y^2 - 2x - 6y - 15 = 0$

Let the points of intersection of tangent be (h , k)

chord of contact is

$hx + ky - (x + h) - 3(y + k) - 15 = h^2 + k^2 - 2h - 6h - 15$

$(h - 1)x + (k - 3)y + h + 3k - h^2 - k^2 = 0$

Now, $\frac{(h-1)}{h+3k-h^2-k^2} = \frac{5}{17}$ ... (i)

and $\frac{k-3}{h+3k-h^2-k^2} = \frac{1}{17}$

$\frac{h-1}{k-3} = 5 \Rightarrow h - 1 = 5(k - 3)$

h = 5(K – 3) + 1

Substituting in 1 we get k = 2

$\therefore$ h = – 4

$\therefore$ Point is (–4, 2)

Sol 23: The equation of circle which touches a given line at a point is

$(x-1)^2 + (y - 1)^2 + \lambda(2x - 3y + 1) = 0$

$\therefore x^2 - y^2 + 2(\lambda - 1)x - (2 + 3\lambda)y + \lambda + 2 = 0$

R = $\sqrt{13}$

$\therefore (\lambda - 1)^2 + \left(\frac{(2+3\lambda)}{2}\right)^2 - \lambda - 2 = 13$

$\therefore \lambda^2 = 4 \therefore \lambda = \pm 2$

$\therefore$ Equation of circles are

$x^2 + y^2 + 2x - 8y + 4 = 0$ or $x^2 + y^2 - 6x + 4y = 0$

Sol 24: Equation of circle touching other. circle is at point is s + λ (L) = 0

Where L is equation of tangent at the point

$x^2 + y^2 + 4x - 6y - 3 + \lambda$ (2x + 3y

+ 2(x + 2) – 3 (y + 3) – 3) = 0

It passes through (1, 1)

$\therefore \lambda = \frac{-(1+1+4-6-3)}{(2+3+6-12-3)} = \frac{3}{-4} = \frac{-3}{4}$

$\therefore$ Equation of circle is

$4x^2 + 4y^2 + 16x - 24y - 12 - 3(4x - 8) = 0$

$4x^2 + 4y^2 + 4x - 24y + 12 = 0$

$x^2 + y^2 + x - 6y + 3 = 0$

Exercise 2

Single Correct Choice Type

Sol 1: (A) Since BAC = 90º

locus of A is the circle with (3, 0), (–3, 0) as diameter

Let A = (h, – k)

$(h - 3)(h + 3) + k^2 = 0$

Now, centroid

$C(x, y) = \left(\frac{h+3-3}{3}, \frac{k+0+0}{3}\right)$

Substituting h, k in terms of (x, y)

$(3x - 3)(3x + 3) + (3y)^2 = 0$

$x^2 + y^2 = 1$ is the equation of centroid

Sol 2: (C) | y | = x + 1 & $(x - 1)^2 + y^2 = 4$

Substituting value of | y |

$(x - 1)^2 + (x + 1)^2 = 4$

$x^2 = 1$

x = ± 1

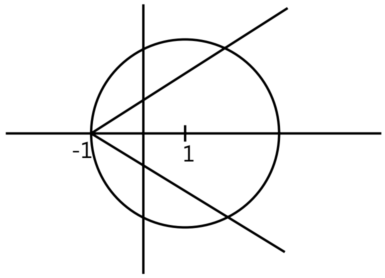
For x = –1; y = 0

For x = +1; | y | = 2 $\therefore$ y ± 2

$\therefore$ Three possible solutions are possible

Alternate method

Plotting the graph of $|y| = x + 1$ and $(x-1)^2 + y^2 = 4$



We can directly see that three possible intersection are possible

Sol 3: (B) Line 1 passes through (3, 1) and

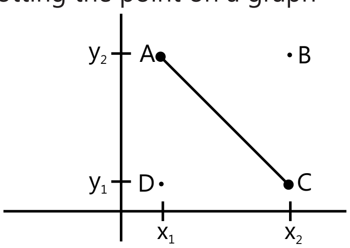
Line 2 passes through (1, 3)

Lines L_1 and L_2 are $\perp$ $\therefore$ locus of point of intersection is a circle with (3, 1) & (1,3) as ends of diameter

Locus of points is $(x-3)(x-1) + (y-1)(y-3) = 0$

$\therefore x^2 + y^2 - 4x - 4y + 6 = 0$

Sol 4: (B) Plotting the point on a graph



It is not necessary that

$|x_2 - x_1| = |y_2 - y_1|$

With (x_2, y_1) & (x_1, y_2) as ends of diameter $\angle ABC = 90^\circ$ and $\angle ADC = 90^\circ$

$\therefore$ ABCD are concyclic

Sol 5: (D) Let A = (0, 6), B = (5, 5) & C = (–1, 1)

Slope of AB = $\frac{-1}{5}$ & $m_{AC} = 5$

$\therefore$ AB $\perp$ AC

Circumcentre is midpoint of BC

O = (2, 3)

And radius = $\frac{1}{2}\sqrt{6^2 + 4^2} = \sqrt{13}$

Now y = mx is tangent to the circle $\therefore \frac{3-2m}{\sqrt{1+m^2}} = \sqrt{13}$

$4m^2 - 12m + 9 = 13m^2 + 13 \Rightarrow 9m^2 + 12m + 4 = 0$

$9m^2 + 6m + 6m + 4 = 0$

$(3m + 2)^2 = 0$

$m = -\frac{2}{3}$

$\therefore$ Equation of line is 3y + 2x = 0

Sol 6: (A) The circumcenter of triangle A,B,C is (0, 0)

Let c ≡ (h , k)

And centroid (c_1) is $\left(\frac{1+h}{3}, \frac{1+k}{3}\right)$

Let the orthocentre be (x, y)

The centroid divides O and C in ratio 2 : 1

$\therefore \left(\frac{1+h}{3}, \frac{1+k}{3}\right) = \left(\frac{x}{3}, \frac{y}{3}\right)$

$\therefore h = (x-1)$ and $k = (y-1)$

$(x-1)^2 + (y-1)^2 = 1$

$\therefore x^2 + y^2 - 2x - 2y + 1 = 0$

Sol 7: (D) Centre of circle is (– 8, –6)

Equation of line is y = 2x + 5

$\therefore$ Q is the foot of perpendicular of (– 8, –6) on 2x – y + 5 = 0

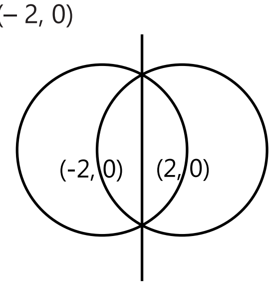
$\therefore \frac{x-(-8)}{2} = \frac{y-(-6)}{-1} = \frac{-(-5)}{5}$

$\therefore x = -6$ & $y = -7$

$\therefore$ Q ≡ (–6, –7)

Sol 8: (A) Centre of C_1 = (2, 0) $R_1 = 4$ & $R_2 = 4$

Centre of C_2 = (– 2, 0)



$\therefore$ The other 2 points of rhombus lie on y axis put in x = 0 we get

$Y = \pm 2\sqrt{3}$

$\therefore$ Length of 1st diagonal is (2 – (–2) = 4 and length of 2nd diagonal = $4\sqrt{3}$

Area of rhombus = $\frac{1}{2}ab = \frac{1}{2}\times 16\sqrt{3} = 8\sqrt{3}$ sq. units

Sol 9: (A) From (3, 4) chords are drawn to

$x^2 + y^2 - 4x = 0$

Let mid points of chord be (b, h)

$\therefore h^2 + k^2 - 4h = xh + yk - 2(h + x)$

Now (3, 4) pass through these chords

$\therefore h^2 + k^2 - 4h = 3h + 4k - 2(h + 3)$

$\therefore$ Locus of mid-point is $x^2 + y^2 - 5x - 4y + 6 = 0$

Sol 10: (B) Let p = (x, y)

$(x, y) = \left(\frac{20\cos\theta + 15}{5}, \frac{20\sin\theta + 15}{5}\right)$

$\cos\theta = \frac{x-1}{4}$ & $\sin\theta = \frac{y-1}{4}$

$(x - 1)^2 + (y - 1)^2 = 16$

This is a circle.

Sol 11: (B) (3, 4) & (–1, –2) are ends of diameter

$(x - 3)(x + 1) + (y - 4)(y + 2) = 0$

$x^2 + y^2 - 2x - 2y - 11 = 0$

Sol 12: (A) Shortest distance from line to circle

= $\perp$ distance – radius

Centre of circle $\equiv$ (3, –4) & radius = 5

$\therefore \perp$ distance = $\left|\frac{9 - 16 - 25}{\sqrt{25}}\right| = \frac{32}{5}$

$\therefore$ shortest distance = $\frac{32}{5} - 5 = \frac{7}{5}$

Sol 13: (A) Slope of the line is 1

$\therefore y = x + c$

The two circle are

$s_1 \equiv x^2 + y^2 = 4$

$c_1 = (0, 0)$ & R = 2

$s_2 \equiv x^2 + y^2 - 10x - 14y + 65 = 0$

$c_2 = (5, 7)$ & R = 3

Length intercepted = $2\sqrt{R^2 - (\perp \text{distance})^2}$

$\therefore \lambda_1 = 2\sqrt{2^2 - \left(\frac{O - O + C}{\sqrt{2}}\right)^2} = 2\sqrt{2^2 - \frac{C^2}{2}}$

$\lambda_2 = 2\sqrt{(3)^2 - \frac{(5 - 7 + C)^2}{2}}$

$\lambda_1 = \lambda_2$

$\therefore 4 - \frac{C^2}{2} = 9 - \frac{(C-2)^2}{2}$

$\therefore C^2 - 4C + 4 - C^2 = 10$

$C = -\frac{3}{2}$

Line is $y = x - \frac{3}{2}$

$2x - 2y - 3 = 0$

Sol 14: (D) Equation of circle is $x^2 + y^2 = r^2$

Let P $\equiv$ (a, b)

Let the midpoint of a point (h, k) on circle & P(a, b) be M(x, y)

$(x, y) = \left(\frac{a+h}{2}, \frac{b+k}{2}\right)$

h = 2x – a ; k = 2y – b

$(2x - a)^2 + (2y - b)^2 = r^2$ is locus of M

$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{r}{2}\right)^2$

Multiple Correct Choice Type

Sol 15: (C, D) Let h, k be the point of intersection

$\therefore$ Slope of lines is $\left(\frac{k}{h-1}\right)$ and $\left(\frac{k}{h+1}\right)$

For point (1, 0) and (–1, 0)

And $\tan(\theta - \theta_1) = \frac{\tan\theta - \tan\theta_1}{1 + \tan\theta\tan\theta_1}$

$\therefore$ The angle between lines is either 45º or 135º

$\theta - \theta_1$ = 45º or 135º

$\pm 1 = \frac{\frac{k}{h-1} - \frac{k}{h+1}}{1 + \frac{k^2}{h^2 - 1}}$

$\pm 1 = \frac{2k}{h^2 + k^2 - 1}$

$\therefore h^2 + k^2 - 2k - 1 = 0$ $\quad C \equiv (0, 1)$ $R = \sqrt{2}$

$\therefore h^2 + k^2 + 2k - 1 = 0$ $\quad C \equiv (0, -1)$ $R = \sqrt{2}$

Sol 16: (B, C, D) $s_1 \equiv x^2 + y^2 + 2x + 4y + 1 = 0$

$s_2 \equiv x^2 + y^2 - 4x + 3 = 0$

$s_3 \equiv x^2 + y^2 + 6y + 5 = 0$

Radical axes of s_1 and s_2 is

$6x + 4y - 2 = 0$

$3x + 2y - 1 = 0$

Radical axes of s_3 and s_2 is

$6y + 4x + 2 = 0$

$3y + 2x + 1 = 0$

$5x + 3y = 0$

$x = 1$ $y = -1$

(1, -1) is the radical centre

It is a known property that circle which is orthogonal to 3 circle has its center equal to radical center & radius = length of tangent from radical center to any circles.

Radices = $\sqrt{1+1+2-4+1} = 1$

Equation of orthogonal circle is $(x-1)^2 + (y+1)^2 = 1$

This circle touches both x & y axis.

Its x & y-intercept are 1

Sol 17: (B, C) $c_1 \equiv x^2 + y^2 - 4x + 6y + 8 = 0$

$c_2 \equiv x^2 + y^2 - 10x - 6y + 14 = 0$

Centre of $c_1 \equiv (2, -3)$

Centre of $c_2 \equiv (5, 3)$

$r_1 = \sqrt{4+9-8} = \sqrt{5}$

$r_2 = \sqrt{25+9-14} = 2\sqrt{5}$

$c_1c_2 = r_1 + r_2$

$c_1c_2 = \sqrt{(5-3)^2 + (6)^2} = 3\sqrt{5}$

$\therefore$ c_1 & c_2 touch each other

$\therefore$ Radical axis is the common tangent and the mid-point of c_1c_2 doesn't lie on radical axis as their radius are not the same.

Sol 18: (B, D) A = (–1, 1); B = (0, 6); C = (5, 5)

$AB \perp BC$

$\therefore$The circle passing through ABC will have AC as a diameter

$S : (x+1)(x-5) + (y-1)(y-5) = 0$

$\therefore x^2 + y^2 - 4x - 6y = 0$

Center c = (2, 3) ; $r = \sqrt{13}$

The line joining origin to center is $y = \frac{3}{2}x$

$\therefore 3x - 2y = 0$

The points will lie on the line $\perp$ to $3x - 2y = 0$ & passing through (2, 3) at a distance of r from (2, 3)

$L : y - 3 = \frac{-2}{3}(x-2)$ $\tan\theta = \frac{-2}{3}$

$2x + 3y - 13 = 0$

Let points be (h, k)

When θ is in 2nd quadrant

$\sin\theta > 0$ & $\cos\theta < 0$

$h = a + r\cos\theta$; $k = a + r\sin\theta$

$\therefore h = 2 + \sqrt{13} \times \frac{-3}{\sqrt{13}}$

$k = 3 + \sqrt{13} \times \frac{2}{\sqrt{13}}$

$\therefore P_1 = (-1, 5)$

When q lies in 4th quadrant

$\sin\theta < 0$ & $\cos\theta > 0$

$h = 2 + \frac{3}{\sqrt{13}} \times \sqrt{13}$

$k = 3 + \left(\frac{-2}{\sqrt{13}} \times \sqrt{13}\right)$

$\therefore P_2 = (5, 1)$

Sol 19: (A, C, D) $s_1 : x^2 + y^2 + 2x + 4y - 20 = 0$

$s_2 \equiv x^2 + y^2 + 6x - 8y + 10 = 0$

$c_1 = (-1, -2)$ & $c_2 = (-3, 4)$

$r_1 = \sqrt{1^2 + 2^2 + 20} = 5$

$r_2 = \sqrt{3^2 + 4^2 - 10} = \sqrt{15}$

$c_1c_2 = \sqrt{2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$

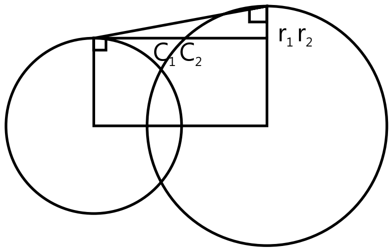
$c_1c_2 = r_1 + r_2$

and $c_1c_2 > | r_1 - r_2 |$

$\therefore$ The two circles intersect each other at 2 points

$2gg_1 + 2ff_1 = 2 \times 3 + 4x - 4 = -10 = c + c_1$

The 2 circle are orthogonal



Length of common tangents

$= \sqrt{(c_1c_2)^2 - (r_2 - r_1)^2} = \sqrt{40 - (5 - \sqrt{15})^2}$

$= \sqrt{10\sqrt{15}} = 5(12/5)^4$

The equation of common chord is $s_1 - s_2$

$\Rightarrow 4x - 12y + 30 = 0$

$\Rightarrow 2x - 6y + 15 = 0$

Perpendicular from c_1 on this $\Rightarrow \frac{-2+12+15}{\sqrt{40}} = \frac{25}{\sqrt{40}}$

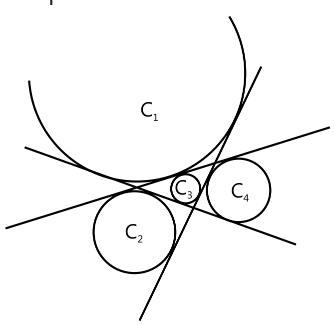
Length of common chord $= 2\sqrt{r^2 - a^2}$

$= 2\sqrt{25 - \left(\frac{25}{\sqrt{40}}\right)^2} = 2\sqrt{25 - \frac{625}{40}}$

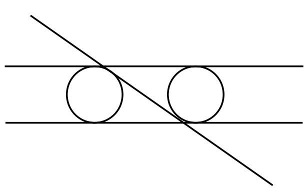
$= 2\sqrt{\frac{75}{8}} = \frac{10}{2}\sqrt{\frac{3}{2}} = 5\sqrt{\frac{3}{2}}$

Sol 20: (A, C, D) Consider 2 lines not parallel to one another and when the third line passes through intersection of both lines, no circle is possible.

When the third line doesnot pass through point of intersection of the lines & is not parallel to either of them 4 circle are possible.



When the 3rd line is parallel to one of the line then



2 circle are possible

When all 3 lines are parallel no circles are possible

Sol 21: (A, B, D) $c_1 = (x + 7)^2 + (y - 2)^2 = 25$

$\therefore r_1 = 5$

c_2 is director circle of c_1

$\therefore r_2 = 5\sqrt{2}$

And c_3 director circle of c_2

$\therefore r_3 = 5\sqrt{2} \times \sqrt{2} = 10$

Area enclosed by $c_3 = \pi r^2 = 100\pi$

Area enclosed of $c_2 = \pi \times (\sqrt{2}\,r)^2 = 2\pi r^2$

= 2 times area enclosed by c_1

Sol 22: (B, C) $S_1 \equiv x^2 + y^2 - 2x - 4y + 1 = 0$ $r_1 = 2$

$G \equiv (1, 2), r_1 = 2$

$S_2 \equiv x^2 + y^2 + 4x + 4y - 1 = 0$

$C_2 \equiv (-2, -2), r_2 = 3$

$C_1C_2 = \sqrt{3^2 + 4^2} = 5$

The two circle touch each other externally and common tangent is $S_2 - S_1 = 0$

$6x + 8y - 2 = 0$

$3x + 4y - 1 = 0$

Sol 23: (A, C, D) $S_1 \equiv x^2 + y^2 - 6x - 6y + 9 = 0$

$S_2 \equiv x^2 + y^2 + 6x + 6y + 9 = 0$

$C_1 = (-g, -t) = (3, 3)$

$r_1 = \sqrt{3^2 + 3^2 - 9} = 3$

and $C_2 = (-3, -3)$

$r_2 = \sqrt{3^2 + 3^2 - 9} = 3$

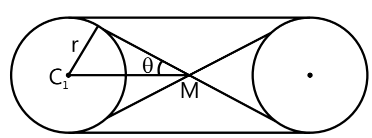
$C_1C_2 = \sqrt{6^2 + 6^2} = 6\sqrt{2}$

$r_1 + r_2 = 6$

They do not intersect with each other

Since their radius are same

$\therefore$ External direct common tangents are parallel



Also, the point of intersection of transverse common tangents is midpoint of C_1 and C_2 (same radii)

M = (0, 0)

$\sin\theta = \dfrac{r}{MC_1} = \dfrac{3}{\sqrt{3^2+3^2}} = \dfrac{1}{\sqrt{2}}$

$\theta = 45^\circ$

Angle between tangents = $2\theta = 90^\circ$

Sol 24: (B, C) $S_1 \equiv x^2 + y^2 + px + py - 7 = 0$

$S_2 \equiv x^2 + y^2 - 10x + 2py + 1 = 0$

S_1 & S_2 are orthogonal

$\therefore 2gg_1 + 2ff_1 = c + c_1$

$\Rightarrow p(-5) + p \cdot p = -6$

$\Rightarrow p^2 - 5p + 6 = 0$

$\Rightarrow$ P =2 or p = 3

Sol 25: (A, B, D) (A) Two circles having the same center. Have infinitely many common normal.

(B) Radical axis is always perpendicular to the line joining center but it does not necessarily bisect the line joining the centres. It bisects only when $r_1 = r_2$

(C) Let the centres of the two circles be C_1 & C_2.

Consider a point O, on radical axis centres which lies on the line C_1C_2

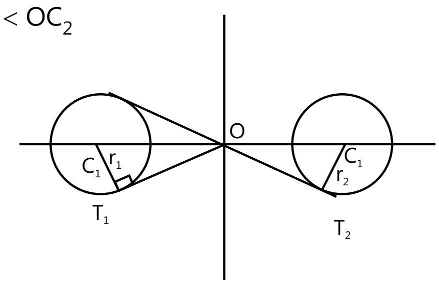
Now $OC_1^2 = r_1^2 + OT_1^2$

$OC_2^2 = r_2^2 + OT_2^2$

Since length of tangent is same

$\therefore OC_1^2 < OC_2^2$ if $r_1 < r_2$

$\Rightarrow OC_1 < OC_2$



(D) Consider two circles having same centre these circles donot have a radical axis

Assertion Reasoning Type

Sol 26: (C) L : $\overbrace{k(x-y-4)}^{L_1} + \overbrace{7x+y+20}^{L_2} = 0$

L are the lines passing through intersection of L_1 & L_2

Point of intersection is (– 2, – 6)

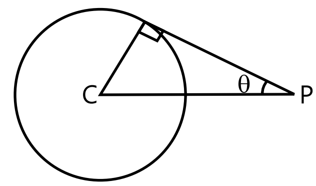
Which is center of circle c

Every line L is normal to circle

Statement-I is true & statement-2 is false

Sol 27: (A) Length of tangent from (13, 6)

$= \sqrt{13^2 + 6^2 - 13\times 6 + 8\times 6 - 75} = 10$



$\therefore$ Radius of circle $= \sqrt{3^2 + (-4)^2 + 75} = 10$

$\therefore \tan\theta = 1$

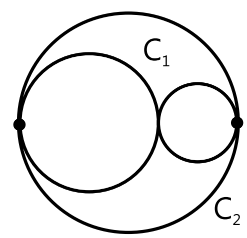
$\therefore \theta = 45^\circ$

Angle between tangents = $2\theta = 2 \times 45 = 90^\circ$

Director circle of a circle S_1 is such that the angle between the tangents drawn from any point on director circle to S_1 is 90°

Sol 28: (D) (1, 5) lies outside the circle

as 1 + 25 – 2 – 7 = 17 > 0



$\therefore$ Two circles shown C_1, C_2 are possible

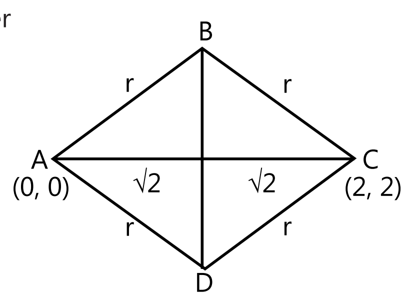
$\therefore$ Statement-I is false

Sol 29: (A) Since x + y – 2 = 0 is $\perp$ bisector of C_1C_2

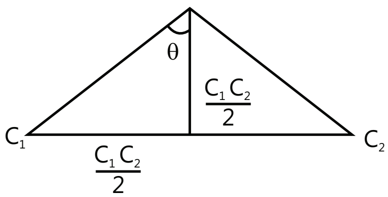
Radius of both the circles is same

Since length of common chord = $2\sqrt{2}$

ABCD is a square since diagonals are equal & $\perp$ to each other



When their centres are mirror image of each other then the common chord bisects C_1C_2 and $\frac{1}{2}\times$ length of common chord = $\frac{1}{2}c_1c_2$



$\tan\theta = 1$

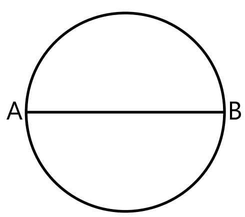
$\theta = 45^o$

The circles are orthogonal

When the centres are mirror image & length of chord = distance between centres then the two circles are orthogonal. The inverse is not true

$\therefore$ Statement-II is wrong

Sol 30: (A) Let AB = diameter



The circle with AB as diameter is

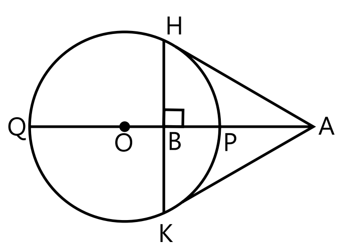
$(x - x_1)(x - x_2) + (y - y_1).(y - y_2) = 0$

when C is obtuse, then C lies inside the circle

$D(x_3, y_3) < 0$

(Power of a point inside a circle < 0)

Sol 31:



Since KPHQ are concyclic

$\therefore$ PB × BQ = HB × BK = $(HB)^2$

$\therefore$ (AB – AP) (AQ – AB) = $(HB)^2$

Also AH^2 = AP × AQ (from property of tangents)

$AH^2 - HB^2 = AP \times AQ - [AB \times AP + AB\times AQ - AB^2 - APAQ]$

$AB^2 = AP \times AQ - [AB(AP + AQ) - AB^2 - APAQ]$

$\therefore AB = \frac{2AP\times AQ}{AP+AQ}$

$\therefore$ Statement-I is true

Statement-II: $AK^2 = AB \times AO$ & $AK^2 = AP \times AQ$

$\therefore AB \times \frac{(AP+AQ)}{2} = AP \times AQ$

$\therefore AB = \frac{2AP\times AQ}{(AP+AQ)}$

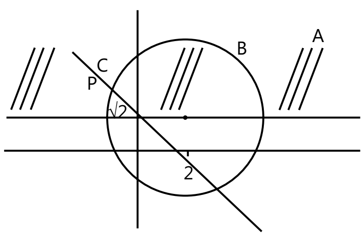
Comprehension Type

Paragraph 1: (32-34)

Sol 32: (B) A : {(x, y) : y ≥ 1 }

B : {(x, y) : $x^2 + y^2 - 4x - 2y - 4 = 0$}

C : {(x, y) : x + y = $\sqrt{2}$



There is only one point P of intersection of region A, B, C

Sol 33: (C) B : $x^2 + y^2 - 4x - 2y - 4 = 0$

$\Rightarrow 2x^2 + 2y^2 - 8x - 4y - 8 = 0$

$\Rightarrow (x - 5)^2 + (x + 1)^2 + (y - 1)^2 + (y - 1)^2 - 36= 0$

$\therefore f(x) = 36$

Sol 34: (C) S is director circle of B

$\therefore B : (x - 2)^2 + (y - 1)^2 = 9$

$s : (x - 2)^2 + (y - 1)^2 = 18$

Arc of B = 9π

Arc of s = 18π

Area of S-Area of B = 9π

Paragraph 2: (35-36)

Sol 35: (D) Let m be slope of tangents

$\therefore$ (y – 2) = m(x – 4) are equation of tangent

$s = x^2 + y^2 = 4$

For tangents $c^2 = a^2(1 + m^2)$

$\therefore (2 - 4m)^2 = 4(1 + m^2)$

$12m^2 - 16m = 0$

$4m(3m - 4) = 0$

$m = 0$ or $m = \frac{4}{3} \Rightarrow \tan\theta = \frac{4}{3}$

$\theta \in (45^\circ, 60^\circ)$ Ans.(D)

Sol 36: (B) the tangents are

y = 2 & 4x – 3y – 10 = 0

$\therefore$ Intercepts made on x axis by 2^{nd} tangent $= \frac{10}{4} = \frac{5}{2}$

Paragraph 3: (37-39)

Sol 37: (D) $s : x^2 + y^2 - 4x - 1 = 0$

L : y = 3x – 1

Centre of circle = (2, 0)

Radius = $\sqrt{5}$

Length of chord AB

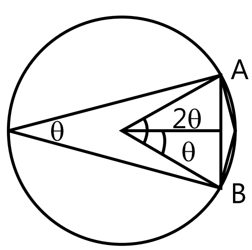
$= 2\sqrt{r^2 - (\text{perpendicular distance from centre})^2}$

Perpendicular distance from centre

$= \frac{6-1}{\sqrt{10}} = \sqrt{\frac{5}{2}}$

$AB = 2\sqrt{5 - \frac{5}{2}} = 2\sqrt{\frac{5}{2}} = \sqrt{10}$

Sol 38: (A)



Angle subtends at minor arc = 180 – angle at major arc

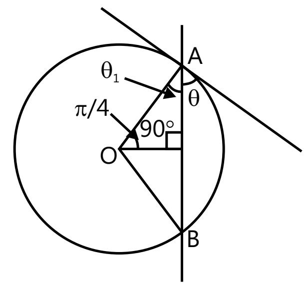
$\tan\theta = \frac{1}{2}\frac{\ell_{AB}}{\pm\text{distance}}$

$= \frac{1}{2} \times \frac{\sqrt{10}}{\sqrt{5/2}} = 1$

$\theta = 45^\circ$

Angle at minor arc $= \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Sol 39: (C)



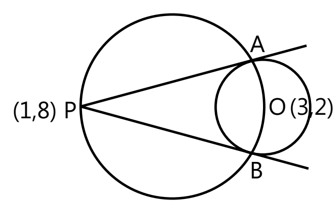
$\theta_1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

$\& \ \theta + \theta_1 = \frac{\pi}{2}$

$\theta = \frac{\pi}{4}$

Previous Years' Questions

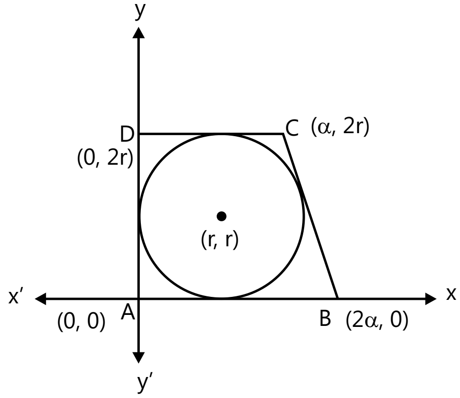
Sol 1: (B) For required circle, P(1, 8) and O(3,2) will be the end point of its diameter.



$\therefore (x-1)(x-3) + (y-8)(y-2) = 0$

$\Rightarrow \quad x^2 + y^2 - 4x - 10y + 19 = 0$

Sol 2: (B) $18 = \frac{1}{2}(3\alpha)(2r) \Rightarrow \alpha r = 6$

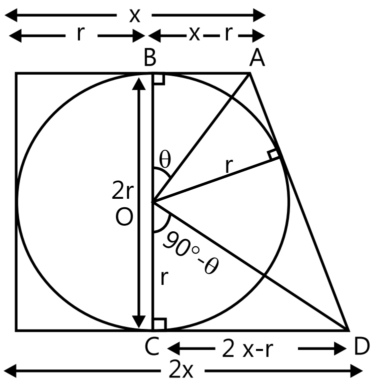


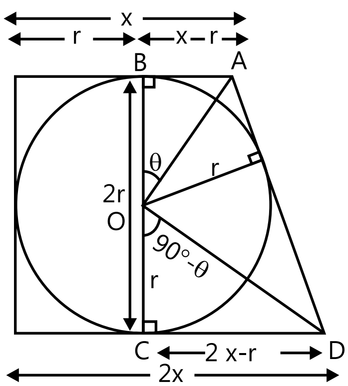
Line, $y = -\frac{2r}{\alpha}(x - 2\alpha)$ is tangent to circle

$(x-r)^2 + (y-r)^2 = r^2$

$2\alpha = 3r$ and $\alpha r = 6$

$r = 2$





Alternate solution

$\frac{1}{2}(x + 2x) \times 2r = 18$

$xr = 6$ ...(i)

In ΔAOB, $\tan\theta = \frac{x-r}{r}$

and in ΔDOC

$\tan(90^o - \theta) = \frac{2x-r}{r}$

$\therefore \frac{x-r}{r} = \frac{r}{2x-r}$

$\Rightarrow \quad x(2x - 3r) = 0$

$\Rightarrow \quad x = \frac{3r}{2}$ ....(ii)

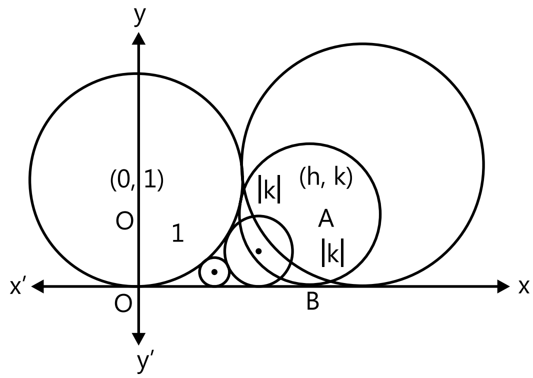
From Eqs. (i) and (ii) we get

r = 2

Sol 3: (A) Let the locus of centre of circle be (h, k) touching

$(y-1)^2 + x^2 = 1$ and x-axis shown as

Clearly, from figure,



Distance between O and A is always 1+|k|,

ie, $\sqrt{(h-0)^2 + (k-1)^2} = 1 + |k|,$

$\Rightarrow \quad h^2 + k^2 - 2k + 1$

$= 1 + k^2 + 2|k|$

$\Rightarrow \quad h^2 = 2|k| + 2k$

$\Rightarrow \quad x^2 = 2|y| + 2y$

where $|y| = \begin{cases} y, & y \ge 0 \\ -y, & y < 0 \end{cases}$

$\therefore \quad x^2 = 2y + 2y, y \ge 0$

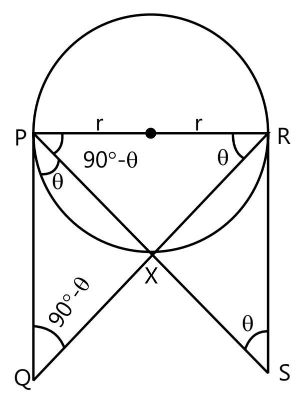
and $\quad x^2 = 2y + 2y, y < 0$

$\Rightarrow \quad x^2 = 4y \quad$ when $\; y \ge 0$

and $\quad x^2 = 0 \quad$ when $\; y < 0$

$\therefore \quad \{(x, y) : x^2 = 4y, \text{ when } y \ge 0\} \cup \{(0, y) : y < 0\}$

Sol 4: (A) From figure it is clear that ΔPRQ and ΔRSP are similar.



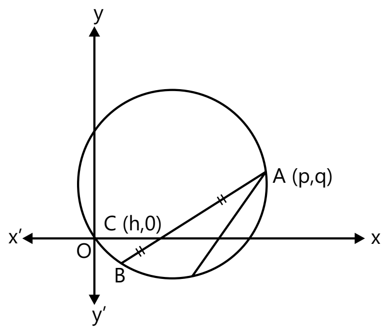
$\therefore \quad \frac{PR}{RS} = \frac{PQ}{RP}$

$\Rightarrow \quad PR^2 = PQ \cdot RS$

$\Rightarrow \quad PR = \sqrt{PQ.RS}$

$\Rightarrow \quad 2r = \sqrt{PQ \cdot RS}$

Sol 5: (B) Choosing OA as x-axis, A=(r, 0), B=(0, r) and any point P on the circle is $(r\cos\theta, r\sin\theta)$. If (x, y) is the centroid of ΔPAB, then



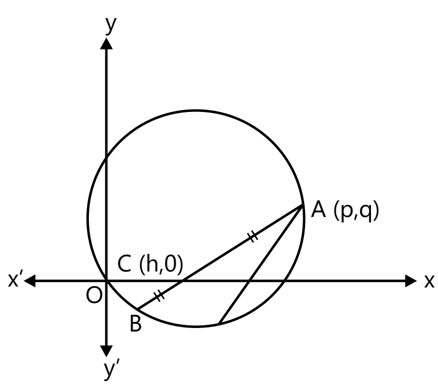
$$3x = r\cos\theta + r + 0$$

and $\quad 3y = r\sin\theta + 0 + r$

$\therefore \quad (3x - r)^2 + (3y - r)^2 = r^2$

Hence, locus of P is a circle.

Sol 6: (D) From equation of circle it is clear that circle passes through origin. Let AB is chord of the circle.



$A \equiv (p,q) \cdot C$ is mid point and coordinate of C is (h, 0)

Then coordinates of B are (-p + 2h, -q) and B lies on the circle $x^2 + y^2 = px + qy$. we have

$$(-p + 2h)^2 + (-q)^2 = p(-p + 2h) + q(-q)$$

$\Rightarrow \quad p^2 + 4h^2 - 4ph + q^2 = -p^2 + 2ph - q^2$

$\Rightarrow \quad 2p^2 + 2q^2 - 6ph + 4h^2 = 0$

$\Rightarrow \quad 2h^2 - 3ph + p^2 + q^2 = 0 \quad$ ....(i)

There are given two distinct chords which are bisected at x-axis then, there will be two distinct values of h satisfying Eq. (i).

So, discriminant of this quadratic equation must be > 0.

$\Rightarrow \quad D > 0$

$\Rightarrow \quad (-3p)^2 - 4 \cdot 2(p^2 + q^2) > 0$

$\Rightarrow \quad 9p^2 - 8p^2 - 8q^2 > 0$

$\Rightarrow \quad p^2 - 8q^2 > 0$

$\Rightarrow \quad p^2 > 8q^2$

Sol 7: Equation of given circle C is

$$(x - 3)^2 + (y + 5)^2 = 9 + 25 - 30$$

ie, $(x - 3)^2 + (y + 5)^2 = 2^2$

Centre = (3, -5)

If L_1 is diameter, then $2(3) + 3(-5) + p - 3 = 0 \Rightarrow p = 12$

$\therefore \; L_1$ is $2x + 3y + 9 = 0$

L_2 is $2x + 3y + 15 = 0$

Distance of centre of circle from L_2 equals

$$\left|\frac{2(3) + 3(-5) + 15}{\sqrt{2^2 + 3^3}}\right| = \frac{6}{\sqrt{13}} < 2 \text{ (radius of circle)}$$

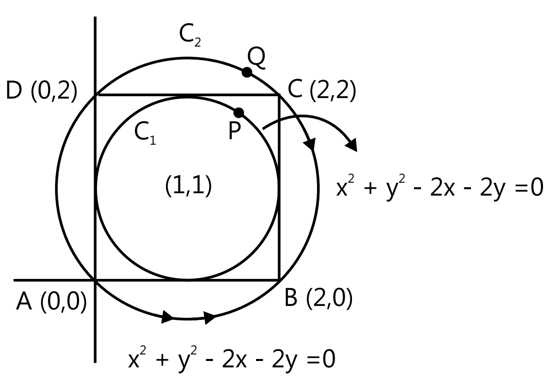
$\therefore \quad L_2$ is a chord of circle C.

Statement-II, false.

Sol 8: (A) Let the, equation of circles are

$$C_1 : (x - 1)^2 + (y - 1)^2 = (1)^2$$

and $\quad C_2 : (x - 1)^2 + (y - 1)^2 = (\sqrt{2})^2$



$\therefore$ Coordinates of $P(1 + \cos\theta, 1 + \sin\theta)$

and $Q(1+\sqrt{2}\cos\theta, 1+\sqrt{2}\sin\theta)$

$\therefore$ $PA^2 + PB^2 + PC^2 + PD^2$

$= \{(1+\cos\theta)^2 + 1 + \sin\theta)^2\} + \{(\cos\theta - 1)^2 + (1+\sin\theta)^2\}$
$+ \{(\cos\theta - 1)^2 + (\sin\theta - 1)^2\}$
$+ \{(1+\cos\theta)^2 + (\sin\theta - 1)^2\}$

$= 12$

Similarly, $QA^2 + QB^2 + QC^2 + QD^2 = 16$

$\therefore$ $\frac{\sum PA^2}{\sum QA^2} = \frac{12}{16} = 0.75$

Sol 9: (C) Let C be the centre of the required circle.

Now, draw a line parallel to L at a distance of r_1 (radius of C_1) from it.

Now, $CC_1 = AC$

$\Rightarrow$ C lies on a parabola.

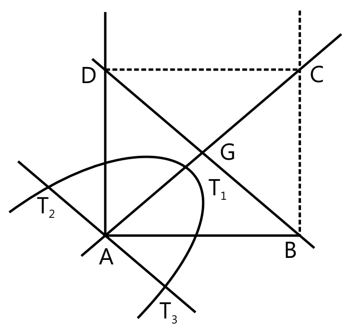
Sol 10: (C)

Since, $AG = \sqrt{2}$

$\therefore$ $AT_1 = T_1G = \frac{1}{\sqrt{2}}$

As A is the focus, T_1 is the vertex and BD is the directrix of parabola.

Also, T_2T_3 is latus reetum.



$\therefore$ $T_2T_3 = 4 \cdot \frac{1}{\sqrt{2}}$

$\therefore$ Area of $\Delta T_1T_2T_3 = \frac{1}{2} \times \frac{1}{\sqrt{2}} \times \frac{4}{\sqrt{2}} = 1$ sq unit

Sol 11: (D) Let centre of circle C be (h, k)

Then, $\left|\frac{\sqrt{3}h + k - 6}{\sqrt{3+1}}\right| = 1$

$\Rightarrow$ $\sqrt{3}h + k - 6 = +2$

$\Rightarrow$ $\sqrt{3}h + k = 4$ ....(i)

(Rejecting '2' because origin and centre of C are on the same side of PQ).

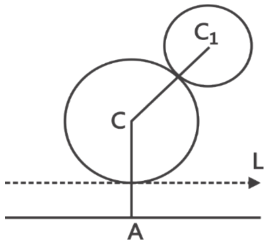
The point $(\sqrt{3}, 1)$ satisfies Eq. (i).

$\therefore$ Equation of circle C is $(x - \sqrt{3})^2 + (y-1)^2 = 1$.

Sol 12: (A) Slope of line joining centre of circle to point D is

$$\tan\theta = \frac{\frac{3}{2} - 1}{\frac{3\sqrt{2}}{2} - \sqrt{3}} = \frac{1}{\sqrt{3}}$$

It makes an angle 30° with x-axis.



$\therefore$ Point E and F will make angle 150° and -90° with x-axis.

$\therefore$ E and F are given by

$$\frac{x - \sqrt{3}}{\cos 150^o} = \frac{y-1}{\sin 150^o} = 1$$

and $$\frac{x - \sqrt{3}}{\cos(-90^o)} = \frac{y-1}{\sin(-90^o)} = 1$$

$\therefore$ $E = \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$ and $F = (\sqrt{3}, 0)$

Sol 13: (D) Clearly, points E and F satisfy the equations given in option (d).

Sol 14:

$2x^2 + y^2 - 3xy = 0$ (given)

$\Rightarrow$ $2x^2 - 2xy - xy + y^2 = 0$

$\Rightarrow$ $2x(x-y) - y(x-y) = 0$

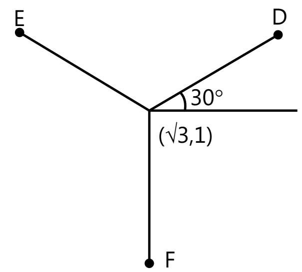
$\Rightarrow$ $(2x-y)(x-y) = 0$

$\Rightarrow$ $y = 2x, y = x$ are the equations of straight lines passing through origin.

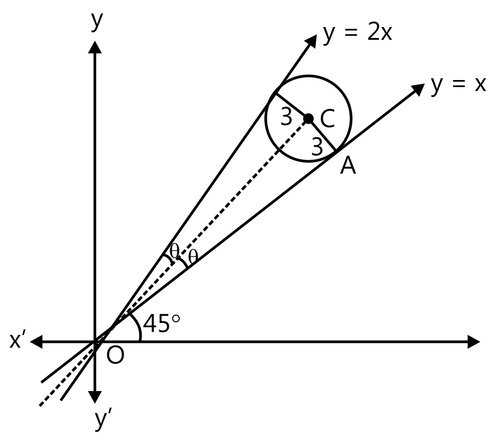
Now, let the angle between the lines be 2θ and the line y = x

Makes angle of 45° with x-axis.

Therefore, $\tan(45^o + 2\theta) = 2$ (slope of the line y =2x)



$\Rightarrow \quad \dfrac{\tan 45^\circ + \tan 2\theta}{1 - \tan 45^\circ \times \tan 2\theta} = 2$



$\Rightarrow \quad \dfrac{1 + \tan 2\theta}{1 - \tan 2\theta} = 2$

$\Rightarrow \quad \dfrac{(1 + \tan 2\theta) - (1 - \tan 2\theta)}{(1 + \tan 2\theta) + (1 - \tan 2\theta)} = \dfrac{2-1}{(2+1)} = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{2\tan 2\theta}{2} = \dfrac{1}{3}$

$\Rightarrow \tan 2\theta = \dfrac{1}{3}$

$\Rightarrow \quad \dfrac{2\tan\theta}{1 - \tan^2\theta} = \dfrac{1}{3}$

$\Rightarrow \quad (2\tan\theta)\cdot 3 = 1 - \tan^2\theta$

$\Rightarrow \quad \tan^2\theta + 6\tan\theta - 1 = 0$

$\Rightarrow \quad \tan\theta = \dfrac{-6 \pm \sqrt{36 + 4 \times 1 \times 1}}{2}$

$\Rightarrow \quad \tan\theta = \dfrac{-6 \pm \sqrt{40}}{2}$

$\Rightarrow \quad \tan\theta = -3 \pm \sqrt{10}$

$\Rightarrow \quad \tan\theta = -3 + \sqrt{10} \qquad \left(\because 0 < \theta < \dfrac{\pi}{4}\right)$

Again, in ΔOCA

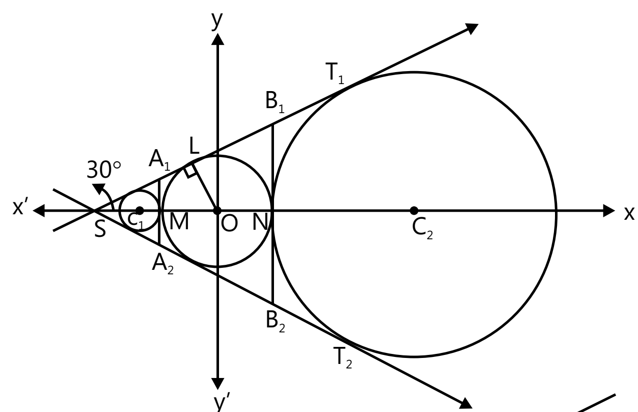
$$\tan\theta = \frac{3}{OA}, OA = \frac{3}{\tan\theta}$$

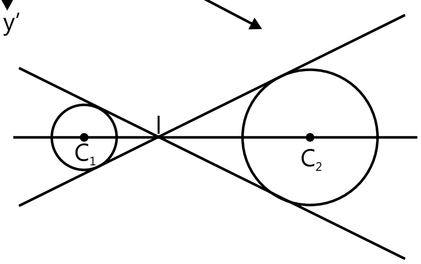
$$= \frac{3}{(-3 + \sqrt{10})}$$

$$\therefore \quad = \frac{3(3 + \sqrt{10})}{(-3 + \sqrt{10})(3 + \sqrt{10})}$$

$$= \frac{3(3 + \sqrt{10})}{(10 - 9)} = 3(3 + \sqrt{10})$$

Sol 15:





From figure it is clear that, triangle OLS is a right triangle with right angle at L.

Also, OL = 1 and OS = 2

$\therefore \quad \sin(\angle LSO) = \dfrac{1}{2} \Rightarrow \angle LSO = 30^\circ$

Since, $SA_1 = SA_2$, ΔSA_1A_2 is an equilateral triangle.

The circle with centre at C_1 is a circle inscribed in the ΔSA_1A_2. Therefore, centre C_1 is centroid of ΔSA_1A_2. This, C_1 divides SM in the ratio 2:1. Therefore, coordinates of C_1 are (-4/3,0) and its radius C_1 M=1/3

$\therefore$ Its equation is $(x + 4/3)^2 + y^2 = (1/3)^2$ ...(i)

The other circle touches the equilateral triangle SB_1B_2

Externally. Its radius r is given by $r = \dfrac{\Delta}{s-a}$,

where $B_1B_2 = a$. But $\Delta = \dfrac{1}{2}(a)(SN) = \dfrac{3}{2}a$

and $\quad s - a = \dfrac{3}{2}a - a = \dfrac{a}{2}$

Thus, $\quad r = 3$

$\Rightarrow$ Coordinates of C_2 are (4,0)

$\therefore$ Equation of circle with centre at C_2 is

$(x-4)^2 + y^2 = 3^2$ ...(ii)

Equations of common tangents to circle (i) and circle C are

$x = -1$ and $y = \pm\frac{1}{\sqrt{3}}(x+2)$ [T_1 and T_2]

Equation of common tangents to circle (ii) and circle C are

$x = -1$ and $y = \pm\frac{1}{\sqrt{3}}(x+2)$ [T_1 and T_2]

Two tangents common to (i) and (ii) are T_1 and T_2 at O. To find the remaining two transverse tangents to (i) and (ii), we find a point I which divides the joint of C_1C_2 in the ratio $r_1 : r_2 = 1/3 : 3 = 1 : 9$

Therefore, coordinates of I are (-4/5,0)

Equation of any line through I is y = m (x+4/5). It will touch (i) if

$$\frac{\left|m\left(\frac{-4}{3}+\frac{4}{5}\right)-0\right|}{\sqrt{1+m^2}} = \frac{1}{3}$$

$\Rightarrow \left|-\frac{8m}{15}\right| = \frac{1}{3}\sqrt{1+m^2}$

$\Rightarrow 64m^2 = 25(1+m^2)$

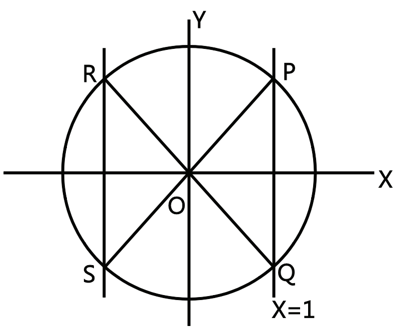
$\Rightarrow 39m^2 = 25$

$\Rightarrow m = \pm\frac{5}{\sqrt{39}}$

Therefore, these tangents are

$$y = \pm\frac{5}{\sqrt{39}}\left(x+\frac{4}{5}\right)$$

Sol 16: Let equation of Circle be $x^2 + y^2 = 4$ and parallel chords are $x = 1$ and -13



$P \equiv (1,13),\ Q \equiv (1,-13)$

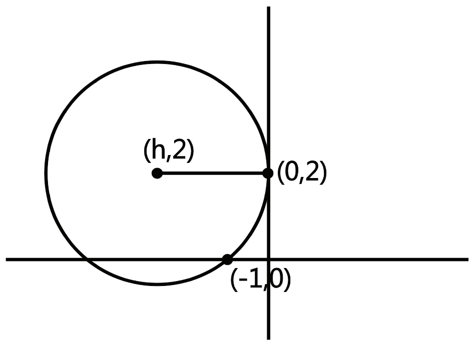
$R \equiv (-\sqrt{3},1), S \equiv (-\sqrt{3},-1)$

$\angle POQ = \frac{2\pi}{3} = \frac{\pi}{k}$

$\angle ROS = \frac{\pi}{3} = \frac{\pi}{k}$

$\Rightarrow k = 3$

Sol 17: (D)



$(x-h)^2 + (y-2)^2 = h^2$

Passes through $(-1,0)$, then

$(-1,-h)^2 + (0-2)^2 = h^2$

$(1+h)^2 - h^2 = -4$

$\Rightarrow (1+h-h)(1+h+h) = -4$

$\Rightarrow (1)(2h+1) = -4$

$h = -5/2$

Circle is

$$\left(x+\frac{5}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$$

Only $(-4,0)$ satisfies the eq. of circle.

D is the Answer.

Sol 18: (D) Any tangent to circle $x^2 + y^2 = 4$ and $(x-3)^2 + y^2 = 1$, then

$$\frac{|3x_1 + 0\times y_1 - 4|}{\sqrt{{x_1}^2 + {y_1}^2}} = 1$$

$$\frac{|3x_1 - 4|}{y} = 1$$

$\Rightarrow |3x_1 - 4| = 2$

$\Rightarrow x_1 = 2, 2/3$

$\Rightarrow (x_1, y_1) \equiv (2,0)\ \&\ \left(\frac{2}{3}, \frac{4\sqrt{2}}{3}\right)$

Tangents

$2 \cdot x + 0 = 4 \Rightarrow x = 2$ and $\frac{2x}{3} + \frac{4\sqrt{2}}{3} = 4$

$\Rightarrow x + 2\sqrt{2} = 6$

Sol 19: (A) The tangent to circle $x^2 + y^2 = 4$ at $\left(\sqrt{3},1\right)4$

$PT \equiv \sqrt{3}x + y = 4$

Eq. of L is $x - \sqrt{3}y = \lambda$

Circle $\left(x-3\right)^2 + y^2 = 1$ is touching L, then

$$\frac{\left|3 - \sqrt{3} \times 0 - \lambda\right|}{\sqrt{1+3}} = 1$$

$\left|3 - \lambda\right| = 2$

$\lambda = 1,5$

Tangents $x - \sqrt{3}y = 1$

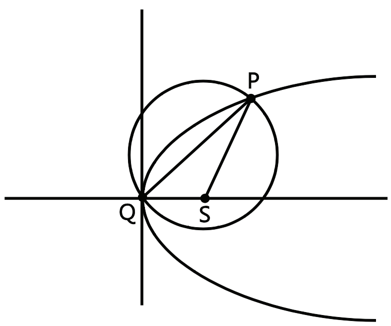
Sol 20: Let P be ($2t^2$, 4t) lies on circle

$4t^4 + 16t^2 - 4t^2 - 16t = 0$

$\Rightarrow t^4 + 4t^2 - t^2 - 4t = 0$

$\Rightarrow t\left(t-1\right)\left(t^2 + t + 4\right) = 0$

$\Rightarrow t = 0,1$



$P \equiv \left(2,4\right)$ $\quad Q \equiv \left(0,0\right)$ $\quad S \equiv \left(2,0\right)$

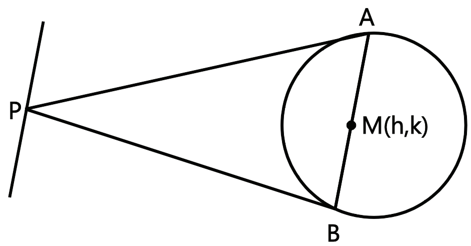
$\Delta = \frac{1}{2} \times 2 \times 4 = 4$ sq units

Sol 21: (A) Let point P be $\left(t, \frac{4t-20}{5}\right)$

Eq. of chord of contact $xt + y\left(\frac{4t-20}{5}\right) = 9$

$\left(5t\right)x + y\left(4t - 20\right) = 45$ ... (i)

If (h,k) mid-point, the eq. of chord of contact $T=S_1$



$xh + yk = h^2 + k^2$ ... (ii)

(i) & (ii) are identical, then

$$\frac{h}{5f} = \frac{k}{4f - 20} = \frac{h^2 + k^2}{45}$$

$$t = \frac{9h}{h^2 + k^2}$$

$$4t - 20 = \frac{45k}{h^2 + k^2}$$

$$\Rightarrow \frac{9h \times 4}{h^2 + k^2} - 20 = \frac{45k}{h^2 + k^2}$$

$\Rightarrow 36h - 20\left(n^2 + k^2\right) = 45k$

$\Rightarrow 20\left(h^2 + k^2\right) - 36h + 45k = 0$

$\Rightarrow 20\left(x^2 + y^2\right) - 36x + 45y = 0$

Sol 22: (C) Let circle touching x-axis be

$\left(x - \alpha\right)^2 + \left(y - k\right)^2 = k^2$ ... (i)

Also for y-axis intercepts

$\left(0 - \alpha\right)^2 = \left(y - k\right)^2 = k^2$

$\Rightarrow \left(y - k\right)^2 = k^2 - \alpha^2$

$\Rightarrow y = k \pm \sqrt{k^2 - \alpha^2}$

Intercept $= 2\sqrt{k^2 - \alpha^2} = 2\sqrt{7}$

$\Rightarrow k^2 = 7 + \alpha^2$

From (i) $\alpha = 3$

$\Rightarrow k^2 = 7 + 9 = 16$

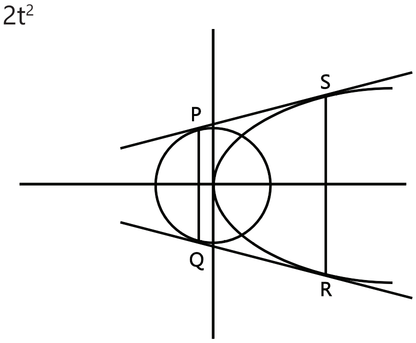
$\Rightarrow k = \pm 4$

Circle: $\left(x-3\right)^2 + \left(y-4\right)^2 = 16$

$\left(x-3\right)^2 + \left(y+4\right)^2 = 16$

Sol 23: (D) Let tangent to parabola $y^2=8x$

Be $ty=x+2t^2$



It is also tangent to circle, then

$$\frac{2t^2}{\sqrt{1+t^2}}=\sqrt{2}$$

$$\Rightarrow 4t^4=2\left(1+t^2\right)$$

$$\Rightarrow 2t^4-t^2-1=0$$

$$\Rightarrow \left(2t^2+1\right)\left(t^2-1\right)=0$$

$$\Rightarrow t=\pm 1$$

$$\Rightarrow S\equiv\left(2,4\right)\,\&\,R\equiv\left(2,-4\right)$$

$$\Rightarrow P\equiv\left(-1,1\right)\ \ \&\ \ Q\left(-1,1\right)$$

$$\text{Area}=\frac{1}{2}\left(2+B\right)\times 3=15 \text{ sq units}$$

Sol 24: (B, C) Let circle be $x^2+y^2+2yx+2y+C=0$

Applying condition for orthogenality

$$2gx-1+2f\times 0=C+\left(-15\right)$$

$$\Rightarrow 2g+c=15 \text{ and } 2g\times 0+2f\times 0=C-1$$

$$\Rightarrow C=1$$

$$\Rightarrow g=7$$

Also,

$$1+2f+C=0$$

$$\Rightarrow f=-1$$

$$\text{Centre}\equiv\left(-g,-f\right)\equiv\left(-7,1\right)$$

$$\text{Radius}=\sqrt{g^2+f^2-C}=\sqrt{49+1-1}=7$$

Hence, B and C are the correct options

Sol 25: (A, C, D) $(x-2)^2+(y-8)^2=4$

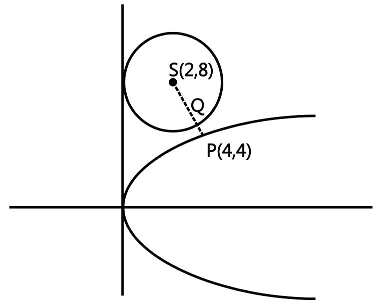
Shortest distance is measured along common normal

The equation of normal to parabola

$y=mx-2am-am^3 \Rightarrow y=mx-2m-m^3$

Passes through (2, 8), then

$8=2m-2m-m^3 \Rightarrow m=-2$



Normal

$y=-2x=12$

$$SP=\sqrt{\left(4-2\right)^2+\left(4-8\right)^2}=\sqrt{4+16}=2\sqrt{5} \text{ units}$$

Let SQ : QP = 1 : λ

$$\frac{1}{S\left(2,8\right)}\cdot\frac{\lambda}{Q\left(h,k\right)P\left(4,4\right)}$$

$$Q\left(h,k\right)\equiv\left(\frac{4+2\lambda}{1+\lambda},\frac{8\lambda+y}{1+\lambda}\right)$$ lies on

Circle, then

$$\left(\frac{4+2\lambda}{1+\lambda}-2\right)^2+\left(\frac{8\lambda+4}{1+\lambda}-8\right)^2=4$$

$$\Rightarrow\left(\frac{2}{1+\lambda}\right)^2+\left(\frac{-4}{1+\lambda}\right)^2=4$$

$$\Rightarrow\frac{20}{\left(1+\lambda\right)^2}=4$$

$$\Rightarrow 1+\lambda=\sqrt{5}$$

$$\Rightarrow \lambda=\sqrt{5}-1$$

$$\frac{SQ}{QP}=\left(\sqrt{5}-1\right)$$

x – intercept of normal at P is 6 slope of tangent at Q is $\frac{1}{2}$

Sol 26: (C) For point of intersection

$2y+y^2=3$

$$\Rightarrow y^2+2y-3=0$$

$$\Rightarrow (y+3)(y-1)=0$$

$\Rightarrow y = 1, -3$

$\Rightarrow = \left(\sqrt{2},1\right)$

The eq. of tangent at $\left(1,\sqrt{2}\right)$

$\sqrt{2}x + y = 3$ (i)

Eqs. of circle C_2 and C_3

$C_2 \equiv x^2+(y-y_2)^2 = 12$

$C_2 \equiv x^2+(y-y_3)^2 = 12$

If line (i) touches circle, then

$$\left|\frac{\sqrt{2}\times 0 + y - 3}{\sqrt{2+1}}\right| = 2\sqrt{3}$$

$\Rightarrow \left|y-3\right| = 6$

$\Rightarrow \left|y-3\right| = \pm 6$

$\Rightarrow y = -3, 9$

$\Rightarrow y_2 = -3$ and $y_3 = 9$

$\Rightarrow$ Centres $Q_2 \equiv (0,-3)$

$Q_3 \equiv (0,9)$

$\Rightarrow Q_2Q_3 = 12$

For point of contact R_2 and R_3

$R_2 \equiv \left(2\sqrt{2},-1\right)$ and $R_3 \equiv \left(-2\sqrt{2},7\right)$

$$R_2R_3 = \sqrt{\left(4\sqrt{2}\right)^2 + (8)^2} = \sqrt{32+64} = \sqrt{96} = \sqrt{16\times 6} = 4\sqrt{6}$$

0(0,0), $R_2(\sqrt{2},-1)$, $R_3(-2\sqrt{2},7)$

$$\text{Area of } \Delta OR_2R_3 = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 2\sqrt{2} & -1 & 1 \\ -2\sqrt{2} & 7 & 0 \end{vmatrix}$$

$$= \frac{1}{2}\left(7\times 2\sqrt{2} - 2\sqrt{2}\right) = \frac{1}{2}\times 6\times 2\sqrt{2}$$

$= 6\sqrt{2}$ sq. units

Now

Area of ΔPQ_2Q_3

$$= \frac{1}{2}\begin{vmatrix} \sqrt{2} & 1 & 1 \\ 0 & -3 & 1 \\ 0 & 9 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[\sqrt{2}\left(-3-9\right)\right] = 6\sqrt{2} \text{ sq units}$$

Sol 27: (A, C) Let point P be $\left(\cos\theta,\sin\theta\right)$, The tangent and normal are

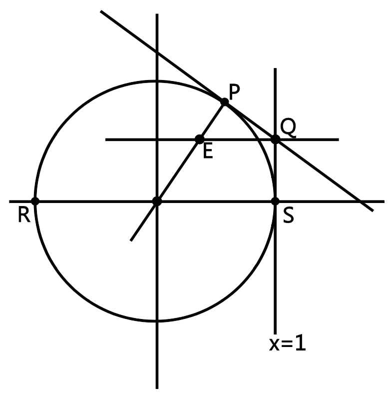
$x\cos\theta + y\sin\theta = 1 \quad x\sin\theta - y\cos\theta = 0$

$$\Rightarrow \theta \equiv \left(1, \frac{1-\cos\theta}{\sin\theta}\right)$$

$$\Rightarrow E \equiv \left(1, \frac{y\cos\theta}{\sin\theta}, y\right) \equiv (h,k) \quad (\text{let})$$

$$\Rightarrow h = \left(\frac{1-\text{Cos}\theta}{\text{Sin}\theta}\right).\frac{\cos\theta}{\sin\theta}$$

$$k = \left(\frac{1-\text{Cos}\theta}{\text{Sin}\theta}\right)$$



$$K = \frac{1-\dfrac{h}{\sqrt{h^2+K^2}}}{1-\dfrac{K}{\sqrt{h^2+K^2}}}$$

$$\Rightarrow K^2 + h = \sqrt{h^2+K^2} \Rightarrow y^2 + x = \sqrt{x^2+y^2}$$